Rectifiers and Filters

Introduction:

For the operation of most of the electronics devices and circuits, a d.c. source is required. So it is advantageous to convert domestic a.c. supply into d.c. voltages. The process of converting a.c. voltage into d.c. voltage is called as rectification.This is achieved with i) Step-down Transformer, ii) Rectifier, iii) Filter and iv) Voltage regulator circuits.

These elements constitute d.c. regulated power supply shown in the figure below.

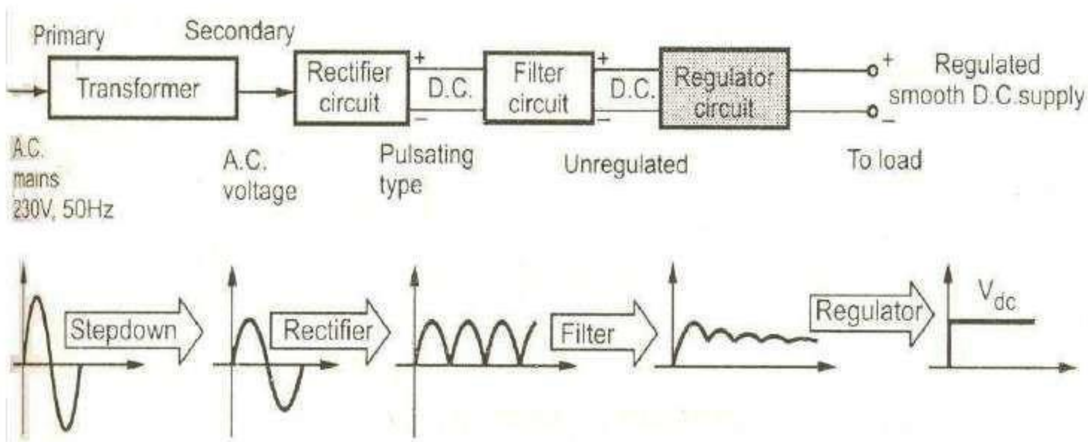


Fig. Block diagram of Regulated D.C. Power Supply

The block diagram of a regulated D.C. power supply consists of step-down transformer, rectifier, filter, voltage regulator and load.

An ideal regulated power supply is an electronics circuit designed to provide a predetermined d.c. voltage Vo which is independent of the load current and variations in the input voltage ad temperature.

If the output of a regulator circuit is a AC voltage then it is termed as voltage stabilizer, whereas if the output is a DC voltage then it is termed as voltage regulator.

The elements of the regulated DC power supply are discussed as follows:

TRANSFORMER:

A transformer is a static device which transfers the energy from primary winding to secondary winding through the mutual induction principle, without changing the frequency. The transformer winding to which the supply source is connected is called the primary, while the winding connected to the load is called secondary.

If N_1,N_2 are the number of turns of the primary and secondary of the transformer then $\alpha = \frac{N_2}{N_1}$ is called the turns ratio of the transformer.

The different types of the transformers are

- Step-Up Transformer
- Step-Down Transformer
- Centre-tapped Transformer

The voltage, current and impedance transformation ratios are related to the turns ratio of the transformer by the following expressions.

Voltage transformation ratio : $\frac{V_2}{V_1} = \frac{N_2}{N_1}$

Current transformation ratio : $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

Impedance transformation ratio : $\frac{Z_L}{Z_{in}} = \left(\frac{N_2}{N_1}\right)^2$

RECTIFIER:

Any electrical device which offers a low resistance to the current in one direction but a high resistance to the current in the opposite direction is called rectifier. Such a device is capable of converting a sinusoidal input waveform, whose average value is zero, into a unidirectional waveform, with a non-zero average component.

A rectifier is a device which converts a.c. voltage (bi-directional) to pulsating d.c. voltage (Uni-directional).

Important characteristics of a Rectifier Circuit:

1. **Load currents:** They are two types of output current. They are average or d.c. current and RMS currents.

   i) **Average or DC current:** The average current of a periodic function is defined as the area of one cycle of the curve divided by the base.

   It is expressed mathematically as $I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t)$; where $i = I_m \sin \omega t$

   6) **Effective (or) R.M.S. current:** The effective (or) R.M.S. current squared of a periodic function of time is given by the area of one cycle of the curve which represents the square of the function divided by the base.

