UNIT-I

Transmission Lines-I

→ Energy can be transmitted in 2 ways:

① wireless transmission:
By the radiation of free electromagnetic (EM) waves, the energy can be transmitted from one place to another.

② Wire transmission:-
By using various conductor arrangements known as transmission line, the energy can be transmitted from one place to another.

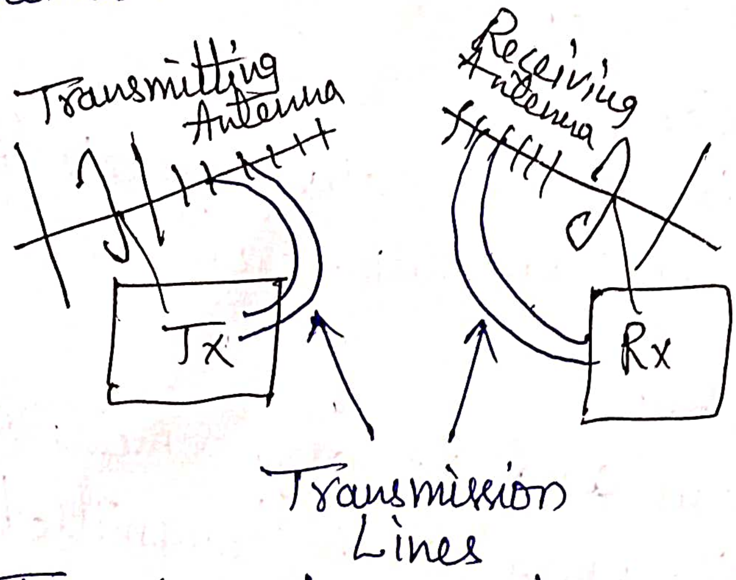
Definition of Transmission line:-
Transmission line is a conductive method for guiding Electromagnetic (EM) energy from one place (Transmitter) to another place (Receiver).

Importance of transmission lines:-

① The generation & transmission of electromagnetic (EM) radiation at low frequencies will require directive antennas of larger dimensions. It is very expensive.

② If the wave of electromagnetic is not guided, so much of power is wasted due to the radiation losses. So, for efficient transmission EM energy should be guided.

③ Transmission lines are used as a link between the Antenna to a transmitter (or) Receiver.



④ Transmission lines used as power distribution section at low frequencies (high power) and communications at higher frequencies (low power).

⑤ Transmission lines acts as circuit elements such as Resistors, capacitors, inductors, resonators etc.

⑥. Transmission lines are used for "impedance matching" for maximum power transfer.

⑦ They are also used as filters, transformers and even insulators at very high frequencies.

⑧ They also used as measuring devices in electrical laboratories.

Z.

Types of Transmission Lines :-

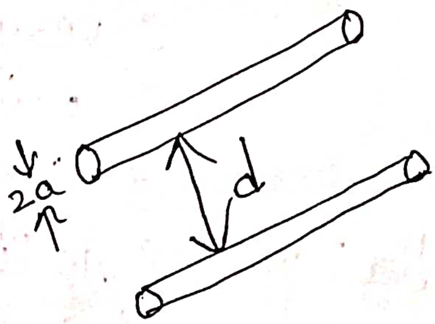
→ Various types of guiding structures are :

① parallel wire line (or) open wire lines
② parallel-plate (or) planar transmission lines
③ Twisted wires
④ wire above conducting plane
⑤ coaxial cables.
⑥ Waveguides
⑦ optical fibers
⑧ Microstrip lines
⑨ strip lines.

① parallel-wire lines (or) open-wire lines:-

→ A transmission line which consists of 2-parallel conductors separated at a uniform distance of 'd', in which each wire has a radius of 'a', as shown below fig①:

→ These lines are most common form of Tx lines, are mounted on posts (or) on towers.



fig①:- Two-wire parallel lines.

→ when a voltage is applied between these lines, the signal current travels down one wire and returns to the source on the other wire, as shown in fig②.

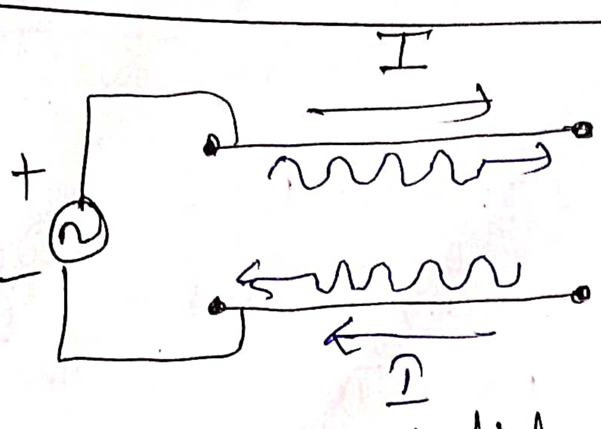


fig ②:- potential difference (V)

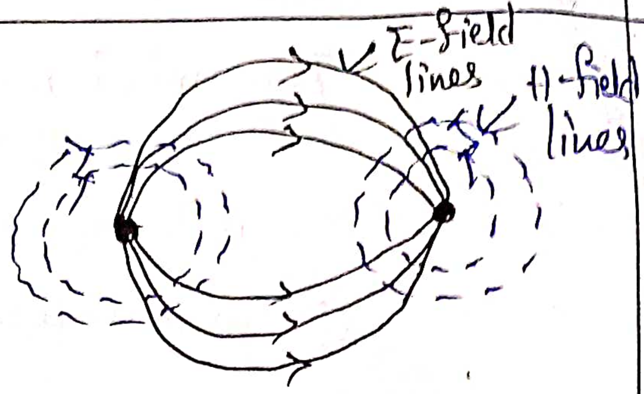


fig ③: Field lines (E&H)

→ when electrical energy propagating through these lines setup electric field (E) between them, in turn creates magnetic field (H), as shown in fig ③.

∴ Both E and H are at right angles to eachother and to the direction of propagation, known as Transverse electro-magnetic (TEM) mode of propagation.

→ These open-wire lines are suitable for frequencies upto 100MHz.

Advantages:-
1. simple structure
2. very easy to construct
3. easy to maintain over short distances
4. cheaper.

Disadvantages:-
1. As it is open transmission line structure, it is capable of getting electromagnetic interference.
2. above 100MHz, there will be more radiation losses (as Tx lines become unstable)
3. As distance increases, cost increases.

Examples/Applications:- power lines, telephone line, telegraph lines etc.

2 Twisted pair of wires :-

→ These are laid underground and consists of 2 (or) more conductors, individually insulated with paper and all these conductors are periodically twisted in pairs & placed in a protective lead shield as shown in fig:

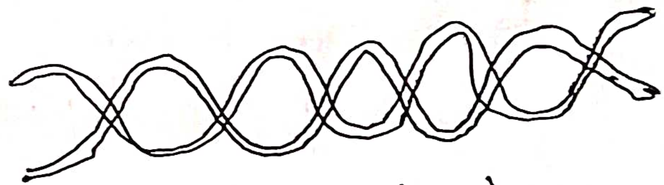


fig:- Twisted wires

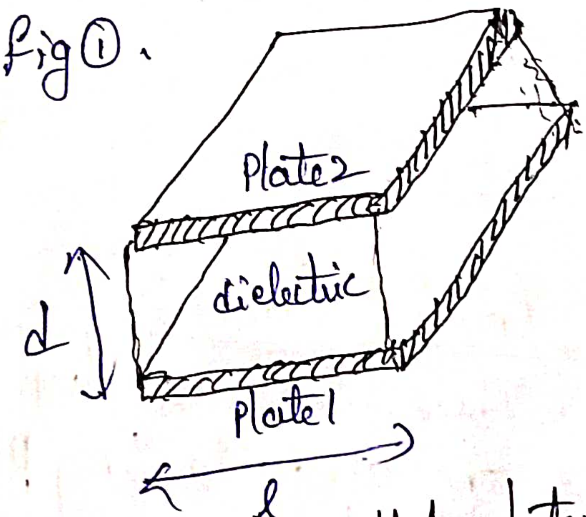
Advantage:- To overcome the electromagnetic interference of open-wire lines, these wires are shielded.

example/Application:- power cables.

3 Parallel-plate transmission lines:-

→ These lines consists of two parallel conducting plates of length (l) separated by a dielectric slab of an uniform thickness (d) as shown below fig ①.

Advantage: These lines are used at microwave frequencies, can be fabricated inexpensively using PCB technology. (Printed-circuit board)



fig①: parallel-plates.

→ Consider, the cross section view as well as the field patterns of a parallel-plate Tx line as shown in fig②.

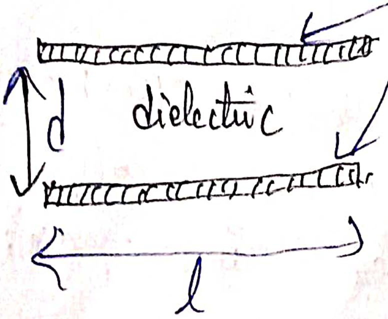
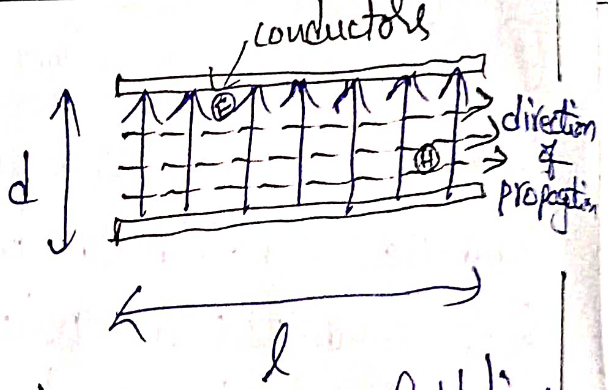


fig (2a): cross-section view of parallel-plates



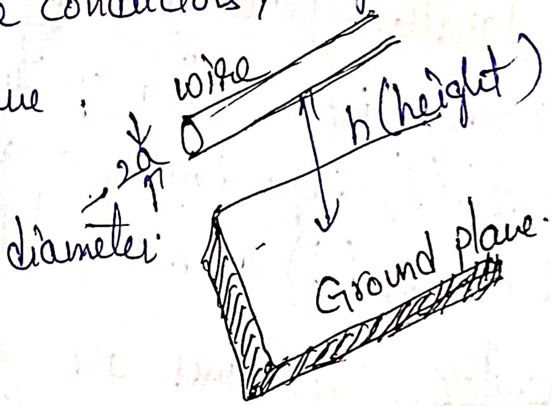
here, ⟶ E-field lines
- - - -> H-field lines

fig (2b): field patterns.

<u>example/Application</u>:- These plates are used to connect the transmitters & antennas.

④ <u>wire above the ground plane</u>:

→ It consists of a single conductor, is place above a conducting (Ground) plane.



