It occurs mainly in b/w energy system

Resonance means intune

Inductor stores energy in form of magnetic field

Capacitor stores energy in form of static electricity

RESONANCE :-

Resonance literally means Intune.

Resonance is a phenomena found in any system involving two independent energy storage systems, they can be mechanical, electrical, huydralic, pNeumatic etc...

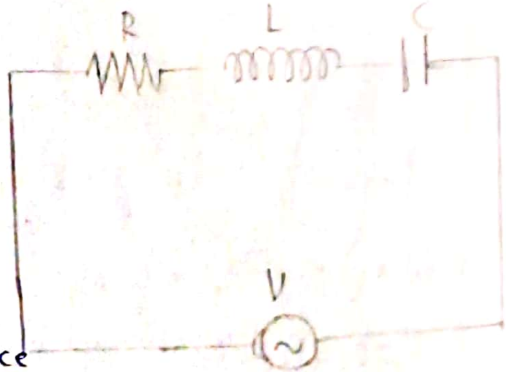
In electrical circuits the resonance occurs when there are energy storage elements in the circuit. The energy storage elements are Inductor and capacitor.

Under resonance condition the energy that is stored in the inductor is transffered to the capacitor and the energy that is stored in the capacitor is again transffered to the inductor. This process goes on and this condition is known as Resonance.

Note:-

For the resonance to be occur there should be two energy storage elements. Resonance will not occurs with one storage element.

In a series RLC circuit the condition for resonance is inductive reactance is equal to capicitive reactance



i.e, $X_L = X_C$

$$\therefore \quad 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

A series RLC circuit can be brought into resonance either by varying

i) magnitude of L, C values.

ii) frequency of supply.

Under resonance condition the series RLC circuit behaves as pure resistive circuit because inductive reactance and capacitive reactance cancels each other.

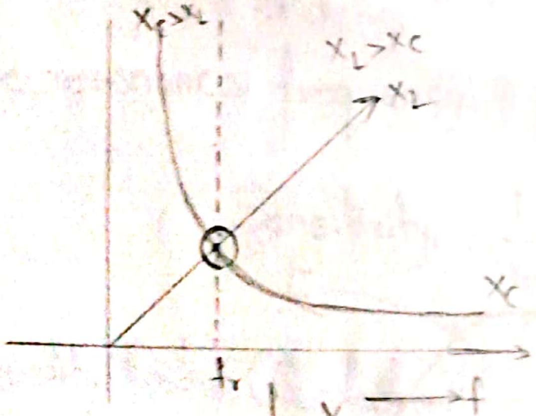
$Z = \sqrt{R^2 + (X_L - X_C)^2}$ $Z = R$

→ The current will be in Inphase with voltage under resonance condition. The power factor is unity and the phase angle is 0.

I V P.f = 1

→ Under resonance condition the current will be maximum.

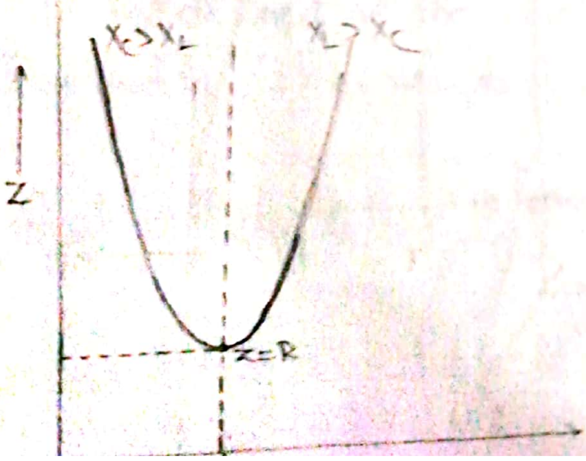
Series Resonance:-

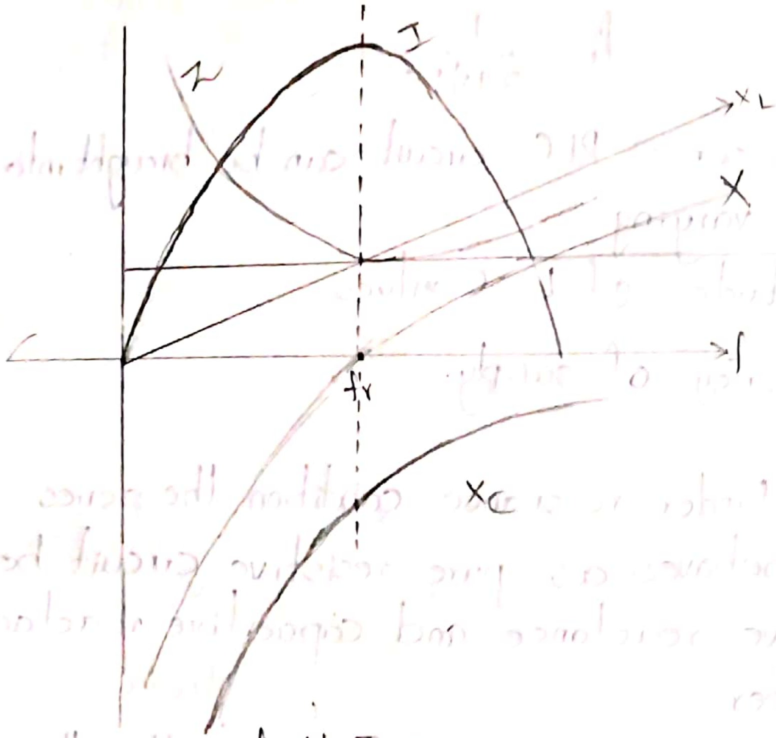


f vs X

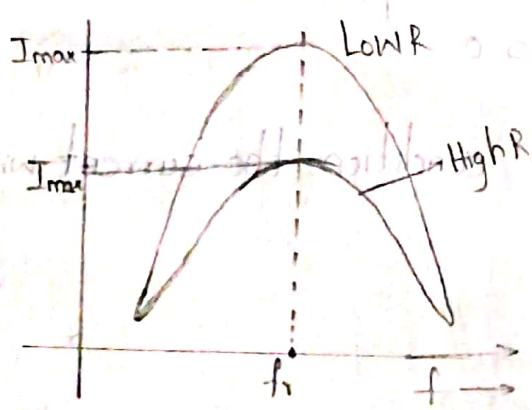
$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

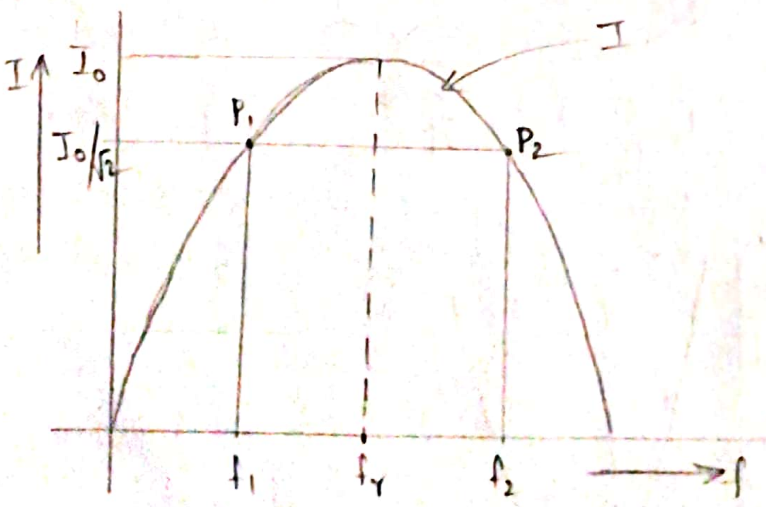




f_r Vs I, Z, X_C, X, X_L



f vs I



f_r = resonance frequency.

f_1 = lower cut-off frequency.

f_2 = Upper Cut-off frequency.

P_1, P_2 = Half Power Points.

Band Width:-

The range of frequencies over which the current is 0.707 times I_0 (or) 70.7% of I_0 is known as Band Width.

(or)

The range of frequencies between f_2 and f_1 of a series RLC circuit is defined as Band Width. It is represented by Δf.

$$B.W \; \Delta f = f_2 - f_1$$

Sensitivity:-

It is defined as the ratio of bandwidth to the resonance frequency.

$$\text{Sensitivity} = \frac{\text{Band width}}{\text{resonance frequency}} = \frac{f_2 - f_1}{f_r}.$$

Quality Factor (Q-factor):-

It is defined as the ratio of resonant frequency to the Bandwidth. It is reciprocal of Sensitivity.

$$\text{Quality factor } Q = \frac{\text{resonance frequency}}{\text{Bandwidth}}$$

$$= \frac{f_r}{f_2 - f_1}$$

Q-factor is defined as the voltage magnification and may be defined as the voltage developed across L and C to the applied voltage.

$$Q = \frac{V_L}{V} = \frac{V_C}{V}$$

$$\frac{V_L}{V} = \frac{\cancel{I} X_L}{\cancel{I} R} = \frac{2\pi f_r L}{R}$$

$$= \frac{\omega_r L}{R}.$$

$$\frac{V_C}{V} = \frac{\cancel{I} X_C}{\cancel{I} R} = \frac{1}{2\pi f_r C R} = \frac{1}{\omega_r RC}.$$

(or)

Quality factor may be defined as interms of energy

$$Q = 2\pi \left[\frac{\text{Max Energy stored}}{\text{Energy dissipated per Cycle}} \right]$$

Maximum energy stored $= \frac{1}{2} L I_0^2 = \frac{1}{2} CV^2$

Energy dissipated per cycle is

Energy = Power x Time

Power $= I_{rms}^2 \cdot R$

$$\left[I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{I_0}{\sqrt{2}} \right]$$

$$\text{Power} = \left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot R.$$

$$Time = \frac{1}{f_r}$$

$$Energy = power \times time$$

$$= \left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot R \times \frac{1}{f_r}$$

$$Q = 2\pi \left[\frac{\frac{1}{2} L I_0^2}{\left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot R \cdot \frac{1}{f_r}} \right]$$

$$= 2\pi \left[\frac{\cancel{2}}{\cancel{2}} \frac{L \cancel{I_0^2} f_r}{\cancel{I_0^2} \cdot R} \right]$$

$$= \frac{2\pi f_r L}{R}$$

$$Q = \frac{\omega_r L}{R}$$

$$\boxed{\therefore Q_L = \frac{\omega_r L}{R}} \quad \boxed{Q_C = \frac{1}{\omega_r RC}}$$

Voltage Magnification In a Series Circuit :-

The voltage is magnified across inductor and capacitor in a series RLC circuit under resonance condition, that voltage magnification is given by, the ratio of voltage develop across the inductor or capacitor to the supply voltage.

$$\cancel{Q = \frac{V_L}{}}$$

$$\text{Voltage Magnification} = \frac{V_L}{V} \cdot \frac{V_C}{V}$$

1) A series RLC circuit has the following parameters resistance = 17Ω, inductance = 38μH capacitance = 44μF. Calculate resonance frequency, current, power voltage across inductor, voltage across capacitor, voltage across resistor under resonance condition, Supply voltage is 70V ?

Given,

$R = 17\Omega$

$V = 70V$

$L = 38\mu H = 38\times10^{-6} H$

$C = 44\mu F = 44\times10^{-6} F$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(38\times10^{-6})(44\times10^{-6})}}$$

$= 3892.2613$

$= 3.89\ KHz$

Ans

$$I_0 = \frac{V}{Z} = \frac{V}{R} = \frac{70}{17} = 4.1176A$$

$P_R = V I_0$

$= 70(4.1176)$

$= 288.285$

$V_L = I_0 X_L$

$= (4.1176)(2\pi f_r L)$

$= (4.1176)(2\pi(3892.2613)(38\times10^{-6}))$

$= 3.8265V$

$V_c = I_0 \times C$

$= (4.1176)\left(\frac{1}{2\pi f_r C}\right)$

$$= (4.1176)\left(\frac{1}{2\pi(3892.2613)(49\times10^{-6})}\right)$$

$$= 3.8265V.$$

$$V_R = I_0 R$$

$$= (4.1176)(17)$$

$$= 69.9992V.$$

$$\simeq 70V.$$

problem-2:-

A series RLC circuit with a resistance of 100Ω, inductance of 0.5H and capacitance of 40μF has an applied voltage of 50v with a variable frequency. Calculate resonant frequency, current, voltage across resistor, inductor and capacitor.

Ans:- Given,

$R = 100\Omega$

$L = 0.5$

$C = 40\mu F = 40\times10^{-6}F$

$V = 50V$

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.5\times40\times10^{-6}}}$$

$$= 35.5881\,Hz.$$

$$I_0 = \frac{V}{Z} = \frac{V}{R} = \frac{\cancel{50}\,1}{\cancel{100}\,2} = 0.5A$$

$$V_R = I_0 R$$

$$= (0.5)(100)$$

$$= 50V.$$

$$V_L = I_0 X_L$$

$$= (0.5)(2\pi(35.588)(0.5))$$

$$= 55.9016$$

$$\cancel{V_C = I_0 X_C}$$

$$\cancel{= (0.5)(2\pi(35.588)(}$$

$$V_C = I_0 X_C$$

$$= \frac{I_0}{2\pi f_r C}$$

$$= \frac{0.5}{2\pi(35.5881)(40\times10^{-6})}$$

$$= 55.901\ V$$

$$Q = \frac{V_L}{V} = \frac{55.901}{50}$$

Problem-3

A series RLC circuit has a Q factor of 5 at 50 rad/sec. The current flowing through the circuit is 10A at resonance and supply voltage is 100 V. The total impedence of the circuit is 20Ω. Find the circuit constants?

Ans:- Given

$Q = 5$

$\omega_r = 50\ rad/sec$

$I = 10\ A.$

$V = 100V$

$Z = 20\Omega = R.$

$$I_0 = \frac{V}{R}$$

$$10 = \frac{100}{R}$$

$$R = 10\ \Omega$$

$$Q_L = \frac{\omega_r L}{R}$$

$$5 = \frac{50\times L}{10}$$

$$L = \frac{5\times10}{50}$$

$$L = 1H$$

$$Q_C = \frac{1}{\omega_r RC}$$

$$5 = \frac{1}{50\times20C}$$

$$C = \frac{1}{5\times50\times10}$$

$$C = 4\times10^{-4}F$$

$$= 400\ \mu F$$

$X_L = 2\pi f L$ $\qquad$ $X_C = \frac{1}{\omega C}$

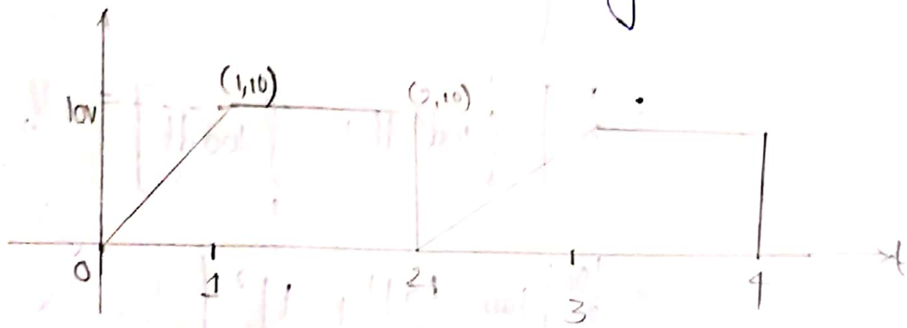
$= \omega L$ $\qquad$ $= \frac{10^{6}}{50 \times 400}$

$= 50 (1H)$

$= 50\Omega$ $\qquad$ $= \frac{10^{6}}{20000} = 50\Omega.$

$\cancel{= 200\Omega}$

Q) Find the form factor for the given wave form.