   It is expressed mathematically as $I_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)\right]^{\frac{1}{2}}$

2. **Load Voltages:** There are two types of output voltages. They are average or D.C. voltage and R.M.S. voltage.

   i) **Average or DC Voltage:** The average voltage of a periodic function is defined as the areas of one cycle of the curve divided by the base.
   It is expressed mathematically as

   $$V_{dc} = \frac{1}{2\pi}\int_0^{2\pi} V\,d(\omega t) \text{ ; Where } V = V_m \sin \omega t$$

   (or) $V_{dc} = I_{dc} \times R_L$

   ii) **Effective (or) R.M.S Voltage:** The effective (or) R.M.S voltage squared of a periodic function of time is given by the area of one cycle of the curve which represents the square of the function divided by the base.

$$V_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} V^2 d(\omega t)\right]^{\frac{1}{2}} \qquad V_{rms} = I_{rms} \times R_L$$

3. **Ripple Factor (γ) :** It is defined as ration of R.M.S. value of a.c. component to the d.c. component in the output is known as "Ripple Factor".

$$\gamma = \frac{V_{rms}}{V_{dc}}$$

$$V_{rms} = \sqrt{V_{rms}^2 - V_{dc}^2}$$

$$\therefore \gamma = \sqrt{\frac{V_{rms}^2}{V_{dc}^2} - 1}$$

4. **Efficiency (η) :** It is the ratio of d.c output power to the a.c. input power. It signifies, how efficiently the rectifier circuit converts a.c. power into d.c. power.

It is given by $$\eta = \frac{P_{dc}}{P_{Ac}}$$

5. **Peak Inverse Voltage (PIV):** It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction.

6. **Regulation:** The variation of the d.c. output voltage as a function of d.c. load current is called regulation. The percentage regulation is defined as

$$\% \text{ Regulation} = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100\%$$

For an ideal power supply, % Regulation is zero.

Using one or more diodes in the circuit, following rectifier circuits can be designed.

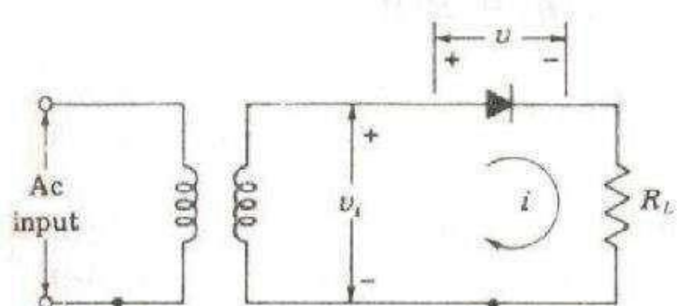
1. Half - Wave Rectifier
2. Full – Wave Rectifier
3. Bridge Rectifier

HALF-WAVE RECTIFIER:

A Half – wave rectifier is one which converts a.c. voltage into a pulsating voltage using only one half cycle of the applied a.c. voltage. The basic half-wave diode rectifier circuit along with its input and output waveforms is shown in figure below.

The half-wave rectifier circuit shown in above figure consists of a resistive load; a rectifying element i.e., p-n junction diode and the source of a.c. voltage, all connected is series. The a.c. voltage is applied to the rectifier circuit using step-down transformer.

The input to the rectifier circuit, $V = V_m \sin \omega t$ Where Vm is the peak value of secondary a.c. voltage



(a)

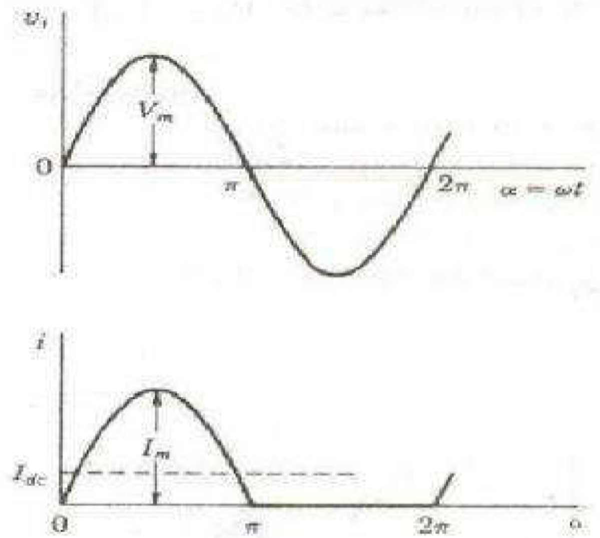


Figure 1.24: Circuit and Waveforms of half wave rectifier

Operation:

For the positive half-cycle of input a.c. voltage, the diode D is forward biased and hence it conducts. Now a current flows in the circuit and there is a voltage drop across R_L. The waveform of the diode current (or) load current is shown in figure.