⑤ <u>Coaxial Cables</u>:-

→ These cables employs 2-conductors placed coaxially i.e one conductor is placed coaxial within another hollow conductor as shown in fig:

→ The volume between these 2-conductors is filled with dielectric material (ex:- polyethylene)

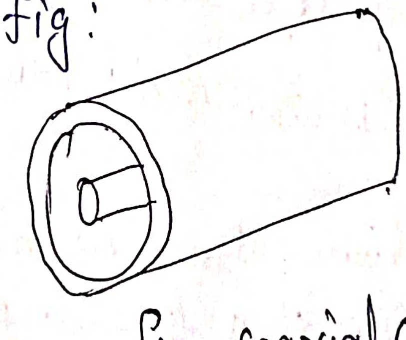


fig: coaxial cable

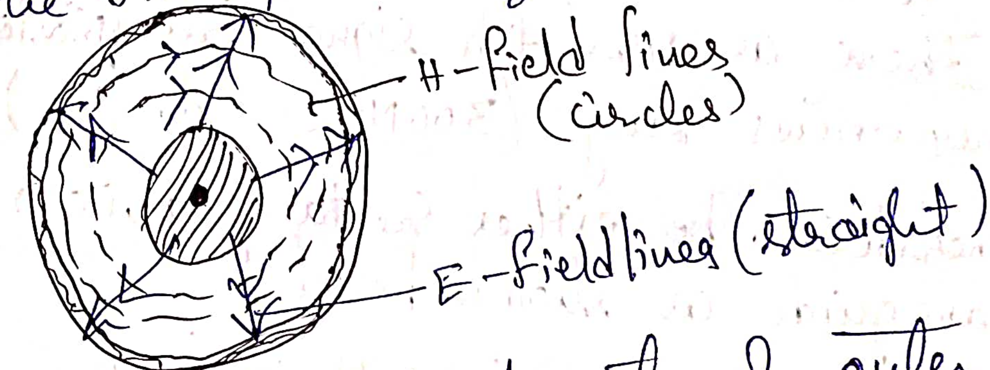
→ These cables are extensively used in the frequency range upto 1GHz.

→ These cables support either TE (or) TM mode of propagation, based on its feeding.

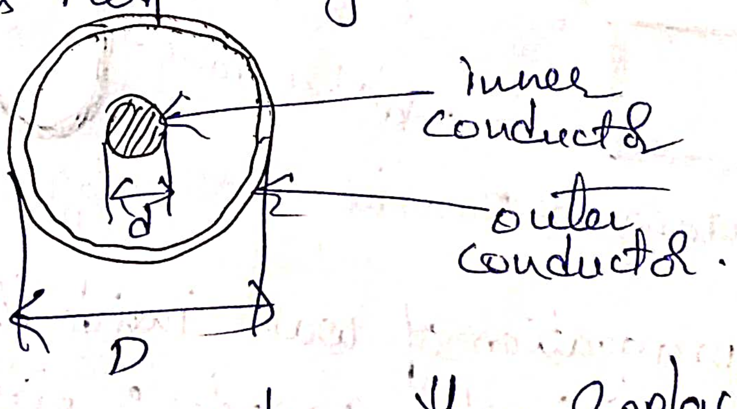
→ The electric & magnetic fields are confined within the dielectric region, cannot leak into freespace

∴ Radiation losses are eliminated.

· Consider, the field patterns of coaxial cable as:



→ Let 'D' and 'd' are the diameters of outer & inner conductors respectively as shown below:



Advantages:

→ Radiation losses are less, thus replaces parallel-wire lines.

Disadvantages: Beyond 1GHz, these cables become unstable.

Applications: ① TV cables, Input cables to high-frequency Precision measuring instruments, used in electrical laboratories

② These are widely used in applications where high voltage levels are needed.

⑥ **Waveguides:**

→ Waveguides are a special category of Tx lines used to guide the waves along the length of the tube.
i.e A hollow metallic tube acts as the medium to transfer EM energy (microwaves) from one end to another.

→ These are used for signal transmission at Microwave frequencies range (300 MHz – 300 GHz).

→ The tube can be either rectangular (or) cylindrical waveguide as shown below:

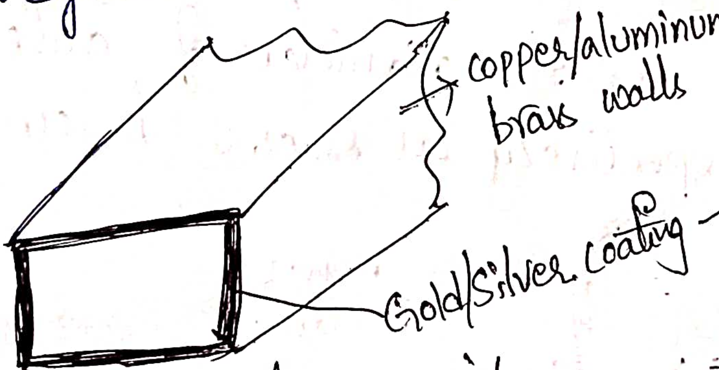


fig ⓐ: Rectangular waveguide.

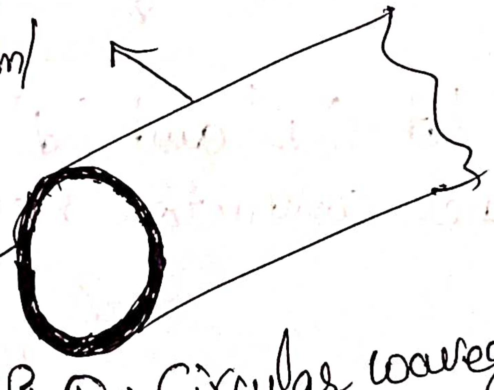


fig ⓑ: Circular waveguide

→ The propagation of wave inside the waveguide originates basically 2 modes: either TE (or) TM mode

Note: TEM mode is not supported in waveguides.

Advantages:- ① power loss during propagation is negligible
② Large power handling capability
↳ larger Bandwidth

③ As it is a simple structure, its installation is easy.

Disadvantages:

① Installation & manufacting cost is high

② waveguides are rigid in nature, thus not flexible

④ larger in size & bulkier.

→ waveguides can operate only above a certain frequency called cutoff frequency and acts as a high-pass filter (HPF).

∴ So, signal frequency must be greater than the cutoff frequency in order to have a proper signal transmission.

⑦ Optical fibers:

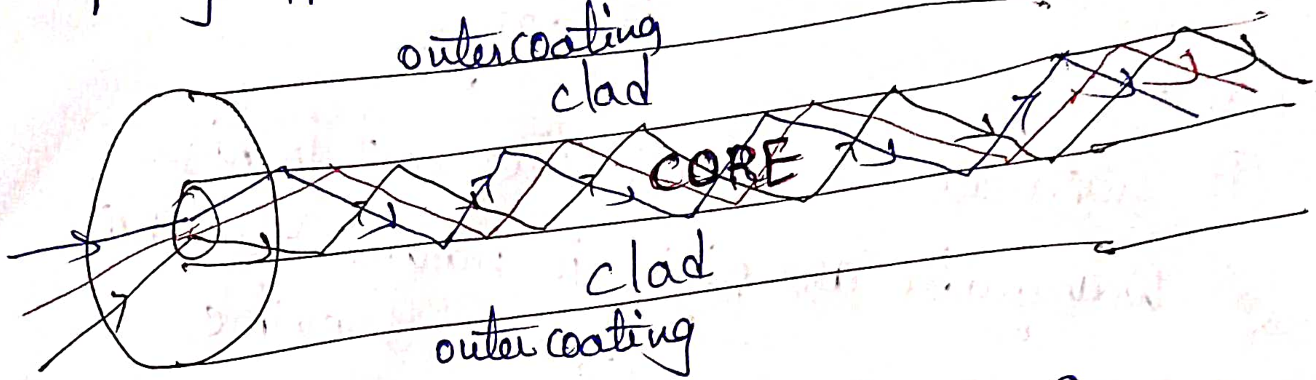
→ These are used for the communication of light signals from one point to another, over long & short distances.

→ In the optical fiber a modulated beam of light are used to carry the information from one place to another on the principle of total internal reflection.

→ Basically optical fibers consist of 2 parts:

① core and cladding: These are made from fused silica glass (SiO_2) and are optically transparent.

② outer coating: protective layers of plastic are uniformly applied to the entire length of the fiber.



core:- The central portion of the optical fibers is called core, in which light rays are guided.

clad:- The surrounding portion of core is called cladding

→ Let n1 – refractive index of core
 n2 – refractive index of clad

. To make the light rays always kept within the core of the optical fibers, the necessary condition is

$$\boxed{n_1 > n_2}$$

Coating:- It gives protection to fibers from external influences, absorbs shear forces, also attenuates any undesirable light in the cladding. (usually colored)

. Let n3 – refractive index of coating

and $\boxed{n_3 > n_1 > n_2}$

Advantages:- ① very high bandwidth ⇒ very high information carrying capacity.

② Small size, light weight

③ low losses ⇒ superior transmission quality ⇒ more efficient

④ Reduced cost & higher security.

⑧ Microstrip lines :-

→ These lines are used to carry EM waves (or) microwave frequency signals.

→ It consists of 3 layers : conducting strip
dielectric
Ground plane.

Structure :- A parallel-plate transmission line consists of an open conducting strip (very thin copper sheet) and a ground plane separated by a dielectric substrate as shown below:

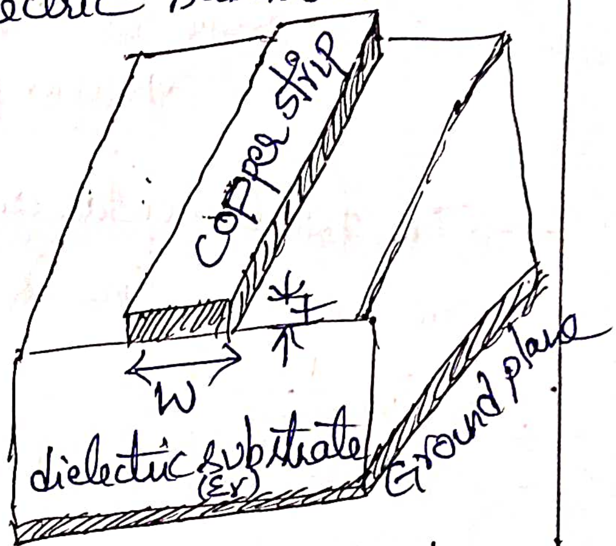


fig : Microstrip line.

→ These lines are fabricated using PCB technology.

advantages :-

① used in Microwave Integrated circuits (MIC)

② used to design and fabricate RF and microwave components such as directional coupler, power divider/combiner, filter, antenna etc.

③ Support all types of modes such as TE (or) TM (or) TEM, to propagate through it.

④ These are cheaper when compared to waveguides.

(9) Strip lines :-

→ These are the planar transmission lines used at frequencies from 100MHz to 100GHz.

→ It consists of central thin conducting strip of width (w) and thickness (t) placed inside dielectric substrate between 2 - wide ground plates.

here, $w > t$

t — same for strip & ground planes.

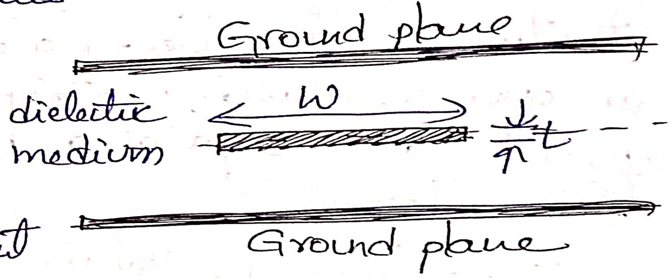


fig : stripline

→ The fundamental and dominant mode in strip line is TEM mode.

Disadvantage :-

→ It is not accessible for adjustment and tuning.

Note :- This problem is avoided in microstrip lines, which allows mounting of active & passive devices, also allows making minor adjustments after circuit fabrication.

Modes of propagation in a waveguide :

→ when an EM wave is transmitted through a waveguide, then it has 2 field components that oscillate mutually perpendicular to each other. (1) Electric field (2) Magnetic field.

→ The propagation of wave inside waveguide originates 3 modes such as :

(1) Transverse Electric Wave (TE) :-
In this mode of wave propagation, the electric field component is totally transverse to the direction of wave propagation, while magnetic field is not transverse.

$$\boxed{E_z = 0 \; ; \; H_z \neq 0}$$

(2) Transverse Magnetic (TM) wave :-
In this mode of wave propagation, the magnetic field component is totally transverse to direction of wave propagation (in Z-direction), while electric field is not transverse.

$$\boxed{E_z \neq 0 \; ; \; H_z = 0}$$

(3) Transverse electromagnetic (TEM) wave :-
In this mode of wave propagation, both field components (E & H) are totally transverse to the direction of wave propagation.

$$\boxed{E_z = H_z = 0}$$

Note :- TEM mode is not supported in waveguides, since TEM mode needs 2 conductors, whereas waveguide is single hollow conductor.

