UNIT-4 BRIDGES

INTRODUCTION

Measurement is defined as a Comparision of an unknown quantity with a known standard value.

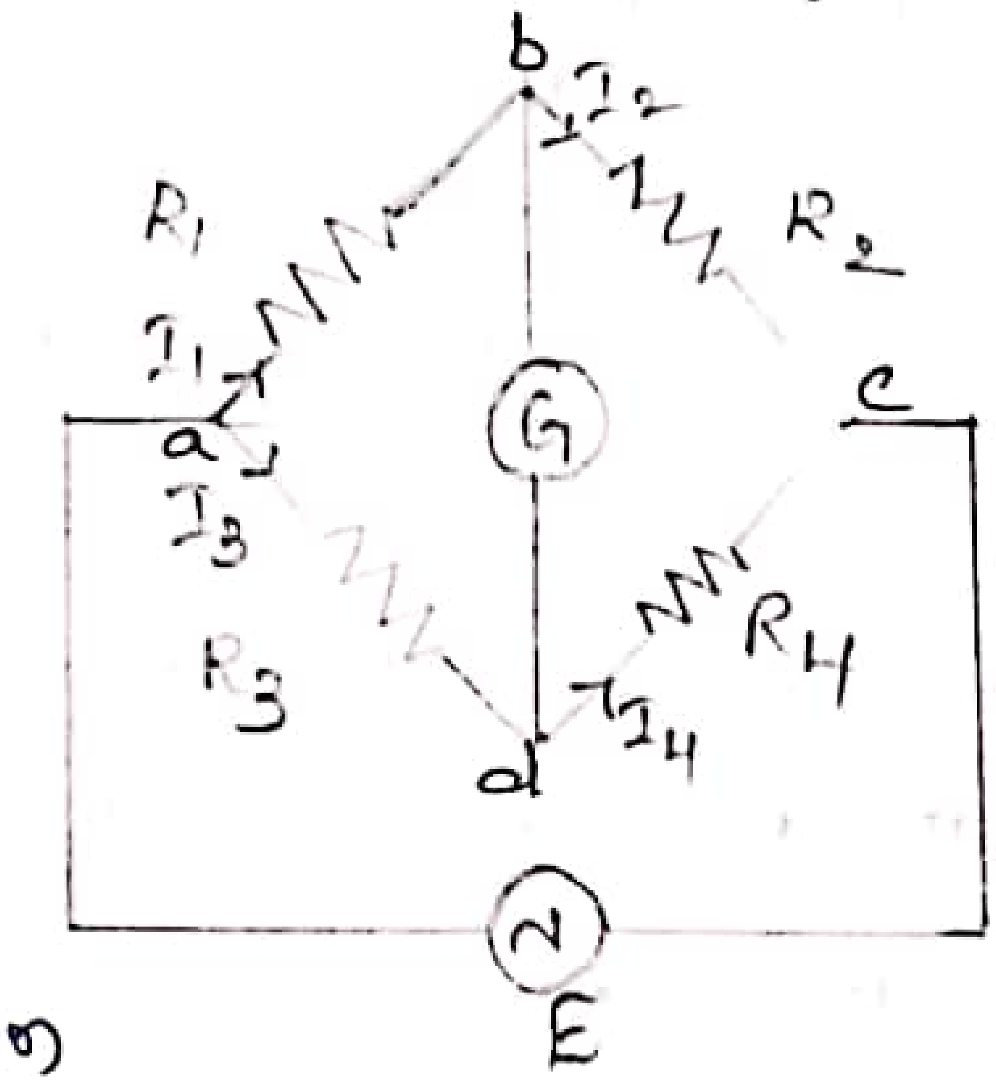
For the Measurement of Resistance, capacitance and inductance values, there are many approachs. One such an approach is the "BRIDGE CIRCUITS"

What is a Bridge Circuit?

A Circuit is the closed path in which the current flows.

A Bridge circuit in the simplest form consists of a network of four resistance or impedances connected to a voltage source and a current detector.

Figure 1 shows a Simple bridge circuit having the arms a, b, c, d with resistors R_1 R_2 R_3 R_4 Of which One is unknown and rest three are known.



- fig 1 -

The unknown value is Calculated with the known parameter values.

The bridge is given the supply voltage to the arms a and c. A Galvanometer is introduced between arms b and d to indicate the variation of current.

Components in the Bridge Circuit.

The major components in the Bridge circuit are the resistors in 4 arms.

Arm ab - unknown values.
Arm bc - known / standard / variable values
Arm cd - known / standard / variable values
Arm ad - known / standard / variable values

power supply - E between pt. a and c
Galvanometer - G between pt. d and b.

Uses of the Bridge Circuits

* Bridge circuits are extensively used for measuring component values such as R, L, C.
* Measurement accuracy of the bridge circuits are very high as the circuit compares the unknown value with a known value.
* It is independent of the characteristics of Null Detector. The accuracy directly related to the components of bridge.
* The basic bridge used for accurate measurement of resistance is the WHEATSTONE BRIDGE.

of Bridge
1. D.C
2.

...s of Bridge Circuits.

Bridge circuits are of two types.

1. D.C Circuits
2. A.C Circuits

D.C Circuits are used to measure Very low Resistance. D.C Circuits are 2 types

1. Wheatstone Bridge
2. Kelvin's Bridge.

A.C Circuits are used to measure Inductance, Capacitance and frequency. A.C Circuits are 4 types.

1. Maxwell's Bridge
2. Hay Bridge → measure Inductance
3. Anderson Bridge
4. Schering Bridge. → measure Capacitance.
5. Wein Bridge → frequency measure.

BASICS OF RESISTOR / RESISTANCE

A Resistor is a passive Component which is used in Various building blocks of The Electronics and Electrical Components & Devices.

The property of the Resistor is Called the Resistance. Resistance is the Property of opposing The flow of Current.

The Symbol of Resistor is R. units are ohms. Ω

Resistance is in the order of 3 form

1. low Resistance → 1Ω under

2. Medium Resistance → 1Ω to 100kΩ 1Ω<R<...

3. High Resistance. → Order of 0.1MΩ >100k...

* Low Resistance values are found in the resistance of connecting leads, resistors of Transformers. Bridge Methods are used to find the Resistance Values.

* Medium Resistance Values are found in the heating elements. → Ammeter-Voltmeter method → Substitution Method.

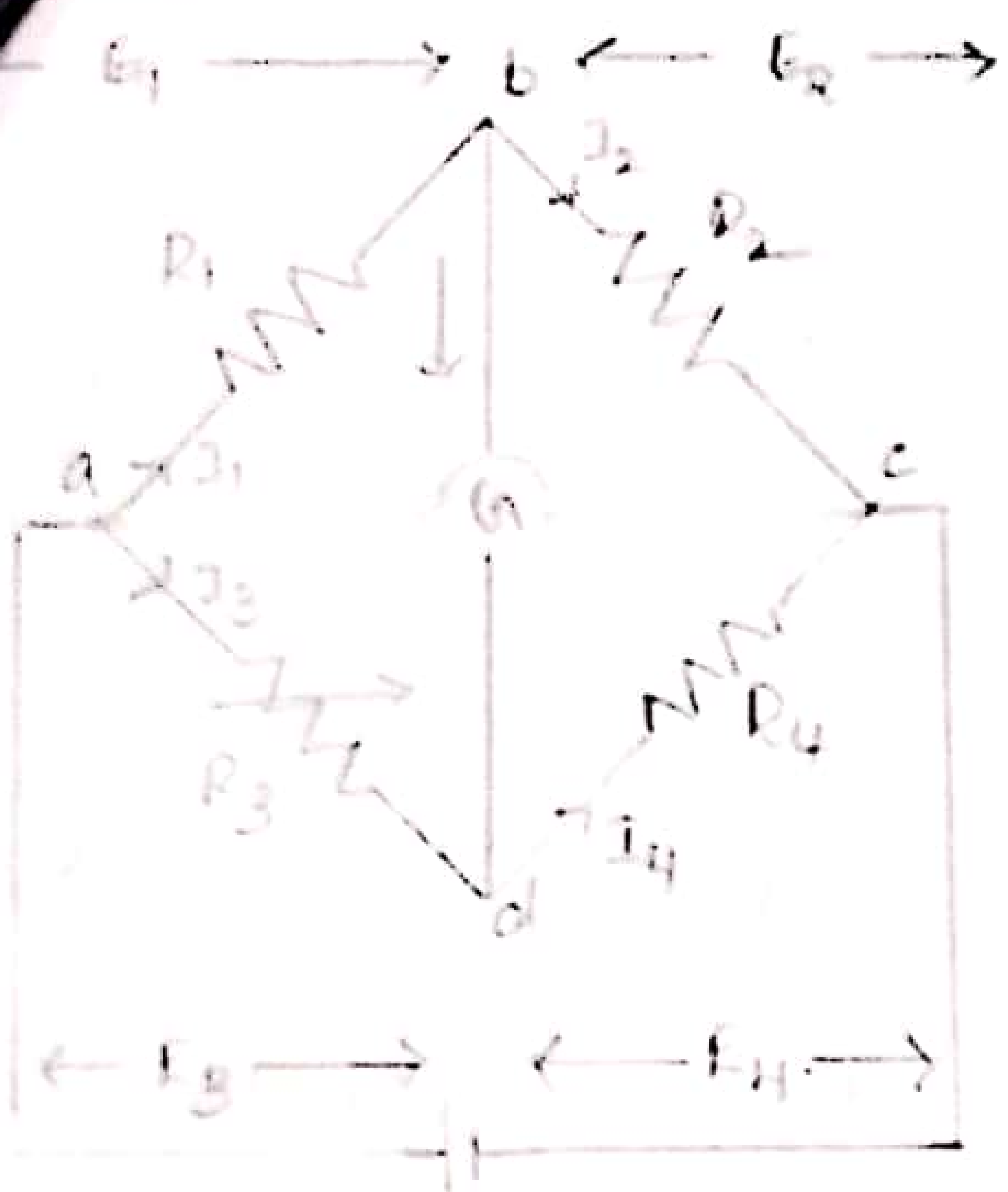
* High Resistance values are found in insulation resistors of Transformer windings. loss of charge methods are used to find the Resistance Values.

D.C BRIDGE CIRCUITS

1. WHEAT STONE BRIDGE

Wheatstone bridge is the most accurate method available for measuring Resistances. Popularly used for laboratory use.

Fig2 shows the circuit diagram of wheat stone bridge.



Components used:

1. Resistors - 4
2. Battery dc voltage - 1
3. Galvanometer - 1

- fig 2 -
Wheatstone Bridge Ckt.

Construction:

* Bridge consists of four arms ab, ad, bc, dc.
* Resistors R_1, R_2, R_3, R_4 are connected across arms ab, bc, ad, dc respectively.
* A DC voltage source is connected at the arm terminals a and c
* A Galvanometer is across the terminals b and d.
* Resistors R_4, R_3 are in series connection
* Resistors R_1, R_2 are in series connection
* The Resistor R_1 & R_2 is parallel to R_3 & R_4.

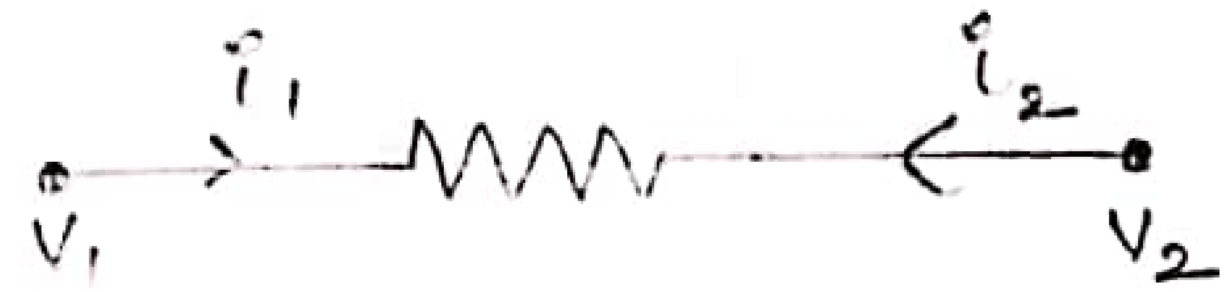
R_1 & R_2 || R_3 & R_4.

* The Resistor R_1 is unknown and the value is Variable.

* A Galvanometer gives the magnitude and direction of current.

Working Principle.

Consider a resistor

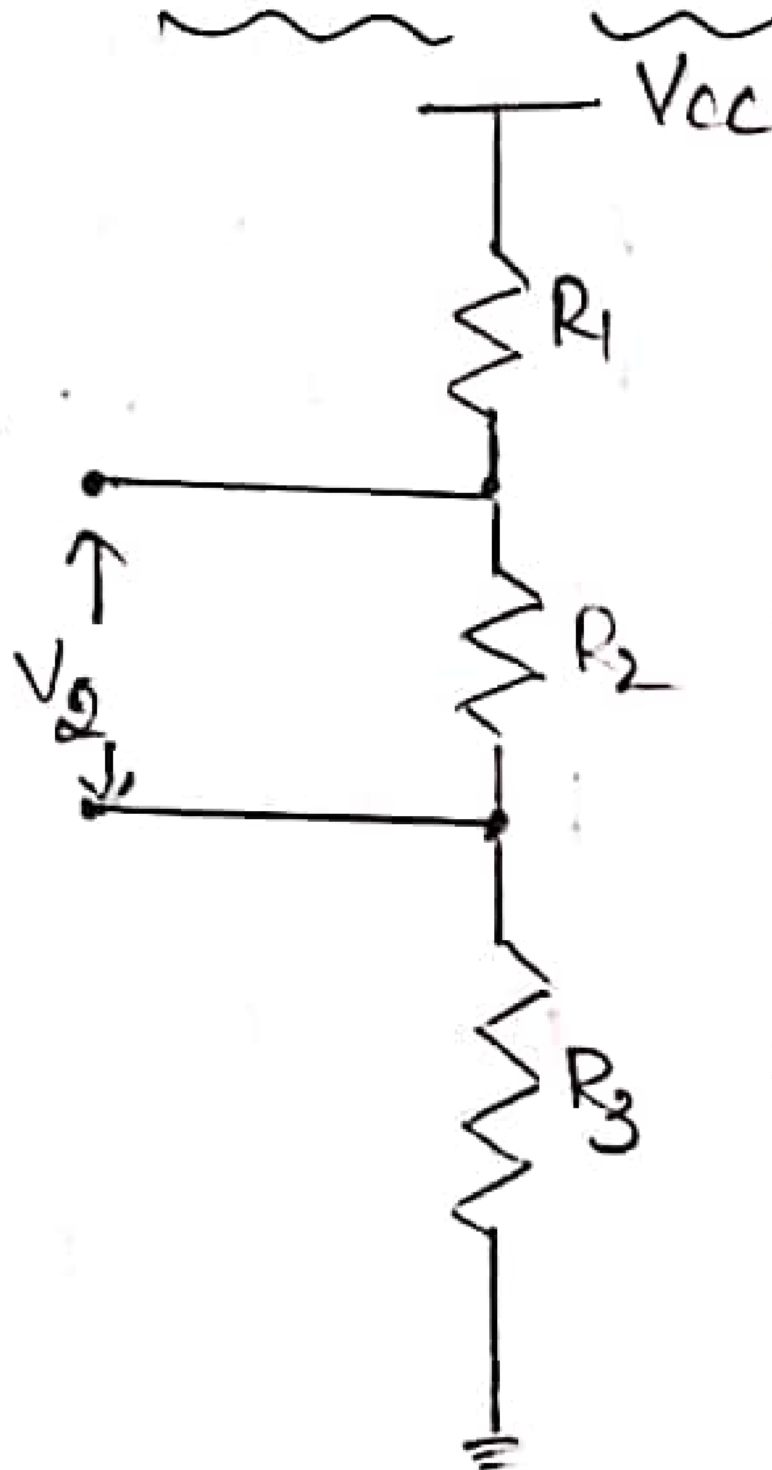


let the currents i_1, i_2 are flowing across resistors because of potentials V_1 and V_2.

i_1 current due to V_1 potential
i_2 Current due to V_2 potential.

Now the condition at which i_1 or i_2 will become zero is that at $V_1 = V_2$.

* The currents flowing into resistors from either side becomes zero under the condition that the potentials on either side will be equal.



Also, the potential Divider Rule.

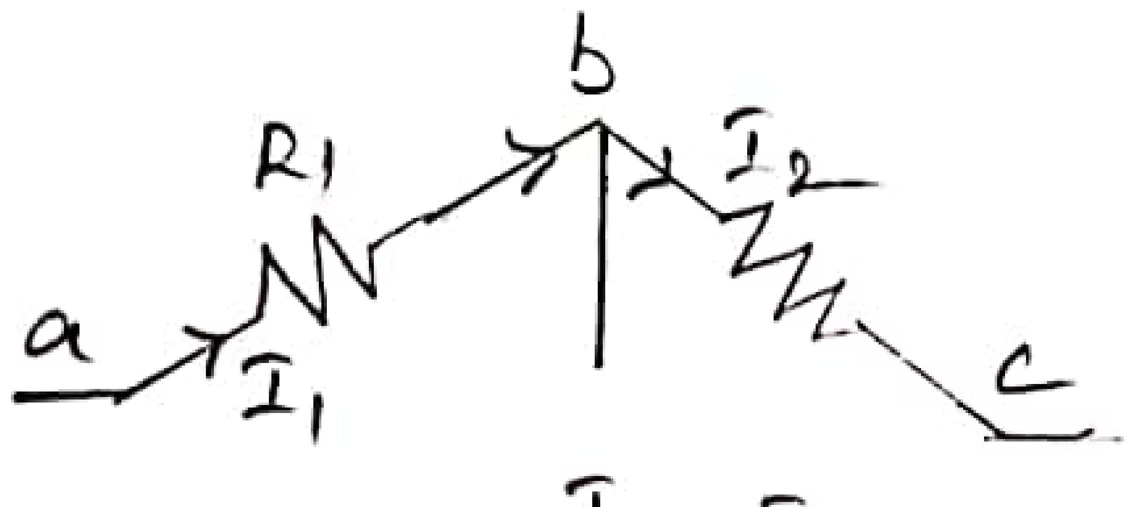
$$V_2 = \frac{R_2 \cdot V_{cc}}{R_1 + R_2 + R_3}.$$

V_{cc} = Supply voltage.

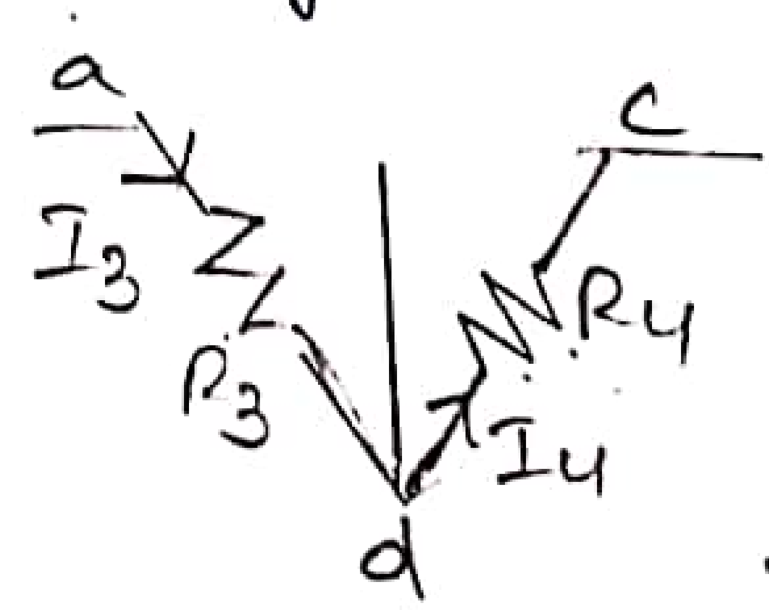
ation of Bridge :

The source of excitation is Vcc.

* Galvanometer indicates the flow of current from b to d.
* To get an indication in Galvanometer point to zero the voltage drops at a and b to be equal.
* changing the value of R_3 (variable) the reading in Galvanometer would pointed to zero.
* The current flows and divides into the two arms at point a. The bridge is balanced when there is no current through the Galvanometer. or when the potential difference at points C and D is equal. i.e, potential across Galvanometer is zero. $V_b = V_d$. $\Rightarrow V_{AB} = V_{AD}$.
* The current I_1 flows along AB arm which will be equal to I_2 along BC arm
* The current I_3 flows along AD arm will be equal to I_4 along DC arm.



$I_1 = I_2$



$I_3 = I_4$

At blancing condition, $V_{AB} = V_{AD}$

By ohms law, $I_1 R_1 = I_3 R_3$.

When no current in the Galvanometer I_1 flows along arm BC as I_2. So, $I_1 = I_2$

$$I_1 = I_2 = \frac{V_{cc}}{R_1 + R_2}.$$

Also, I_3 flows along arm CD as I_4.

$$I_3 = I_4 = \frac{V_{cc}}{R_3 + R_4}.$$

We have $I_1 R_1 = I_3 R_3$

$$\frac{V_{cc} R_1}{R_1 + R_2} = \frac{V_{cc} \cdot R_3}{R_3 + R_4}$$

$$\frac{R_1}{R_1 + R_2} = \frac{R_3}{R_3 + R_4}$$

$$R_1 (R_3 + R_4) = R_3 (R_1 + R_2)$$

$$R_1 R_3 + R_1 R_4 = R_3 R_1 + R_3 R_2$$

$$\boxed{R_1 R_4 = R_3 R_2}$$

This is the bridge Balancing Condition.

$$R_1 = \frac{R_3 R_2}{R_4}$$

where R_1 = unknown Resistance.

Advantages of Wheat Stone bridge.

* This is the basic bridge upon which any bridge circuit is understood.
* The unknown resistance R_1 is independent of the characteristics or the calibration of the null-detecting galvanometer. Hence high accuracy can be obtained.

Disadvantages of wheat stone bridge

* The null detector has to be sensitive enough to indicate the balance position of the bridge with the required degree of precision.

Applications of wheat stone Bridge

* Wheatstone bridge is used to measure the resistance of various types of wire either for purpose of quality control of the wire or for assembly in which it is used.

  eg: the resistance of motor windings, transformers, relay coils can be measured.

* It is also used by Telephone Companies to locate cable faults.

Sensitivity of Wheat stone Bridge.

* When the bridge is unbalanced, current flows through Galvanometer. That means it causes a deflection in the Galvanometer.

* The amount of deflection is the function of Sensitivity of the Galvanometer.

$$\boxed{\text{Sensitivity} = \frac{\text{Deflection}}{\text{unit current}}}$$

units $\frac{mm}{\mu A}$.

$$D = S \times I$$

UNBALANCED WHEATSTONE BRIDGE (or) Thevenin Equivalent Circuit

* To determine whether or not the galvanometer has the required Sensitivity to detect an Unbalance condition, it is necessary to Calculate the galvanometer current.

* Different Galvanometers possess different Current Sensitivities.

* The Sensitivity Can be calculated by Solving the bridge circuit for a Small unbalance

* This Can be achieved by the Thevenin's Equivalence of the Wheatstone bridge.

Thevenin's theorem

"For any linear Electrical network containing only Voltage Sources, Current Sources and resistances Can be replaced at terminals A-B by an equivalent combination of a votage Source V_{Th} in a Series Connection with a Resistance R_{Th}."

To find Thevenin's Equivalent of Wheatstone Bridge,

(i) Find the equivalent voltage appearing at terminals of Galvanometer when it is removed from Circuit.

(ii) find equivalent Resistance from the same terminals of Galvanometer

The Circuit in figure indicates the Thevenins Equivalence.