For the negative half-cycle of input, the diode D is reverse biased and hence it does not conduct. Now no current flows in the circuit i.e., i=0 and Vo=0. Thus for the negative half-cycle no power is delivered to the load.

Analysis:

In the analysis of a HWR, the following parameters are to be analyzed.

i) DC output current
ii) DC Output voltage
iii) R.M.S. Current
iv) R.M.S. voltage
v) Rectifier Efficiency (η)
vi) Ripple factor (γ)
vii) Regulation
viii) Transformer Utilization Factor (TUF)
ix) Peak Factor (P)

Let a sinusoidal voltage V_i be applied to the input of the rectifier.

Then $V = V_m \sin \omega t$ Where Vm is the maximum value of the secondary voltage.

Let the diode be idealized to piece-wise linear approximation with resistance R_f in the forward direction i.e., in the ON state and R_r (=∞) in the reverse direction i.e., in the OFF state.

Now the current 'i' in the diode (or) in the load resistance R_L is given by

$$i = I_m \sin \omega t \quad \text{for} \quad 0 \le \omega t \le \pi$$

$$i = 0 \quad \text{for} \quad \pi \le \omega t \le 2\pi$$

$$\text{where } I_m = \frac{V_m}{R_f + R_L}$$

i) Average (or) DC Output Current (I_{av} or I_{dc}):

The average dc current I_{dc} is given by

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t)$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t\,d(\omega t) + \int_{\pi}^{2\pi} 0 \times d(\omega t)$$

$$= \frac{1}{2\pi} I_m (-\cos\omega t)_0^{\pi}$$

$$= \frac{1}{2\pi} I_m (+1-(-1))$$

$$= \frac{I_m}{\pi} = 0.318\, I_m$$

Substituting the value of I_m, we get $I_{dc} = \frac{V_m}{\pi(R_f + R_L)}$

If $R_L >> R_f$ then $I_{dc} = \frac{V_m}{\pi R_L} = 0.318\frac{V_m}{R_L}$

ii) Average (or) DC Output Voltage (V_{av} or V_{dc}):

The average dc voltage is given by

$$V_{dc} = I_{dc} \times R_L = \frac{I_m}{\pi} \times R_L = \frac{V_m R_L}{\pi(R_f + R_L)}$$

$$\Rightarrow V_{dc} = \frac{V_m R_L}{\pi(R_f + R_L)}$$

If $R_L >> R_f$ then $V_{dc} = \frac{V_m}{\pi} = 0.318\, V_m \quad \therefore V_{dc} = \frac{V_m}{\pi}$

iii) RMS output current (I_{rms}):

The value of the R.M.S. current is given by

$$I_{rms} = \left[\frac{1}{2\pi}\int_0^{2\pi} i^2 d(\omega t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t.d(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0 . d(\omega t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{2\pi}\int_0^{\pi}\frac{1-\cos 2\omega t}{2}\, d(\omega t)\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{4\pi}\left[(\omega t)-\frac{1}{2}\sin 2\omega t\right]_0^{\pi}\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{4\pi}\left[\pi - 0 - \left(\frac{\sin 2\pi}{2} - \sin 0\right)\right]\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{4}\right]^{\frac{1}{2}} = \frac{I_m}{2}$$

$$\therefore I_{rms} = \frac{I_m}{2} \qquad I_{rms} = \frac{V_m}{2(R_f + R_L)}$$

iv) R.M.S. Output Voltage (V_{rms}):

R.M.S. voltage across the load is given by

$$V_{rms} = I_{rms} \times R_L = \frac{V_m R_L}{2(R_f + R_L)} = \frac{V_m}{2\left(1+\frac{R_f}{R_L}\right)}$$

If $R_L >> R_f$ then $V_{rms} = \frac{V_m}{2}$

v) Rectifier efficiency(η) :