Equivalent circuit of a transmission line :-

→ Consider a 2-wire transmission line as shown in fig ①:

- When a voltage is applied across the two conducting wires, current flows through the line.

(i) The voltage produces an electric field lines between the conductors, contain shunt capacitance (C) & shunt conductance (G).

(ii) The current produces magnetic field lines around the conductors and voltage drop along them, indicates series resistance (R) and inductance (L).

→ A unit section of the line may be represented by fig ②: an equivalent circuit shown below:

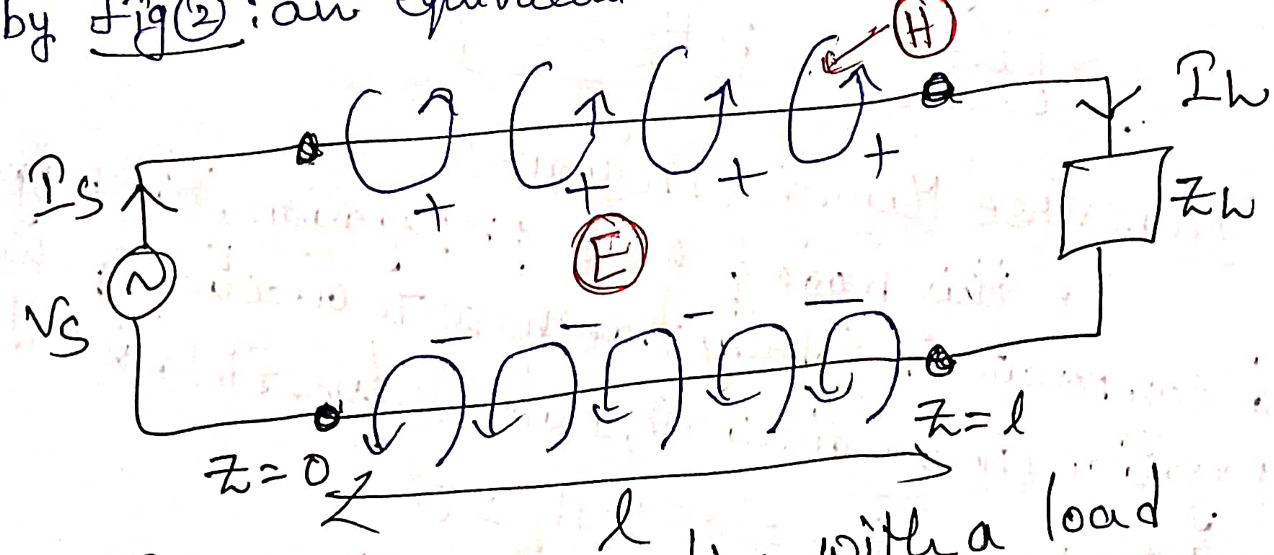


fig ① : Transmission line with a load.

Equivalent circuit of 2-wire Tx line :-

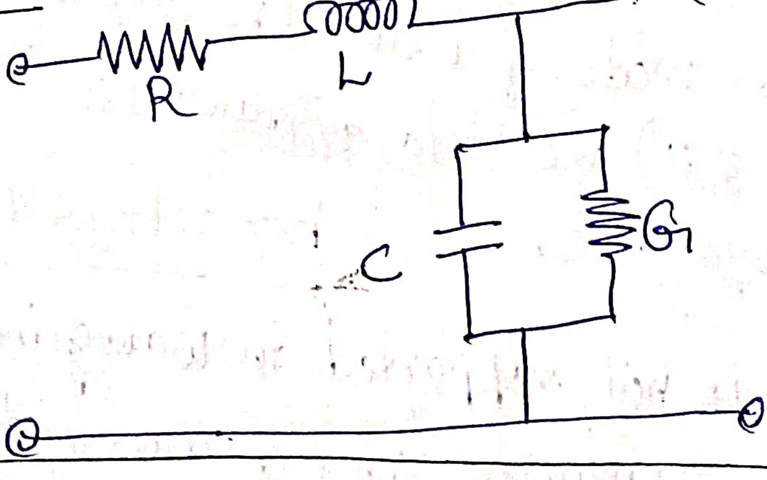


fig ②: equivalent circuit of 2-wire Tx line.

Transmission Line Parameters:- (primary constants)

→ The performance of the transmission line depends on the line parameters,

• Also, It is essential and convenient to describe a transmission line in terms of its line parameters, which are:

(i) Resistance (R):-

→ A transmission line is made up of conductors, each conductor has certain length & diameter.

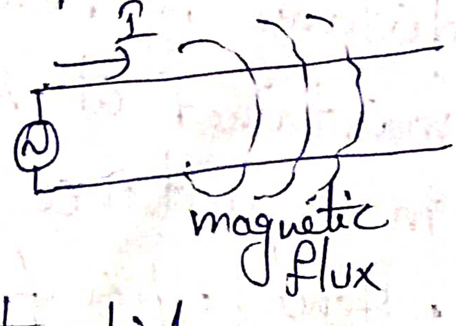
→ when the current is flowing through the conductor, it must have resistance, uniformly distributed all along the length of the conductors.

∴ Resistance per unit length of the transmission line is given by $\boxed{R = \frac{\rho l}{a} \text{ ohms/Km}}$

where l — length of the conductor/wire.
a — radius of the wire.

(ii) Inductance (L):-

→ when the current flows through the conductor, it induces magnetic flux surrounding to it.



→ when the current changes, magnetic flux also varies due to which an electro-magnetic force (emf) induces in the transmission line.

→ This induced emf in the transmission line resists the flow of current, measured as the inductance of the line.

→ The flux linkage per unit current is called as inductance, distributed all along the length of the line.

∴ Inductance per unit length of the transmission line is given by $\boxed{L = \frac{N\phi}{I} \ H/km}$

where N — Number of turns of coil
ϕ — Magnetic flux.

• Also, $\boxed{L = \frac{\psi}{I} \ H/km}$ where $\boxed{\psi = N\phi}$

(iii) Capacitance (C) :-

→ Two metal conductors separated through some distance by dielectric material (eg:- air) and maintained at some potential difference results in capacitance.

• Hence, effect of capacitance distributed along the entire length of lines.

∴ Capacitance associated with the transmission line per unit length is given by,

$$\boxed{C = \frac{\varepsilon A}{d} \ F/km}$$

where ε — dielectric constant
A — Area of cross-section
d — distance between the 2 conductors

• Also, $\boxed{C = \frac{q}{V} \ F/km}$; q – charge
V – applied voltage.

(iv) Conductance (G):-

→ Due to the imperfections of the dielectric medium, also as capacitance being lossy always, a small amount of current flows through the dielectric medium [called leakage current).

• This gives rise to a leakage conductance associated with the Transmission line.

• Also, each capacitance has some shunt conductance, distributed all along the entire length of the transmission line.

∴ conductance per unit length is measured in mhos/km.

Important Points regarding transmission line parameters that:

① The line parameters R, L, C and G are the **primary constants** of the transmission line.

- All these constants are assumed to be independent of frequency.

② The line parameters are not discrete (or) lumped. Rather, they are distributed elements since these are uniformly distributed along the entire length of the line as shown below:

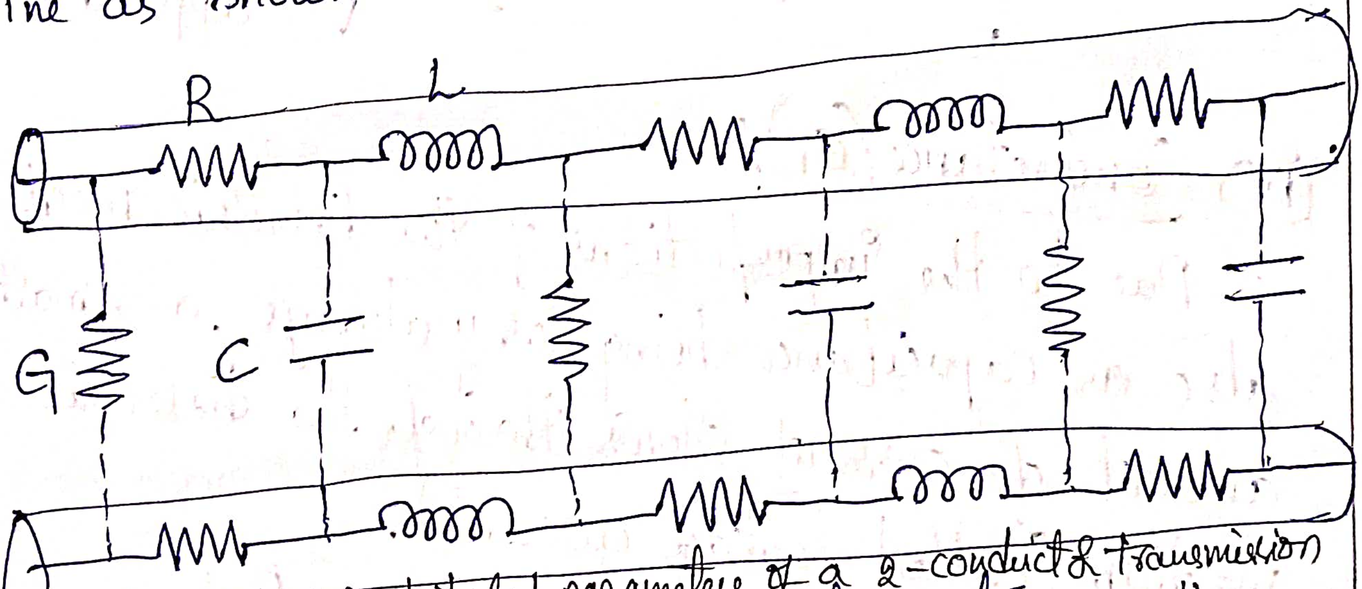


fig: Distributed parameters of a 2-conductor transmission line.

here, R and L — series elements

G and C — shunt elements

i.e series elements form series impedance

$$\boxed{Z = R + j\omega L}$$

and shunt elements form shunt admittance

$$\boxed{Y = G + j\omega C}$$

(3) For each line, the conductors are characterized by $\sigma_c, \mu_c, \varepsilon_c = \varepsilon_0$ and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

(4) $\boxed{G \neq \frac{1}{R}}$ where,

R — ac resistance per unit length of conductors of a transmission line.

G — conductance per unit length due to dielectric medium separating the conductors.

(5) Generally, the value of inductance represents the external inductance per unit length

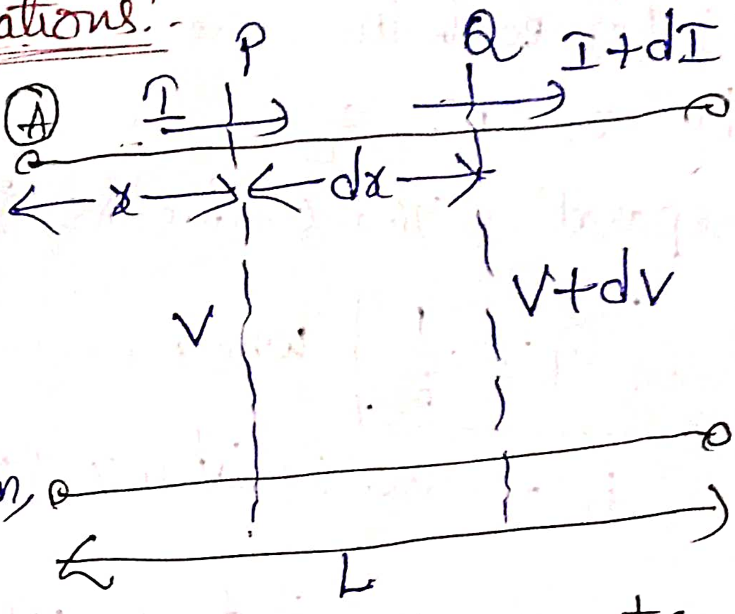
i.e $\boxed{L = L_{ext}}$

- Since, the effects of internal inductance are negligible at high frequencies at which most communication systems operate. $\boxed{L_{in} = \frac{R}{\omega}}$

(6) For each transmission line,

$$\boxed{\begin{gathered} LC = \mu\varepsilon \\ \text{and} \quad \frac{G}{C} = \frac{\sigma}{\varepsilon} \end{gathered}}$$

Transmission Line Equations:-



→ Consider a short section (PQ) of length 'dx' at a distance 'x' from the sending end A.