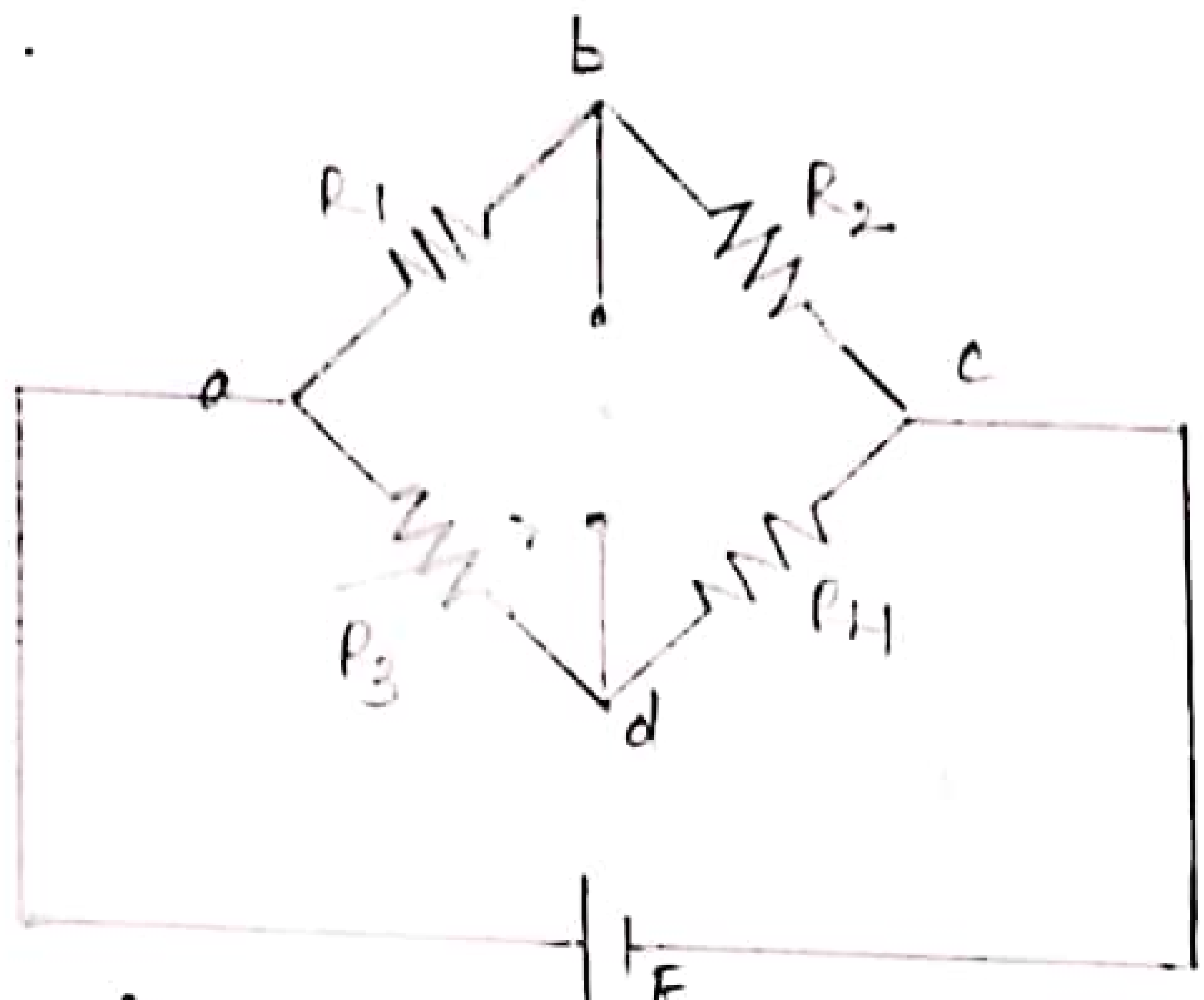


fig: Thevenin's Equivalent Ckt.

* The thevenin or open circuit voltage is found by E_{bd}. Referring to the figure,

$$E_{bd} = E_{ab} - E_{ad} = I_1 R_1 - I_3 R_3$$

where $I_1 = \frac{E}{R_1+R_2}$ and $I_3 = \frac{E}{R_3+R_4}$

Substitute values of I_1, I_3 we get

$$E_{bd} = I_1 R_1 - I_3 R_3$$

$$= \frac{E R_1}{R_1+R_2} - \frac{E R_3}{R_3+R_4}$$

$$E_{Th} = E_{bd} = E\left[\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}\right]$$

* The resistance of the Thevenin's Equivalent Circuit is found by replacing the battery by it's internal Resistance. The figure represents the Thevenins Resistance:

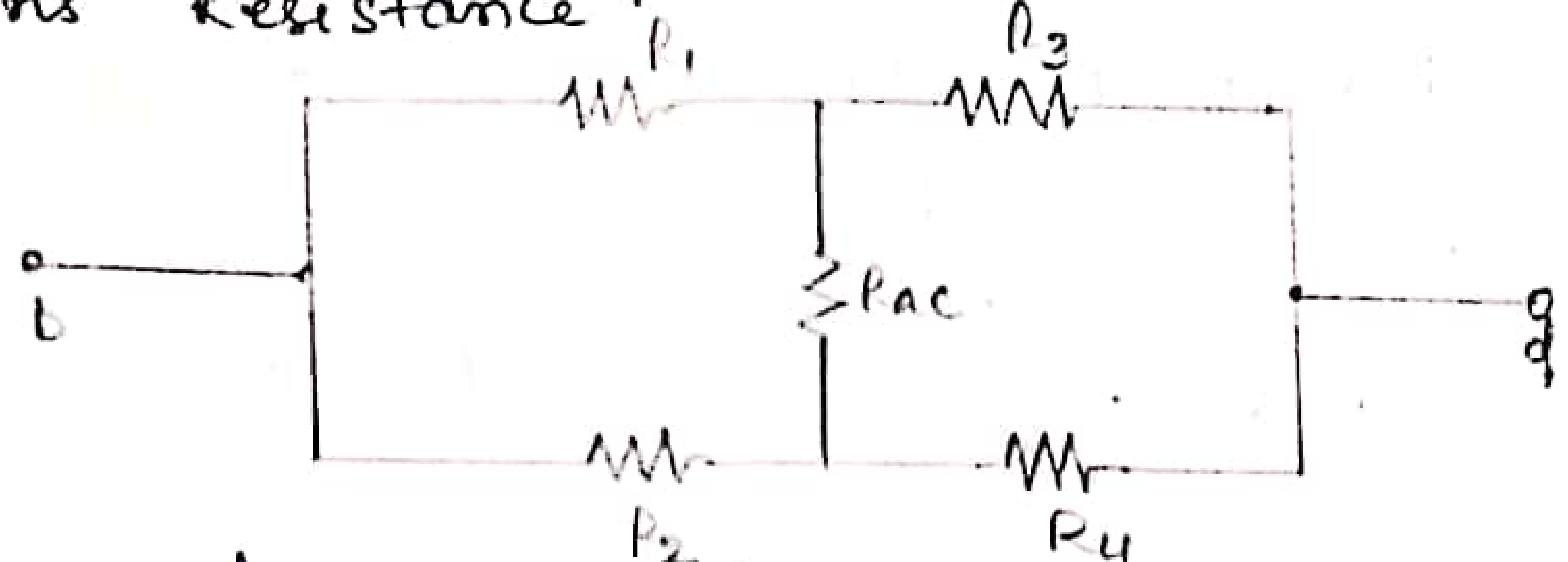


fig: Thevenins Resistance Equivalence.

The Thevenins Resistance is given by $(R_1 \parallel R_2)$ in series $(R_3 \parallel R_4)$

$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4}$$

* The thevenin equivalent of the Wheatstone bridge circuit is reduced as shown in figure.

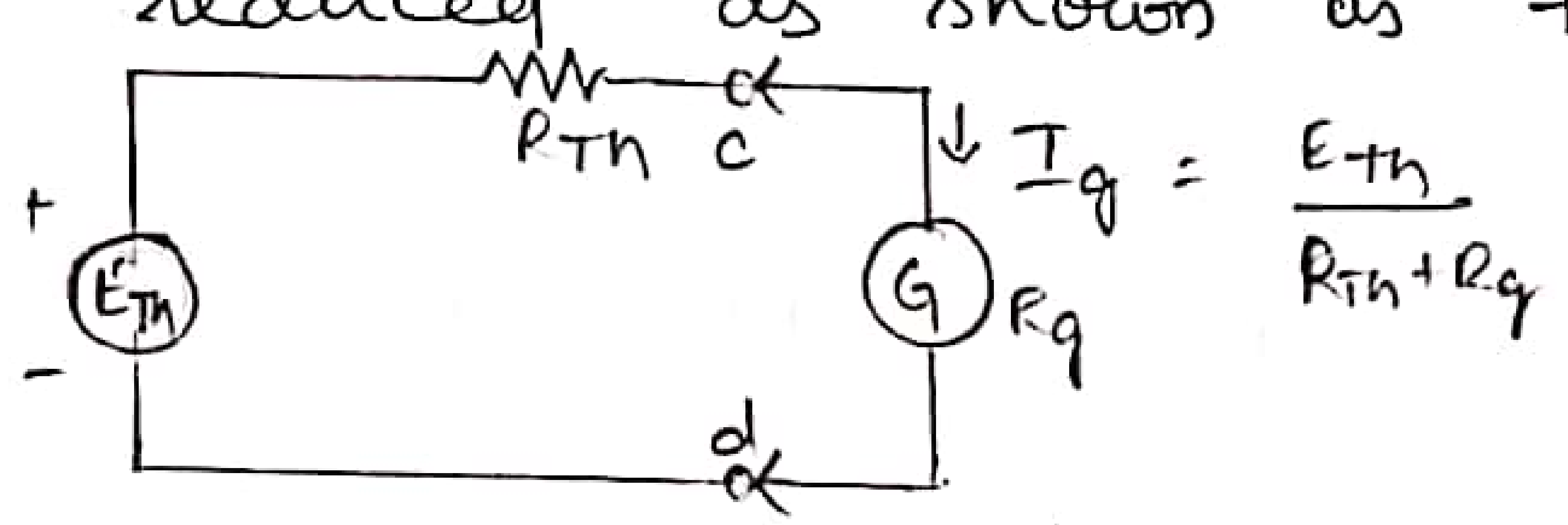


fig: Thevenins Equivalent

* When the null detector is now connected to the output terminals of the Thevenin's Equivalent Circuit, the Galvanometer current is found to be

$$I_g = \frac{E_{Th}}{R_{Th} + R_g}.$$

I_g : Galvanometer Current

R_g : Resistance of Galvanometer

Errors in wheatstone Bridge

1. **Limiting Error** : In wheatstone bridge, the percentage limiting error in measurand R_1 is equal to percentage limiting errors in R_2, R_3, R_4

$$\Rightarrow \quad \% R_1 = \% R_2 + \% R_3 + \% R_4 .$$

2. **Error due to heating of Elements in the bridge arms**

The accuracy of the instrument decreases if the instrument is heated for long time.
The change in resistance due to variation in temperature R_t is given by $R_t = R_0[1+\alpha t]$

R_0 - initial Resistance
α - Coefficient of temperature
t - Temperature.

Power $P = I^2R$ watts

Heat $= I^2Rt$ Joules.

The I^2R loss occuring in the resistors of each arm might tend to increase in temperature which in turn can result in change in resistance values.

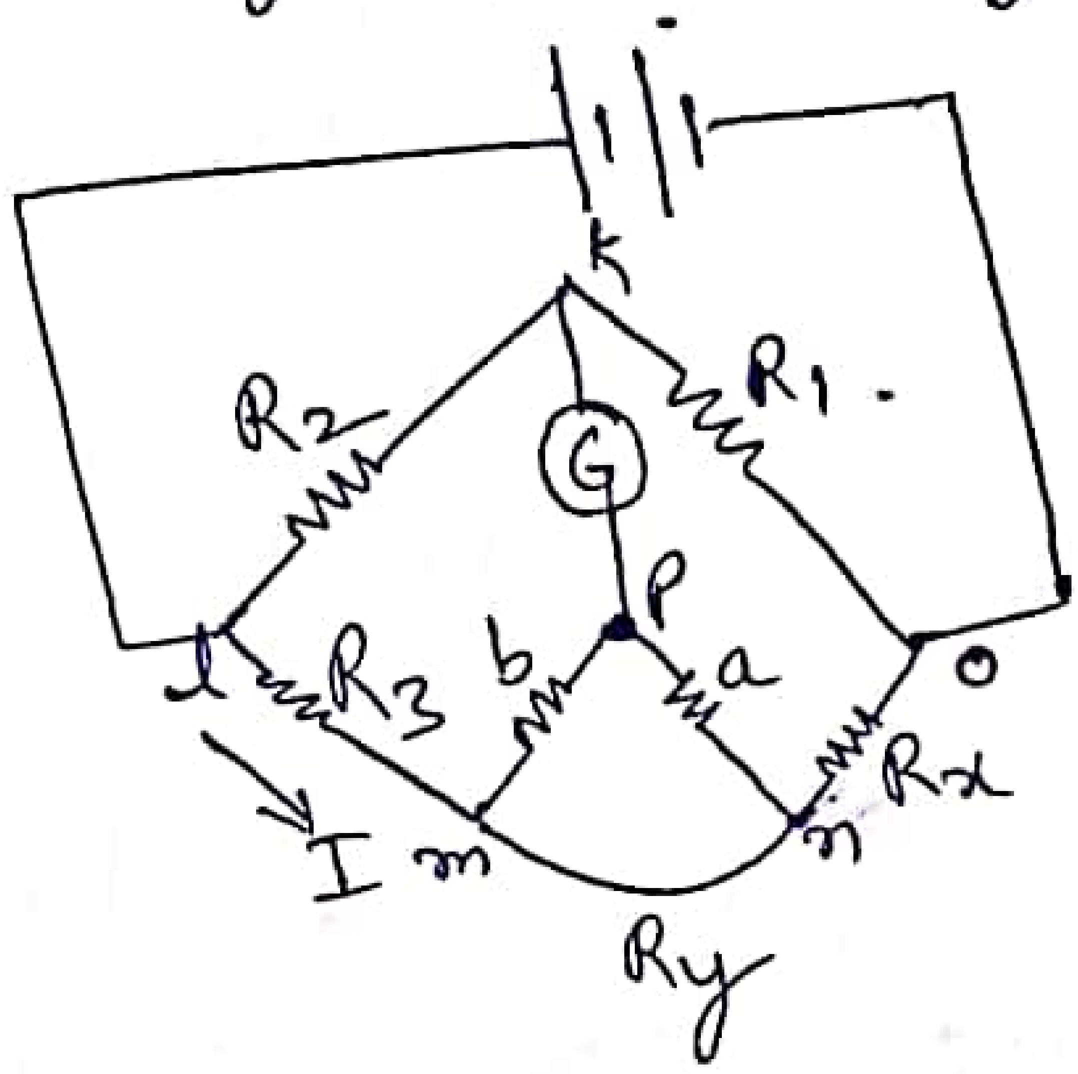
3. **Error due to the effect of the Connecting wires and lead resistors.**

The resistance of the Connecting leads will affect the value of the unknown resistance

④ Contact Resistance errors

The contact resistance of leads also affect the value of the measurand Resistances. It depends on the cleaniness of the contact surfaces and the pressure applied to the circuit.

Kelvin bridge : It is a modification of wheatstone bridge and provides greatly increased accuracy in the measurement of low-value resistances below 1Ω;
Consider the Kelvin Double bridge where R_y represents the resistance of the connecting lead from R_3 to R_x.



The term double bridge is used because the circuit contains a second set of ratio arms and second set of arms labeled a and b in the diagram, connects the galvanometer to a point p at the appropriate potential between m and n and iteliminates the effect of the yoke resistance R_y.
The resistance ratio of a and b is the same as the ratio of R_1 and R_2.
The galvanometer indication will be zero when the potential at k equals the potential p or when $E_{kl} = E_{lmp}$

$$E_{kl} = \frac{E R_2}{R_1+R_2} = \frac{R_2}{R_1+R_2} I \left(R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right)$$

$$E_{lmp} = I \left[R_3 + \frac{b}{\cancel{a+b}} \frac{\cancel{(a+b)} R_y}{a+b+R_y} \right]$$

$$= I \left[R_3 + \frac{bR_y}{a+b+R_y} \right]$$

$$\frac{R_2}{R_1+R_2}\cancel{I}\left[R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}\right]=\cancel{I}\left[R_3+\frac{bR_y}{a+b+R_y}\right]$$

$$R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}=\frac{R_1+R_2}{R_2}\left(R_3+\frac{bR_y}{a+b+R_y}\right)$$

$$=\left(1+\frac{R_1}{R_2}\right)\left(R_3+\frac{bR_y}{a+b+R_y}\right)$$

$$\cancel{R_3}+R_x+\frac{(a+b)R_y}{a+b+R_y}=\cancel{R_3}+\frac{bR_y}{a+b+R_y}+\frac{R_1R_3}{R_2}+\frac{R_1}{R_2}\frac{bR_y}{a+b+R_y}$$

$$R_x=\cancel{\frac{bR_y}{a+b+R_y}}+\frac{R_1R_3}{R_2}+\frac{R_1}{R_2}\frac{bR_y}{a+b+R_y}-\frac{aR_y}{a+b+R_y}-\cancel{\frac{bR_y}{a+b+R_y}}$$

$$=\frac{R_1R_3}{R_2}+\frac{bR_y}{a+b+R_y}\left(\frac{R_1}{R_2}-\frac{a}{b}\right)$$

Here $\frac{R_1}{R_2}=\frac{a}{b}$

$$\therefore\quad R_x=\frac{R_1R_3}{R_2}+\frac{bR_y}{a+b+R_y}\times 0$$

Hence $R_x=\dfrac{R_1R_3}{R_2}$

A.C BRIDGE CIRCUITS.

Introduction

* Impedances at Audio frequency or Radio frequency are commonly determined by means of an ac Wheatstone Bridge.

* Figure shows an ac bridge.

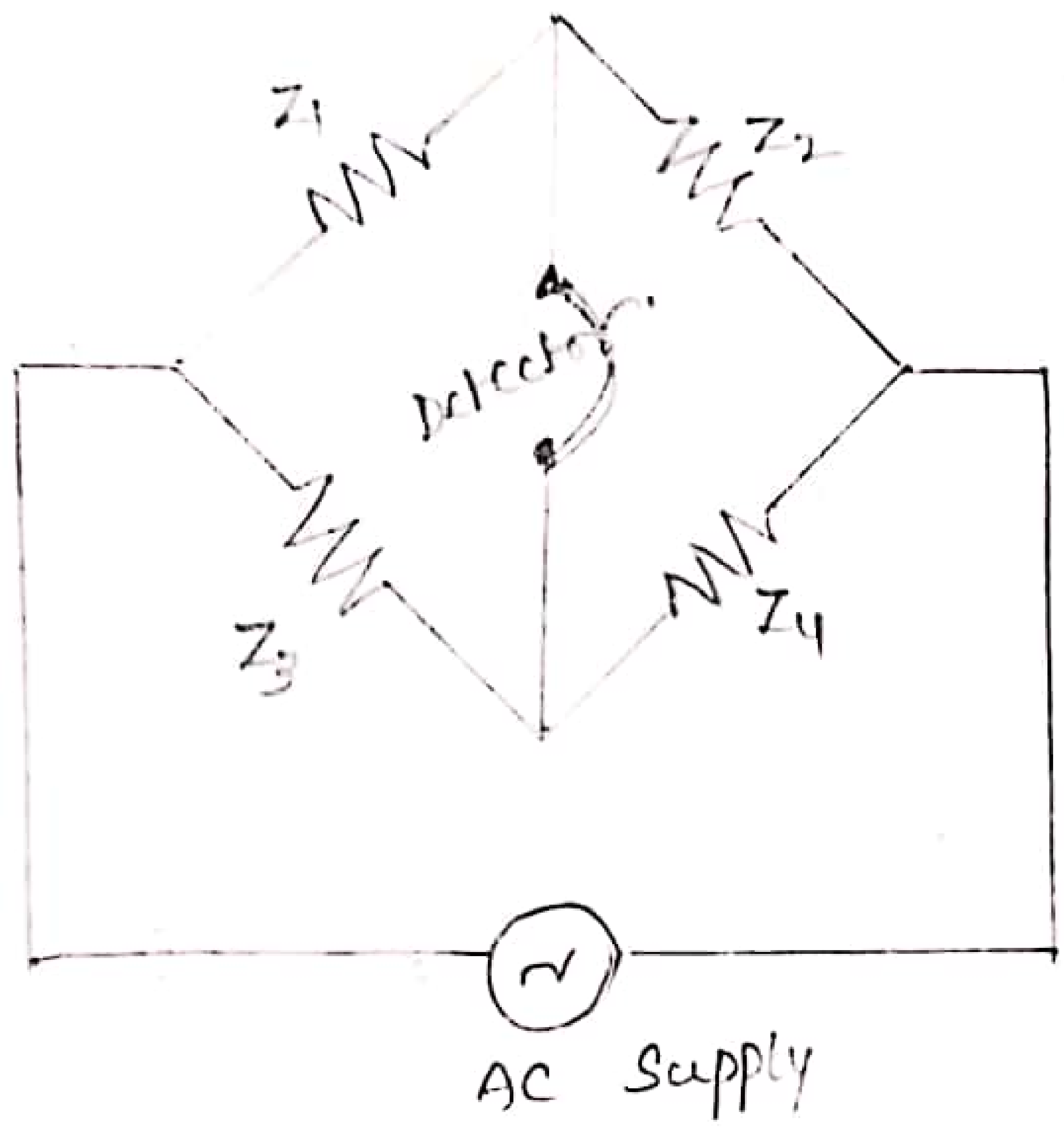


fig: A model of Ac Bridge.

* The bridge is similar to a dc bridge. Except (1) the bridge arms are impedances
(2) the bridge is excited by an a.c source rather than d.c source.
(3) The Galvanometer is replaced by a detector; such as pair of headpho

* when the bridge is balanced,

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}.$$

Z_1, Z_2, Z_3, Z_4 are the impedances of the arms and are vector complex quantities that possess phase angles. It is necessary to adjust both the magnitude and phase angle of the impedance arms to achieve balance. i.e the bridge must be balanced for both reactance and resistive components.