The rectifier efficiency is defined as the ration of d.c. output power to the a.c. input power i.e.,

$$\eta = P_{dc}/P_{ac}$$

Theoretically the maximum value of rectifier efficiency of a half-wave rectifier is 40.6%

vi) **Ripple Factor (γ) :**

The ripple factor γ is given by

$$\gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \quad \text{(or)} \quad \gamma = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}$$

$$\therefore \gamma = \sqrt{\left(\frac{I_m / 2}{I_m / \pi}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2}\right)^2 - 1} = 1.21$$

$$\Rightarrow \gamma = 1.21$$

vii) **Regulation:**

The variation of d.c. output voltage as a function of d.c. load current is called *regulation.*

The variation of V_{dc} with I_{dc} for a half-wave rectifier is obtained as follows:

$$I_{dc} = \frac{I_m}{\pi} = \frac{V_m / \pi}{R_f + R_L}$$

But $V_{dc} = I_{dc} \times R_L$

$$V_{dc} = \frac{V_m}{\pi} \frac{R_L}{R_f + R_L} = \frac{V_m}{\pi}\left(1 - \frac{R_f}{R_f + R_L}\right)$$

$$= \frac{V_m}{\pi} - I_{dc} R_f$$

$$\therefore V_{dc} = \frac{V_m}{\pi} - I_{dc} R_f$$

This result shows that V_{dc} equals $\frac{V_m}{\pi}$ at no load and that the dc voltage decreases linearly with an increase in dc output current. The larger the magnitude of the diode forward resistance, the greater is this decrease for a given current change.

viii) Transformer Utilization Factor (UTF):

The d.c. power to be delivered to the load in a rectifier circuit decides the rating of the transformer used in the circuit. So, transformer utilization factor is defined as

$$\therefore TUF = \frac{P_{dc}}{P_{ac\,(rated)}}$$

The factor which indicates how much is the utilization of the transformer in the circuit is called Transformer Utilization Factor (TUF).

The a.c. power rating of transformer $= V_{rms} I_{rms}$

The secondary voltage is purely sinusoidal hence its rms value is $\frac{1}{\sqrt{2}}$ times maximum while the current is half sinusoidal hence its rms value is $\frac{1}{2}$ of the maximum.

$$\therefore P_{ac\,(rated)} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{2} = \frac{V_m I_m}{2\sqrt{2}}$$

The d.c. power delivered to the load $= I_{dc}^2 R_L = \left(\frac{I_m}{\pi}\right)^2 R_L$

$$\therefore TUF = \frac{P_{dc}}{P_{ac\,(rated)}}$$

$$= \left(\frac{I_m}{\pi}\right)^2 R_L = \frac{2\sqrt{2}}{V_m I_m}$$

$$= \frac{I_m^2 \cdot R_L \cdot 2\sqrt{2}}{\pi^2 \, I_m \cdot R_L} \qquad (\because V_m \approx I_m R_L)$$

= 0.287

$$\therefore TUF = 0.287$$

The value of TUF is low which shows that in half-wave circuit, the transformer is not fully utilized.

If the transformer rating is 1 KVA (1000VA) then the half-wave rectifier can deliver 1000 X 0.287 = 287 watts to resistance load.

ix) Peak Inverse Voltage (PIV):

It is defined as the maximum reverse voltage that a diode can withstand without destroying the junction. The peak inverse voltage across a diode is the peak of the negative half-cycle. For half-wave rectifier, PIV is V_m.

x) Form factor (F):

The Form Factor F is defined as

F = rms value / average value

$$F = \frac{Im/2}{Im/\pi} = \frac{0.5\ Im}{0.318\ Im} = 1.57$$

xi) Peak Factor (P):

The peak factor P is defined as

P= Peak Value / rms value $= \frac{V_m}{V_m / 2} = 2$ **P = 2**

Disadvantages of Half-Wave Rectifier:

1. The ripple factor is high.
2. The efficiency is low.
3. The Transformer Utilization factor is low.

Because of all these disadvantages, the half-wave rectifier circuit is normally not used as a power rectifier circuit.

Problems from previous external question paper:

1. A diode whose internal resistance is 20J is to supply power to a 100J load from 110V(rms) source pf supply. Calculate (a) peak load current (b) the dc load current (c) the ac load current (d) the percentage regulation from no load to full load.

