

UNIT-IV

Bridges

→ It is a simplest form consist of a network of four resistance arms forming a closed circuit, with a dc source of current applied to two opposite junctions and a current detector connected to the other two junctions.

→ Bridge circuits are used for measuring component values such as R, L and C. Bridge circuit compares the value of an unknown component with that of a known component, High accuracy is achieved by using bridges.

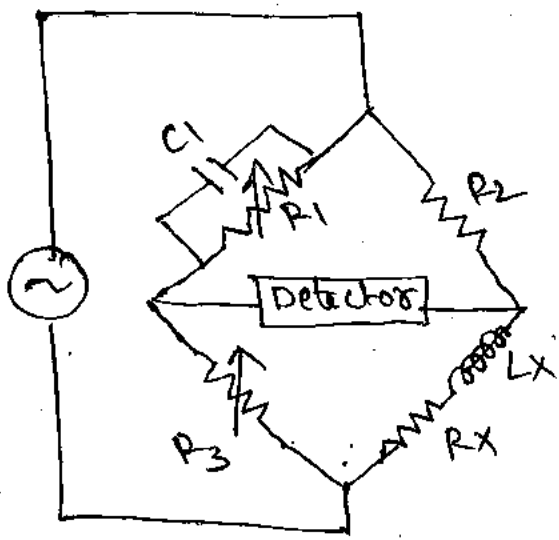
Types of Bridges

- 1) DC bridges → used for measuring resistance
- 2) AC bridges → used for measure capacitance and inductance.

DC bridges are ⇒ Wheatstone bridge, Kelvin bridge.

- AC bridges are ⇒
- ① Maxwell's bridge
 - ② Hay's bridge
 - ③ Anderson bridge
 - ④ Schering bridge
 - ⑤ Wein bridge.

① Maxwell's Bridge: C Inductance Capacitance bridge



→ It measures unknown inductance in terms of a known capacitor.

→ The resistor R1 is parallel with C1 and hence it is easy to write balance equation using arm 1.

Bridge balancing condition is

$$Z_1 Z_X = Z_2 Z_3 ; Z_X = \frac{Z_2 Z_3}{Z_1} = Z_2 Z_3 Y_1 \quad \text{--- (1)}$$

$Z_1 = R_1$ in parallel with C_1 , i.e. $Y_1 = \frac{1}{Z_1}$

$$\frac{1}{Z_1} = Y_1 = \frac{1}{R_1} + j\omega C_1$$

$\rightarrow Z_2 = R_2, Z_3 = R_3, Z_X = R_X$ in series with

$$Z_X = R_X + j\omega L_X$$

Z_X substitute in (1) i.e. L_X

$$R_X + j\omega L_X = \frac{Z_2 Z_3}{Z_1}$$

$$R_X + j\omega L_X = R_2 R_3 \left[\frac{1}{Z_1} \right]$$

$$R_X + j\omega L_X = R_2 R_3 \left[\frac{1}{R_1} + j\omega C_1 \right] \quad (\because Y_1 = \frac{1}{Z_1})$$

$$R_X + j\omega L_X = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

Equating real terms & imaginary terms

$$\boxed{R_X = \frac{R_2 R_3}{R_1}}$$

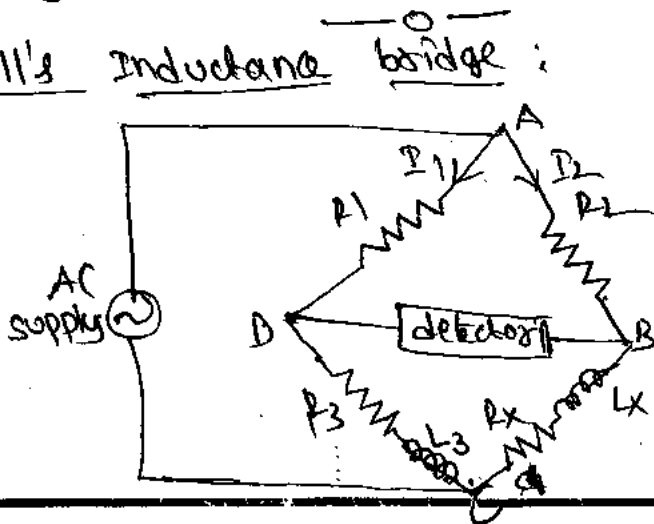
$$\boxed{L_X = C_1 R_2 R_3}$$

$$Q = \frac{\omega L_X}{R_X} = \frac{\omega C_1 R_2 R_3 \times R_1}{R_2 R_3} = \omega C_1 R_1$$

$$\boxed{Q = \omega C_1 R_1}$$

\rightarrow It is limited to measurement of low Q values (1-10). The measurement is independent of excitation frequency.

Maxwell's Inductance bridge:



$R_1, R_2,$
 R_3, L_3
 R_4, L_4

At Balance condition

$$\frac{R_1}{(R_3 + \delta) + j\omega L_3} = \frac{R_2}{R_x + j\omega L_x}$$

R_3 having internal resistance δ

$$R_1 [R_x + j\omega L_x] = R_2 ((R_3 + \delta) + j\omega L_3)$$

$$R_1 R_x + R_1 j\omega L_x = R_2 (R_3 + \delta) + j\omega L_3 R_2$$

equating imaginary term

$$R_1 j\omega L_x = j\omega L_3 R_2$$

$$R_1 L_x = L_3 R_2$$

$$L_x = \frac{L_3 R_2}{R_1}$$

it is unknown inductance value in Maxwell's induction bridge.

→ This bridge having two separate arms R_1 & R_2 third arm is resistance R_3 is series with inductance L_3 . Fourth arm is unknown inductance L_x and series resistance R_x .

$$R_1 R_x = R_2 (R_3 + \delta)$$

$$R_x = \frac{R_2}{R_1} (R_3 + \delta)$$

Advantages of Maxwell's bridge

- ① The balanced equation is independent of losses associated with inductance.
- ② The balanced equation is independent of frequency of measurement.
- ③ The scale of the resistance can be calculated to read the inductance directly.



Example problem

① A Maxwell bridge is used to measure an inductive impedance. The bridge constant at balance are $C_1 = 0.01 \mu F$, $R_1 = 470 k\Omega$, $R_2 = 5.1 k\Omega$, $R_3 = 100 k\Omega$. Find the series equivalent of the unknown impedance.

Sol!

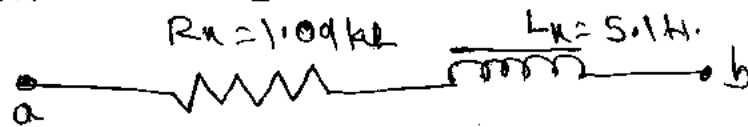
Find R_x, L_x

$$R_x = \frac{R_2 R_3}{R_1} ; L_x = C_1 R_2 R_3$$

$$R_x = \frac{100 k \times 5.1 k}{470 k} ; L_x = 5.1 k \times 100 k \times 0.01 \mu F$$

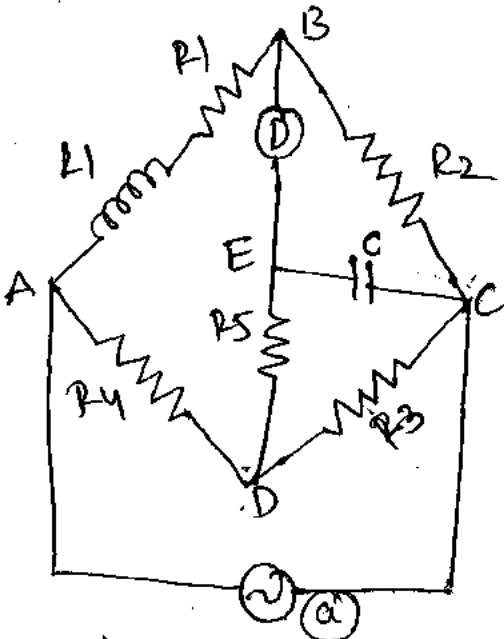
$$R_x = 1.09 k\Omega ; L_x = 5.1 H$$

equivalent series circuit is

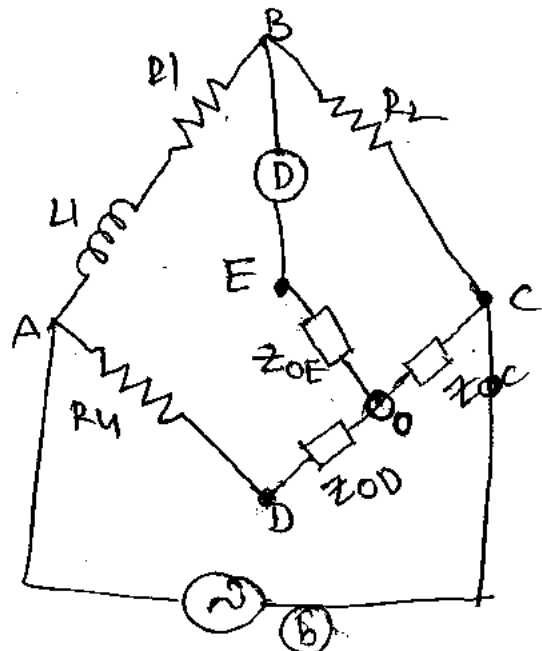


ANDERSON BRIDGE

→ It is very important and useful modification of the Maxwell Wien bridge as shown below.