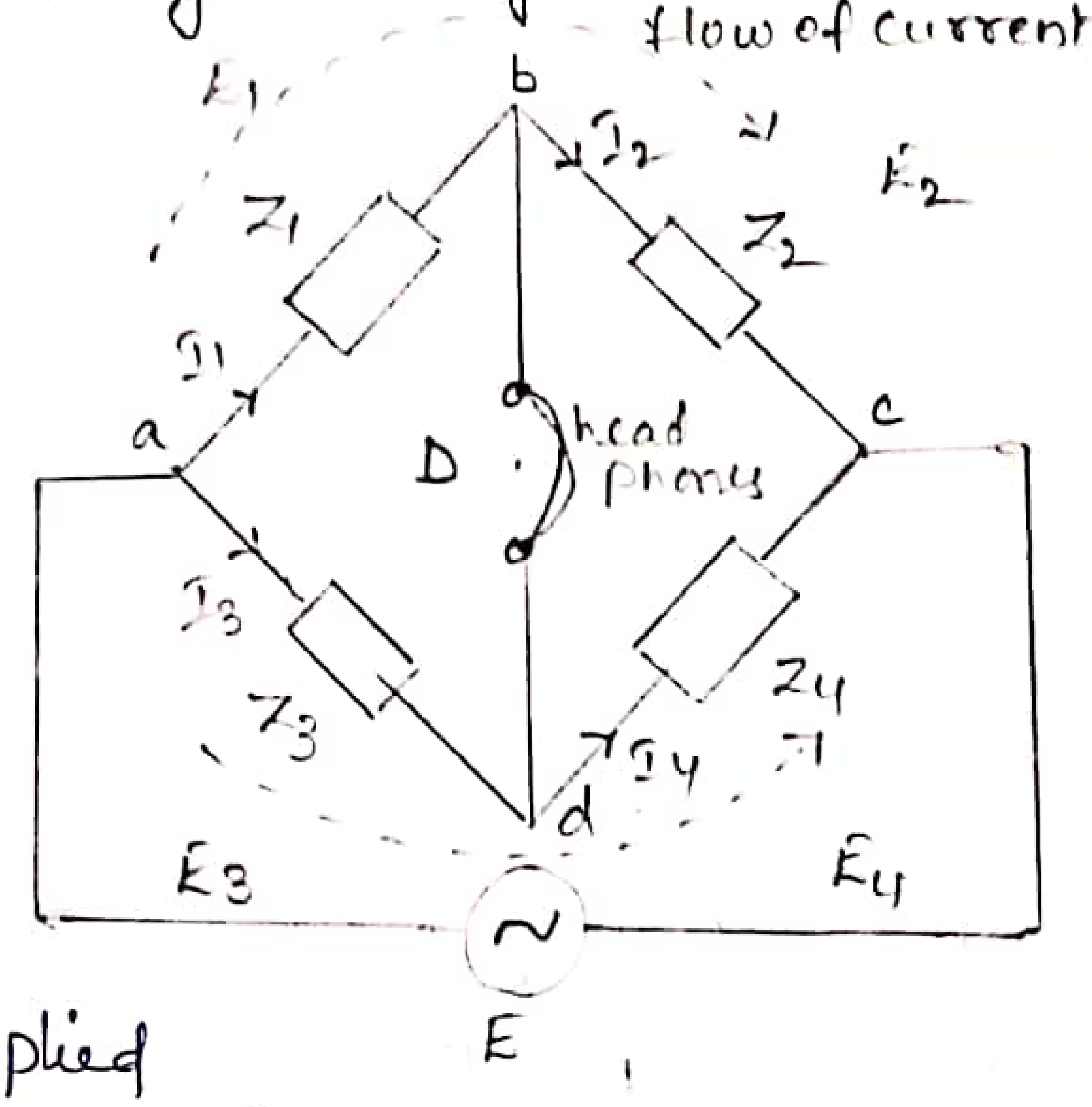
Derivation of Bridge Balancing Equation

Consider the following Bridge Circuit.

Z_1, Z_2, Z_3, Z_4 } impedances on four arms.

E - ac voltage source

detector - D-headphones



The potential E is applied to the circuit.

fig:

According to the potential divider rule,

voltage across ab = $E_{ab} = \frac{E \cdot Z_1}{Z_1 + Z_2}$

Voltage across ad $= E_{ad} = \frac{E}{Z_3+Z_4} \cdot Z_3$.

At bridge balance,

$$E_{ab} = E_{ad}.$$

$$\frac{E}{Z_1+Z_2} \cdot Z_1 = \frac{E}{Z_3+Z_4} \cdot Z_3.$$

$$\cancel{E} Z_1 (Z_3+Z_4) = \cancel{E} Z_3 (Z_1+Z_2).$$

$$\cancel{Z_1 Z_3} + Z_1 Z_4 = \cancel{Z_3 Z_1} + Z_3 Z_2$$

$$\boxed{Z_1 Z_4 = Z_2 Z_3}$$

The product of impedances of the opposite arms are equal.

Considering the phase angles,

$$|Z_1 Z_4| = |Z_2 Z_3|$$

$$\boxed{\angle Z_1 + \angle Z_4 = \angle Z_2 + \angle Z_3}.$$

∴ The Bridge Balance Conditions equations are

$Z_1 Z_4 = Z_2 Z_3$ and $\angle Z_1 + \angle Z_4 = \angle Z_2 + \angle Z_3$.

Types of BRIDGE CIRCUITS.

1. For Measurement of Inductance
   - * Maxwells Bridge
   - * Hay Bridge
   - * Anderson Bridge

2. For Measurement of Capacitance
   - * Schearing Bridge

3. For Measurement of Frequency.

* Wein's Bridge.

MEASUREMENT OF INDUCTANCE

1. Maxwell's Bridge

* Maxwell's bridge is a type of Alternating Current bridge circuit.
* It is named after the Scientist Maxwell
* This bridge is used to measure inductance by comparing it with a standard Capacitance

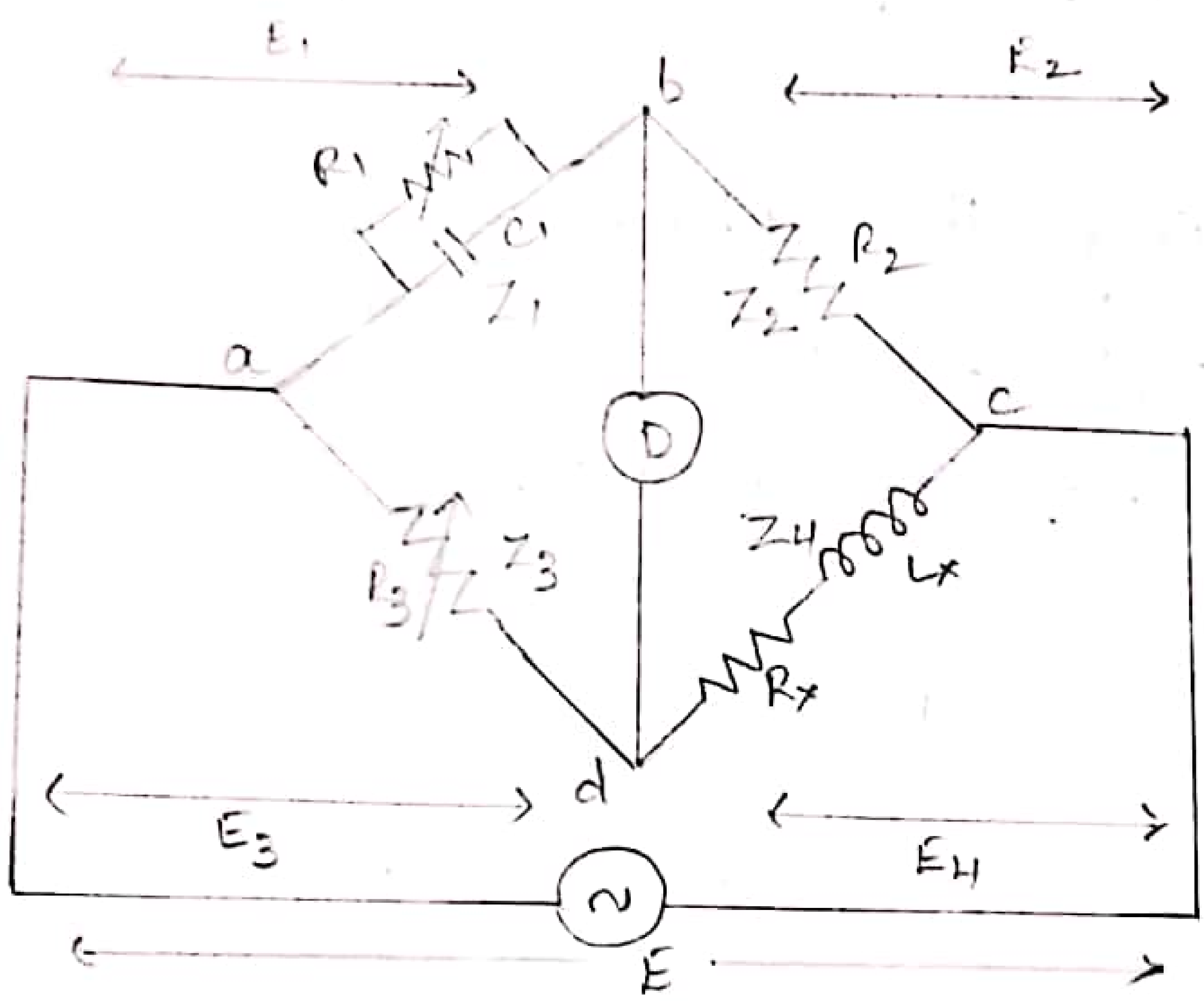


fig: Circuit diagram of Maxwell's Bridge.

Construction

* The Bridge Consists of four arms ab, bc, cd and da. Each arm has an impedance
* Of the four impedances, three are known and One will be unknown.

* The parts of the bridge Comprises of
  * Four impedances at four arms
  * A detector Connecting the terminals b, d
  * An AC power supply between terminals a c.
* Detector helps to obtain balance condition of the bridge

Working Principle.

* The bridge measures an unknown inductance in terms of a known capacitor.
* The use of standard arm offers the advantage of compactness and easy Shielding. The Capacitor is almost a loss-less Component.
* The four impedances of the bridge arms are

Arm dc - known impedance R, L_1 in series

$$Z_1 = R_x + j\omega L_1$$

Arm bc - Resistance $Z_1 = R_2$

Arm ab - Resistor (variable) in parallel with Capacitor

( )

$$Z_1 = R_4 \parallel X C_4$$

$$Z_1 = \frac{R_1}{1+R_1 j\omega C_1}$$

$$\frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}$$

$$= \frac{R_1 / j\omega C}{(R_1 j\omega C + 1)/j\omega C_1}$$

Arm da - variable Resistor

$$Z_3 = R_3 .$$

* The impedences Z_1, Z_2, Z_3, Z_4 are :

$$Z_4 = R_x + j\omega L_x \quad Z_2 = R_2 \quad Z_3 = R_3 \quad Z_4 = \frac{R_4}{1+j\omega C_4 R_4} .$$

The General Equation of bridge balance is

$$Z_1 Z_4 = Z_2 Z_3$$

Substituting the values of Z_1 Z_2 Z_3 Z_4 in $Z_1Z_4 = Z_2Z_3$

$$(R_4 + j\omega L_4)\frac{R_1}{1+j\omega C_1 R_1} = R_2 R_3$$

$$(R_4 + j\omega L_4) R_1 = R_2 R_3 (1 + j\omega C_1 R_1)$$

$$R_1 R_4 + j\omega L_4 R_1 = R_2 R_3 + j\omega C_1 R_1 R_2 R_3$$

Equating the real and imaginary parts on bothsides

$$R_1 R_4 = R_2 R_3$$

$$\boxed{R_4 = \frac{R_2 R_3}{R_1}}$$

$$j\omega L_4 R_1 = j\omega C_4 R_4 R_2 R_3$$

$$L_4 = \frac{C_1 R_2 R_3 R_1}{R_1}$$

$$\boxed{L_4 = C_1 R_2 R_3}$$

The unknown inductance in terms of known Capacitance.

The unknown impedance

$$Z_4 = R_4 + j\omega L_4$$

$$\boxed{Z_4 = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3.}$$

* Inductance is having a coil. Coil having Self inductance and Quality factor.

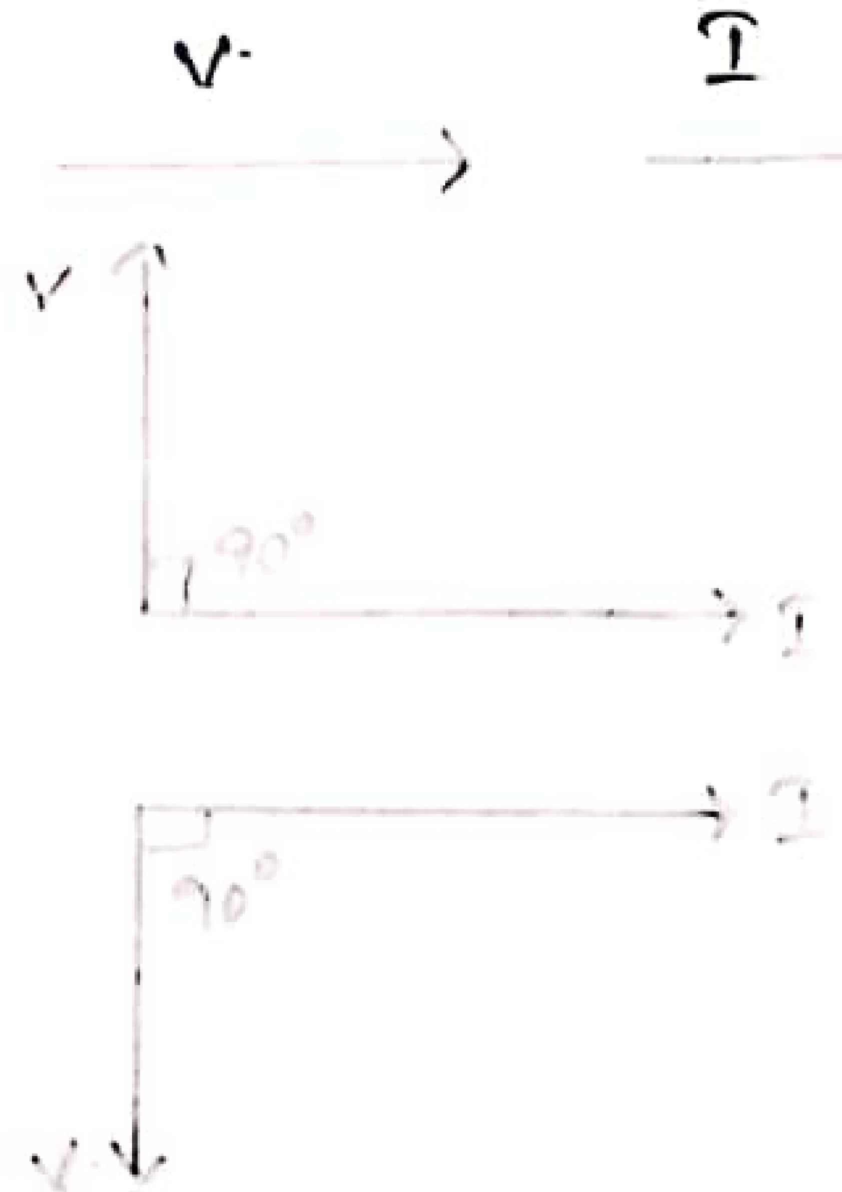
Quality factor is given by $Q = \frac{\omega L_4}{R_4}$

$$Q = \frac{\omega C_1 R_2 R_3}{\frac{R_2 R_3}{R_1}} = \omega C_1 R$$

$$\boxed{Q = \omega C_1 R_1.}$$

Phasor Diagram

Phasor Diagram represent the relation between voltages and currents flowing across the circuit. The standard phase relations between votages and currents for components R, L, C are



Resistor — V, I are in phase

Inductor — V leads I by 90°

Capacitor — V lags I by 90°.

By the basic standard values or relationships we draw the phasor circuit for maxwells bridge

Consider a reference phasor I_1. All other V, I relations are considered in reference to I_1.

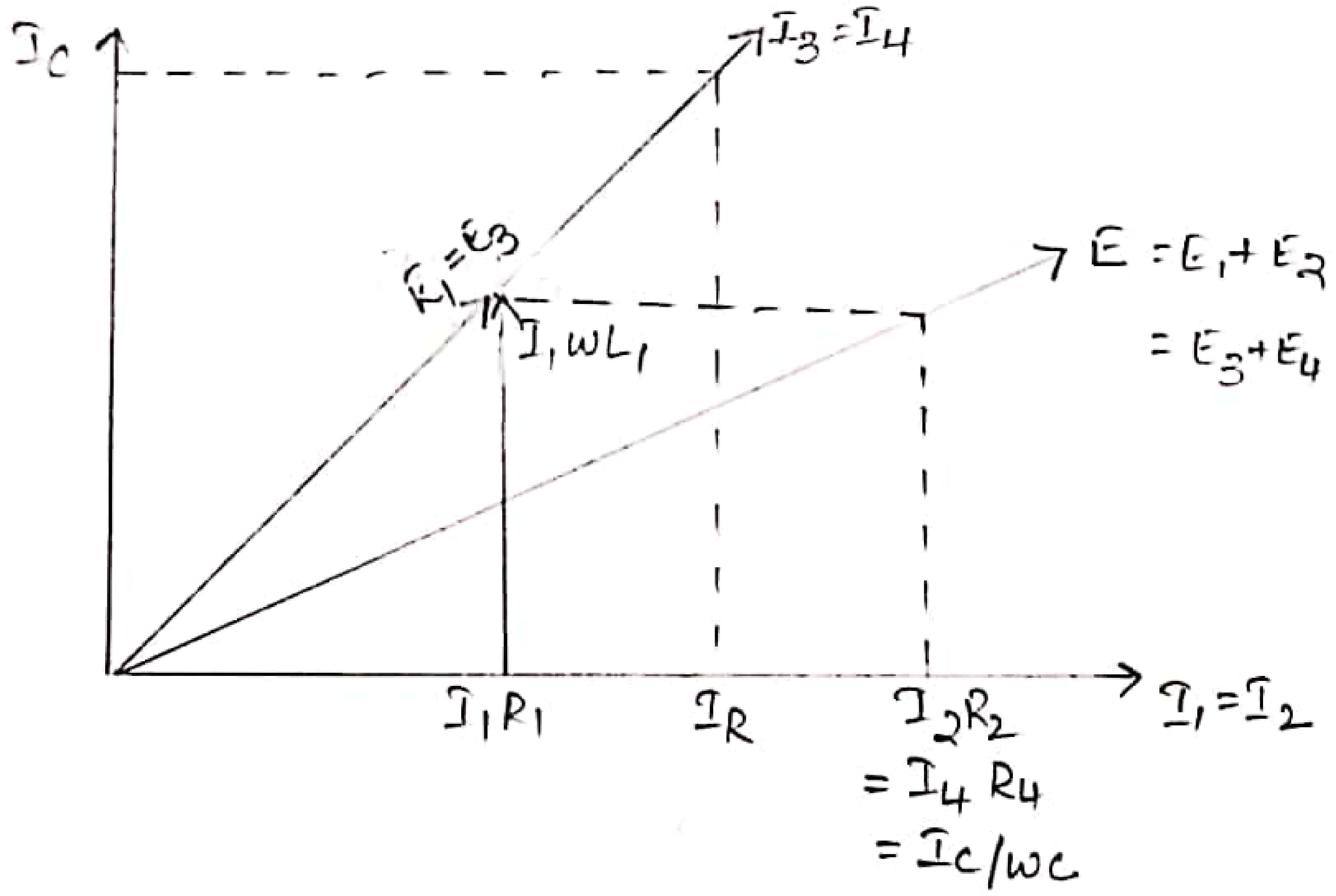


fig: Phasor diagram of maxwell's bridge.

Advantages of Maxwell's bridge

1. The Maxwell's bridge is limited to the measurements of low Q values (1 - 10)

2. The measurement is independent of excitation frequency.

3. The Scale of the resistance can be calibrated to read Inductance directly.

4. The Balance Equation is independent of losses associated with inductance.

5. The Bridge is useful for measurement of inductances at power and audio frequency.

Disadvantages of maxwell's bridge

1. The bridge uses a fixed Capacitor and so. there exists an interaction between the resistance and reactance balance.

   * To avoid this disadvantage instead variable resistors, variable Capacitors can be used

2. The bridge limits for $1 < Q \leq 10$

   If $Q > 10$, Resistance unknown becomes very large and it is impractical to obtain the values in the specified range.

Applications.

* The bridge is used for inductance measurements
* Comparison with capacitor is ideal than with inductance

* Commercial bridges measure from 1-1000 H with ± 2% error.

Problems on maxwell's bridge.

Prob1 A maxwell's bridge is used to measure an inductive impedance. The bridge constants at balance are:

$C_1 = 0.01 \mu f$ $R_1 = 470 k\Omega$ $R_2 = 5.1 k\Omega$ $R_3 = 100 k\Omega$

Find the series equivalent of the unknown impedence

Sol. The unknown impedance is given by

$$Z_4 = \frac{R_2 R_3}{R_1} + j\omega C_1 R_2 R_3$$

$$= \frac{5.1 \times 10^3 \times 100 \times 10^3}{470 \times 10^3} + j\omega \frac{0.01 \times 10^{-6} \times 5.1 \times 10^3 \times}{100 \times 10^3}$$

$$= 1085.106 + j\omega (5.1)$$

$$Z_4 = 1085.106 + j5.1\omega$$

2. HAY'S BRIDGE

The Hay's Bridge is used for the measurement of self-inductance.

* This bridge is a modification of maxwells bridge
* This bridge is named after the scientist, who invented it.

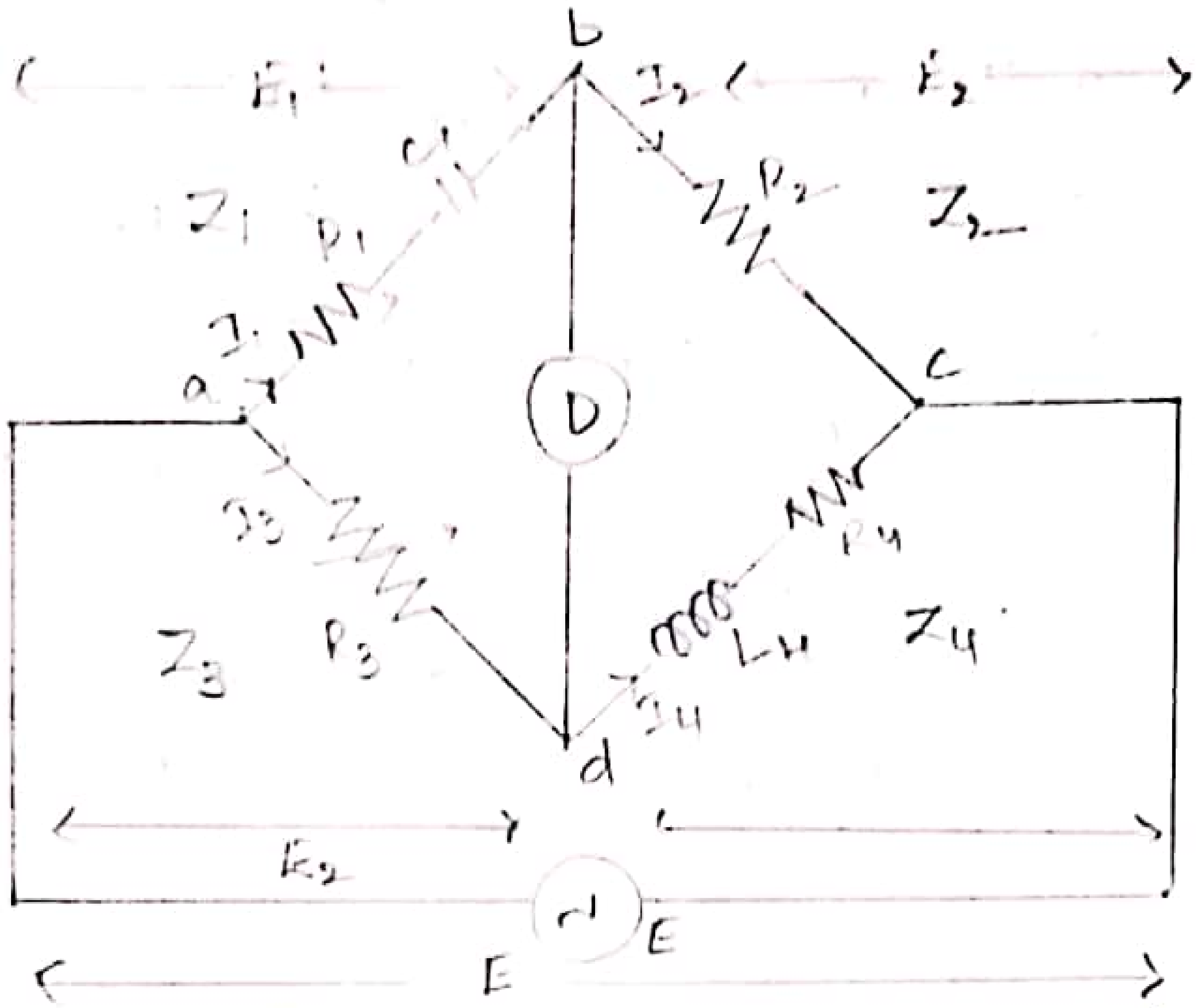


fig: Hay Bridge.