→ Let dx - very small then,

- current is constant for voltage calculations
- voltage is constant for current calculations.

fig:- short section PQ, at a distance 'x' from sending end of a transmission line.

→ At P, let the voltage be V and I
At Q, let the voltage will be (V+dV) and current (I+dI)

→ series impedance:- For a small section dx, it will be $(R+j\omega L)dx$

shunt admittance:- For a small section dx, it will be $(G+j\omega C)dx$.

(i) Potential difference:- Since dx is very small,

- The voltage drop from P to Q is due to the current (I) flowing through the series impedance $(R+j\omega L)dx$.
- Thus, the potential difference between P and Q is given by :

$$V-(V+dV) = I(R+j\omega L)dx$$

$$-dV = (R+j\omega L)I\,dx$$

$$\boxed{-\frac{dV}{dx} = (R+j\omega L)I} \quad \text{——— (1)}$$

(ii) <u>current difference</u>: Since dx is very small,

- The current difference between P and Q is due to the shunt admittance $(G+j\omega C)dx$ voltage (V) applied to the shunt admittance.

- Thus, $I-(I+dI) = V(G+j\omega C)dx$

$$-dI = (G+j\omega C)V\,dx$$

$$\boxed{-\frac{dI}{dx} = (G+j\omega C)V} \quad \text{——— (2)}$$

→ differentiating (1) w.r.to 'x' and substituting for $\frac{dI}{dx}$

we get, $$-\frac{d^2V}{dx^2} = (R+j\omega L)\frac{dI}{dx}$$

$$= (R+j\omega L)\left[-(G+j\omega C)V\right]$$

$$\boxed{\therefore \frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)V} \quad \text{——— (3)}$$

- Similarly, differentiating (2) w.r.to 'x' & substituting for $\frac{dV}{dx}$

we get, $$-\frac{d^2I}{dx^2} = (G+j\omega C)\frac{dV}{dx}$$

$$= (G+j\omega C)\left[-(R+j\omega L)I\right]$$

$$\boxed{\therefore \frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C)I} \quad \text{——— (4)}$$

→ Let $(R+j\omega L)(G+j\omega C) = P^2$,
where P — propagation constant / complex constant for a given frequency.

$$\boxed{\therefore \quad \frac{d^2V}{dx^2} = P^2 V \qquad \text{and} \qquad \frac{d^2I}{dx^2} = P^2 I} \quad \text{—— (5)}$$

- The equations (5) are referred to as <u>differential equations</u> of the transmission lines. Also, called as <u>wave equations</u> of the transmission line.

<u>Solution</u> of <u>transmission line equations</u>:-

→ The equations (5) are homogeneous equations (2^{nd} order) with constant coefficients, whose <u>solutions</u> are:

$$\boxed{\begin{aligned} V &= ae^{Px} + be^{-Px} \\ I &= ce^{Px} + de^{-Px} \end{aligned}} \quad \text{—— (6)}$$

where a and b are constants with dimensions of voltage.
c and d are constants with dimensions of current.

→ We know that: $e^{Px} = \cosh px + \sinh px$
$$e^{-Px} = \cosh px - \sinh px$$

i.e exponentials are replaced by <u>hyperbolic functions</u>.

→ Substituting, these in Eq^n (6), we get

$$V = a(\cosh px + \sinh px) + b(\cosh px - \sinh px)$$

$\Rightarrow \quad V = (a+b)\cosh px + (a-b)\sinh px$

$= A\cosh px + B\sinh px$ —— (7a)

where $\left.\begin{array}{l} A = (a+b) \\ B = (a-b) \end{array}\right\}$ are new constants.

• Similarly, $I = c(\cosh px + \sinh px) + d(\cosh px - \sinh px)$

$= (c+d)\cosh px + (c-d)\sinh px$

$= C\cosh px + D\sinh px$ —— (7b)

where $\left.\begin{array}{l} C = c+d \\ D = c-d \end{array}\right\}$ are new constants.

∴ Equations (7) are:

$$\boxed{\begin{array}{l} V = A\cosh px + B\sinh px \\ I = C\cosh px + D\sinh px \end{array}} \quad \text{—— (7)}$$

∴ These equations are very useful form for the voltage & current values at any point on a Tx line.

→ Further, to simplify Eqⁿs (7):

• consider Eqⁿ (1): $\frac{-dV}{dx} = (R+j\omega L)I$

Now, substitute $V = A\cosh px + B\sinh px$

$\therefore \quad -\frac{d}{dx}(A\cosh px + B\sinh px) = (R+j\omega L)I$

$\times \quad -(Ap\cosh px + Bp\sinh px) = (R+j\omega L)I$

· As we know: $P^2 = (R+j\omega L)(G+j\omega C)$.

substituting in above,

$$-P(A\sinh px + B\cosh px) = (R+j\omega L)I$$

$$-\frac{\sqrt{(R+j\omega L)(G+j\omega C)}}{R+j\omega L}(A\sinh px + B\cosh px) = I$$

$$-\sqrt{\frac{G+j\omega C}{R+j\omega L}}(A\sinh px + B\cosh px) = I$$

$$\Rightarrow \quad I = \frac{-1}{Z_0}(A\sinh px + B\cosh px)$$

where $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$ — complex constant for a given frequency.

∴ Eqn's (7) may be written in the form,

$$\boxed{\begin{aligned} V &= A\cosh px + B\sinh px \\ I &= \frac{-1}{Z_0}(A\sinh px + B\cosh px) \end{aligned}} \quad \text{— (8)}$$

· Again, these Eqn's (7) can also be expressed in exponential form as:

$$\boxed{\begin{aligned} V &= ae^{Px} + be^{-Px} \\ I &= \frac{1}{Z_0}(be^{-Px} - ae^{Px}) \end{aligned}} \quad \text{since} \quad \boxed{\begin{aligned} \cosh px &= \frac{e^{Px}+e^{-Px}}{2} \\ \sinh px &= \frac{e^{Px}-e^{-Px}}{2} \end{aligned}}$$

Note:- relations between old & new constants are: $\boxed{a = \frac{A+B}{2} \ \& \ b = \frac{A-B}{2}}$

Determination of constants A and B :-

(1) Sending end :- when conditions at the sending end are known :

- Let I_S and V_S — current & voltage at sending end respectively.
- At the sending end : $\boxed{x = 0 \text{ and } V = V_S,\ I = I_S}$

→ substituting in Eq^n (8) will be :

$$V_S = A\cosh p(0) + B\sinh p(0) = (A \times 1) + (B \times 0)$$
$$= A.$$

Similarly $I_S = \frac{-1}{Z_0}\left(A\sinh p(0) + B\cosh p(0)\right) = \frac{-B}{Z_0}.$

i.e
$$\boxed{\begin{aligned} V_S &= A \\ I_S &= \frac{-B}{Z_0} \Rightarrow B = -I_S Z_0 \end{aligned}}$$

→ Thus, A and B are expressed in terms of voltage & current at the sending end.

- Substituting these values of A and B in Eq^n (8), we get

$$\boxed{\begin{aligned} V &= V_S\cosh px - I_S Z_0 \sinh px \\ I &= \frac{-1}{Z_0}\left(V_S \sinh px - I_S Z_0 \cosh px\right) \\ &= I_S \cosh px - \frac{V_S}{Z_0}\sinh px \end{aligned}} \quad (9)$$

→ Eqⁿ's (9) are general line equations of voltage & current at a point, distance 'x' from the sending end in terms of sending voltage (V_S) & current (I_S).

(2) Receiving End :- when the conditions at receiving end are known :

$$\left.\begin{array}{l} I = A\cosh px + B\sinh px \\ V = -BZ_0\cosh px - AZ_0\sinh px \end{array}\right\} \text{(10)}$$

- At $\boxed{x = l}$, length of line and Eqⁿ's (10) will become

$$\boxed{\begin{array}{l} V = V_R = -BZ_0\cosh pl - AZ_0\sinh pl \\ I = I_R = A\cosh pl + B\sinh pl \end{array}}$$

where I_R and V_R are the current & voltage at the receiving end.

- rearranging above Eqⁿ's : $I_R - A\cosh pl = B\sinh pl$

$$V_R + AZ_0\sinh pl = BZ_0\cosh pl$$

& dividing : $$\frac{I_R - A\cosh pl}{V_R + AZ_0\sinh pl} = \frac{B\sinh pl}{-BZ_0\cosh pl}$$

$$\Rightarrow -I_R Z_0\cosh pl + AZ_0\cosh^2 pl = V_R\sinh pl + AZ_0\sinh^2 pl.$$

$$\Rightarrow AZ_0 = V_R\sinh pl + I_R Z_0\cosh pl$$

$$\Rightarrow \boxed{A = \frac{V_R}{Z_0}\sinh pl + I_R\cosh pl}$$

→ Similarly, $$\frac{I_R - B\sinh pl}{V_R + BZ_0\cosh pl} = \frac{A\cosh pl}{-AZ_0\sinh pl}$$

$$\Rightarrow -I_R Z_0 \sinh pl + B Z_0 \sinh^2 pl = V_R \cosh pl + B Z_0 \cosh^2 pl$$

$$\Rightarrow -B Z_0 = V_R \cosh pl + I_R Z_0 \sinh pl$$

$$\Rightarrow \boxed{B = -\frac{V_R}{Z_0}\cosh pl - I_R \sinh pl}$$

$\therefore$ substituting A & B expressions in Eqn (10), we get

$$I = \left[\frac{V_R}{Z_0}\sinh pl + I_R \cosh pl\right]\cosh px - \left[\frac{V_R}{Z_0}\cosh pl + I_R \sinh pl\right]\sinh px$$

$$\therefore \boxed{I = I_R \cosh p(l-x) + \frac{V_R}{Z_0}\sinh p(l-x)}$$

and similarly,

$$V = \left[V_R \cosh pl + I_R Z_0 \sinh pl\right]\cosh px - \left[V_R \sinh pl + I_R Z_0 \cosh pl\right]\sinh px$$

$$\therefore \boxed{V = V_R \cosh p(l-x) + I_R Z_0 \sinh p(l-x)}$$

Secondary Constants of Transmission Line:

→ The electrical properties of the transmission line can be represented by the transmission line equations through two complex constants, termed as the secondary constants of the line which are as follows:

① characteristic Impedance (Z_0)

② Propagation constant (P)

→ Generally,

$$\boxed{\begin{aligned} Z_0 &= \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}} \\ P &= \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{ZY} \end{aligned}}$$

since $\underset{\text{(series impedance)}}{Z = R+j\omega L}$ and $\underset{\text{(shunt admittance)}}{Y = G+j\omega C}$.

→ Although, they are referred to as constants but generally, they will vary if the frequency is changed.

→ The above two equations gives the relationship between primary constants R, L, C, G and secondary constants Z_0 & P.

→ These equations used to calculate secondary constants if primary constants are known and vice-versa.

