UNIT-III

Electrostatic Fields

→ Electrostatics is a branch of science that deals with the static (or in rest) electric charges.

→ It is a branch that studies the time invarient electric fields in a space (or) vacuum produced by various types of static charge distributions i.e line/surface/volume configurations.

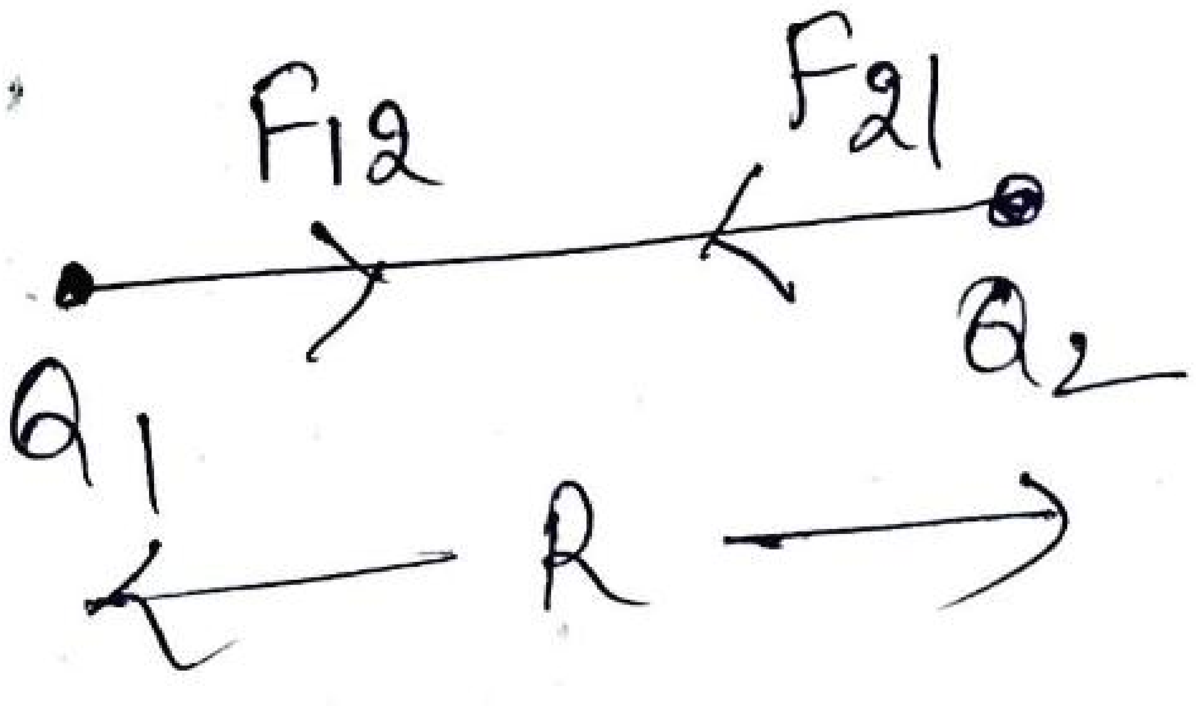
Coulomb's Law:

It deals with the force that one point charge exerts on another point charge. The polarity of charges may be positive (or) negative.

Statement:- It states that the force (F) between the 2 point charges Q_1 and Q_2,

(i) lies along the line joining the charges
(ii) directly proportional to product of the charges
(iii) Inversely proportional to square of the distance (R) between them.

Mathematically, $F = \frac{KQ_1Q_2}{R^2}$



where $K = \frac{1}{4\pi\varepsilon}$ - proportinality constant

where $\varepsilon = \varepsilon_0 \varepsilon_r$; ε_0 - permittivity in free space
ε_r - permittivity in any medium.

$$\therefore \boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}} \text{ Newton}$$ (since $\varepsilon_r = 1$ for freespace (or) vacuum)

Note:- $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m $= \frac{1}{36\pi} \times 10^{-9}$ F/m

$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}$ m/F

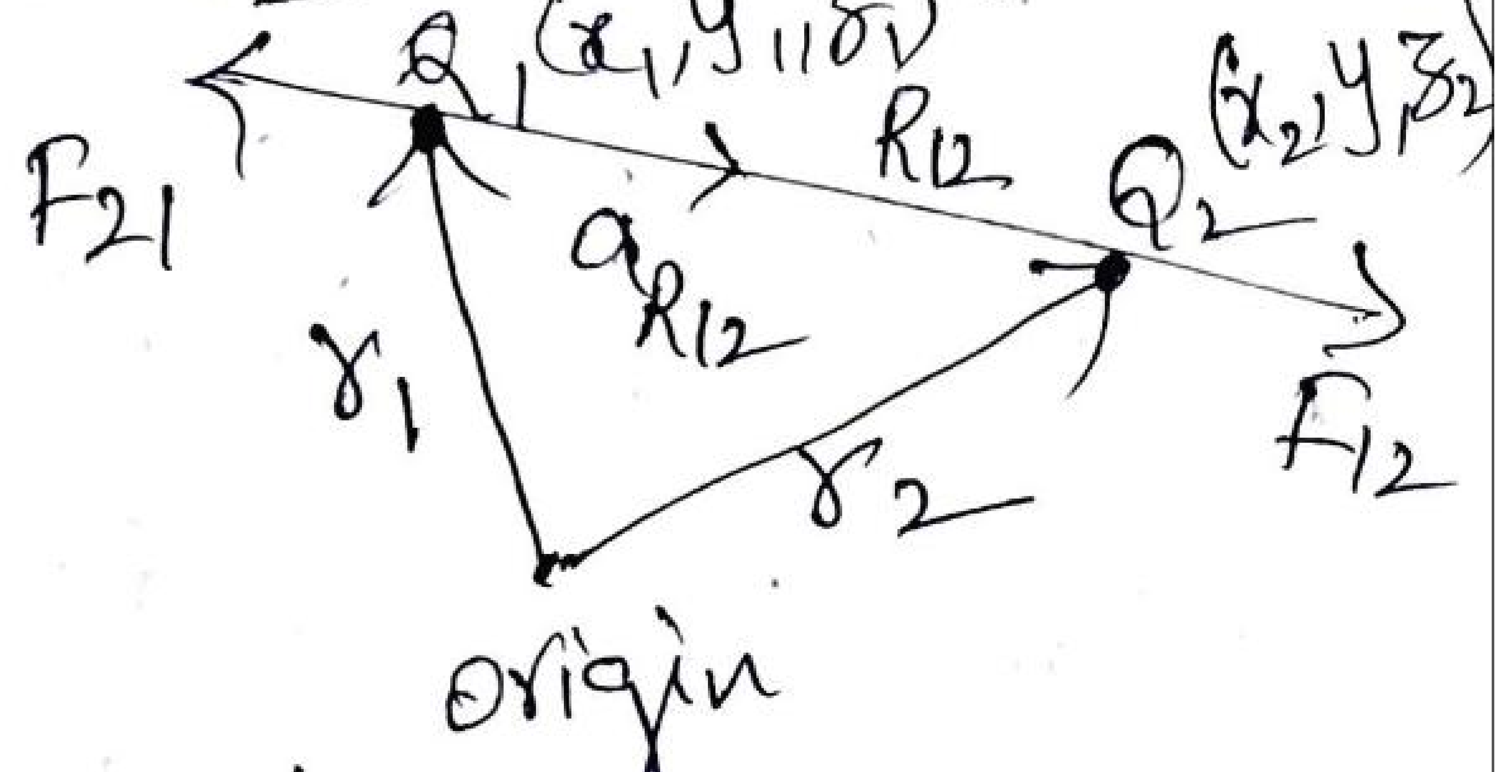
$1 C = 6 \times 10^{18} e^-$

$1 e^- = 1.602 \times 10^{-19} C$

Typical values for problems.

Force in vector form:-

→ If 2 point charges Q_1 and Q_2 are located at points from origin as having position vectors r_1 and r_2 shown below:



• The position vectors are given as:

$$\bar{r_1} = x_1 a_x + y_1 a_y + z_1 a_z$$
$$\bar{r_2} = x_2 a_x + y_2 a_y + z_2 a_z$$

$\therefore$ Radial distance $\overline{R_{12}} = (r_2 - r_1)$

$$= (x_2 - x_1)a_x + (y_2 - y_1)a_y + (z_2 - z_1)a_z$$

Also, $\boxed{F_{12} = -F_{21}}$

→ The force F_{12} on Q_2 due to Q_1 is:

$$F_{12} = \pm \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R_{12}|^2} a_{R_{12}}$$

where, $\overline{R_{12}} = r_2 - r_1$ and unit vector $a_{R_{12}} = \frac{\overline{R_{12}}}{|R_{12}|}$

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$$\therefore\ F_{12} = \pm \frac{Q_1 Q_2}{4\pi\varepsilon_0 |R_{12}|^2} \cdot \frac{\overline{R_{12}}}{|R_{12}|}$$

$$= \pm \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\varepsilon_0 |R_{12}|^3} = \pm \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\varepsilon_0 |r_2 - r_1|^3} \text{ Newton}$$

Note:- unit vector always indicates the direction of forces.

- if Force → positive value → Force of repulsion
- if Force → negative value → Force of attraction.

Force for N-charges:-

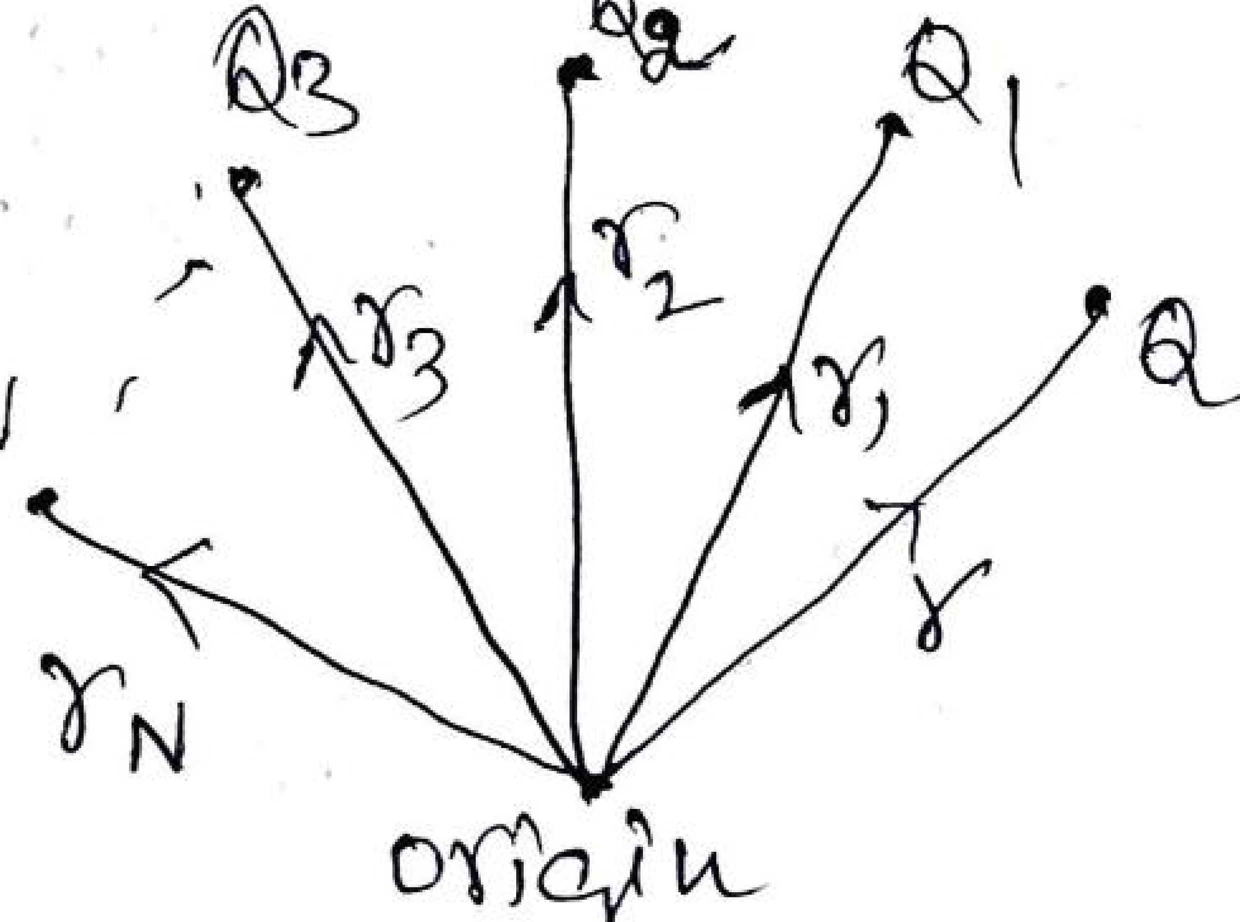
→ If there are more than 2 points, we use principle of superposition (since Force is linear quantity) to determine force on a particular charge.

Note:- coulomb's law is linear, obeys superposition theorem

→ Let $Q_1, Q_2 \dots Q_N$ be N-charges located at points with position vectors $r_1, r_2 \dots r_N$ as shown in fig.