Ans:-

$y - 0 = \frac{10-0}{1-0}(x-0).$ $\qquad$ $(y-1) = 0$

$y = 10x$ $\qquad$ $y = 10$

$v(t) = 10t. \quad 0 < t < 1$ $\qquad$ $v(t) = 10 \quad 1 < t < 2$

$V_{avg} = \frac{\int_0^t v(t)\,dt}{T}$ $\qquad$ $V_{rms}^2 = \frac{\int_0^t v^2(t)\,dt}{T}$

$= \frac{1}{2}\int_0^2 v(t)\,dt$ $\qquad$ $= \frac{\int_0^1 v^2(t)\,dt + \int_1^2 v^2(t)\,dt}{2}$

$= \frac{1}{2}\int_0^1 10t\,dt + \int_1^2 10\,dt$ $\qquad$ $= \frac{1}{\sqrt{2}}\left[100\int_0^1 t^2\,dt + 100\int_1^2 dt\right]$

$= \frac{1}{2}\left[\frac{10}{2} + 10\right]$ $\qquad$ $= \frac{50}{\sqrt{2}}\left[\frac{1}{3} + \frac{1}{2}\right]$

$= 15/2 = 7.5$ $\qquad$ $= 50\left[\frac{4}{3}\right]$

$= 50\sqrt{2}\left[\frac{4}{3}\right]$

$= 8.169$

~~form factors~~ = rms value

$$V_{rms}^2 = \frac{\int_0^t v^2(t)\,dt}{T}$$

$$= \frac{1}{2}\left[\int_0^1 v^2(t)\,dt + \int_1^2 v^2(t)\,dt\right]$$

$$= \frac{1}{2}\left[\int_0^1 100t^2\,dt + \int_1^2 100\,dt\right]$$

$$= \frac{100}{2}\left[\cancel{100}\ \frac{t^3}{3}\Big|_0^1 + t\Big|_1^2\right]$$

$$= 50\left[\frac{1}{3} + 1\right] = \frac{50 \times 4}{3} = \frac{200}{3}.$$

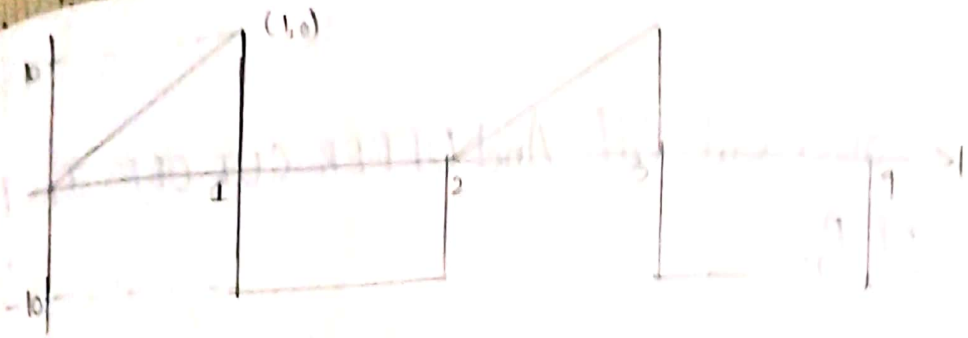
$$V_{rms} = \sqrt{\frac{200}{3}}$$

$$= 10\sqrt{\frac{2}{3}} = 8.164$$

$$\text{form factor} = \frac{V_{rms}}{V_{avg}} = \frac{8.164}{7.5}$$

$$= 1.0885$$

(2)



Ans:

$0 < t < 1$ $\quad\quad$ $1 < t < 2$

$v(t) = 10t$ $\quad\quad$ $v(t) = -10$

$$V_{avg} = \frac{\int_0^1 v(t)\,dt}{T}$$

$$= \frac{1}{2}\left[\int_0^2 v(t)\,dt\right]$$

$$= \frac{1}{2}\left[\int_0^1 10t\,dt + \int_1^2 10\,dt\right]$$

$$= \frac{1}{2}\left[\frac{10}{2} + -10\right]$$

$$= -5/2 = -2.5$$

$V_{avg} = -2.5$

$$V_{rms}^2 = \frac{\int_0^1 v^2(t)\,dt}{1}$$

$$= \frac{1}{2}\left[\int_0^2 v^2(t)\,dt\right]$$

$$= \frac{1}{2}\left[\int_0^1 100t^2\,dt + \int_1^2 100\,dt\right]$$

$$= \frac{100}{2}\left[1/3 + 1\right]$$

$$= \frac{200}{3}$$

$$V_{rm} = 10\sqrt{\frac{2}{3}} = 8.169.$$

$V_{rms} = 8.169$

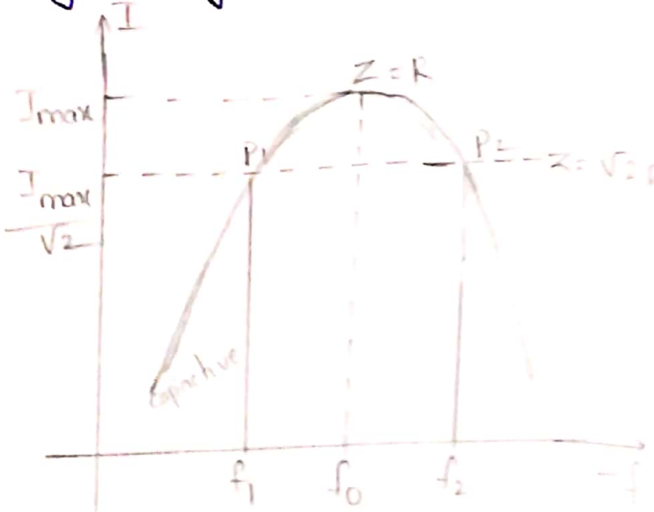
Lower Cut-Off And UPPER CUT-OFF IN TERMS Of f_0:

The impedence under resonance condition is given by ~~by $Z=\sqrt{2}R$~~ at half power points is given by

$Z = \sqrt{2}\,R$.

Lower Cut-Off And Upper Cut-Off Frequency In Terms Of f_0 :-

The impedence under resonance condition of half power points is given by $z = \sqrt{2}R$.



Now,

$$i_{rms} = \frac{V_{rms}}{z}$$

at resonance $z = R$

$$i_{max} = \frac{V_{rms}}{R}$$

at $p = P/2 \Rightarrow i_{rms} = \frac{i_{max}}{\sqrt{2}}$

So,

$$\frac{V_{rms}}{z} = \frac{V_{rms}}{\sqrt{2}R}$$

$$\Rightarrow z = \sqrt{2}R$$

$$\sqrt{R^2+X^2} = \sqrt{2}R$$

$$R = \pm X.$$

f_1, f_2 interms of f_0 :-

$$z = \sqrt{2}R$$

At higher Half power frequency f_2 :-

$$X_L - X_C = R$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- (1)}$$

At lower cut off frequency f_1

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{--- (2)}$$

(1) + (2).

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = R - R$$

$$(\omega_1 + \omega_2) L = \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) C$$

$$\omega_1 \omega_2 = \frac{1}{LC}$$

(1) - (2)

$$\Rightarrow \quad \omega_2 L - \frac{1}{\omega_2 C} - \left(\omega_1 L - \frac{1}{\omega_1 C}\right) = R - (-R)$$

$$\omega_2 L - \frac{1}{\omega_2 C} - \omega_1 L + \frac{1}{\omega_1 C} = 2R$$

$$L(\omega_2 - \omega_1) + \left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2 C}\right) = 2R.$$

$$(\omega_2 - \omega_1)\left[L + \frac{1}{\omega_1 \omega_2 C}\right] = 2R.$$

$$(\omega_2 - \omega_1)\left[\frac{LC\omega_1\omega_2 + 1}{\omega_1 \omega_2 C L}\right] = \frac{2R}{L}$$

$$(\omega_2 - \omega_1)\left(\frac{\cancel{2}}{1}\right) = \frac{\cancel{2}R}{L}$$

$$\omega_2 - \omega_1 = \frac{R}{L}$$

$$2\pi(f_2 - f_1) = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L} \quad \text{——} \ (4)$$

W.k.T

$$f_0 = \frac{f_1 + f_2}{2} \quad \text{——} \ (5)$$

$$f_1 + f_2 = 2f_0.$$

$$f_2 = 2f_0 - f_1 \quad \text{——} \ (6)$$

substitute (6) in (4) we get,

$$2f_0 - f_1 - f_1 = \frac{R}{2\pi L}$$

$$f_0 - f_1 = \frac{R}{4\pi L}$$

$$f_0 = f_1 + \frac{R}{4\pi L}.$$

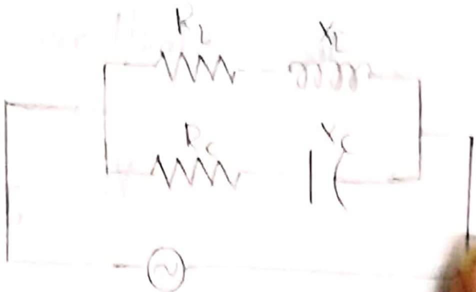
$$f_2 = f_0 + \frac{R}{4\pi L}$$

$$f_1 = f_0 - \frac{R}{4\pi L}.$$

PARALLEL RESONANCE :-

Considering, internal resistance of both 'L' and 'C'

The total admittance of the circuit can be given as



$$Y = Y_1 + Y_2$$

The impedence of the inductor is

$$Z_L = R_L + jX_L$$

The impedence of the capacitor is

$$Z_C = R_L - jX_C.$$

Here

Z_L : impedence of the branch containing inductor

Z_C : impedence of the branch containing capacitor.

Now, the admittance of the inductor can be given as

$$Y_L = \frac{1}{Z_L} = \frac{1}{R_L + jX_L}$$

$$= \frac{R_L - jX_L}{(R_L + jX_L)} \times \frac{1}{(R_L - jX_L)}$$

$$Y_L = \frac{R_L - jX_L}{R_L^2 + X_L^2} = \frac{R_L - jX_L}{R_L^2 + \omega^2 L^2}$$

The addmittance of the capacitor is

$$Y_C = \frac{1}{R_C - jX_C}$$

$$= \frac{1}{R_C - \frac{j}{\omega C}}$$

$$= \frac{1}{R_C - \frac{j}{\omega C}} \times \frac{R_C + (j/\omega C)}{R_C + (j/\omega C)}$$

$$Y_C = \frac{R_C + (j/\omega C)}{R_C^2 + 1/\omega^2 C^2}$$

The total admittance of the resonance circuit is

$$Y = Y_L + Y_C$$

$$Y = Y_L + Y_C$$

$$= \frac{R_L - jX_L}{R_L^2 + \omega^2 L^2} + \frac{R_C + (j/\omega C)}{R_C^2 + 1/\omega^2 C^2}$$

$$= \left[\frac{R_L}{R_L^2 + \omega^2 L^2} + \frac{R_C}{R_C^2 + 1/\omega^2 C^2}\right] + j\left[\frac{-\omega L}{R_L^2 + \omega^2 L^2} + \frac{1/\omega C}{R_C^2 + 1/\omega^2 C^2}\right]$$

Under resonance condition the imaginary part becomes zero and equals the imaginary part and find ω_0.

$$-\frac{\omega L}{R_L^2+\omega^2 L^2} + \frac{1/\omega C}{R_C^2 + 1/\omega^2 C^2} = 0.$$

$$\frac{\omega L}{R_L^2+\omega^2 L^2} = \frac{\omega^2 C^2}{\omega C}\left[\frac{1}{R_C^2\omega^2 C^2+1}\right]$$

$$\frac{L}{C}(R_C^2\omega^2 C^2+1) = R_L^2+\omega^2 L^2$$

$$R^2\omega^2 C^2 L + L = R^2 C + \omega^2 L^2 C$$

$$R^2\omega^2 C^2 L - \omega^2 L^2 C = R^2 C - L$$

$$\omega^2 LC\,\cancel{(R^2 C - L)} = \cancel{R^2 C - L}$$

$$\omega^2 LC = 1$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$y_L = \frac{R_L^* - jX_L}{R_L^2 + \omega^2 L^2}$$

$$= G - jB$$

$$y_C = \frac{R_C + (j/\omega C)}{R_C^2 + (1/\omega^2 C^2)}$$

$$= G + jB$$

G : Conductance

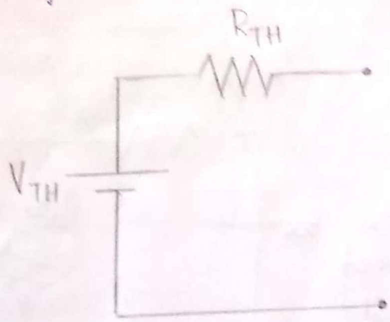
B : Suspectence

4. Network Theorem's

Thevinin's Theorem:-

In any bilateral network consisting of no. of sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance.

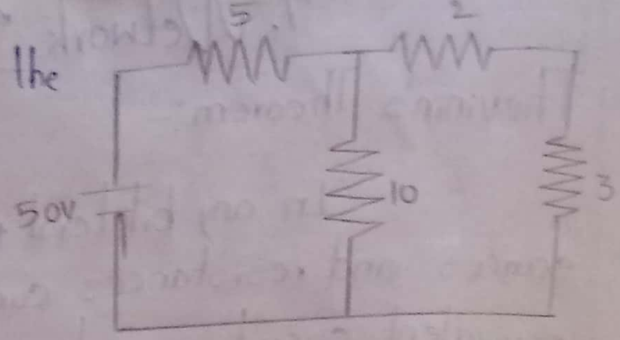
Where voltage source is the open circuit voltage across the terminals of the network (whose value is to be findout) and resistance is equal to equivalent network resistance measured between the terminals with all energy sources replaced by internal resistances.



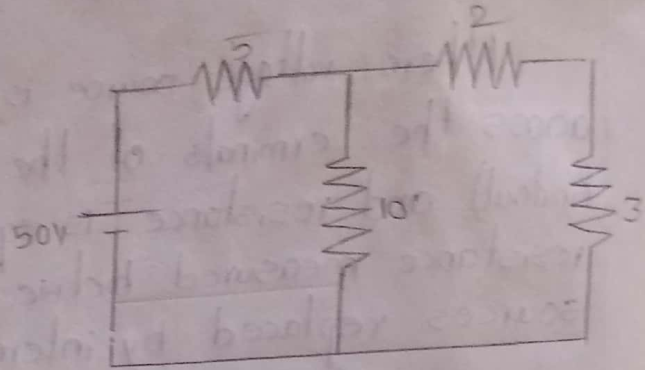
Procedure:-

1. Temporarily remove the load resistance (R_L) whose voltage (or) current (or) power is to be measured (or) is to be find out.
2. Find Open circuit voltage V_{oc} (or) V_{TH} by removing the load resistance (R_L)
3. Find R_{TH} across the terminals of the load by replacing all energy sources with their internal resistance.
4. Replace entire network by V_{TH} and R_{TH}.
5. Find load current I_L by using $I_L = \dfrac{V_{TH}}{R_{TH}+R_L}$

Q) Find the current through the 3-Ω resistance using thevinins theorem? Find the V_{TH} and R_{TH}



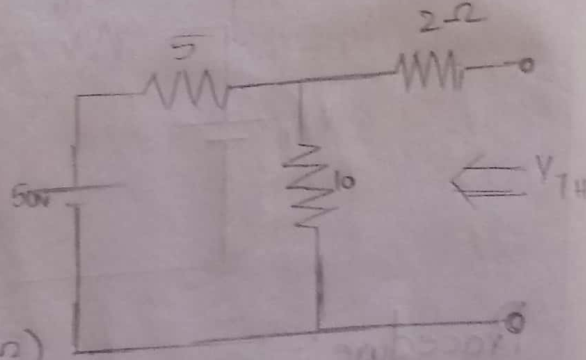
Ans:-



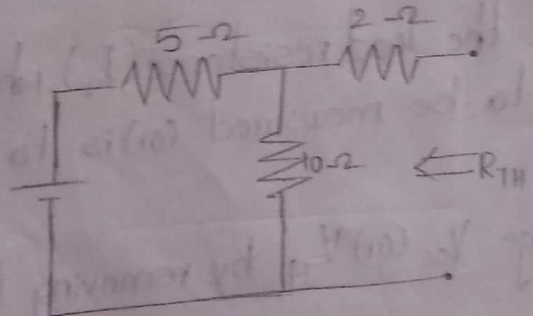
$$I = \frac{50}{15} = 3.333$$

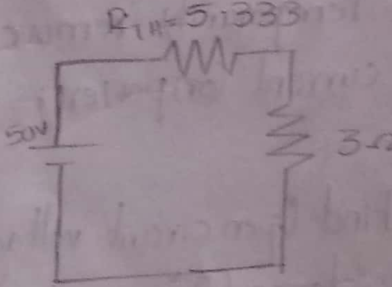
$$V_{TH} = 10 \times 3.333$$

$$= 33.33V$$



Here current Across open circuit is 0. So, in 2Ω resistance there is no flow of current it is S.C. So total equivalent resistance is (15Ω)





$$R_{TH} = (5 \| 10) + 2$$

$$= \frac{5 \times 10}{5+10} + 2$$

$$= \frac{50}{15} + 2$$

$$= 5.333\Omega$$

$$I_3 = \frac{V}{R_{eq}}$$

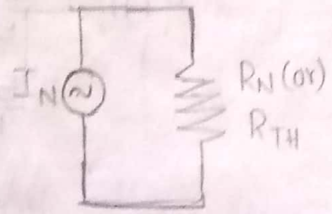
$$I_3 = \frac{33.3}{5.33+3}$$

$$I_3 = 4.0012A$$

Norton's Theorem:-

Any Bilateral circuit (or) Network consisting of no.of sources and resistances can be replaced by an equivalent circuit consisting of current source in parallel with a resistance. The value of current source is equal to the short circuit current between the terminals of the network and resistance is the equivalent resistance measured between the terminals of the network with all energy sources replaced by their internal resistances.

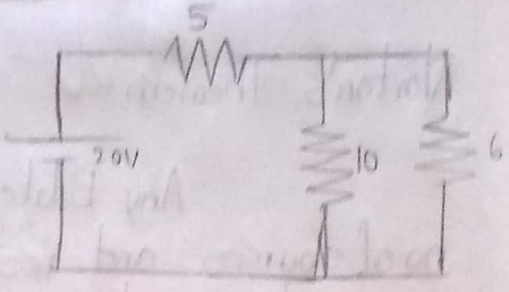
Norton's Equivalent current I_N (or) I_{sc}.



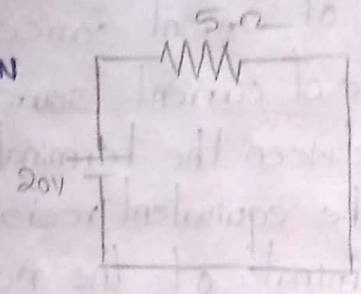
Procedure:-

1. Temporarily remove the load resistance "R_L" and put a short circuit across it.
2. Find the short circuit current I_s (or) Norton's current.
3. Find R_{TH} (or) R_N
4. Replace the entire circuit with Norton current I_N and Norton's equivalent circuit resistance (R_N (or) R_{TH}).
5. Connect R_L to the equivalent circuit.
6. Calculate load current "I_L".

Q) Find I_N, R_{TH} (or) R_N and current through 6 Ω resistor in the given circuit.

