UNIT – II

Transmission Lines – II

Input Impedance Relations :-

Consider, a transmission line of length (l) terminated by a load impedance (Z_R) is shown in fig:

From fig: V_S – voltage source

Z_0 – characteristic impedance of transmission line

Z_{in} – input impedance of given Tx line i.e $Z_{in} = \frac{V_S}{I_S}$

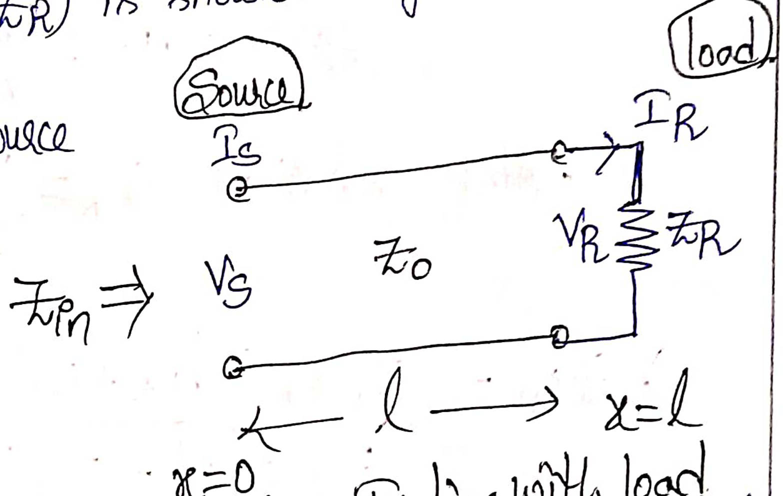


fig: Tx line with load

→ The voltage and current expressions from sending end at any distance (x) are interms of V_R and I_R as:

$$\boxed{\begin{aligned} V &= V_R \cosh P(l-x) + I_R Z_0 \sinh P(l-x) \\ I &= I_R \cosh P(l-x) + \frac{V_R}{Z_0} \sinh P(l-x) \end{aligned}}$$

At sending/Source end: $\boxed{x=0 \; ; \; V = V_S \; ; \; I = I_S}$

substituting these conditions in above expressions, we get

$$V_S = V_R \cosh Pl + I_R Z_0 \sinh Pl$$

$$I_S = I_R \cosh Pl + \frac{V_R}{Z_0} \sinh Pl$$

$\therefore$ input impedance $Z_{in} = \frac{V_S}{I_S} = \frac{V_R \cosh Pl + I_R Z_0 \sinh Pl}{I_R \cosh Pl + \frac{V_R}{Z_0} \sinh Pl}$

multiplying both Numerator & denominator by $\frac{Z_0}{I_R}$, we get:

$$Z_{in} = \frac{\frac{Z_0 V_R}{I_R}\cosh \gamma l + \cancel{I_R} Z_0 \frac{Z_0}{\cancel{I_R}}\sinh \gamma l}{\frac{Z_0 \cancel{I_R}}{\cancel{I_R}}\cosh \gamma l + \frac{V_R}{\cancel{Z_0}}\cdot\frac{\cancel{Z_0}}{I_R}\sinh \gamma l}$$

. Also substitute $Z_R = \frac{V_R}{I_R}$ (load / termination impedance)

$$\Rightarrow Z_{in} = \frac{Z_0 Z_R \cosh \gamma l + Z_0^2 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}$$

$$\therefore \boxed{Z_{in} = Z_0\left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}\right]}$$

$\Rightarrow$ Input impedance of a Tx line having length (l) & characteristic impedance (Z_0).

<u>Simplification:</u>

Further simplify the above Z_{in} expression by dividing both numerator & denominator by $\underline{\cosh \gamma l}$.

$$Z_{in} = Z_0\left[\frac{Z_R \frac{\cosh \gamma l}{\cosh \gamma l} + Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}}{Z_0 \frac{\cosh \gamma l}{\cosh \gamma l} + Z_R \frac{\sinh \gamma l}{\cosh \gamma l}}\right]$$

$$\Rightarrow \boxed{Z_{in} = Z_0\left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]}$$

$\Rightarrow$ simplified expression of Z_{in}.

→ By varying the load of the Tx line, there exists various special cases such as:

1. short-circuited line ($Z_R=0$)
2. open-circuited line ($Z_R=\infty$)
3. Matched line ($Z_R=Z_0$)
4. Any other termination (Z_R)

(1) Short-circuited line:-

→ The short circuit is formed when the load impedance is characterized by zero value ($Z_R=0$) since $V=0$ for short circuit as shown in fig:

Let Z_{SC} → input impedance of the short-circuited Tx line.

i.e $Z_{SC}=\frac{V_S}{I_S}$

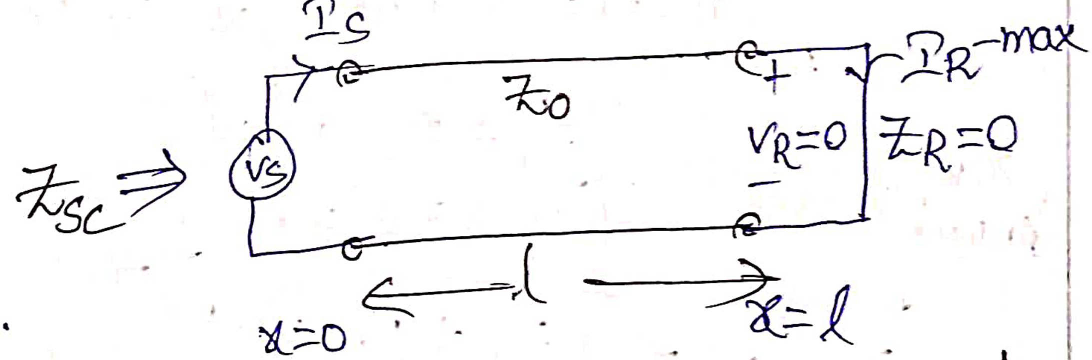


fig: A short-circuited Tx line.

→ The voltage & current expressions at any distance 'x' from sending end in terms of V_S & I_S are:

$$V=V_S\cosh px-I_S Z_0\sinh px \quad \text{—— (1)}$$

$$I=I_S\cosh px-\frac{V_S}{Z_0}\sinh px$$

At $x=l$: $\boxed{V=0}$ substitute in eqn (1)

$\therefore 0 = V_S \cosh \gamma l - I_S Z_0 \sinh \gamma l$

$\Rightarrow V_S \cosh \gamma l = I_S Z_0 \sinh \gamma l$

$$\Rightarrow Z_{SC} = \frac{V_S}{I_S} = Z_0 \frac{\sinh \gamma l}{\cosh \gamma l} = Z_0 \tanh \gamma l.$$

$\therefore \boxed{Z_{SC} = Z_0 \tanh \gamma l}$ — input impedance of short-circuited Tx line.

For lossless line: $\tanh \gamma l = j \tan \beta l$

$\therefore \boxed{Z_{SC} = j Z_0 \tan \beta l}$ — input impedance of short-circuited lossless Tx line.

<u>Another approach for Z_{SC}:</u>

As we know: $$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right] \quad \text{— (A)}$$

since load is <u>short-circuited</u>: $\boxed{Z_R = 0}$

substituting in above eq^n (A)

$$Z_{SC} = Z_0 \left[\frac{\cancel{Z_0} \tanh \gamma l}{\cancel{Z_0}} \right] = Z_0 \tanh \gamma l$$

$\therefore \boxed{Z_{SC} = Z_0 \tanh \gamma l}$

Similarly, for <u>lossless line</u>:

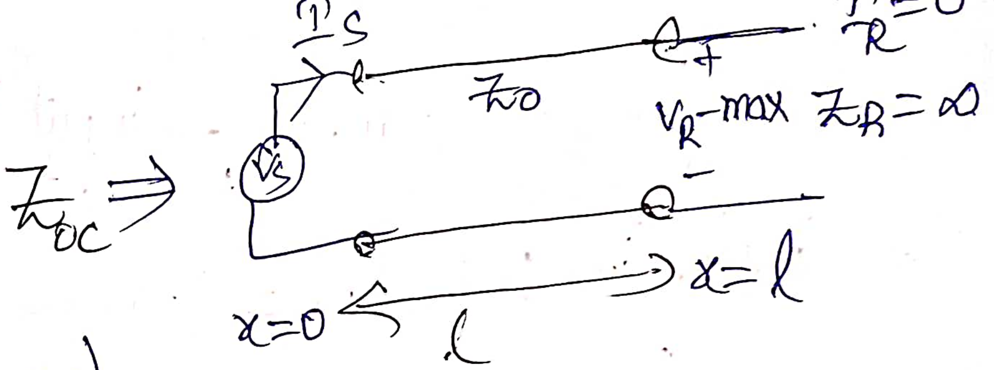
$\boxed{Z_{SC} = j Z_0 \tan \beta l}$

2 Open-Circuited Line :-

→ An open-circuited line is formed when the load impedance has infinite value ($Z_R = \infty$) since $I = 0$ for open circuit as shown below:

Let

Z_{oc} = input impedance of the open-circuited Tx line (load is open).



We know : $V = V_S \cosh px - I_S Z_0 \sinh px$

$I = I_S \cosh px - \frac{V_S}{Z_0} \sinh px$ —— (2)

At $x = l$: $\boxed{I = 0}$ substitute in Eqn (2)

$$0 = I_S \cosh pl - \frac{V_S}{Z_0} \sinh pl$$

$$\Rightarrow \frac{V_S}{Z_0} \sinh pl = I_S \cosh pl$$

$$\therefore Z_{oc} = \frac{V_S}{I_S} = Z_0 \frac{\cosh pl}{\sinh pl} = Z_0 \coth pl$$

$$\therefore \boxed{Z_{oc} = Z_0 \coth pl}$$ — input impedance of open-circuited Tx line

For lossless line :- $\tanh pl = j \tan \beta l$

$$\therefore Z_{oc} = \frac{Z_0}{\tanh pl} = \frac{Z_0}{j \tan \beta l} = -j Z_0 \cot \beta l .$$

$$\therefore \boxed{Z_{oc} = -j Z_0 \cot \beta l}$$ — input impedance of OC lossless Tx line.

Another approach of Z_{oc}:-

As we know: $Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$

since load is open-circuited: $\boxed{Z_R = \infty}$

• on substitution:

$$Z_{oc} = Z_{in} = Z_0 \left[\frac{1 + \cancel{\frac{Z_0}{Z_R}}^{0} \tanh \gamma l}{\cancel{\frac{Z_0}{Z_R}}^{0} + \tanh \gamma l} \right]$$

(dividing both Nr & Dr by Z_R)

$$= \frac{Z_0}{\tanh \gamma l} = Z_0 \coth \gamma l$$

$$\therefore \boxed{Z_{oc} = Z_0 \coth \gamma l}$$

Similarly, for lossless line $\boxed{Z_{oc} = -j Z_0 \cot \beta l}$.

Important Relations:-

① $Z_{sc} \times Z_{oc} = Z_0 \tanh \gamma l \times Z_0 \coth \gamma l$

$$= Z_0 \cancel{\tanh \gamma l} \times Z_0 \frac{1}{\cancel{\tanh \gamma l}} = Z_0^2$$

$$\boxed{\therefore Z_0 = \sqrt{Z_{oc} Z_{sc}}}$$

② $\frac{Z_{sc}}{Z_{oc}} = \frac{Z_0 \tanh \gamma l}{Z_0 \coth \gamma l} = \tanh^2 \gamma l$

$$\therefore \boxed{\tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}}$$

③ Matched line:-

when the Tx line is terminated by the characteristic impedance, the load of the line is equal to characteristic impedance ($Z_R = Z_0$) and it is called as matched line.

→ As we know: $Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$

substituting the condition: $\boxed{Z_R = Z_0}$

$$Z_{in} = Z_0 \left[\frac{Z_0 + Z_0 \tanh \gamma l}{Z_0 + Z_0 \tanh \gamma l} \right] = Z_0 .$$

$\therefore \boxed{Z_{in} = Z_0}$ i.e input impedance is equal to characteristic impedance for a matched line.

Note:- Totally there are 4 types of terminations. They are ① short ckt ② open ckt ③ with Z_0 ④ Any other

① Short ckt ($Z_R = 0$):- If the line is terminated with shorted load then total reflection of energy takes place.

② open ckt ($Z_R = \infty$): If the line is terminated with open load then total reflection of energy takes place.

③ Matched termination ($Z_R = Z_0$):- If the line is terminated with Z_0 (characteristic impedance, there is no reflection of energy.

④ Any other termination:- If the line is terminated with Z_R then there is a partial reflection.

Input Impedance relations for lossless Tx line :-

→ For lossless Tx line : $\boxed{\alpha = 0}$

$$\boxed{P = j\beta}$$ (since $P = \alpha + j\beta$)

Let, consider the term:

$$\tanh Pl = \tanh(j\beta l) = j\tan\beta l \quad (\text{since } \tanh(jx) = j\tan x)$$

$$\therefore \boxed{\tanh Pl = j\tan\beta l} \rightarrow \text{for lossless line}$$

① Input impedance :-

As we know: $Z_{in} = Z_0 \left[\dfrac{Z_R + Z_0 \tanh Pl}{Z_0 + Z_R \tanh Pl} \right]$

$$\therefore \boxed{Z_{in} = Z_0 \left[\dfrac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l} \right]}$$

② Input impedance under short-circuit load :-

As we know : $Z_{SC} = Z_0 \tanh Pl$

$$\therefore \boxed{Z_{SC} = jZ_0 \tan\beta l}$$

③ Input impedance under open-circuit load :-

As we know: $Z_{OC} = Z_0 \coth Pl$

$$= \frac{Z_0}{\tanh Pl} = \frac{Z_0}{j\tan\beta l}$$

$$\therefore \boxed{Z_{OC} = -jZ_0 \cot\beta l}$$

④ Input impedance under matched load : $\boxed{Z_{in} = Z_0}$

Short-circuit (SC) and open-circuit (OC) Lines:-

Consider, the fig: shows the variations of Z_{OC} and Z_{SC} as a function of the physical length (or) electrical length of the Tx lines.