→ The balancing can be obtained



Condition for this bridge by connecting the mesh

Impedances C, R_3, R_5 (boom EDC loop) to an equivalent star by using star/delta transformation.

$$Z_{OD} = \frac{R_3 R_5}{(R_3 + R_5 + \frac{1}{j\omega C})} ; Z_{OC} = \frac{R_3 / j\omega C}{(R_3 + R_5 + \frac{1}{j\omega C})} = Z_3$$

$$\Rightarrow Z_1 = (R_1 + j\omega L_1)$$

$$\Rightarrow Z_2 = R_2$$

$$\Rightarrow Z_3 = Z_{OC} = \frac{R_3 / j\omega C}{(R_3 + R_5 + \frac{1}{j\omega C})}$$

$$\Rightarrow Z_4 = R_4 + Z_{OD}$$

For balance condition

$$Z_1 Z_3 = Z_2 Z_4$$

$$(R_1 + j\omega L_1) Z_{OC} = R_2 (R_4 + Z_{OD})$$

$$\Rightarrow (R_1 + j\omega L_1) \times \left[\frac{R_3 / j\omega C}{(R_3 + R_5 + \frac{1}{j\omega C})} \right] = R_2 \left[R_4 + \frac{R_3 R_5}{(R_3 + R_5 + \frac{1}{j\omega C})} \right]$$

$$\Rightarrow (R_1 + j\omega L_1) \times \left[\begin{array}{c} \text{simplifying} \\ \frac{R_3 / j\omega C}{(R_3 + R_5 + \frac{1}{j\omega C})} \end{array} \right] = R_2 R_4 (R_3 + R_5 + \frac{1}{j\omega C}) + \left(\frac{R_2 R_3 R_5}{(R_3 + R_5 + \frac{1}{j\omega C})} \right)$$

$$\Rightarrow (R_1 + j\omega L_1) \times \frac{R_3}{j\omega C} = R_2 R_4 (R_3 + R_5 + \frac{1}{j\omega C}) + R_2 R_3 R_5$$

$$\Rightarrow \frac{R_1 R_3}{j\omega C} + \frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{R_2 R_4}{j\omega C} + R_2 R_3 R_5$$

$$\Rightarrow \frac{j R_1 R_3}{j^2 \omega C} + \frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{j R_2 R_4}{j^2 \omega C} + R_2 R_3 R_5$$

(-j is added for j terms) ($j^2 = -1$)

$$\Rightarrow \frac{-R_1 R_3}{\omega C} + \frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + \frac{j R_2 R_4}{\omega C} + R_2 R_3 R_5$$

Equating real terms A imaginary term

$$\frac{L_1 R_3}{C} = R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5$$

$$L_1 = \frac{C}{R_3} (R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5)$$

$$L_1 = \cancel{C R_2} \left(L_1 = \frac{C R_2 R_3 R_4}{R_3} + \frac{C R_2 R_4 R_5}{R_3} + \frac{C R_2 R_3 R_5}{R_3} \right)$$

$$L_1 = C R_2 R_4 + \frac{C R_2 R_4 R_5}{R_3} + C R_2 R_5$$

$$L_1 = C R_2 \left[R_4 + \frac{R_4 R_5}{R_3} + R_5 \right] ; \therefore C R_2 \left[R_4 + R_5 + \frac{R_4 R_5}{R_3} \right]$$

$$\text{Now } \frac{-j R_1 R_3}{\omega C} = \frac{-j R_2 R_4}{\omega C}$$

$$R_1 R_3 = R_2 R_4$$

Anderson
bridge

$$R_1 = \frac{R_2 R_4}{R_3}$$

$$L_1 = C R_2 \left[R_4 + R_5 + \frac{R_4 R_5}{R_3} \right]$$

Example Problem

① For an Anderson bridge, arm AB unknown inductance having resistance R_1 and inductance L_1 . arm BC, CD, DA are resistors having 1000Ω , 1000Ω and 2000Ω respectively. A Capacitor of $10\mu\text{F}$ and resistance 400Ω are connected between CE and ED. Source between A and C, $\gamma = 496$.
find L_1 & R_1 .

Sol! Given $R_2 = 2000\Omega$, $R_3 = 1000\Omega$, $R_4 = 1000\Omega$,
 $C = 10\mu\text{F}$, $\gamma = 496$

$$\Rightarrow R_1 = \frac{R_2 R_4}{R_3} = \frac{2000 \times 1000}{1000} = 2000\Omega$$

$$\Rightarrow L_1 = C R_2 \left[R_4 + R_5 + \frac{R_4 R_5}{R_3} \right]$$

$$= 10 \times 10^{-6} \times$$

Given $AB \rightarrow R_2 = 200 \Omega$
 $CD \rightarrow R_3 = 1000 \Omega$
 $DA \rightarrow R_4 = 1000 \Omega$
 $C = 10 \mu F, \delta = 496$

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{200 \times 1000}{1000} = 200 \Omega$$

$$L = \frac{CR_3}{R_4} (\delta R_4 + R_2 R_4 + \delta R_2)$$

$$= \frac{10 \times 10^{-6} \times 1000}{1000} (496 \times 10^3 + 200 \times 10^3 + 496 \times 200)$$

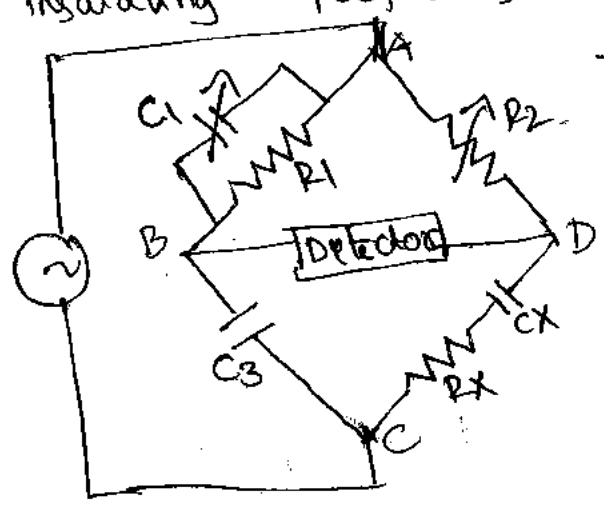
$$= 7.952 H$$

Advantages of Anderson bridge

- 1) These bridges are used for accurate measurement of Inductance
- 2) In the other bridges we use variable capacitors but in anderson bridge fixed capacitors used.
- 3) The bridge is easy to balance.

SCHERING'S BRIDGE or C measurement of capacitance (schering bridge)

→ A very important bridge used for the precision measurement of capacitors and their insulating properties is the Schering bridge.



→ The standard capacitor C_3 is high quality mica capacitor for general measurement or an air capacitor for insulation measurement.

→ For balance general equation

$$Z_1 Z_X = Z_2 Z_3 \quad \left(\begin{array}{l} C_3 \text{ has} \\ \text{internally} \\ Z_3 \end{array} \right)$$

$$Z_X = \frac{Z_2 Z_3}{Z_1}$$

→ $Z_X = Z_2 Z_3 Y_1$ (∵ $Y_1 = \frac{1}{Z_1}$)

where $Z_X = R_X + \frac{j}{\omega C_X} \Rightarrow R_X + \frac{j}{\omega C_X} = R_X - \frac{j}{\omega C_X}$ (∵ $j^2 = -1$)

$Z_2 = R_2$

$Z_3 = \frac{1}{j\omega C_3} = \frac{j}{j^2 \omega C_3} = \frac{-j}{\omega C_3}$

$Y_1 = \frac{1}{R_1} + j\omega C_1$

→ a) $Z_X = Z_2 Z_3 Y_1$

substitute then

$\underbrace{\left(R_X - \frac{j}{\omega C_X} \right)}_{Z_X} = R_2 \underbrace{\left[\frac{-j}{\omega C_3} \right]}_{Z_3} \underbrace{\left[\frac{1}{R_1} + j\omega C_1 \right]}_{Y_1}$

⇒ $\left[R_X - \frac{j}{\omega C_X} \right] = \frac{R_2 (-j)}{R_1 (\omega C_3)} + \frac{R_2 (-j) (j\omega C_1)}{\omega C_3} = \left[(j^2 = -1) \right]$

$\left[R_X - \frac{j}{\omega C_X} \right] = \frac{-R_2 j}{R_1 \omega C_3} + \frac{R_2 C_1}{C_3}$

equating real & imaginary term

$R_X = \frac{R_2 C_1}{C_3}$ and

$C_X = \frac{R_1}{R_2} C_3$ $\frac{-j}{\omega C_X} = \frac{-R_2 j}{R_1 \omega C_3}$

$\frac{1}{\omega C_X} = \frac{R_2}{R_1 \omega C_3}$

$R_2 C_X = R_1 C_3$

$C_X = \frac{R_1}{R_2} C_3$

Dissipation factor (D) = $\frac{R_X}{X_X} = \omega C_X R_X$

also D is reciprocal of quality factor

factor $D = \frac{1}{Q}$ means quality of capacitor

Example problem

- ① An Schering bridge has the following constants
arm AB - capacitor of $0.5 \mu\text{F}$ in parallel with $1 \text{ k}\Omega$
resistance, Arm AD - resistance of $2 \text{ k}\Omega$, Arm BC -
capacitor of $0.5 \mu\text{F}$, arm CD - unknown capacitor C_x and
 R_x in series frequency - 1 kHz , Determine the unknown
capacitance and dissipation factor.

Sol:

$$\text{From Schering bridge } R_x = \frac{R_2 C_1}{C_3}$$

$$C_x = \frac{R_1}{R_2} C_3$$

$$R_x = \text{Given: arm AB} \rightarrow C_1 = 0.5 \mu\text{F}$$

$$R_1 = 1 \text{ k}\Omega$$

$$\text{arm AD} \rightarrow R_2 = 2 \text{ k}\Omega$$

$$\text{arm BC} \rightarrow C_3 = 0.5 \mu\text{F}$$

$$\text{arm CD} \rightarrow \text{find } C_x, f = 1 \text{ kHz}$$

$$R_x = \frac{2 \text{ k} \times 0.5 \times 10^{-6}}{0.5 \times 10^{-6}}$$

$$\boxed{R_x = 2 \text{ k}\Omega}$$

$$C_x = \frac{1 \text{ k}\Omega}{2 \text{ k}\Omega} \times 0.5 \times 10^{-6}$$

$$\boxed{C_x = 0.25 \mu\text{F}}$$

Dissipation factor is given by

$$D = \omega C_x R_x$$

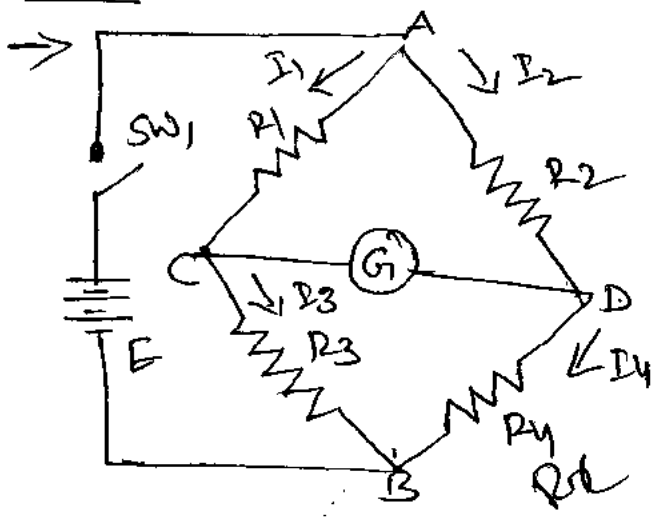
$$= 2\pi f \times C_x R_x$$

$$= 2 \times 3.14 \times 1 \text{ k} \times 0.25 \times 10^{-6} \times 2 \times 10^3$$

$$\boxed{D = 3.1416}$$

—————o—————

*Wheat stone bridge (Measurement of resistance)



① Balanced wheatstone bridge

→ This is a dc bridge used for accurate measurement of resistance and it is called wheatstone bridge.