- then the resultant force is

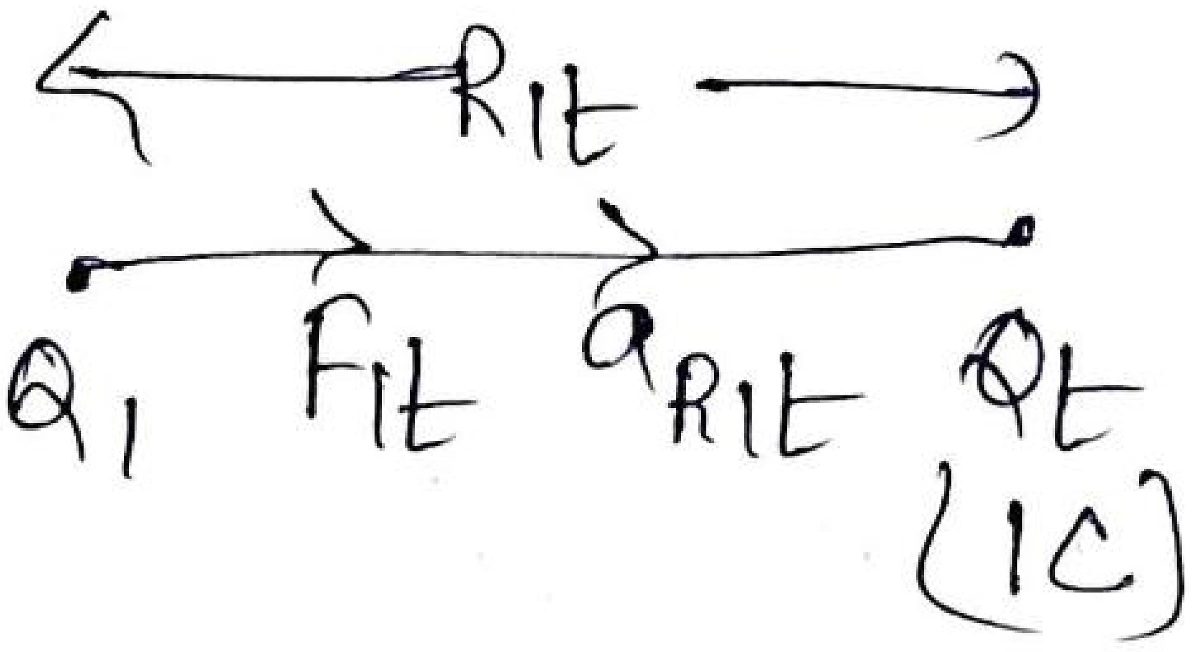
$$F_T = F_1 + F_2 + F_3 + \dots + F_N$$



$$\therefore \boxed{F = \frac{Q}{4\pi\varepsilon_0} \sum_{m=1}^{N} \frac{Q_K (r - r_m)}{|r - r_m|^3} \text{ Newton}}$$

Electric Field Intensity ($\bar{E}$):-

→ The electrostatic force acting on a unit charge placed at that point.



$$F_{1t} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_t}{|R_{1t}|^2} a_{R1t}$$

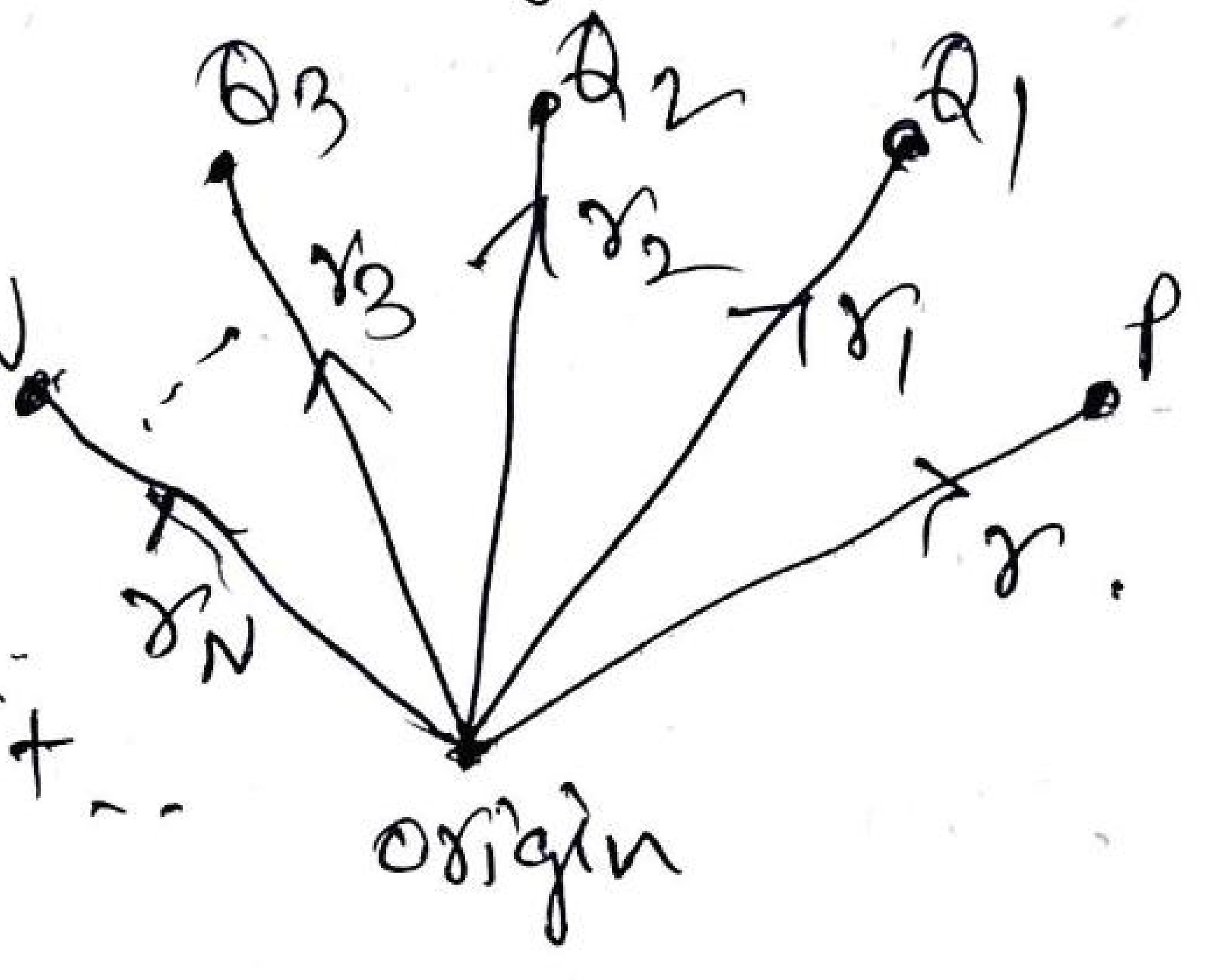
$$\Rightarrow \frac{F_{1t}}{Q_t} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1}{|R_{1t}|^2} a_{R1t}$$

$$\Rightarrow \boxed{\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{|R|^2} a_R} \text{ V/m} \text{ — for point charge.}$$

where $\bar{E} = \frac{F_{1t}}{Q_t} = \frac{F}{Q}$ N/C. (Standard definition)

$\bar{E}$ for N-charges:-

→ Electric field intensity ($\bar{E}$) is linear, obeys the law of superposition.



$$\therefore \bar{E}_T = \bar{E}_1 + \bar{E}_2 + \bar{E}_3 + \cdots$$

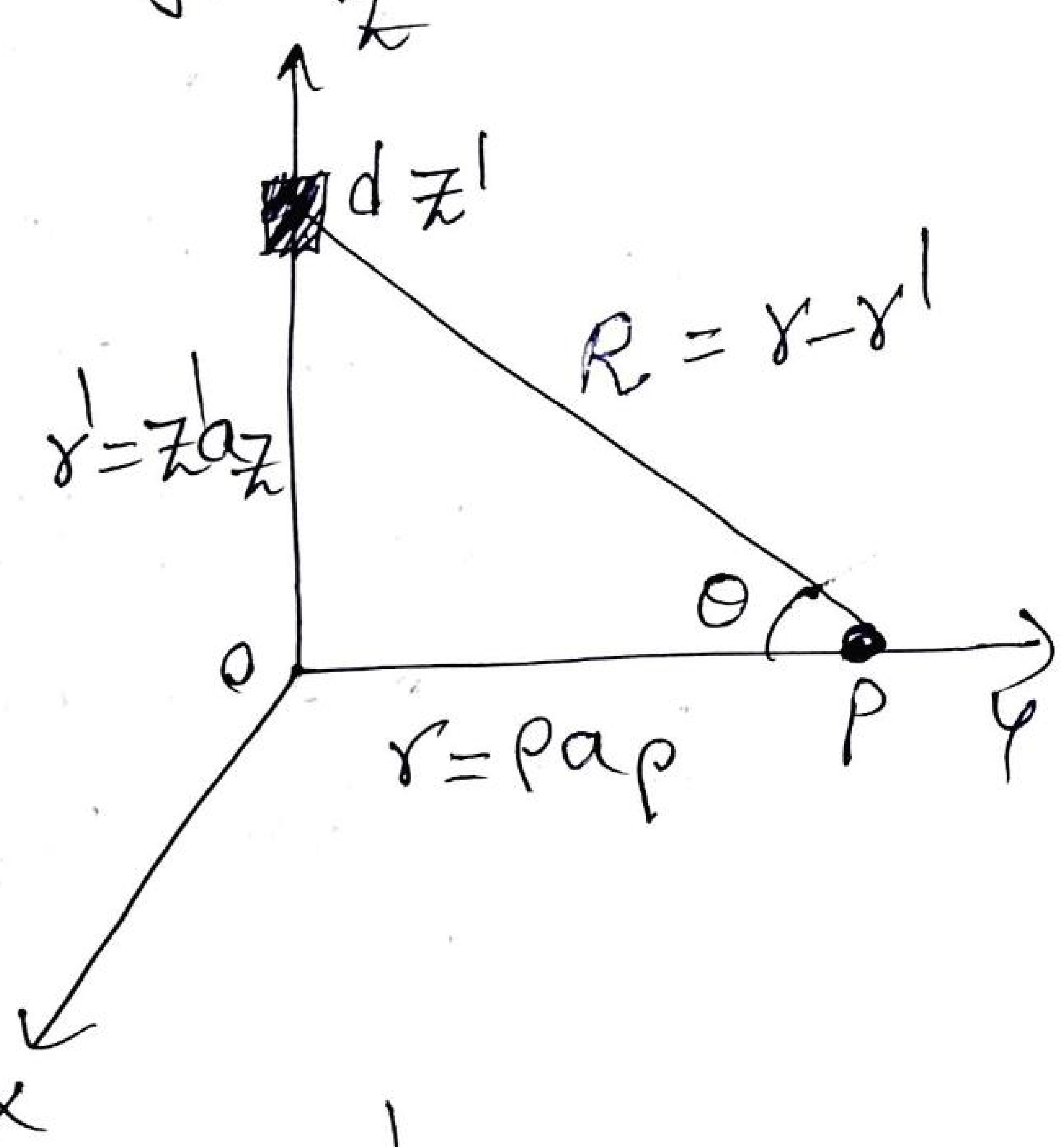
$$\bar{E}_T = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|^2} a_1 + \frac{Q_2}{4\pi\varepsilon_0 |r - r_2|^2} a_2 + \cdots$$

$$\therefore \boxed{\bar{E} = \frac{1}{4\pi\varepsilon_0} \sum_{m=1}^{N} \frac{Q_m}{|r - r_m|^2} a_m} \text{ V/m.}$$

Electric Field Intensity due to Infinite Line charge: (3)

→ Consider, a line charge (sequence of point charges) as a placed on z-axis, having charge density ρ_L (c/m).

- Also, consider a small differential length dl carrying a charge dQ along z-axis, hence $dl = dz'$



(i) $\therefore dQ = \rho_L \cdot dz'$

(ii) Let P — observation point

& R — distance b/w source & point P.

$\therefore R = r - r'$

$\bar{R} = \rho a_\rho - z' a_z$

- $|\bar{R}| = \sqrt{\rho^2 + z'^2}$
- direction $a_{\bar{R}} = \dfrac{R}{|R|} = \dfrac{\rho a_\rho - z' a_z}{\sqrt{\rho^2 + z'^2}}$ —— (1)

→ For every charge on +ve z-axis, there is equal charge present on −ve z-axis.

$\therefore \bar{E}$ cancel each other in z-direction.

$\therefore E_z = 0 \Rightarrow$ no z-component of $\bar{E}$

Eq (1) becomes,

$\therefore a_{\bar{R}} = \dfrac{\rho a_\rho}{\sqrt{\rho^2 + z'^2}}$ (since $a_z = 0$)

→ We know that: $\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2} a_R$ (for point charge).

$$dE = \frac{dQ}{4\pi\varepsilon_0 |R|^2} a_{\bar{R}}$$

$$= \frac{\rho_L \cdot dz'}{4\pi\varepsilon_0 \left|\sqrt{\rho^2+z'^2}\right|^2} \cdot \left(\frac{\rho\, a_\rho}{\sqrt{\rho^2+z'^2}}\right).$$

$$dE = \frac{\rho_L \cdot dz' \rho\, a_\rho}{4\pi\varepsilon_0 (\rho^2+z'^2)^{3/2}}$$

Integrating on both sides

$$\int_{-\infty}^{\infty} dE = \int_{-\infty}^{\infty} \frac{\rho_L \cdot \rho\, a_\rho}{4\pi\varepsilon_0 (\rho^2+z'^2)^{3/2}} dz'$$

$$\Rightarrow \bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{\rho\, a_\rho}{(\rho^2+z'^2)^{3/2}} dz'$$

Let $z' = \rho\tan\theta \Rightarrow dz' = \rho\sec^2\theta\, d\theta$

$$\left.\begin{array}{l} -\infty = \rho\tan\theta \Rightarrow \theta = -\pi/2 \\ +\infty = \rho\tan\theta \Rightarrow \theta = \pi/2 \end{array}\right\} \text{limits.}$$

$$\therefore \bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho\, a_\rho}{(\rho^2+\rho^2\tan^2\theta)^{3/2}} \rho\sec^2\theta\, d\theta$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho^2\sec^2\theta\, d\theta\, a_\rho}{\rho^3\sec^3\theta}$$

$\boxed{\text{Since } \rho^2(1+\tan^2\theta) = \rho^2\sec^2\theta}$

$$\Rightarrow \bar{E} = \frac{\rho_L a_\rho}{4\pi\varepsilon_0 \rho}\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{\rho_L a_\rho}{4\pi\varepsilon_0 \rho}(+\sin\theta)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{\rho_L a_\rho}{\cancel{4}_2\pi\varepsilon_0 \rho}(\cancel{2}) = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho$$

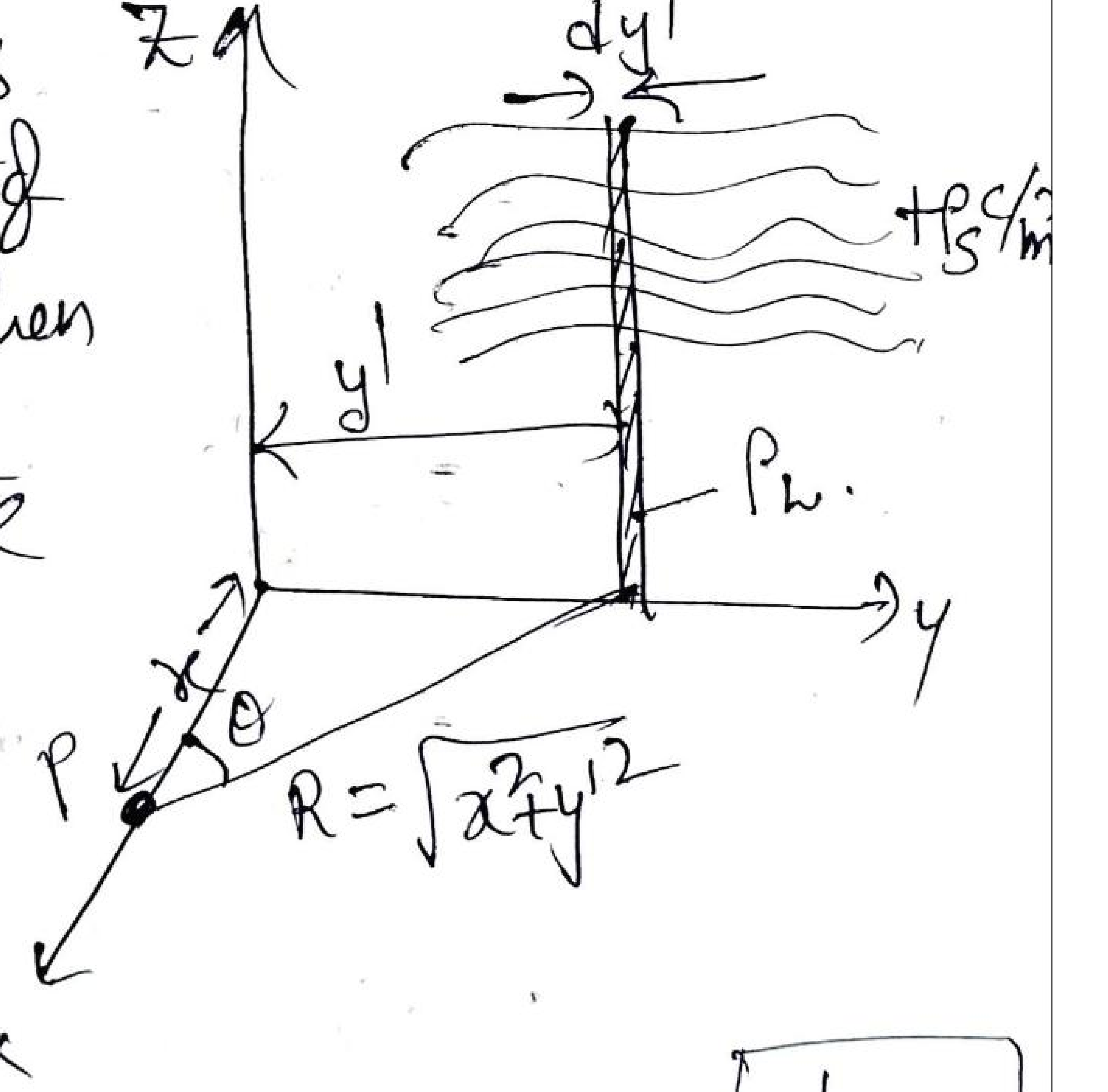
$$\therefore \boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho \ \ V/m}$$ for line charge.