* The Hay bridge can be constructed as shown in fig
* This bridge differs from Maxwells Bridge in its construction by having a resistance in Series with a capacitor C_1 instead of a parallel.
* For large phase angles, R_1 needs to be low. So, it is convinient for measuring high Q values

$Q = \frac{\omega L}{R}$ R is low, ω is high $\Rightarrow$ Q is high

* Hays bridge is prefered for coils with high Q. whereas maxwells bridge is for coils with low Q.

Construction

* The bridge consists of four arms ab, bc, cd, da.
* On the four arms impedances of known and unknown exist. Three of them are known and one is unknown.
* The parts of the bridge are as follows.
  * Arm cd - unknown inductance L_4 in series with a Resistance R_4.
  * Arm bc - known non-inductive Resistance R_3
  * Arm ab - Standard capacitor c_1 is connected in series with Resistor R_1. (variable)
  * Arm ad - a variable Resistor R_3.
* The terminals a, c are connected to an AC Power supply
* A Detector is connected between points b, d.

Working

* The bridge gives us the unknown values of impedance at bridge balance for high Q values
* When the bridge is ON, the e.m.f is induced in the bridge due to AC supply. and current flows through the circuit.
* Adjusting the variable resistor at arm ad i.e R_3 such that no current flows through detector,

i.e., the detector shows a null value at the bridge balance.

* The bridge is said to be balanced when no current flows through terminals b and d.

* At such using the balance conditions the unknown impedance can be found using the known impedances.

General Balance Condition $Z_1 Z_4 = Z_2 Z_3$.

The impedances on the four arms are as follows

arm cd	Z_4	: $R_4 + j\omega L_4$	unknown	
arm bc	Z_2	: R_2	known	
arm ab	Z_1	: $R_1 - \dfrac{j}{\omega C_1}$	known	
arm da	Z_3	: R_3	known	

Substituting these values of impedances in the bridge balance condition equation.

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_4 + j\omega L_4)\left(R_1 - \frac{j}{\omega C_1}\right) = R_2 R_3$$

$$R_4\left(R_1 - \frac{j}{\omega C_1}\right) + j\omega L_4\left(R_1 - \frac{j}{\omega C_1}\right) = R_2 R_3$$

$$R_1 R_4 - \frac{jR_4}{\omega C_1} + j\omega L_4 R_1 - \frac{j^2 \omega L_4}{\omega C_1} = R_2 R_3$$

$$R_1R_4 - \frac{jR_4}{\omega C_1} + j\omega L_1 R_1 + \frac{L_4}{C_1} = R_2R_3 \qquad \because j^2 = (\sqrt{-1})^2 = -1$$

consider real and imaginary parts and compare on bothsides.

$$R_1R_4 + \frac{L_4}{C_1} + j\omega L_1 R_1 - j\frac{R_1}{\omega C_1} = R_2R_3$$

Real part :

$$R_1R_4 + \frac{L_4}{C_1} = R_2R_3 \rightarrow ①$$

Substitute the value of R_4 obtained from Equating imaginary parts in the above equation

i.e put ② in ①

$$R_1R_4 + \frac{L_1}{C_1} = R_2R_3$$

$$R_4 = \omega^2 L_4 R_1 C_1$$

$$(\omega^2 L_4 R_1 C_1)R_1 + \frac{L_1}{C_1} = R_2R_3$$

$$\frac{\omega^2 L_4 R_1^2 C_1^2 + L_4}{C_1} = R_2R_3$$

$$L_4((\omega R_1 C_1)^2 + 1) = R_2R_3C_1$$

$$L_4 = \frac{R_2R_3C_1}{1 + (\omega R_1 C_1)^2} \rightarrow ③$$

imaginary part :

$$j\omega L_4 R_1 - j\frac{R_4}{\omega C_1} = 0$$

$$\omega L_4 R_1 = \frac{R_1}{\omega C_1}$$

$$R_1 = \omega^2 L_4 R_1 C_1 \rightarrow ②$$

Substitute the value of L_4 in eqn ③ in above eqn ②

$$R_1 = \omega^2 L_4 R_1 C_1$$

$$L_4 = \frac{R_2R_3C_1}{1+(\omega R_1 C_1)^2}$$

$$R_4 = \omega^2 R_1 C_1 \cdot \frac{R_2R_3C_1}{1+(\omega R_1 C_1)^2}$$

$$R_4 = \frac{\omega^2 R_2 R_3 R_1^2 C_1^2}{1+(\omega R_1 C_1)^2}$$

The value of L_4 measured in terms of capacitor C_1

Quality factor

* Q-factor or Quality-factor indicates the goodness of coil.

* It is a type of figure of merit.

* Quality factor is defined as the ratio of power stored in the coil to the power lost or dissipiated by the coil.

$$Q = \frac{\omega L_4}{R_4}$$

$$L_4 = \frac{R_2 R_3 C_1}{1+(\omega R_1 C_1)^2} \qquad R_4 = \frac{\omega^2 R_2 R_3 R_1^2 C_1^2}{1+(\omega R_1 C_1)^2}$$

$$Q = \frac{\omega \cdot \left[\frac{R_2 R_3 C_1}{1+(\omega R_1 C_1)^2}\right]}{\left[\frac{\omega^2 R_2 R_3 R_1^2 C_1^2}{1+(\omega R_1 C_1)^2}\right]} = \frac{\omega R_2 R_3 C_1}{\omega^2 R_2 R_3 C_1^2 R_1^2} = \frac{1}{\omega C_1 R_1}$$

$$\boxed{Q = \frac{1}{\omega C_1 R_1}}$$

Phasor Diagram:

Phasor Diagram is the representation of the relation ship between voltages and currents of the circuit. The fig: gives the phasor diagram of Hay's bridge.

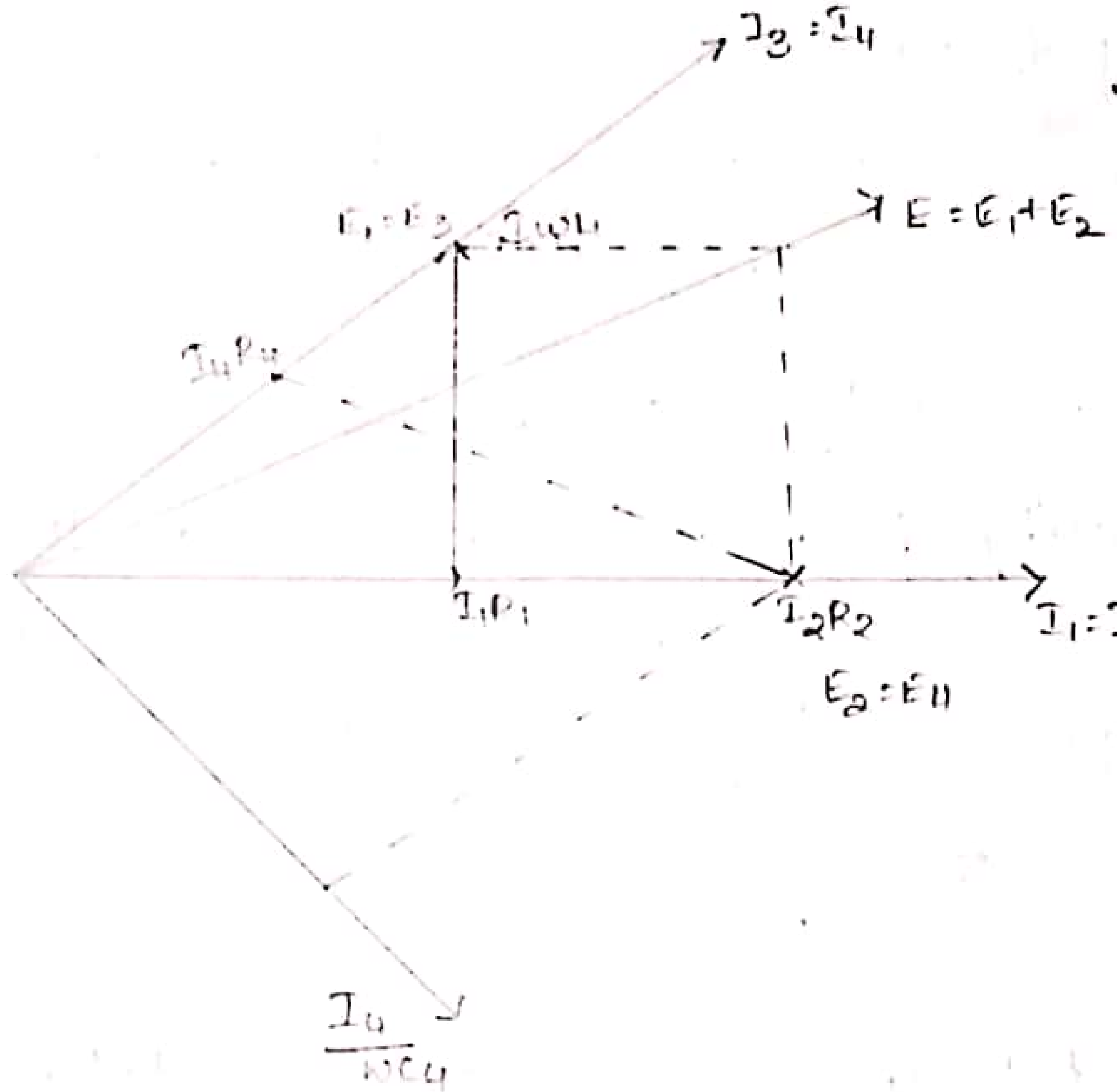


fig: Phasor diagram of Hay's Bridge

Advantages of Hay's Bridge

* Hays Bridge is preferred for coils with high Q
* The expression to find Quality factor $Q = \frac{1}{\omega C_4 R_4}$ is Simple.
* The inductance balance depends on its Q and frequency. $L = \frac{R_2 R_3 C_4}{1 + (1/Q)^2}$ where $Q = \frac{1}{\omega C_4 R_4}$.

For a value of $Q > 10$ the term $\frac{1}{Q^2}$ will be Smaller than 1/100 and can be therefore neglected.

Disadvantages of Hay's Bridge

* Hay's bridge is not suited for measurements of coils having $Q < 10$.

Applications of Hay's Bridge

* Hays bridge is used in the measurement of incremental ~~impedance~~. inductance
* A Commercial bridge measures from $1 \mu H$ to 100 H with $\pm 2\%$ error.

3. ANDERSON'S BRIDGE

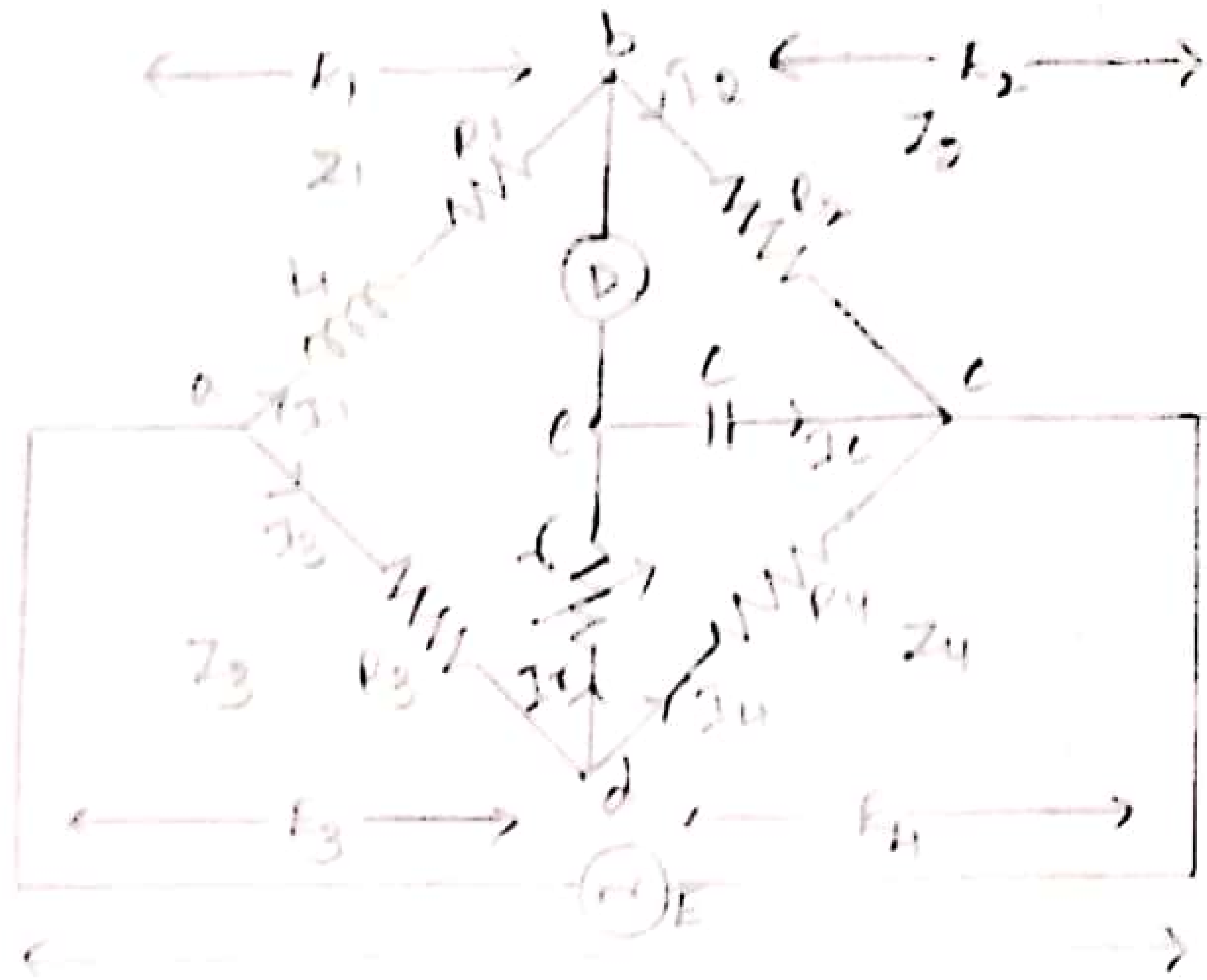


fig: Anderson's Bridge.

* Anderson's Bridge is one of the bridge that is used for the measurement of Self-inductance.
* The unknown value of self-inductance is measured in terms of Standard fixed Capacitor.
* The figure shows the Andersen bridge.

Construction

* the bridge consists of four arms ab, bc, cd ea which impedances are connected.
* Apart from those four arms another arm ce is present. for which a capacitor is connected.
* L_1 is the self inductance which is to be measured and R_1 is the resistance. L_1, R_1 are connected in series. on arm ab.
* R_2, R_3, R_4 are non-inductive resistances whose

values are known. C is a standard fixed capacitor.

Working operation

* When the bridge is ON, the e.m.f is induced in the bridge due to AC supply and the current flows through the circuit.

* At the bridge balance when no current flows through Detector, the Detector shows null value.

* As such the unknown impedance can be found using the known impedances.

General Balance Condition $Z_1 Z_4 = Z_2 Z_3$

$Z_1 = R_1 + j\omega L_1$

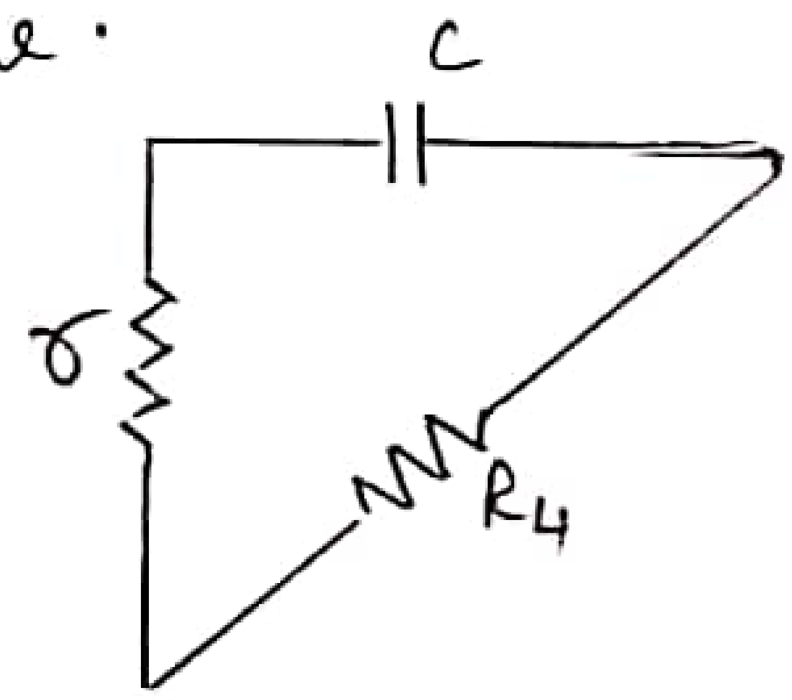
$Z_2 = R_2$

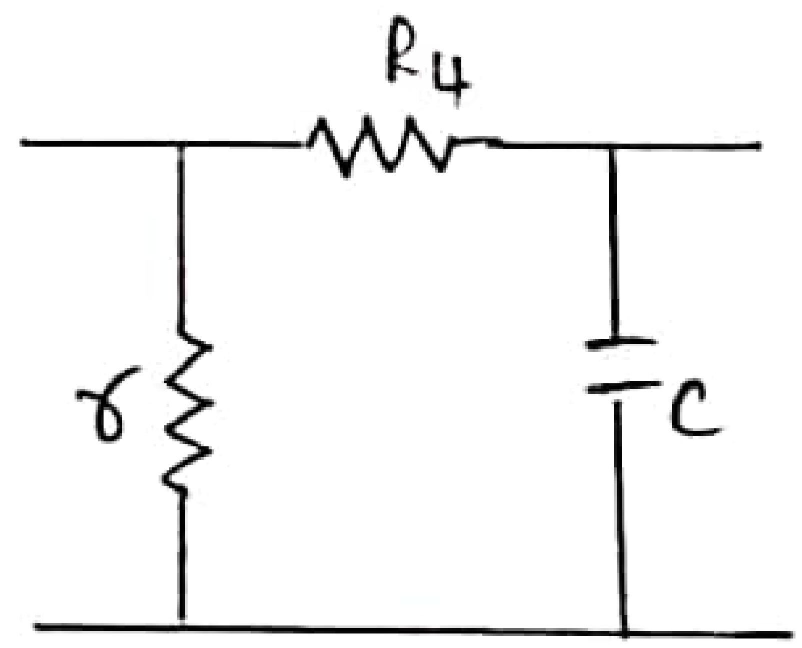
$Z_3 = R_3$

$Z_4 = R_4$

* Inorder to balance the circuit, the circuit is to be reconstructed.

* the connection at arm dc, de, ce resembles a π network or delta connection. It is to be transformed to a T network or star connection as shown in figure.





π network

* Conversion of a π to T Network.

fig: π Network

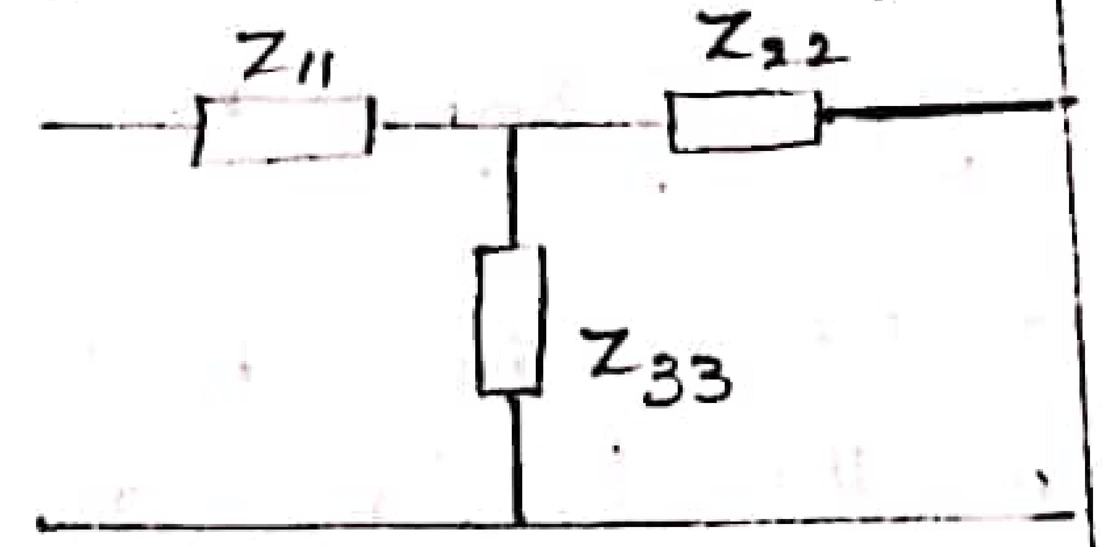
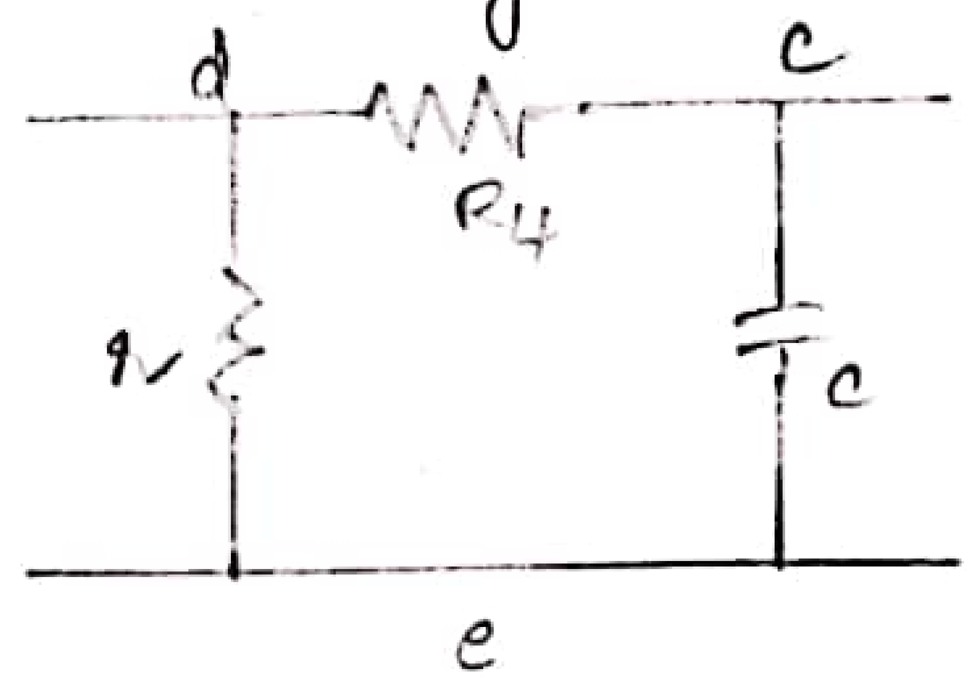