→ The source of emf and switch is connected to point A & B, galvanometer is connected between point

→ #when switch SW1 is closed, current C/D flows & divides into two arms at point 'A', i.e. I_1 & I_2 . The bridge is balanced when no current through the galvanometer.

→ To obtain bridge balance equation

$$I_1 R_1 = I_2 R_2$$

Galvanometer current to be zero,

$$I_1 = I_3 = \frac{E}{R_1 + R_3}$$

$$I_2 = I_4 = \frac{E}{R_2 + R_4}$$

$$\text{Now } \left(\frac{E}{R_1 + R_3} \right) R_1 = \left(\frac{E}{R_2 + R_4} \right) R_2 \Rightarrow (I_1 R_1 = I_2 R_2)$$

$$R_1 (R_2 + R_4) = R_2 (R_1 + R_3)$$

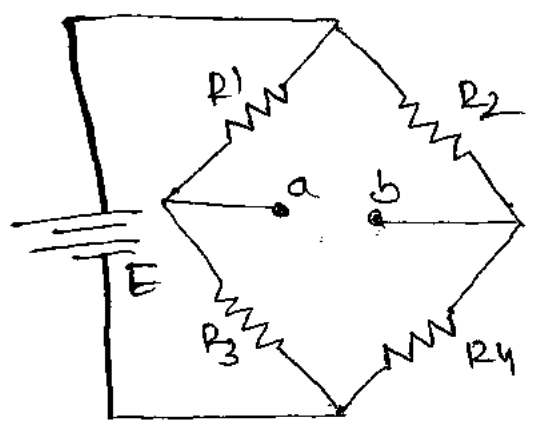
$$R_1 R_2 + R_1 R_4 = R_1 R_2 + R_2 R_3$$

$$R_1 R_4 = R_2 R_3$$

$$R_4 = \frac{R_2 R_3}{R_1}$$

$$\text{Now } R_x = \frac{R_2 R_3}{R_1}$$

Unbalanced Wheatstone Bridge



→ For obtaining the unbalanced wheatstone conditions, use the Thevenin's theorem.

→ For determining the current through the galvanometer, we have to find the thevenin's equivalent.

→ Thevenin's equivalent voltage is found by disconnecting the galvanometer from bridge circuit. From the circuit determining the open circuit voltage between terminals a and b.

→ Voltage at point a

$$E_a = \frac{E \times R_3}{R_1 + R_3}$$

at point b

$$E_b = \frac{E \times R_4}{R_2 + R_4}$$

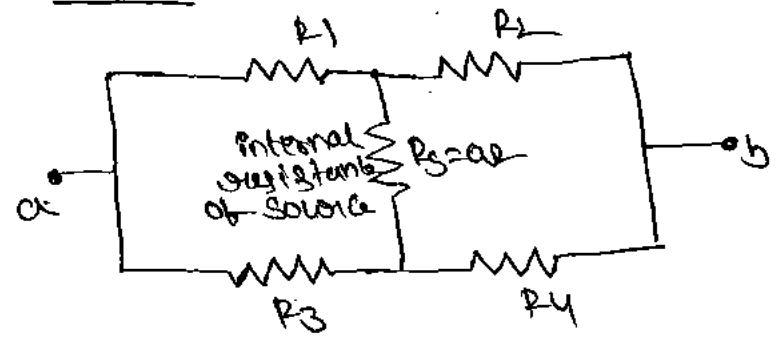
→ Thevenin's equivalent

$$E_{th} = E_{ab} = E_a - E_b$$

$$E_{ab} = \frac{E \times R_3}{R_1 + R_3} - \frac{E \times R_4}{R_2 + R_4}$$

$$E_{ab} = E \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]$$

→ Thevenin's resistance



→ Now $R_1 // R_3$ in series with $R_2 // R_4$ is

$$\underline{R_1 // R_3 + R_2 // R_4}$$

$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

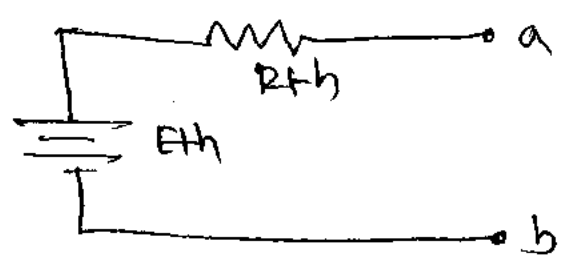


Fig: Thevenin's equivalent

→ The deflection in the galvanometer is.

$$I_g = \frac{E_{Th}}{R_{Th} + R_g}$$

(∵ R_g is galvanometer resistance)

* Sensitivity of wheatstone Bridge

→ when the bridge is in unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer. The amount of deflection depends to the sensitivity of galvanometer.

→ sensitivity is defined as deflection per unit current.

$$\text{Sensitivity (S)} = \text{mm}/\mu\text{A}$$

$$\text{Deflection (D)} = S \times I$$

I is current in micro amperes.

* Applications of wheatstone bridge:

- 1) used to measure the dc resistance of various wires
- 2) used in telephone companies for locate cable fault.

Limitations of wheatstone bridge

- 1) For low resistance measurements, the resistance contact becomes significant and

introduces an error. This can be eliminated by Kelving double bridge.

4) For high resistance measurement, the resistance in the bridge becomes so large and galvanometer is insensitive to imbalance. If this high resistance in mega ohms, wheatstone bridge is can't be used.

Example problems

① The wheatstone ^{balance} bridge have $R_1 = 10k$, $R_2 = 15k \Omega$, $R_3 = 40k$, find the unknown resistance R_x .

→ For balanced wheatstone bridge

$$R_1 R_x = R_2 R_3 \quad \text{ie } R_1 R_x = R_2 R_3$$

$$\Rightarrow R_x = \frac{R_2 R_3}{R_1} = \frac{15k \times 40k}{10k} = 60k \Omega$$

② An unbalanced wheatstone bridge have the parameter $R_1 = 1k$, $R_2 = 2.5k$, $R_3 = 3.5k$, $R_4 = 10k$, $R_g = 300 \Omega$, Resistance in galvanometer = R_g - voltage source $E = 6V$. find current through galvanometer.

Sol:

$$E_{th} = E_a - E_b = E_b - E_a$$

$$E_{th} = E \left[\frac{R_4}{R_2 + R_4} - \frac{R_3}{R_1 + R_3} \right]$$

$$E_{th} = 6 \left[\frac{10k}{2.5k + 10k} - \frac{3.5k}{1k + 3.5k} \right] = 0.132V$$

Thevening equivalent resistance R_{th} is

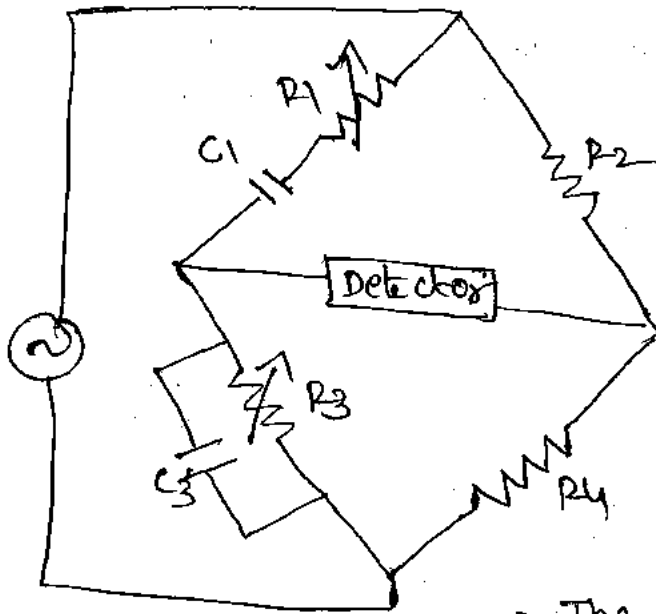
$$R_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$R_{th} = \frac{1k \times 3.5k}{1k + 3.5k} + \frac{2.5k \times 10k}{2.5k + 10k}$$

$$R_{th} = 2.778k \Omega$$

$$\text{Current } (I_g) = \frac{E_{th}}{R_{th} + R_g} = \frac{0.132V}{2.778k + 300\Omega} = 42.88 \mu A$$

Wien's Bridge



→ Wien bridge has a series RC combination in one arm and parallel combination in adjoining arm.