Electric Field intensity due to infinite sheet charge :-

→ Consider, a sheet charge is placed in yz-plane & point of observation (P) on x-axis, then

(i) $\rho_L = \rho_S \, dy'$ — line charge

(ii) $R = \sqrt{x^2+y'^2}$ — distance



→ Let $\bar{E}$ due to line charge is

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho = \frac{\rho_S \, dy'}{2\pi\varepsilon_0 \sqrt{x^2+y'^2}} \cos\theta$$

$\boxed{\text{here, } \rho = R}$

here, $\cos\theta$ — it can give the contribution of 'P' in any direction.

$$\therefore d\bar{E} = \frac{\rho_S \, dy' \cos\theta}{2\pi\varepsilon_0 \sqrt{x^2+y'^2}}$$

$$= \frac{\rho_S \, dy'}{2\pi\varepsilon_0 \sqrt{x^2+y'^2}} \cdot \frac{x}{\sqrt{x^2+y'^2}}$$ (since $\cos\theta = \frac{x}{\sqrt{x^2+y'^2}}$ from figure).

$$d\bar{E} = \frac{\rho_S \, dy' \, x}{2\pi\varepsilon_0 (x^2+y'^2)}$$

Integrate on both sides.

$$\bar{E} = \frac{\rho_S}{2\pi\varepsilon_0} \int_{-\infty}^{\infty} \frac{x}{x^2+y'^2} \, dy'$$

$$= \frac{\rho_S}{2\pi\varepsilon_0} \left(\tan^{-1} \frac{y'}{x} \right) \Big|_{-\infty}^{\infty}$$

$$= \frac{\rho_S}{2\pi\varepsilon_0} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right)$$

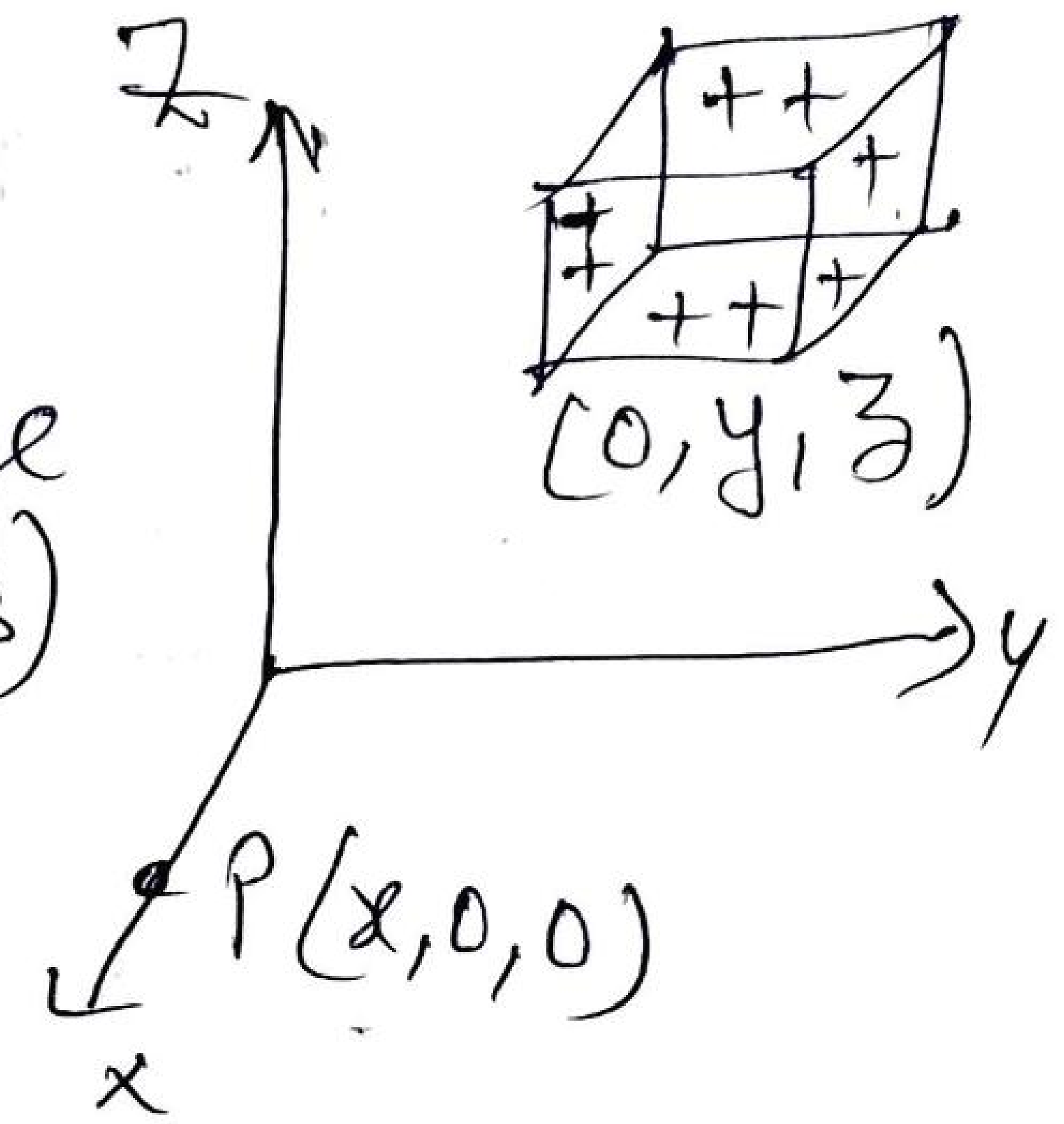
$$= \frac{\rho_S}{2\cancel{\pi}\varepsilon_0} (\cancel{\pi}) = \frac{\rho_S}{2\varepsilon_0}$$ (in x-direction)

$$\therefore \boxed{\bar{E}_x = \frac{\rho_S}{2\varepsilon_0}} \; V/m$$

<u>Note:</u>- $\bar{E}$ due to surface charge is independent of distance.

Electric Field intensity due to infinite volume charge.

→ Consider infinite volume charge cube in yz-plane and P (point of observation) on x-axis, where $\bar{E}$ is to be found.



here, (i) $\rho_v = \dfrac{dQ}{dv}$ → volume charge density (C/m^3)

$\Rightarrow dQ = \rho_v \, dv$

$\Rightarrow Q = \int_v \rho_v \, dv.$

(ii) distance b/w source & point (P) is

$$\bar{R} = x a_x - y a_x - z a_z.$$

$$|\bar{R}| = \sqrt{x^2 + y^2 + z^2}$$

Let $\bar{E} = \dfrac{Q}{4\pi\varepsilon_0 |R|^2} a_R$ (for point charge)

$$= \frac{\int_v \rho_v \, dv}{4\pi\varepsilon_0 |R|^2} \cdot \frac{\bar{R}}{|R|} = \int_v \frac{\rho_v \, dv \cdot \bar{R}}{4\pi\varepsilon_0 |R|^3}$$

$$\bar{E} = \int_v \frac{\rho_v \, dv (x a_x - \cancel{y a_y}^{0} - \cancel{z a_z})^{0}}{4\pi\varepsilon_0 |R|^3}$$

Eliminating y and z-components of $\bar{E}$

$$\bar{E} = \int_v \frac{\rho_v \, dv \, x a_x}{4\pi\varepsilon_0 |R|^3}$$

$$\Rightarrow \vec{E} = \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} \frac{\rho_v (\cancel{r^2} \sin\theta \, dr \, d\theta \, d\phi) \, x a_x}{4\pi\varepsilon_0 r^{\cancel{3}}}$$

since $dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$ (for spherical coordinate system.)

& Let $R = r$

$$\Rightarrow \vec{E} = \frac{\rho_v}{4\pi\varepsilon_0}\left[\int_0^{r} \frac{1}{r} dr \cdot \int_0^{\pi} \sin\theta \, d\theta \cdot \int_0^{2\pi} d\phi \; x a_x\right]$$

$$= \frac{\rho_v}{4\pi\varepsilon_0}\left(\ln r\Big|_0^{r}\right)\left(-\cos\theta\Big|_0^{\pi}\right)\left(\phi\Big|_0^{2\pi}\right) x a_x$$

$$= \frac{\rho_v}{\cancel{4\pi}\varepsilon_0} \cdot (\ln r - 1) \cdot \cancel{2} \cdot \cancel{2\pi} \cdot x a_x$$

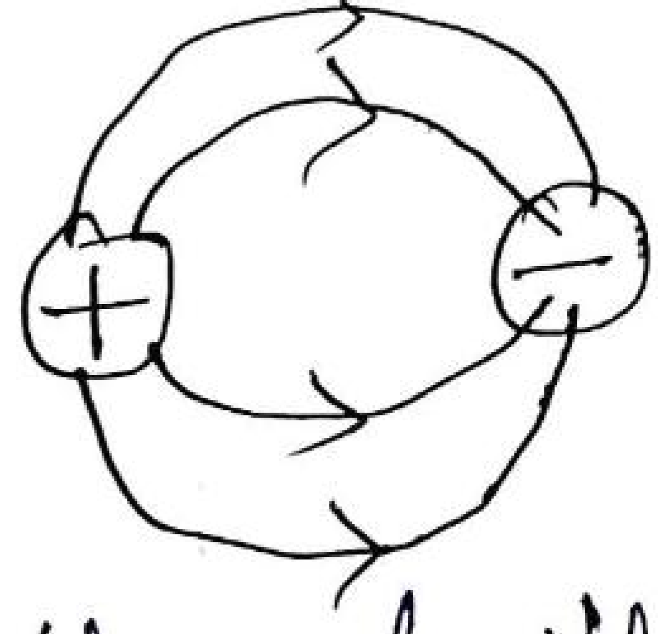
$$\therefore \boxed{\vec{E} = \frac{\rho_v}{\varepsilon_0}(\ln r - 1) \, x a_x} \; V/m.$$

=

Electric Flux : It is the total number of lines of force in any direction, denoted by (Ψ), measured in coulomb (c).

→ The flux lines start from positive charge and terminate on negative charge.

- The flux lines are parallel and never cross each other.



Electric flux density :

→ The number of flux lines passing through the unit surface area is called electric flux density, denoted by ($\overline{D}$). Units are C/m^2. $\overline{D} = \frac{\text{flux}}{\text{unit surface area}} = \frac{d\psi}{dS}$

- This is also called displacement flux density (or) displacement density.

Gauss's Law : It constitutes one of the fundamental laws of electromagnetism.

→ It states that the total electric flux (ψ) through any closed surface is equal to the total charge enclosed by that surface.

i.e $\boxed{\psi = Q}$

Relation between $\overline{E}$ and $\overline{D}$:-

→ We know that : $\overline{D} = \frac{d\psi}{ds}$

$$\Rightarrow d\psi = \overline{D} \cdot ds \Rightarrow \psi = \int \overline{D} \cdot ds$$

. From Gauss law $\psi = Q \Rightarrow \boxed{Q = \int \overline{D} \cdot ds}$

$\Rightarrow Q = \overline{D} \int ds$ (since $\overline{D}$ — same at all points of Gaussian surface). $\therefore \overline{D}$ — constant).

$$= D \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi \quad (\because ds = r^2 \sin\theta \, d\theta \, d\phi)$$

$$= D r^2 \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, . \int_{\phi=0}^{2\pi} d\phi$$

$$\therefore Q = D r^2 (2)(2\pi) = 4\pi D r^2$$

$$\Rightarrow \boxed{\overline{D} = \frac{Q}{4\pi r^2} a_r} \; C/m^2 \text{ — for point charge.}$$

we know: $\overline{E} = \frac{Q}{4\pi \varepsilon_0 r^2} a_r \; V/m$ — for point charge.