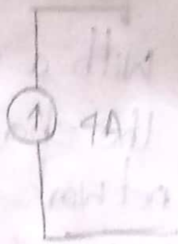


Ans:- i) I_N

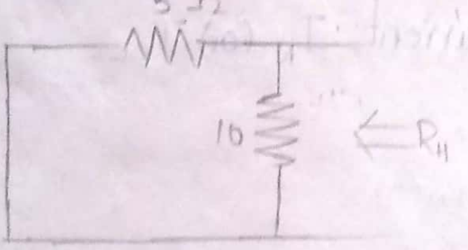


I_{sc} (or) $I_N = \frac{20}{5}$

$= 4A.$



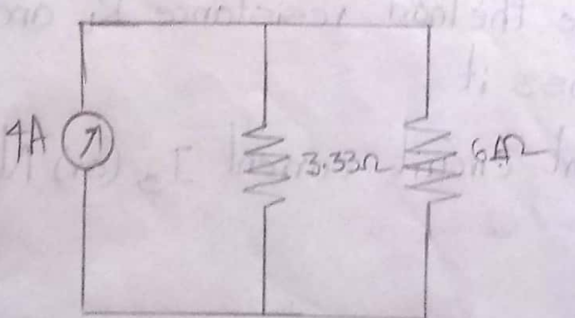
ii) R_{TH} (or) R_N



$R_{TH} = \frac{5 \times 10}{5+10}$

$= \frac{50}{15} = 3.333\Omega.$

iii) $I_{6\Omega}$



$I_6 = \frac{4 \times 3.33}{3.33+6}$

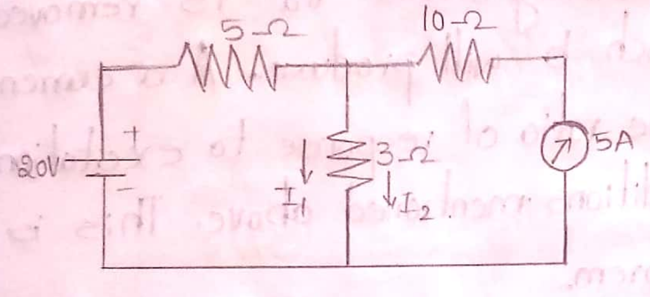
$= 1.42A.$

$\therefore I_6 = 1.42A$

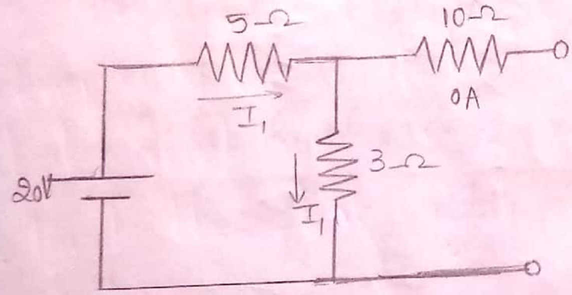
Super Position Theorem :-

The superposition theorem states that in any labour linear bilateral network consists of two or more sources the response in any element is equal to the algebriac sum of the responses caused by individual sources acting alone. While the non-operating voltage sources and current sources in the network are replaced by short circuit and open circuit across the terminals. It is valid only for linear systems.

Q)



Case-(i):- 20V voltage source acting alone.



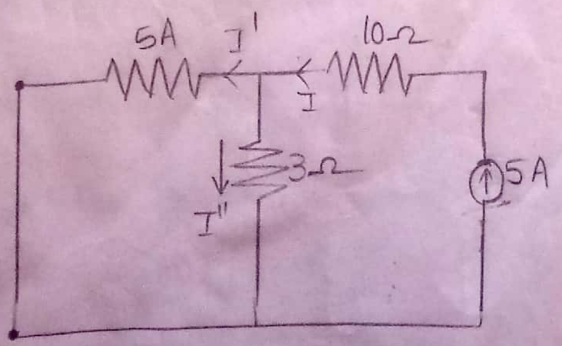
$$20 = 5I + 3I$$

$$8I = 20$$

$$I = \frac{\cancel{20}\,5}{\cancel{8}\,2} = 2.5A.$$

$\cancel{= 2.5}$

Case-ii:- 5A current source acting alone.



Current Division rule

$$I'' = I\left(\frac{R_2}{R_1+R_2}\right)$$

$$= 5\left(\frac{5}{8}\right)$$

$$= 3.125A.$$

Resultant response across 3-Ω resistor.

$$I = I_1 + I_2$$

$$= 2.5 + 3.125$$

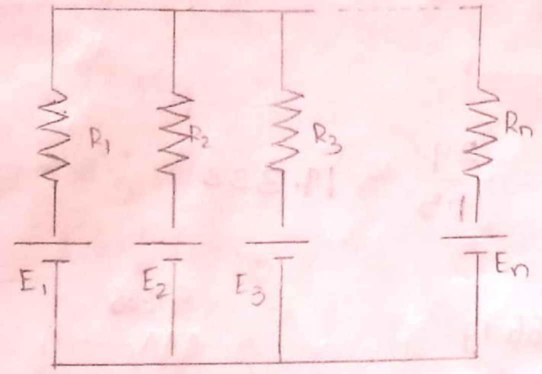
$$= 5.625A.$$

Reciprocity Theorem:-

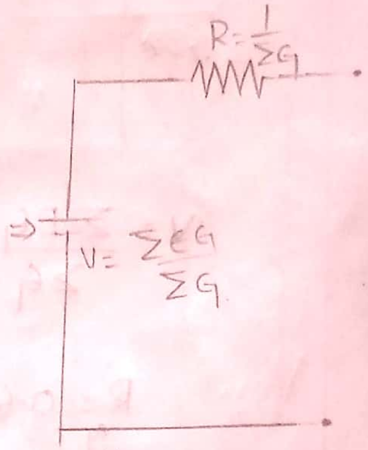
In a linear bilateral network, if a single voltage source V_a in branch 'a' produces a current I_b in branch b the if the ~~se~~voltage source 'V_a' is removed and inserted in branch b will produce ~~A~~ a current I_b in branch a. The ratio of response to excitation is same for the two conditions mentioned above. This is called Reciprocity Theorem.

Milliman's Theorem:-

Milliman's theorem states that any parallel circuit which consists of One voltage source in series with internal resistances in each branch can be converted into an equivalent circuit which consists of one voltage source in series with the equivalent resistance.



Circuit



Milliman's Equivalent Circuit.

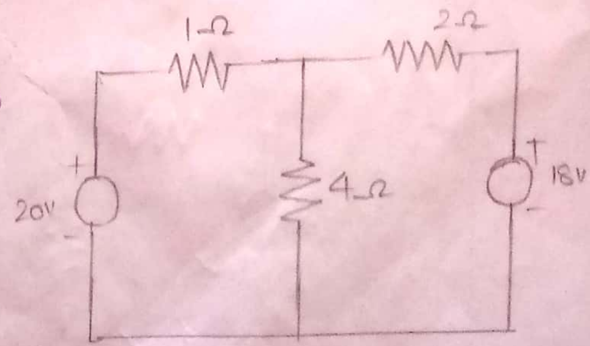
$$\Sigma EG = E_1G_1 + E_2G_2 + E_3G_3 + \ldots + E_nG_n$$

$$\Sigma G = G_1 + G_2 + G_3 + \ldots + G_n$$

$$= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

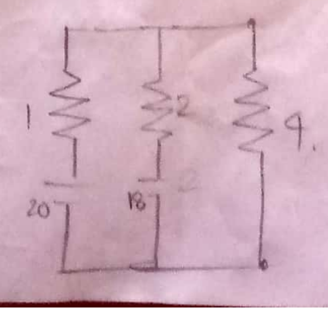
$$V = \frac{\Sigma EG}{\Sigma G}.$$

Q) Verify Milli-man's Theorem and find $I_{4\Omega}$



Ans.

$$V = \frac{\Sigma EG}{\Sigma G}.$$



$$\Sigma EG = E_1G_1 + E_2G_2 + E_3G_3$$

$$= 20 \times \frac{1}{1} + 18 \times \frac{1}{2}$$

$$= 29.$$

$\Sigma G = G_1 + G_2$

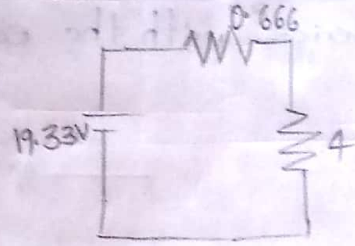
$= \frac{1}{R_1} + \frac{1}{R_2}$

$= \frac{1}{1} + \frac{1}{2}$

$= 1.5$

$R = \frac{1}{\Sigma G} = \frac{1}{1.5} = \frac{10^2}{15_3}$

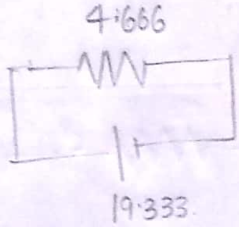
$= 0.666$



$V = \frac{\Sigma EG}{\Sigma G} = \frac{29}{1.5} = 19.333$

$R_{eq} = 0.666 + 4$

$= 4.666.$



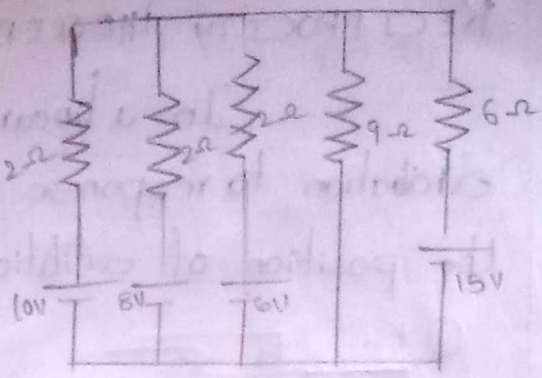
$i = \frac{V}{R} = \frac{19.333}{4.666.}$

$= 4.143.$

$i_{4\Omega} = \cancel{4} \frac{19.333}{4}$

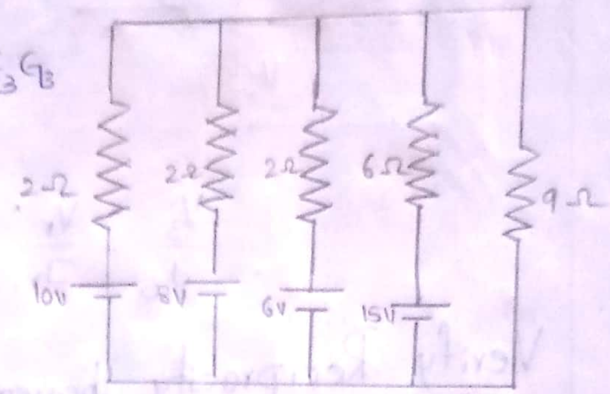
$= 4.$

$I_{9\Omega}$



Now,

$$V = \frac{\sum EG}{\sum G}$$



$$\sum EG = E_1G_1 + E_2G_2 + E_3G_3 + E_4G_4$$

$$= \cancel{10}^{5} \times \frac{1}{\cancel{2}} + \cancel{8}^{4} \times \frac{1}{\cancel{2}} + \cancel{6}^{3} \times \frac{1}{\cancel{2}} + \cancel{15}^{5} \times \frac{1}{\cancel{6}_{2}}$$

$$= 5 + 4 + 3 + 5/2$$

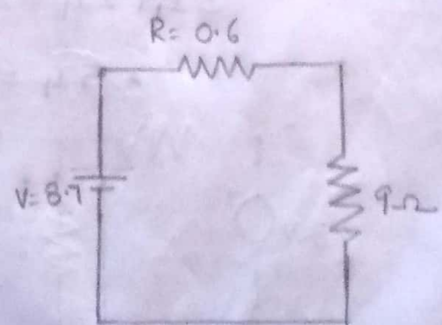
$$\sum EG = 29/2$$

$$\sum G = G_1 + G_2 + G_3 + G_4$$

$$\Rightarrow \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{6}$$

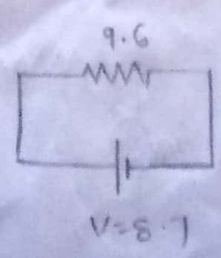
$$= \frac{3}{2} + \frac{1}{6} = \frac{\cancel{10}^{5}}{\cancel{6}_{3}} = \frac{5}{3}.$$

$$R = \frac{1}{\sum G} = \frac{3}{5} = 0.6$$



$$V = \frac{\sum EG}{\sum G} = \frac{29/2}{5/3} = \frac{87}{10} = 8.7$$

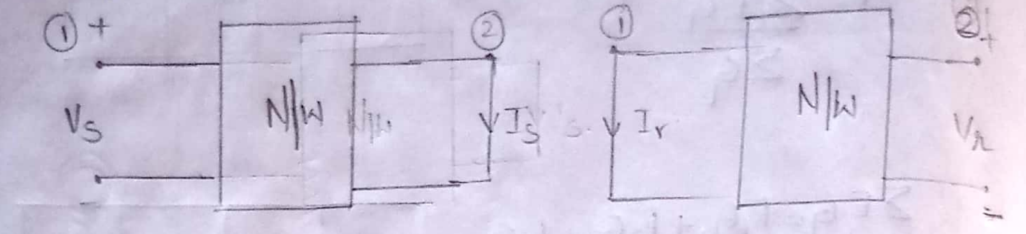
$R_{eq} = 9.6$ $\quad i = \frac{V}{R} = \frac{8.7}{9.6} = 0.9062$



$$i_{9\Omega} = \frac{8.7}{9} = 0.966$$

RECIPROCITY THEOREM:-

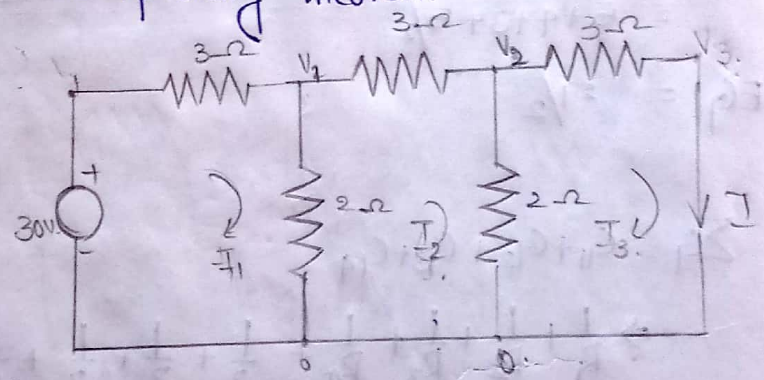
In a linear bilateral network, the ratio of exitabation to response is equal in the case. Even the position of excitation and response are interchanged



$$\frac{V_s}{I_s} \qquad \frac{V_r}{I_r}$$

$$\frac{V_s}{I_s} = \frac{V_r}{I_r}.$$

Verify Reciprocity Theorem.

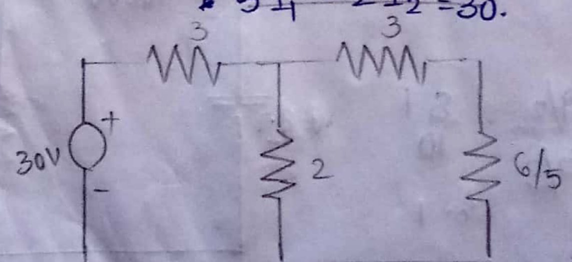


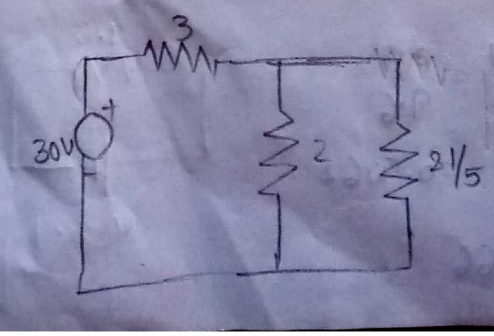
in first loop

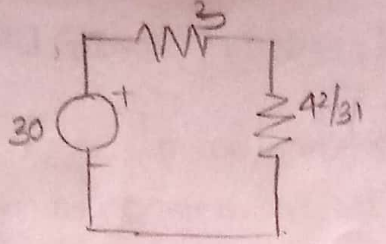
$$30 - 3I_1 - 2(I_1 - I_2) = 0.$$

$$3I_1 + 2I_1 - 2I_2 = 30.$$

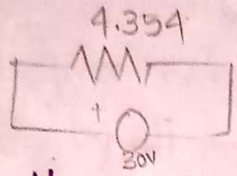
$$5I_1 - 2I_2 = 30.$$







$$R = \frac{\frac{21\times 2}{5}}{\frac{21}{5}+2} = \frac{42/\cancel{5}}{31/\cancel{5}} = \frac{42}{31}$$



$$R = \frac{42}{31} + 3$$

$$R_{eq} = 4.354$$

$$i = \frac{V}{R} = \frac{30}{4.354}$$

$$= 6.8902\,A.$$

$$i = \frac{V_1 - 30}{3} = \frac{V_1}{2} + \frac{V_1 - V_2}{3}$$

$$\frac{V_1 - V_2}{3} = \frac{V_2}{3} + \frac{V_2}{2}.$$

$$2V_1 - 60 = 3V_1 + 2(V_1 - V_2)$$

$$\cancel{2V_1} - 60 = 3V_1 + \cancel{2V_1} - 2V_2$$

$$-60 = 3V_1 - 2V_2 \rightarrow (1).$$

$$2V_1 - 2V_2 = 5V_2$$

$$2V_1 = 7V_2$$

$$V_1 = 7\frac{V_2}{2}$$

$$\boxed{I = 0.8 \text{ to } 0.89}$$

TELLIGEN'S THEOREM:-

In any arbitary lumped network is the algebriac sum of powers in all branches at any instant is zero.