Computation of primary constants:

• when multiplying & dividing both Z_0 & P, we get

① $Z_0 \times P = R+j\omega L$

② $\frac{P}{Z_0} = G+j\omega C$

Characteristic Impedance :-

→ The characteristic impedance of a uniform transmission line may be defined as the ratio of the voltage to the current at the input of an <u>infinite line</u>.

- Thus, characteristic impedance is the input impedance of an infinite line.

(or)

→ It is defined as the impedance looking into an infinite length of the line. Also known as surge impedance.

→ Let :

V_{si} – Sending end voltage

I_{si} – sending end current of an infinite line shown in fig.

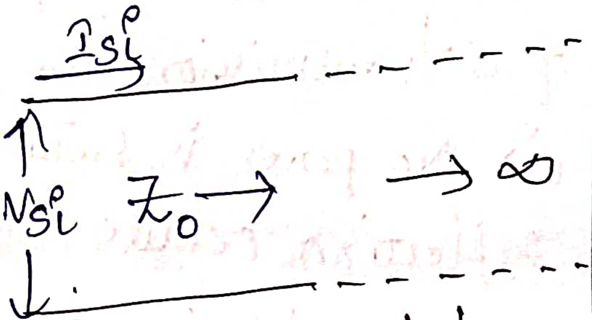


fig : Infinite line

→ We know that : $\boxed{-\frac{dV}{dx} = (R+j\omega L)\,I}$

where,

V – voltage at any point of an infinite line $= V_{si}\,e^{-Px}$

I – current at any point of an infinite line $= I_{si}\,e^{-Px}$

$$\therefore \; -\frac{d}{dx}\left(V_{si}\,e^{-Px}\right) = (R+j\omega L)\left(I_{si}\,e^{-Px}\right)$$

$$-\left[V_{si}\left(\cancel{e^{-Px}}\right)(-P)\right] = (R+j\omega L)\left(I_{si}\,\cancel{e^{-Px}}\right)$$

$$P\,V_{si} = (R+j\omega L)\,I_{si}$$

$$\Rightarrow \frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{P} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

∴ By definition, input impedance $\frac{V_{Si}}{I_{Si}} = Z_0$

∴ $\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$ — expression for characteristic impedance.

Note:- Z_0 doesnot involve length of line (or) the character of the terminating load, but it is determined only by the characteristics of the line per unit length (R, L, C, G).

→ characteristic impedance is a fine & useful concept of a transmission line.

(i) No power is returned from an infinite line, no reflection occurs.

• Thus, no loss of power except I^2R losses.

(ii) when a line is terminated by its Z_0 behaves as an infinite line, also have no reflection.

(or)

when a line is terminated in its characteristic impedance (Z_0), it is said to be correctly terminated / properly terminated (or) non-resonant line.

→ characteristic impedances of various transmission lines are:

Type of the Line	characteristic impedance (units - ohms)	
① Two wire (in dielectric medium)	$\frac{276}{\sqrt{K}} \log_{10} \frac{d}{a}\ \Omega$	d - spacing b/w 2-wires; a - radius of wire.
② Two wire (in air) (K = 1)	$276 \log_{10} \frac{d}{a}\ \Omega$	
③ Coaxial cable (filled with dielectric medium)	$\frac{138}{\sqrt{K}} \log_{10} \frac{D}{d}\ \Omega$	D - outer conductor inner diameter; d - inner conductor diameter.
④ Coaxial Cable (filled with air)	$138 \log_{10} \frac{D}{d}\ \Omega$	

Propagation Constant:-

→ propagation constant is defined as the natural logarithm of the ratio of the sending end current/voltage to the receiving end current/voltage of the line.

→ It is used to govern the manner in which the wave is propagated along a line and specifies the variation of voltage & current in the line as a function of distance.

$$\therefore \quad P = \log_e\left(\frac{V_S}{V_R}\right) \text{ (or) } \log_e\left(\frac{I_S}{I_R}\right)$$

Derivation:-

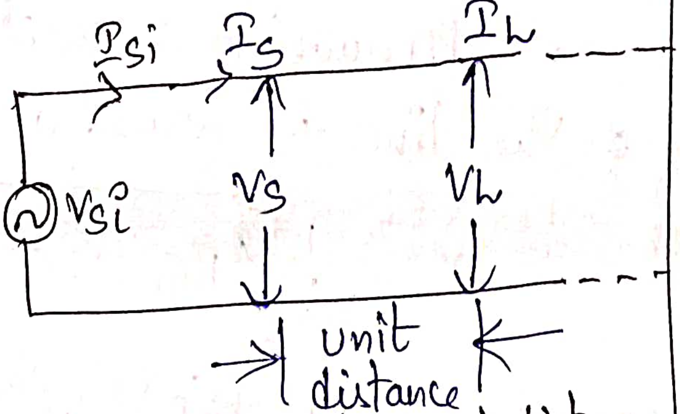


fig: V & I at unit distance apart.

As we know: $I_x = I_{si}\, e^{-Px}$

$$\Rightarrow e^{Px} = \frac{I_{si}}{I_x}$$

Taking $\log_e$ on both sides

$$\log_e e^{Px} = \log_e \frac{I_{si}}{I_x}$$

$$Px = \log_e \frac{I_{si}}{I_x}$$

• Let, if I_{si} (sending end current) & I_x (receiving end current) are unit distance ($x=1$) apart,

then $$\boxed{P = \log_e \frac{I_{si}}{I_x} \text{ (or) } \ln \frac{I_{si}}{I_x}}$$ — expression for propagation constant.

Note:- Since, it is the ratio, it has no units. but normally expressed in $\boxed{\text{per km unit}}$.

Attenuation & phase constants:-

→ propagation constant (P) is usually a complex quantity and can be expressed as: $\boxed{P = \alpha + j\beta}$

where,
(i) Real part is called attenuation constant (α)
(ii) Imaginary part is called phase/wave constant (β)

① attenuation constant (α):- It determines the reduction in voltage & current along the line, measured in neper per km.

- Attenuation constant when multiplied by the length of the line is termed as total attenuation/line attenuation.

$$\therefore \alpha = \sqrt{\frac{1}{2}\left[(RG - \omega^2 LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]} \text{ neper/Km}$$

② phase constant (β): It determines the variation in phase position of voltage & current along the line, measured in radians per km.

- phase constant when multiplied by the length of the transmission line is termed as electrical length of the line.

$$\therefore \beta = \sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]} \text{ rad/Km}$$

Note:- 1 neper = 8.686 db
1 rad = 57.3°

Derivations of α and β:-

$$P = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

squaring on both sides

$$(\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = RG + j\omega(LG+RC) - \omega^2 LC$$

$$(\alpha^2-\beta^2) + j(2\alpha\beta) = (RG-\omega^2 LC) + j[\omega(LG+RC)]$$

By equating real part of equations, we get,

$$\alpha^2 - \beta^2 = RG - \omega^2 LC \quad \text{——— (1)}$$

we know: $|P| = \sqrt{\alpha^2+\beta^2} = \sqrt{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$

$$\Rightarrow \alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \quad \text{——— (2)}$$

Adding Eq^{n}s (1) & (2), we get

$$\alpha^2 - \cancel{\beta^2} + \alpha^2 + \cancel{\beta^2} = (RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$2\alpha^2 = (RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\therefore \boxed{\alpha = \sqrt{\frac{1}{2}\left[(RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}} \text{ nepers/Km}$$

Subtracting Eq^{n}s (1) & (2), we get

$$-2\beta^2 = \left[(RG-\omega^2LC) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]$$

$$\therefore \boxed{\beta = \sqrt{\frac{1}{2}\left[(\omega^2LC-RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}} \text{ rad/Km}$$

Parameters of a Transmission Line :-

In addition to primary & secondary constants, there are 3 more characteristics of a transmission line which are:

(i) **wavelength (λ)**:- It is the distance that a wave travels along the line to have phase of 2π radians, measured in meters.

- By definition, $\beta\lambda = 2\pi \Rightarrow \boxed{\lambda = \frac{2\pi}{\beta}}$ meter

where β - Phase shift

Note:- ① In case of air dielectric ; $\boxed{\lambda \simeq \lambda_f}$ (free space wavelength)

② In case of dielectric having dielectric constant (K) ; $\boxed{\lambda = \frac{\lambda_f}{\sqrt{K}}}$

(ii) **phase velocity (v_p)**:- The velocity with which a plane wave of constant phase is propagating in a transmission line is called the phase velocity (v_p).

(or)

Rate at which the wave changes its phase in terms of guide wavelength, measured in km per second.

→ Since, change of 2π in phase angle represents one cycle in time (t) and occurs in a distance of one wavelength (λ), then

$$\lambda = v_p \times t \Rightarrow v_p = \frac{\lambda}{t} = \lambda f$$

$$\Rightarrow v_p = \frac{2\pi}{\beta} \times f \quad \left(\text{since } \lambda = \frac{2\pi}{\beta}\right)$$

$$\therefore \boxed{v_p = \frac{\omega}{\beta}} \text{ km/sec} \quad (\text{since } \omega = 2\pi f)$$

Note:- ① In case of air dielectric ; $\boxed{v_p \simeq c} = 3\times10^8$ m/s

② In case of solid dielectric insulation (with dielectric constant ε_r) ; $\boxed{v_p = \frac{c}{\sqrt{\varepsilon_r}}}$ km/s

→ In general, phase velocity (v_p) can be greater, equal to (or) less than the velocity of light in free space.

- It is virtual velocity, does not determine the velocity of transmission of energy (or) signal.

Transit time/ time delay :-

It is the time elapsed for the wave to travel from one end to the other, $\boxed{t_d = \frac{l}{v_p}}$ sec

(iii) Group velocity (v_g) :-

→ In lossless systems, group velocity is the velocity at which a pulse (or) energy travel, less than the velocity of light in free space.

→ If β is not proportional to ω, and

- If the wave components travel with different velocities, the envelope of the wave travels with a velocity known as group velocity (v_g).

(or)

- It is the velocity of the envelope of a complex signal. example:- AM signal, pulses transmission, transmission through waveguides.

→ Let ω_1 and ω_2 — angular frequencies being transmitted (close to each other)

β_1 and β_2 — corresponding phase constants.

Then, group velocity $\boxed{v_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1} = \frac{d\omega}{d\beta}}$

Note :- $\frac{d\omega}{d\beta}$ — evaluated at carrier of centre frequency.

Relation between group velocity (v_g) and phase velocity (v_p):

→ phase velocity : $v_p = \frac{\omega}{\beta}$

differentiating w.r.to 'ω', we get,

$$\frac{dv_p}{d\omega} = \frac{\beta - \omega \frac{d\beta}{d\omega}}{\beta^2} = \frac{1 - \frac{\omega}{\beta}\left(\frac{d\beta}{d\omega}\right)}{\beta}$$

$$\beta \frac{dv_p}{d\omega} = 1 - v_p\left(\frac{1}{v_g}\right)$$

$$\Rightarrow \frac{v_p}{v_g} = 1 - \beta \frac{dv_p}{d\omega}$$

$$\Rightarrow \boxed{v_g = \frac{v_p}{1 - \beta\left(\frac{dv_p}{d\omega}\right)}} = \frac{v_p}{1 - \frac{\omega}{v_p}\left(\frac{dv_p}{d\omega}\right)}$$

(since $\beta = \frac{\omega}{v_p}$)

→ If $\boxed{\frac{dv_p}{d\omega} = 0}$

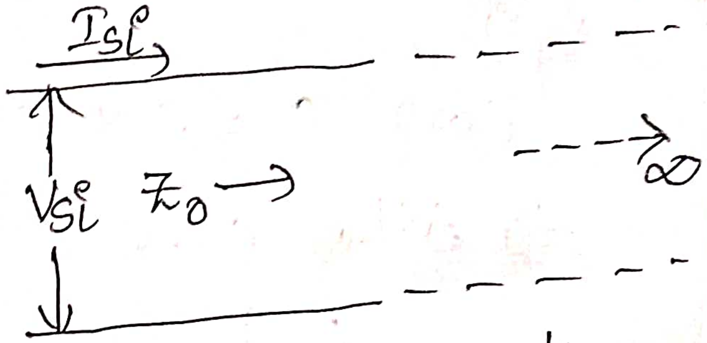
$\Rightarrow \boxed{v_g = v_p}$ This is consistent with the definition of group velocity.