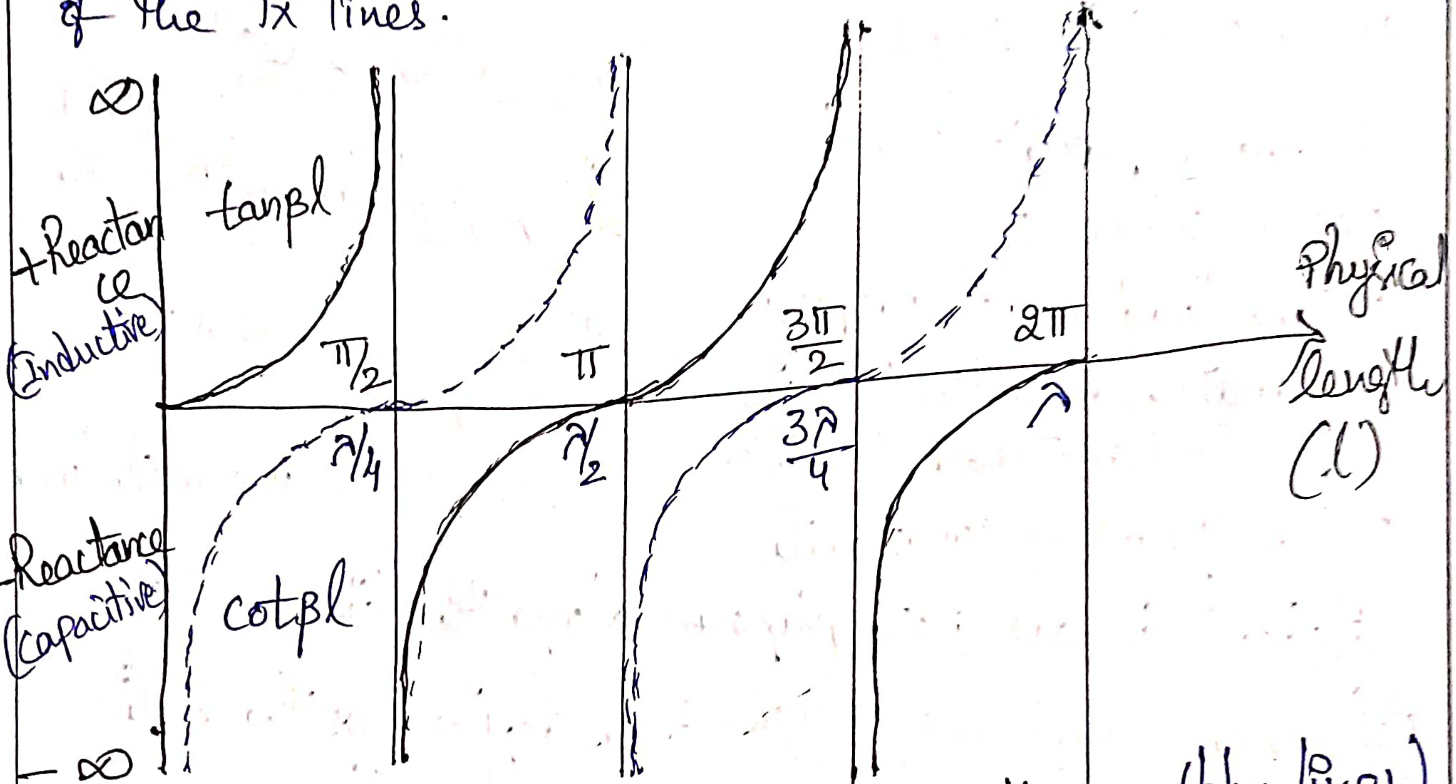


fig: variations of the input impedance with Z_{OC} (blue lines) & Z_{SC} (black lines).

① Consider the variation of Z_{SC} only:-

(i) At even multiples of $\lambda/4$:

The line offers zero reactance ⇒ behaves like series resonant circuit.

(ii) At odd multiples of $\lambda/4$:

The line offers infinite reactance ⇒ behaves as a parallel resonant circuit.

② Consider the variation of Z_{OC} only:.

(i) At even multiples of $\lambda/4$:

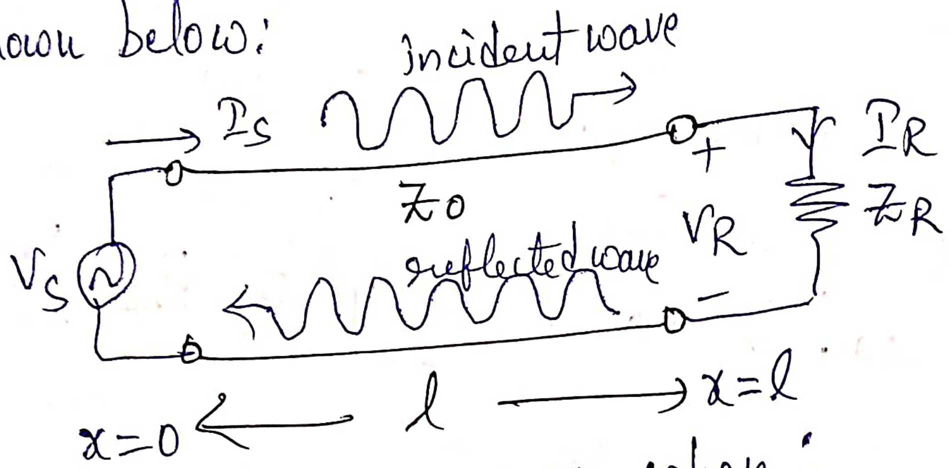
The line offers infinite reactance ⇒ behaves as parallel resonant circuit.

(ii) At odd multiples of $\lambda/4$:

The line offers zero reactance ⇒ behaves as series resonant circuit.

Reflection Coefficient :-

→ Consider, a Tx line of length l, having characteristic impedance (Z_0) is connected with source & load (Z_R) as shown below:



→ The reflection may occur when :

① The load is not properly matched to the Tx line (Mismatched load)

② The primary constants (R, L, C, G) are not uniformly distributed along the length of line.

Definition: The reflection coefficient is defined as the ratio of reflected voltage to the incident voltage (or) −ve ratio of reflected current to incident current.

i.e $$\boxed{\Gamma = \frac{V_r}{V_i} = \frac{-I_r}{I_i}}$$

Derivation: Consider the voltage & current expressions as:

$$V = A\cosh px + B\sinh px$$

$$I = \frac{-1}{Z_0}(A\sinh px + B\cosh px)$$

• Replace sine & cosine terms using exponentials such as :

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$$\boxed{\begin{aligned} V &= ae^{Px} + be^{-Px} \\ I &= \frac{-1}{Z_0}\left(ae^{Px} - be^{-Px}\right) \end{aligned}}$$

where e^{-Px} term represents forward travelling wave
e^{Px} term represents reflected wave.

Assume: Load is placed at $\boxed{y=0}$ then $\boxed{x=-y}$

$\therefore$ V & I expressions are rewrite as

$$\boxed{\begin{aligned} V &= ae^{-Py} + be^{Py} \\ I &= \frac{-1}{Z_0}\left(ae^{-Py} - be^{Py}\right) \end{aligned}}$$

where, e^{Py} — forward wave
e^{-Py} — reflected wave.

$\therefore$ reflection coefficient $(\Gamma) = \dfrac{ae^{-Py}}{be^{Py}} = \dfrac{a}{b}e^{-2Py}$

$\therefore \boxed{\Gamma = \dfrac{a}{b}e^{-2Py}}$ → expression of reflection coefficient at a distance (y) from load end.

determine Constants (a & b):-

Let: Load conditions: $\boxed{y=0 \; ; \; V = V_R \; ; \; I = I_R}$

. substitute in above expressions:

$\boxed{V_R = a+b}$ — ①

$I_R = \dfrac{-1}{Z_0}(a-b) = \dfrac{1}{Z_0}(b-a) \Rightarrow \boxed{I_R Z_0 = b-a}$ — ②.

① + ② $\Rightarrow \boxed{b = \dfrac{V_R + I_R Z_0}{2}}$

(i) - (2) $\Rightarrow$ $\boxed{a = \frac{V_R - I_R Z_0}{2}}$.

$\therefore$ At load $(y=0)$:-

reflection coefficient $(\Gamma_l) = \frac{a}{b} e^{-2P(0)} = \frac{a}{b}$

$$\Rightarrow \quad \Gamma_l = \frac{\left(\frac{V_R - I_R Z_0}{2}\right)}{\left(\frac{V_R + I_R Z_0}{2}\right)} = \frac{V_R - I_R Z_0}{V_R + I_R Z_0}$$

$$= \frac{\frac{V_R}{I_R} - Z_0}{\frac{V_R}{I_R} + Z_0}$$

$$\therefore \boxed{\Gamma_l = \frac{Z_R - Z_0}{Z_R + Z_0}} \quad \left(\text{since } \frac{V_R}{I_R} = Z_R\right).$$

reflection coefficient of load in terms of impedances.

various load conditions :-

(1) short-circuited load :- $Z_R = 0$ then $\Gamma_l = \frac{-Z_0}{Z_0} = -1$

$\therefore \boxed{\Gamma_l = -1} \rightarrow$ Maximum reflections with 180° phaseshift

(2) open-circuited load :- $Z_R = \infty$ then $\Gamma_l = \frac{1 - Z_0/Z_R}{1 + Z_0/Z_R}$

$$= \frac{1-0}{1+0} = 1$$

$\therefore \boxed{\Gamma_l = 1} \rightarrow$ Maximum reflections with same phase.

(3) Matched load :- $Z_R = Z_0$ then $\Gamma_l = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$.

$\boxed{\Gamma_l = 0} \rightarrow$ No reflections.

Note:- The range of reflection coefficient : $\boxed{0 \text{ to } 1}$

Input impedance in terms of reflection coefficient :-

We know that : $Z_{in} = Z_0 \left[\dfrac{Z_R \cosh Pl + Z_0 \sinh Pl}{Z_0 \cosh Pl + Z_R \sinh Pl} \right]$

$$= Z_0 \left[\frac{Z_R\left(\frac{e^{Pl}+e^{-Pl}}{\cancel{2}}\right) + Z_0\left(\frac{e^{Pl}-e^{-Pl}}{\cancel{2}}\right)}{Z_0\left(\frac{e^{Pl}+e^{-Pl}}{\cancel{2}}\right) + Z_R\left(\frac{e^{Pl}-e^{-Pl}}{\cancel{2}}\right)} \right]$$

$$= Z_0 \left[\frac{(Z_R+Z_0)e^{Pl} + (Z_R-Z_0)e^{-Pl}}{(Z_R+Z_0)e^{Pl} - (Z_R-Z_0)e^{-Pl}} \right]$$

divide Nr & Dr by $(Z_R+Z_0)e^{Pl}$

$$= Z_0 \left[\frac{1 + \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-2Pl}}{1 - \left(\frac{Z_R-Z_0}{Z_R+Z_0}\right)e^{-2Pl}} \right]$$

$$= Z_0 \left[\frac{1 + \Gamma_l e^{-2Pl}}{1 - \Gamma_l e^{-2Pl}} \right] \quad \left(\text{since } \Gamma_l = \frac{Z_R-Z_0}{Z_R+Z_0} \right)$$

$$\boxed{\therefore Z_{in} = Z_0 \left[\frac{1 + \Gamma_l e^{-2Pl}}{1 - \Gamma_l e^{-2Pl}} \right]}$$

input impedance in terms of reflection coefficient (Γ_l).

$\lambda/4$, $\lambda/2$ and $\lambda/8$ Lines :-

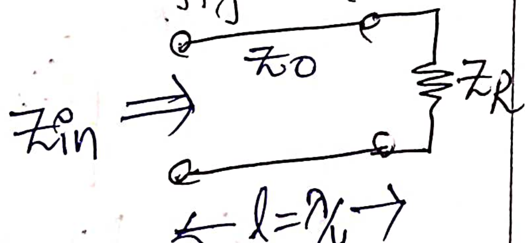
The input impedance of lossless transmission line

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l} \right] \quad ; \quad \text{where, } \beta = \frac{2\pi}{\lambda}$$

$\lambda/4$ Line (Quarter wave transformer (or) Impedance transformer)

→ A Tx line of length $\boxed{l = \lambda/4}$ is called as Quarter wave transformer, used for "Impedance matching"

fig: $\lambda/4$ line



$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{4}\right)}{Z_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \times \lambda/4\right)} \right]$$

$$= Z_0 \left[\frac{Z_R + jZ_0 \tan \pi/2}{Z_0 + jZ_R \tan \pi/2} \right] = Z_0 \left[\frac{\cancel{\frac{Z_R}{\tan \pi/2}}^{0} + jZ_0}{\cancel{\frac{Z_0}{\tan \pi/2}}^{0} + jZ_R} \right]$$

$$= Z_0 \left[\frac{jZ_0}{jZ_R} \right] = \frac{Z_0^2}{Z_R} \quad \left(\text{since } \tan \pi/2 = \infty\right)$$

$$\therefore Z_{in} = \frac{Z_0^2}{Z_R} \Rightarrow \boxed{Z_0 = \sqrt{Z_{in} Z_R}}$$

various load conditions

1. SC load: if $Z_R = 0 \Rightarrow Z_{in} = \infty$ (open circuited)
2. OC load: if $Z_R = \infty \Rightarrow Z_{in} = 0$ (short circuited)

} – it transforms short into open & vice-versa

3. Inductive load: if $Z_R = jX \Rightarrow Z_{in} = -jX$ (capacitive)
4. Capacitive load: if $Z_R = -jX \Rightarrow Z_{in} = jX$ (inductive)

∴ A quarter-wave impedance transformer is a Tx line of length one-quarter wavelength ($\lambda/4$), terminated with some known impedance (Z_R).

- It presents at it's input the dual of the impedance with which it is terminated.