(or)

In any given network the algebriac sum of powers delivered by all sources is equal to algebrias sum of powers absorbed.

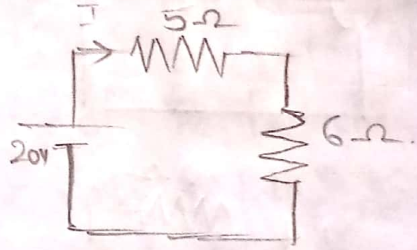
i.e.,

$$\sum_{K=1}^{n} V_K \, i_K = 0.$$

where

n = no. of elements.

Q) Verify Telligin's theorem:-



$R_{eq} = 11\Omega$.

$I = \frac{20}{11\Omega} = 1.818A$.

$P_{delivered} = 20 \times 1.818$

$= 36.3636W$.

$P_{absorbed} = \left(\frac{20}{11}\right)^2 \times 5 + \left(\frac{20}{11}\right)^2 \times 6$

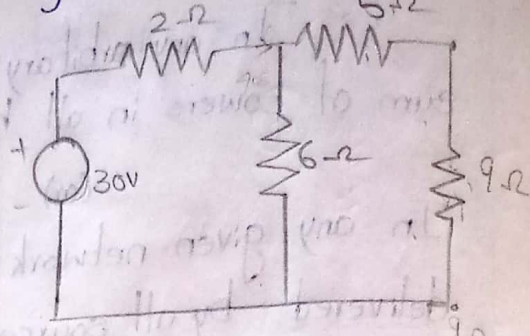
$= 36.3636W$.

SUBSTITUTION THEOREM:-

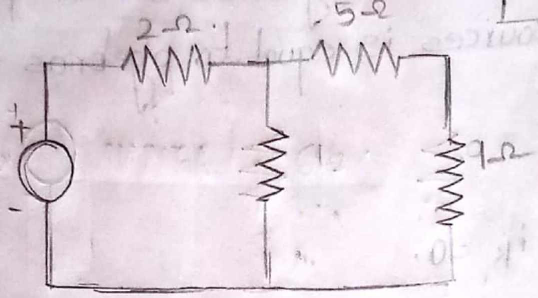
This theorem states that any branch of a network can be replaced by a new branch without changing the voltages and currents in the other branches. The current in the new branch and the voltage across the new branch are the same as in the original branch.

This theorem is useful when there is a need to replace one branch by the other desired circuit element.

Q) Replace 9Ω with a voltage Source.



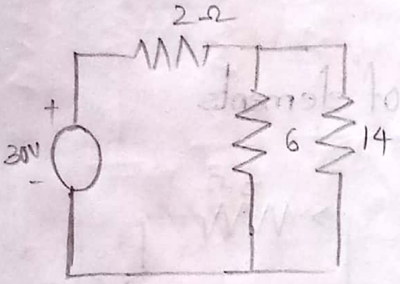
$\frac{14\times}{14+6}$ Ans:



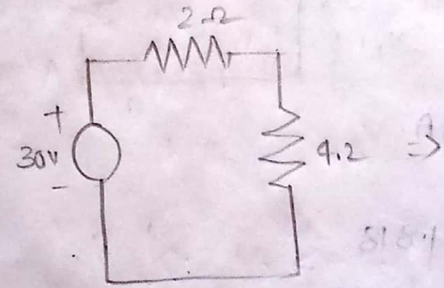
$\frac{84}{20}$

$\frac{124}{20}$

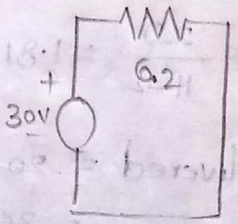
$R_{eq} = 5+9$
$= 14\Omega$.



$R = \frac{14\times6}{14+6} = \frac{84}{20}$.
$= 4.2$



$R = 2+4.2 = 6.2$



$I = \frac{V}{R} = \frac{30}{6.2}$

$I = 4.838$

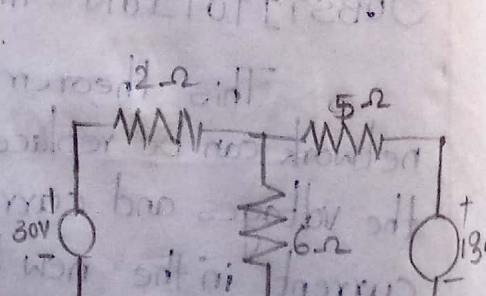
$I_{9\Omega} = 4.83 \times \frac{6}{6+14}$

$= 4.83 \times \frac{6}{20}$

$= 1.449.$

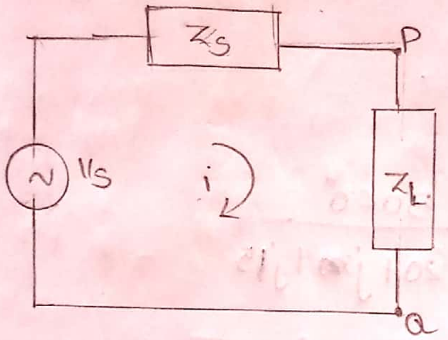
$V_9 = 1.449 \times \cancel{4.83}9$

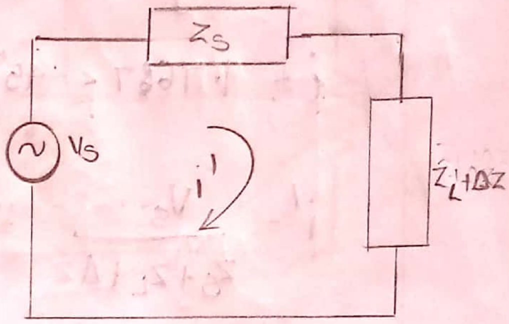
$= 13.041\Omega$



Compensation Theorem:-

This theorem is the combination of substitution and super position theorems. This theorem is used when it is desired to calculate the change in magnitude of current and voltage when there is a small change in the impedence of one of the branches.





$$i = \frac{V_s}{Z_s + Z_L} \qquad i' = \frac{V_s}{Z_s + Z_L + \Delta Z}$$

Let the impedence of the branches PQ changes from Z_L to $Z_L + \Delta Z$ and the new current be i', therfore change in current

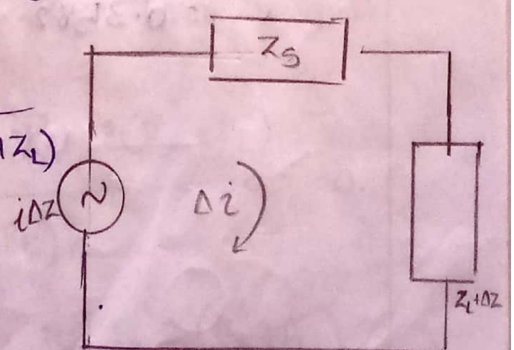
$\therefore$ change in current $\Delta i = i - i'$

$$\Delta i = i - i'$$

$$= \frac{V_s}{Z_s + Z_L} - \frac{V_s}{Z_s + Z_L + \Delta Z}$$

$$\Delta i = \frac{V_s \Delta Z}{(Z_s + Z_L + \Delta Z)(Z_s + Z_L)}$$

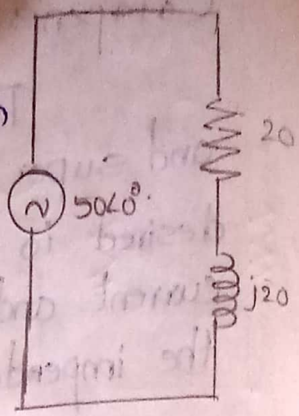
$$\Delta i = \frac{i \Delta Z}{Z_s + Z_L + \Delta Z}.$$



$\therefore$ As per the theorem the small change in curent due to small change in impedence is given by

$$\Delta i = \frac{V_s \Delta Z}{Z_s + Z_L + \Delta Z}.$$

Q) Calculate the change in current for the network when X_L is change from $j20\Omega$ to $j15\Omega$.



Ans: $$i = \frac{50\angle 0^\circ}{20+j20}$$

$$i = 1.7667 \angle -45^\circ$$

$$i' = \frac{V_s}{Z_S+Z_L+\Delta Z} = \frac{50\angle 0^\circ}{20+j\cancel{20}+j15}$$

$$= \frac{50\angle 0^\circ}{20+15j}$$

$$\cancel{= 1.56\angle -51.34} = 2\angle -36.869$$

$$\cancel{\Delta i = i - i'}$$

$$\cancel{= (1.767\angle -45) - (1.56\angle -51.34)}$$

$$\cancel{= 0.276\angle -6.496}$$

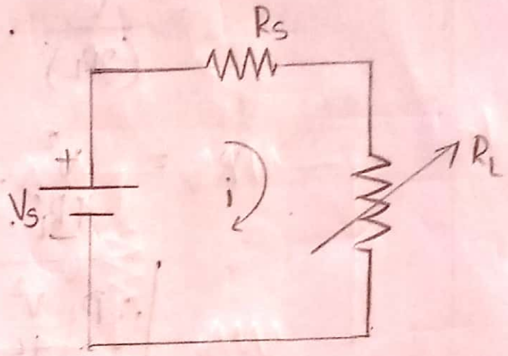
$$\Delta i = i - i'$$

$$= (1.767\angle -45^\circ) - (2\angle -36.869)$$

$$= 0.3582 \angle -172.86$$

Maximum Power Transfer Theorem:-

This theorem states that the power delivered by an active network to a load connected across its terminals is maximum when the impedence of the load is the complex conjugate of the active network impedence.



$R_S = R_L$.

$$i = \frac{V_S}{R_S + R_L}.$$

Maximum power deliverd to load is

$$P = i^2 R_L.$$

$$= \left(\frac{V_S}{R_S+R_L}\right)^2 R_L$$

$$P = \frac{V_S^2 R_L}{(R_S+R_L)^2}$$

To get maximum power $\frac{dP}{dR_L} = 0.$

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left(\frac{V_S^2 R_L}{(R_S+R_L)^2}\right) = 0.$$

$$\frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L 2(R_S+R_L)}{(R_S+R_L)^2} = 0.$$

$$(R_S+R_L)^{\cancel{2}} \cancel{V_S^2} = \cancel{V_S^2} R_L 2\cancel{(R_S+R_L)}$$

$$R_S + R_L = 2R_L$$

$$\boxed{R_S = R_L}$$

power transfferd at maximum power condition is

$$P = \frac{V^2}{(R_s+R_L)^2} \cdot R_L$$

$$= \frac{V^2}{(2R_L)^2} \cdot R_L$$

$$P = \frac{V^2}{4R_L^{\cancel{2}}} \cdot \cancel{R_L} = \frac{V^2}{4L}.$$

$$\boxed{\therefore P = \frac{V^2}{4L}}$$

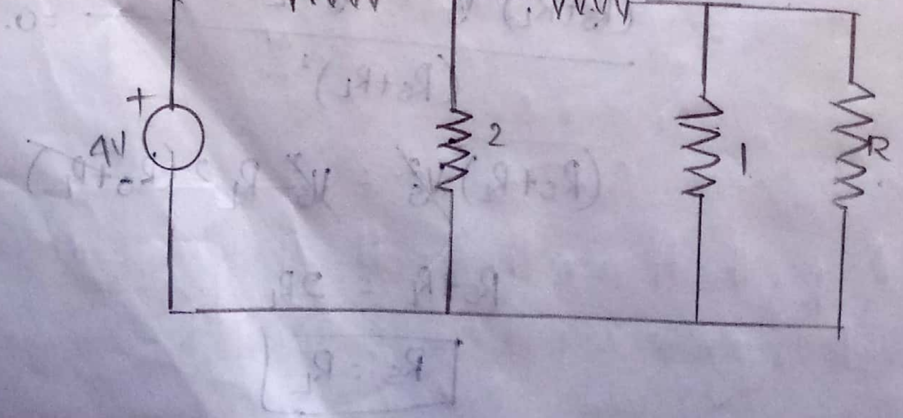
Procedure To Verify MPTT:-

1. The network is to be reduced to a single source, single network impedence and load impedence. i-e-> V_{TH}, R_{TH} are to be find out.

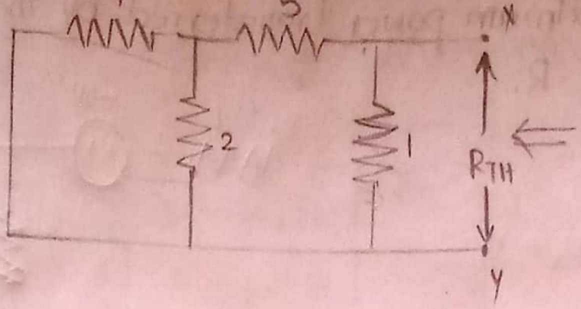
2. To verify the maximum power transfor theorem condition.

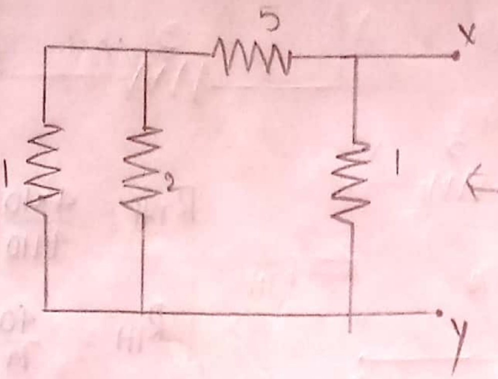
problem:-

Find the value of resistance ~~at~~ at the which maximum power is transffered?

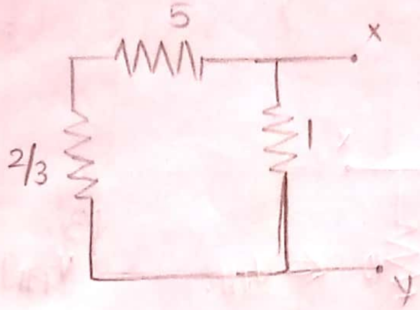


Sol:



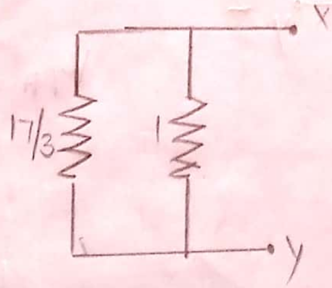


$R = \frac{1\times 2}{1+2} = \frac{2}{3}$.



$R = 5 + 2/3$

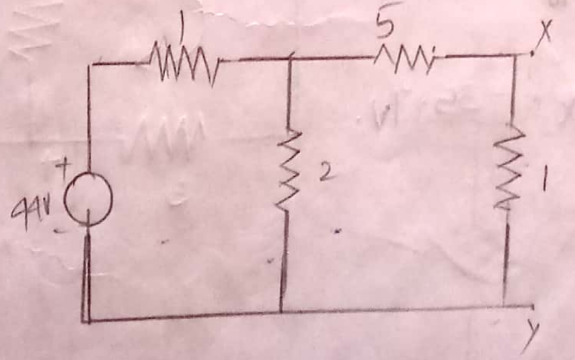
$= 17/3$.



$R_{TH} = \frac{\frac{17}{3}\times 1}{\frac{17}{3}+1}$

$= \frac{17/\cancel{3}}{20/\cancel{3}} = 17/20$

$R_{TH} = 0.85$



$Req = \frac{2\times 5}{2+5} = \frac{5}{6}.\ \frac{12}{8}$

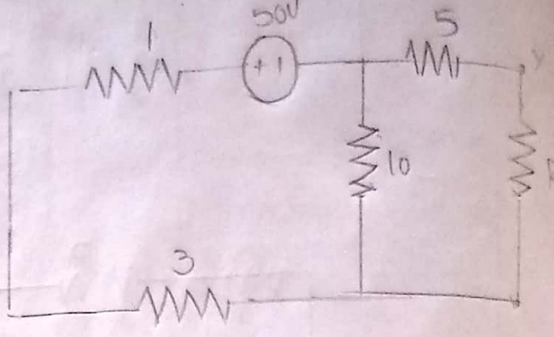
$= 3/2$

$= 1.5$

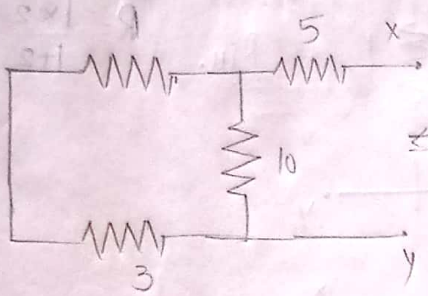
$+1 = 2.5$

$I = \frac{4}{2.5} = 1.6$.

Find the Maximum power transferred by the source to the resistance R.



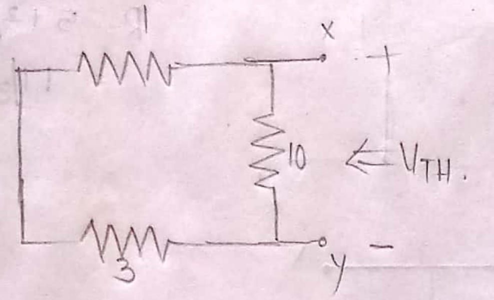
Ans:



$R_{TH} = \frac{4\times10}{4+10} = \frac{40}{14}$

$R_{TH} = \frac{40}{14} + 5$

$= 7.857\,\Omega$.



13

$I = \frac{50}{13}$

$= 3.846$.