Z.

Infinite Line :-

→ An infinite line is a hypothetical line in which the length of the transmission line is infinite shown in fig①.

→ when an A.C voltage applied to the sending end of infinite line, a finite amount of current will flow (due to C & G b/w 2 wires)



fig①: Infinite line

∴ The ratio of applied voltage to current gives the input impedance of an infinite line, known as characteristic impedance (Z_0) is given by

$$\boxed{Z_0 = \frac{V_{si}}{I_{si}}}$$

where V_{si} — sending end voltage of an infinite line.
I_{si} — sending end current of an infinite line.

→ current at any point distance (x) from the sending end is given by $\boxed{I = c e^{Px} + d e^{-Px}}$ —— ①

where c, d —— constants

- At sending end of an infinite line: $x = 0$ and $I = I_{si}$

substitute these conditions in Eqn ①,

$$I_{si} = c e^{0} + d e^{-0} = c + d \quad (\text{since } e^{0} = 1) \quad \text{—— ②}$$

• At Receiving end of infinite line : $x = \infty$ and $I = 0$

substituting in Eq^n ① $\quad 0 = c \times \infty + d \times 0 \quad \left(\text{since } e^{\infty} = \infty,\ e^{-\infty} = 0\right)$

(or) $\quad 0 = c \times \infty$

$\Rightarrow$ either $c = 0$ (or) $\cancel{\infty = 0}$ → never possible.

$\therefore \boxed{c = 0}$

• substituting in Eq^n ②, $\quad I_{si} = c + d = 0 + d$

$\Rightarrow \boxed{d = I_{si}}$

$\therefore$ substituting c & d values in Eq^n ①,

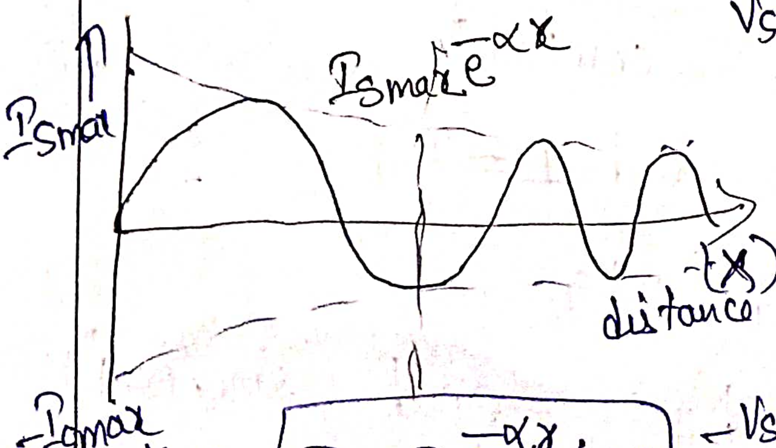
$$\boxed{I = I_{si}\, e^{-Px}}$$

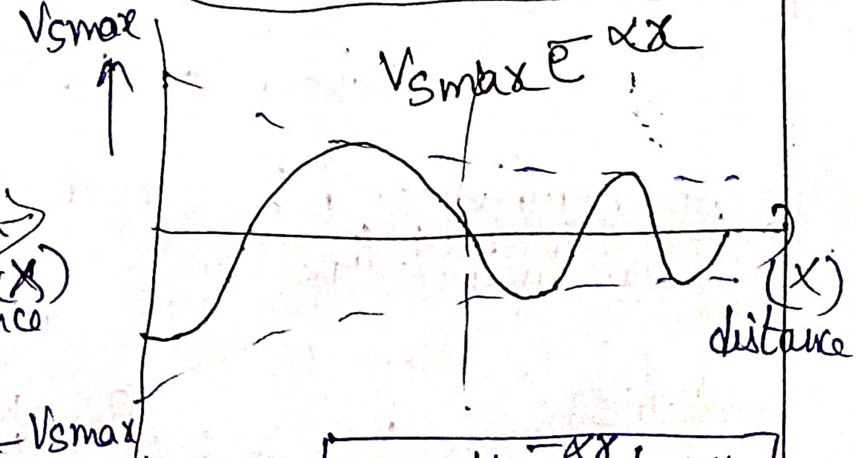
This equation gives current at any point of an infinite line.

Similarly, voltage at any point of an infinite line is

$$\boxed{V = V_{si}\, e^{-Px}}$$

$\boxed{\text{since } P = \alpha + j\beta \qquad \dfrac{I_S}{I_R} = e^{Px} = e^{\alpha x} \angle \beta x}$





$\boxed{I_R = I_S e^{-\alpha x} \angle -\beta x}$ $\qquad$ $\boxed{V_R = V_S e^{-\alpha x} \angle -\beta x}$

Infinite line is equivalent to a finite line terminated in its characteristic impedance (Z_0):-

→ If a finite length of line is joined with an infinite line, their total input impedance is same as that of infinite itself. i.e together they make one infinite line.

- However, the infinite line presents an impedance (Z_0) at AB as shown in fig (1a), because input impedance of an infinite line is Z_0.

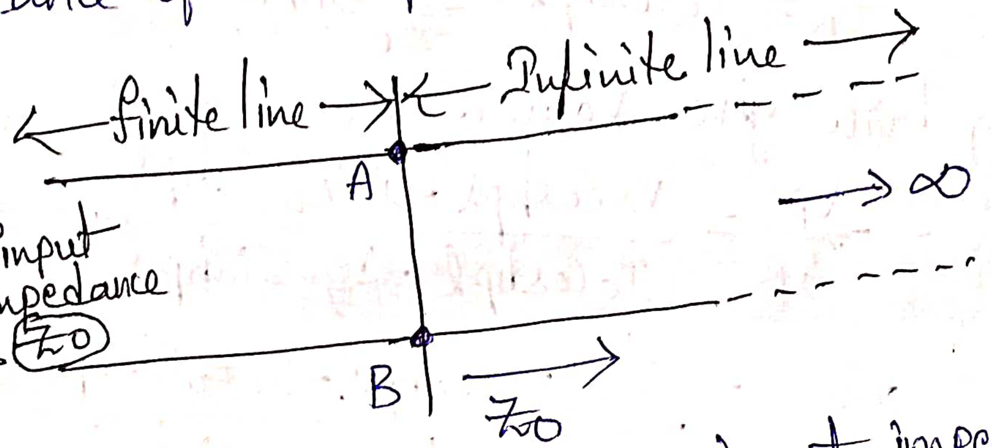


fig (1a). Infinite line
input impedance is (Z_0)

Conclusion:- A finite line has an input impedance Z_0 when it is terminated in Z_0, as shown in fig (1b).

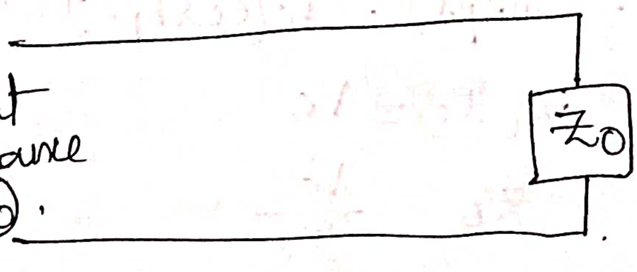


fig (1b). Equivalent of an infinite line
input impedance is (Z_0).

Statement:- A finite line terminated by its characteristic impedance (Z_0) behaves as an infinite line.

Proof: Consider a line of, length - l
termination - Z_0 (characteristic impedance)

Let voltage & current at termination — V_R and I_R.

$$\therefore \frac{V_R}{I_R} = Z_0.$$

→ We know that : $V = V_S \cosh px - I_S Z_0 \sinh px$

$$I = I_S \cosh px - \frac{V_S}{Z_0} \sinh px$$

• Substitute $x = l$, $V = V_R$ and $I = I_R$ then we get,

$$V_R = V_S \cosh pl - I_S Z_0 \sinh pl$$

$$I_R = I_S \cosh pl - \frac{V_S}{Z_0} \sinh pl$$

. Dividing both gives value of Z_0.

$$\therefore Z_0 = \frac{V_R}{I_R} = \frac{V_S \cosh pl - I_S Z_0 \sinh pl}{I_S \cosh pl - \frac{V_S}{Z_0} \sinh pl}$$

$$\cancel{Z_0} = \frac{\cancel{Z_0}(V_S \cosh pl - I_S Z_0 \sinh pl)}{I_S Z_0 \cosh pl - V_S \sinh pl}$$

$$Z_0 I_S \cosh pl - V_S \sinh pl = V_S \cosh pl - I_S Z_0 \sinh pl$$

$$\Rightarrow Z_0 I_S (\cosh pl + \cancel{\sinh pl}) = V_S(\cosh pl + \cancel{\sinh pl})$$

$$Z_0 I_S = V_S$$

$$Z_0 = \frac{V_S}{I_S}$$

$\Rightarrow \boxed{Z_0 = Z_{in}}$ since $Z_{in} = \frac{V_S}{I_S}$ – input impedance of the line

Conclusion :- Thus, input impedance of a finite line terminated in its characteristic impedance (Z_0) is the characteristic impedance of the line.

Losses in Transmission Lines:

→ when a signal is transmitted through a transmission line suffer from losses due to various factors.

· Consider, major losses that occur in transmission lines are:

① **Copper losses:-** There are 2 types of copper losses.

(i) **I^2R losses:-** These are due to the dissipation of energy in the form of heat, by the current flow through the conductors ($R \neq 0$). **Remedy:-** make sure $R=0$.

(ii) **Losses due to skin effect:-** Due to skin effect, the operation frequency of the transmission line increased which in turn increases power losses in line.

Remedy:- To prevent these losses plate the line with silver.

② **Dielectric losses:-** These losses are due to heating effect on dielectric material between the conductors.

Remedy:- use polyethylene as a dielectric material.

③ **Radiation and Induction losses:-** These losses are caused by the fields surrounding the conductors.

- Induction losses occur when electro-magnetic field around a conductor, induces current in nearby metallic objects.
- Radiation losses occur because some magnetic lines of force of a conductor do not return to conductor when cycle alternates (i.e escapes into space).

→ Depending on amount of losses present, transmission lines can be divided into 2 categories:

① Lossless Line
② Low-loss Line

Lossless Line :-

→ while designing a transmission line, the loss of the line must be minimized to prevent loss of information.

→ A transmission line is said to be lossless, when

① conductors of the line are perfect: $\boxed{\sigma_c = \infty}$ → high conductivity

② dielectric medium between the conductors has zero conductivity: $\boxed{\sigma_d = 0}$

→ Thus, for a lossless line:

• The line parameters become $\boxed{\begin{matrix} R = 0 \\ G = 0 \end{matrix}}$ ⇒ Necessary Condition

characteristics :- A lossless transmission line is characterized by the following parameters:

① propagation constant: $P = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\Rightarrow \alpha + j\beta = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC}$$

• real part:- Equating real part gives $\boxed{\alpha = 0}$

⇒ No attenuation

• imaginary part:- Equating imaginary part gives phase constant $\boxed{\beta = \omega\sqrt{LC}}$

② phase velocity (v_p): $\boxed{v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$ independent of frequency.

③ characteristic impedance (Z_0):

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$$

$\therefore \boxed{Z_0 = \sqrt{\frac{L}{C}}}$ where $R_0 = \sqrt{\frac{L}{C}}$ and $X_0 = 0$.

Note:- phase velocity (v_p) and Z_0 both independent of frequency and depend only on its distributed line parameters.