Solution:

Given a half-wave rectifier circuit R_f=20J, R_L=100J

Given an ac source with rms voltage of 110V, therefore the maximum amplitude of sinusoidal input is given by

$$V_m = \sqrt{2} \times V_{rms} = \sqrt{2} \times 110 = \mathbf{155.56V}.$$

(a) Peak load current : $I_m = \frac{V_m}{R_f + R_L} \Rightarrow I_m = \frac{155.56}{120} = \mathbf{1.29A}$

(b) The dc load current : $I_{dc} = \frac{I_m}{\pi} = \mathbf{0.41A}$

(c) The ac load current : $I_{rms} = \frac{I_m}{2} = \mathbf{0.645A}$

(d) $V_{no\text{-}load}$: $\frac{V_m}{\pi} = \frac{155.56}{\pi} = \mathbf{49.51\ V}$

$V_{full\text{-}load}$: $\frac{V_m}{\pi} - I_{dc} R_f = \mathbf{41.26\ V}$

$$\%\ \text{Regulation} = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100 = \mathbf{19.97\%}$$

2. A diode has an internal resistance of 20J and 1000J load from 110V(rms) source pf supply. Calculate (a) the efficiency of rectification (b) the percentage regulation from no load to full load.

Solution:

Given a half-wave rectifier circuit R_f=20J, R_L=1000J

Given an ac source with rms voltage of 110V, therefore the maximum amplitude of sinusoidal input is given by

$$V_m = \sqrt{2} \times V_{rms} = \sqrt{2} \times 110 = 155.56V.$$

(a) % Efficiency (η) = $\frac{40.6}{1+\frac{20}{100}} = \frac{40.6}{1.02}$ = **39.8%.**

(b) Peak load current : $I_m = \frac{V_m}{R_f + R_L} = \frac{155.56}{1020}$ = 0.1525 A

= **152.5 mA**

The dc load current : $I_{dc} = \frac{I_m}{\pi}$ = **48.54 mA**

$V_{no\text{-}load} = \frac{V_m}{\pi} = \frac{155.56}{\pi}$ = **49.51 V**

$V_{full\text{-}load} = \frac{V_m}{\pi} - I_{dc} R_f$ = **49.51 – (48.54 x10^{-3} x 20)**

= **49.51 – 0.97 = 48.54 V**

% Regulation = $\frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100$

= $\frac{49.51 - 48.54}{48.54} \times 100$ = **1.94 %**

3. An a.c. supply of 230V is applied to a half-wave rectifier circuit through transformer of turns ration 5:1. Assume the diode is an ideal one. The load resistance is 300J.

Find (a) dc output voltage (b) PIV (c) maximum, and (d) average values of power delivered to the load.

Solution: (a) The transformer secondary voltage = 230/5 = 46V.

Maximum value of secondary voltage, $V_m = \sqrt{2} \times 46$ = 65V.

Therefore, dc output voltage, $V_{dc} = \frac{V_m}{\pi} = \frac{65}{\pi}$ = **20.7 V**

(b) PIV of a diode : V_m = **65V**

(c) Maximum value of load current, $I_m = \frac{V_m}{R_L} = \frac{65}{300}$ = **0.217 A**

Therefore, maximum value of power delivered to the load,

$\mathbf{P_m} = I_m^2 \times R_L = (0.217)^2 \times 300$ = **14.1W**

(d) The average value of load current, $I_{dc} = \frac{V_{dc}}{R_L} = \frac{20.7}{300}$ = **0.069A**

Therefore, average value of power delivered to the load,

$\mathbf{P_{dc}} = I_{dc}^2 \times R_L = (0.069)^2 \times 300$ = **1.43W**

FULL – WAVE RECTIFIER

A full-wave rectifier converts an ac voltage into a pulsating dc voltage using both half cycles of the applied ac voltage. In order to rectify both the half cycles of ac input, two diodes are used in this circuit. The diodes feed a common load R_L with the help of a center-tap transformer.

A center-tap transformer is the one which produces two sinusoidal waveforms of same magnitude and frequency but out of phase with respect to the ground in the secondary winding of the transformer. The full wave rectifier is shown in the figure below.