∴ Relating these (2) equations ;

$$\boxed{\overline{D} = \varepsilon_0 \overline{E}} \text{ (or) } \boxed{\overline{D} = \varepsilon \overline{E}}$$

since $\varepsilon = \varepsilon_0 \varepsilon_r$

proof of Gauss Law:-

$$\psi = \int D \cdot ds = \int \frac{Q}{4\pi r^2} ds \quad \left(\because D = \frac{Q}{4\pi r^2} \right)$$

$$= \int_0^{2\pi} \int_0^{\pi} \frac{Q}{4\pi \cancel{r^2}} \cancel{r^2} \sin\theta \, d\theta \, d\phi$$

$$= \frac{Q}{4\pi} \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

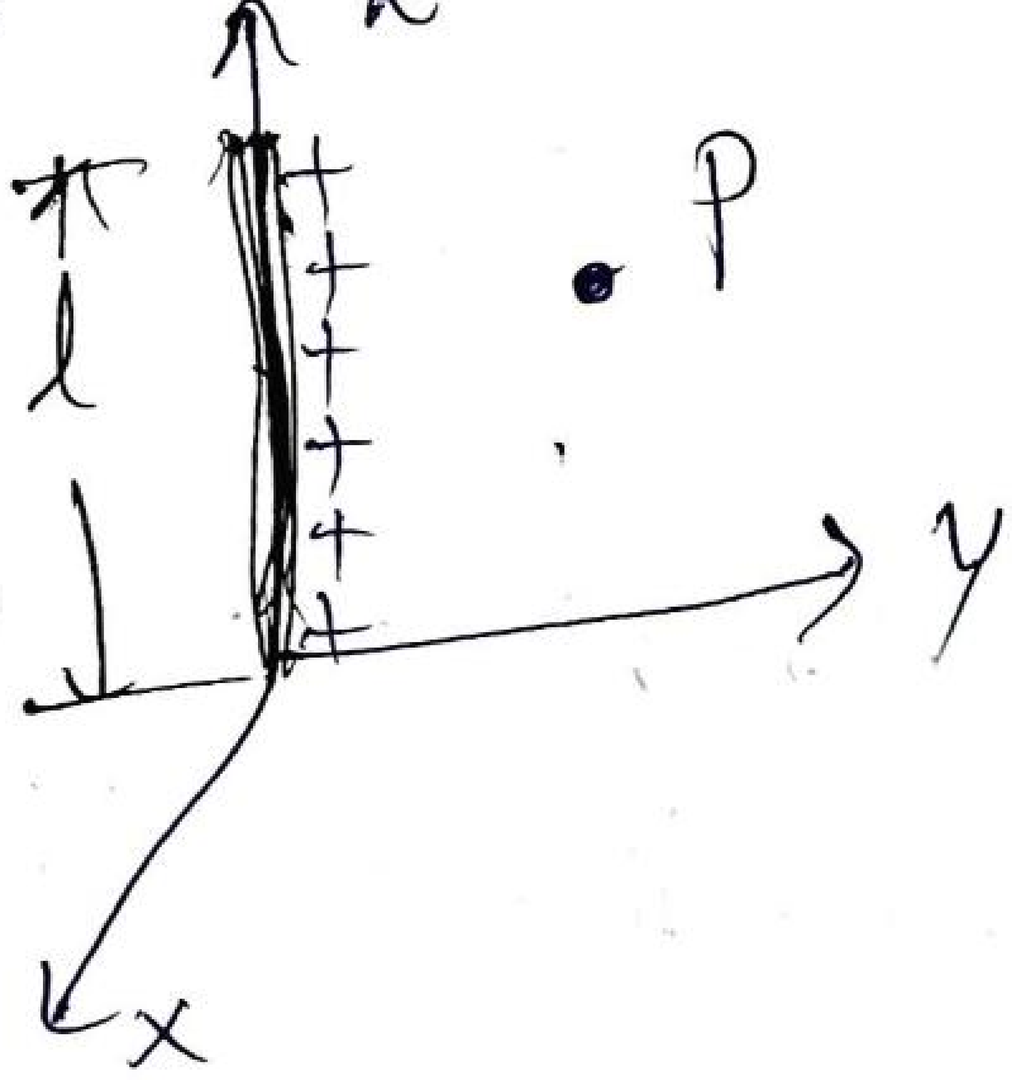
$$= \frac{Q}{\cancel{4\pi}} (\cancel{2}) (\cancel{2\pi}) = Q.$$

$$\therefore \boxed{\psi = Q}$$

Gauss Law Applications:-

(i) Electric flux density due to infinite line charge:-

Consider, infinite line charge along z-axis as shown, find the electric flux density at a point (P).



→ According to Gauss law:

$$Q = \int D \cdot ds$$

$$\int \rho_L \, dL = \int D \cdot ds$$

$$\rho_L \cdot l = D \int ds$$

$$= D \int_0^{2\pi} \int_{-\infty}^{\infty} d\phi \, dz \, \rho$$

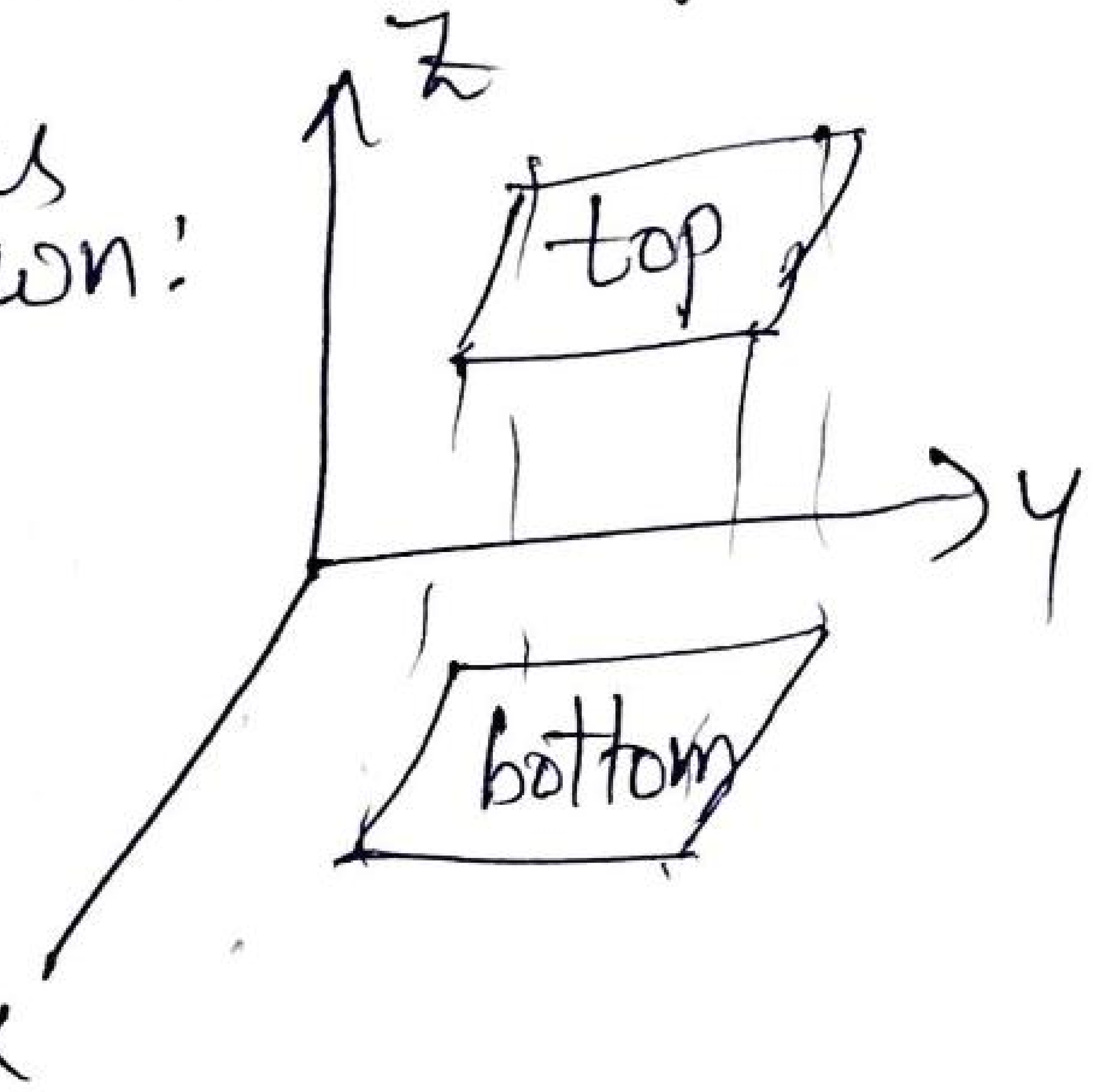
$$\Rightarrow \rho_L \cdot l = D\rho \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} dz$$

$$\Rightarrow \rho_L \cdot \cancel{l} = \bar{D}\rho\,(2\pi)(\cancel{l})$$

$$\Rightarrow \boxed{\bar{D} = \frac{\rho_L}{2\pi\rho}\, a_\rho} \; C/m^2$$

(ii) <u>Electric flux density due to sheet charge</u>:-

Consider, the sheet in XY plane as shown:



→ According to Gauss law:

$$Q = \int \bar{D} \cdot ds$$

$$\int \rho_S \, ds = \int \bar{D} \cdot ds$$

$$\rho_S \int ds = \int_{top} \bar{D} \cdot ds + \int_{bottom} \bar{D} \cdot ds$$

$$\Rightarrow \rho_S \int \cancel{ds} = 2\bar{D} \int \cancel{ds}$$

$$\Rightarrow \boxed{\bar{D} = \frac{\rho_S}{2}} \; C/m^2.$$

(iii) <u>Electric flux density due to volume charge</u>:-

According to Gauss law:

$$Q = \int \bar{D} \cdot ds$$

$$\int \rho_v \, dv = \int \bar{D} \cdot ds$$

(8)

$$\rho_V \int_0^{\infty}\int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \, dr\, d\theta\, d\phi = \bar{D}\int_0^{\infty}\int_0^{\pi} r\, dr\, d\theta$$

$$\rho_V \int_0^{r} r^2 dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi = \bar{D}\int_0^{a} r\, dr \int_0^{\pi} d\theta.$$

$$\rho_V \cdot \frac{r^3}{3} \times 2 \times 2\pi = \bar{D}\,\frac{a^2}{2} \times \pi$$

$$\therefore \boxed{\bar{D} = \frac{8\rho_V r^3}{3a^2}} \; C/m^2.$$

Statement of Gauss's law (Integral Form):

The integral of the normal component of the flux density evaluated over a closed surface must be equal to the charge enclosed by the closed surface.

$$\boxed{Q_{en} = \oint_S \bar{D}_n \cdot ds}$$

Limitations of Gauss's Law:

(1) charges should be uniformly distributed.

(2) It can be applied to closed surface (Gaussian surface).

Gaussian surface: An imaginary closed surface of arbitrary shape used to find $\bar{E}$.

(3) For line charge distribution — cylindrical
spherical charge distribution — concentric sphere.
Surface charge distribution — cylindrical.

(4) Field point must lies on gaussian surface.

(5) charge must be inside gaussian surface.

(6) It satisfies only when all charges are symmetrical

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Gauss Divergence Theorem:

Divergence: It converts the integral form of Gauss law into point form.

- Gauss law integral form: $\boxed{Q_{en} = \oint_S \bar{D} \cdot ds = \int_V \rho_v \, dv}$

- Divergence is of 3 types:-
  1. positive divergence :- The flow of flux from inside to outside & the volume becomes zero. ex:- baloon.
  2. Negative divergence:- The flow of flux from outside to inside is negative divergence. ex:- vacuum created room.
  3. zero divergence:- The net flow of flux is zero when the inward flux = outward flux. ex:- water tank

→ Mathematically, divergence can be expressed as:

$$\text{div } \bar{D} = \lim_{\Delta v \to 0} \frac{\oint_S \bar{D} \cdot ds}{\Delta v} = \lim_{\Delta v \to 0} \frac{Q_{en}}{\Delta V} = \rho_v \; C/m^3 \quad \text{---} (1)$$

- Also, it can be expressed using mathematical operator.

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

Let $\bar{D} = D_x a_x + D_y a_y + D_z a_z$ (component form of vector)

$$\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \quad \text{---} (2)$$

Equating equations (1) & (2), $\boxed{\nabla \cdot \bar{D} = \rho_v}$ Point form of Gauss law.

↳ Maxwell's 1st equation

Note:- unit of divergence:- C/m^3. (and ∇ unit $- 1/m$)

Statement of divergence theorem:-

→ The integral of normal component of the flux density evaluated over a closed surface must be equal to the integral of the divergence of flux density throughout the volume enclosed by the closed surface.

We know from Gauss law: Maxwell's 1st equation.

$$\boxed{\begin{aligned} Q_{en} = \oint_S \bar{D} \cdot ds = \int_V \rho_V \, dv \quad &\text{— (1) (Integral form)} \\ \nabla \cdot \bar{D} = \rho_V \quad &\text{— (2) (point form)} \end{aligned}}$$

substituting (2) in (1),

$$\therefore \boxed{\oint_S \bar{D} \cdot ds = \int_V (\nabla \cdot \bar{D}) \, dv}$$ Gauss divergence theorem

i.e surface integral converted into volume integral.

Note:- $\nabla \cdot \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$ (cartesian)

divergence formulae

$$\nabla \cdot \bar{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \text{ (cylindrical)}$$

$$\nabla \cdot \bar{D} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta D_\theta) + \frac{1}{r \sin\theta} \frac{\partial D_\phi}{\partial \phi}$$

(spherical)

Electric potential:

→ It is defined as the amount of work done required to bring/move a unit of electric charge from a reference point to the specific point against an electric field. i.e $\boxed{V = \frac{W}{Q_t}}$

→ It is a scalar, used to develop vector ($\bar{E}$).

→ Consider a positive charge (Q_m) placed in an external electric field, let a test charge (Q_t) is placed at a point A.

• Due to $\bar{E}$ around Q_m, Q_t will experience an electrostatic force F_e directed away from the charge.

• Since both charges are same, repulsive force exerted

i.e $F_{ext} = -F_e$

→ As per coulomb's law:

$$F_e = \bar{E}Q_t \qquad \left(\text{since } \bar{E} = \frac{F}{Q_t}\right)$$

As we know: workdone = force x distance

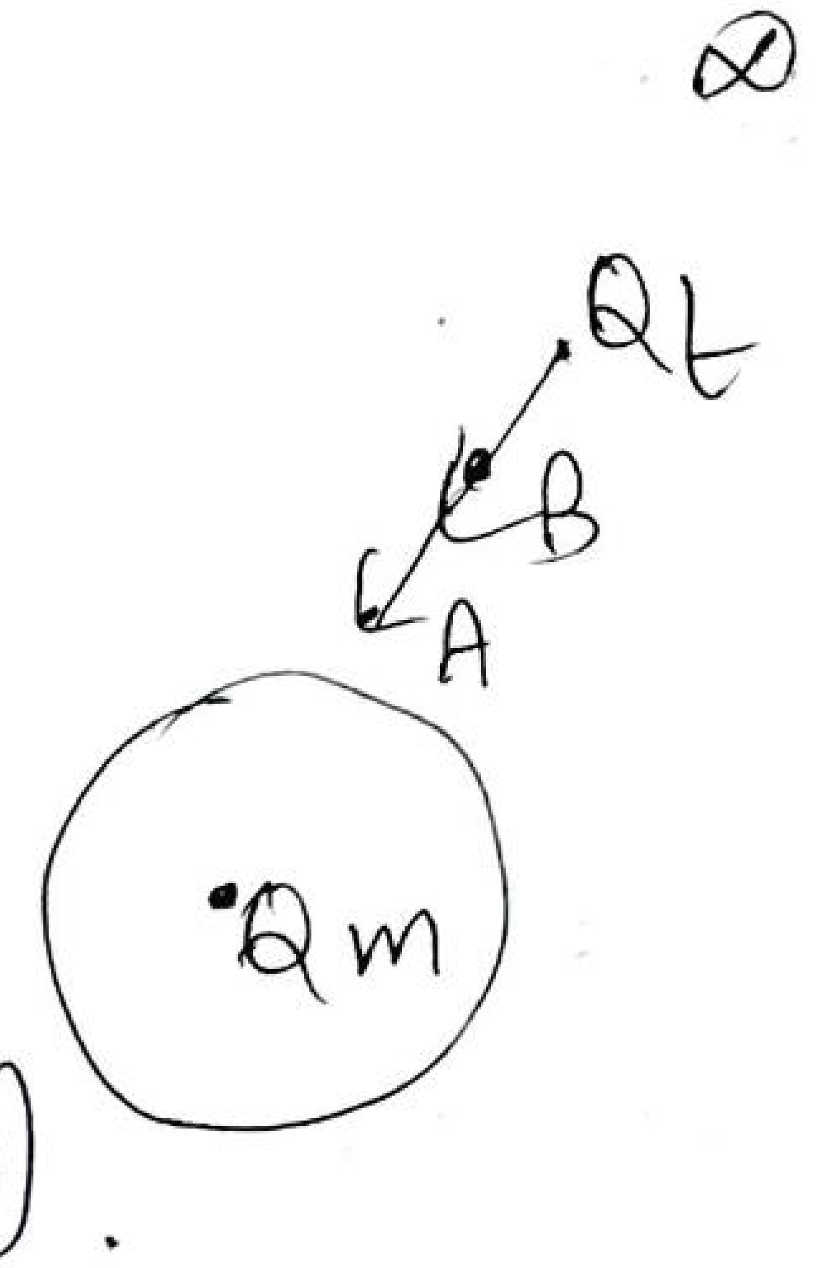
$$dw = EQ_t \times dh$$

$$\int dw = -\int EQ_t \cdot dh$$ (since −ve sign is due to workdone by test charge).

$$w = -Q_t \int_{\text{initial}}^{\text{final}} \bar{E} \cdot dh$$

Now, potential $\boxed{V = \frac{W}{Q_t} = -\int_{\text{initial}}^{\text{final}} \bar{E} \cdot dh}$ J/C (or) volts.