Low-Loss line :-

→ The low-loss condition is easily satisfied at very high frequencies, i.e $\boxed{\omega L >> R \text{ and } \omega C >> G}$

characteristics:

→ A low-loss line is characterized by following parameters.

① propagation constant: $P = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\Rightarrow \alpha + j\beta = \sqrt{j\omega L\left(1+\frac{R}{j\omega L}\right) j\omega C\left(1+\frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

• Since $\omega L >> R$ & $\omega C >> G$, by substituting

$$\alpha + j\beta = j\omega\sqrt{LC}\left(1+\frac{1}{2}\frac{R}{j\omega L}\right)\left(1+\frac{1}{2}\frac{G}{j\omega C}\right).$$

since, expanding the brackets and retaining only first order terms

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{LC}\left[1 + \frac{G}{2j\omega C} + \frac{R}{2j\omega L} + \frac{1}{4}\cancel{\left(\frac{R}{j\omega L}\right)\left(\frac{G}{j\omega C}\right)}\right]$$

neglect as it is a second-order term

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{LC}\left[1 + \frac{1}{2j\omega}\left(\frac{R}{L} + \frac{G}{C}\right)\right]$$

$$\Rightarrow \alpha + j\beta = j\omega\sqrt{LC} + \frac{\sqrt{LC}}{2}\left(\frac{R}{L} + \frac{G}{C}\right)$$

- attenuation constant: equating real part,

$$\alpha = \frac{1}{2}\left(\sqrt{LC}\,\frac{R}{L} + \sqrt{LC}\,\frac{G}{C}\right)$$

$$\therefore \boxed{\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right)}$$

- phase constant:- equating imaginary part gives,

$$\boxed{\beta = \omega\sqrt{LC}} \rightarrow \text{is a linear function of } \omega.$$

(2) phase velocity (v_p):- $\boxed{v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$

i.e independent of frequency, depends on line parameters.

(3) characteristic impedance (Z_0):-

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}\left(1 + \frac{R}{j\omega L}\right)^{1/2}\left(1 + \frac{G}{j\omega C}\right)^{1/2}$$

$$= \sqrt{\frac{L}{C}}(1+0)(1+0) = \sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Z_0 = \sqrt{\frac{L}{C}}} \text{ where } R_0 = \sqrt{\frac{L}{C}} \text{ and } X_0 = 0$$

- Thus, Z_0 is a pure resistance determined by L & C, Also it is independent of frequency. $\Rightarrow$ Same as lossless line.

Distortion :-

→ Signals transmitted over transmission lines are normally complex and consists of many frequency components.

→ For ideal transmission, the waveform at the line receiving end must be same as waveform of original input signal at sending end.

Condition:

- All frequencies should have the same attenuation and same delay caused by a finite phase velocity / velocity of propagation.
- When these conditions are not satisfied, distortion exist.

Types: Distortion on transmission lines is usually of 2 types:

① Frequency distortion :- If various frequency components of the signal undergoes different attenuations, then frequency distortion exist on transmission lines.

Remedy: ① attenuation constant (α) should not a function of frequency. ② Use equalizers to minimize it.

② Delay distortion / phase distortion : If the time required to transmit various frequency components over the line and the consequent delay is not a constant, then delay distortion is said to be exist on transmission lines.

Remedy: ① phase velocity (v_p) should be independent of frequency.

② phase constant (β) should be frequency dependent

Since $\boxed{v_p = \frac{\omega}{\beta} = \text{constant}}$ ⇒ valid only when
$\beta = \text{constant} \times \omega$ (frequency dependent)

• This type of distortion also known as phase distortion.

Conclusion:

→ The transmission line will neither have delay distortion nor frequency distortion,

only if: ① α is independent of ω

② β is a constant multiplied by ω. (i.e frequency dependent)

Note:- from α, β expressions (as we know earlier):

① α is dependent of ω ⇒ introduce frequency distortion

② β is not constant multiplied by ω ⇒ introduces delay distortion

Distortionless Line :-

→ When the transmission line is free from the frequency & delay distortions, it is said to be "distortion-less transmission line".

Condition:- The line is distortion-less, when:

① attenuation constant (α) is independent of frequency

② phase constant (β) is linearly dependent on frequency.

→ The distortion-less transmission line exits when the distributed parameters of the line are related as:

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

Analysis: we know that:

- propagation constant $P = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$= \sqrt{L\left(\frac{R}{L}+j\omega\right) C\left(\frac{G}{C}+j\omega\right)}$$

$$= \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)}$$

- using distortion-less condition,

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

Now, $P = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{R}{L}+j\omega\right)} = \sqrt{LC}\left(\frac{R}{L}+j\omega\right)$

(or)

$$P = \sqrt{LC}\left(\frac{G}{C}+j\omega\right).$$

$\Rightarrow \alpha + j\beta = \sqrt{LC}\left(\frac{R}{L}+j\omega\right) = \sqrt{LC}\left(\frac{G}{C}+j\omega\right)$

Equating real and imaginary parts, we get

① attenuation constant (α):

$$\alpha = \sqrt{LC}\,\frac{R}{L} \text{ (or) } \sqrt{LC}\,\frac{G}{C}$$

$$\therefore \boxed{\alpha = R\sqrt{\frac{C}{L}} \text{ (or) } G\sqrt{\frac{L}{C}}}$$

$\therefore$ Thus, α is made as frequency independent.

② phase constant (β): $\boxed{\beta = \omega\sqrt{LC}}$

Thus, β is made frequency dependent by multiplying a constant ($\sqrt{LC}$) with ω.

③ phase velocity (v_p):- $\boxed{v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$

∴ v_p is not change with frequency ⇒ No dispersion
(i.e independent of frequency)

④ characteristic impedance (Z_0):- $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$\Rightarrow Z_0 = \sqrt{\frac{L\left(\frac{R}{L}+j\omega\right)}{C\left(\frac{G}{C}+j\omega\right)}} = \sqrt{\frac{L}{C}}.$$

∴ $\boxed{Z_0 = \sqrt{\frac{L}{C}}}$ where $R_0 = \sqrt{\frac{L}{C}}$ and $X_0 = 0$

Note:- for a distortion-less line:
- phase velocity (v_p) and characteristic impedance (Z_0) are same as those for loss-less line.
- The characteristic impedance is purely real.

Condition for Minimum Attenuation:-

statement:- proof for the condition $\boxed{\frac{R}{L} = \frac{G}{C}}$

proof:- We know that,

$$\text{Attenuation constant } (\alpha) = \sqrt{\frac{1}{2}(RG-\omega^2 LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

- It is observed that, α depends on: ① 4 primary constants
② frequency

→ ∴ Therefore, analyse the condition for minimum attenuation separately for R, G, L and C.

(i) Value of 'L' for minimum attenuation:

While determining value of 'L' for minimum attenuation, assume that other three line constants C, G and R including ω are constant. i.e only 'L' should be varied.

∴ differentiating 'α' expression w.r.to 'L' and equating it to zero, we get

$$\frac{d\alpha}{dL} = 0 \quad \text{let } x = \sqrt{\tfrac{1}{2}(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

$$\Rightarrow \frac{d\alpha}{dL} = \frac{1}{2\sqrt{x}}\frac{d}{dL}\left[\tfrac{1}{2}(RG-\omega^2LC)+\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right] = 0$$

$$\Rightarrow \frac{1}{2\sqrt{x}}\left(-\omega^2C + \frac{1}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\cdot 2(G^2+\omega^2C^2)\omega^2L\right) = 0$$

$$\Rightarrow \frac{1}{2\sqrt{x}}\left(\frac{-\omega^2C\,2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}+2(G^2+\omega^2C^2)\omega^2L}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right) = 0$$

$$\Rightarrow -\cancel{\omega^2}C\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + (G^2+\omega^2C^2)\cancel{\omega^2}L = 0$$

$$\Rightarrow (G^2+\omega^2C^2)L = C\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\Rightarrow \frac{L}{C} = \frac{\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}{(G^2+\omega^2C^2)} = \sqrt{\frac{R^2+\omega^2L^2}{G^2+\omega^2C^2}}$$

$$\Rightarrow L^2G^2+\cancel{\omega^2L^2C^2} = C^2R^2+\cancel{\omega^2L^2C^2}$$

$$\Rightarrow L^2G^2 = C^2R^2 \Rightarrow \boxed{LG = RC}$$

$$\Rightarrow \boxed{\frac{R}{L} = \frac{G}{C}}$$ distortion less condition.

(or) $\boxed{L = \frac{RC}{G}}$ henries/km

Note:- Normally L value less than desired values and hence 'α' can be reduced by increasing 'L' artificially.

(ii) Value of C for minimum attenuation:

→ Now, only 'C' is varied & all other three line constants L, G, R including ω are assumed constant.

. Similar to previous, $\frac{d\alpha}{dC} = 0$ then we get

same condition $LG = RC$

$\Rightarrow \boxed{C = \frac{LG}{R}}$ farad/km

Note:- In practice 'C' value is greater than the desired value and hence 'α' can be reduced by decreasing 'C'.

(iii) Value of R and G for minimum attenuation:-

→ If either R or G is the only variable →

in such case no minimum is found by differentiating & equating to zero.

→ However, when $\boxed{R = 0 \text{ and } G = 0}$ ⇒ $\boxed{\alpha = 0}$

∴ Therefore R and G values should be kept as small as possible for minimum attenuation.

T & π Equivalent Circuits of a Transmission Line :-

→ Consider, a transmission line terminated with a load is shown as:

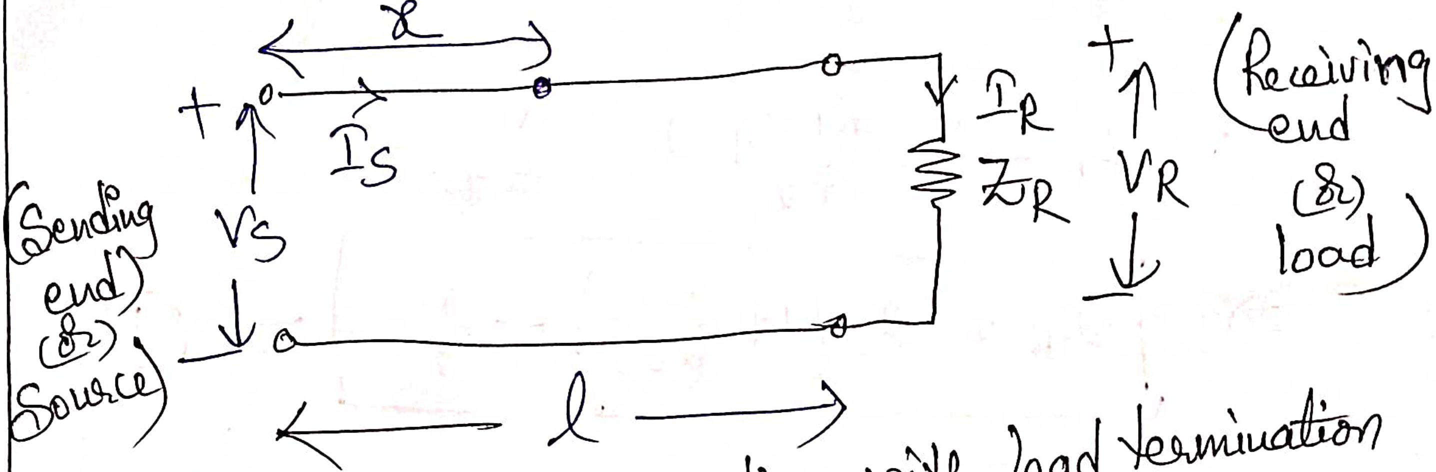


fig ①: Transmission line with load termination

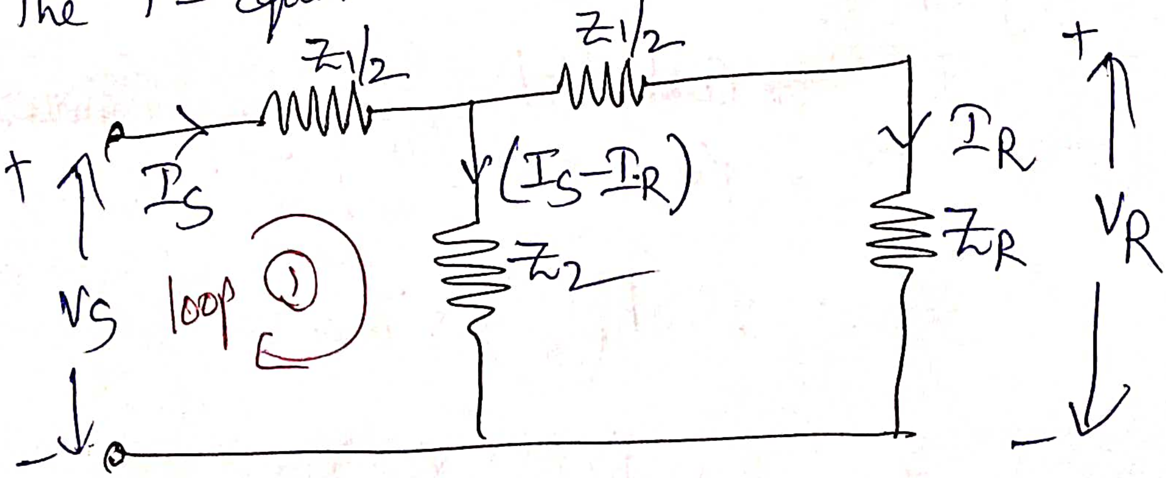
→ We know that :