$\lambda/8$ Line :-

A Tx line of length $\boxed{l = \lambda/8}$ having characteristic impedance (Z_0) is terminated with load impedance (Z_R) is shown below:

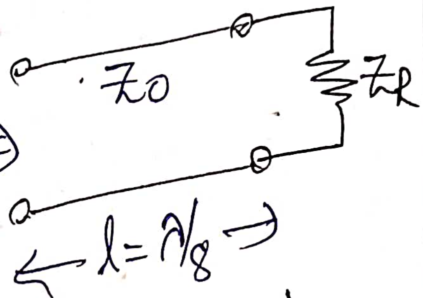


fig: $\lambda/8$ line

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{8}\right)}{Z_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \cdot \lambda/8\right)} \right]$$

$$= Z_0 \left[\frac{Z_R + jZ_0 \tan \pi/4}{Z_0 + jZ_R \tan \pi/4} \right]$$

(since $\tan \pi/4 = 1$)

$$\boxed{\therefore Z_{in} = Z_0 \left[\frac{Z_R + jZ_0}{Z_0 + jZ_R} \right]}$$

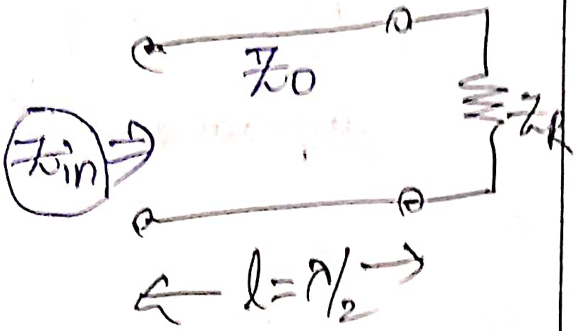
→ Now, $|Z_{in}| = Z_0 \left[\frac{\sqrt{Z_R^2 + Z_0^2}}{\sqrt{Z_0^2 + Z_R^2}} \right] = Z_0$

$$\therefore \boxed{|Z_{in}| = Z_0}$$

Note:- ① A $\lambda/8$ line can transform a real impedance into a complex impedance.

② A $\lambda/4$ line can transform a real impedance into another real impedance.

$\lambda/2$ Line: A Tx line of length $[l = \lambda/2]$ having the characteristic impedance (Z_0) is terminated with the load impedance (Z_R) is shown below:



$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)}{Z_0 + jZ_R \tan\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2}\right)}\right]$$

$$= Z_0\left[\frac{Z_R + jZ_0 \tan\pi}{Z_0 + jZ_R \tan\pi}\right]$$

(since $\tan\pi = 0$)

$$= Z_0\left[\frac{Z_R}{Z_0}\right] = Z_R$$

$\therefore \boxed{Z_{in} = Z_R} \Rightarrow$ input impedance is equal to load impedance.

Standing Waves in OC & SC Lines :-

→ A standing wave results from two waves (incident & reflected) travelling in opposite directions between the input end and the load end.

(i) At some points in the line, the 2 waves will always be in phase and will add.

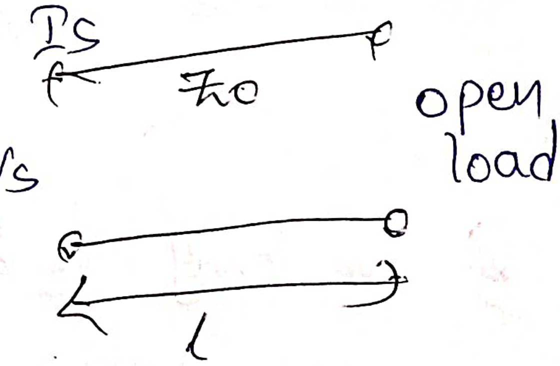
∴ The place where 2 waves add will be points of maximum voltage, termed as anti-nodes.

(ii) At other points, the 2 waves will always be out of phase and will cancel.

∴ The points of cancellation will have minimum voltage, termed as nodes.

→ As the positions of maxima & minima (or) anti-nodes & nodes — voltage remain motionless, a standing wave is said to exist on the line.

① open-circuited lines :-



- At the open-end termination, there exist a
  - maximum voltage
  - minimum (zero) current

∴ $\boxed{Z = \infty}$

Let quarter-wavelength ($l = \lambda/4$) from the open end,

- Incident wave will be 90° earlier
- Reflected wave 90° later at the end, and

thus both waves will be 180° out of phase.

$\therefore$ At this point $\boxed{\text{voltage} = 0}$ (minimum) $\Rightarrow$ current maximum occurs.

Note:- ① standing wave pattern repeated for every half-wave lengths.

② Maxima are spaced half-wavelength apart

③ Minima are also spaced half-wavelength apart on Tx line.

④ distance b/w maximum & minimum is quarter wavelength

V & I distributions along oc lines:-

① In high frequency lossless line:

The values of the different maximum are equal as shown below.

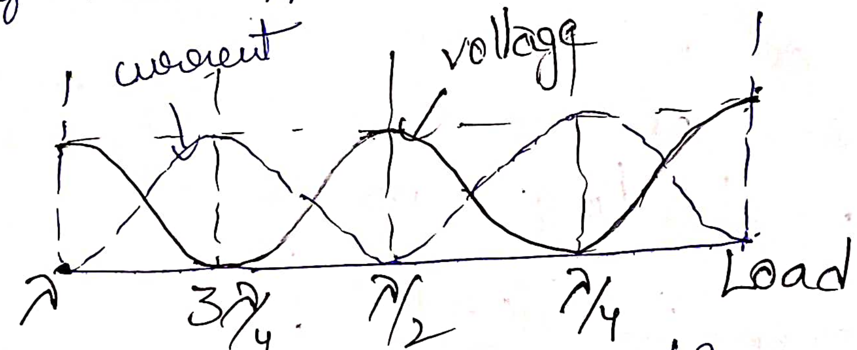
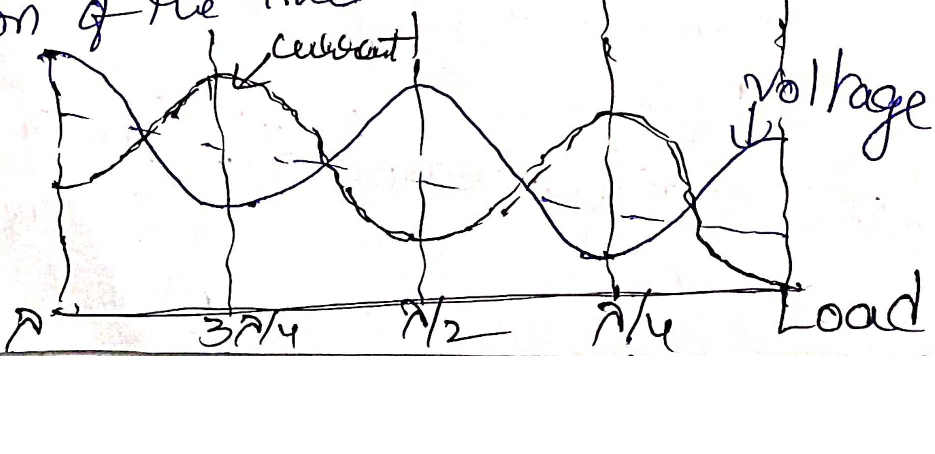


fig ⓐ : Lossless Line.

② In lossy line:-

The values of different maxima goes on decreasing due to attenuation of the line as shown below:



(2) Short-circuited Lines:

→ At short-circuited termination, current is maximum & V = 0 ⇒ $\boxed{Z = 0}$.

→ The standing waves thus has a node (or) minimum at the short-circuited end and at every $\lambda/2$ length from end.

Note:- (1) V & I distributions differs from OC lines. i.e (V & I are interchanged).

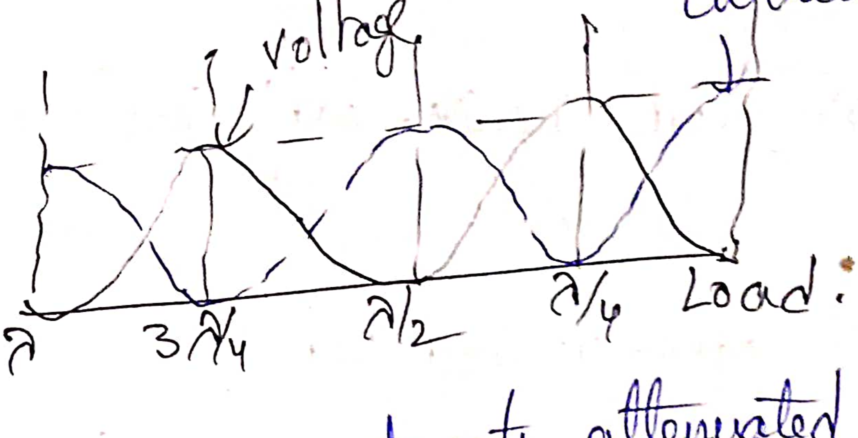
(2) The voltage on the line goes through:

(i) Minima at distances from load at even multiples of $\lambda/4$.

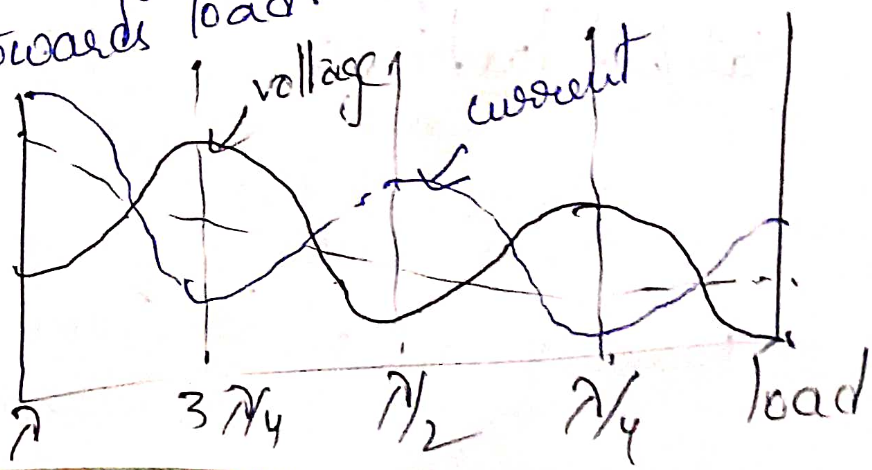
(ii) Maxima that are odd multiples of $\lambda/4$.

→ Consider V & I distributions along SC lines are:

(1) Lossless line:



(2) Lossy line: The voltage & current gets attenuated as they travel towards load.



20

Voltage standing wave ratio (VSWR):-

→ VSWR is the ratio of maximum voltage to minimum voltage of a standing wave.

$$\boxed{VSWR(=\rho) = \frac{V_{max}}{V_{min}}}$$

also, $\boxed{CSWR = \frac{I_{max}}{I_{min}}}$

where $V_{max} = |V_i| + |V_R|$ → Maximum voltage when both waves add in phase.

$V_{min} = |V_i| - |V_R|$ → Minimum voltage when both waves add in opposite phase.

$$\therefore VSWR = \frac{|V_i| + |V_R|}{|V_i| - |V_R|} = \frac{1 + \left|\frac{V_R}{V_i}\right|}{1 - \left|\frac{V_R}{V_i}\right|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\therefore \boxed{VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

various load conditions:-

① Short-circuited load:- $\boxed{Z_R = 0}$ then $\Gamma_l = -1$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$

② open-circuited load:- $\boxed{Z_R = \infty}$ then $\Gamma_l = 1$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1}{1-1} = \infty$$

③ Matched load:- $\boxed{Z_R = Z_0}$ then $\Gamma_l = 0$

$$VSWR = \frac{1+0}{1-0} = 1$$

Note:- Range of VSWR is from $\boxed{1 \text{ to } \infty}$

UHF lines as Circuit Elements (SC & OC Lines)

(or)

Different lengths of Tx Lines with SC & OC load:-

→ At high frequencies (> 150 MHz), ordinary lumped circuit elements become difficult to construct. Thus Tx lines can be used as circuit elements as their size becomes very small.

→ A Tx lines operated at a frequency range from 300 MHz to 3 GHz are known as ultra-high frequency (UHF) lines.

→ At radio frequencies (RF) lines, UHF lines :

$$\boxed{\begin{array}{l} \omega L >> R \\ \omega C >> G \end{array}}$$

characteristics of RF/UHF lines:-

→ From low-loss Tx lines : (refer unit-1)

① propagation constant (P):-

attenuation constant $\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)$

Phase constant $\beta = \omega\sqrt{LC}$

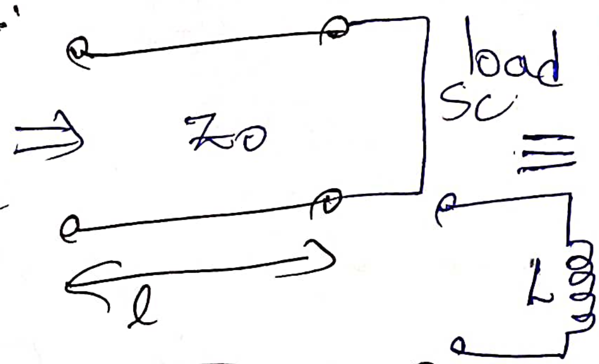
Note:- G-neglected for air, dielectric lines.

② characteristic impedance (Z_0):-

$Z_0 = \sqrt{L/C}$ — purely resistive

(1) Under Short-circuited (sc) load:

Assume:- line to be lossless line.



$\therefore \boxed{Z_{SC} = jZ_0 \tan\beta l}$

$= jZ_0 \tan\left(\frac{2\pi}{\lambda} \cdot l\right)$

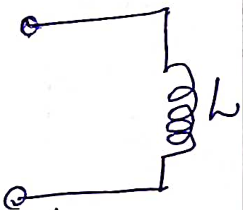
Note:- βl – electrical length.

Case (i): $0 < l < \lambda/4$: $l = 0$; $\beta l = \frac{2\pi}{\lambda} \cdot 0 = 0^\circ$

$l = \lambda/4$; $\beta l = \frac{2\pi}{\lambda} \cdot \lambda/4 = \pi/2 = 90^\circ$

Thus, the short-circuited lines can provide inductive reactance of any value ~~& length~~ by selecting suitable length.