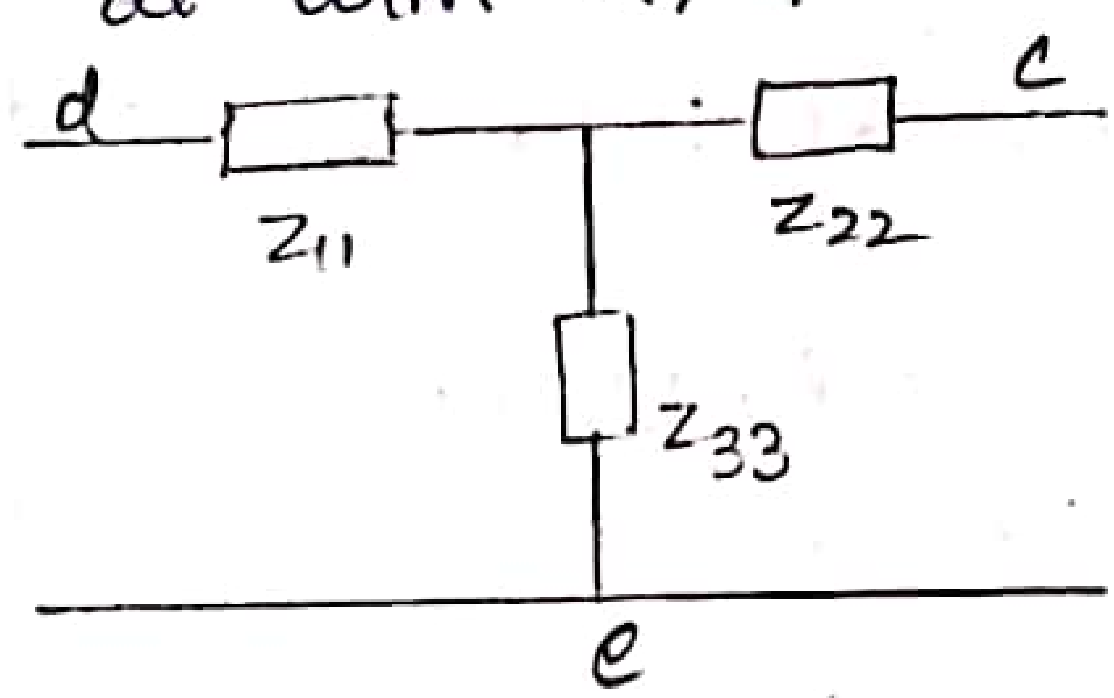
fig:- T- Network

The transformed impedances are :

$$Z_{11} = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} \; ; \quad Z_{22} = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} \; ; \quad Z_{33} = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

* Considering the π network at arm d, c, e





Here from fig, $Z_A = r$ $\quad Z_B = R_4 \quad Z_C = X_C = \frac{1}{j\omega C}$.

$$Z_{11} = \frac{Z_A Z_B}{Z_A + Z_B + Z_C} = \frac{r R_4}{r + R_4 + \frac{1}{j\omega C}}$$

$$Z_{22} = \frac{Z_B Z_C}{Z_A + Z_B + Z_C} = \frac{R_4 \cdot \frac{1}{j\omega C}}{r + R_4 + \frac{1}{j\omega C}}$$

$$Z_{33} = \frac{Z_A Z_C}{Z_A + Z_B + Z_C} = \frac{r \cdot \frac{1}{j\omega C}}{r + R_4 + \frac{1}{j\omega C}}.$$

* The bridge circuit can be reconstructed as

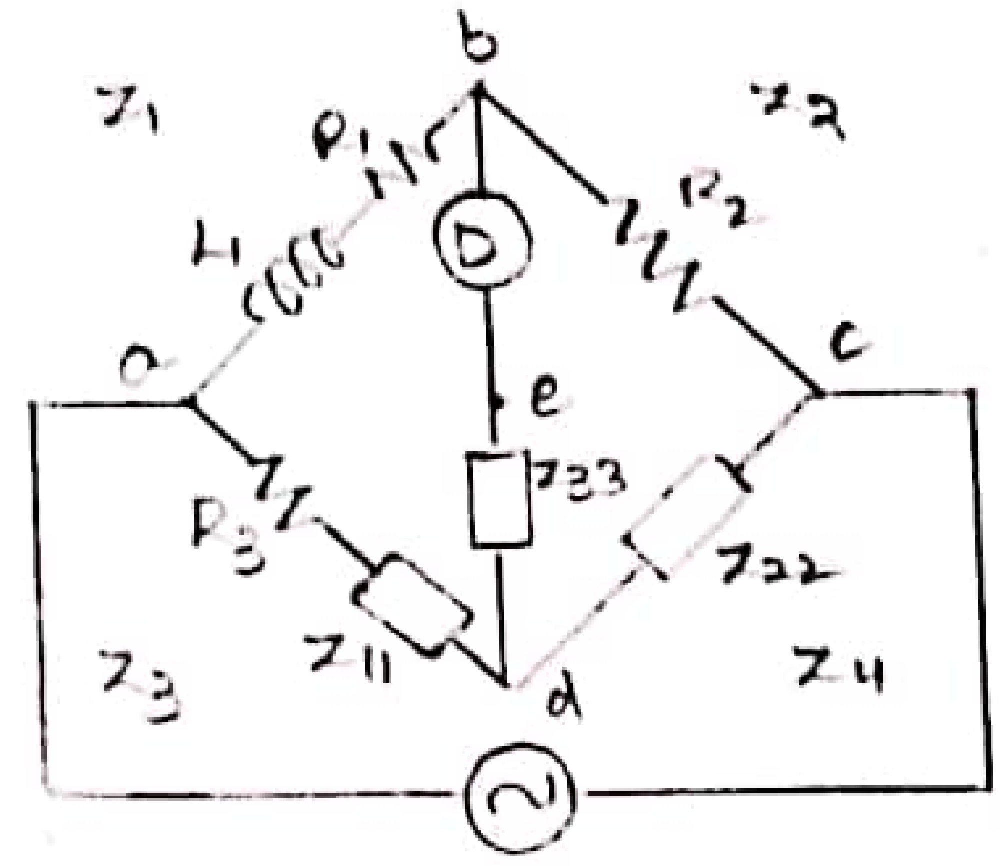


fig:

* The impedances of circuit are

$Z_1 = R_1 + j\omega L_1$

$Z_2 = R_2$

$Z_3 = R_3 + Z_{11}$

$Z_4 = Z_{22}$

$$Z_{11} = \frac{rR_4}{r + R_4 + \frac{1}{j\omega C}}$$

$$Z_{22} = \frac{\frac{R_4}{j\omega C}}{r + R_4 + \frac{1}{j\omega C}}$$

Substitute the above in General Balance Equation

$$Z_1 Z_4 = Z_2 Z_3 .$$

$$(R_1 + j\omega L_1) Z_{22} = R_2 (R_3 + Z_{11})$$

$$(R_1 + j\omega L_1) \frac{\frac{R_4}{j\omega C}}{r + R_4 + \frac{1}{j\omega C}} = R_2 \left(R_3 + \frac{rR_4}{r + R_4 + \frac{1}{j\omega C}} \right)$$

$$(R_1 + j\omega L_1) \frac{R_4}{j\omega C} = \left(r + R_4 + \frac{1}{j\omega C} \right) R_2 \left(R_3 + \frac{rR_4}{r + R_4 + \frac{1}{j\omega C}} \right)$$

$$\frac{R_1 R_4}{j\omega C} + \frac{\cancel{j\omega} L_1 R_4}{\cancel{j\omega} C} = R_2 R_3 \left(r + R_4 + \frac{1}{j\omega C} \right) + \frac{rR_4R_2 \cancel{\left(r + R_4 + \frac{1}{j\omega C} \right)}}{\cancel{\left(r + R_4 + \frac{1}{j\omega C} \right)}}$$

$$\frac{R_1 R_4}{j\omega C} + \frac{L_1 R_4}{C} = R_2 R_3 r + R_2 R_3 R_4 + \frac{R_2 R_3}{j\omega C} + r R_4 R_2$$

$$\frac{L_1 R_4}{C} - \frac{j R_1 R_4}{\omega C} = R_2 (r R_3 + R_3 R_4 + R_4 r) - \frac{j R_2 R_3}{\omega C}$$

Equating real and imaginary parts on bothsides

$$\frac{L_1 R_4}{C} = R_2 (r R_3 + R_3 R_4 + r R_4) \qquad \frac{R_1 R_4}{\cancel{\omega C}} = \frac{R_2 R_3}{\cancel{\omega C}}$$

$$L_1 = \frac{C R_2}{R_4} (r R_3 + R_3 R_4 + r R_4) \qquad R_1 = \frac{R_2 R_3}{R_4}$$

$$L_1 = C R_2 \left(r \frac{R_3}{R_4} + R_3 + r \right) \qquad \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

the unknown impedance $Z_1 = R_1 + j\omega L_1$

$$Z_1 = \frac{R_2 R_3}{R_4} + j\omega C R_2 \left(r \frac{R_3}{R_4} + R_3 + r \right)$$

Quality factor

The Quality factor is given by $Q = \frac{\omega L_1}{R_1}$

$$Q = \frac{\omega C \cancel{R_2} \left(r \frac{R_3}{R_4} + R_3 + r \right)}{\frac{\cancel{R_2} R_3}{R_4}}$$

$$= \frac{\omega C R_4}{R_3} \left(r \frac{R_3}{R_4} + R_3 + r \right) = \omega C \left(\frac{R_4}{\cancel{R_3}} \cdot r \cdot \frac{\cancel{R_3}}{\cancel{R_4}} + \frac{R_4}{\cancel{R_3}} \cdot \cancel{R_3} + r \cdot \frac{R_4}{R_3} \right)$$

$$Q = \omega C \left(r + R_4 + r \frac{R_4}{R_3} \right)$$

Phasor Diagram of Anderson's Bridge

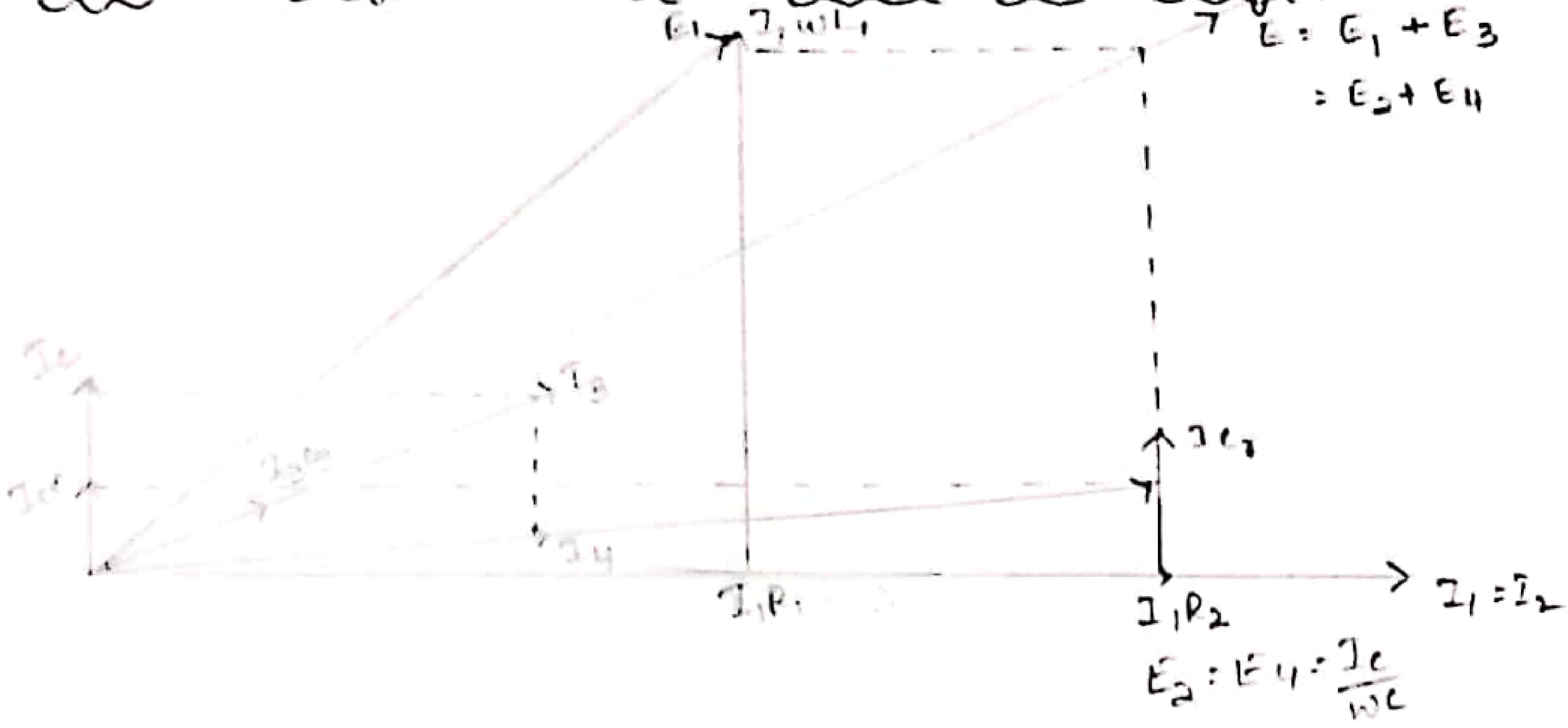


fig: phasor diagram of Andersons Bridge.

Advantages of Anderson's Bridge

* The Anderson's Bridge measures inductance over a wide range of values.

* A fixed Capacitor is used hence the Circuit is inexpensive.

* The bridge can be used for measurement of Capacitance in terms of inductance.

* This bridge is used for $Q < 1$.

Disadvantages of Anderson's Bridge

* The Andersons Bridge is very complicated bridge circuit

* Many components are required to build this circuit.

* This will have difficult scheilding due to junction point. e.

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4. SCHERING'S BRIDGE

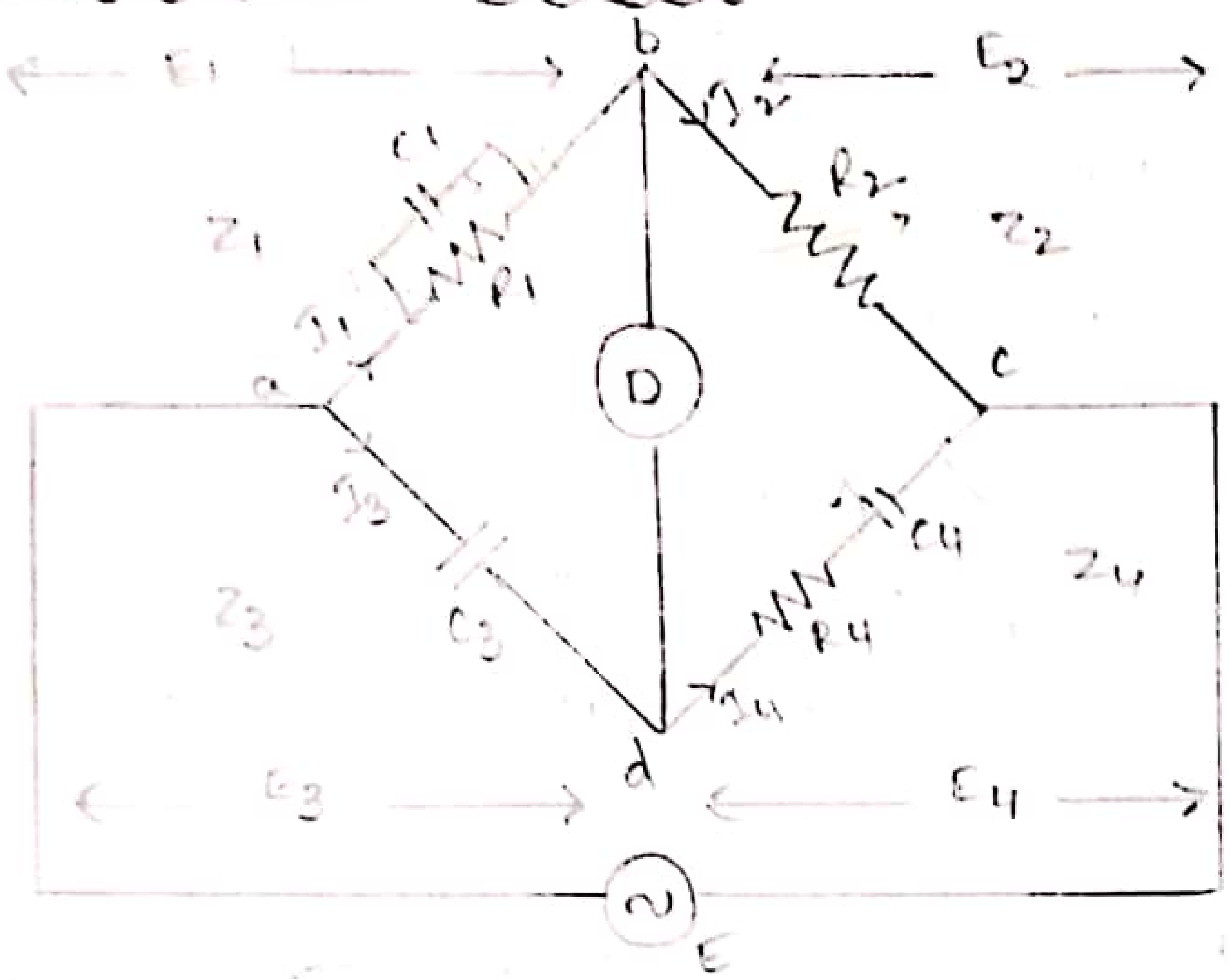


fig: Schering's Bridge Circuit.

A very important bridge used for the precision measurement of Capacitors and their insulating Properties is the Schering Bridge.

* The figure shows the basic Circuit of the Schering Bridge.

Construction

* The bridge circuit Consists of four arms of impedances. of which one is unknown.
* The unknown impedance is to be measured by the known Values of impedance.
* The arm 4 - cd has a Resistance in Series with a Capacitor. (unknown)
* The arm2 - bc contain a Variable Resistor.
* The arm-3 ad has a Capacitor variable connected in parallel to a resistor R_1.

* The standard capacitor C_3 is a high quality mica capacitor (low-loss) for general measurements.

An air capacitor (a very small electric field) for insulation measurements.

Working operation

* When the bridge is ON, the e.m.f is induced in the bridge due to AC supply and current flows through the circuit.

* Adjusting the variable components (C_4, R_2) in the arms such that no current flows through detector.

i.e. the detector shows a null value at the bridge balance.

* The bridge is said to be balanced when no current flows through terminals b and d.

* At such using the bridge balance conditions, the unknown impedance can be found using the known impedances.

General Balance Condition $Z_1 Z_4 = Z_2 Z_3$.

The impedances on the four arms are as follows.

arm cd Z_4 : $R_4 - \frac{j}{\omega C_4}$ (unknown)

$R_4 + \frac{1}{j\omega C_4} = R_4 - \frac{j}{\omega C_4}$.

arm bc Z_2 : R_2 (variable)

arm ab : $Z_1 = R_1 \parallel C_1$

$$= \frac{R_1 \times \frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}$$

$$= \frac{\frac{R_1}{\cancel{j\omega C_1}}}{\frac{R_1 j\omega C_1 + 1}{\cancel{j\omega C_1}}} = \frac{R_1}{1 + j\omega C_1 R_1}.$$

arm da : $Z_3 = \frac{1}{j\omega C_3}$

$$= \frac{-j}{\omega C_3}$$

Substituting the values of $Z_1\ Z_2\ Z_3\ Z_4$ in the General balance Equation, $Z_1 Z_4 = Z_2 Z_3$.

$$\left(R_4 - \frac{j}{\omega C_4}\right)\left(\frac{R_1}{1 + j\omega C_1 R_1}\right) = R_2\left(\frac{-j}{\omega C_3}\right)$$

$$\left(\frac{R_4 \omega C_4 - j}{\omega C_4}\right)\left(\frac{R_1}{1 + j\omega C_1 R_1}\right) = \frac{-R_2 j}{\omega C_3}$$

On Cross multiplication,

$$(R_4 \omega C_4 - j) R_1 (\omega C_3) = -j R_2 (\omega C_4)(1 + j\omega C_1 R_1)$$

$$\omega^2 C_4 C_3 R_1 R_4 - j\omega R_1 C_3 = -j\omega C_4 R_2 - j^2 \omega^2 R_2 C_4 C_1 R_1$$

$\because j = \sqrt{}$
$j^2 = -$

$$\omega^2 C_4 C_3 R_1 R_4 - j\omega R_1 C_3 = -j \cdot \omega C_4 R_2 + \omega^2 R_2 C_1 C_4 R_1$$

Equating Real terms and imaginary Terms.

Real

$$\cancel{\omega^2}\cancel{C_4} C_3 R_4 \cancel{R_1} = \cancel{\omega^2} R_2 \cancel{C_4} C_1 \cancel{R_1}$$

$$C_3 R_4 = R_2 C_1$$

$$\boxed{R_4 = \frac{R_2 C_1}{C_3}.}$$

imaginary

$$\cancel{\omega} R_1 C_3 = \cancel{\omega} C_4 R_2$$

$$R_1 C_3 = C_4 R_2$$

$$\boxed{C_4 = \frac{R_1 C_3}{R_2}}$$

Substituting values of R_4, C_4 in Z_4.

$$Z_4 = R_4 - \frac{j}{\omega C_4} \qquad R_4 = \frac{C_1 R_2}{C_3} \qquad C_4 = \frac{R_1 C_3}{R_2}$$

$$Z_4 = \frac{C_1 R_2}{C_3} - \frac{j}{\frac{\omega R_1 C_3}{R_2}}$$

$$Z_4 = \frac{C_1 R_2}{C_3} - \frac{j R_2}{\omega C_3 R_1} \qquad \boxed{Z_4 = \frac{R_2}{C_3}\left[C_1 - \frac{j}{\omega R_1}\right]}$$

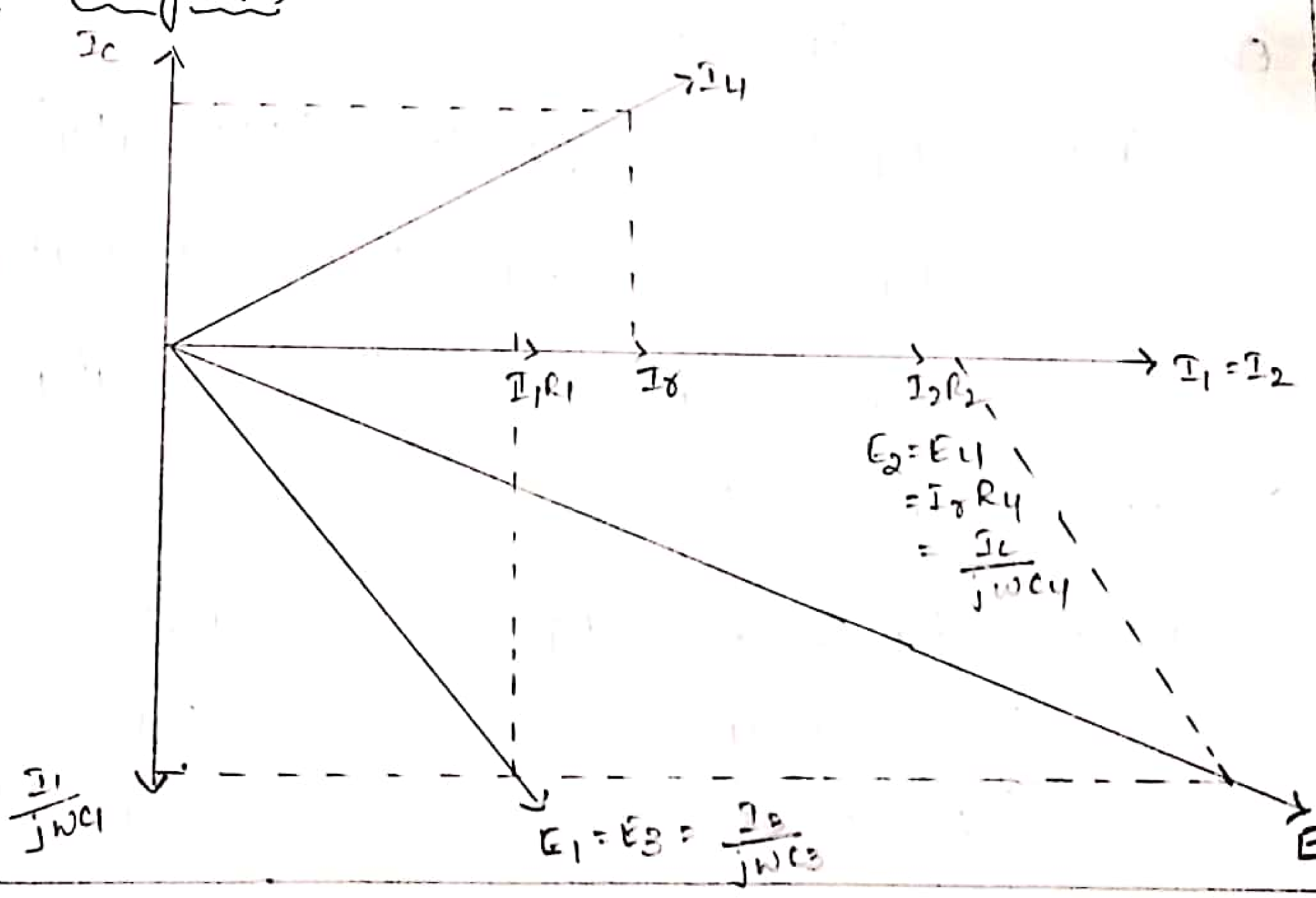
Quality factor

The Quality factor is the measure of power Stored in coil to power dissipiated by coil.