- The voltage & current expressions at any distance 'x' from sending end in terms of V_S, I_S are:

$$\boxed{\begin{aligned} V &= V_S \cosh px - I_S Z_0 \sinh px \\ I &= I_S \cosh px - \frac{V_S}{Z_0} \sinh px \end{aligned}}$$

T-Equivalent circuit :-

→ The T-Equivalent circuit of the fig ① is shown as:



→ Applying KVL to loop ① :

$$V_S = I_S \frac{Z_1}{2} + (I_S - I_R) Z_2$$

$$I_R Z_2 = I_S \frac{Z_1}{2} + I_S Z_2 - V_S$$

$$\Rightarrow I_R = I_S \frac{Z_1}{2Z_2} + I_S - \frac{V_S}{Z_2}$$

$$\Rightarrow \boxed{I_R = I_S\left(1 + \frac{Z_1}{2Z_2}\right) - \frac{V_S}{Z_2}} \quad \text{--- ①}$$

at $x = l$, $\boxed{I_R = I_S \cosh pl - \frac{V_S}{Z_0} \sinh pl}$ --- ②

• By comparing Eqⁿs ① & ②, we get :

$$\boxed{\cosh pl = 1 + \frac{Z_1}{2Z_2}} \quad \text{and} \quad \text{--- ③}$$

$$\frac{1}{Z_2} = \frac{\sinh pl}{Z_0} \Rightarrow \boxed{Z_2 = \frac{Z_0}{\sinh pl}} \quad \text{--- ④}$$

rewritting Eqⁿ ③ as :

$$\frac{Z_1}{2Z_2} = \cosh pl - 1$$

$$= \cancel{1} + 2\sinh^2\left(\frac{pl}{2}\right) - \cancel{1}$$

(since, $\cosh 2x = 1 + 2\sinh^2 x$)

$$\frac{Z_1}{2} = 2\sinh^2\left(\frac{pl}{2}\right) \times Z_2$$

$$\Rightarrow Z_1 = 2\sinh^2\left(\frac{pl}{2}\right) \times 2Z_2$$

substituting ④, we get

$$Z_1 = 2\sinh^2\left(\frac{pl}{2}\right) \times 2 \times \frac{Z_0}{\sinh pl}$$

(Since, $\sinh 2x = 2\sinh x \cosh x$)

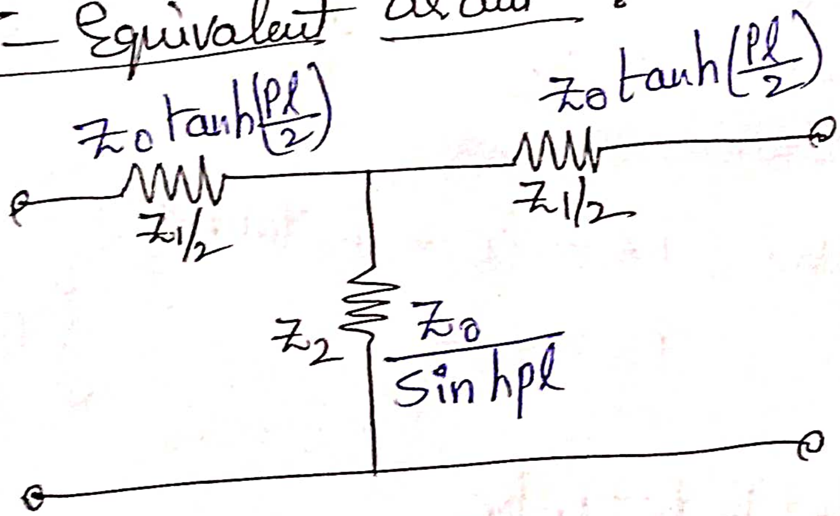
$$= \frac{\cancel{2}\sinh^{\cancel{2}}\left(\frac{pl}{2}\right) \times 2 \times Z_0}{\cancel{2}\,\cancel{\sinh\left(\frac{pl}{2}\right)}\cosh\left(\frac{pl}{2}\right)} = \frac{2Z_0\sinh\left(\frac{pl}{2}\right)}{\cosh(pl/2)}$$

$$= 2Z_0\tanh\left(\frac{pl}{2}\right)$$

$$\therefore Z_1 = 2Z_0\tanh\left(\frac{pl}{2}\right)$$

$$\Rightarrow \boxed{\frac{Z_1}{2} = Z_0\tanh\left(\frac{pl}{2}\right)} \quad — ⑤.$$

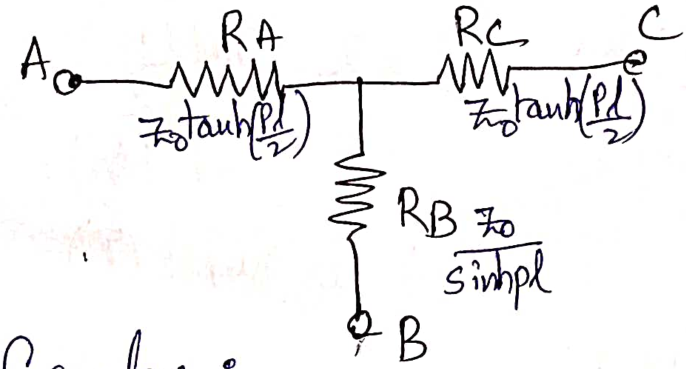
$\therefore$ T – Equivalent circuit :



Z

π – Equivalent Circuit:-

→ From star to delta conversion formulae, convert T – Equivalent circuit to π – equivalent circuit.



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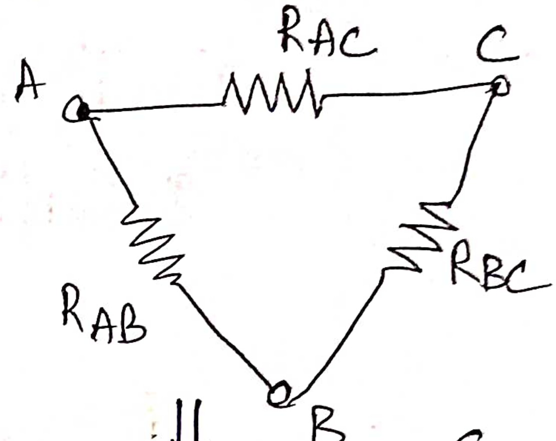
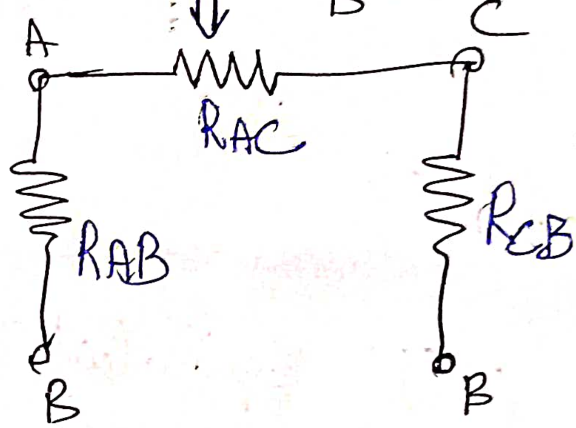


⇓



formulae:

$$R_{AB} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_C}$$

$$R_{BC} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_A}$$

$$R_{AC} = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}.$$

→ As we know: $R_A = R_C = Z_0 \tanh\left(\frac{Pl}{2}\right)$, $R_B = \frac{Z_0}{\sinh pl}$ } from T–equivalent circuit.

Let,

Numerator: $R_A R_B + R_B R_C + R_A R_C$

$$= Z_0 \tanh\left(\frac{Pl}{2}\right)\frac{Z_0}{\sinh pl} + Z_0 \tanh\left(\frac{Pl}{2}\right)\frac{Z_0}{\sinh pl} + Z_0^2 \tanh^2\left(\frac{Pl}{2}\right)$$

$$= 2\left[Z_0 \tanh\left(\frac{Pl}{2}\right)\frac{Z_0}{\sinh pl}\right] + Z_0^2 \tanh^2\left(\frac{Pl}{2}\right)$$

$$= \cancel{2} Z_0 \frac{\sinh\left(\frac{Pl}{2}\right)}{\cosh\left(\frac{Pl}{2}\right)} \frac{Z_0}{\cancel{2}\cancel{\sinh\left(\frac{Pl}{2}\right)}\cosh\left(\frac{Pl}{2}\right)} + Z_0^2 \frac{\sinh^2\left(\frac{Pl}{2}\right)}{\cosh^2\left(Pl/2\right)}$$

$$= \frac{Z_0^2}{\cosh^2\left(\frac{Pl}{2}\right)} + Z_0^2 \frac{\sinh^2\left(\frac{Pl}{2}\right)}{\cosh^2\left(\frac{Pl}{2}\right)}$$

$$= \frac{Z_0^2}{\cosh^2\left(\frac{Pl}{2}\right)}\left[1 + \sinh^2\left(\frac{Pl}{2}\right)\right]$$

$$= \frac{Z_0^2}{\cancel{\cosh^2\left(\frac{Pl}{2}\right)}}\left[\cancel{\cosh^2\left(\frac{Pl}{2}\right)}\right] = Z_0^2.$$

$\therefore$ Shunt elements

$$R_{AB} = \frac{R_AR_B + R_BR_C + R_AR_C}{R_C} = \frac{Z_0^{\cancel{2}}}{\cancel{Z_0}\tanh\left(\frac{Pl}{2}\right)}$$

$$= Z_0 \coth\left(\frac{Pl}{2}\right)$$

$$R_{CB} = \frac{R_AR_B + R_BR_C + R_AR_C}{R_A} = \frac{Z_0^{\cancel{2}}}{\cancel{Z_0}\tanh\left(\frac{Pl}{2}\right)}$$

$$= Z_0 \coth\left(\frac{Pl}{2}\right)$$

Series element

$$R_{AC} = \frac{R_AR_B + R_BR_C + R_AR_C}{R_B} = \frac{Z_0^{\cancel{2}}}{\left(\frac{\cancel{Z_0}}{\sinh pl}\right)} = Z_0 \sinh pl$$

$\therefore$ π – equivalent circuit :-