$V_{TH} = I_{10} \times R_{10}$

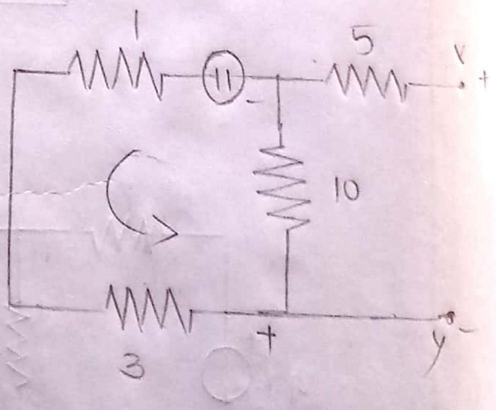
$= 3.57\times10$

$= 35.7$

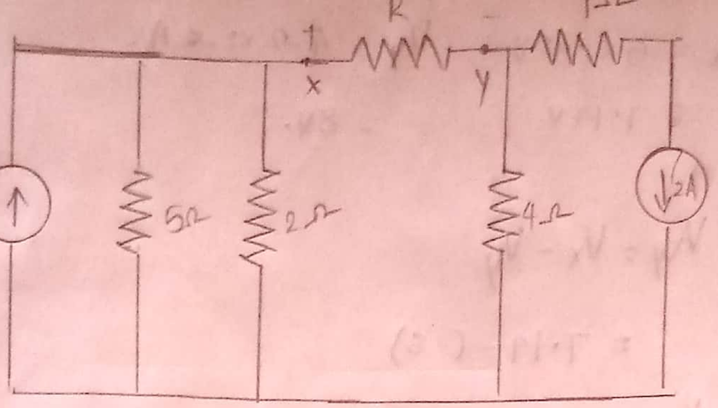
$V_{10} = 35.7$

$V_{xy} = -V_{10}$

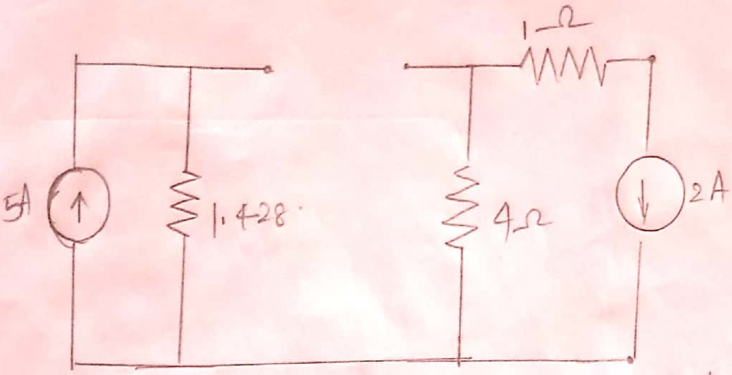
$V_{TH} = V_{oc} = -35.7\,V$.

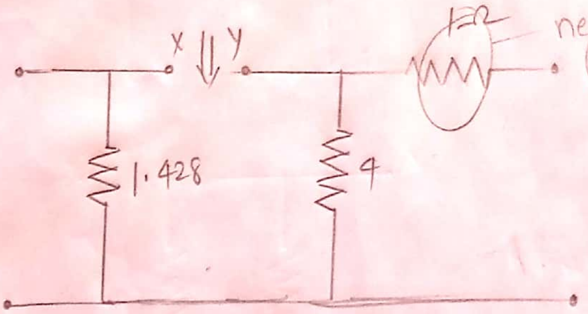


Q)

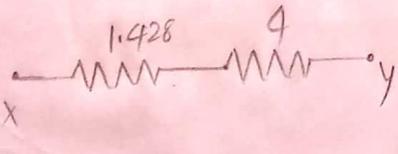


Ans:- $\underline{R_{TH}}$



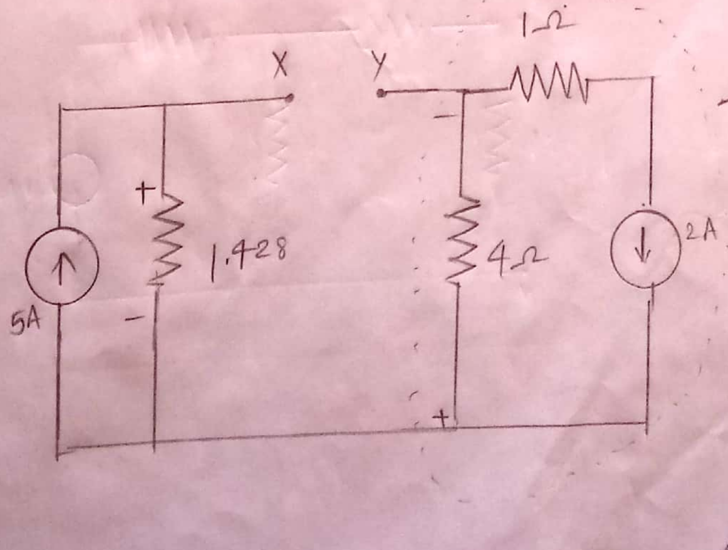


neglected ∵ the end of resistor isn't contact with any and.



$R_{TH} = 5.428.$

$\underline{V_{TH}}$



→ Here the current flows upto y but at y the circuit is open So, current only flow across the loop So 1,4 are in series.

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$V_x = 5 \times 1.428$ $\qquad$ $V_y = 4\Omega \times 2A.$

$= 7.14V$ $\qquad$ $= 8V.$

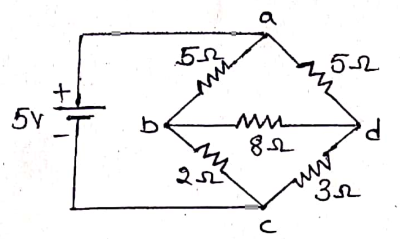
$V_{xy} = V_x - V_y$

$= 7.14 - (-8)$

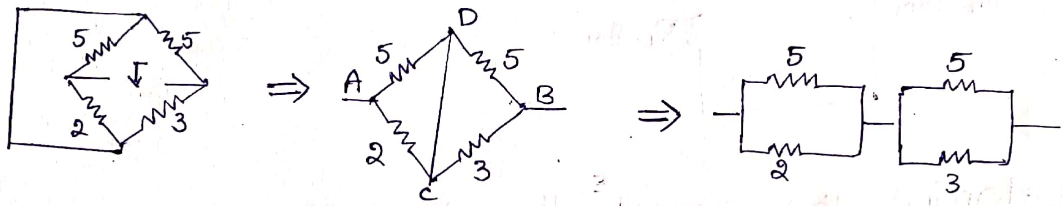
$V_{xy} = V_{TH} = 15.14$

UNit-4 - Network theorems

① Find the current through 8Ω resistance for the network shown using Thevenin's theorem.



Calculation of R_{th}:

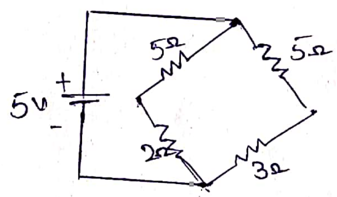


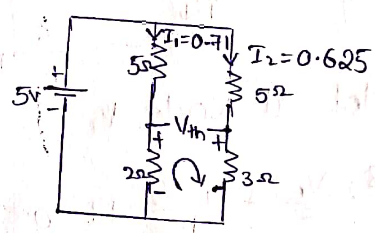
$(5 \| 2) + (5 \| 3)$

$$R_{th} = \frac{5 \cdot 2}{5+2} + \frac{5 \cdot 3}{5+3}$$

$$= \frac{10}{7} + \frac{15}{8} = \boxed{R_{th} = 3.30\,\Omega}$$

Calculation of V_{th}:





Here 5Ω & 2Ω are in series then current flowing through them is

$$I_1 = \frac{V}{R} \Rightarrow I = \frac{5}{7} = 0.71A$$

The current flowing through $(5+3)\Omega$ is I_2

$$I_2 = \frac{5}{8} = 0.625A$$

The voltage drop across 2Ω & 3Ω respectively are

$$V_{2\Omega} = I_1 R \Rightarrow V_{2\Omega} = 0.71 \times 2 = 1.42$$

$$V_{3\Omega} = I_2 R_{3\Omega}$$

$$= 0.625 \times 3$$

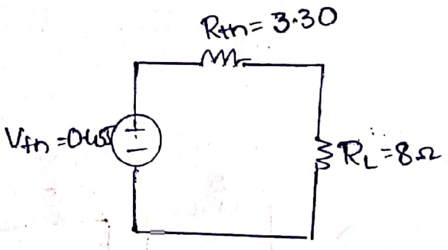
$$V_{3\Omega} = 1.875$$

Apply Mesh to below loop of V_{th}

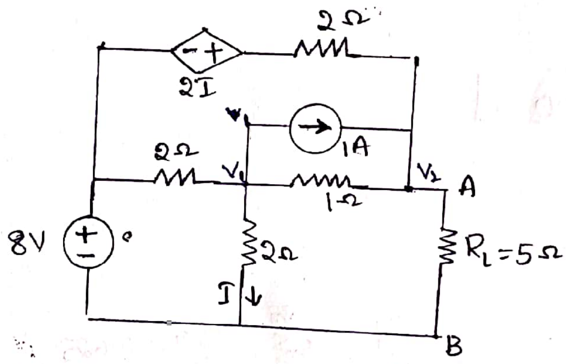
$$V_{2\Omega} + V_{th} - V_{3\Omega} = 0$$

$$1.42 + V_{th} - 1.875 = 0$$

$$\boxed{V_{th} = 0.455}$$



② Determine the current through load resistance R_L in the figure 5 using Thevenin's theorem. Also find maximum power transfer to resistance RL



Apply Nodal at Node V_1

$$\frac{V_1 - 8}{2} + \frac{V_1}{2} + \frac{V_1 - V_2}{1} + 1 = 0$$

$$V_1 - 8 + V_1 + 2V_1 - 2V_2 + 2 = 0$$

$$4V_1 - 2V_2 - 6 = 0 \rightarrow ①$$

Apply Nodal to Node V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 2I - 8}{2} - 1 = 0$$

$$3V_2 - 2V_1 - 2 - 2I - 8 = 0$$

$$3V_2 - 2V_1 - 2I = 10$$

$3V_2 - 2V_1 - 2(\frac{V_1}{2}) = 10 \quad [\because I = \frac{V_1}{2}]$ ②

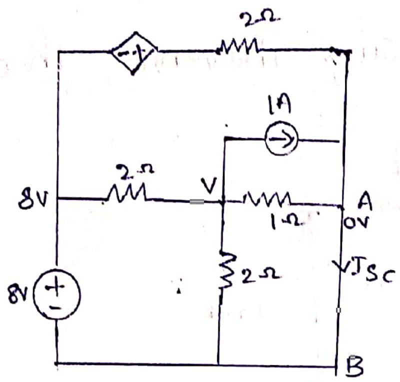
$-3V_1 + 3V_2 = 10 \rightarrow$ ②

Solving Eq ① & ② we get

$V_1 = 6.33$

$\boxed{V_2 = V_{th} = 9.66}$

Calculation of I_{sc}:



Apply Nodal at Node V

$$\frac{V-8}{2} + \frac{V}{2} + \frac{V}{1} + 1 = 0$$

$$V - 8 + V + 2V + 2 = 0$$

$$4V = 6$$

$$V = \frac{6}{4} = 1.5$$

$$\boxed{V = 1.5}$$

Apply nodal at '0 V'

$$I_{sc} + \frac{0-V}{1} - 1 + \frac{0-2I-8}{2} = 0$$

$$2I_{sc} - 2V - 2 + 0 - 2I - 8 = 0$$

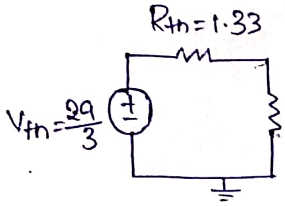
$$2I_{sc} - 2V - 2 - 2(\frac{V}{2}) = 10$$

$$2I_{sc} - 3V = 10$$

$$2I_{sc} - 3(1.5) = 10$$

$$\boxed{I_{sc} = 7.25\ A}$$

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{9.66}{7.25} = 1.33\,\Omega$$

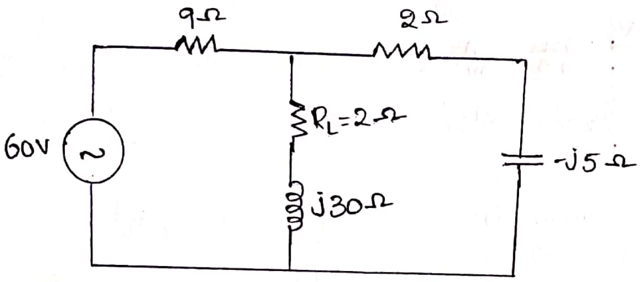


Current through 5 Ω resistance

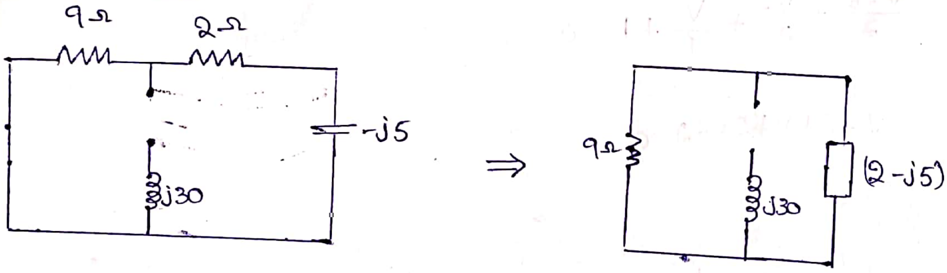
$$I_{5\Omega} = \frac{V}{R_{eq}} = \frac{\frac{29}{3}}{\left(\frac{4}{3}+5\right)} = \frac{29}{19}\ A$$

$$\boxed{I_{5\Omega} = 1.52\ A}$$

③ Find current through RL using Thevenin's theorem for the circuit shown .

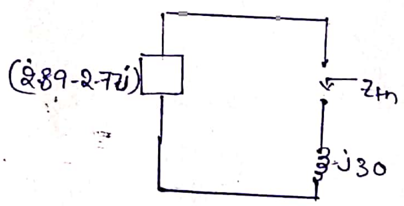


<u>calculation of Zth</u>:-



$$(9\Omega) \,//\, (2-j5) \Rightarrow \frac{9\times(2-j5)}{11-j5} = \frac{18-45j}{11-j5}$$

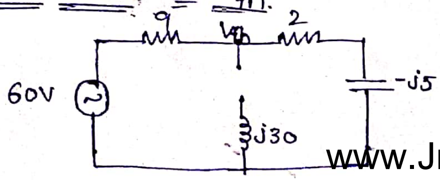
$$(2.89-2.77j)\ \Omega$$



$$Z_{th} = 2.89 - j2.77 + j30$$

$$Z_{th} = (2.89 + j27.23)\ \Omega.$$

<u>calculation of Vth</u>:



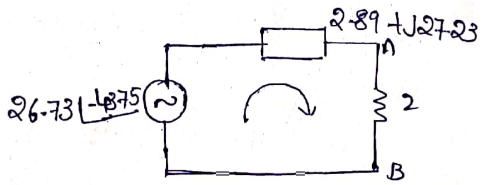
Apply Nodal at Node V_{th} (3)

$$\frac{V_{th}-60}{90}+\frac{V-0}{2-j5}=0$$

$$V_{th}\left[\frac{1}{9}+\frac{1}{2-j5}\right]=\frac{60}{9}$$

$$V_{th}=19.31-18.49j$$

$$V_{th}=26.73\angle 43.75 \text{ volts.}$$

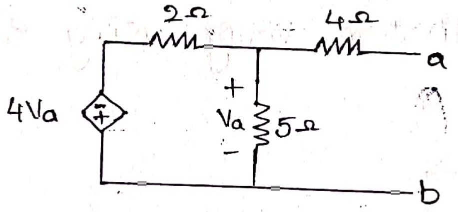


$$I=\frac{V}{Z}=\frac{26.7\angle -43.7}{4.89+j27.23}$$

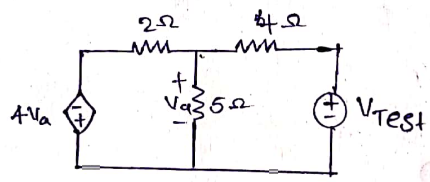
$$I_{2\Omega}=-0.53-0.80j=0.96\angle -123.5.$$

Nortons theorem.

(4) For the circuit shown in the figure, find Norton's equivalent circuit



when ever only dependent sources are present in the N/w we have to connect a Test source voltage source to the load terminals & calculate the ratio. $\frac{V_{Test}}{I_{Test}}$



Apply Nodal at Node Va

$$\frac{V_a+4V_a}{2}+\frac{V_a}{5}-I_{Test}=0$$

$$\frac{5V_a}{2}+\frac{V_a}{5}-I_{Test}=0$$

$$\frac{5V_a}{2}+\frac{V_a}{5}=I_{Test}$$

$$I_{Test}=\frac{27V_a}{10}\rightarrow (1)$$

Apply KVL to loop ①

$$V_{Test} - 4I_{Test} - V_a = 0.$$

$$V_a = V_T - 4I_T \rightarrow ②$$

using Eq ② & ①

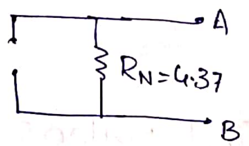
$$\frac{27V_a}{10} = I_{Test}$$

$$\frac{27(V_{Test} - 4I_{Test})}{10} = I_{Test}$$

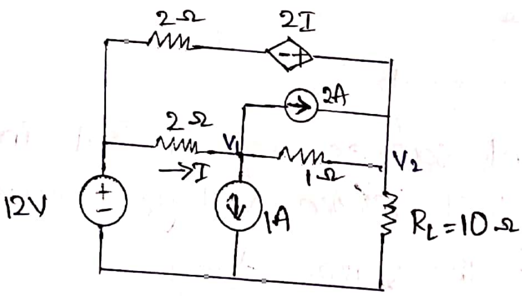
$$27V_{Test} - 108I_{Test} = 10I_{Test}$$

$$27V_{Test} = 118I_{Test}$$

$$\frac{V_{Test}}{I_{Test}} = \frac{118}{27}, \quad R_N = 4.37\,\Omega.$$



⑤ Determine the current through $R_L = 10\,\Omega$ resistor as shown in fig using Thevenin's theorem. Verify using same with Nortons theorem.