(i) potential at any (single) point:-



• At point (B), potential is

$$V_B = -\int_{\infty}^{B} \bar{E} \cdot d\bar{l}$$

whereas ∞ – reference point where potential is zero (i.e $V_\infty = 0$).

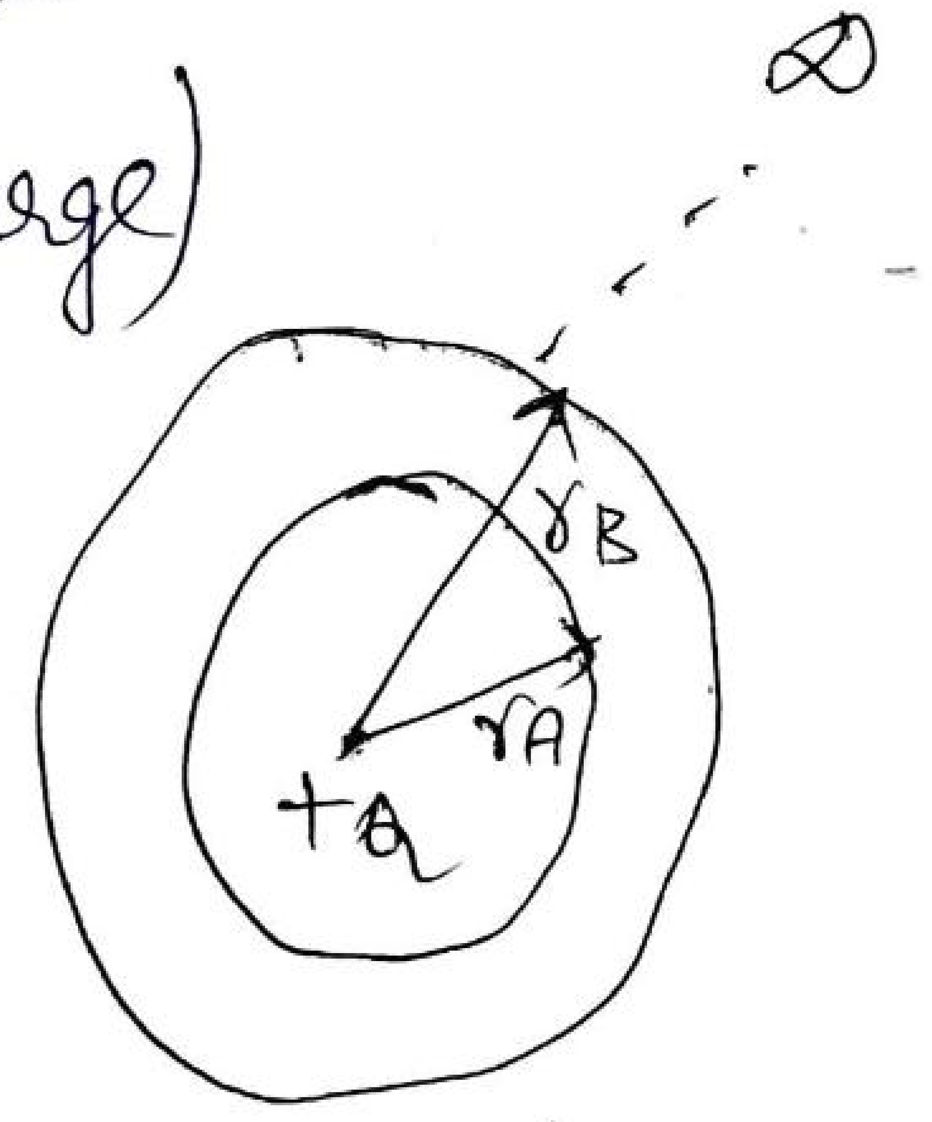
(ii) potential difference between 2 points:-

• potential difference between A & B is given as

$$V_{AB} = -\int_{B}^{A} \bar{E} \cdot d\bar{l} = V_A - V_B .$$

① potential of a point charge:- In general, $V = -\int_{\text{initial}}^{\text{final}} \bar{E} \cdot d\bar{l}$

where $\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} a_r$ (for point charge)



$$\therefore V = -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0 r^2} a_r \cdot dr\, a_r$$

$$= \frac{-Q}{4\pi\epsilon_0} \int_{r_B}^{r_A} \frac{1}{r^2} dr \qquad (a_r \cdot a_r = 1)$$

$\ast\ d\bar{l} = dr\, a_r$ moving in radial direction

$$= \frac{-Q}{4\pi\epsilon_0} \left[\left(\frac{-1}{r}\right) \Big|_{r_B}^{r_A} \right]$$

$$= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_A} - \frac{1}{r_B} \right]$$

if $r_B \to \infty$, $V = \frac{Q}{4\pi\epsilon_0 r_A}$

In general $\boxed{V = \frac{Q}{4\pi\epsilon_0 r}}$ volts.

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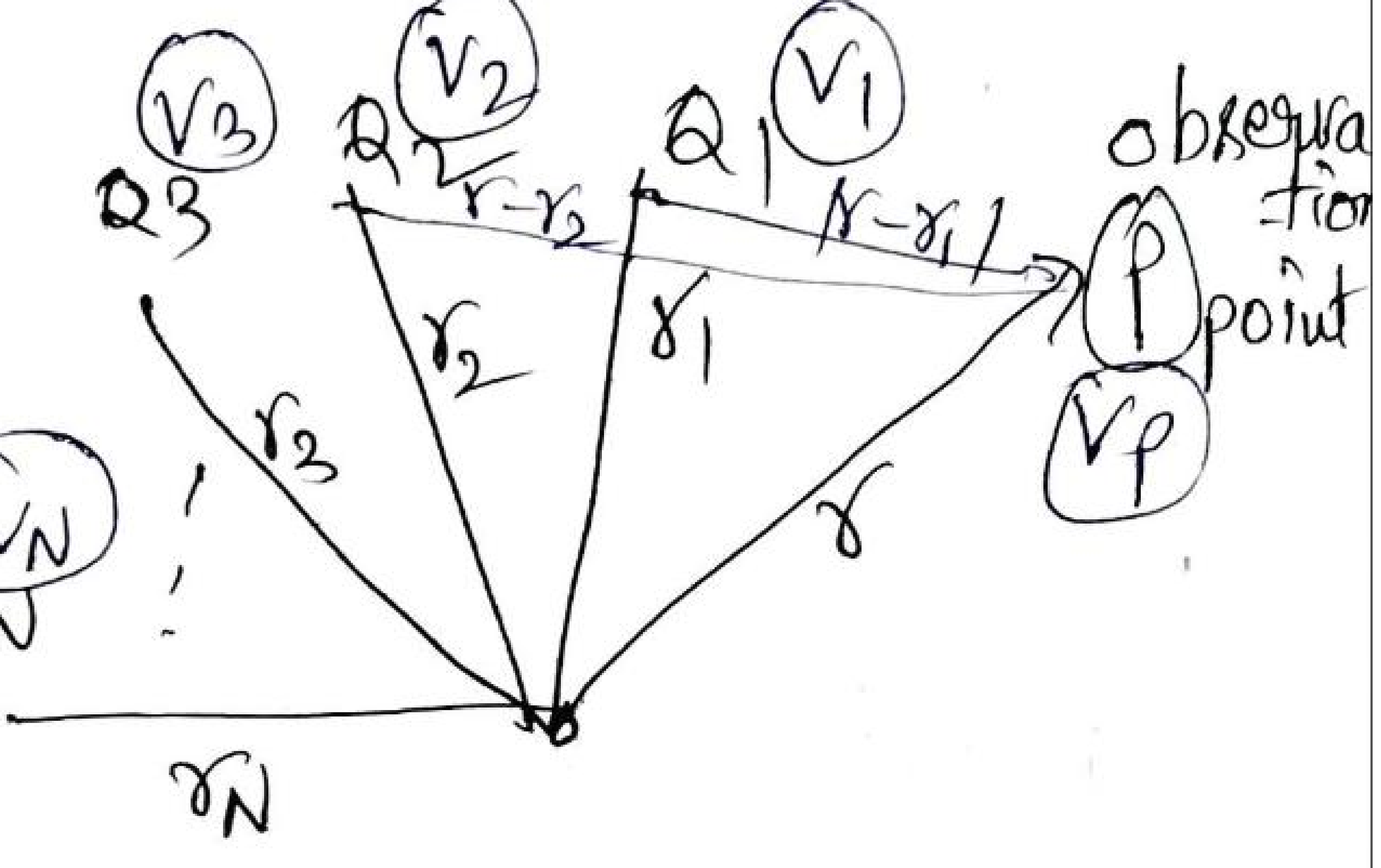
Potential of a system comprising N-point charges: (11)

→ As potential is linear, it obeys superposition theorem. hence, it can be apply for N-no. of charges system.

∴ According to superposition theorem:



$$V_P = V_1 + V_2 + \cdots + V_N$$

i.e

$$V_P = \frac{Q_1}{4\pi\varepsilon_0 |r - r_1|} + \frac{Q_2}{4\pi\varepsilon_0 |r - r_2|} + \cdots$$

$$\therefore \boxed{V_P = \frac{1}{4\pi\varepsilon_0} \sum_{m=1}^{N} \frac{Q_m}{|r - r_m|}} \text{ volts.}$$

Law of conservative field: (KVL in EMF Theory)

→ The algebraic sum of all the potentials around a closed loop must be zero.

i.e $V = -\int \bar{E} \cdot dL = 0$

Let:

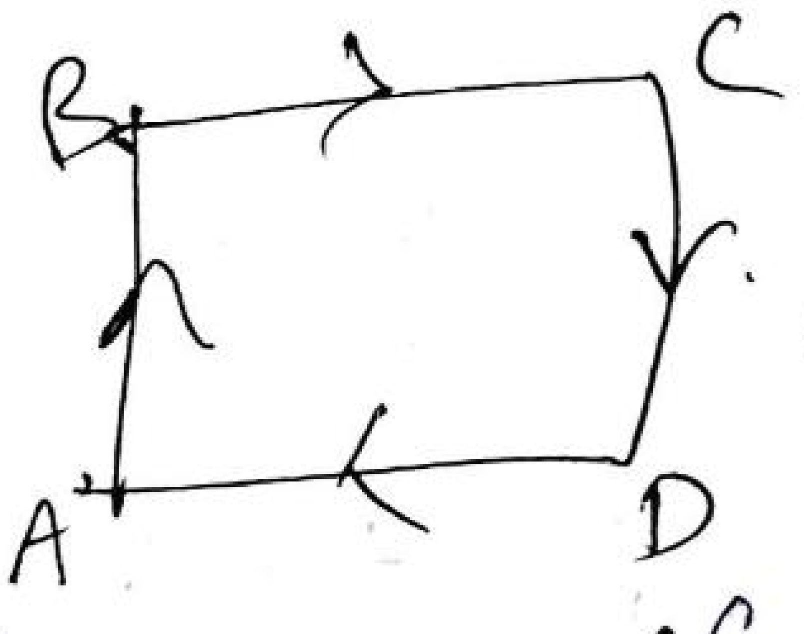


fig: closed path (or loop)

$$\text{if } \left(\int_A^B \bar{E} \cdot dL + \int_B^C \bar{E} \cdot dL + \int_C^D \bar{E} \cdot dL + \int_D^A \bar{E} \cdot dL \right) = 0.$$

i.e if $\boxed{\oint_L \bar{E} \cdot dL = 0}$ → conservative law in electrostatics.

→ Maxwell's 2nd equation. (Integral form)

Curling: cross-product

→ $\nabla \times E$ denotes how much vector E curls around the point.

→ The curl of E is an axial vector whose magnitude is the maximum circulation of E per unit area tends to zero whose direction is the normal direction of area when the area is oriented, so as to make circulation maximum.

$$\nabla \times E = \text{curl } E = \left[\lim_{\Delta S \to 0} \frac{\oint_L \bar{E} \cdot dL}{\Delta S} \right] a_{n\,max}$$

where ΔS — area bounded by the curve L.
a_n — unit vector normal to surface ΔS
— determined by right hand rule.