$\therefore$ The equivalent inductance impedance is obtained by equating, $j\omega L_{eq} = jZ_0 \tan\beta l$



$\Rightarrow \boxed{L_{eq} = \frac{Z_0}{\omega} \tan\beta l}$ – it acts as an inductor.

Note:- tan value $\rightarrow$ +ve ; $Z_{SC} \rightarrow$ inductive reactance.

Case (ii):- $\lambda/4 < l < \lambda/2$: $l = \lambda/4$; $\beta l = \frac{2\pi}{\lambda} \cdot \lambda/4 = \pi/2 = 90^\circ$

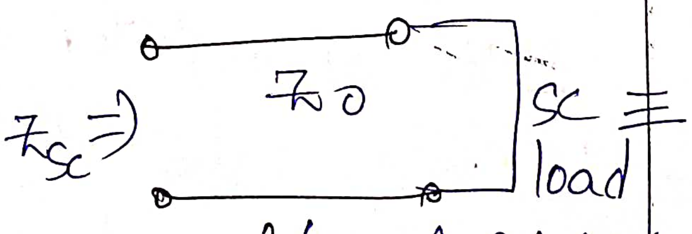
$-j\frac{1}{\omega C_{eq}} = jZ_0 \tan\beta l$; $l = \lambda/2$; $\beta l = \frac{2\pi}{\lambda} \cdot \lambda/2 = \pi = 180^\circ$.

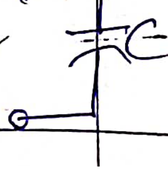
$C_{eq} = \frac{1}{\omega Z_0 \tan\beta l}$.

$\tan(90^\circ \text{ to } 180^\circ) \rightarrow$ –ve value

$\Rightarrow Z_{SC}$ is –ve.

$\therefore$ It behaves as a capacitor.



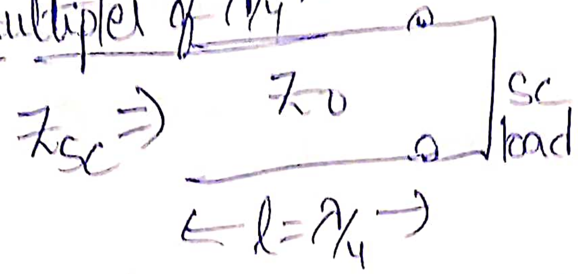


Case(iii): $l = \lambda/4$:

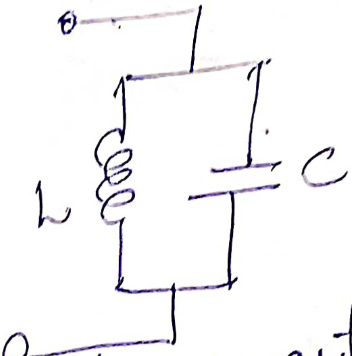
It behaves as Quarter-wave transformer.

$$Z_{in} = \frac{Z_0^2}{Z_R} = \infty \quad (\text{since } Z_R = 0 \text{ for SC}).$$

odd multiples of $\lambda/4$:

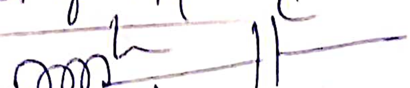


$\equiv$



Parallel resonant circuit.

even multiples of $\lambda/4$:



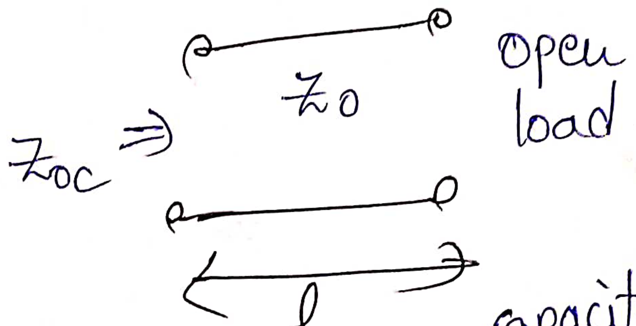
series resonant circuit

(2) open-circuited load (OC):

→ For lossless line:

$$\boxed{Z_{OC} = -j Z_0 \cot \beta l}$$

$$= -j Z_0 \cot\left(\frac{2\pi}{\lambda} \cdot l\right)$$



Case(i): $0 < l < \lambda/4$:

→ Thus, the open-circuited lines can provide capacitive reactances of any value by selecting suitable length.

∴ The equivalent capacitance impedance is given by

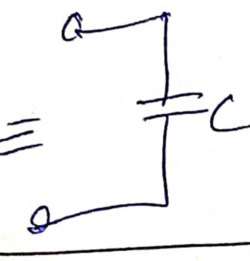
$$Z_{OC} = -j Z_0 \cot \beta l$$

(angle 0 to 90° cot is +ve)

$$-j \frac{1}{\omega C_{eq}} = -j Z_0 \cot \beta l.$$

$Z_{OC} \rightarrow$ -ve reactance

$\equiv$



$$\boxed{C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}}$$

it acts as capacitor.

Case ②: $\lambda/4 < l < \lambda/2$:- Angle $\underbrace{90^\circ \text{ to } 180^\circ}_{\cot \text{ is } -ve}$

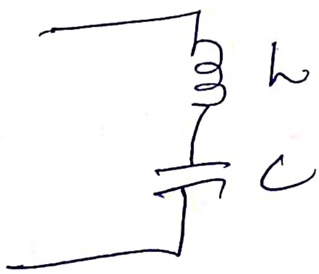
$j\omega L_{eq} = -jZ_0 \cot \beta l$

$\boxed{L_{eq} = \frac{-Z_0 \cot \beta l}{\omega}}$ $Z_{OC} \Rightarrow$ +ve reactance $\rightarrow$ acts as inductor.

Case ③: $l = \lambda/4$:- it is quarter wave transformer.

$$Z_{IN} = \frac{Z_0^2}{Z_R} = 0 \quad (\text{since } Z_R = \infty \text{ for open load})$$

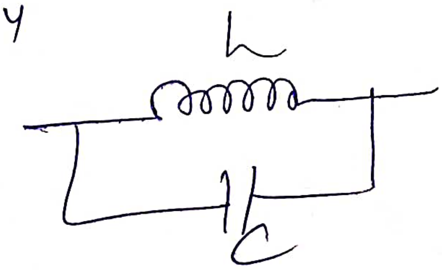
odd multiples of $\lambda/4$:- $\equiv$



series resonant circuit.

even multiples of $\lambda/4$

$\equiv$



parallel resonant circuit.

Infinite length of Tx line :-

$$Z_{in} = Z(l) = Z_0 \left[\frac{1 + \Gamma_L e^{-2Pl}}{1 + \Gamma_L e^{-2Pl}} \right]$$

$$Z(l = \infty) = Z_{in} = Z_0 \left[\frac{1 + \frac{\Gamma_L}{e^{2Pl}}}{1 - \frac{\Gamma_L}{e^{2Pl}}} \right]$$

$$\boxed{\therefore Z_{in} = Z_0} = Z_0 \left[\frac{1 + \frac{\Gamma_L}{e^{2P\infty}}}{1 - \frac{\Gamma_L}{e^{2P\infty}}} \right] = Z_0 \qquad (e^{2P\infty} = \infty)$$

Impedance Matching techniques:-

→ when the Tx line is terminated with a load impedance which is not equal to characteristic impedance of the Tx line, mismatch occurs; thus reflections exist on the line.

- Mismatch — 1. Reduces efficiency
  2. increases power loss

Remedy:-

To avoid mismatching, it is necessary to add impedance matching device between the load and the line.

2 techniques:- 1. Quarter-wave transformer
2. Stub Matching.

1. Quarter-wave impedance matching (or) Quarter wave transformer:-

→ when the Tx line is mismatched, a quarter wave line is inserted between the line and load to match load impedance to the line as shown in fig:

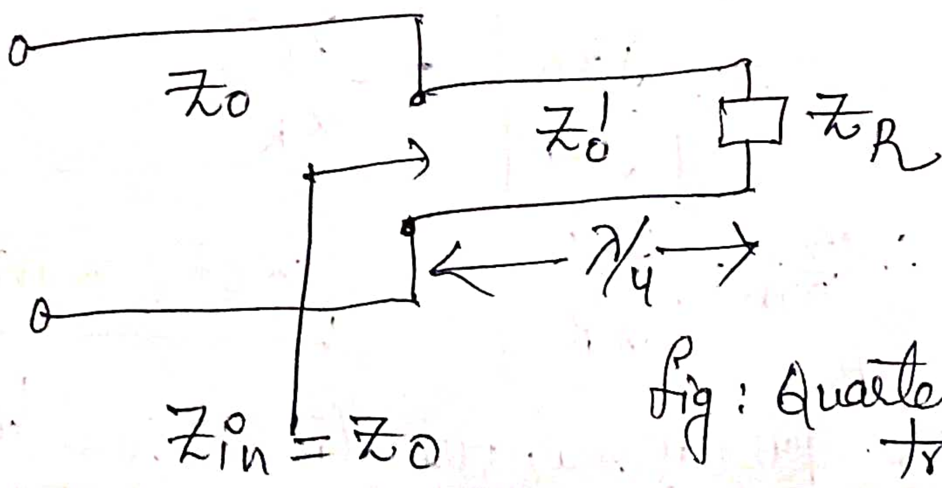


fig: Quarter wave transformer.

→ Z_0' is selected such that $Z_{in} = Z_0$

$$\therefore \boxed{Z_0' = \sqrt{Z_0 Z_L}}$$

∴ The quarter-wave transformer is also called $\lambda/4$ line.

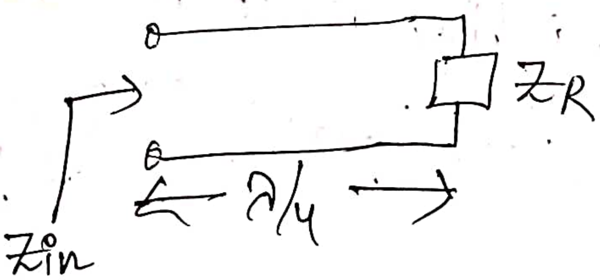
Advantages:- ① Simple to design.

Disadvantages:- ① Need to cut the line to insert a quarter-wave transformer in between the line and the load.

② It is frequency sensitive.

Analysis:- $l = \lambda/4$

$$\beta l = \frac{2\pi}{\lambda} \times \lambda/4 = \pi/2.$$



We know that:

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \pi/2}{Z_0 + jZ_R \tan \pi/2} \right] \quad \text{for lossless line}$$

$$= Z_0 \left[\frac{\cancel{\frac{Z_R}{\tan \pi/2}}^{\,0} + jZ_0}{\cancel{\frac{Z_0}{\tan \pi/2}}^{\,0} + jZ_R} \right] \quad \left(\text{since } \tan \pi/2 = \infty\right)$$

$$= Z_0 \left[\frac{jZ_0}{jZ_R} \right] = \frac{Z_0^2}{Z_R}$$

$$\therefore \boxed{Z_{in} = \frac{Z_0^2}{Z_R}} \Rightarrow \boxed{Z_0 = \sqrt{Z_{in} Z_R}}$$

Note:- As Z_R – purely real, use this for real loads but not for complex loads

Steps for smith chart :- (Quarter wave transformer)

① Normalize the load impedance.

$Z_R = 100 + j100\,\Omega$ $\quad z_r = \dfrac{100+j100}{50} = 2+j2\,\Omega$

$Z_0 = 50\,\Omega$

· Point z_r on smith chart & extend the line upto periphery (on periphery mark l ($\simeq ?\lambda$)).

② Draw constant VSWR circle.
OZ_r as radius, draw a circle from centre.
· it will intersect, positive real axis at $\underline{4.2}$.
l 0.25λ i.e here, convert complex load impedance into real : exact value $= 4.2 \times 50 = 210\,\Omega$.

③ Find Z_0' of $\lambda/4$ line :

$Z_0' = \sqrt{Z_{in} Z_R} = \sqrt{50 \times 210} = 102.46\,\Omega \simeq 102\,\Omega$.

④ Move point (4.2) to (2.04) on centre line.
(due to new Z_0')

∴ Normalized impedance $= \dfrac{210}{102} = 2.04 \simeq 2$.

⑤ Draw constant VSWR circle and read the intersection point value after $\lambda/4$ length.

$[0.49] \simeq 0.5$.

⑥ Denormalize with char. imp of $\lambda/4$ line : $0.49 \times 102 = 50\,\Omega$.

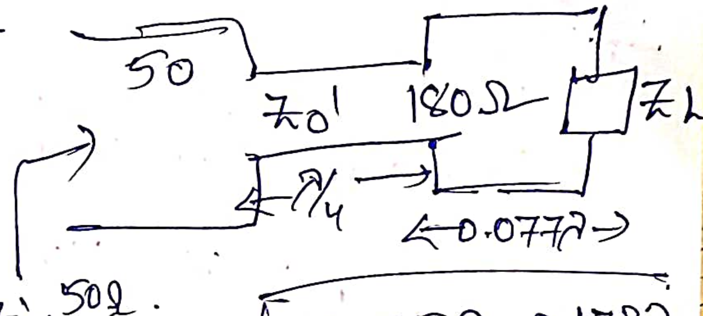
example 2:-

① $Z_L = 50 + j70\ \Omega$

$Z_0 = 50\ \Omega$

$z_l = 1 + 1.4j$

∴ Mark $l = 0.173\lambda$.



$0.25\lambda - 0.173\lambda$

0.077λ

② real $Z_{in} = 3.6$ @ 0.25λ.
draw circle
exact value $= 3.6 \times 50 = 180\ \Omega$.

③ $Z_0' = \sqrt{180 \times 50} = 95\ \Omega$.

④ $\frac{\cancel{180}}{95} \sim 2$. — Mark it on chart.

⑤ draw another (new) circle from 0 to 2 point as radius.
move $\lambda/4$ length. reach point (0.53)

⑥ denormalize
$0.53 \times 95 = 50.35\ \Omega$.

2

Single stub Impedance Matching:-

Stub:- It is a short section of the line which is connected to the Tx line to reduce reflections along the line.