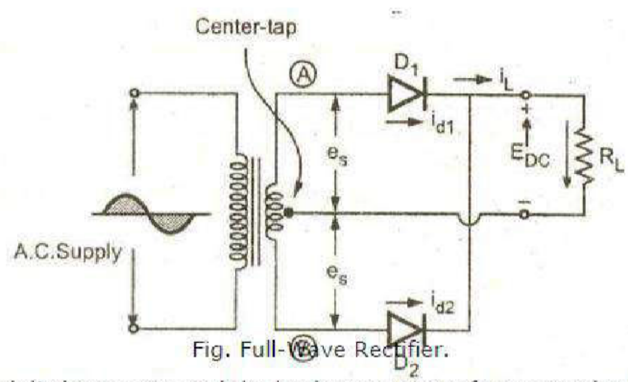


Fig. Full-Wave Rectifier.

The individual diode currents and the load current waveforms are shown in figure below:

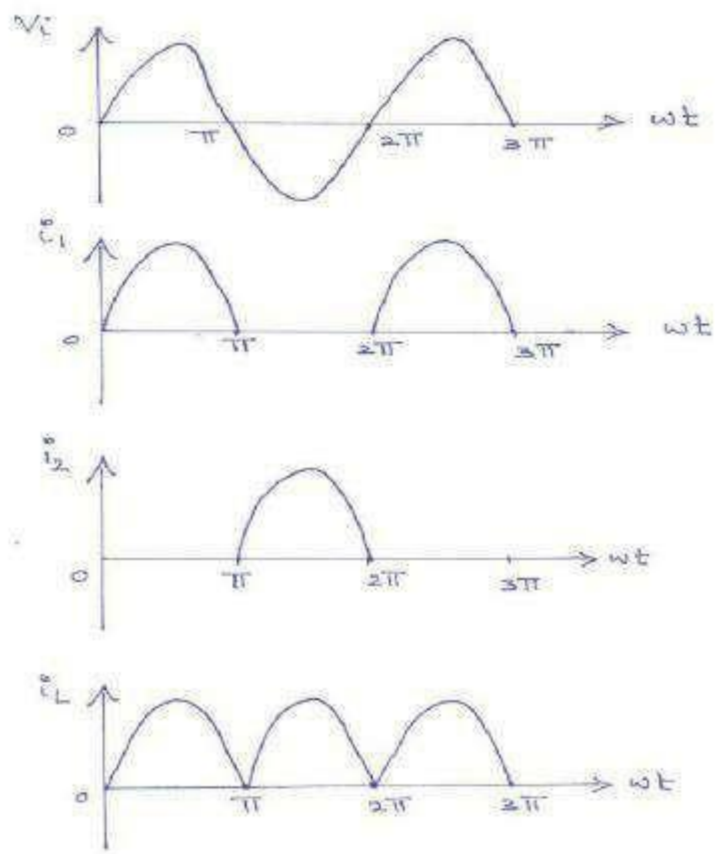


Figure 1.25: Input & Output waveforms of Full wave rectifier

Operation:

During positive half of the input signal, anode of diode D_1 becomes positive and at the same time the anode of diode D_2 becomes negative. Hence D_1 conducts and D_2 does not conduct. The load current flows through D_1 and the voltage drop across R_L will be equal to the input voltage.

During the negative half cycle of the input, the anode of D_1 becomes negative and the anode of D_2 becomes positive. Hence, D_1 does not conduct and D_2 conducts. The load current flows through D_2 and the voltage drop across R_L will be equal to the input voltage.

It is noted that the load current flows in the both the half cycles of ac voltage and in the same direction through the load resistance.

Analysis:

Let a sinusoidal voltage V_i be applied to the input of a rectifier. It is given by $V_i = V_m \sin\omega t$
The current i_1 though D_1 and load resistor R_L is given by

$i_1 = I_m \sin\omega t$ for $0 \le \omega t \le \pi$

$i_1 = 0$ for $\pi \le \omega t \le 2\pi$ Where $I_m = \frac{V_m}{R_f + R_L}$

Similarly, the current i_2 through diode D_2 and load resistor R_L is given by

$i_2 = 0$ for $0 \le \omega t \le \pi$

$i_2 = I_m \sin\omega t$ for $\pi \le \omega t \le 2\pi$

Therefore, the total current flowing through R_L is the sum of the two currents i_1 and i_2.

i.e., $i_L = i_1 + i_2$.