Apply Nodal at Node V_1

$$\frac{V_1 - 12}{2} + \frac{V_1 - V_2}{1} + 1 + 2 = 0$$

$$V_1 - 12 + 2V_1 - 2V_2 + 2 + 4 = 0$$

$$3V_1 - 2V_2 = 6 \rightarrow ①$$

Apply Nodal at Node V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 2I - 12}{2} - 2 = 0$$

$$2V_2 - 2V_1 + V_2 - 2I - 12 - 4 = 0$$

$$-2V_1 + 3V_2 - 2\left(\frac{V_1 - 12}{2}\right) - 16 = 0$$

$-3V_1 + 3V_2 - 4 = 0$

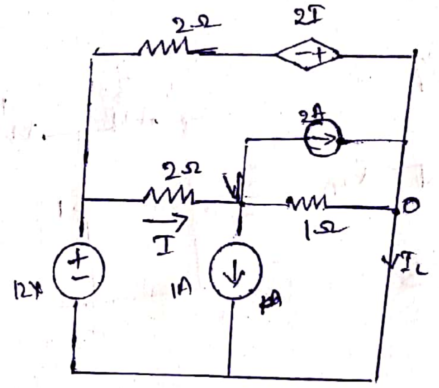
$-3V_1 + 3V_2 = 4 \rightarrow (2)$

By solving eq (1) & eq (2) we get

$V_1 = 8.66$

$V_2 = V_{th} = 10$

<u>calculation of I_{sc}:</u>



Apply Nodal at Node V

$$\frac{V-12}{2} + \frac{V-0}{1} + 2 + 1 = 0$$

$$V - 12 + 2V + 4 + 2 = 0$$

$$3V = 6$$

$$V = \frac{6}{3}$$

$$\boxed{V = 2}$$

Apply nodal at '0V'

$$I_{sc} + \frac{0-V}{1} + \frac{0-2I-12}{2} - 2 = 0$$

$$-2V - 2I - 12 - 4 = 0$$

-2 .

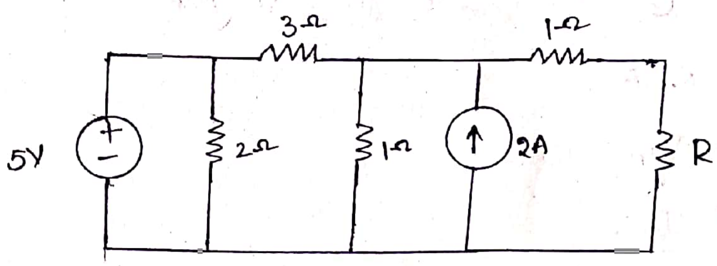
$$I_{sc} - 2(2) - 2\left(\frac{2-12}{2}\right) - 12 - 4 = 0$$

$$I_{sc} + -4 - 2 + 12 - 12 - 4 = 0$$

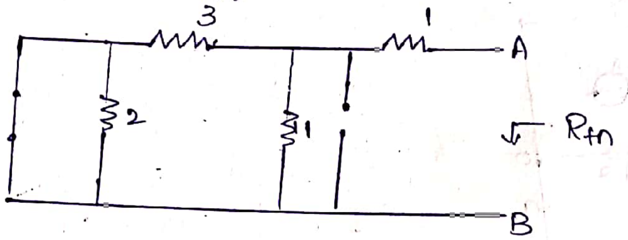
$$I_{sc} = 10 \text{ Amps}$$

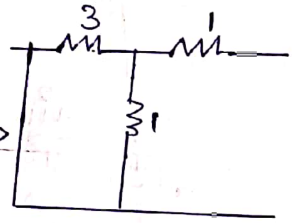
$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{2}{10} = \frac{1}{5} = 0.2\,\Omega$$

(5) Find the value of R in the circuit shown in figure such that maximum power transfer takes place. What is the amount of this power.

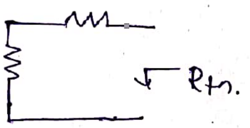


Calculation of R_{th}





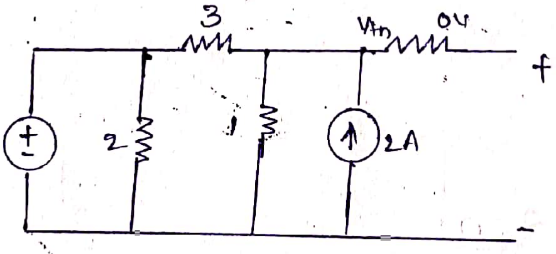
$(3 \| 1)\Omega$



$R_{th} = \frac{3}{4}\Omega + 1\Omega$

$R_{th} = 1.75\,\Omega$

Calculation of V_{th}:

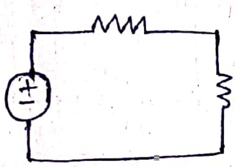


Apply Nodal at Node V_{th}

$$-2 + 0 + \frac{V_{th}}{1} + \frac{V_{th} - 5}{3} = 0$$

$$-6 + 3V_{th} + V_{th} - 5 = 0$$

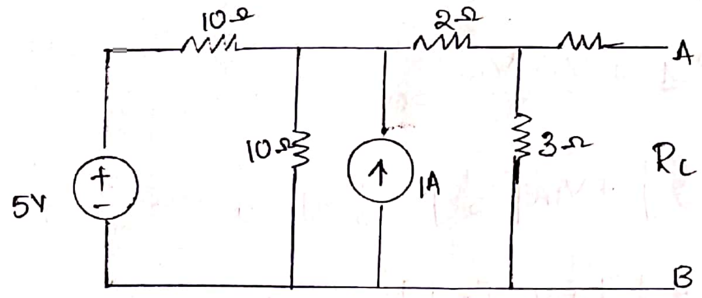
$$V_{th} = 2.75 \text{ Volt.}$$



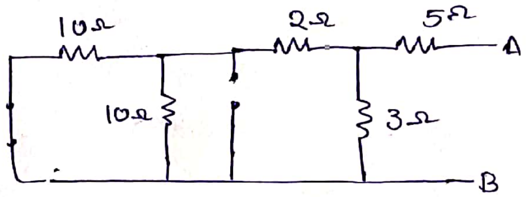
$$P_{max} = \frac{V^2}{4R_L} = \frac{(2.75)^2}{4 \times 1.75}$$

$P_{max} = 1.08$ watt

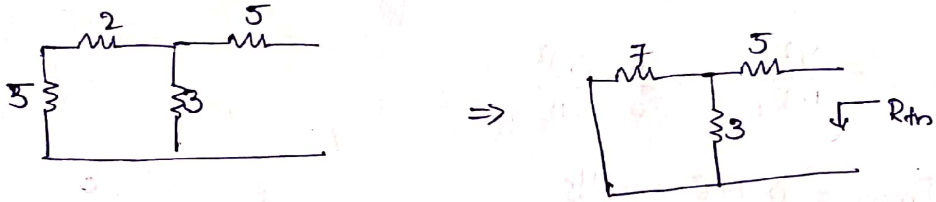
(7) In the network shown in figure. Find the resistance RL to be connected b/w the terminals A & B so that maximum power is developed across RL. What is the maximum power delivered? (5)



Calculation of R_{th}:

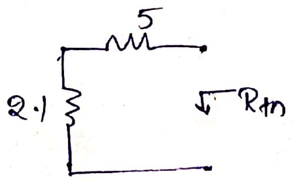


$$10 \| 10 = \frac{10 \times 10}{10+10} = \frac{100}{20} = 5$$



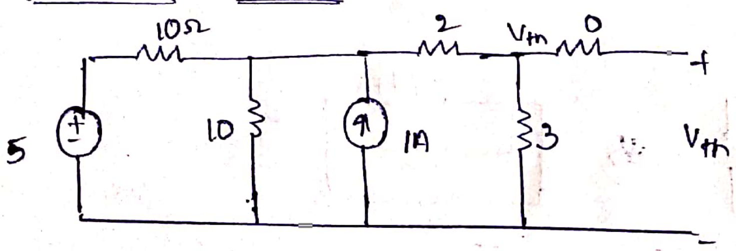
$7 \| 3$

$$\Rightarrow \frac{7 \times 3}{7+3} = \frac{21}{10} = 2.1\,\Omega$$



$$R_{th} = 5 + 2.1 = 7.1\,\Omega$$

Calculation of V_{th}:



Apply KCL:

$$\frac{V_{th} - V_1}{2} + \frac{V_{th} - 0}{3} = 0$$

$$\frac{V_{th}}{2} - \frac{V_1}{2} + \frac{V_{th}}{3} = 0$$

$$V_{th}\left[\frac{1}{2} + \frac{1}{3}\right] + V_1\left[\frac{-1}{2}\right] = 0$$

Apply kvl at V_1

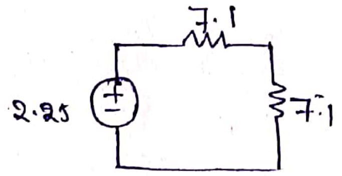
$$\frac{V_1 - 5}{10} + \frac{V_1 - 10}{10} - 1 + \frac{V_1 - V_{th}}{2} = 0$$

$$V_1\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{2}\right] + V_{th}\left[\frac{-1}{2}\right] - \frac{1}{2} - 1 = 0$$

$$V_1\left[\frac{1}{10} + \frac{1}{10} + \frac{1}{2}\right] + V_{th}\left[\frac{-1}{2}\right] = 1 + \frac{1}{2} = \frac{3}{2}$$

$V_1 = 3.75$

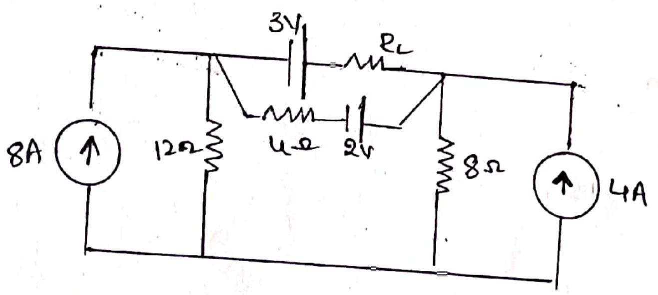
$V_{th} = 2.25$



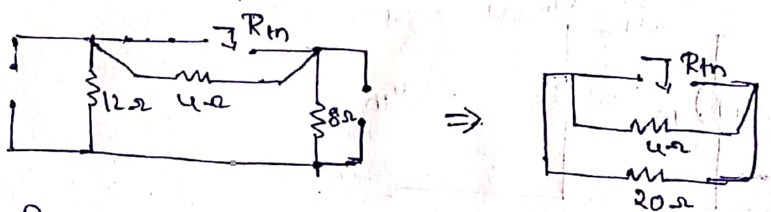
$$P_{max} = \frac{V^2}{4R_L} = \frac{(2.25)^2}{4(7.1)}$$

$P_{max} = 0.178$ watts

⑧ Obtain the maximum amount of power transferred to RL from the sources using Maximum power transfer theorem in the circuit shown



calculation of R_{th}:

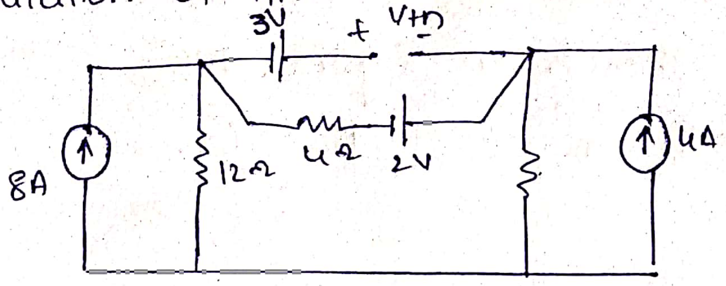


$R_{th} = 4\Omega \,||\, 20\,\Omega$

$$R_{th} = \frac{80}{24} = \frac{10}{3}\,\Omega$$

$$R_{th} = \frac{10}{3}\,\Omega$$

Calculation of V_{th}:



Apply Nodal at Node V_1

$$-8 + \frac{V_1}{12} + \frac{V_1 + 2 - V_2}{4} + 0 = 0$$

$$\frac{-96 + V_1 + 3V_1 + 6 - 3V_2}{12} = 0$$

$$4V_1 - 3V_2 = 90 \longrightarrow (1)$$

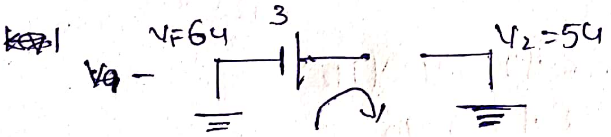
Apply Nodal at Node "V_2"

$$-4 + \frac{V_2}{8} + \frac{V_2 - 2 - V_1}{4} + 0 = 0$$

$$\frac{-32 + V_2 + 2V_2 - 4 - 2V_1}{8} = 0$$

$$-2V_1 + 3V_2 = 36 \rightarrow (2)$$

Solving Eq (1) & (2)
$V_1 = 63$ volts
$V_2 = 54$ volts

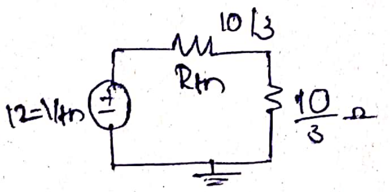


KVl

$$V_1 + 3 - V_{th} - V_2 = 0$$

$$63 + 3 - V_{th} - 54 = 0$$

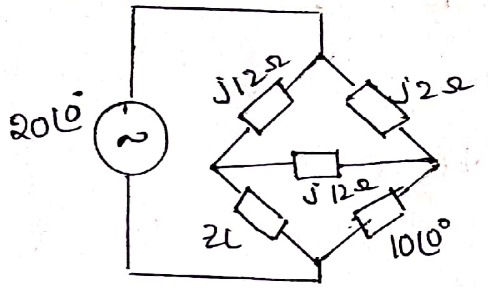
$$V_{th} = 12 \text{ volts}$$



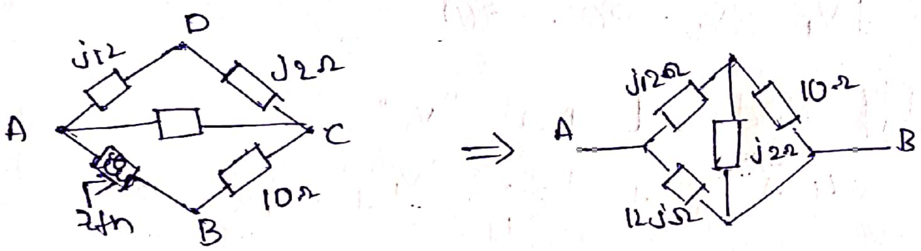
$$P_{max} = \frac{V^2}{4R_L} = \frac{(12)^2}{4 \times \frac{10}{3}}$$

$$P_{max} = \frac{144 \times 3}{40} = 10.8 \text{ watts}$$

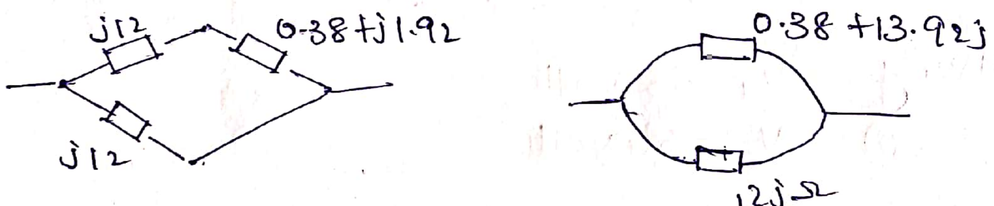
9) Calculate Z_L for the maximum power transfer to it from the network shown in figure. Also calculate the amount of maximum power transfer.