→ The stub should be positioned at input admittance to be $1+jx$. (since stub is connected in parallel (or) shunt to Tx line).

→ The length of short circuited stub should be such that ~~sus~~ susceptance of stub should be $-jx$.

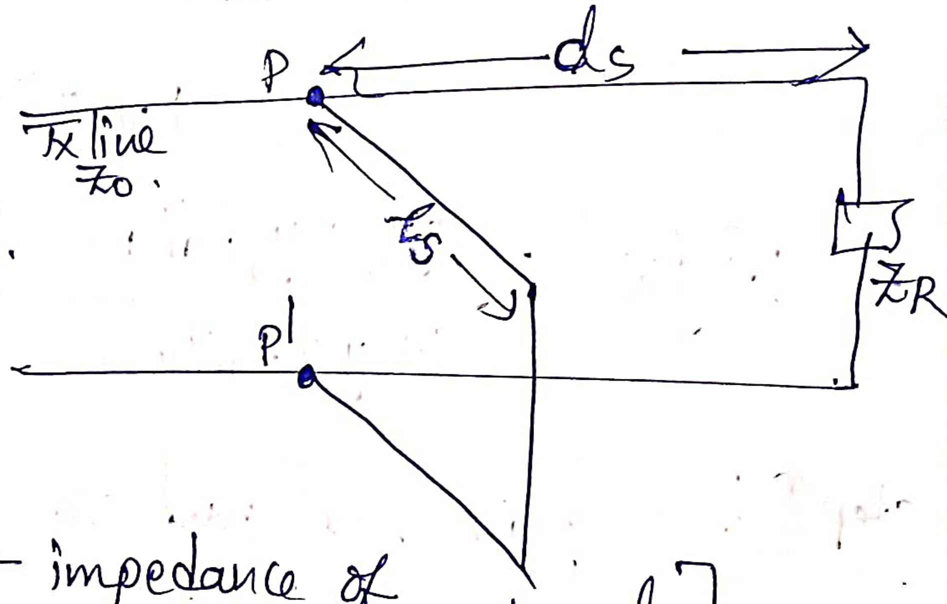
So, Total admittance at PP' : $y = y_1 + y_2 = 1 + jx - jx = 1$

From fig:

where,

d_s – distance of the stub from load.

l_s – length of the stub.



→ Consider, the input impedance of lossless Tx line

$$Z_{IN} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l} \right]$$

Let $z_{in} = \frac{Z_{IN}}{Z_0}$ → normalized input impedance

$z_r = \frac{Z_R}{Z_0}$ → normalized load impedance.

$$\frac{Z_{IN}}{Z_0} = \frac{\frac{Z_R}{Z_0} + j\tan\beta l}{1 + j\frac{Z_R}{Z_0}\tan\beta l}$$

$$z_{in} = \frac{z_r + j\tan\beta l}{1 + jz_r\tan\beta l}$$ — Normalize i/p impedance

Consider, admittance (since parallel stub)

$$\frac{1}{y_{in}} = \frac{\frac{1}{y_r} + j\tan\beta l}{1 + j\frac{1}{y_r}\tan\beta l} = \frac{1 + jy_r\tan\beta l}{y_r + j\tan\beta l}.$$

$$\Rightarrow y_{in} = \frac{(y_R + j\tan\beta l)}{(1 + jy_R\tan\beta l)} \times \frac{(1 - jy_r\tan\beta l)}{(1 - jy_r\tan\beta l)}$$

$$= \frac{y_R - jy_r^2\tan\beta l + j\tan\beta l + y_r\tan^2\beta l}{1 + y_R^2\tan^2\beta l}$$

$$\Rightarrow 1 + jX = \frac{y_R + y_r\tan^2\beta l}{1 + y_R^2\tan^2\beta l} + \frac{j(\tan\beta l - y_R^2\tan\beta l)}{1 + y_r^2\tan^2\beta l}.$$

step ①:- To find location of stub:- Equate real part to 1 ($l = d_s$) (d_s - location of stub).

$$\frac{y_R + y_R\tan^2\beta d_s}{1 + y_R^2\tan^2\beta d_s} = 1$$

$$y_R + y_R\tan^2\beta d_s = 1 + y_R^2\tan^2\beta d_s.$$

$$y_R\tan^2\beta d_s(1 - y_R) = (1 - y_R)$$

$$y_R \tan^2 \beta d_s = 1$$

$$\Rightarrow \tan^2 \beta d_s = \frac{1}{y_R} = z_R = \frac{Z_R}{Z_0}$$

$$\Rightarrow \tan \beta d_s = \sqrt{\frac{Z_R}{Z_0}}$$

$$\Rightarrow \beta d_s = \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

$$\Rightarrow \boxed{d_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}} \rightarrow \text{location of stub expression.}$$

step(2): To find length of stub:-

→ Consider SC input impedance $Z_{SC} = jZ_0 \tan \beta l$ (since stub is short circuited at its end).

$$\frac{Z_{SC}}{Z_0} = j \tan \beta l = z_{SC} \text{ — normalized SC input impedance.}$$

$$\therefore \frac{1}{z_{SC}} = \frac{1}{j \tan \beta l} \Rightarrow \boxed{y_{SC} = -j \cot \beta l}$$

Equate this admittance of stub to imaginary part of main Tx line

$$\cot \beta l_s = \frac{\tan \beta l_s - y_R^2 \tan \beta l_s}{1 + y_R^2 \tan^2 \beta l_s}$$

$$\cot \beta l_s = \frac{\tan \beta l_s (1 - y_R^2)}{1 + y_R^2 \tan^2 \beta l_s}$$

$$\Rightarrow \cot\beta l_s = \frac{\sqrt{\frac{Z_R}{Z_0}}\left(1-\frac{Z_0^2}{Z_R^2}\right)}{1+\frac{Z_0^2}{Z_R^2}\times\frac{\cancel{Z_R}}{\cancel{Z_0}}} = \frac{\cancel{Z_R}\sqrt{\frac{Z_R}{Z_0}}\left(\frac{Z_R^2-Z_0^2}{Z_R^2}\right)}{Z_R+Z_0}$$

$$\Rightarrow \cot\beta l_s = \frac{\frac{\cancel{(Z_R+Z_0)}(Z_R-Z_0)}{\sqrt{Z_0 Z_R}}}{\cancel{(Z_R+Z_0)}}$$

since
$\tan\beta l_s = \sqrt{\frac{Z_R}{Z_0}}$

$z_r = \frac{Z_R}{Z_0}$

$y_r = \frac{1}{z_r} = \frac{Z_0}{Z_R}$

$$\therefore \cot\beta l_s = \frac{Z_R - Z_0}{\sqrt{Z_R Z_0}}$$

$$\Rightarrow \tan\beta l_s = \frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}$$

$$\Rightarrow \beta l_s = \tan^{-1}\left(\frac{\sqrt{Z_R Z_0}}{(Z_R - Z_0)}\right)$$

$$\Rightarrow \boxed{l_s = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}\right)} \rightarrow \text{length of the stub.}$$

Note:-

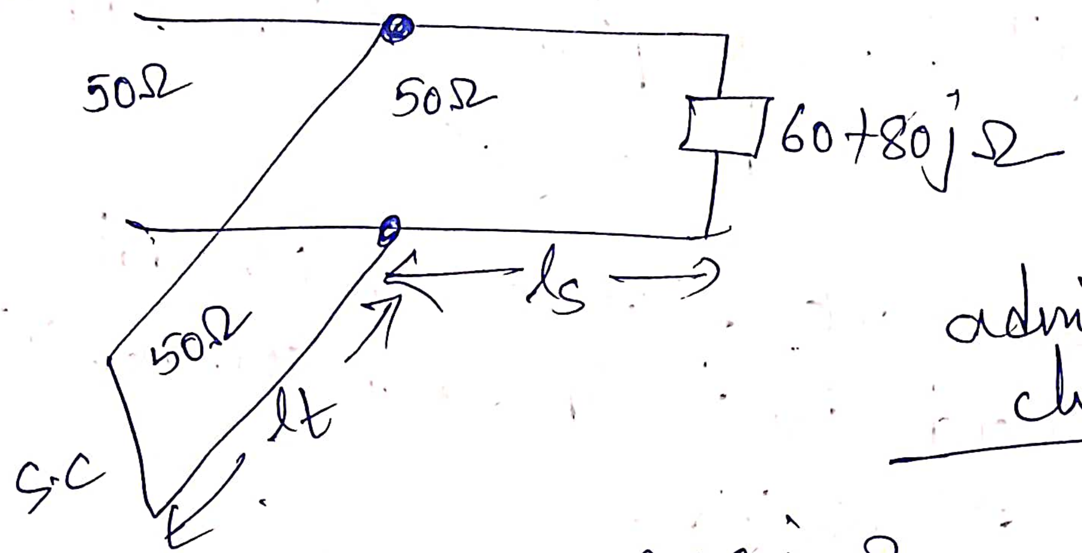
Position of stub $d_s = \frac{\lambda}{2\pi}\left[\phi + \pi - \cos^{-1}(|\Gamma|)\right]$

length of stub $l_s = \frac{\lambda}{2\pi}\tan^{-1}\left(\frac{\sqrt{1-|\Gamma|^2}}{2|\Gamma|}\right)$

in terms of reflection coefficient.

Single stub Matching using smith chart: ⑤

Given.



admittance chart

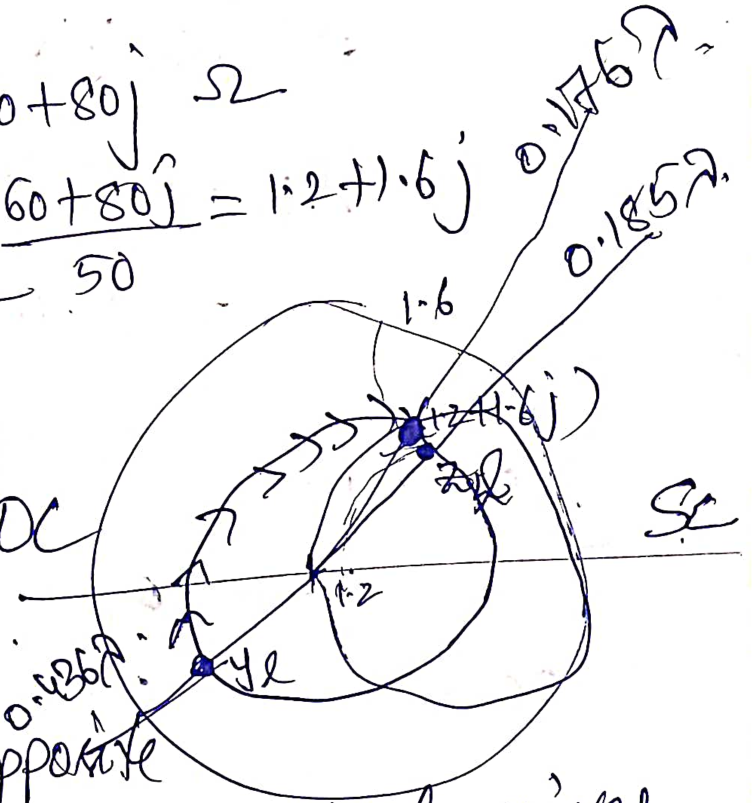
① Normalize load $Z_L = 60+80j\ \Omega$

$$z_l = \frac{60+80j}{50} = 1.2+1.6j$$

Mark it on smith chart

② draw a line o to z_l

③ Draw a VSWR circle OC from o to ($0z_L$) as radius.



- Now extend $0z_l$ line opposite direction which intersects VSWR circle gives load admittance (y_l).

④ From load point (y_l) move towards generator along the circle untill it intersects unit circle

$l_s = 0.176 - 0.436\lambda = -0.260\lambda$
$= -0.260 + 0.5\lambda$
$= 0.240\lambda$.

$l_s = 0.5\lambda - 0.436\lambda$
$= 0.064\lambda$ (or)
$= 0.064\lambda + 0.176\lambda = 0.240\lambda$

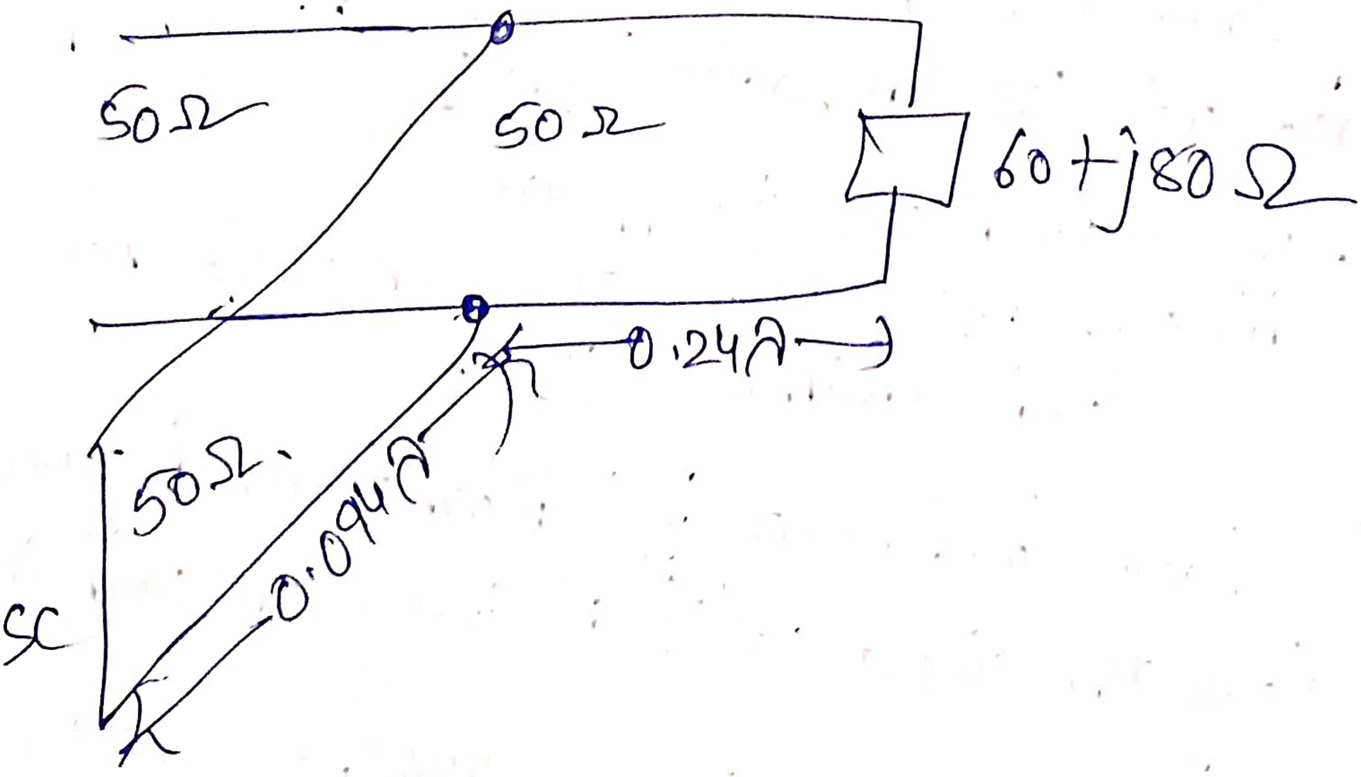
(5) Now we reached a point $1+1.5j$

To get 1, susceptance of the stub should be $-j1.5$.