→ ∴ By using curling, the integral form of law of conservative field is converted to point form (or) differential form as:

$$\boxed{\nabla \times \bar{E} = 0} \rightarrow \text{point form of Maxwell's } 2^{nd} \text{ Equation.}$$

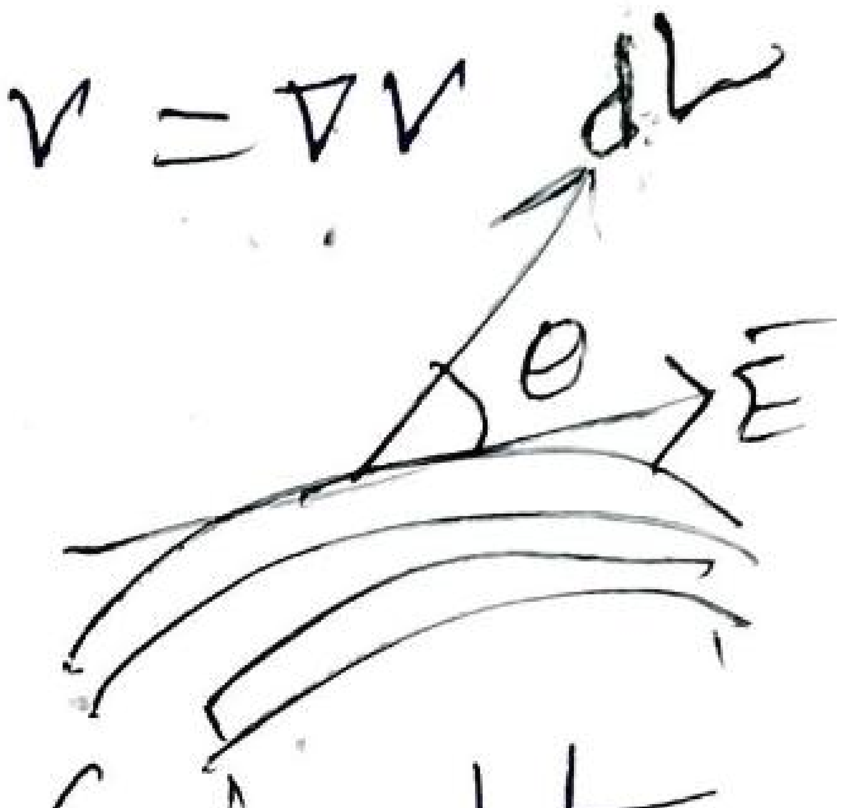
Maxwell's Equations for electrostatic fields:

	Integral form	Point form
① Maxwell's 1^{st} Equation: (Gauss law)	$\oint_S \bar{D} \cdot dS = \int_V \rho_V dV$	$\nabla \cdot \bar{D} = \rho_V$
② Maxwell's 2^{nd} Equation: (law of conservation of electrostatic field)	$\oint_L \bar{E} \cdot dL = 0$	$\nabla \times \bar{E} = 0$

Potential Gradient :-

* The gradient of a scalar quantity (v) gives a vector that represents both magnitude & direction of a Maximum rate of change of potential (v).

$\boxed{E = -\nabla V}$ where grad $v = \nabla V$



proof:- we know that: $V = -\int \bar{E} \cdot dl$

$$dv = -\bar{E} \cdot dl$$

$$dv = -E\,dl\cos\theta \quad \text{(using dot product)}$$

$$\Rightarrow \frac{dv}{dl} = -E\cos\theta$$

(i) if $\theta = 90^{\circ} \Rightarrow \frac{dv}{dl} = 0 \Rightarrow$ No workdone
$\Rightarrow$ equipotential surface.

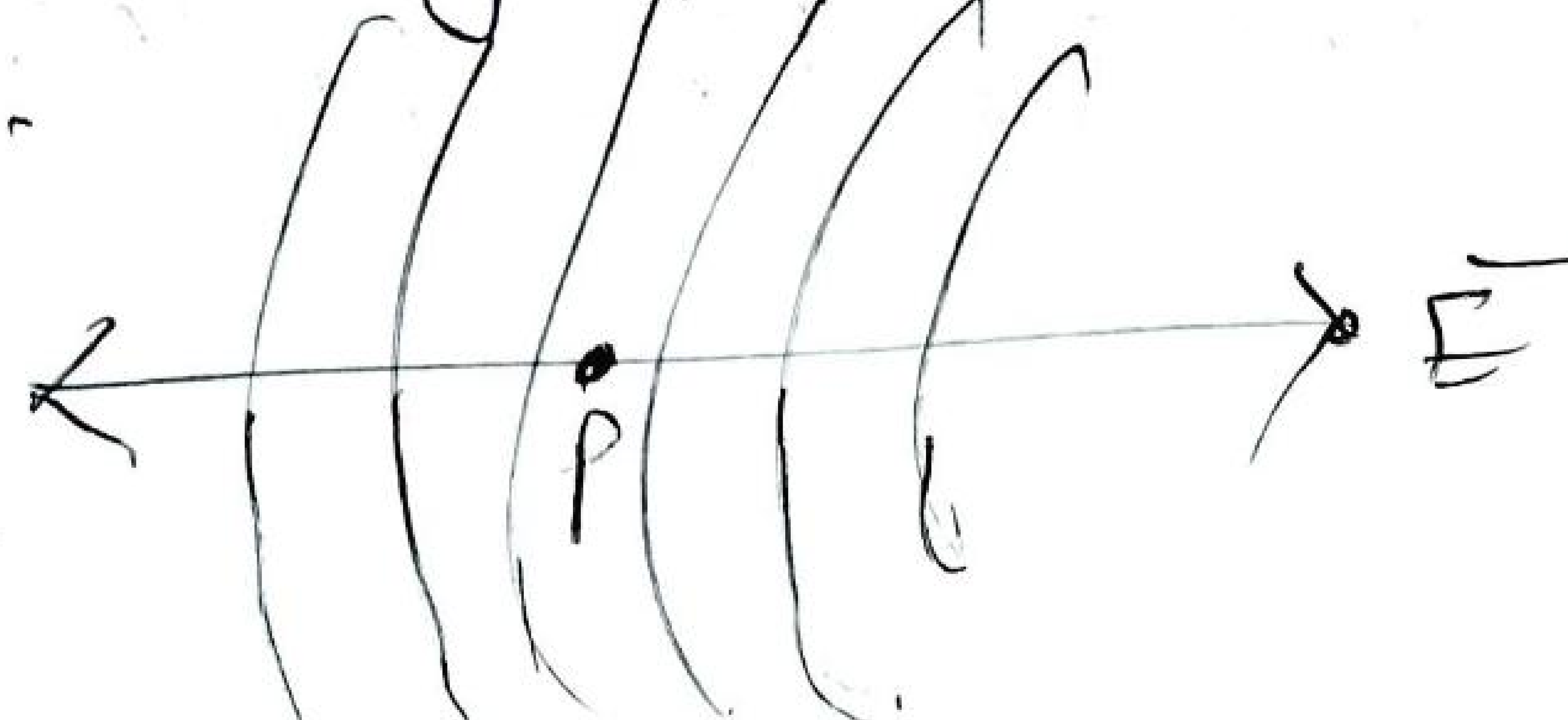
(ii) if $\theta = 180^{\circ} \Rightarrow \frac{dv}{dl} = \bar{E} \Rightarrow \left.\frac{dv}{dl}\right|_{max} = \bar{E}_{max}$

- This indicates maximum rate of change of potential with distance gives maximum electric field intensity.

$$\therefore \boxed{\bar{E} = -\text{grad}\,V = -\nabla V}$$

i.e Maximum value is obtained for E when potential is increasing most rapidly in opposite direction.

$\frac{dv}{dl}$



Mathematical proof:-

→ Let V is a function of (x, y, z) then

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz \quad \text{———} \quad (1)$$

we know: $V = -\int E \cdot dL$

$$\Rightarrow dV = -E \cdot dL$$

$$= -\left[E_x a_x + E_y a_y + E_z a_z\right] \cdot \left[dx a_x + dy a_y + dz a_z\right]$$

$$\Rightarrow dV = -\left[E_x dx + E_y dy + E_z dz\right] \quad \text{———} \quad (2)$$

Equating (1) & (2) ⇒ $E_x = \frac{-\partial V}{\partial x}$; $E_y = \frac{-\partial V}{\partial y}$; $E_z = \frac{-\partial V}{\partial z}$

$$\therefore \bar{E} = -\left[\frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z\right]$$

$$= -\left[\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right]V$$

$$\therefore \boxed{\bar{E} = -\nabla V}$$

Note:- Formulas for Gradient in 3 coordinate systems:

(1) $\nabla V = \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z$ (cartesian coordinate system)

(2) $\nabla V = \frac{\partial V}{\partial \rho}a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}a_\phi + \frac{\partial V}{\partial z}a_z$ (cylindrical ")

(3) $\nabla V = \frac{\partial V}{\partial r}a_r + \frac{1}{r}\frac{\partial V}{\partial \theta}a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}a_\phi$ (spherical)

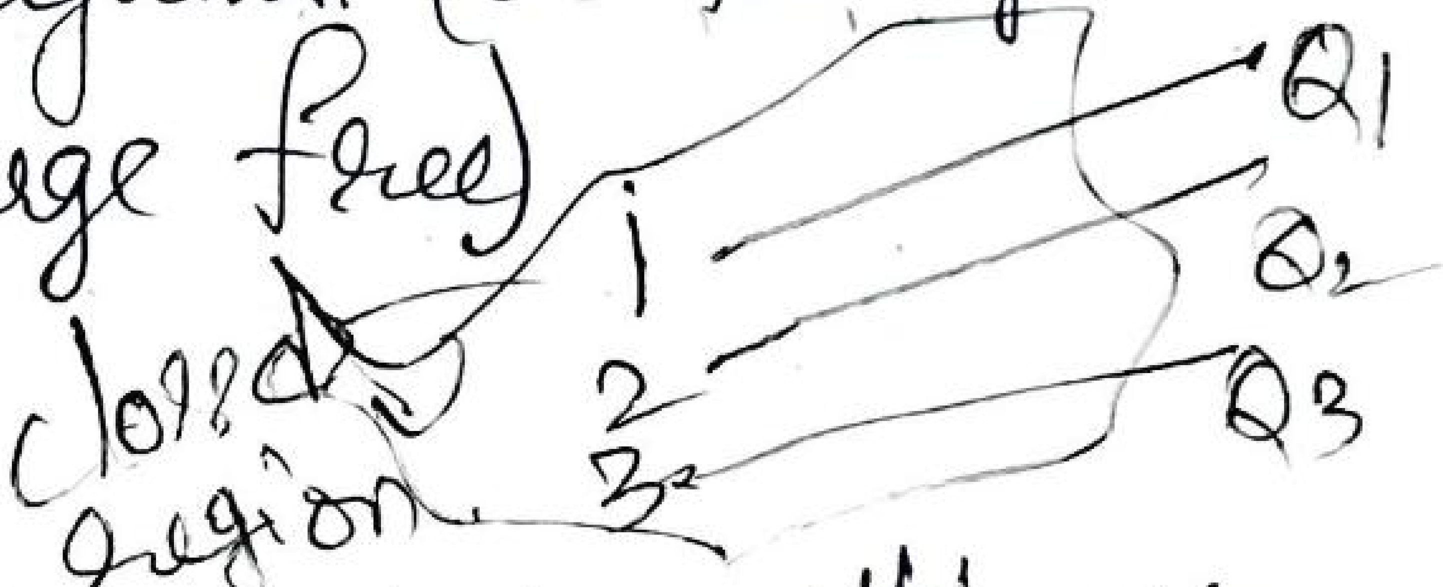
* Energy & Energy Density :-

→ Energy density is the amount of energy stored in a given system (or) region of space per unit volume.

→ Consider,

(i) a point charge Q_1 transferred from infinity to position r_1 in the system.

- It takes no work to bring the first charge since there is no electric field against (as the system is initially empty i.e charge free)



$\therefore W_1 = 0\ J$

(ii) another point charge Q_2, to bring it to position r_2 from infinity.

- Do some work against the electric field generated by Q_1.

$\left(\text{since } V = \frac{W}{Q_t} \Rightarrow W = VQ\ J\right)$.

$\therefore W_2 = Q_2 V_{21}\ J$

where V_{21} — electrostatic potential at position r_2 due to Q_1.

(iii) Similarly, the work required to bring Q_3 is

$$W_3 = Q_3 V_{31} + Q_3 V_{32} = Q_3(V_{31} + V_{32})\ J$$

where V_{31} & V_{32} — potentials at r_3 due to Q_1 & Q_2.

→ Thus, the total work done in assembling the 3 charges is given as $W_E = W_1 + W_2 + W_3$

$$= 0 + Q_2 V_{21} + Q_3 (V_{31} + V_{32}) \quad \text{--- (1)}$$

→ if the charges were positioned in reverse order such as: Let

$$Q_2 V_{21} = Q_2 \frac{Q_1}{4\pi\varepsilon_0 R_{21}} = Q_1 \frac{Q_2}{4\pi\varepsilon_0 R_{12}} = Q_1 V_{12}$$

$$\therefore \boxed{Q_2 V_{21} = Q_1 V_{12}}$$

substituting this reverse logic, Eqn (1) becomes,

$$W_E = Q_1 V_{12} + Q_1 V_{13} + Q_2 V_{23} \quad \text{--- (2)}$$

→ Adding Eqn (1) & (2) ⇒ $2W_E = Q_1 (V_{12} + V_{13}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32})$

$$\therefore 2W_E = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$\Rightarrow W_E = \frac{1}{2}\left[Q_1 V_1 + Q_2 V_2 + Q_3 V_3\right]$$ where

V_1, V_2, V_3 — potentials at r_1, r_2 & r_3.

for N-point charges :- $\boxed{W_E = \frac{1}{2} \sum_{m=1}^{N} Q_m V_m}$ Joules

for single charge : $\boxed{W_E = \frac{1}{2} Q V}$ J

ex:- energy storage in capacitor is $\boxed{W_E = \frac{1}{2} C V^2}$ J

since capacitance $C = \frac{Q}{V}$

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Note: (i) Energy equation represents potential energy of the system.

(ii) This is the workdone in bringing static charges from infinity & assembling them in the system.

(iii) when charges return back to $\infty \Rightarrow$ system gets dissolved $\Rightarrow$ kinetic energy released.

Continuous charge configurations :-

→ if the system has continuous charge distribution (in place of point charge) then energy equation becomes:

① $Q = \int_L \rho_L \, dL$; $W_E = \frac{1}{2}\int_L \rho_L \cdot V \, dL$ J (line charge)

② $Q = \int_S \rho_S \, ds$; $W_E = \frac{1}{2}\int_S \rho_S \cdot V \, ds$ J (Surface charge)

③ $Q = \int_V \rho_V \, dv$; $W_E = \frac{1}{2}\int_V \rho_V \cdot V \, dv$ J (Volume charge)

For Most practical configuration (volume):-

$$Q = \int_V \rho_V \, dv \Rightarrow W_E = \frac{1}{2}\int_V \rho_V \cdot V \, dv .$$

Energy density in terms of $\overline{D}$ & $\overline{E}$:-

$$W = \frac{1}{2}\int_V (\nabla \cdot \overline{D}) \, V \, dv \qquad \text{since } \rho_V = \nabla \cdot \overline{D}$$

. From vector identity : $\boxed{\nabla \cdot V\overline{D} \equiv V(\nabla \cdot \overline{D}) + \overline{D} \cdot (\nabla V)}$

$$\therefore W_E = \frac{1}{2}\int_V (\nabla \cdot V\bar{D})\, dv - \frac{1}{2}\int_V (\bar{D} \cdot \nabla V)\, dv$$

- As per divergence theorem:

$$\int_S V\bar{D} \cdot ds = \int_V (\nabla \cdot V\bar{D})\, dv$$

$$\therefore W_E = \frac{1}{2}\int_S V\bar{D} \cdot ds - \frac{1}{2}\int_V (\bar{D} \cdot \nabla V)\, dv$$

$$= 0 - \frac{1}{2}\int_V (\bar{D} \cdot \nabla V)\, dv$$

$$= -\frac{1}{2}\int_V \bar{D} \cdot (-\bar{E})\, dv \quad (\text{since } \bar{E} = -\nabla V)$$

$$\boxed{W_E = \frac{1}{2}\int_V \bar{D} \cdot \bar{E}\, dv} \text{ Joules.}$$

Also $\boxed{W_E = \frac{1}{2}\int_V \varepsilon_0 \bar{E}^2\, dv}$ Joules (since $\bar{D} = \varepsilon_0 \bar{E}$)

Energy Density: Energy density $= \dfrac{\text{energy}}{\text{unit volume}}$

$$\therefore W_E = \frac{1}{2}\int_V \bar{D} \cdot \bar{E}\, dv$$

$$\Rightarrow \boxed{\frac{dW_E}{dv} = \frac{1}{2}(\bar{D} \cdot \bar{E}) = \frac{1}{2}(\varepsilon_0 \bar{E}^2) = \frac{1}{2}\frac{\bar{D}^2}{\varepsilon_0}} \ J/m^3$$

Energy density $\left(\dfrac{dW_E}{dv}\right)$, measured in J/m^3.