Calculation of Z_{th}:



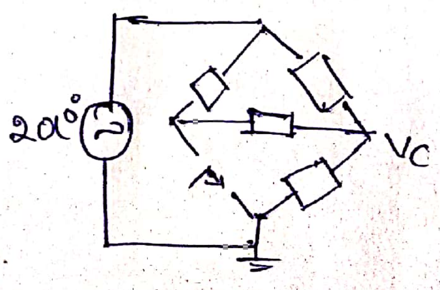
$$J2\Omega \parallel 10 = \frac{j20}{10+j2} = 0.38 + j1.92$$



$$Z_{AB} = Z_{th} = \frac{(0.38+j13.92)(j12)}{0.38+j25.92}$$

$$Z_{th} = 0.081 + 6.44j$$

Calculation of V_{th}



Apply nodal at node V_{th}

$$\frac{V_{th}-20}{j12}+\frac{V_{th}-V}{j12}+0=0$$

$$2V_{th}-V=20 \rightarrow ①$$

Apply Nodal at Node 'v'

$$\frac{V-V_{th}}{12j}+\frac{V-0}{10}+\frac{V-20}{12j}$$

$$-\frac{V_{th}}{12j}+V\left[\frac{1}{j12}+\frac{1}{10}+\frac{1}{j2}\right]=\frac{20}{j2}$$

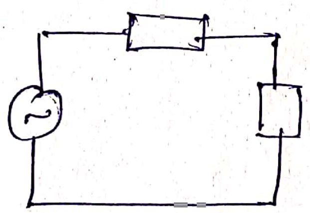
$$-\frac{V_{th}}{12j}+\left[(2V_{th}-20)\left[0.1-j0.58\right]\right]=-10j$$

$$(0.08j)V_{th}+V_{th}(0.2-j1.16)=-10j+20(0.1-j0.58)$$

$$V_{th}\left[0.2-j1.08\right]=2-11.6j-10j$$

$$V_{th}=\frac{2-21.6j}{0.2-j1.08}=19.66-1.79j$$

$$V_{th}=19.74\angle -5.2^{\circ}\ \text{volt}$$

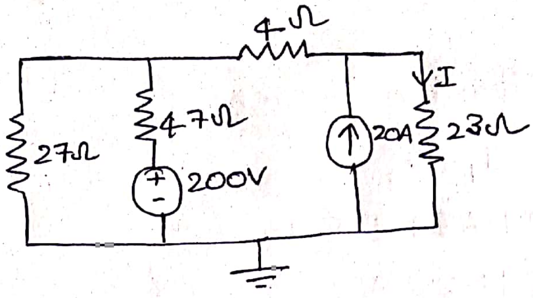


$$Z_L=0.08-6.44\Omega$$

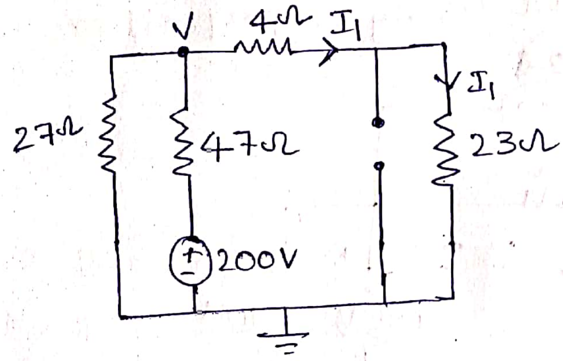
$$P_{max}=\frac{V^2}{4R_L}=\frac{(19.74)^2}{4\times 0.08}=1217.7\ \text{watt.}$$

SUPER POSITION THEOREM

1, Compute the current in 23Ω resistor using superposition theorem for the circuit.



Response due to 200V source (I_1).



$$\frac{V-0}{27} + \frac{V-200-0}{47} + \frac{V-0}{27} = 0$$

$$\frac{2V}{27} + \frac{V}{47} = \frac{200}{47}$$

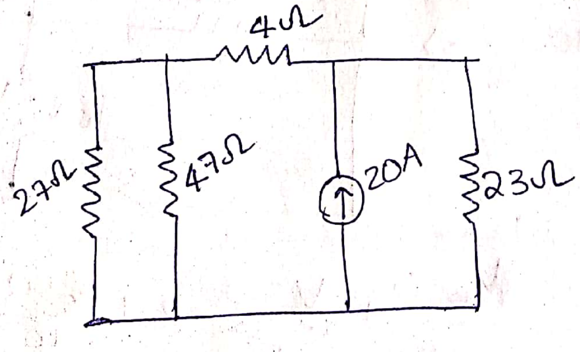
$$\frac{94V+27V}{27\times\cancel{47}} = \frac{200}{\cancel{47}}$$

$$121V = 5400$$

$$V = \frac{5400}{121} = 44.62V$$

$$I_1 = \frac{44.62}{27} = 1.65A.$$

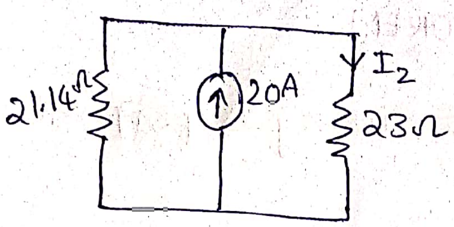
Response due to 20A source (I_2).



$= (27 \| 47) + 4$

$= 17.14 + 4$

$= 21.14\,\Omega$.



$$I_2 = \frac{I_1 \times 21.14}{21.14+23} = \frac{20\times 21.14}{44.14} = 9.57A.$$

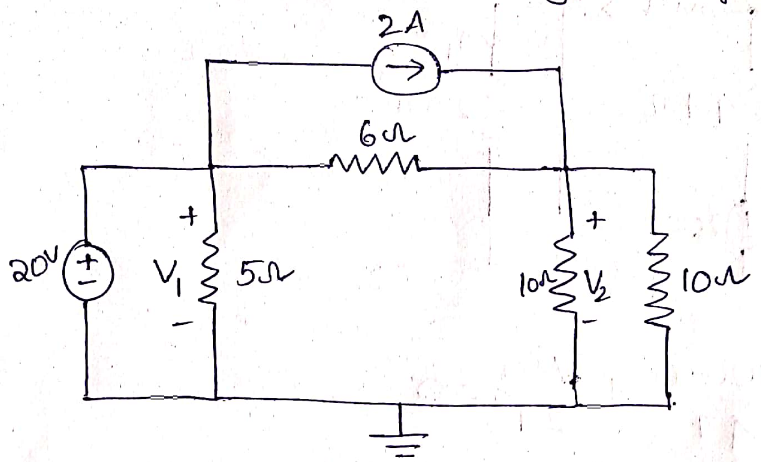
According to Super Position Theorem,

$$I = I_1 + I_2$$

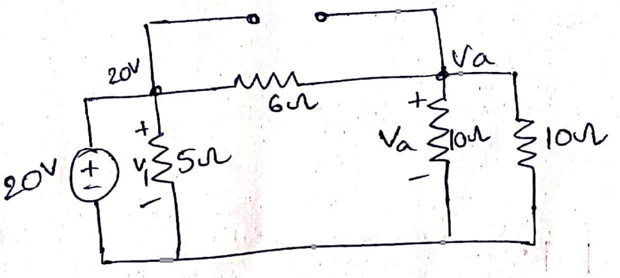
$$I = 1.65 + 9.57$$

$$= 11.22A.$$

2, find voltage across 10Ω using superposition Theorem.



Response due to 20V,



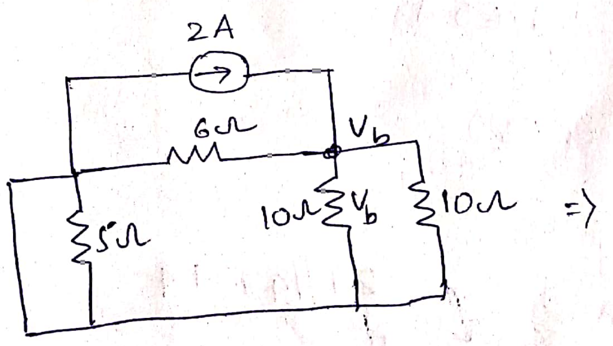
$$\frac{Va-0}{10} + \frac{Va-0}{10} + \frac{Va-20}{6} = 0$$

$$Va\left(\frac{1}{10}+\frac{1}{10}+\frac{1}{6}\right) = \frac{20}{6}$$

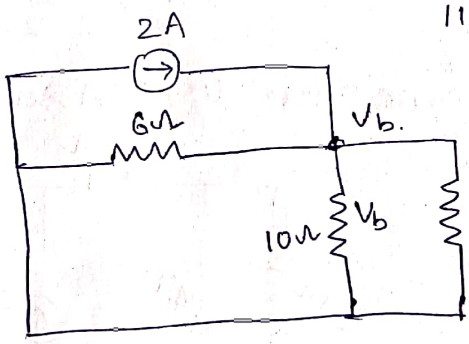
$$Va\left(\frac{1}{5}+\frac{1}{6}\right) = \frac{20}{6} \Rightarrow Va\left(\frac{11}{30}\right) = \frac{20}{6}$$

$$Va = \frac{100}{11} = 9.09V$$

Response due to 2A,



$\Rightarrow$



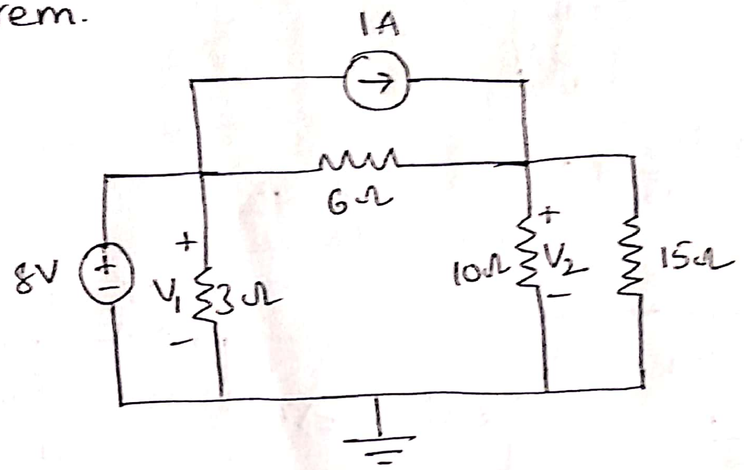
$$\frac{V_b-0}{6} + \frac{V_b-0}{10} - 2 + \frac{V_b-0}{10} = 0$$

$$V_b\left(\frac{1}{6}+\frac{1}{5}\right) = 2$$

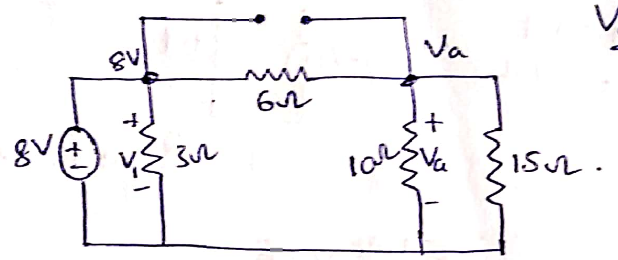
$$V_b = \frac{60}{11} = 5.45V$$

$$V_2 = V_a + V_b$$
$$= 9.09 + 5.45$$
$$= 14.54V.$$

④ Find voltage across 10Ω resistance using superposition Theorem.



Response due to 8V,



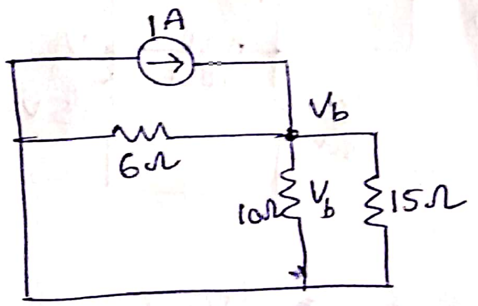
$$\frac{V_a - 8}{6} + \frac{V_a - 0}{10} + \frac{V_a}{15} = 0$$

$$V_a\left(\frac{1}{6} + \frac{1}{10} + \frac{1}{15}\right) = \frac{8}{6}$$

$$V_a\left(\frac{1}{3}\right) = \frac{8}{6}$$

$$V_a = 4V.$$

Response due to 1A,



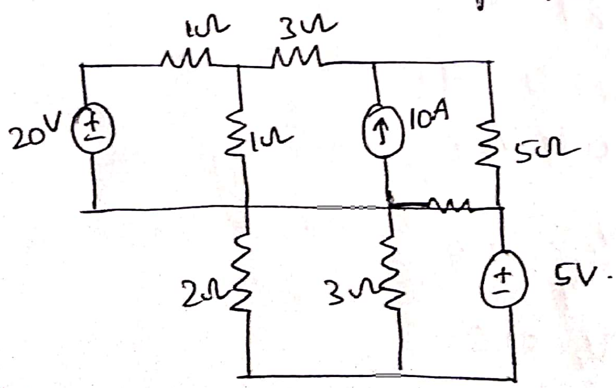
$$-1 + \frac{V_b - 0}{6} + \frac{V_b}{10} + \frac{V_b}{15} = 0.$$

$$V_b\left(\frac{1}{3}\right) = 1$$

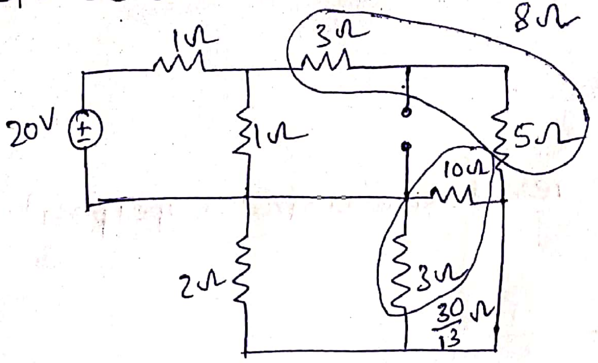
$$V_b = 3$$

$$V_2 = V_a + V_b = 4 + 3 = 7V.$$

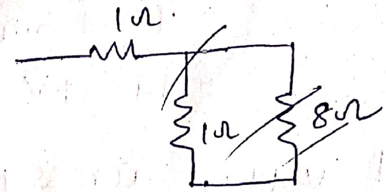
⑤ Determine I in circuit using Super Position Theorem.



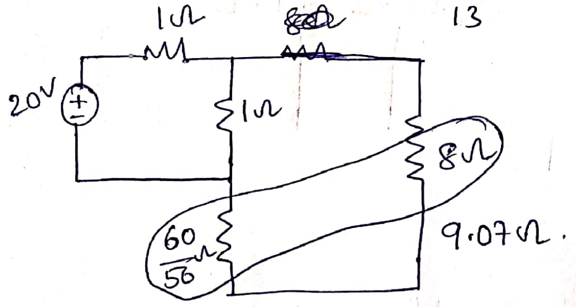
Response due to 20V,



⇒

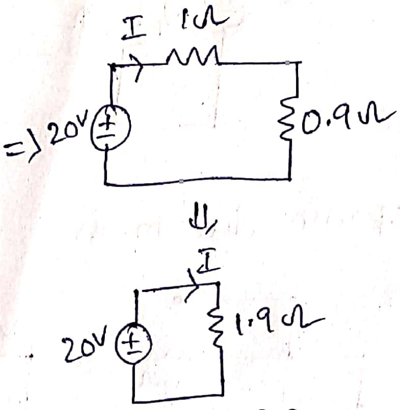


$$2\Omega \,||\, \frac{30}{13}\,\Omega = \frac{2\times\frac{30}{13}}{2+\frac{30}{13}} = \frac{60}{56} \text{ then}$$



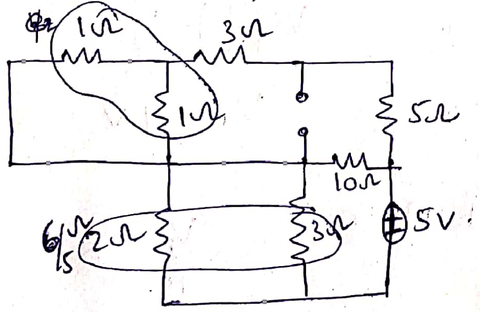
then $1\Omega \,||\, 9.07\Omega$
$= 0.9\Omega$.

⇒

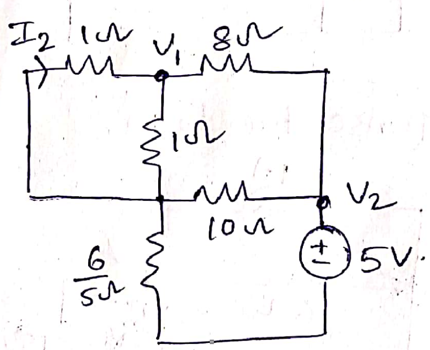


$$I = \frac{20}{1.9} = 10.52A.$$

Response due to 5V,



⇒



Apply nodal at node V_1,

$$-I_2 + \frac{V_1}{1} + \frac{V_1-V_2}{8} = 0 \Rightarrow -\left(\frac{0-V_1}{1}\right) + V_1 + \frac{V_1-V_2}{8} = 0$$

$$\Rightarrow 2V_1 + \frac{V_1}{8} + V_2\left(\frac{-1}{8}\right) = 0$$

$$17V_1 - V_2 = 0 \quad \text{—(1)}$$

Apply nodal at node V_2,

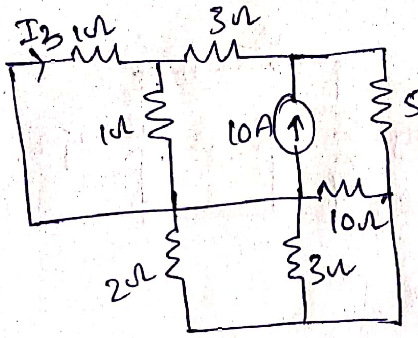
$$\frac{V_2-V_1}{8} + \frac{V_2}{10} + \frac{V_2-5}{6/5} = 0$$

$$\Rightarrow \frac{V_2-V_1}{8} + \frac{V_2}{10} + \frac{(V_2-5)\,5}{6} = 0$$

$$-15V_1 + 127\,V_2 = 500 \quad \text{—(2)}$$

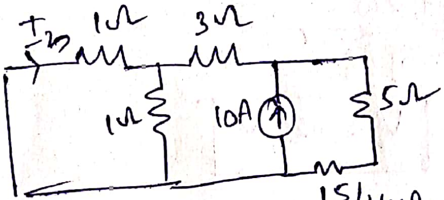
$V_1 = 0.23\,V,\; V_2 = 3.96\,V \Rightarrow I_2 = \frac{-V_1}{1} = -0.23A$

Response due to 10A source (I_2).