(6) Now mark $-j1.5$ on smith chart & extend line to it from center, cutting at 0.344λ.

$$l_t = 0.344\lambda - 0.25\lambda$$
$$= 0.094\lambda.$$

Solution:-

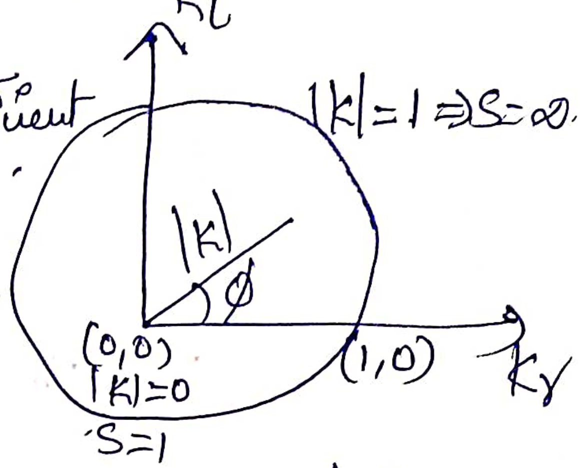


Z.

Smith chart:-

→ In 1939, phillip Hagar smith — an electrical engineer at bell telephone laboratories invented smith chart.

→ It is a graphical representation of reflection coefficient in complex plane. (or) It is a Polar plot of r & x-circles in the complex reflection coefficient plane.



Normalized impedance $(Z_r) = \dfrac{Z_R}{Z_0} = \dfrac{50+j100}{100}$

$= 0.5+j1$

$= r+jx$

$K = |K|\angle\phi$

$|K| \leq 1$.

∴ it consists 2 orthogonal circles

(i) r – circles (resistance)

(ii) x – circles (reactance)

Let $K = \dfrac{Z_R - Z_0}{Z_R + Z_0} = \dfrac{\frac{Z_R}{Z_0} - 1}{\frac{Z_R}{Z_0} + 1} = \dfrac{Z_r - 1}{Z_r + 1}$

$\Rightarrow Z_r = \dfrac{1+K}{1-K}$ (since K is complex quantity)

Let $K = K_r + jK_x$

$$\boxed{\therefore Z_r = \frac{1+K_r+jK_x}{1-K_r-jK_x}}$$ — Basis for construction of r & x-circles.

Constant Resistance Circles:-

$$Z_r = \frac{1+K}{1-K} = \frac{(1+K_r+jK_x)}{(1-K_r-jK_x)} \times \frac{(1-K_r+jK_x)}{(1-K_r+jK_x)}$$

$$= \frac{1-\cancel{K_r}+jK_x+\cancel{K_r}-K_r^2+\cancel{jK_xK_r} + jK_x-\cancel{jK_rK_x}-K_x^2}{(1-K_r)^2+K_x^2}$$

$$r+jx = \frac{1-K_r^2-K_x^2+2jK_x}{(1-K_r)^2+K_x^2}$$

equating real parts & imaginary parts

$$\boxed{r = \frac{1-K_r^2-K_x^2}{(1-K_r)^2+K_x^2}} \qquad \boxed{x = \frac{2K_x}{(1-K_r)^2+K_x^2}}$$

Consider, r - expression & multiply crossly

$$r\left[(1+K_r^2-2K_r)+K_x^2\right] = 1-K_r^2-K_x^2$$

$$r+rK_r^2-2rK_r+rK_x^2 = 1-K_r^2-K_x^2$$

$$r+(r+1)K_r^2+(r+1)K_x^2-2rK_r = 1$$

$$\Rightarrow (r+1)K_r^2+(r+1)K_x^2-2rK_r = 1-r$$

divide by $(r+1)$ term

$$K_r^2+K_x^2-\frac{2r}{r+1}K_r = \frac{1-r}{r+1}$$

$$\left(K_r - \frac{r}{r+1}\right)^2 - \left(\frac{r}{r+1}\right)^2 + K_x^2 = \frac{1-r}{r+1}$$

$$\left(K_r - \frac{r}{r+1}\right)^2 + K_x^2 = \frac{1-r}{r+1} + \frac{r^2}{(r+1)^2}$$

$$= \frac{1-\cancel{r}+\cancel{r^2}}{(r+1)^2} = \frac{1}{(r+1)^2}$$

$$\boxed{\therefore \left(K_r - \frac{r}{r+1}\right)^2 + K_x^2 = \frac{1}{(r+1)^2}}$$

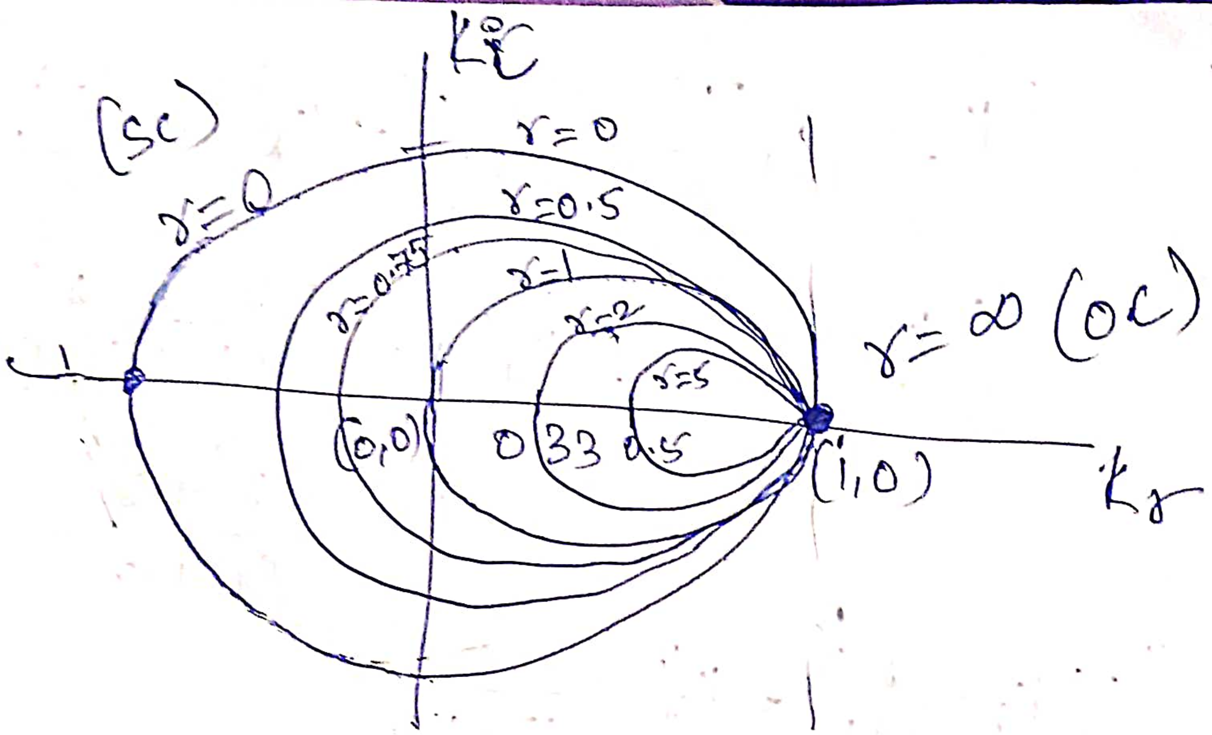
This is equation of the circle of standard form

$(x-h)^2 + (y-k)^2 = a^2$, with centre (h, k) & radius a

$$\boxed{\begin{aligned} \text{Center} &= \left(\frac{r}{r+1}, 0\right) \\ \text{Radius} &= \frac{1}{r+1} \end{aligned}}$$

→ For different values of 'r' constant various circles:

r	Centre $\left(\frac{r}{r+1}, 0\right)$	radius $\left(\frac{1}{r+1}\right)$
0	(0, 0)	1
0.5	(1/3, 0)	2/3
1	(1/2, 0)	1/2
2	(2/3, 0)	1/3
5	(5/6, 0)	1/6
∞	(1, 0)	0 — point/dot



Constant Reactance Circles:

Take $x = \dfrac{2K_x}{(1-K_r)^2 + K_x^2}$

$$(1-K_r)^2 + K_x^2 = \frac{2K_x}{x}$$

$$(1-K_r)^2 + K_x^2 - \frac{2K_x}{x} = 0$$

$$(1-K_r)^2 + \left(K_x - \frac{1}{x}\right)^2 - \frac{1}{x^2} = 0.$$

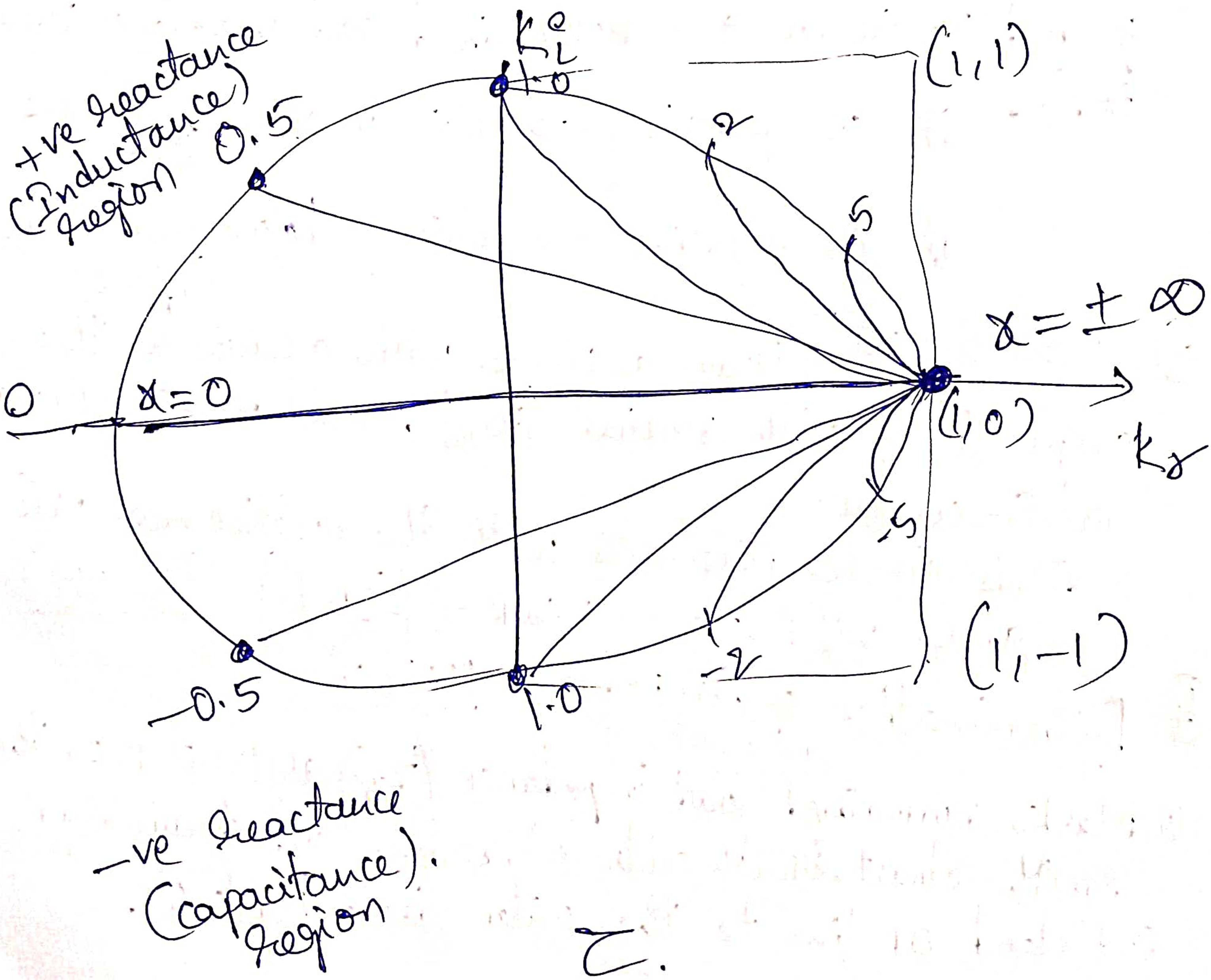
$$(1-K_r)^2 + \left(K_x - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \longrightarrow \text{Equation of circle.}$$

$$\boxed{\begin{aligned} \text{Center} &= \left(1, \frac{1}{x}\right) \\ \text{radius} &= \frac{1}{x} \end{aligned}}$$

Note:- 'x' may be either +ve (or) −ve.

→ For different values of x, various circles are:

x	centre $(1, \frac{1}{x})$	radius $(\frac{1}{x})$
0	$(1, \infty)$	∞
$\pm 1/2$	$(1, \pm 2)$	2
± 1	$(1, \pm 1)$	1
± 2	$(1, \pm 1/2)$	$1/2$
± 5	$(1, \pm 1/5)$	$1/5$
$\pm \infty$	$(1, 0)$	0



Z.