→ It is designed to measure the frequency, and also used for measurement of unknown frequency capacitor

→ The impedance of one arm is

$$Z_1 = R_1 + \frac{1}{j\omega C_1} = R_1 + \frac{j}{\omega C_1}$$

$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

→ admittance of parallel arm is

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

→ bridge balance condition

$$Z_1 Z_4 = Z_2 Z_3 \Rightarrow Z_2 = \frac{Z_1 Z_4}{Z_3}$$

$$Z_2 = \frac{Z_1 Z_4}{Y_3} \Rightarrow Z_2 = Z_1 Z_4 Y_3$$

$$\rightarrow R_2 = R_4 \left(R_1 - \frac{j}{\omega C_1} \right) \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_4 R_1}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\omega C_3 R_4 R_1 +$$

$$R_2 = \left(R_4 R_1 - \frac{R_4 j}{\omega C_1} \right) \left(\frac{1}{R_3} + j\omega C_3 \right)$$

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{R_4 R_1}{j\omega C_1 R_3} - \frac{R_4 j}{R_3 \omega C_1} - \frac{R_4 j^2 \omega C_3}{\omega C_1}$$

$$R_2 = \left[\frac{R_4 R_1}{R_3} + \frac{R_4 C_3}{C_1} \right] - \frac{R_4 j}{R_3 \omega C_1} + \frac{j R_4 R_1}{\omega C_1}$$

$$R_2 = \left[\frac{R_4 R_1}{R_3} + \frac{R_4 C_3}{C_1} \right] - j \left[\frac{R_4}{\omega C_1 R_3} + \frac{R_4 R_1}{\omega C_1} \right]$$

$$R_2 = \frac{R_4 R_1}{R_3} + R_4 R_1 j \omega C_3 - \frac{R_4 j}{\omega C_1 R_3} - \frac{j^2 \omega C_3 R_4}{\omega C_1} \quad (j^2 = -1)$$

$$R_2 = \left[\frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1} \right] + j \left[\frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 \right]$$

equating real & imaginary terms

$$\Rightarrow R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \quad \times \quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0$$

$$\Rightarrow R_2 = R_4 \left(\frac{R_1}{R_3} + \frac{C_3}{C_1} \right)$$

$$\Rightarrow \frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1} \quad \text{--- (1)}$$

$$\Rightarrow \frac{R_4}{\omega C_1 R_3} = \omega C_3 R_1 R_4 \Rightarrow \frac{1}{\omega C_1 R_3} = \omega C_3 R_1$$

$$\omega^2 C_1 C_3 R_1 R_3 = 1$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3} \Rightarrow \omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

$$\omega = 2\pi f$$

$$f = \frac{1}{2\pi \sqrt{C_1 C_3 R_1 R_3}}$$

Now find for R_3 & C_3 .

$$\omega^2 C_1 C_3 R_1 R_3 = 1$$

$$C_3 = \frac{1}{\omega^2 C_1 R_1 R_3}, \quad R_3 = \frac{1}{\omega^2 C_3 R_1}$$

C_3 substitute in eqn (1)

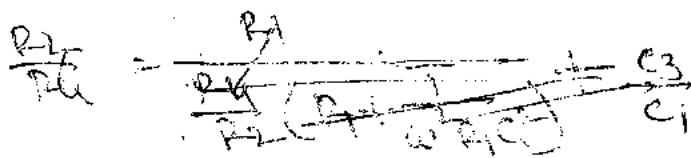
$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{1}{\omega^2 C_1^2 R_1 R_3}$$

$$\frac{R_2}{R_4} = \frac{1}{R_3} \left(R_1 + \frac{1}{\omega^2 C_1^2 R_1} \right)$$

$$R_3 = \frac{R_4}{R_2} \left(R_1 + \frac{1}{\omega^2 C_1^2 R_1} \right)$$

Now R_3 is substitute in eqn (1) for C_3

$$\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}$$



$$R_4 \left(\frac{R_1}{R_3} + \frac{C_3}{C_1} \right) = \frac{R_2}{R_4} = \frac{R_1}{1/\omega^2 C_1 C_3 R_3} + \frac{C_3}{C_1}$$

$$\frac{R_2}{R_4} = \omega^2 C_1 C_3 R_1^2 + \frac{C_3}{C_1}$$

$$\frac{R_2}{R_4} = C_3 \left(\frac{1}{C_1} + \omega^2 C_1 R_1^2 \right)$$

$$C_3 = \frac{R_2}{R_4} \left(\frac{C_1}{1 + \omega^2 C_1 R_1^2} \right)$$

Example problem

① A Wien bridge circuit consists of $R_1 = 4.7 \text{ k}\Omega$, $C_1 = 5 \text{ nF}$, $R_2 = 20 \text{ k}\Omega$, $C_3 = 10 \text{ nF}$, $R_3 = 10 \text{ k}\Omega$, $R_4 = 100 \text{ k}\Omega$ find freq. of component.

Sol:

$$f = \frac{1}{2\pi \sqrt{C_1 R_1 R_3 C_3}}$$

$$f = \frac{1}{2\pi \sqrt{5 \times 10^{-9} \times 4.7 \times 10^3 \times 10 \times 10^{-9} \times 10 \times 10^{-9}}}$$

$$f = 3.283 \text{ kHz}$$

② Find the equivalent parallel resistance and capacitance that causes a Wien bridge to null with the following component values $R_1 = 3.1 \text{ k}\Omega$, $C_1 = 5.2 \text{ nF}$, $R_2 = 25 \text{ k}\Omega$, $R = 2.5 \text{ kHz}$, $R_4 = 100 \text{ k}\Omega$

Solution :

$$R_3 = \frac{R_4}{R_2} \left(R_1 + \frac{1}{\omega^2 R_1 C_1^2} \right)$$

$$\begin{aligned} \omega &= 2\pi f \\ &= 2 \times 3.14 \times 2500 \\ \omega &= 15.71 \text{ k rad/s} \end{aligned}$$

$$R_3 = \frac{100\text{k}}{25\text{k}} \left(3.1\text{k} + \frac{1}{(15.71\text{k})^2 \times 3.1\text{k} \times (5.2 \times 10^{-6})^2} \right)$$

$$R_3 = 12.6\text{k}\Omega$$

$$C_3 = \frac{R_2}{R_4} \left(\frac{C_1}{1 + \omega^2 R_1^2 C_1^2} \right)$$

$$C_3 = \frac{25\text{k}}{100\text{k}} \left(\frac{5.2 \times 10^{-6}}{1 + (15.71\text{k})^2 \times (3.1\text{k})^2 \times (5.2 \times 10^{-6})^2} \right)$$

$$C_3 = 20.3\text{ pF}$$

Q meter :

→ It is an instrument designed to measure some electrical properties of coils and capacitors.

→ The voltage drop across the coil or capacitor is Q times the applied voltage.

$$Q = \frac{\text{Reactance (XL)}}{\text{Resistance (R)}}$$

→ If a fixed voltage is applied to the circuit, a voltmeter across the capacitor can be calibrated to read Q directly.

→ At resonance $X_L = X_C$
& $E_L = I X_L$

$$E_C = I X_C \rightarrow X_C = \frac{E_C}{I}$$

$$E = I R \rightarrow R = \frac{E}{I}$$

- E is applied voltage
- E_L is Inductive Voltage
- X_C is Capacitive reactance
- I is Circuit Current
- E_C is Capacitor voltage

X_L is inductive reactance

R is coil resistance.

$$\rightarrow Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C / I}{E / I} = \frac{E_C}{E}$$

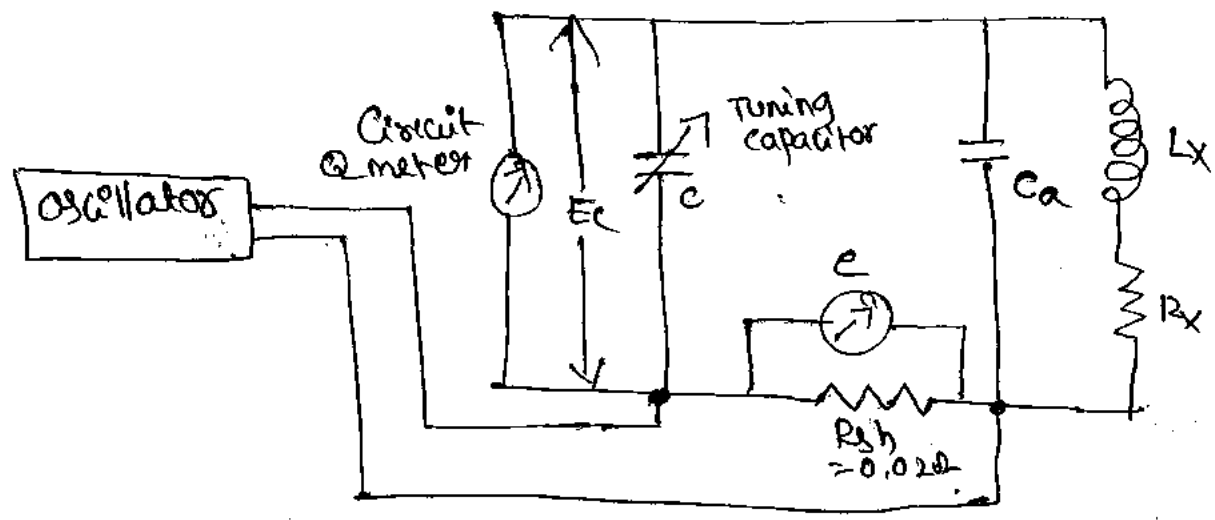


Fig: Q-meter

→ From the circuit, oscillator have the frequency range from 50KHz to 50MHz, delivers current to resistance R_{sh} having a value of 0.02Ω .

→ The voltage across shunt is measured with a thermocouple meter.

→ The voltage across capacitor is measured by an electronic voltmeter corresponding to E_C and calibrated directly to read Q .

→ The circuit is tuned to resonance by varying C until the electronic voltmeter reads the maximum value.

→ The resonance output voltage E corresponding to E_C is $E = Q \times e$

i.e. $Q = \frac{E}{e}$ e is known

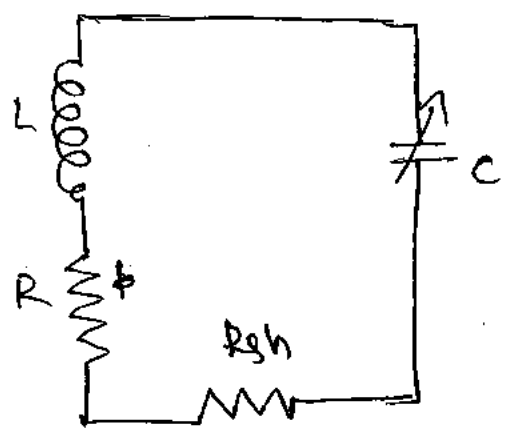
- The inductance of the coil determined by connecting it to the terminals of instrument. The circuit is tuned to resonance by varying either the capacitance or the oscillator frequency.
- If the capacitance is varied the oscillator frequency is adjusted to given frequency and resonance is obtained. If the capacitance set to desired value, oscillator frequency is varied until resonance occurs.
- The inductance of the coil can be calculated from known values of the coil frequency and resonating capacitance.

$$X_L = X_C \text{ at } \phi = 0$$

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ or } L = \frac{1}{(2\pi f)^2 C}$$

→ The actual Q of coil is greater than the indicated Q.