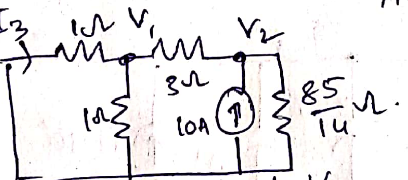
$2\Omega, 3\Omega, 10\Omega$ are in parallel.

$$R_{eq} = \frac{60}{6+30+20} = \frac{15}{14}\Omega.$$



$\frac{15}{14}\Omega$ & 5Ω are in series.

$$R_{eq} = \frac{15}{14}+5 = \frac{85}{14}\Omega.$$



Apply nodal at V_1,

$$-I_2 + V_1 + \frac{V_1 - V_2}{3} = 0$$

$$\Rightarrow -\left(\frac{-V_1}{1}\right) + V_1 + \frac{V_1 - V_2}{3} = 0$$

$$7V_1 - V_2 = 0 \quad \text{—①}$$

Apply nodal at V_2,

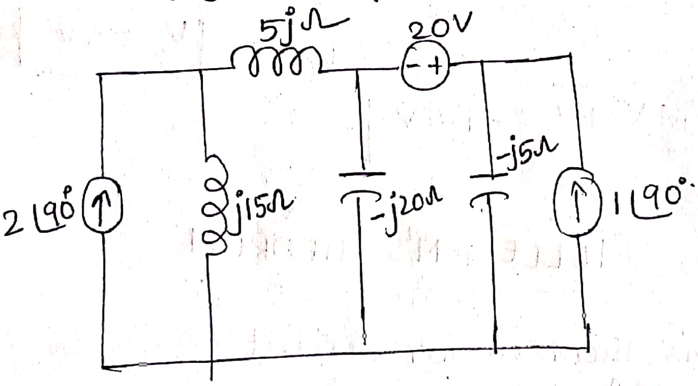
$$\frac{V_2 - V_1}{3} + \frac{V_2}{85/14} - 10 = 0 \Rightarrow \left(\frac{-1}{3}\right)V_1 + V_2\left(\frac{127}{255}\right) = 10 \quad \text{—②}$$

solving ① & ②

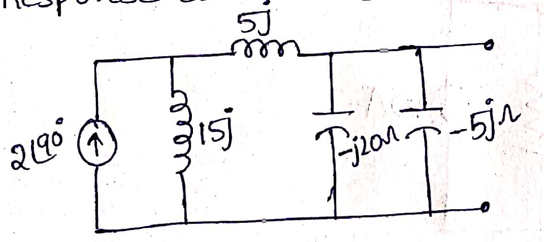
$I_3 = \frac{-V_1}{1} = -3.17A$

$I_3 = -3.17A$

6). Find the voltage across $-j20\Omega$ capacitor using superposition theorem in below. All impedences are in ohms.



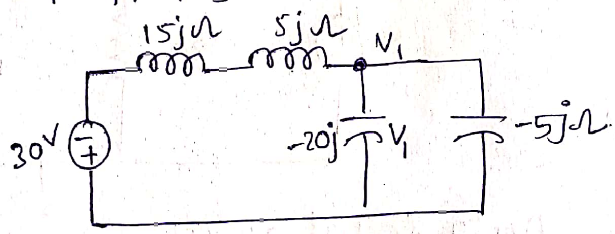
Response due to $2\angle 90°$



$V = 2\angle 90° (j15) = -30V$

$Z = 15j\,\Omega$

By applying source transformation.



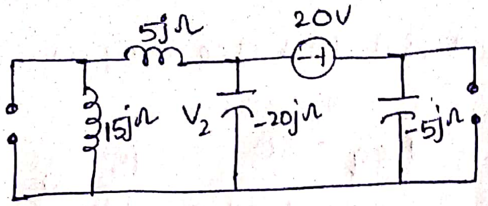
Apply nodal at V_1,

$$\frac{V_1 + 30}{j20} + \frac{V_1}{-20j} + \frac{V_1}{-5j} = 0$$

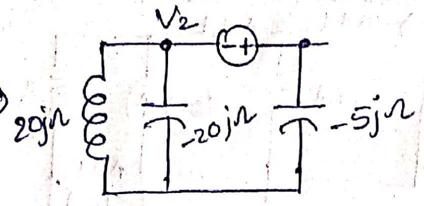
$$V_1\left(\frac{1}{20j} - \frac{1}{20j} - \frac{1}{5j}\right) = \frac{-30}{20}j$$

$$V_1 = \frac{15}{2} = 7.5V$$

Response due to 20V source V_2.



$\Rightarrow$

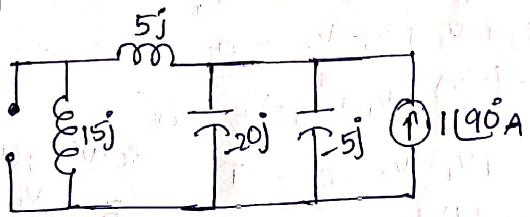


Apply V_2,

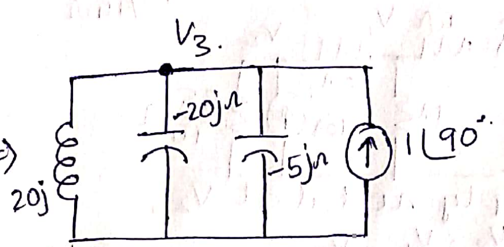
$$\frac{V_2-0}{20j} + \frac{V_2}{-20j} + \frac{V_2+20}{-5j} = 0$$

$$\boxed{V_2 = -20V}$$

Response due to $1\angle 90°$



$\Rightarrow$



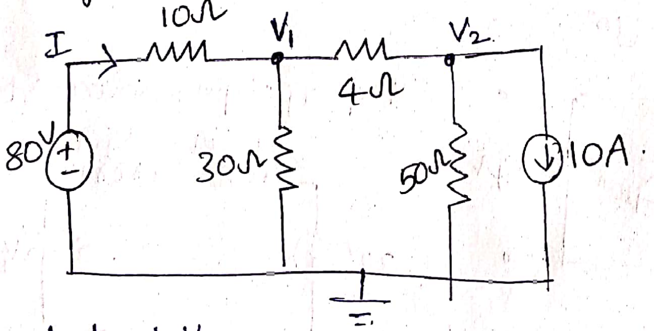
$$\frac{V_3}{j20} + \frac{V_3}{-20j} + \frac{V_3}{-5j} - 1\angle 90° = 0$$

$$\boxed{V_3 = 5V}$$

$$\boxed{V = V_1 + V_2 + V_3 = -7.5V}$$

TELLEGEN'S THEOREM

1) Verify Tellegen's theorem for circuit.



Apply nodal at V_1,

$$\frac{V_1-80}{10} + \frac{V_1}{30} + \frac{V_1-V_2}{4} = 0 \Rightarrow V_1\left(\frac{1}{10}+\frac{1}{30}+\frac{1}{4}\right) - \frac{1}{4}V_2 = 8$$

$$\frac{23}{60}V_1 - \frac{1}{4}V_2 = 8 \quad (1)$$

Apply nodal at V_2,

$$\frac{V_2-V_1}{4} + \frac{V_2}{50} - 10 = 0 \Rightarrow \frac{-1}{4}V_1 + \frac{27}{100}V_2 = -10 \quad (2)$$

$V_1 = 8.29V$, $V_2 = -44.71V$.

$$I_{10\Omega} = \frac{80+8.29}{10} = 8.829A.$$

$$I_{4\Omega} = \frac{-8.29+44.71}{4} = 9.105A$$

$P_{10\Omega} = (8.829)^2 \times 10 = 779.512\,W\,(Abs).$

$P_{30\Omega} = \frac{(-8.89)^2}{30} = 2.598\,W\,(Abs)$

$P_{4\Omega} = (9.105)^2 4 = 331.604\,W\,(Abs).$

$P_{50\Omega} = \frac{(-44.71)^2}{50} = 39.97\,W\,(Abs).$

$P_{80V} = (80)(8.829) = 706.32\,W\,(del).$

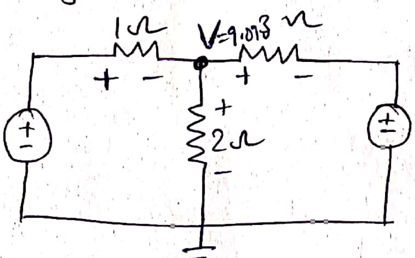
$P_{10A} = (44.71)(10) = 447.1\,W\,(del).$

$P_{del} = 1153.42\,W$

$P_{Abs} = 1153\,W$

hence $P_{delivered} = P_{Absorbed}$.

2) Verify the Tellegens Theorem for the circuit



Apply nodal at node V,

$$\frac{V-10}{1} + \frac{V}{2} + \frac{V-20}{3} = 0$$

$$V\left(1+\frac{1}{2}+\frac{1}{3}\right) = 10+\frac{20}{3}$$

$$V\left(\frac{11}{\cancel{6}_{2}}\right) = \frac{50}{\cancel{3}} \Rightarrow V = \frac{100}{11} = 9.09\,V.$$

$$I_{1\Omega} = \frac{10-9.09}{1} = 0.91A$$

$P_{1\Omega} = (0.91)^2 1 = 0.821\,(abs)$

$P_{10V} = (0.821)\,10 = 9.1\,W\,(del).$

$$I_{2\Omega} = \frac{9.09}{2} = 4.54A.$$

$P_{2\Omega} = (4.54)^2 2 = 41.22\,(abs).$

$$I_{3\Omega} = \frac{20-9.09}{3} = 3.63A$$

$P_{20\Omega} = 20 \times 3.63 = 72.6\,(del)$

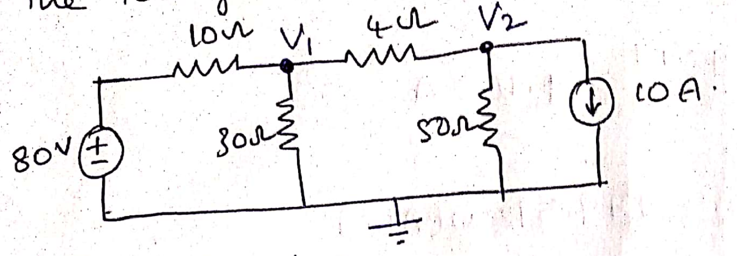
$P_{3\Omega} = (3.63)^2 \times 3 = 39.53$

$\therefore P_{del} = 81.7\,W$

$P_{Abs} = 81.57\,W$

hence $\boxed{P_{del} = P_{Abs}}$

3) Verify the Tellegen's Theorem for circuit



Apply Nodal at node V_1,

$$\frac{V_1-80}{10}+\frac{V_1}{3}+\frac{V_1-V_2}{4}=0$$

$$V_1\left(\frac{1}{10}+\frac{1}{3}+\frac{1}{4}\right)-V_2\left(\frac{1}{4}\right)=8 \quad \text{--- (1)}$$

Apply Nodal at node V_2.

$$\frac{V_2-V_1}{4}+\frac{V_2}{50}+10=0$$

$$V_1\left(\frac{-1}{4}\right)+V_2\left(\frac{27}{100}\right)=-10 \quad \text{--- (2)}$$

Solving (1) & (2) $V_1=-2.78V$ $V_2=-39.6V$.

$$I_{10\Omega}=\frac{80-(-2.78)}{10}=8.27A.$$

$P_{10\Omega}=8.27^2\times 10=683.92W.$ (abs)

$P_{80V}=8.27\times 80=661.6W$ (del)

$$I_{4\Omega}=\frac{-2.78+39.6}{4}=\cancel{33}9.2A.$$

$P_{4\Omega}=9.2^2\times 4=338.56W$ (abs).

$$I_{50\Omega}=\frac{-39.6}{50}=-0.792A$$

$P_{50\Omega}=(-0.792)^2\times 50=31.20W$ (abs).

$P_{10A}=39.6\times 10=396W$ (del)

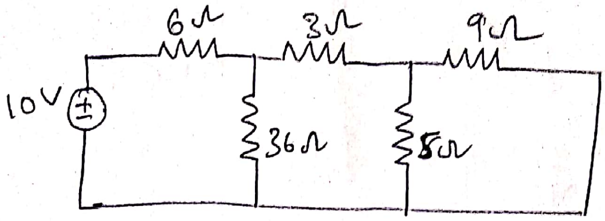
$P_{del}=1057.6W$

$P_{abs}=1053.6W.$

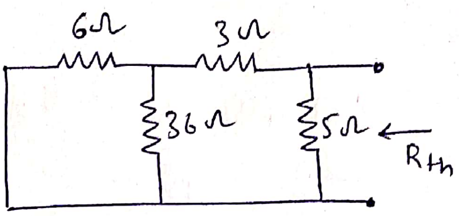
Hence $P_{del}=P_{abs}$.

COMPENSATION THEOREM.

1) Find current in 9Ω in circuit, when 5Ω resistor is changed to 6Ω resistor using compensation theorem.



R_{th}:



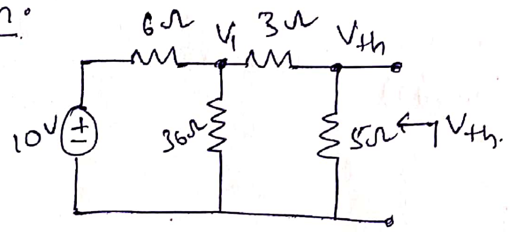
$$(6 \| 36 + 3) \| 5$$

$$\left(\frac{6\times 36}{42} + 3\right) \| 5$$

$$\frac{\frac{57}{7}\times 5}{\frac{57}{7}+5} = \frac{\frac{285}{7}}{\frac{92}{7}} = 3.09\Omega$$

$\therefore R_{th} = 3.09\Omega$.

V_{th}:



Apply nodal at V_1,

$$\frac{V_1 - 10}{6} + \frac{V_1}{36} + \frac{V_1 - V_{th}}{3} = 0$$

$$V_1\left(\frac{1}{6} + \frac{1}{36} + \frac{1}{3}\right) - \frac{1}{3}V_{th} = \frac{10}{6}$$

$$V_1\left(\frac{6+1+12}{36}\right) - \frac{1}{3}V_{th} = \frac{10}{6}.$$

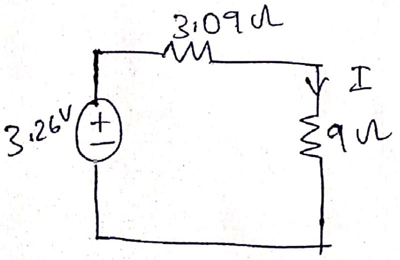
$$\frac{19}{36}V_1 - \frac{1}{3}V_{th} = \frac{5}{3} \quad \text{(1)}$$

Apply nodal at V_{th},

$$\frac{V_{th} - V_1}{3} + \frac{V_{th}}{5} = 0$$

$$-\frac{1}{3}V_1 + \frac{8}{15}V_{th} = 0 \quad \text{(2)}$$

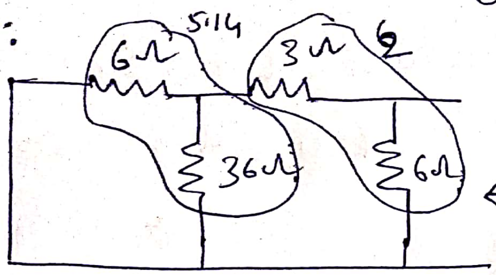
$$\boxed{\begin{aligned} V_1 &= 5.21V \\ V_{th} &= 3.26V \end{aligned}}$$



$$I = \frac{3.26}{(3.09 + 9)} = 0.26A$$

when 5Ω resistor change to 6Ω.

Rth:

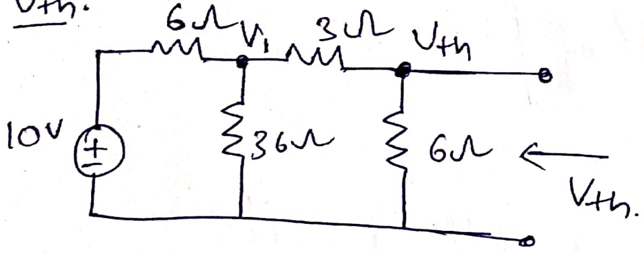


5.14Ω series with 2.

$R_{th} = 5.14 + 2$

$= 7.14\,\Omega$.

Vth:



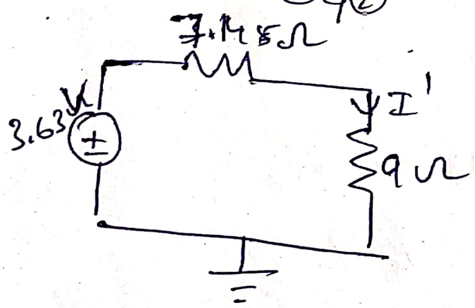
$$\frac{V_1 - 10}{6} + \frac{V_1}{36} + \frac{V_1 - V_{th}}{3} = 0$$

$$\frac{19}{36}V_1 - \frac{1}{3}V_{th} = \frac{10}{6} \quad \text{--- (1)}$$

Apply nodal at V_{th},

$$-\frac{1}{3}V_1 + \frac{9}{18}V_{th} = 0 \quad \text{--- (2)}$$

Solving (1) & (2), $V_1 = 5.45V$ $\quad V_{th} = 3.63V$.



$$I' = \frac{3.63}{(7.14+9)} = 0.22\,A.$$