Properties of Smith chart :-

① **Normalisation of impedance** :- The smith chart represents the normalised values of R and X circles.

if $Z_R = R_R + jX_R$ — load impedance } of a lossless line
Z_0 = characteristic impedance

then normalized values are $r = \frac{R_R}{Z_0}$ & $x = \frac{X_R}{Z_0}$

- Again, to obtain actual values, normalized values should be multiplied by Z_0.

② **Load Impedance plot** :- The intersection points of R and X-circles give normalised load impedance values (Z_R).

if X-positive → point is above real axis ($\Gamma_R = 0$).

if X-negative → point is below real axis. ($\Gamma_R = 0$)

③ **VSWR plot** :- Draw a circle with a centre at the origin (0, 0) with radius $|Z_R|$, called as VSWR circle & S-circles.

- This circle intersects with the positive real axis gives VSWR. $\left(VSWR = \left|\frac{Z_R}{Z_0}\right|\right)$ $\boxed{S = OM}$

④ **Determination of an input impedance point** :-

① Mark normalised load impedance (Z_R) as point P on the smith chart. ② With centre (O), radius (OP) draw VSWR circle.
③ Extend OP line to the outer circle (Mark Q).

④ Rotate towards the generator (clockwise) upto a length l/λ. (Mark T)

⑤ Draw a line OT which cuts VSWR circle at point gives z_{in} (normalised input impedance) (Mark N)

⑥ The angle NOP gives electric length (βl) of the line.

⑤ **Determination of admittance point:-**

$$\frac{Z_{in}}{Z_0} \times \frac{Z_R}{Z_0} = 1 \quad \text{(or)} \quad z_{in} \times z_r = 1$$

$$\Rightarrow \boxed{y_r = z_{in}} \quad \left(\text{since } y_r = \frac{1}{z_r}\right)$$

- After locating z_{in} (impedance point), rotate it to a distance $\lambda/4$ (quarter wave) towards generator (clockwise)

∴ As the $\lambda/4$ distance gives opposite point on chart, point opposite to impedance point on circle gives the admittance point.

⑥ **Determination of the load impedance:-** Given VSWR & location of 1st Vmin from load.

Consider a Tx line of length (l), Then,

- To locate the load impedance:

→ Draw S-circle with centre 'O' and radius VSWR.

→ Locate point (A) on the outer circle at the left side end of the horizontal axis as position of Vmin.

→ Move towards the load (clockwise) to a given length l/λ on the wavelength scale and locate the point P'. Draw the line OP' which cuts S-circle at P.

→ The location of point P gives normalised load impedance

⑦ Input impedance & admittance of a SC (short-circuited) line:

→ Z_{in} of an SC line is purely reactive & R = 0.

→ The SC termination represents position of V_{min} i.e point A on outer circle at left side end.

→ From point A, move towards generator (anti-clock wise) to a given length l/λ on wavelength scale & locate the point P, gives normalised input impedance of the SC line.

→ The opposite point Q gives the normalised input admittance.

⑧ Input impedance & admittance of an OC (open-circuited) line

→ Z_{in} of OC line is purely reactive & $R = \infty$.

→ The OC termination represents position of V_{max}. i.e point B on outer circle at right side end.

→ From point B, move towards generator to a given length l/λ on wavelength scale & locate point (P)

→ P gives normalised input impedance of OC line

→ The opposite point Q gives normalised input admittance.

⑨ Determination of locations & lengths of stubs:

→ Impedance matching can be easily done by using smith charts.

→ locations & lengths of stubs (single/double) can be obtained by locating the admittances on the chart.

→ Since the stubs are connected in parallel, it is much easier to combine admittance in parallel than impedances.

→ Also, SC stubs are preferred over OC stubs to avoid radiation losses.

Applications of Smith Chart:-

① Smith chart as an admittance diagram:-

→ Generally, smith chart is used as an impedance diagram, Z obtained from intersection of R & X circles.

→ Also, smith chart is used for admittance:

- Normalised admittance $Y = G + jB$

where $G = \frac{1}{R}$ normalised conductance.

$B = \frac{1}{X}$ normalised susceptance.

∴ At termination, $y_r = \frac{1}{Z_r} = \frac{1}{R+jX} = \frac{R-jX}{R^2+X^2}$

$$\therefore G - jB = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2}$$

- Also $y_r = \frac{1}{Z_r} = \frac{1-\Gamma}{1+\Gamma}$

$$\therefore G - jB = \frac{1-\Gamma}{1+\Gamma}$$

The G and B circles on Γ-plane can be drawn similar to R and X circles.

→ A Smith chart with G and B circles is called an admittance diagram.

∴ The admittance diagram is the mirror image of the impedance diagram ; all measurements will be taken in reverse direction.

Converting impedance into admittance:

→ For a lossless quarter wave transformer, the input impedance is $Z_{in} = \frac{Z_0^2}{Z_R}$.

② Reflection coefficient values:

→ Draw the line OP & extend it to outer circle, cuts at Q.

∴ Magnitude of reflection coefficient is given by

$$|\Gamma| = \frac{OP}{OQ}$$ (OP, OQ measure using scale).

(or)

$|\Gamma|$ can also be obtained from the Γ-scale provided in the chart.

→ The angles are indicated on the outer circle.

• The angle (ϕ) of the line OP (w.r.t +ve real axis) gives angle of reflection coefficient.

③ Location of Vmax & Vmin :-

There are 2 intersection point of VSWR circle with the horizontal axis/Real axis AB.

∴ V_{min} → The point at the left side of the centre represents voltage minima (V_{min}). (Point L)

V_{max} → The point at right side of the centre represents voltage maxima (V_{max}) (Point M).

→ whereas, the locations can be obtained from wavelength scale on outer circle.

arc AQ → gives distance of V_{min} from the load.

arc QAB → gives distance of 1st V_{max} from load.

④ Open & short-circuited lines:-

→ At point 'B' on right side end of horizontal axis, both $R = X = \infty$ ⇒ OC line.

→ At point 'A' on left side end of horizontal axis both $R = X = 0$ ⇒ SC line.

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UNIT II

Problems :

Given that

Open-circuited impedance, $Z_{oc} = 750\ \Omega$.

Short-circuited impedance, $Z_{sc} = 500\ \Omega$.

Find characteristic Impedance (Z_o) :

The characteristic impedance of a line is given by

$$Z_0 = \sqrt{Z_{oc} \times Z_{sc}} = \sqrt{750 \times 500} = \sqrt{375000}.$$

The characteristic impedance is $Z_0 = 612.37\ \Omega$.

Example 10.13 $Z_{oc} = 900\angle{-30°}\Omega$, $Z_{sc} = 400\angle{-10°}\Omega$. Calculate the Z_0 and γ of a 12 km long line.

Solution Given: $Z_{oc} = 900\angle{-30°}\Omega$, $Z_{sc} = 400\angle{-10°}\Omega$.

a) The characteristic impedance is given by

$$Z_0 = \sqrt{Z_{oc} \times Z_{sc}} = \sqrt{900\angle{-30} \times 400\angle{-10}} = 600\angle{-20°}\ \Omega.$$

Hence, the characteristic impedance, $Z_0 = 600\angle{-20°}\ \Omega$.

We know that $\tanh \gamma l = \sqrt{\dfrac{Z_{sc}}{Z_{oc}}}$.

$$\tanh \gamma l = \sqrt{\frac{400\angle{-10}}{900\angle{-30}}} = 0.67\angle 10.$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = 0.66 - j0.116.$$

By componendo and dividendo principle,

$$\frac{2e^{\gamma l}}{2e^{-\gamma l}} = \frac{1 + 0.66 - j0.116}{1 - (0.66 - j0.116)}.$$

$$e^{2\gamma l} = 4.269 - j1.798 = 4.632\angle{-22.8}.$$

Componendo & dividendo rule:

if $\dfrac{a}{b} = \dfrac{c}{d}$ then

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}.$$

Taking ln of both sides,

$$2\gamma l = \ln(4.632\angle{-22.8°}).$$

$$\gamma = \frac{1}{2l}(\ln(4.632) + j(-22.8°))$$

or $$\gamma = \frac{1}{2 \times 12}(1.533 - j0.398).$$

$$\gamma = \alpha + j\beta = 0.0638 - j0.01658 \text{ napers/km}.$$

Example 10.14 A two wire line has a characteristic impedance of 300 Ω and is fed to a 90 Ω resistor at 200 MHz. A quarter wave line is used as a tube 0.25 cm in diameter. Find the centre-to-centre spacing in air.

Solution Given: $Z_1 = 300\ \Omega$, $Z_2 = 90\ \Omega$, tube diameter = 0.25 cm, radius r = 0.125 cm.

For a quarter wave line, we know that

$$Z_0 = \sqrt{Z_1 \times Z_2} = \sqrt{300 \times 90} = 164.32\ \Omega.$$

Also, we know that the characteristic impedance of a parallel wire line is,

$$Z_0 = 276 \log (d/r) \text{ ohms},$$

where d = centre-to-centre spacing.

$$\therefore \quad 164.32 = 276 \log \left[\frac{d}{0.125}\right].$$

$$\log \left[\frac{d}{0.125}\right] = 0.595.$$

$$\frac{d}{0.125} = 3.936 \quad \text{or} \quad d = 0.49 \text{ cm}.$$

Example 10.15 A 60 ohm lossless line is connected to a source with 10 V, $Z_g = 50 - j40$ and terminated with a load of $j40$ ohms. If the line is 100 m long and $\beta = 0.25$ rad/m, calculate Z_{in} and voltage at (i) the sending end, (ii) the receiving end, (iii) 4 m from the load end and (iv) 3 m from the source.

Solution Given:
Length, $l = 100$ m, characteristic impedance, $Z_0 = 60$ ohms, termination impedance, $Z_R = j40$ ohms, source resistance, $Z_g = 50 - j40$ ohms, source voltage, $V_g = 10$ V, $\alpha = 0$, $\beta = 0.25$ rad/m;

$$\text{input current is } I_s = \frac{V_g}{Z_0 + Z_g} = \frac{10}{60 + 50 - j40} = 85.4\angle 20 \text{ mA}.$$

We know that the source impedance for a lossless transmission line is

$$Z_s = Z_0 \left(\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}\right).$$

(i) At the sending end, $l = 100$ m, $\beta l = 0.25 \times 100 = 25 \text{ rad} = 1432.4°$.

$$Z_s = 60 \left(\frac{j40 + j60 \tan 1432.4}{60 + j^2 40 \tan 1432.4}\right) = j29.38\ \Omega.$$

$$\text{Input voltage is } V_s = Z_s I_s = j29.38 \times (85.4\angle 20) \times 10^{-3} = 2.5\angle 110 \text{ V}.$$

(ii) At the receiving end, $l = 0$.

$$Z_s = Z_R = j40\ \Omega.$$

$$V_R = Z_R I_s = j40 \times (85.4\angle 20) \times 10^{-3} = 3.416\angle 110 \text{ V.}$$

(iii) At 4 m from the load end: $l = 4$, $\beta l = 0.25 \times 4 = 1$ rad $= 57.3°$.

$$Z_1 = 60\left(\frac{j40 + j60\tan 57.3}{60 + j^2 40\tan 57.3}\right) = -j435.53\ \Omega.$$

$$V_1 = Z_1 I_s = -j435.53 \times 85.4\angle 20 \times 10^{-3} = 37.2\angle{-70°} \text{ V.}$$

(iv) At 3 m from the source end: $l = 97$, $\beta l = 025 \times 97. = 2425$ rad $= 1389.4°$.

$$Z_2 = 60\left(\frac{j40 + j60\tan 1389.4}{60 + j^2 40\tan 1389.4}\right) = -j0.303\ \Omega.$$

$$V_2 = Z_2 I_s = -j0.303 \times (85.4\angle 20) \times 10^{-3} = 0.0258\angle{-70°} \text{ V.}$$

Example 10.16 An open wire unloaded line, 75 km long, is operated at a frequency of 1000 Hz. The open circuit impedance is found to be $330\angle{-30°}\ \Omega$ and the short circuit impedance is $540\angle 7°\ \Omega$. Calculate the parameters of line.

Solution Given: Length of the unloaded line, $l = 75$ km, $f = 1000$ Hz, $Z_{oc} = 330\angle{-30°}\ \Omega$, $Z_{sc} = 540\angle 7°\ \Omega$.

We know that $Z_0 = Z_{sc} \times Z_{oc}$

$$= \sqrt{540\angle 7° \times 330\angle{-30°}} = 422.14\angle{-11.5}.$$

Also, $Z_{sc} = Z_0 \tanh \gamma l$.

$$\tanh \gamma l = \frac{Z_{sc}}{Z_0} = \frac{540\angle 7°}{422.14\angle{-23°}}.$$

$$= 1.28\angle 30° = 1.108 + j0.64.$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = 1.108 + j0.64.$$

By componendo and dividendo principle,

$$\frac{2e^{\gamma l}}{2e^{-\gamma l}} = \frac{1 + 1.108 - j0.64}{1 - 1.108 - j0.64}.$$

$$e^{2\gamma l} = \frac{2.108 + j0.64}{-0.108 - j0.64}.$$

$$e^{2\gamma l} = \frac{2.203\angle 16.89°}{0.649\angle{-99.58°}} = 3.394\angle 116.47°.$$

Taking ln of both sides,

$$2\gamma l = \ln(3.394\angle 116.47^\circ).$$

Since, $\ln(r\angle\theta) = \ln r + j\theta^\circ$,

$$\gamma = \frac{1}{2l}(\ln(3.394) + j(116.47^\circ))$$

$$= \frac{1}{2l}(1.222 + j2.033) = \frac{1.222 + j2.033}{150} = 0.00815 + j0.0135.$$

$$\gamma = 0.0157\angle 58.88^\circ.$$

$$\beta = 0.00815 \text{ rad/km} \quad \text{and} \quad \alpha = 0.0135 \text{ nepers/km}.$$

Also we know that $= R + j\omega L$.

$$R + j\omega L = 0.0157\angle 58.88^\circ \times 422.14\angle -10.5^\circ = 6.627\angle 47.38$$

$$R + j\omega L = 4.487 + j4.877.$$

$$R = 4.487 \ \Omega/\text{km} \quad \text{and} \quad \omega L = 4.877.$$

$$2\pi f \times L = 4.877.$$

$$L = \frac{4.877}{2\pi f} = \frac{4.877}{2 \times \pi \times 1000} = 0.776 \text{ mH/km}.$$

$$\frac{\gamma}{Z_0} = G + j\omega C.$$

$$G + j\omega C = \frac{0.0157\angle 58.88^\circ}{422.14\angle -10.5^\circ} = 3.72 \times 10^{-5}\angle 70.38^\circ$$

$$G + j\omega C = 12.5 \times 10^{-6} + j35 \times 10^{-6}.$$

$$G = 12.5 \ \mu\text{℧/km and} \quad \omega C = 3.5 \times 10^{-5}.$$

$$2\pi f C = 3.5 \times 10^{-5}.$$

$$C = \frac{3.5 \times 10^{-5}}{2\pi f} = \frac{3.5 \times 10^{-5}}{2 \times \pi \times 1000}$$

$$\therefore \quad C = 5.576 \text{ nF/km}.$$

Example 10.17 The input impedance of a short circuited lossy transmission line of length 2 m and characteristic impedance 75 Ω is $45 + j225$ Ω.