Effect of R_{sh} on Q:



$$Q_{act} = \frac{\omega L}{R}$$

$$Q_{obs} = \frac{\omega L}{R + R_{sh}}$$

$$\text{Now } \frac{Q_{act}}{Q_{obs}} = \frac{\omega L / R}{\omega L / (R + R_{sh})}$$

$$\frac{Q_{act}}{Q_{obs}} = \frac{R + R_{sh}}{R} = 1 + \frac{R_{sh}}{R}$$

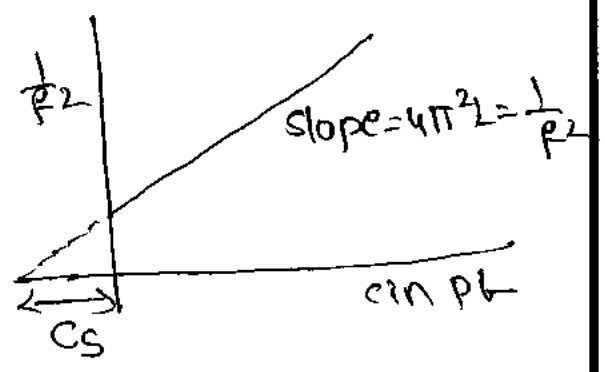
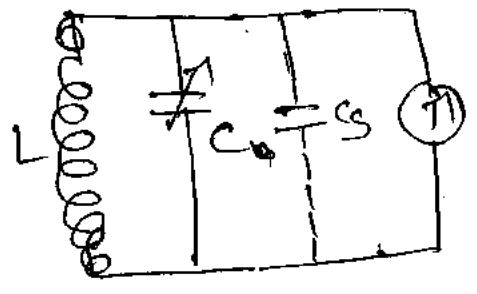
$$\Rightarrow Q_{act} = Q_{obs} \left[1 + \frac{R_{sh}}{R} \right]$$

Q_{act} is actual Q

Q_{obs} is observed Q

→ To make the Q_{obs} value as close as possible to Q_{act} , R_{sh} should be made as small as possible. An R_{sh} value of $0.02 - 0.04 \Omega$ introduces negligible errors.

Measurement of stray capacitance (C_s) distributed capacitance (C_s).



$$f = \frac{1}{2\pi\sqrt{LC}}$$

Total capacitance of circuit $C = C_1 + C_s$

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_s)}}$$

→ After oscillator, tuning capacitor are varied for new value of resonance, now

$$C = C_2 + C_s$$

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_s)}}$$

$$\text{But } f_2 = 2f_1$$

→ oscillator frequency of Q meter is increased to twice the original frequency

$$\frac{1}{2\pi\sqrt{L(C_2 + C_s)}} = 2 \cdot \frac{1}{2\pi\sqrt{L(C_1 + C_s)}}$$

$$2\pi\sqrt{L(C_2 + C_s)} = \pi\sqrt{L(C_1 + C_s)}$$

$$2\sqrt{L(C_2 + C_s)} = \sqrt{L(C_1 + C_s)}$$

squaring both side

$$4(L(C_2 + C_s)) = L(C_1 + C_s)$$

$$4(C_2 + C_s) = C_1 + C_s$$

$$4C_2 + 4C_s = C_1 + C_s$$

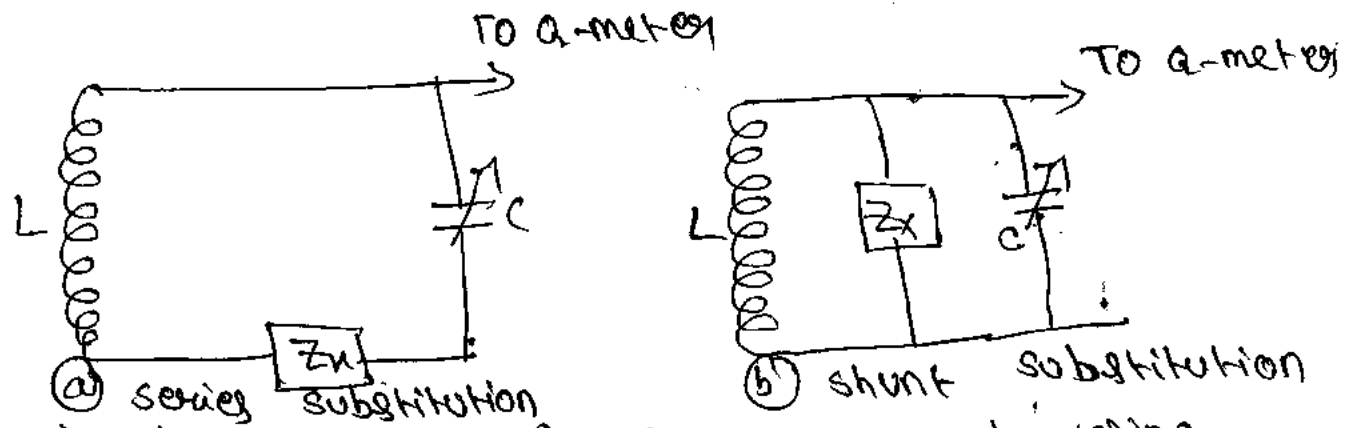
$$4C_2 + 4C_s - C_1 - C_s = 0$$

$$4C_2 - C_1 + 3C_s = 0$$

$$3C_s = C_1 - 4C_2$$

$$C_s = \frac{C_1 - 4C_2}{3}$$

Impedance Measurement using Q-meter



→ An unknown impedance measured using Q-meter, either by series or shunt substitution method.

→ The unknown impedance Z_x is determined by individually determining the components R_x & L_x .

→ From series substitution unknown impedance Z_x is not connected and tuned circuit is adjusted for resonance at oscillator frequency. The value of Q & C are noted.

→ The unknown impedance is connected then capacitor is varied for resonance, and new values Q' & C' are noted.

From fig (a) $\omega L = \frac{1}{\omega C}$ — (1)

fig (b) $\omega L + X_x = \frac{1}{\omega C'}$ — (2)

subtracting (2) - (1)

$$\omega L + X_k - \omega L = \frac{1}{\omega c'} - \frac{1}{\omega c}$$

$$X_k = \frac{1}{\omega c} \left(\frac{c}{c'} - \frac{c'}{c} \right)$$

$$X_k = \frac{1}{\omega c} \left(\frac{c-c'}{c'} \right) = \frac{1}{\omega} \left(\frac{c-c'}{cc'} \right)$$

Since $R' = R + R_k$

$$R_k = R' - R$$

→ R is resistance of auxiliary coil

$$\Rightarrow R_k = R' - R = \frac{\omega L}{Q'} \text{ and } R = \frac{\omega L}{Q}$$

$$R_k = \frac{\omega L}{Q'} - \frac{\omega L}{Q} = \omega L \left[\frac{1}{Q'} - \frac{1}{Q} \right]$$

$$R_k = \frac{\omega L}{\omega L} \left[\frac{Q-Q'}{Q Q'} \right]$$

→ The unknown impedance $Z_k = R_k + jX_k$

→ Y_k represents the sum admittance of unknown impedance. It consists of two shunt elements Conductance G_k and Susceptance B_k .

$$Y_k = G_k + jB_k$$

$$G_k = \frac{1}{\omega L} \left(\frac{Q-Q'}{Q Q'} \right)$$

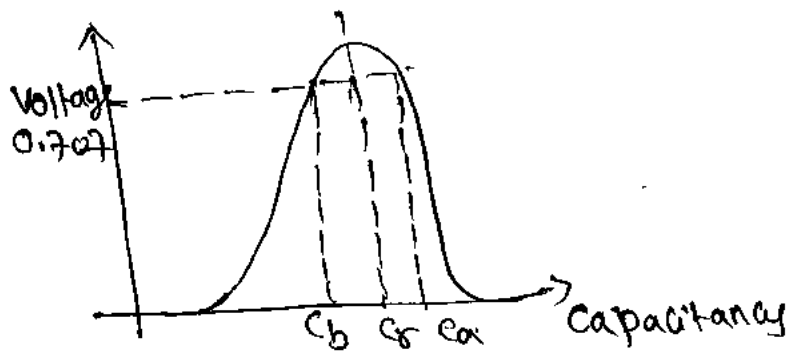
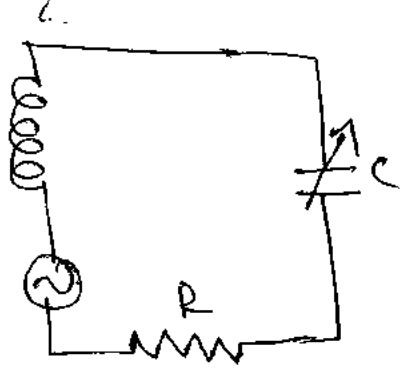
$$B_k = \omega c - \omega c'$$

$$Y_k = \frac{Q-Q'}{\omega L(Q Q')} + j(\omega c - c')$$

Measurement of Q by Susceptance method

→ The coil under test is connected in series with a capacitor. meter is an indicator, the circuit is tuned to resonance to the oscillator frequency, by tuning the variable capacitor to a

Value C_0 .



a) Susceptance method of a measurement

b) Resonance curve

→ The capacitor is then detuned to a value C_b on the resonant voltage, which meter reading falls to 70.7% of the resonant voltage. Now capacitor is set to high capacitance value of C_a and the deflection drops to 70.7% of resonant voltage.

→ Capacitance at resonance

$$X_{Ca} = \frac{1}{\omega C_a} \quad \& \quad X_{Cb} = \frac{1}{\omega C_b}$$

At half power point

$$\omega L - \frac{1}{\omega C_a} = R \quad \text{and} \quad \frac{1}{\omega C_b} - \omega L = R$$

adding two equations

$$\omega L - \frac{1}{\omega C_a} + \frac{1}{\omega C_b} - \omega L = 2R$$

$$\Rightarrow \frac{1}{\omega C_b} - \frac{1}{\omega C_a} = 2R$$

but $C_0^2 = C_a C_b$.