$$Q = \frac{R}{X_C} = \frac{R}{\frac{-j}{\omega C}} = j\omega C R.$$

$$\boxed{Q = j\omega C R}$$

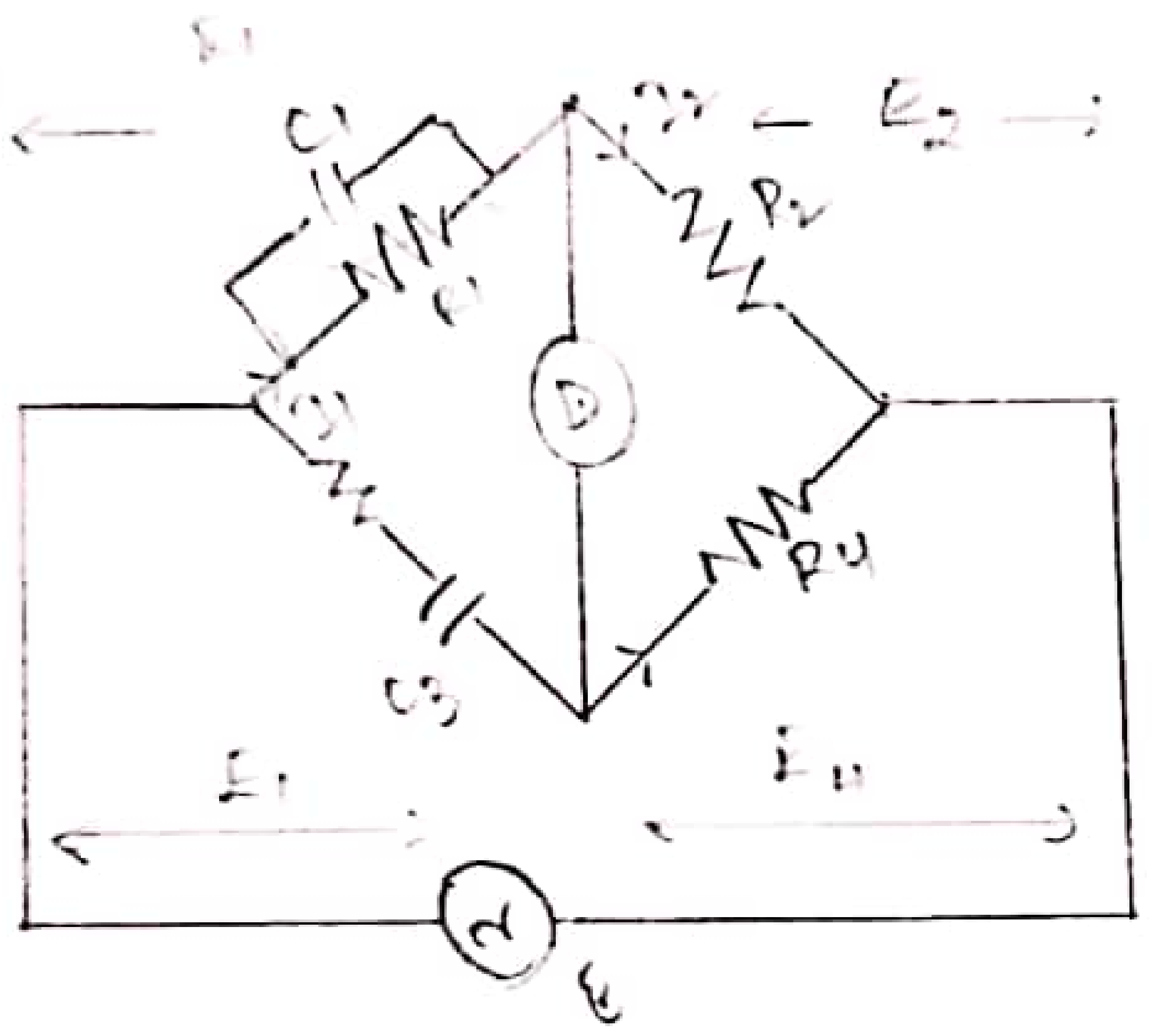
Phasor Diagram



5. Wein's Bridge

* this Bridge is used for the measurement of frequency in the range from 100 Hz to 100k Hz
* the accuracy of the bridge is between 0.1% to 0.5%.
* this bridge is named after the Scientist Wein who invented it.
* It is an originating current Bridge.
* It is used to measure frequency.
* It can also be used for the measurement of an unknown Capacitance with great accuracy.

Construction



* the figure shows the basic form of a Wein Bridge
* It has a series RC combination in one arm and a parallel combination in the adjoining arm.

* The arms bc and cd have a resistor. An AC voltage source is connected between the arms a & c.

* The arm terminals b and d have a detector to detect the current flow.

Working

* When the bridge is ON, the e.m.f is induced in the bridge due to AC supply and current flows through the circuit.

* Adjusting the variable components in the arms such that no current flows through detector.

* The bridge is said to be balanced when no current flows through terminals b and d.

* At such using the bridge balance conditions the frequency can be determined.

At balance

The impedances are $Z_1 = \dfrac{R_1}{1+j\omega C_1 R_1}$

$$Z_2 = R_2$$

$$Z_3 = R_3 + \frac{1}{j\omega C_3}$$

$$Z_4 = R_4$$

$$Z_1 Z_4 = Z_2 Z_3 .$$

$$\left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = R_2\left(R_3 + \frac{1}{j\omega C_3}\right)$$

$$\frac{R_1 R_4}{1+j\omega C_1 R_1} = R_2 R_3 + \frac{R_2}{j\omega C_3}$$

$$R_1 R_4 = (1+j\omega C_1 R_1)\left(R_2 R_3 + \frac{R_2}{j\omega C_3}\right)$$

$$R_1 R_4 = (1+j\omega C_1 R_1) R_2 R_3 + \frac{R_2 (1+j\omega C_1 R_1)}{j\omega C_3}$$

$$R_1 R_4 = R_2 R_3 + j\omega C_1 R_1 R_2 R_3 + \frac{R_2}{j\omega C_3} + \frac{j R_2 \omega C_1 R_1}{j\omega C_3}$$

$$R_1 R_4 = R_2 R_3 + j\omega C_1 R_1 R_2 R_3 - \frac{j R_2}{\omega C_3} + \frac{R_2 C_1 R_1}{C_3}$$

Equating real and imaginary parts.

Real part

$$R_1 R_4 = R_2 R_3 + \frac{R_1 R_2 C_1}{C_3}$$

$$\frac{R_1 R_4}{R_2} = R_3 + \frac{R_1 C_1}{C_3}$$

$$\frac{R_4}{R_2} = \frac{R_3}{R_1} + \frac{C_1}{C_3}$$

imaginary

$$\frac{R_2}{\omega C_3} = \omega C_1 R_1 R_2 R_3$$

$$\omega^2 = \frac{1}{C_1 C_3 R_1 R_3}$$

$$\omega = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}} \text{ rad/s.}$$

$$\omega = 2\pi f .$$

$$2\pi f = \frac{1}{\sqrt{C_1 C_3 R_1 R_3}}$$

$$f = \frac{1}{2\pi\sqrt{C_1 C_3 R_1 R_3}} \text{ Hz.}$$

Chosing Values R, C Such that $R_1 = R_3 = R$
$C_1 = C_3 = C$

$$f = \frac{1}{2\pi\sqrt{R^2C^2}}$$

$$f = \frac{1}{2\pi RC}.$$

Applications of weinbridge

* The bridge is used for measuring frequency in the audio range. The audio range is divided into 20-200-2K-20KHz ranges.

* The bridge is also used for measuring Capacitances. In Such a case frequency is to be known.

* The bridge is also used in harmonic distortion analyser; as a Notch filter.

* The bridge is also used in audio frequency and radio frequency oscillator as a frequency determining element.

Errors and Precautions in using Bridge Circuits

Errors

1. Stray Conductance effect, due to imperfect insulation

2. Mutual inductance effects due to magnetic Coupling between various Components of the bridge.

3. Stray capacitance effects due to electrostatic fields between Conductors at different potentials.

4. Residuals in components for example the presence of small magnitudes of series inductance or shunt capacitance in non-reactive resistors.

Precautions

1. Hight Quality components must be used for the elements of the bridge.

2. The layout of the bridge must be made to avoid interaction of bridge arms.

3. Sensitivity of bridge must be more.

4. The bridge components and other pieces must be mounted on insulating stands to prevent stray conductance effects.

5. Residual error can be avoided by identifying the nature, evaluating them and Compensating them.

Q meter

The Q meter is an instrument designed to measure some of the electrical properties of coils and capacitors. It is based on the familiar characteristics of a series-resonant ckt. namely that the voltage across the coil or the capacitor is equal to the applied voltage times the Q of the circuit.

If a fixed voltage is applied to the circuit, a voltmeter across the capacitor can be calibrated to read Q directly. At series resonance

$$X_C = X_L$$

$$\frac{1}{\omega C} = \omega L$$

$$\omega^2 = \frac{1}{LC} \quad \therefore \omega = \frac{1}{\sqrt{LC}}$$

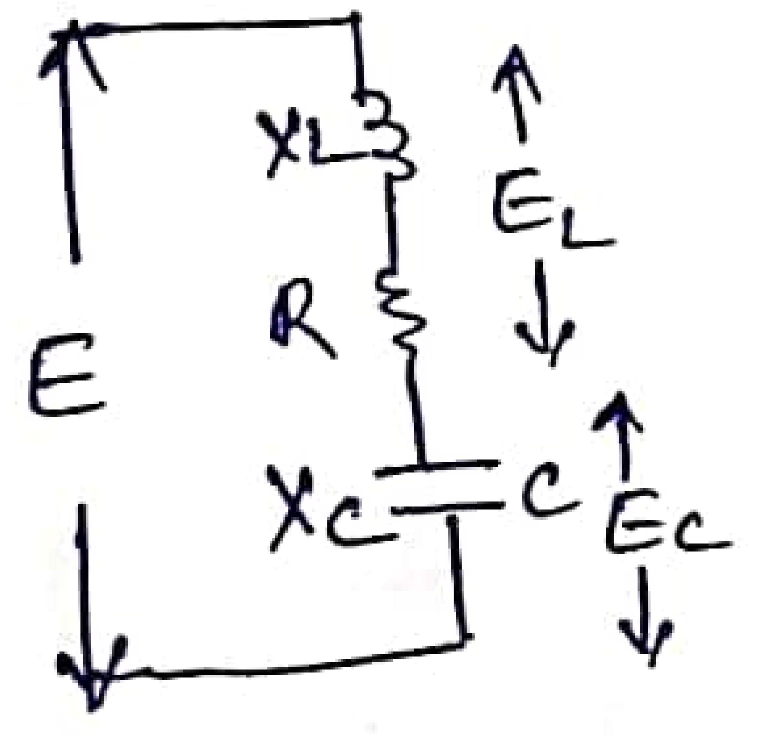
$$2\pi f = \frac{1}{\sqrt{LC}}$$

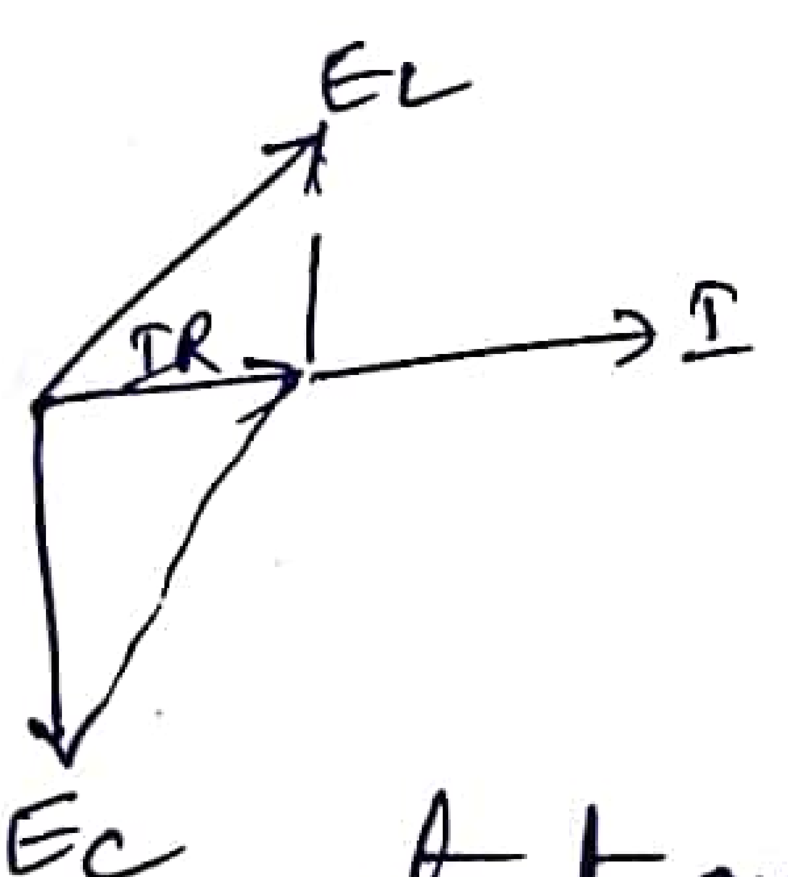
$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$E_C = I X_C = I X_L$$

$$E = IR$$

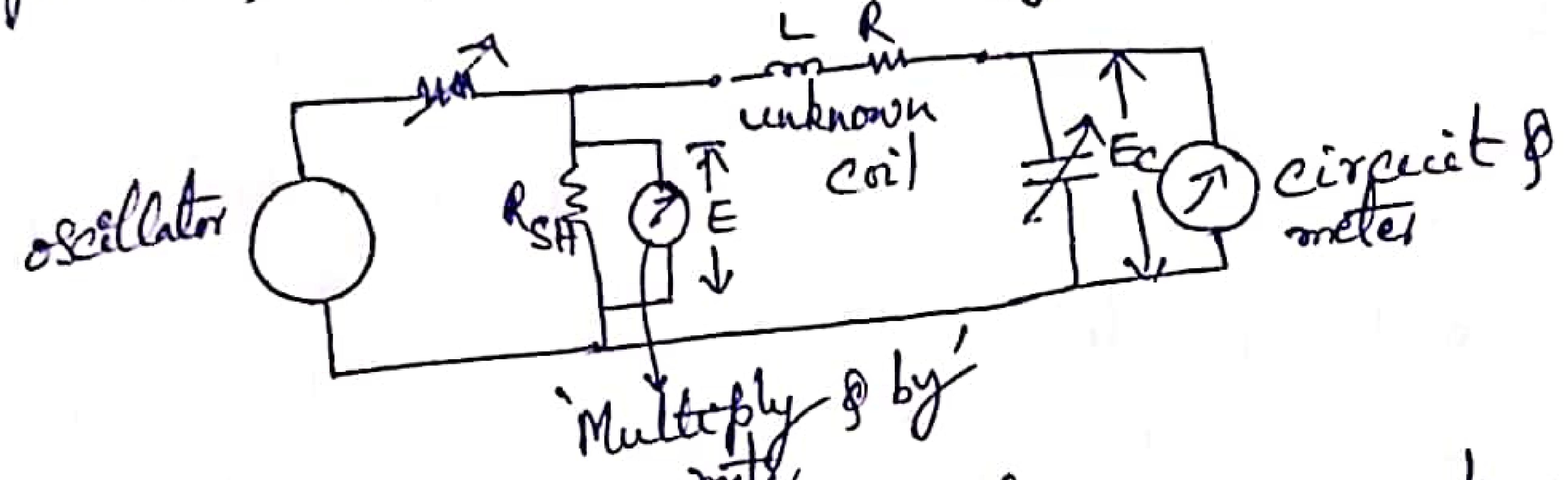
$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$





Therefore if E is maintained at a constant and known level, a voltmeter connected across the capacitor can be calibrate directly in terms of the circuit Q.

A practical Q meter circuit is shown in fig.



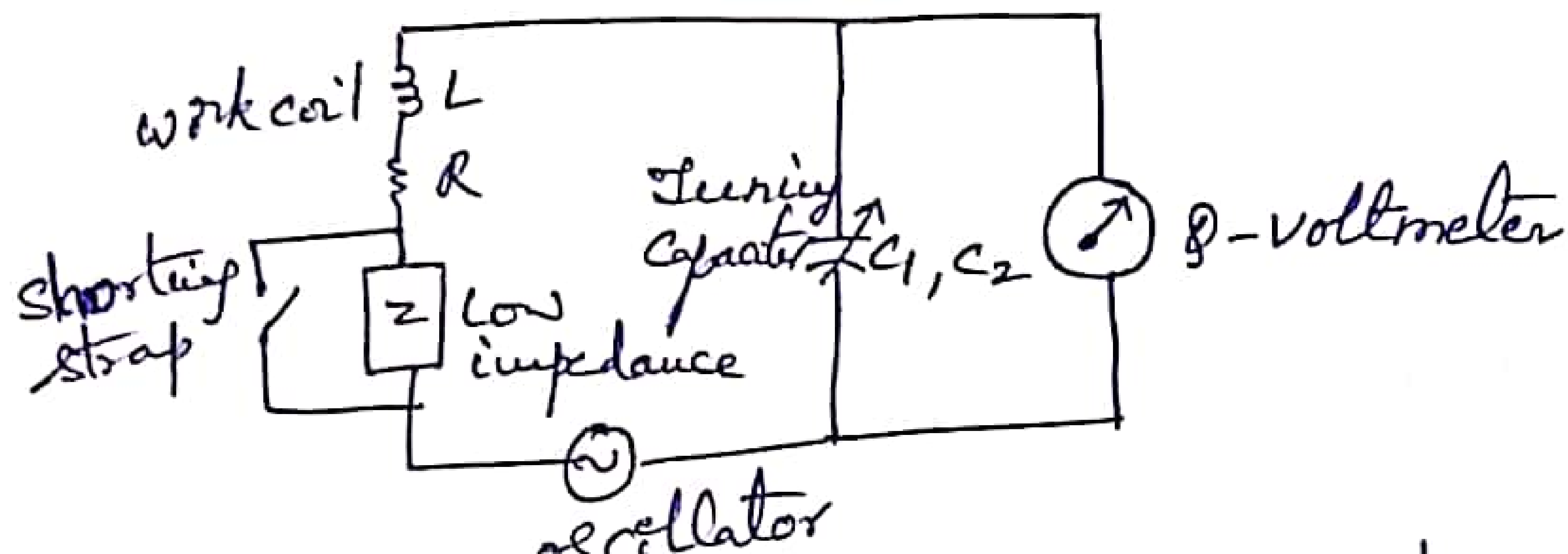
The wide range oscillator with a frequency range from 50 KHz to 50 MHz delivers current to a low value shunt resistance R_{SH}. The value of this shunt is very low of the order of 0.02 Ω. The voltage across the shunt ie E is measured with a thermocouple meter. The voltage across the variable capacitor ie E_c is measured with an electronic voltmeter whose scale is calibrated directly in Q values.

To make a measurement, the unknown coil is connected to the test terminals of the instrument and the circuit is tuned to resonance either by setting the oscillator to a given frequency and varying the resonating capacitor or by presetting the capacitor to a desired value and adjusting the frequency of the oscillator. The Q reading on the o/p meter must be multiplied by the "Multiply Q by" meter to obtain the actual value. The inductance of the coil can be calculated from the equation $L = \frac{1}{(2\pi f)^2 C}$ $[\because X_L = X_C]$

Measurement methods :

Direct connection : Most coils can be connected directly across the test terminals. The circuit is resonated by adjusting either the oscillator frequency or the resonating capacitor. The indicated Q is read directly from the "circuit Q" meter multiplied by the setting of "Multiply Q by" meter. When the last meter is set at the unity mark, the "circuit Q" meter reads the correct value of Q directly.

Series Connection: Low impedance components such as low-value resistors, small coils and large capacitors are measured in series with the measuring circuit.



The component to be measured, here indicated by [Z], is placed in series with a stable work coil across the test terminals. Two measurements are made. In the first measurement the unknown is short-circuited by a small shorting strap and the circuit is resonated, establishing a reference condition. The values of the tuning capacitor (C_1) and the indicated Q (Q_1) are noted. In the second measurement the shorting strap is removed and the circuit is retuned, giving a new value for the tuning capacitor (C_2) and a change in the Q value from Q_1 to Q_2.

For the reference condition

$$X_{C_1} = X_L \quad \text{or} \quad \frac{1}{\omega C_1} = \omega L$$

and neglecting the resistance of the measuring ckt

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

For the Second measurement, the reactance of the unknown can be expressed in terms of the new value of the tuning capacitor (C_2) and the in-circuit value of the inductor L. This yields

$$X_S = X_{C_2} - X_L$$

$$X_S = \frac{1}{\omega C_2} - \frac{1}{\omega C_1}$$

So that

$$X_s = \frac{C_1 - C_2}{\omega C_1 C_2}$$

X_s is inductive if $C_1 > C_2$ and capacitive if $C_1 < C_2$.
The resistive component of the unknown impedance can be found in terms of X_S and the indicated value of circuit Q.

$$R_1 = \frac{X_1}{Q_1} \quad \text{and} \quad R_2 = \frac{X_2}{Q_2}$$

$$R_S = R_2 - R_1 = \frac{1}{\omega C_2 Q_2} - \frac{1}{\omega C_1 Q_1}$$

$$R_S = \frac{C_1 Q_1 - C_2 Q_2}{\omega C_1 C_2 Q_1 Q_2}$$

If the unknown is purely resistive $C_1 = C_2$

$$R_S = \frac{\cancel{C_1}(Q_1 - Q_2)}{\omega C_1^{\cancel{2}} Q_1 Q_2} = \frac{Q_1 - Q_2}{\omega C_1 Q_1 Q_2}$$

If the unknown is a small inductor

$$L_S = \frac{C_1 - C_2}{\omega^2 C_1 C_2}$$

The Q of the coil is found from

$$Q_S = \frac{X_S}{R_S} = \frac{\dfrac{C_1 - C_2}{\omega C_1 C_2}}{\dfrac{C_1 Q_1 - C_2 Q_2}{C_2 \omega C_1 Q_1 Q_2}}$$

$$= \frac{C_1 - C_2}{\cancel{\omega C_1 C_2}} \times \frac{\cancel{\omega C_1 C_2} Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}$$

$$= \frac{(C_1 - C_2) Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}$$

If the unknown is a large capacitor

$$\frac{1}{\omega C_S} = \frac{C_1 - C_2}{\omega C_1 C_2}$$

$$C_S = \frac{C_1 C_2}{C_1 - C_2}$$

The Q of the capacitor is found from

$$Q_S = \frac{(C_1 - C_2) Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}$$

<u>parallel connection</u> :- High impedance components such as high value resistors, certain inductors and small capacitors are measured by connecting them in parallel with the measuring circuit. Before the unknown is connected, the circuit is resonated by using a suitable work coil to establish reference values for Q and C (Q_1 and C_1). Then, when the component under test is connected to the circuit, the capacitor is readjusted for resonance and a new value for the tuning capacitance (C_2) is obtained and a change in the value of circuit (ΔQ) from Q_1 to Q_2.