(a) Find α and β of a line.

(b) Determine the input impedance if the short circuit is replaced by a load impedance of $67.5 - j45$ Ω.

Solution Given: Input impedance of a short circuited line, $Z_{sc} = 45 + j225\ \Omega$, characteristic impedance, $Z_0 = 75\ \Omega$, length $l = 2$ m.

(a) We know that $Z_{sc} = Z_0 \tanh \gamma l$.

$$\tanh \gamma l = \frac{Z_{sc}}{Z_0} = \frac{45 + j225}{75} = 0.6 + j3.$$

$$\frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = 0.6 + j3.$$

By componendo and dividendo principle,

$$\frac{2e^{\gamma l}}{2e^{-\gamma l}} = \frac{1 + 0.6 - j3}{1 - 0.6 - j3}.$$

$$e^{2\gamma l} = \frac{1.6 + j3}{0.4 - j3} = 1.12\angle 114.3.$$

Taking ln of both sides,

$$2\gamma l = \ln(1.12\angle 144.3°).$$

Since, $\ln(r\angle\theta) = \ln r + j\theta°$,

$$\gamma = \frac{1}{2l}(\ln(1.12) + j(144.3°)) = \frac{0.1133 + j2.518}{2 \times 2} = 0.028 + j0.63.$$

$$\gamma = 0.0157\angle 58.88°;\ \beta = 0.63 \text{ rad/km and } \alpha = 0.028 \text{ nepers/km}.$$

(b) Given: Load impedance $Z_R = 67.5 - j45\ \Omega$.
We know that the input impedance is

$$Z_{in} = Z_0 \left(\frac{1 + Ke^{-2\gamma l}}{1 - Ke^{-2\gamma l}} \right),$$

where $K = \dfrac{Z_R - Z_0}{Z_R + Z_0} = \dfrac{67.5 - j45 - 75}{67.5 - j45 + 75} = 0.3\angle -82°.$

$$e^{2\gamma l} = 1.12\angle 114.3.$$

$$Z_{in} = 75 \left(\frac{1 + \dfrac{0.3\angle -82}{1.12\angle 144.3}}{1 - \dfrac{0.3\angle -82}{1.12\angle 144.3}} \right) = 75 \left(\frac{1 + 0.2678\angle 226.3}{1 - 0.2678\angle 226.3} \right) = 52.35\angle 22.66.$$

Input impedance is $Z_{in} = 48.3 + j20\ \Omega$.

Example 10.18 A dipole antenna is fed by a lossless transmission line having $Z_o = 60\ \Omega$. The source impedance is $600\ \Omega$. If the length of the line is 0.1λ, determine antenna impedance.

Solution Given: Length of the line $l = 0.1\lambda$, characteristic impedance, $Z_0 = 60\ \Omega$, source impedance $Z_S = 600\ \Omega$.

Let Z_R be the antenna impedance. We know that the source impedance for a lossless transmission line is

$$Z_S = Z_0\left(\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}\right),$$

or $$Z_R = Z_0\left(\frac{Z_S - jZ_0 \tan \beta l}{Z_0 - jZ_S \tan \beta l}\right).$$

Now $$\tan \beta l = \tan\left[\frac{2\pi}{\lambda}(0.1\lambda)\right] = \tan(0.2\pi) = 0.726.$$

$$\therefore \quad Z_R = 60\left[\frac{600 - j60(0.726)}{60 - j600(0.726)}\right] = 60\left[\frac{600 - j43.56}{60 - j435.6)}\right]$$

$$= 17.06 + j80.3 = 82.09\angle 78^\circ.$$

$\therefore$ Antenna impedance is $Z_R = 82.09\angle 78^\circ$.

Example 10.19 A transmission line of 100 Ω characteristic impedance is connected to a load of 400 Ω. Calculate the reflection coefficient and standing wave ratio.

Solution Given: Characteristic impedance, $Z_0 = 100\ \Omega$, load, $Z_R = 400\ \Omega$.

We know that reflection coefficient K is given by

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{400 - 100}{400 + 100}.$$

$\therefore$ Reflection coefficient, $K = 3/5 = 0.6$.

The standing wave ratio is $\text{VSWR} = \dfrac{1 + |K|}{1 - |K|} = \dfrac{1 + 0.6}{1 - 0.6} = \dfrac{1.6}{0.4} = 4.$

Example 10.20 An UHF transmission line of $Z_0 = 150$ ohms is terminated with an unknown load. The VSWR measured in the line is 5 and the position of minimum current nearest the load is one-fifth wavelength away. Calculate the value of the load impedance.

Solution Given: Characteristic impedance, $Z_0 = 150$, VSWR $= S = 5$, position of minimum current $y_{max} = \lambda/5$.

Position of the first minimum current or voltage maximum can be obtained by

$$2\beta y_{max} - \phi = 0.$$

$$2.\frac{2\pi}{\lambda}.\frac{\lambda}{5} - \phi = 0$$

or $\quad \phi = \dfrac{4\pi}{5} = 2.51$ radians or 144°.

We know that the magnitude of the reflection coefficient is

$$|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = 0.666.$$

The reflection coefficient K is

$$K = |K|e^{j\phi} = 0.667\angle 144°.$$

Also we know that $K = \dfrac{Z_R - Z_0}{Z_R + Z_0}$.

$$0.667\angle 144° = \frac{Z_R - 150}{Z_R + 150}$$

$$-0.5388 + j0.3916 = \frac{Z_R - 150}{Z_R + 150}$$

$$Z_R(-0.5388 + j0.3916 - 1) = (-150 + 150 \times 0.5388 - j150 \times 0.3916)$$

$$Z_R(-1.5388 + j0.3916) = -69.18 - j58.74$$

$$Z_R = \frac{90.753\angle -139.66}{1.588\angle 165.72}.$$

The load impedance, $\quad Z_R = 57.15\angle -305.38$ or $Z_R = 57.15\angle 54.62\ \Omega$.

Example 10.21 An open wire transmission line having $Z_0 = 650\angle -12°\ \Omega$ is terminated in Z_0 at the receiving end. If this line is supplied from a source of internal resistance 300 Ω, calculate the reflection factor and reflection loss at the sending end terminals.

Solution Given: $Z_i = 300\ \Omega$, $Z_0 = 650\angle -12°\ \Omega$.
We know that the reflection factor is

$$K_f = \frac{2\sqrt{Z_i Z_0}}{(Z_i + Z_0)}.$$

$$\therefore \quad K_f = \frac{2\sqrt{300 \times 650\angle -12°}}{(300 + 6358 - j135)} = \frac{883\angle -6}{945.5\angle -8.22} = 0.934\angle 2.22°.$$

The reflection loss,

$$K_e = 20\log_{10}\left|\frac{1}{K_f}\right| = 20\log\left(\frac{1}{0.934}\right).$$

$$K_e = 0.59 \text{ dB}.$$

$\therefore$ The reflection loss is 0.59 dB.

Example 10.22 A 80 Ω distortionless line connects a signal of 50 kHz to a load of 140 Ω. The load power is 75 mW. Calculate:

(i) Voltage reflection coefficient,

(ii) VSWR,

(iii) Position of V_{max}, I_{max}, V_{min} and I_{min}.

Solution Given: $Z_R = 140\ \Omega$, $Z_0 = 80\ \Omega$.

(i) Voltage reflection coefficient

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{140-80}{140+80} = 0.273.$$

$$\therefore \quad K = 0.273, \quad \phi = 0.$$

(ii) VSWR

$$\text{VSWR} = \frac{1+|K|}{1-|K|} = \frac{1+0.273}{1-0.273} = 1.75.$$

(iii) Position of V_{max}, I_{max}, V_{min} and I_{min}

The condition for maximum voltage occurs at y_{max} from the load, i.e.,

$$2\beta y_{max} - \phi = 2n\pi.$$

For first maximum, $n = 0$,

$$2\beta y_{max} - \phi = 0$$

$$2\beta y_{max} - 0 = 0, \quad y_{max} = 0.$$

Therefore, the first voltage maximum, V_{max}, and current minimum, I_{min}, occur at the load position.

The first voltage minimum, V_{min}, occurs at a distance of $\lambda/4$ from V_{max}.

$$\therefore \quad y_{min} = y_{max} + \lambda/4 = 0 + \lambda/4 = \lambda/4.$$

We know that the wavelength, $\lambda = \dfrac{v_0}{f} = \dfrac{3\times 10^8}{50\times 10^3} = 6$ km.

$$\therefore \quad y_{min} = \frac{v_0}{4\times f} = \frac{3\times 10^8}{4\times 50\times 10^3} = 1.5 \text{ km}.$$

Therefore, the first voltage minimum and current maximum, I_{max} occur at a distance of 1.5 km from the load. These values repeat every $\dfrac{\lambda}{2} = 3$ km distance from the load.

Given: The power at the load is $P = 75$ mW.

We know that $P = \frac{V_{max}^2}{ZR}$ and $I_{min} = \frac{V_{max}}{Z_R}$.

V_{max} at the load is $\sqrt{P \times Z_R} = \sqrt{75 \times 10^{-3} \times 140} = 3.24$ volts and $I_{min} = \frac{3.24}{140} = 0.23$ mA.

Example 10.23 Design a quarter wave transformer to match a line having impedance of 300 Ω to load of 600 Ω.

Solution Given: $Z_0 = 300\ \Omega$, $Z_L = 600\ \Omega$.

The quarter wave transformer should have a sending end impedance of

$$Z_S = \frac{Z_0^2}{Z_L} = \frac{300 \times 300}{600} = 150\ \Omega.$$

The impedance will be matched if the Z_S of a $\lambda/4$ transformer is 150 Ω.

Example 10.24 Calculate the reflection coefficient and VSWR of a 50 Ω line terminated with (i) matched load, (ii) short circuit, (iii) $+j50\ \Omega$ load, (iv) $-j50\ \Omega$ load.

Solution

(i) Matched load

$$Z_0 = 50\ \Omega, \quad Z_R = 50\ \Omega.$$

Reflection coefficient $K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{50-50}{50+50} = 0$

and VSWR $S = \frac{1+|K|}{1-|K|} = \frac{1+0}{1-0}$

$\therefore \quad S = 1$

(ii) Short circuit

$$Z_R = 0,\ Z_0 = 50\ \Omega.$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{0-50}{0+50} = -1.$$

$$S = \frac{1+|K|}{1-|K|} = \frac{1}{0} = \infty.$$

(iii) $+j50\ \Omega$ load

$\therefore \quad Z_R = +j50\ \Omega,\ Z_0 = 50\ \Omega.$

$$K = \frac{+j50 - 50}{j50 + 50} = \frac{-1+j}{1+j} = \frac{1.414\angle 135^\circ}{1.414\angle 45^\circ}$$

$\therefore \quad K = 1\angle 90°.$

$$S = \frac{1+|K|}{1-|K|} = \frac{1+1}{1-1} = \frac{2}{0} = \infty.$$

(iv) $-j50\ \Omega$ load

$\therefore \quad Z_R = -j50\ \Omega,\ Z_0 = 50\ \Omega.$

$$K = \frac{-j50-50}{j50+50} = \frac{1.414\angle -135°}{1.414\angle -45°} = 1\angle -90°.$$

$$\therefore \quad S = \frac{1+1}{1-1} = \frac{2}{0} = \infty.$$

Example 10.25 An aerial of $(200 - j300)\ \Omega$ is to be matched with $500\ \Omega$ lines. The matching is to be done by means of a low loss $600\ \Omega$ stub line. Find the position and length of the stub line used if the operating wave length is 20 metres.

Solution Given: $Z_R = 200 - j300\ \Omega = 360.55\angle -56.31,\ Z_0 = 500\ \Omega,\ \lambda = 20$ m.

We know that the reflection coefficient is

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{200 - j300 - 500}{200 - j300 + 500} = \frac{-300 - j300}{700 - j300} = 0.2068 - j0.5172.$$

$\therefore \quad K = 0.557\angle -110.8.$

So, $|K| = 0.557$ and $\phi = -110.8°.$

We know that the position of the stub line l_s in terms of the reflection coefficient is

$$l_s = \frac{\lambda}{2\pi}(\phi + \pi - \cos^{-1}(|K|).$$

Substituting the values of λ, ϕ and $|K|$, we get,

$$l_s = \frac{20}{2\pi}(-110.8 + \pi - \cos^{-1}(0.557))$$

$$= \frac{10}{180}(-110.8 + 180 - 56.15) = \frac{10 \times 12.05}{180} = 0.669.$$

The stub position from the aerial is

$$l_s = 0.669 \text{ metres.}$$

Also the length of the stub line l_s in terms of the reflection coefficient $|K|$ is

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}.$$