↳ value of capacitance at resonance.

$$\Rightarrow \frac{-\omega C_a - \omega C_b}{\omega^2 C_a C_b} = 2R$$

$$\frac{C_a - C_b}{\omega C_a C_b} = 2R$$

$$\Rightarrow \frac{C_a - C_b}{\omega C_0^2 R} = 2$$

but $Q = \frac{1}{\omega C R}$

Now $\Rightarrow \frac{Q(C_a - C_b)}{C_b} = 2$

$Q(C_a - C_b) = 2 C_b$

$Q = \frac{2 C_b}{C_a - C_b}$

(41)

* Precautions to be taken when using a bridge

- Assuming that a suitable method of measurement has been selected and that source & detector are given, therefore some precautions must be observed to obtain accurate readings.
- The leads should be carefully laid out in such way that no loops with magnetic flux are produced
- With large L, self-capacitance of leads is more important than their inductance,
- In measuring a capacitor, lead capacitance as low as possible. For this reason leads should not be too close together and should be made of fine wire.
- In inductive and capacitive measurement, leads are encased in metal tubes to shield them from mutual electromagnetic action.

(42)

* Sources of Error — o —

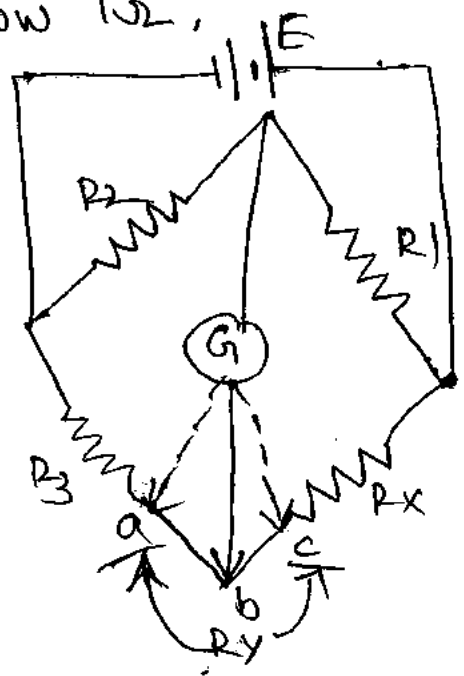
- 1) In sufficient knowledge of parameters and design conditions
- 2) poor design
- 3) change in process parameters
- 4) poor maintenance.

- 5) Errors caused by person operating the instrument
- 6) certain design limitations.

Kelvin's Bridge

→ A Modified form of Wheatstone's bridge is the kelvin's bridge.

→ It is used to measure the value of resistance below 1Ω.



→ Ry represents the resistance of the connecting leads from R3 to Rx

→ galvanometer connected to point a and point c and b.

→ when it is connected to a point a, Ry is added to Rx

→ when it is connected to point c, Ry is added to bridge arm R3.

→ when it is connected to point b in between c and a, then

The ratio of resistance from c to b (Rcb) is equal to ratio of from resistance from a to b of R1/R2 then

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$

$$R_{cb} \rightarrow (R_1 + R_{cb})$$

$$R_{ab} \rightarrow (R_3 + R_{ab})$$

$$\rightarrow \frac{R_1 + R_{cb}}{R_3 + R_{ab}} = \frac{R_1}{R_2}$$

$$(R_1 + R_{cb}) = \frac{R_1}{R_2} (R_3 + R_{ab}) \quad \text{--- (1)}$$

~~R_{cb}~~ but $R_{ab} + R_{cb} = R_y$ & $\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$

add '1' to the above equation

$$\frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \Rightarrow \frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

but $R_{cb} + R_{ab} = R_y$ — (a)

$$\Rightarrow \frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$R_{ab} = \frac{R_2 R_y}{R_1 + R_2}$$

\Rightarrow Now sub R_{ab} in eq'n (a)

$$R_{cb} + \frac{R_2 R_y}{R_1 + R_2} = R_y$$

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$R_{cb} = \frac{R_1 R_y + \cancel{R_2 R_y} - \cancel{R_2 R_y}}{R_1 + R_2} = \frac{R_1 R_y}{R_1 + R_2}$$

$$R_{cb} = \frac{R_1 R_y}{R_1 + R_2}$$

Now sub R_{ab} & R_{cb} in eq'n (b).

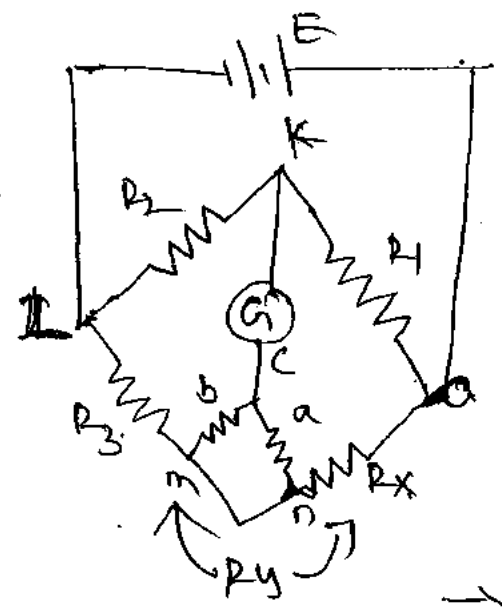
$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2} \left(R_3 + \frac{R_2 R_y}{R_1 + R_2} \right)$$

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_2 R_y}{R_2 (R_1 + R_2)}$$

$$R_x = \frac{R_1 R_3}{R_2}$$

— 0 —

Kelvin's Double bridge



→ ^{army} a, b connects the galvanometer to a point c at the potential between m & n connection i.e Ry. The ratio of resistance of arms a & b is same as the ratio of R1 & R2. The galvanometer indication is zero when the potentials at k & c are equal

$$\rightarrow E_{LK} = E_{Lmc}$$

But $E_{LK} = \frac{R_2}{R_1 + R_2} \times E$

$$E = I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right]$$

Now sub E in E_{LK}

$$E_{LK} = \frac{R_2}{R_1 + R_2} \left[I \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \right]$$

Similarly $E_{Lmc} = I \left[R_3 + \frac{b}{a+b} \left[\frac{(a+b)R_y}{a+b+R_y} \right] \right]$

$$E_{LK} = E_{Lmc}$$

$$= \frac{R_2}{R_1 + R_2} \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] = \left[R_3 + \frac{b}{a+b} \frac{(a+b)R_y}{(a+b+R_y)} \right]$$

$$= R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2} \left[R_3 + \frac{b R_y}{a+b+R_y} \right]$$

$$= R_x + \frac{(a+b)R_y}{a+b+R_y} + \frac{R_3}{R_2} = \frac{R_1 R_3}{R_2} + \frac{R_3}{R_2} + \frac{b R_y}{R_2(a+b+R_y)} + \frac{b R_y}{(a+b+R_y)}$$

$$R_3 + R_1 + \frac{(a+b)R_1}{a+b+R_1} = \left(\frac{R_1}{R_2} + 1\right)$$

$$R_N = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_1}{R_2 (a+b+R_1)} + \frac{b R_1}{(a+b+R_1)} - \frac{(a+b) R_1}{a+b+R_1}$$

$$R_N = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_1}{R_2 (a+b+R_1)} + \frac{\cancel{b R_1} - a R_1 - \cancel{b R_1}}{(a+b+R_1)}$$

$$R_N = \frac{R_1 R_3}{R_2} + \frac{b R_1 R_1}{R_2 (a+b+R_1)} - \frac{a R_1}{(a+b+R_1)}$$

$$R_N = \frac{R_1 R_3}{R_2} + \frac{b R_1}{(a+b+R_1)} \left[\frac{R_1}{R_2} - \frac{a}{b} \right]$$

$$\text{but } \frac{R_1}{R_2} = \frac{a}{b}$$

$$R_N = \frac{R_1 R_3}{R_2}$$

— 0 —

COUNTERS

* Counters are the ~~first~~ electronic circuits designed to count the frequency values using the clock pulses.

FREQUENCY COUNTER

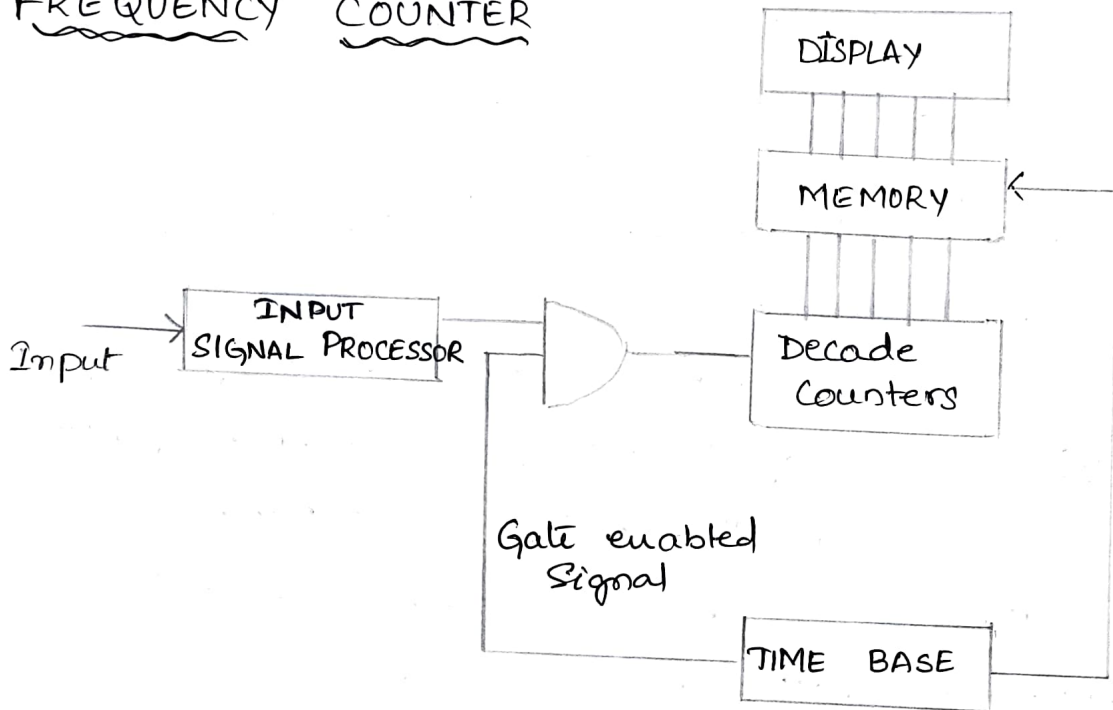


fig: Basic block of Frequency Counters.