In a parallel circuit, computation of the unknown impedance is best approached in terms of its parallel components X_p and R_p as indicated in fig

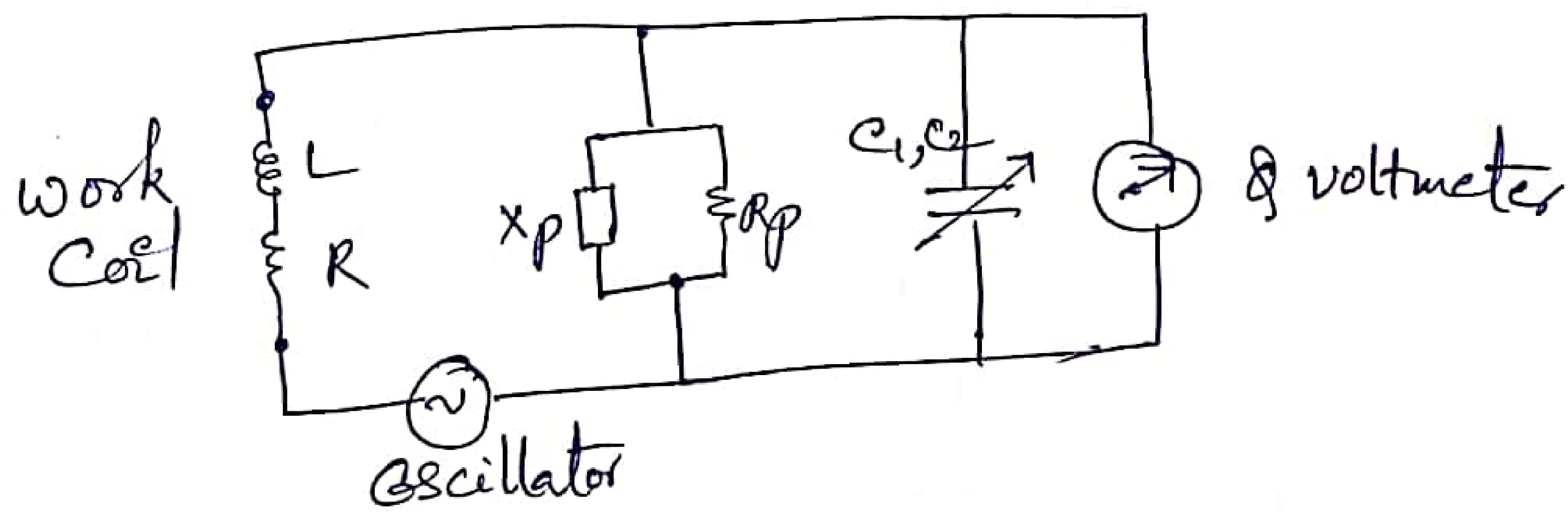
At the initial resonance condition, when the unknown is not yet connected into the circuit, the working coil (L) is tuned by the capacitor (C_1). Therefore

$$\omega L = \frac{1}{\omega C_1}$$

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

When the unknown impedance is now connected into the ckt. and the capacitor is tuned for resonance the reactance of the working coil (X_L) equals the parallel reactances of the tuning capacitor X_{C_2} and X_P.

Therefore $\quad X_L = \dfrac{X_{C_2} X_P}{X_{C_2} + X_P}$



$$\frac{X_{C_2} + X_P}{X_{C_2} X_P} = \frac{1}{X_L}$$

$$\frac{1}{X_P} + \frac{1}{X_{C_2}} = \frac{1}{\omega L}$$

$$\frac{1}{X_P} = \omega C_1 - \frac{1}{X_{C_2}}$$

$$= \omega C_1 - \omega C_2$$

$$X_P = \frac{1}{\omega C_1 - \omega C_2} = \frac{1}{\omega (C_1 - C_2)}$$

If the unknown is inductive

$$X_P = \omega L_P$$

$$\omega L_P = \frac{1}{\omega(C_1 - C_2)}$$

$$L_P = \frac{1}{\omega^2(C_1 - C_2)}$$

If the unknown is capacitive

$$X_P = \frac{1}{\omega C_P}$$

$$\frac{1}{\cancel{\omega} C_P} = \frac{1}{\cancel{\omega}(C_1 - C_2)}$$

$$C_P = C_1 - C_2$$

In a parallel resonant circuit the total resistance at resonance is equal to the product of the circuit Q and the reactance of the coil.

$$\therefore \quad R_T = Q_2 X_L$$

$$R_T = Q_2 X_{C_1} = \frac{Q_2}{\omega C_1}$$

The resistance (R_P) of the unknown impedance is found by computing the conductances in the ckt of fig.

Let G_T = total conductance of resonant ckt

G_P = conductance of the unknown impedance

G_L = conductance of working coil

$$\therefore \quad G_T = G_P + G_L$$

$$G_P = G_T - G_L$$

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$$G_T = \frac{1}{R_T} = \frac{\omega C_1}{Q_2}$$

$$\frac{1}{R_p} = \frac{\omega C_1}{Q_2} - \frac{R}{R^2+\omega^2L^2} \qquad \left[\because Z_L = R+j\omega L \quad G_L = \frac{1}{R+j\omega L} = \frac{R-j\omega L}{R^2+\omega^2L^2}\right]$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R}\,\frac{1}{1+\frac{\omega^2L^2}{R^2}}$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R Q_1^2} \qquad \left[\because Q_1 = \frac{\omega L}{R}\right]$$

$$\frac{1}{R_p} = \frac{\omega C_1}{Q_2} - \frac{1}{R Q_1 \cdot Q_1}$$

$$= \frac{\omega C_1}{Q_2} - \frac{\omega C_1}{Q_1} \qquad \left[\because Q_1 = \frac{1}{\omega C_1 R} \quad \omega C_1 = \frac{1}{Q_1 R}\right]$$

$$= \frac{\omega C_1 Q_1 - \omega C_1 Q_2}{Q_1 Q_2}$$

$$R_p = \frac{Q_1 Q_2}{\omega C_1 (Q_1 - Q_2)} = \frac{Q_1 Q_2}{\omega C_1 \Delta Q}$$

The Q of the unknown is found by

$$Q_p = \frac{R_p}{X_p} = \frac{Q_1 Q_2}{\cancel{\omega} C_1 \Delta Q} \cdot \cancel{\omega}(C_1 - C_2)$$

$$= \frac{(C_1 - C_2) Q_1 Q_2}{C_1 \Delta Q}$$

Problem:- A maxwell's bridge is used to measure an inductive impedance. The bridge constants at balance are $C_1 = 0.01\ \mu F$, $R_1 = 470\,k\Omega$, $R_2 = 5.1\,k\Omega$ and $R_3 = 100\,k\Omega$ find L_x, R_x.

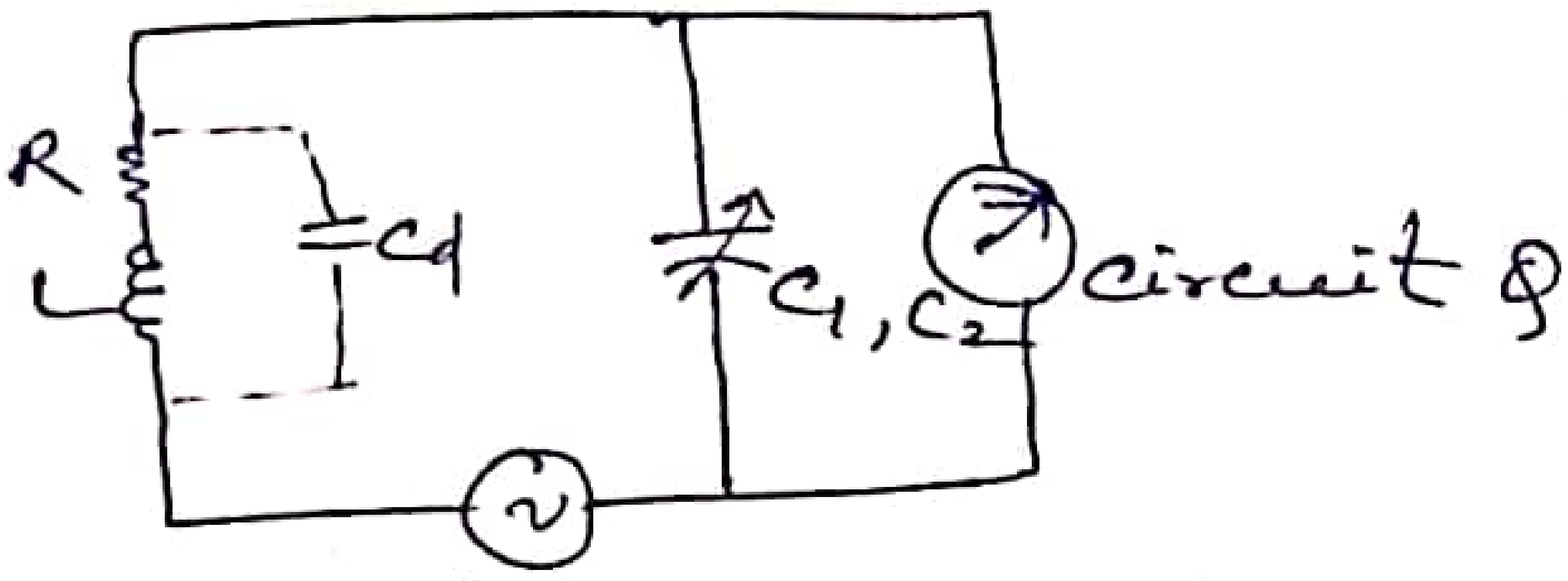
$$L_x = R_2 R_3 C_1$$

$$R_x = \frac{R_2 R_3}{R_1}$$

Sources of error

probably the most important factor affecting measurement accuracy is the distributed capacitance or self-capacitance of the measuring circuit. The presence of distributed capacitance in a coil modifies the actual or effective Q and the inductance of the coil. At the frequency at which the self-capacitance and the inductance of the coil are resonant, the circuit exhibits a purely resistive impedance. This characteristic may be used for measuring the distributed capacitance.

One simple method of finding the distributed capacitance (Cd) of a coil involves making two measurements at different frequencies. The coil under test is connected directly to the test terminals of the Q meter. The tuning capacitor is set to a high value preferably to its maximum position and the circuit is resonated by adjusting the oscillator frequency. Resonance is indicated by maximum deflection on the 'circuit Q' meter. The value of the tuning capacitor C_1 and the oscillator frequency (f_1) are noted. The frequency is then increased to twice its original value ($f_2 = 2f_1$) and the circuit is returned by adjusting the resonating capacitor (C_2).



f_1, f_2

At the initial resonance condition, the capacitance of the circuit equals $C_1 + C_d$ and the resonant frequency equals

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1+C_d)}}$$

After the oscillator and the tuning capacitor are adjusted the capacitance of the circuit is $C_2 + C_d$ and the resonant frequency equals

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2+C_d)}}$$

$$f_2 = 2f_1$$

$$\frac{1}{\cancel{2\pi}\sqrt{\cancel{L}(C_2+C_d)}} = \frac{2}{\cancel{2\pi}\sqrt{\cancel{L}(C_1+C_d)}}$$

$$\frac{1}{C_2+C_d} = \frac{4}{C_1+C_d}$$

$$C_1 + C_d = 4C_2 + 4C_d$$

$$C_1 - 4C_2 = 3C_d$$

$$C_d = \frac{C_1 - 4C_2}{3}$$

COUNTERS

* Counters are the ~~inst~~ electronic circuits designed to count the frequency values using the clock pulses.

FREQUENCY COUNTER

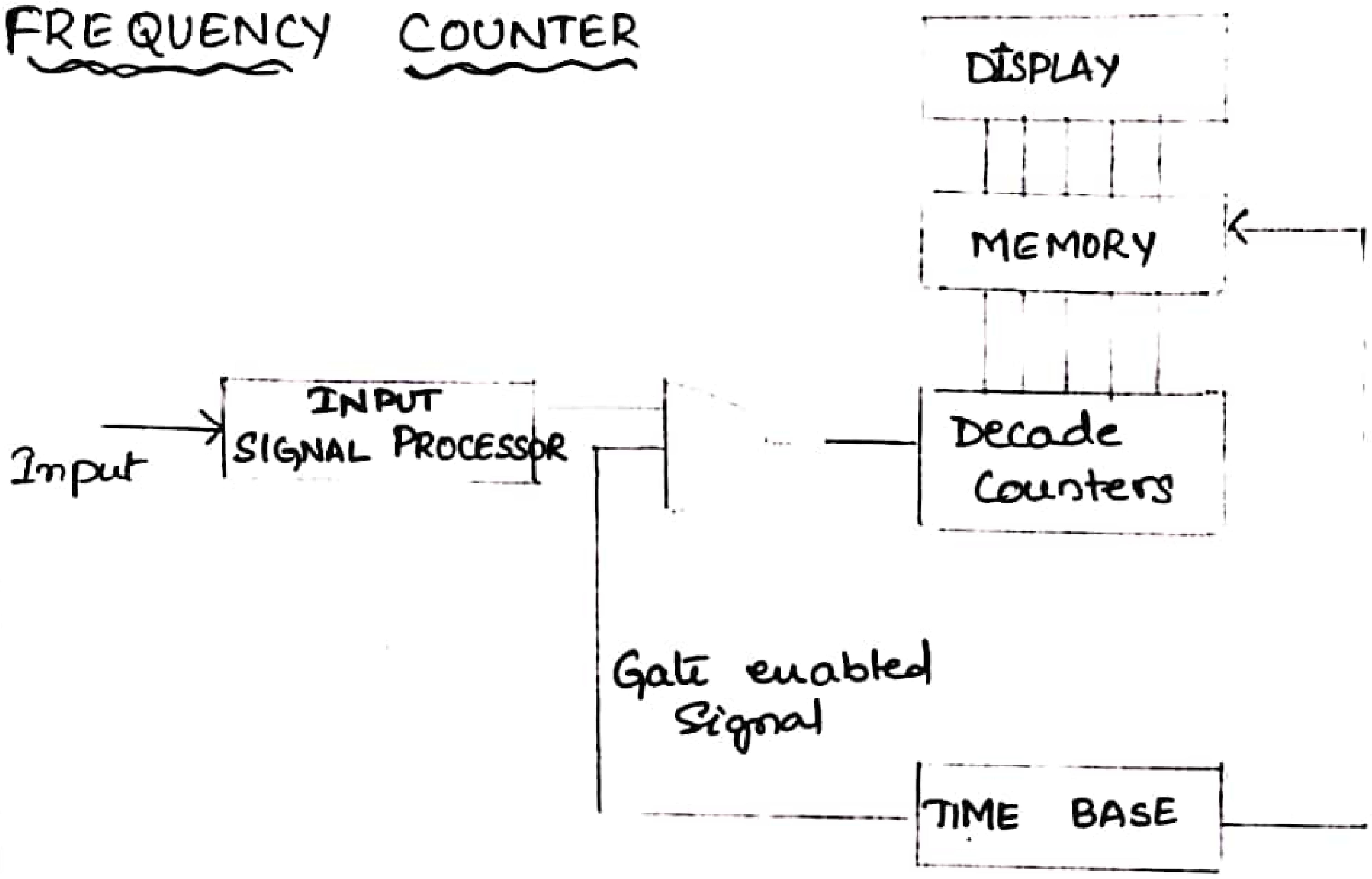


fig1: Basic block of Frequency Counters.

The figure1 shows the block diagram of a Simple frequency Counter.

Principle of operation

* The frequency Counter operates on the Principle of "gating the input frequency into the Counter for a pre-determined time".

Eg:- If an unknown frequency was gated into the Counter for an exact 1second, the number of Counts allowed into Counter would be the frequency of the input.

* The term "gated" stems from the fact that an AND ~~&~~ an OR gate is used in circuit

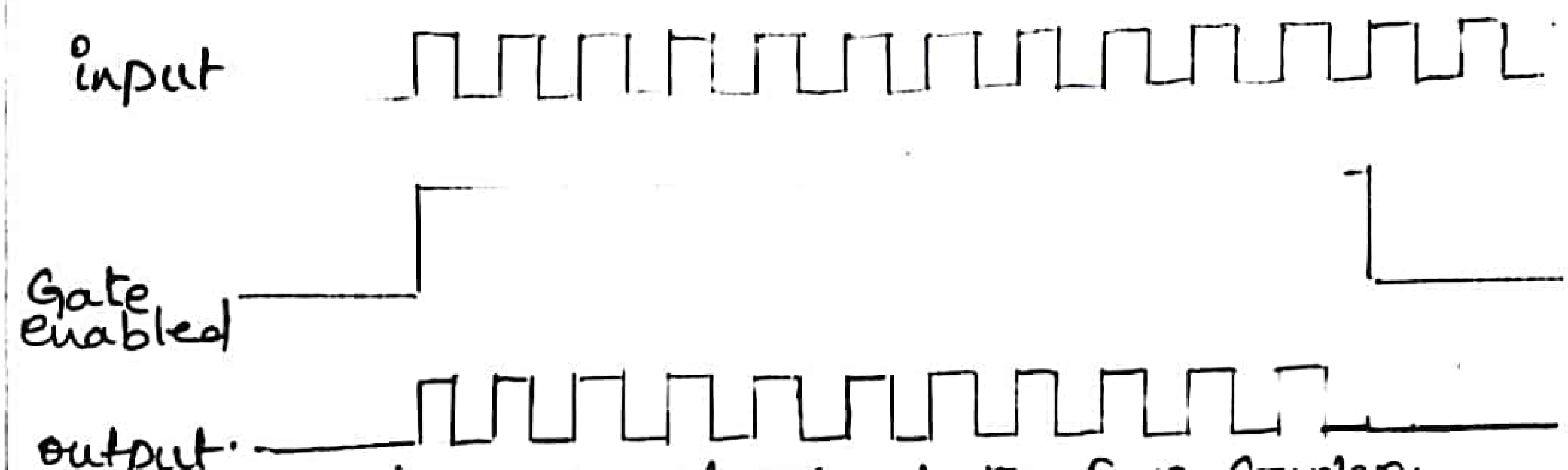


fig:- waveforms of the Freq. Counter.

The waveforms in fig shows the use of AND gate.

working of the Counter.

* As long as the Gate enabled signal is of 1-sec pulse and of logic 1, the output of AND gate is same as the unknown input.

* when the 1-sec pulse returns to logic 0, The output of AND gate is 0.

* Therefore, exactly 1sec of unknown input pulses is allowed at the, output of AND gate.

* These pulses are to be counted and displayed.

* when the Gate enabled signal is of more than 1sec duration, then the decimal point of the display is switched with the gate time selector switch to ~~Count~~ 'Correct the frequency display.

eg: Gate was open for 10 sec, the accumulated Count would be average frequency in 0.1Hz.

i. DECADE COUNTERS

* The heart of the Frequency Counter is the decade Counter.

* The output of the decade Counters follows the Sequence in fig.

Clock	Counter state			
	D	C	B	A
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

fig: Binary coded decimal Counting sequence.

Eg: decimal no 138 is Coded as
0001 0011 1000 in BCD.

* Each BCD Counter allows one decade of Counting. So, BCD Counters must be Cascaded.

Eg: To Count between 0-999 three BCD Counters are required.

* There are two methods of cascading BCD Counters.

1. Ripple Cascading
2. Synchronous Cascading

1. Ripple Cascading:

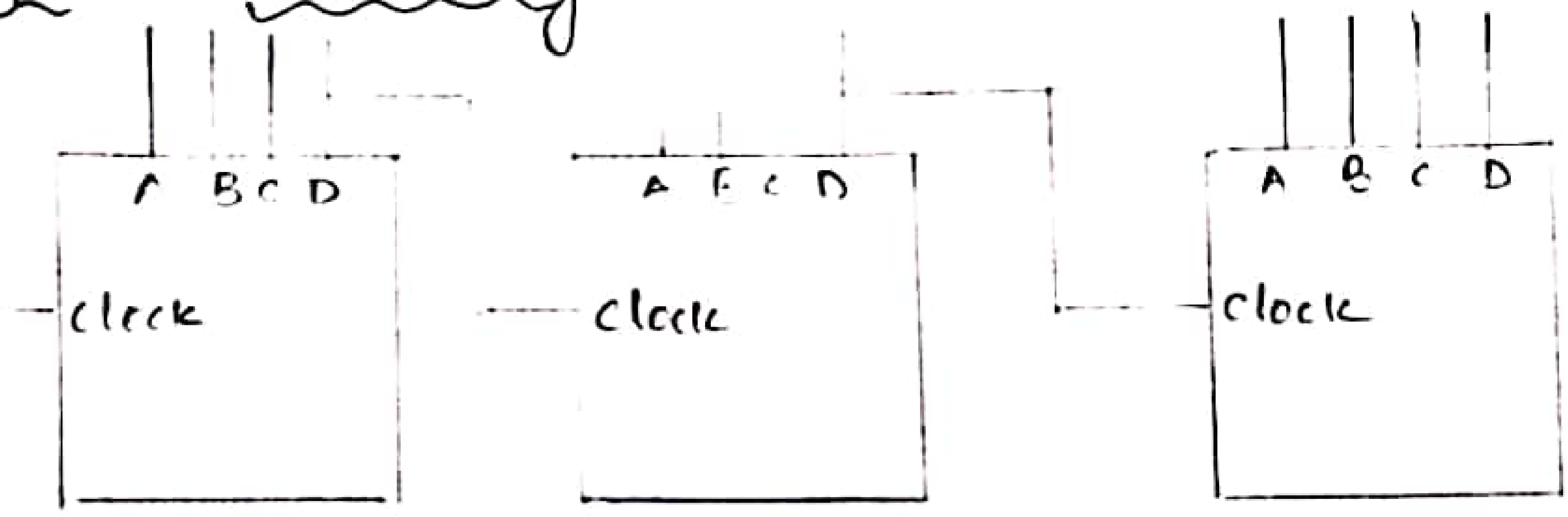


fig4: Cascading Ripple Counters.

* This type of Cascading is very slow.
* This is used only for low-frequency measurements. It is not used for serious frequency measurement equipment.
* In this type of cascading, the last output of the least Significant Counter to drive the clock input of the next Significant Counter.