Equation of continuity (Electrostatic fields):

→ We know that:

(i) current — rate of change of charge. $I = \frac{dQ}{dt}$ A

(ii) current density — current per unit area.

(i) $J = \frac{I}{A}$ A/m^2 (valid for uniform current on the surface)

(ii) $J = \frac{dI}{ds}$ A/m^2 (practically, current is non-uniform throughout the surface).

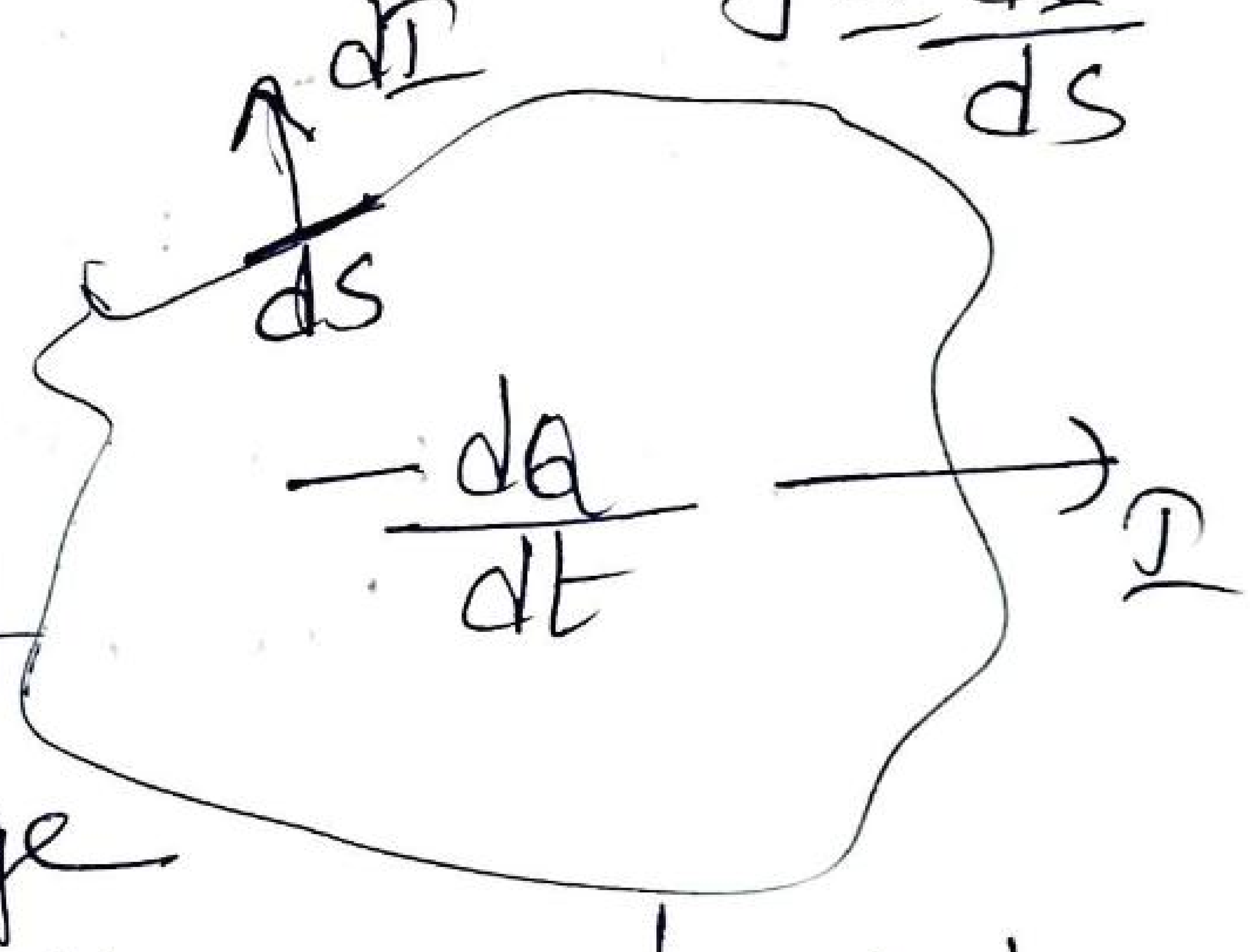
$\therefore dI = J \cdot ds$

$\Rightarrow \boxed{I = \oint_S J \cdot ds}$ A

Statement for equation of continuity:

The net outward flow of charges must be equal to continuous decrease of positive charges inside the closed surface.

$J = \frac{dI}{ds}$



i.e $I = -\frac{dQ}{dt}$

$\oint_S J \cdot ds = -\frac{dQ}{dt}$; Q-point charge

. for most practical configuration i.e volume:

$Q = \int_V \rho_v \, dv$

$\therefore \oint_S J \cdot ds = -\int_V \frac{\partial \rho_v}{\partial t} dv$

Use divergence theorem: $\oint_S \bar{D} \cdot ds = \int_V (\nabla \cdot \bar{D}) \, dv$

$\therefore \int_V (\nabla \cdot \overline{J})\, dV = -\int_V \frac{\partial \rho_v}{\partial t}\, dV$

$\therefore \boxed{\nabla \cdot \overline{J} = -\frac{\partial \rho_v}{\partial t}}$ → for time-varying fields

- But, for static fields : ρ_v — constant $\therefore \frac{\partial \rho_v}{\partial t} = 0$

$\therefore \boxed{\nabla \cdot \overline{J} = 0}$ → for static fields.

Conduction Current Density:

→ If the current flows through the conductors, it is called as conduction current.

→ A conductor is characterized by large amount of free electrons that provide conduction current due to an impressed electric field.

$$\therefore F = -e\overline{E}$$

- An electron having mass 'm', $\overline{E}$, mobility μ.

Thus, $\frac{m\mu}{\tau} = -e\overline{E}$ — (2)

$$\mu = \frac{-e\tau}{m}\overline{E} \quad \text{— (3)}$$

where τ — average time interval b/w the collisions.

- If there are 'n' no. of electrons per unit volume,

$$\rho_v = -ne \quad \text{—— (4)}$$

$\therefore$ current density $(\overline{J}) = \rho_v \mu = (-ne)\left(\frac{-e\tau}{m}\overline{E}\right)$

$$\Rightarrow J = \frac{ne^2\tau}{m} \bar{E}$$

$$\therefore \boxed{J = \sigma E}$$ — ohm's law for electrostatic fields. (point form of ohm's law)

where $\sigma = \frac{ne^2\tau}{m}$ (conductivity of the material)

J – conduction current density (A/m^2).

$$\therefore \boxed{\bar{J} = \sigma \bar{E}}$$ — Equation of conduction current density.

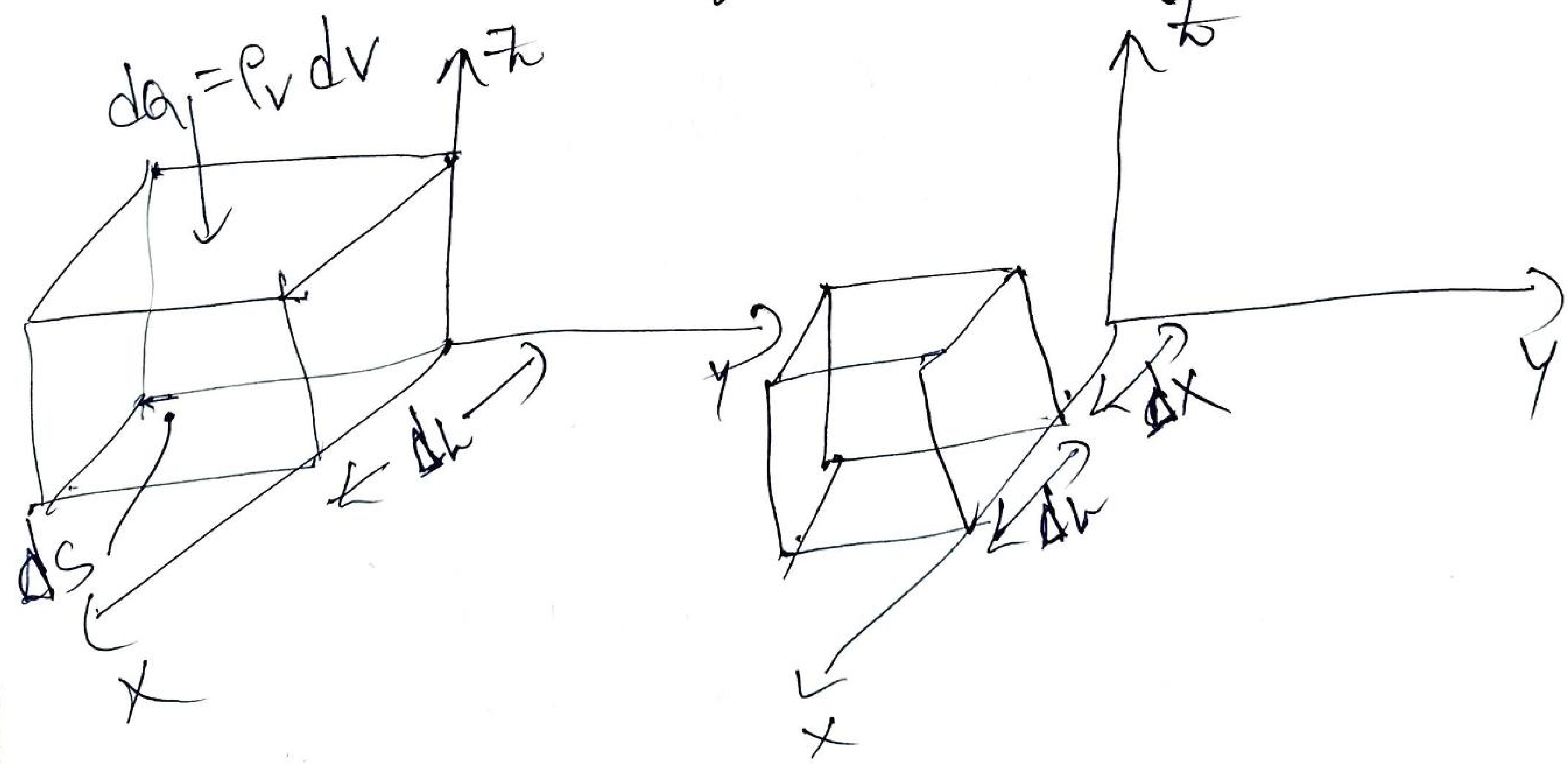
Convection current density :-

→ if the current flows in insulators like liquids, rarefied gases etc.

→ It does not require any conductors

→ It does not obey the ohm's law.

ex:- current flow in vacuum tube by using beam of electrons (electron bunches).



→ consider the bunch of charge element

$$dQ = \rho_v \, dv = \rho_v \, ds \, dL$$

. if the block moves dx in time dt

$$dI = \frac{dQ}{dt} = \frac{\rho_v \, ds \, dx}{dt} \quad (\text{since } dL = dx)$$

$$dI = \rho_v \, ds \cdot v_x$$ (where velocity in x-direction $v_x = \frac{dx}{dt}$).

$$\Rightarrow \frac{dI}{ds} = \rho_v \cdot v_x$$

$$\therefore \boxed{J_v = \rho_v \cdot v_x} \rightarrow \text{convection current density.}$$

Dielectric constant: (or) Relative permittivity:

(i) If two charges q_1 and q_2 separated by a distance (r) in vacuum, then electrostatic force between them is:

$$F_v = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \quad \text{— (1)} \quad (\text{since Coulomb's law}).$$

(ii) Now, when the same charges with same separation placed in a medium of absolute permittivity (ε), then electrostatic force between them is:

$$F_m = \frac{1}{4\pi\varepsilon} \frac{q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2} \quad \text{— (2)}.$$

$\therefore$ Let

$$\frac{F_v}{F_m} = \frac{\left(\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}\right)}{\left(\frac{1}{4\pi\varepsilon_0\varepsilon_r} \frac{q_1 q_2}{r^2}\right)} = \varepsilon_r \qquad \boxed{\therefore \varepsilon_r = \frac{F_v}{F_m}}$$

definition: dielectric constant of a medium is defined as the ratio of electrostatic force between 2 point charges when placed in vacuum to the force between the same charges placed in a medium.

Poisson's & Laplace Equations :-

→ point form : $\nabla \cdot \bar{D} = \rho_v$ —— ①

$\bar{D} = \varepsilon \bar{E}$ —— ②

$\bar{E} = -\nabla V$ —— ③

substituting ② & ③ in ①

$$\nabla \cdot \varepsilon \bar{E} = \rho_v$$

$$\nabla \cdot \varepsilon(-\nabla V) = \rho_v$$

$$\nabla \cdot \nabla V = \frac{-\rho_v}{\varepsilon}$$

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon}}$$ → poisson's equation.

if $\rho_v = 0 \Rightarrow \boxed{\nabla^2 V = 0}$ → Laplace's equation.

Note :- ① $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ (cartesian)

② $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$ (cylindrical)

③ $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$ (spherical)

Note :- ① $\nabla \cdot \bar{D} = \rho_v$ — Maxwell's 1st equation

② $\nabla \times \bar{E} = 0$ — Maxwell's 2nd equation

③ $\nabla V = -E$ — potential gradient.

④ $\nabla^2 V = \frac{-\rho_v}{\varepsilon_0} = 0$ ⇒ Laplace's equation ($\rho_v = 0$).

Capacitors & its types :-

→ Capacitance: It is the charge required to develop the potential between the two oppositely charged surfaces/plates. i.e $\boxed{C = \frac{Q}{V}}$ mF/μF/nF.