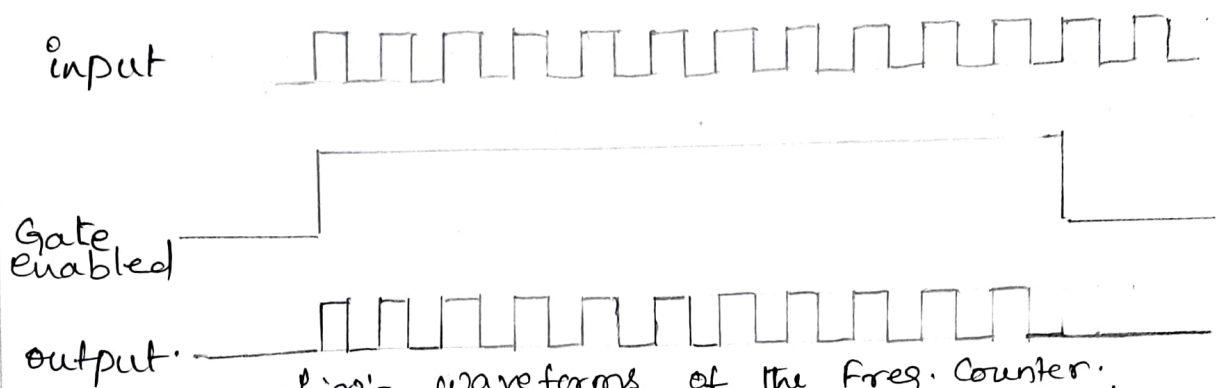
The figure 1 shows the block diagram of a simple frequency counter.

Principle of operation

* The frequency counter operates on the principle of "gating the input frequency into the counter for a pre-determined time".

Eg:- If an unknown frequency was gated into the counter for an exact 1 second, the number of counts allowed into counter would be the frequency of the input.

* The term "gated" stems from the fact that an AND or an OR gate is used in circuit



figs - waveforms of the Freq. Counter.
The waveforms in figs shows the use of AND gate. working of the Counter.

* As long as the Gate enabled signal is of 1-sec pulse and of logic 1, the output of AND gate is same as the unknown input.

* When the 1-sec pulse returns to logic 0, The output of AND gate is 0.

* Therefore, exactly 1 sec of unknown input pulses is allowed at the output of AND gate.

* These pulses are to be counted and displayed.

* When the Gate enabled signal is of more than 1 sec duration, then the decimal point of the display is switched with the gate time selector switch to ~~count~~ correct the frequency display.

Eg: Gate was open for 10 sec, the accumulated count would be average frequency is 0.1 Hz.

i. DECADE COUNTERS

* The heart of the Frequency Counter is the decade counter.

* The output of the decade counter follows the sequence in fig.

clock	Counter State			
	D	C	B	A
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

fig: Binary coded decimal counting sequence.

Eg: decimal no 138 is coded as
0001 0011 1000 in BCD.

* Each BCD counter allows one decade of counting. So, BCD counters must be cascaded.

Eg: To count between 0-999 three BCD counters are required.

* There are two methods of cascading BCD counters.

1. Ripple cascading
2. Synchronous cascading

1. Ripple Cascading:

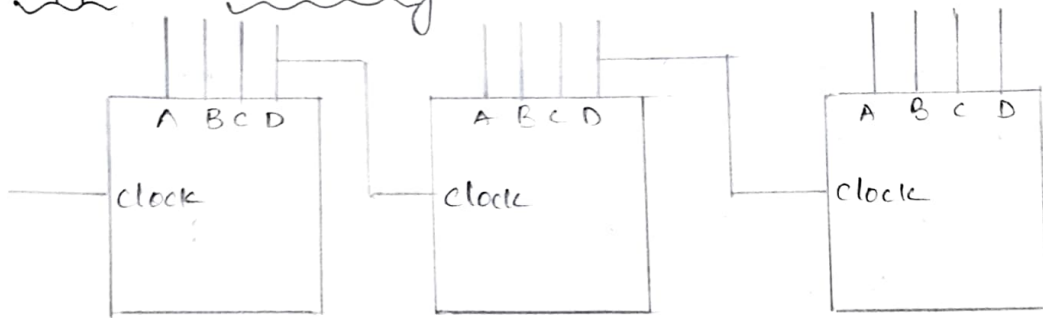


fig 4: Cascading Ripple Counters.

- * This type of cascading is very slow.
- * This is used only for low-frequency measurements. It is not used for serious frequency measurement equipment.
- * In this type of cascading, the last output of the least significant counter is used to derive the clock input of the next significant counter.

2. Synchronous Cascading:

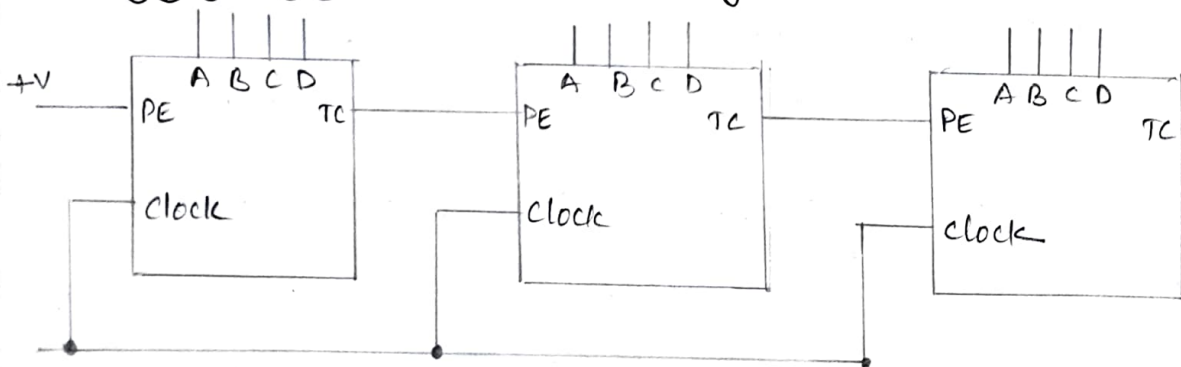


fig 5: Cascaded Synchronous Counters.

- * A Terminal Count (TC) or Carry output is used for the purpose of cascading.
- * This output is used to enable the following counter to be incremented on the next clock pulse.

* This insures that the state of the next counter is coincident with the clock.

Logic diagram of Binary Synchronous Counter

The Binary Synchronous Counter requires that all the flip-flop clocks be connected together which greatly reduces the propagation delay and allows higher counting speeds.

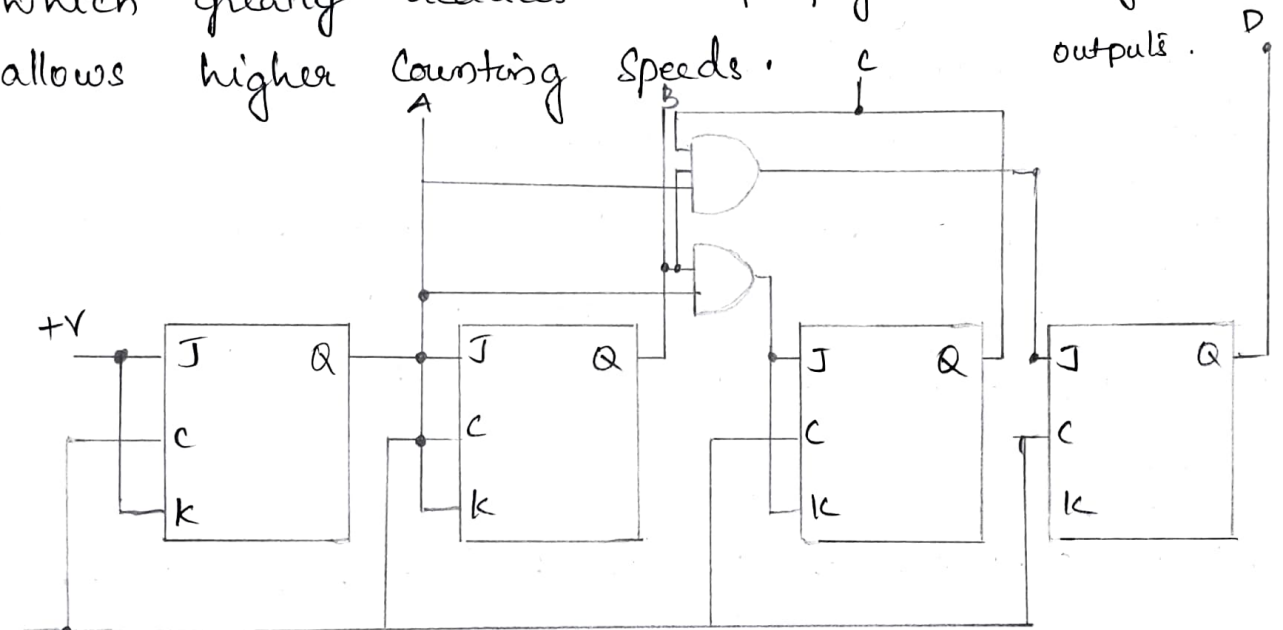


Fig: Binary Synchronous Counter.

Logic diagram of Ripple BCD Counter.

The Ripple BCD Counter is constructed by four flip-flops and an AND gate. The clock of one flip-flop is derived from the output of the previous flip-flop.

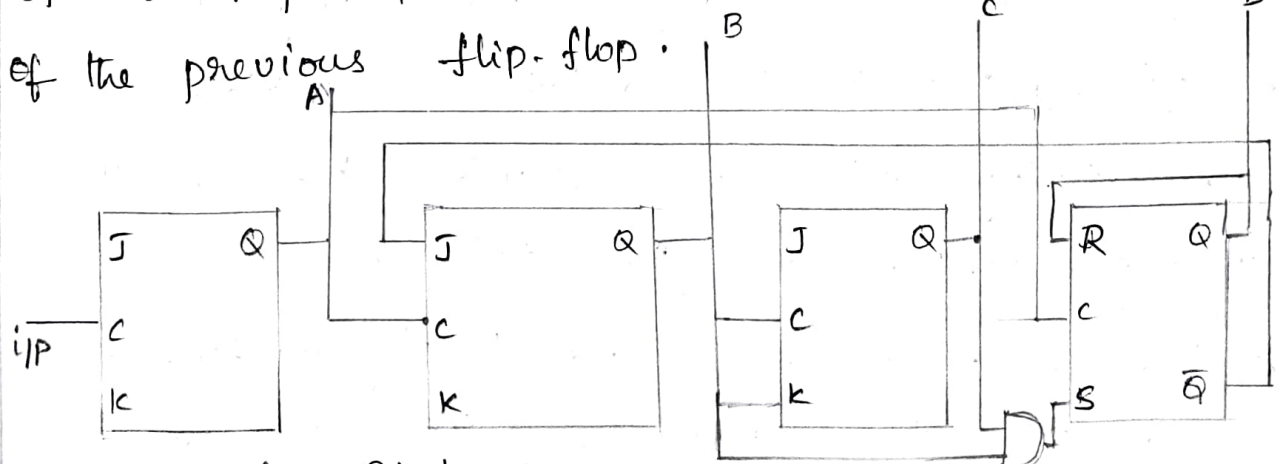


Fig: Ripple BCD Counter

2. DISPLAY

- * The BCD information available at the output of the Counter must be converted to some form of visible display.
- * This conversion depends on the type of display desired.
- * It is desirable in a frequency counter to display the count continuously.
- * The counter is reset to zero and allowed to count during the gate period. During this time the output of the counter is constantly changing.
- * The output of the counter cannot be displayed during this period.
- * The count at the end of the measurement period is stored in a simple memory and displayed.
- * Digital logic usually cannot supply the required current for driving a display.
- * A display driver is included between the decade counter and the displays.
- * This technique is called Display Multiplexing and reduces the number of drivers and decoders required to implement large counters.