2. Synchronous Cascading:

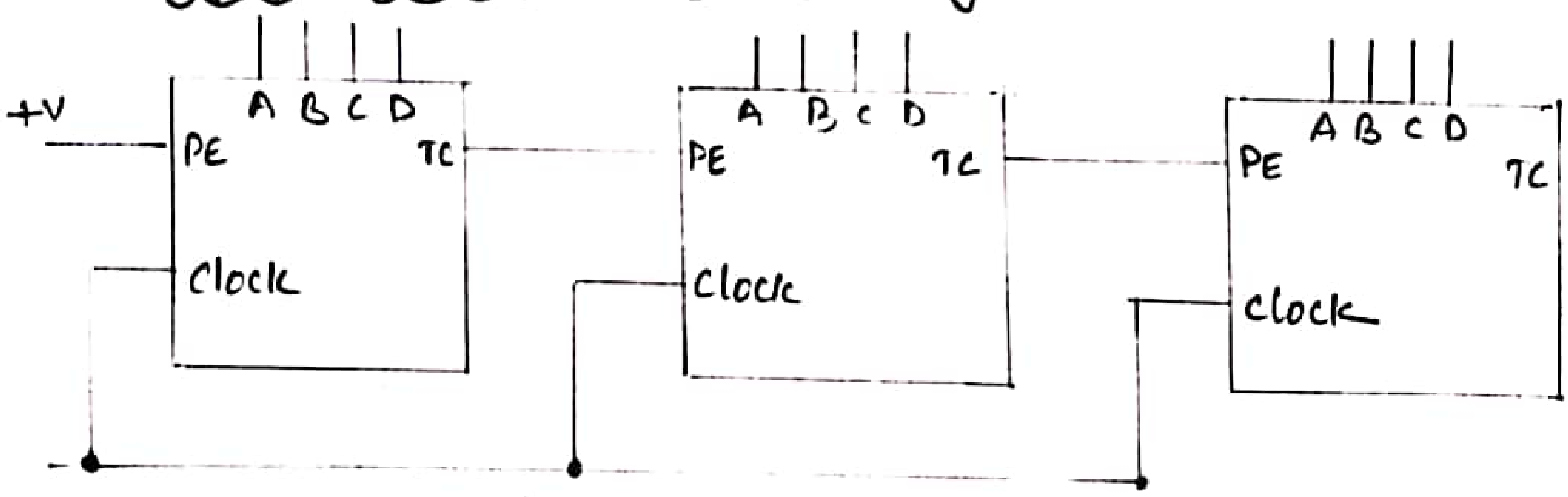


fig5: Cascaded Synchronous Counters.

* A Terminal Count (TC) or Carry output is used for the purpose of Cascading.
* This output is used to enable the following Counter to be incremented on the next clockpulse.

* This insures that the State of the next Counter is coincident with the clock.

Logic diagram of Binary Synchronous Counter

The Binary Synchronous Counter requires that all the flip-flop clocks be connected together which greatly reduces the propogation delay and allows higher Counting Speeds.

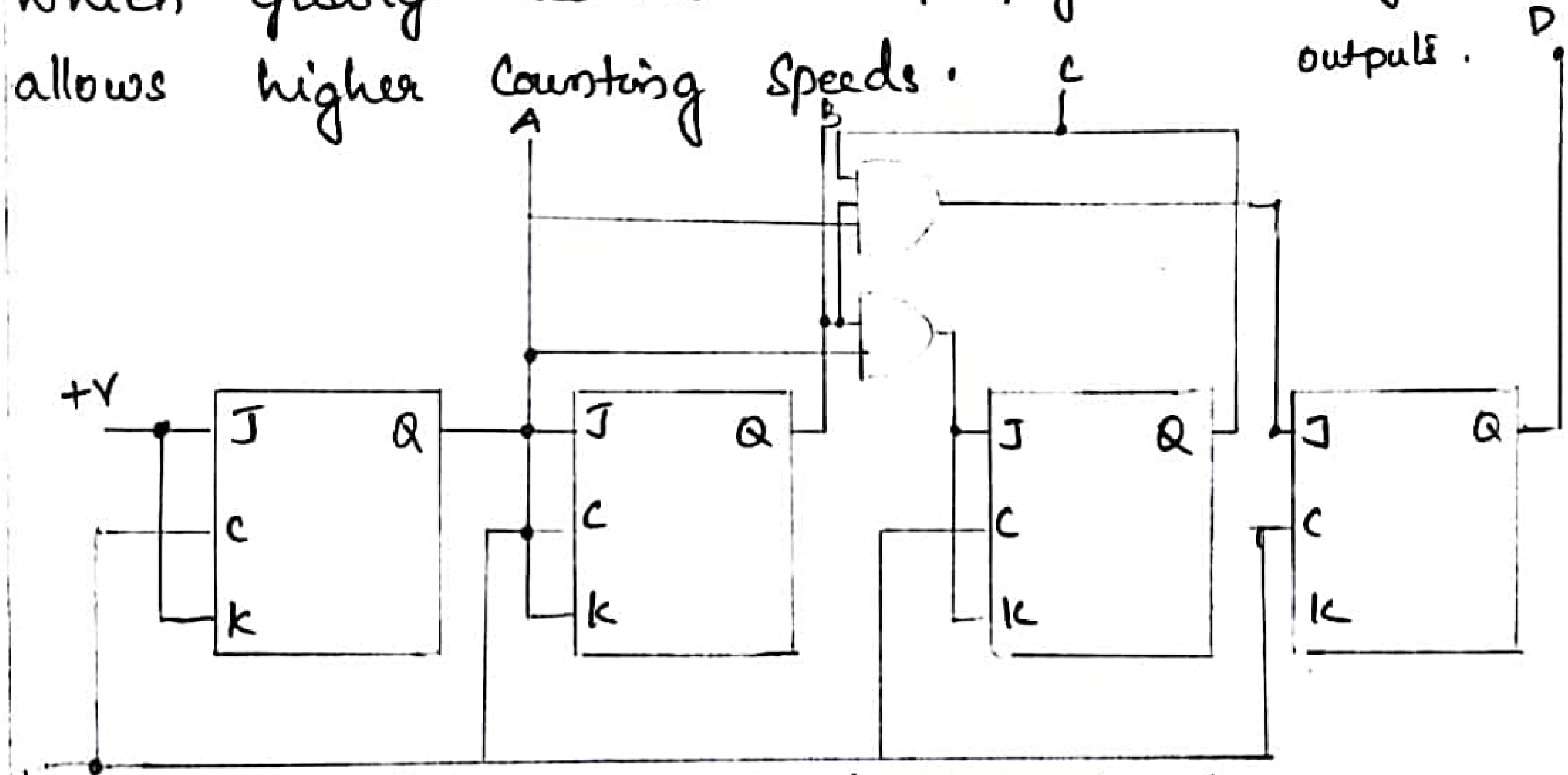


fig6: Binary Synchronous Counter.

Logic diagram of Ripple BCD Counter.

The Ripple BCD Counter is constructed by four flip-flops and an AND gate. The clock of one flip-flop is derived from the output of the previous flip-flop.

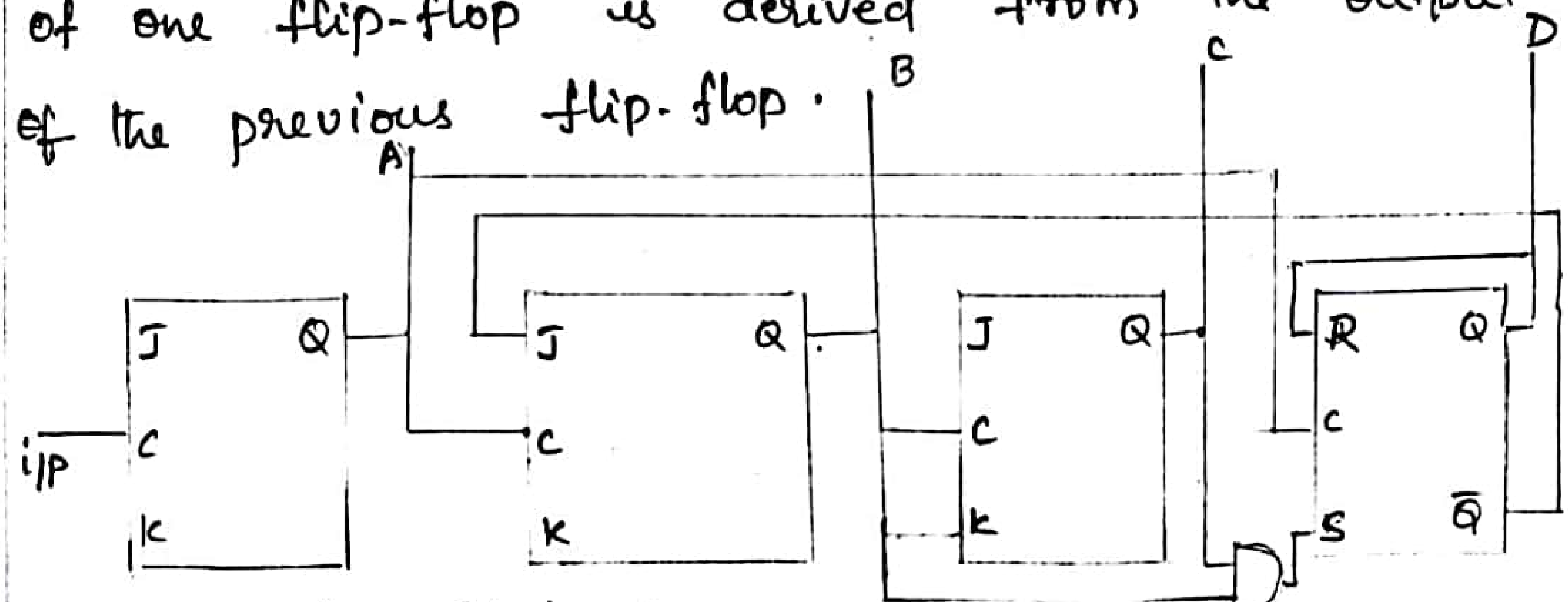


fig7 Ripple BCD counter

2. DISPLAY

* The BCD information available at the output of the Counter must be Converted to some form of visible display.
* This Conversion depends on the type of display desired.
* It is desirable in a frequency Counter to display the Count continuously.
* The Counter is reset to zero and allowed to Count during the gate period. During this time the output of the Counter is Constantly changing.
* The output of the Counter cannot be displayed during this period.
* The Count at the end of the measurement Period is Stored in a simple memory and displayed. ~~dur~~
* Digital logic usually cannot supply the required current for driving a display.
* A display driver is included between the decade Counter and the displays.
* This Technique is called Display multiplexing and reduces the number of drivers and decoders required to implement large Counters.

3. Time Base

* The Sequence of events within the frequency Counter is Controlled by the time base, which must provide the timing for the following events.

→ resetting the counter
→ Opening the count gate
→ Closing the Count gate
→ Storing the counting frequency.

* The opening and closing of the count gate determine the accuracy of Time Base signal. frequency Counter.

* So, the accuracy of the frequency counter depends directly on the accuracy of the Time Base Signals.

* These Signals are driven from Crystal-Controlled Oscillator.

4. Input Signal Processing

* The unknown frequency is not the Correct logic level to drive the frequency Counter.

* A processing circuit is required.

* This processing circuit may be
  → an amplifier to increase signal level
  → an attenuator to adjust variations in Input amplitudes.
  → a Comparator

ELECTRONIC COUNTER

* The decade Counter can be easily incorporated in a Commercial Test instrument Called an Electronic Counter.
* A decade Counter by itself behaves as a totaliser by totalling the pulses applied to it during the time interval that a gate pulse is present.

Modes of operation

→ Totalising Mode
→ Frequency mode
→ Period Mode
→ Ratio mode
→ Time Interval mode.

1. Totalising Mode.

* In the totalising mode the input pulses are counted (totalised) by the decade counter as long as switch is closed.
* If the Count pulse exceeds the capacity of the decade Counter, the overflow indicator is activated and the Counter starts Counting again.

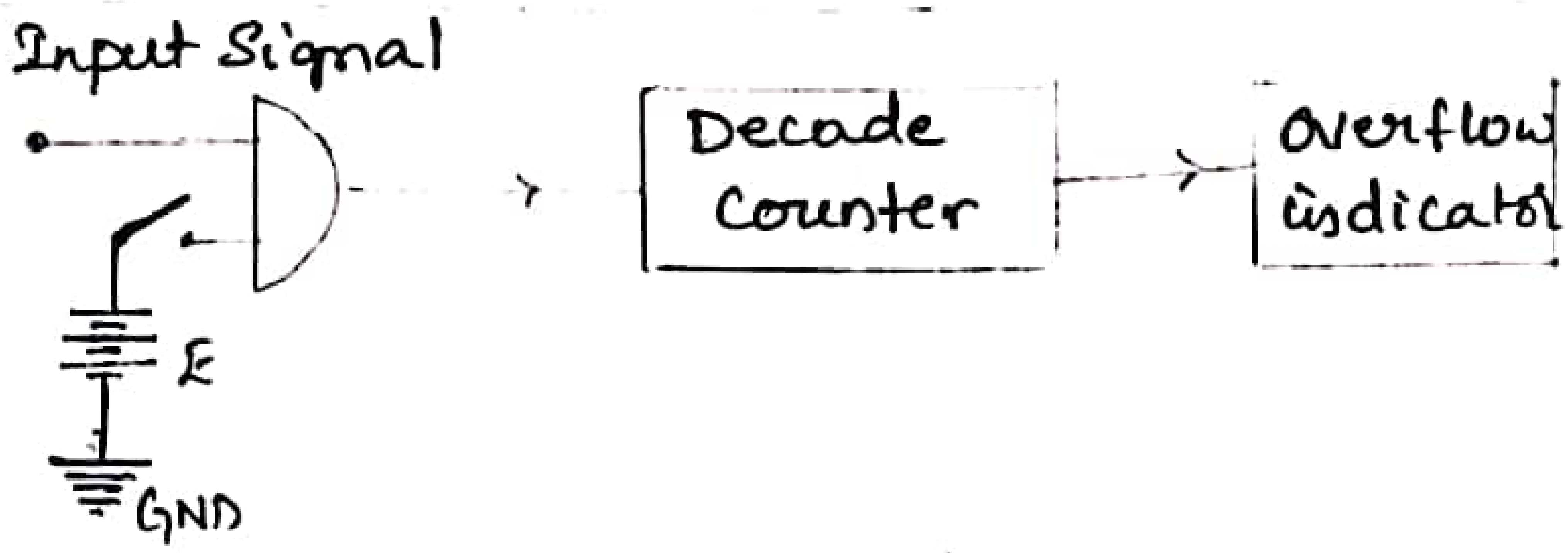


fig8: Block diagram of the Totalising Mode of an Electronic Counter.

2. Frequency Mode

* If the time interval in which the pulses are being totalised is accurately controlled the counter operates in the frequency mode.

* Accurate control of the time interval is achieved by applying a rectangular pulse of known duration to the AND gate in place of dc voltage source. This is called gating of the counter.

* The frequency of the input signal is computed as $f = \frac{N}{t}$.

where f : frequency of input signal
N : pulse counted
t : duration of the gate pulse.

3. Period Mode

* In some applications it is desirable to

measure the period of the signal rather than its frequency. Since period is the reciprocal of the frequency, it can easily measured by using the input signal as a gating pulse and counting the clock pulses.

* The period of the input signal is determined from the number of pulses of known frequency

$$T = \frac{N}{f}$$

N : pulse counted
f : freq. of clock.

4. Ratio Mode

* The ratio mode of operation simply displays the numerical value of the ratio of the frequency of the two signals.

* The low frequency signal is used in place of the clock to provide a gate pulse. The number of cycles of the high frequency signal which are stored in the decade counter during the presence of an externally generated gate pulse is read directly as a ratio of the frequency.

5. Time interval Mode.

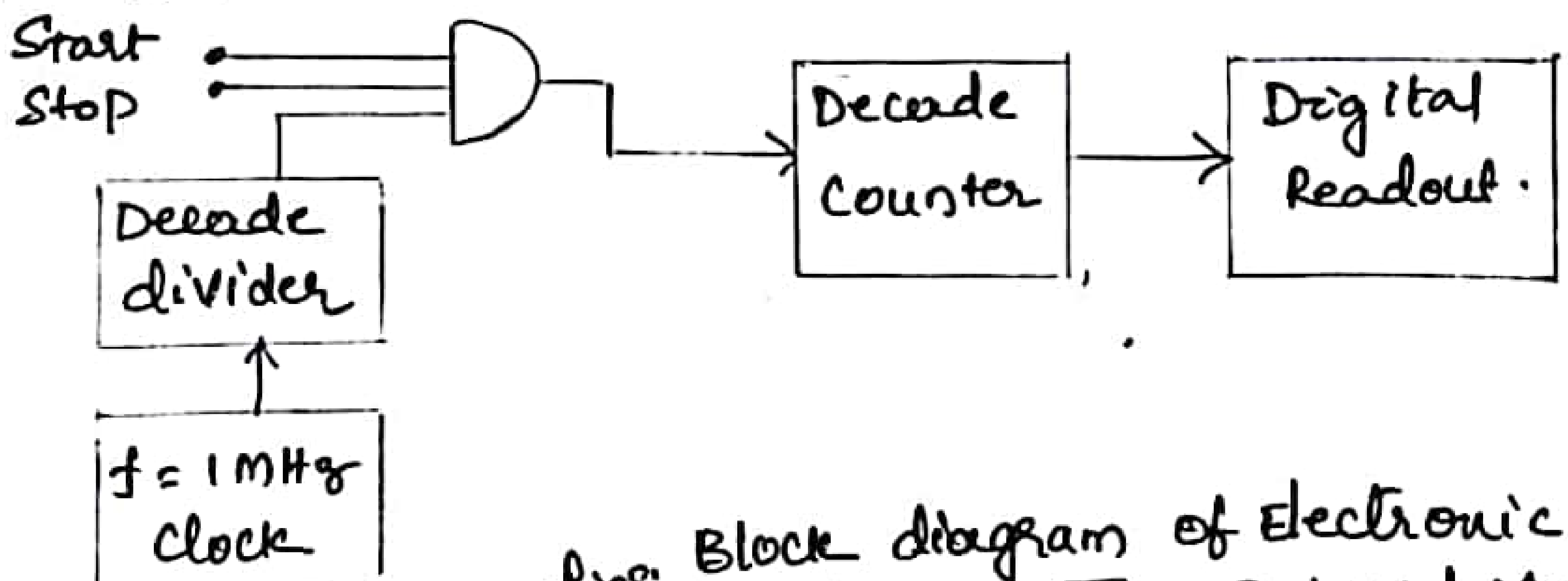


fig: Block diagram of Electronic Counter in Time Interval Mode.

* The time interval mode of operation measures the time elapsed between two events.
* The measurement can be done using the circuit of figure 9
* The gate is controlled by two independent inputs. → The start input, which enables the gate
→ The stop input, which disables the gate
* During the time interval between the start and stop signal, clock pulses accumulate in the Register, providing an indication of the time interval between the start and stop of the event.

Applications.

→ Research and Development laboratories
→ in standard Laboratories
→ On Service benches
→ in everyday operations of many electronic installations.

* Counters are used (i) in Communication to measure the carrier frequency,
(ii) in the digital system to measure clock frequency.

MEASUREMENT ERRORS / SOURCES OF ERRORS

1. Gating error

* occurs with frequency and period measurements.
* For frequency measurement the main gate is opened and closed by oscillator output pulse.
  * this allows the input signal to pass through the gate and be counted by the decade counters.
  * The gating pulse is not synchronised with the input signal. They are in fact two totally unrelated signals.

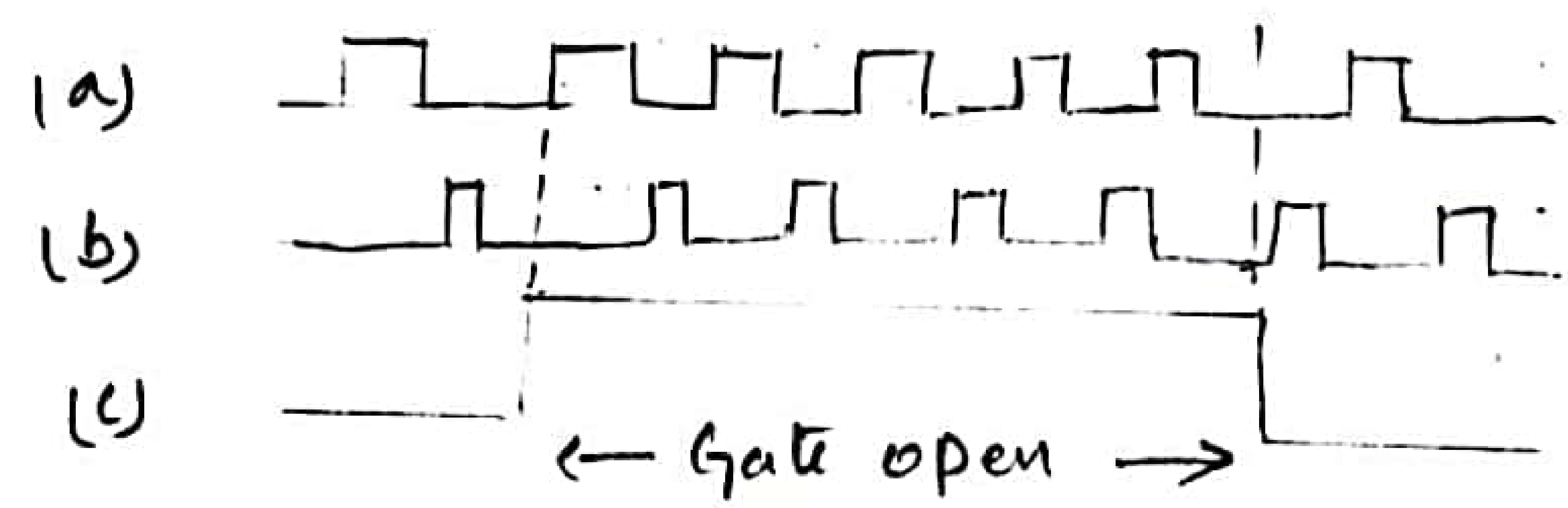


fig 10: Gating error.

* Fig 10 shows the waveforms of two input signals (a) and (b) out of phase with respect to the gating signal (c)
  * In (a) 5 pulses can be counted

    (b) 4 pulse can be only allowed
  * Therefore ± 1 count ambiguity in the measurement occurs.

* In measuring low frequencies, this error shows major effect on the result.
  for eg: where $f = 10\,Hz$ to be measured
  gating time = 1 sec.
  The decade counter indicates 10 ± 1 count.
  Hence an inaccuracy of 10 %.

* Period measurements are preferred over frequency measurements at lower frequencies

* In period measurements,

  the no. of pulses counted $N_p = \frac{f_c}{f_x}$.

  f_c : clock frequency

  f_x : unknown frequency of input signal.

* In frequency measurement, with 1 sec time

  No of pulses counted $N_f = f_x$.

* The cross over frequency f_o at $N_p = N_f$ is

$$\frac{f_c}{f_x} = f_x \quad \Rightarrow \quad f_x = \sqrt{f_c}$$

$$f_o = \sqrt{f_c}$$

* Signals with a frequency lower than f_o should therefore be measured in the period mode

* Signals with a frequency above f_o should be measured in frequency mode.

2. Time Base Error :

* Inaccuracies in the time base also cause errors in the measurement.

* In frequency measurements the time base determines the opening and closing of the signal gate and it provides the pulses to be counted.

* Time Base error consists of

→ Oscillator Calibration Errors

→ Short term crystal stability errors

→ long term Crystal Stability errors.

* Improved Calibration accuracy can be obtained by using very low Frequency Stations rather High Frequency because the Transmission paths for very low frequencies is shorter than high frequency Transmission.

* Short term crystal errors are caused by monetary frequency variations due to voltage transients, shock and vibration, electrical interference etc

* These errors can be minimised by taking frequency measurements over long gate times 10s to 100s

* Long-term stability errors are due to aging and deterioration of the crystal

3. Trigger Level Error

* In frequency Measurements the Signal gate is opened and closed by the input signal.

* The accuracy with which the gate is opened and closed is a function of the trigger level error.

* These errors can be minimised with large signal amplitudes and fast rise times.

Maximum accuracy can be obtained if the following suggestions are followed.

(a) The effect of the one-count gating error can be minimized by making frequency measurements above $\sqrt{f_c}$ and period measurements below $\sqrt{f_c}$ where f_c : clock frequency of counter

(b) Since long term stability has a cumulative effect the accuracy of measurement is mostly a function of the time since the last calibration against a primary or secondary standard.

(c) The accuracy of time measurements is greatly affected by the slope of the incoming signal controlling the signal gate. Large signal amplitude and fast rise time assure maximum accuracy.