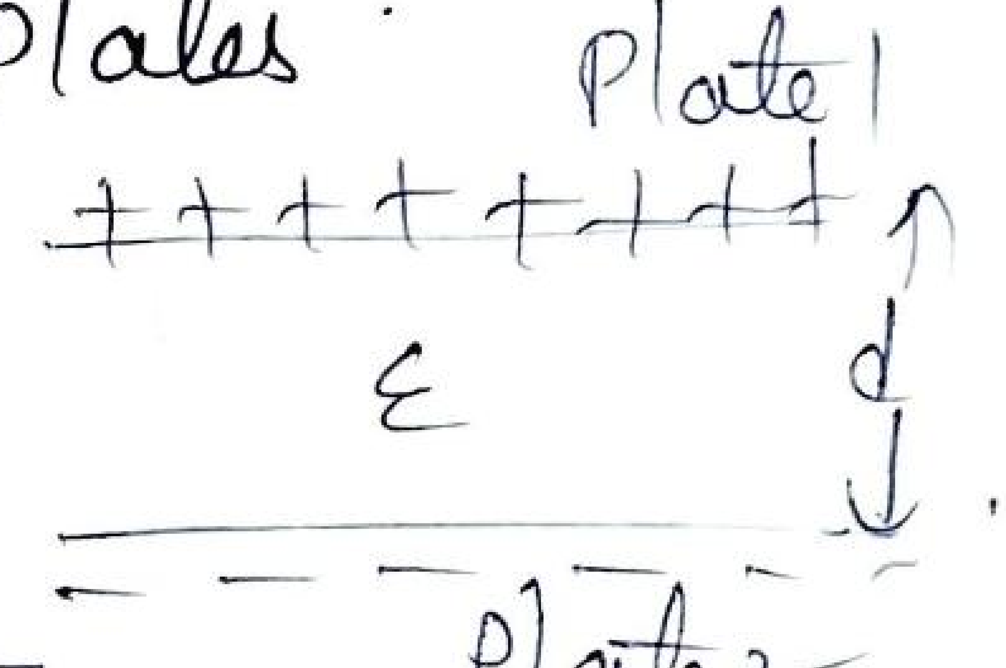
since $1F = \frac{1C}{1V}$.

(i) parallel plate capacitor:

→ It consists of 2 parallel plates separated by a distance (d).

· The space between plates is filled with dielectric of permittivity (ε).

· Let A – area of cross section of plates.



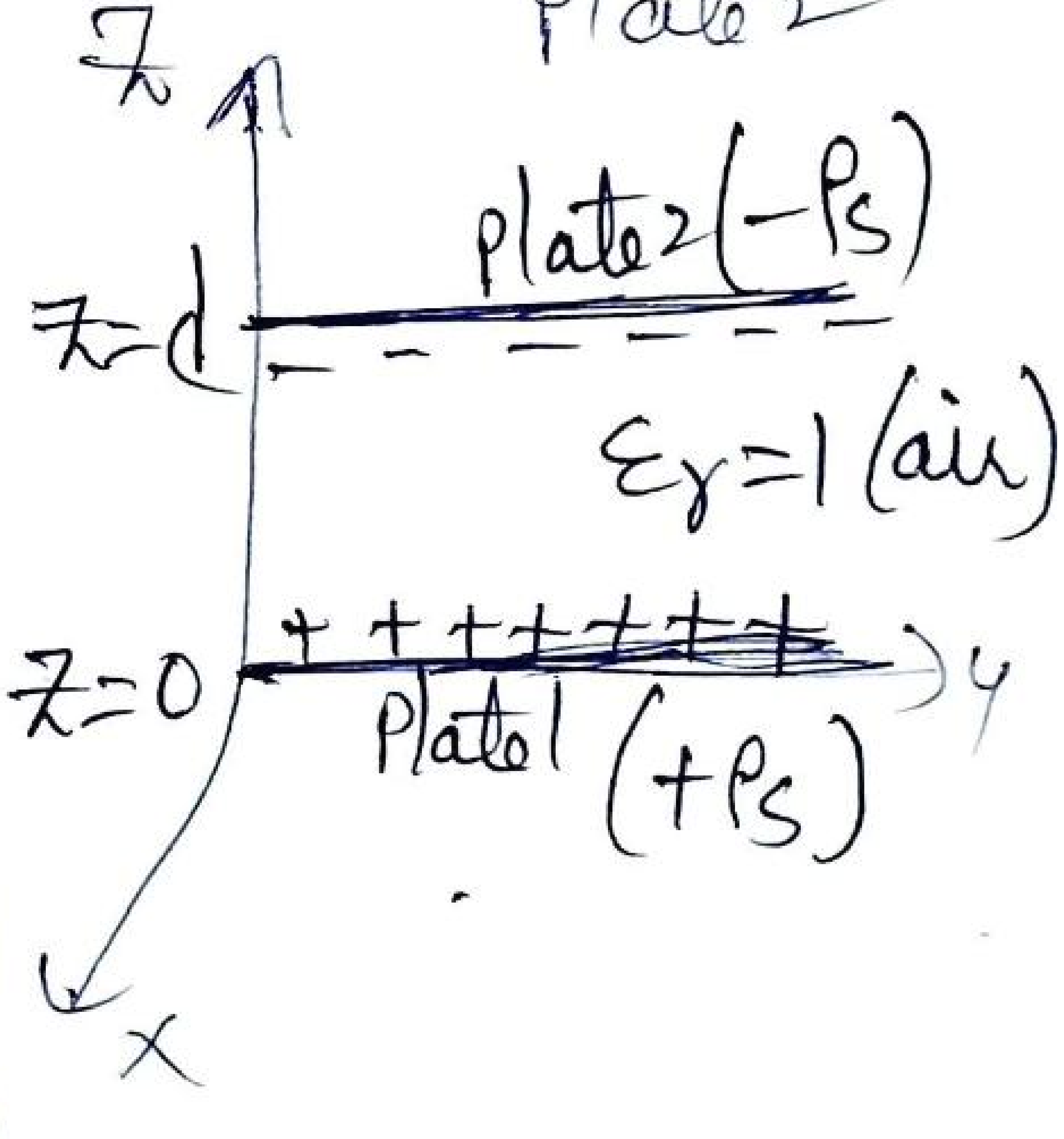
→ As $C = \frac{Q_{encl}}{V}$ ——— ①

(i) Q_{en} (charge enclosed) :-

· Using Gauss's law: $Q_{en} = \oint_S \overline{D} \cdot ds$

$Q_{en} = \rho_S \cdot A$ ——— ②

where ρ_S – surface charge density of plate (C/m^2)



(ii) potential :- we know: $V = -\int_{initial}^{final} \overline{E} \cdot dL$

where $\overline{E} = \overline{E}\Big|_{plate1} + \overline{E}\Big|_{plate2} = \frac{\rho_S}{2\varepsilon_0} + \frac{\rho_S}{2\varepsilon_0} = \frac{\rho_S}{\varepsilon_0}\ V/m$

(since $\bar{E}$ for surface charge configuration $= \frac{\rho_s}{2\varepsilon_0}$ V/m)

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$\therefore$ potential $V = -\int_d^0 \frac{\rho_s}{\varepsilon_0} a_z \cdot dz\, a_z$ (since $d\bar{l} = dz\, a_z$)

$$= -\frac{\rho_s}{\varepsilon_0}(0-d) = \frac{\rho_s d}{\varepsilon_0} \quad \text{--- (3)}$$

- substitute (2) & (3) in (1) gives,

capacitance $C = \frac{\cancel{\rho_s} \cdot A}{\frac{\cancel{\rho_s} d}{\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$

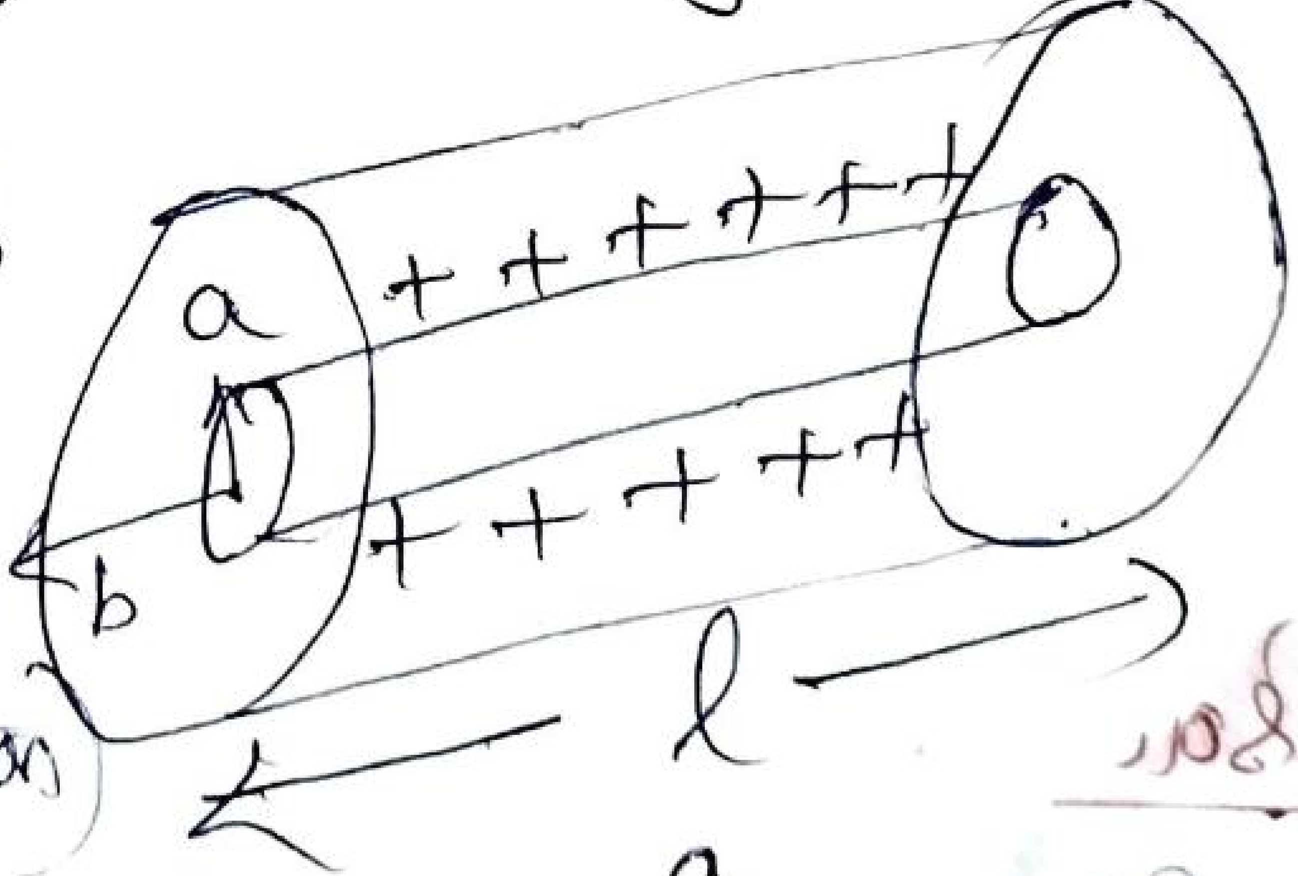
$\therefore \boxed{C = \frac{\varepsilon_0 A}{d}}$ farads (or) $\boxed{C = \frac{\varepsilon A}{d}}$ farads

where $\varepsilon = \varepsilon_0 \varepsilon_r$.

(ii) Coaxial Cable capacitor:

→ Consider length (l) of two coaxial conductors of inner radius (a) and outer radius (b).

- The space between the conductors is filled with homogeneous dielectric with permittivity (ε).



(i) $Q = \int_L \rho_L\, dL$ (Since, it is of line charge configuration)

$= \rho_L \cdot L$ --- (2)

(ii) $V = -\int_{\text{initial}}^{\text{final}} \bar{E} \cdot d\bar{l}$

$= -\int_b^a \frac{\rho_L}{2\pi\varepsilon_0 \rho} a_\rho \cdot d\rho\, a_\rho = -\frac{\rho_L}{2\pi\varepsilon_0}\int_b^a \frac{1}{\rho} d\rho$

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considel) ($\bar{E}$-for line charge)

$$= \frac{-\rho_L}{2\pi\varepsilon_0}\left[\ln\rho\Big|_b^a\right]$$

$$= \frac{-\rho_L}{2\pi\varepsilon_0}\left(\ln\left(\frac{a}{b}\right)\right) = \frac{\rho_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right) \longrightarrow (3)$$

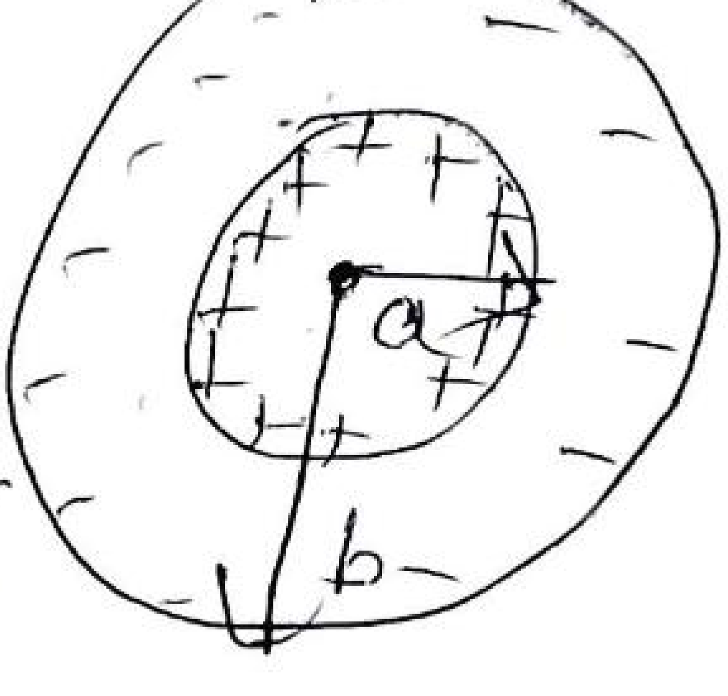
substitute (2) & (3) in (1)

$$C = \frac{Q}{V} = \frac{\rho_L \cdot L}{\frac{\rho_L}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \text{ Farads.}$$

Spherical Capacitor:

→ Consider, 2 concentric spherical conductors with inner sphere radius (a) & outer sphere radius (b), seperated by dielectric medium with permittivity ε.

Let
$$C = \frac{Q_{en}}{V} = \frac{Q}{-\int_{initial}^{final}\bar{E}\cdot d\bar{l}} = \frac{Q}{-\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r \cdot dr\bar{a}_r}$$



($\bar{E}$ — for point charge)

$$= \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi\varepsilon_0}\int_a^b \frac{1}{r^2}dr}$$

$$= \frac{4\pi\varepsilon_0}{\left(\frac{-1}{r}\Big|_a^b\right)} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)} = \frac{4\pi\varepsilon_0 ab}{b-a} \text{ Farads}$$

Isolated Capacitor:

→ For an isolated capacitor i.e $\boxed{b \longrightarrow \infty}$ (subcase of spherical capacitor)

$$\therefore \boxed{C = 4\pi\varepsilon_0 a} \text{ Farads.}$$