3. Time Base

* The sequence of events within the frequency counter is controlled by the time base, which must provide the timing for the following events:

- resetting the counter
- opening the count gate
- closing the count gate
- storing the counting frequency.

* The opening and closing of the count gate determine the accuracy of Time Base signal. Frequency Counter.

* So, the accuracy of the frequency counter depends directly on the accuracy of the time base signals.

* These signals are driven from crystal-controlled oscillator.

4. Input Signal Processing

* The unknown frequency is not the correct logic level to drive the frequency counter.

* A processing circuit is required.

* This processing circuit may be

- an amplifier to increase signal level
- an attenuator to adjust variations in input amplitudes.
- a comparator

ELECTRONIC COUNTER

* The decade Counter can be easily incorporated in a Commercial Test instrument called an Electronic Counter.

* A decade Counter by itself behaves as a totaliser by totalling the pulses applied to it during the time interval that a gate pulse is present.

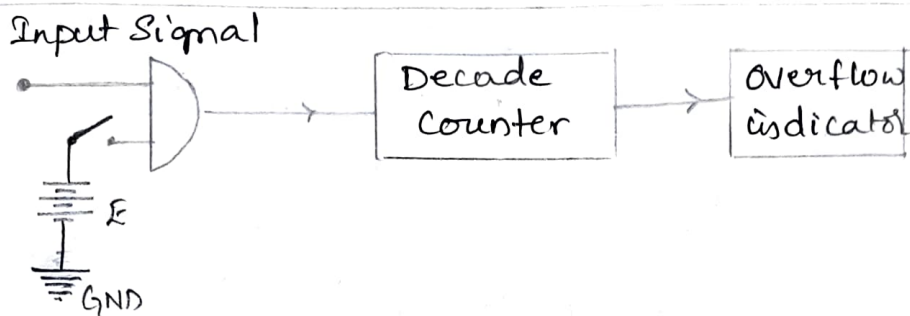
Modes of operation

- Totalising Mode
- Frequency mode
- Period Mode
- Ratio mode
- Time Interval mode.

1. Totalising Mode.

* In the totalising mode the input pulses are counted (totalised) by the decade counter as long as switch is closed.

* If the count pulse exceeds the capacity of the decade counter, the overflow indicator is activated and the counter starts counting again.



figs: Block diagram of the Totalling Mode of an Electronic Counter.

2. Frequency Mode

* If the time interval in which the pulses are being totalised is accurately controlled the counter operates in the frequency mode.

* Accurate control of the time interval is achieved by applying a rectangular pulse of known duration to the AND gate in place of dc voltage source. This is called gating of the counter.

* The frequency of the input signal is computed as $f = \frac{N}{t}$.

where f : frequency of input signal

N : pulse counted

t : duration of the gate pulse.

3. Period Mode

* In some applications it is desirable to

measure the period of the signal rather than its frequency. Since period is the reciprocal of the frequency, it can easily be measured by using the input signal as a gating pulse and counting the clock pulses.

* The period of the input signal is determined from the number of pulses of known frequency

$$T = \frac{N}{f}$$

N : pulse counted

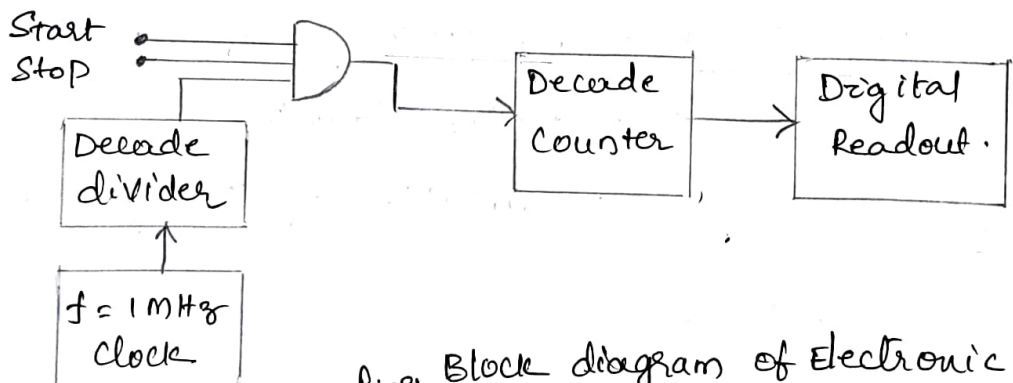
f : freq. of clock.

4. Ratio Mode

* The ratio mode of operation simply displays the numerical value of the ratio of the frequency of the two signals.

* The low frequency signal is used in place of the clock to provide a gate pulse. The number of cycles of the high frequency signal which are stored in the decade counter during the presence of an externally generated gate pulse is read directly as a ratio of the frequency.

5. Time Interval Mode



figs: Block diagram of Electronic Counter in Time Interval Mode.

* The time interval mode of operation measures the time elapsed between two events.

* The measurement can be done using the circuit of figure 9

* The gate is controlled by two independent inputs .
→ The start input, which enables the gate
→ The stop input, which disables the gate

* During the time interval between the start and stop signal, clock pulses accumulate in the register, providing an indication of the time interval between the start and stop of the event.

Applications.

- Research and Development laboratories
- in standard laboratories
- On service benches
- in everyday operations of many electronic installations.

* Counters are used (i) in communication to measure the carrier frequency,
(ii) in the digital system to measure clock frequency.

MEASUREMENT ERRORS / SOURCES OF ERRORS

1. Gating error

* Occurs with frequency and period measurements.
* For frequency measurement the main gate is opened and closed by oscillator output pulse.

* This allows the input signal to pass through the gate and be counted by the decade counters.

* The gating pulse is not synchronised with the input signal. They are in fact two totally unrelated signals.

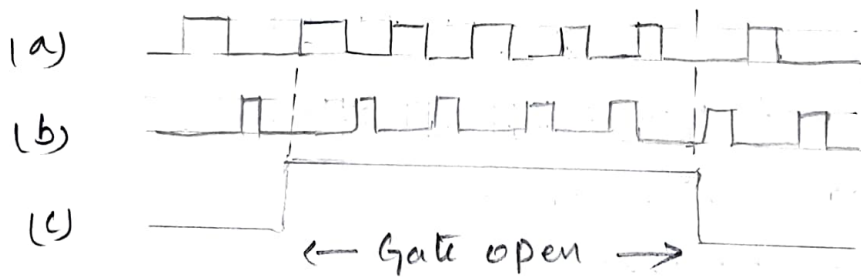


Fig 10: Gating error.

* Fig 10 shows the waveforms of two input signals (a) and (b) out of phase with respect to the gating signal (c)

* In (a) 5 pulses can be counted
(b) 4 pulse can be only allowed

* Therefore ± 1 count ambiguity in the measurement occurs.

* In measuring low frequencies, this error shows major effect on the result.

for eg: where $f = 10 \text{ Hz}$ to be measured
gating time = 1 sec.

The decade counter indicates 10 ± 1 count.
Hence an inaccuracy of 10%.

* Period measurements are preferred over frequency measurements at lower frequencies

* In period measurements,

the no. of pulses counted $N_p = \frac{f_c}{f_x}$.

f_c : clock frequency

f_x : unknown frequency of input signal.

* In frequency measurement, with 1 sec time

no of pulses counted $N_f = f_x$.

* The cross over frequency f_0 at $N_p = N_f$ is

$$\frac{f_c}{f_x} = f_x \quad \Rightarrow \quad f_x = \sqrt{f_c}$$
$$f_0 = \sqrt{f_c}$$

* Signals with a frequency lower than f_0 should therefore be measured in the period mode

* Signals with a frequency above f_0 should be measured in frequency mode.

2. Time Base Error :

- * Inaccuracies in the time base also cause errors in the measurement.
- * In frequency measurements the time base determines the opening and closing of the signal gate and it provides the pulses to be counted.
- * Time Base error consists of
 - Oscillator Calibration Errors
 - Short term crystal stability errors
 - long term crystal stability errors.
- * Improved calibration accuracy can be obtained by using very low frequency stations rather than high frequency because the transmission path for very low frequencies is shorter than high frequency transmission.
- * Short term crystal errors are caused by momentary frequency variations due to voltage transients, shock and vibration, electrical interference etc.
- * These errors can be minimised by taking frequency measurements over long gate times 10s to 100s.
- * Long-term stability errors are due to ageing and deterioration of the crystal.

3. Trigger Level Error

- * In frequency measurements the signal gate is opened and closed by the input signal.
- * The accuracy with which the gate is opened and closed is a function of the trigger level error.
- * These errors can be minimised with large signal amplitudes and fast risetimes.

Maximum accuracy can be obtained if the following suggestions are followed.

- The effect of the one-count gating error can be minimized by making frequency measurements above $\sqrt{f_c}$ and period measurements below $\sqrt{f_c}$ where f_c : clock frequency of counter.
- Since long term stability has a cumulative effect the accuracy of measurement is mostly a function of the time since the last calibration against a primary or secondary standard.
- The accuracy of time measurements is greatly affected by the slope of the incoming signal controlling the signal gate. Large signal amplitude and fast rise time assure maximum accuracy.