i) **Average (or) DC Output Current (I_{av} or I_{dc}):**

The average dc current I_{dc} is given by

$$I_{dc} = \frac{1}{2\pi}\int_0^{2\pi} i_1\, d(\omega t) + \frac{1}{2\pi}\int_0^{2\pi} i_2\, d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\omega t\, d(\omega t) + 0 + 0 + \int_{\pi}^{2\pi} I_m \sin\omega t\, d(\omega t)\right]$$

$$= \frac{I_m}{\pi} + \frac{I_m}{\pi}$$

$$= \frac{2I_m}{\pi} = 0.318\, I_m$$

$$\therefore I_{dc} = \frac{2I_m}{\pi}$$

2 V_m

Substituting the value of I_m, we get $I_{dc} = \frac{2 V_m}{\pi (R_f + R_L)}$

This is double that of a Half-Wave Rectifier.

ii) Average (or) DC Output Voltage (V_{av} or V_{dc}):

The dc output voltage is given by

$$V_{dc} = I_{dc} \times R_L = \frac{2 I_m R_L}{\pi}$$

$$\Rightarrow V_{dc} = \frac{2 V_m R_L}{\pi (R_f + R_L)}$$

If $R_L >> R_f$ then $V_{dc} = \frac{2V_m}{\pi}$

iii) R.M.S. Output Current (I_{rms}):

The value of the R.M.S. current is given by

$$I_{rms} = \left[\frac{1}{2\pi} \int_0^{2\pi} i_L^2 \, d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi} \int_0^{\pi} i_1^2 \, d(\omega t) + \frac{1}{2\pi} \int_{\pi}^{2\pi} i_2^2 \, d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, d(\omega t) + \frac{1}{2\pi} \int_{\pi}^{2\pi} I_m^2 \sin^2 \omega t \, d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{2\pi} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) + \frac{I_m^2}{2\pi} \int_{\pi}^{2\pi} \frac{1 - \cos 2\omega t}{2} d(\omega t) \right]^{\frac{1}{2}}$$

$$= \left[\frac{I^2}{4\pi} \left[\omega t - \frac{\sin 2\omega t}{\omega t} \right]_0^{\pi} + \frac{I^2}{4\pi} \left[\omega t - \frac{\sin 2\omega t}{\omega t} \right]_{\pi}^{2\pi} \right]^{\frac{1}{2}}$$

iv) **R.M.S. Output Voltage (V_{rms}):**

R.M.S. voltage across the load is given by

$$V_{rms} = I_{rms} \times R_L = \frac{V_m}{\sqrt{2}(R_f + R_L)} \times R_L$$

$$\Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}\left(1 + \frac{R_f}{R_L}\right)}$$

If $R_L >> R_f$ then $V_{rms} = \frac{V_m}{\sqrt{2}}$

$$= \left[\frac{I_m^2}{4\pi}[(\pi - 0) - (0)] + \frac{I_m^2}{4\pi}[(2\pi - 0) - (\pi - 0)]\right]^{\frac{1}{2}}$$

$$= \left[\frac{I_m^2}{4\pi} \times \pi + \frac{I_m^2}{4\pi} \times \pi\right]^{\frac{1}{2}} = \left[2 \times \frac{I_m^2}{4}\right]^{\frac{1}{2}} = \frac{I_m}{\sqrt{2}}$$

$$\therefore I_{rms} = \frac{I_m}{\sqrt{2}} \text{ (or) } I_{rms} = \frac{V_m}{\sqrt{2}(R_f + R_L)}$$

v) **Rectifier efficiency(η) :**

The rectifier efficiency is defined as the ration of d.c. output power to the a.c. input power i.e.,

$$\eta = P_{dc}/P_{ac}$$

Theoretically the maximum value of rectifier efficiency of a full-wave rectifier is 81.2%

vi) **Ripple Factor(γ) :**

The ripple factor, γ is given by

$$\therefore \gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1} \quad \text{(or)} \quad \therefore \gamma = \sqrt{\left(\frac{V_{rms}}{V_{dc}}\right)^2 - 1}$$

$$\therefore \gamma = \sqrt{\left(\frac{I_m}{\sqrt{2}} \times \frac{\pi}{2 I_m}\right)^2 - 1} = \sqrt{\left(\frac{\pi}{2\sqrt{2}}\right)^2 - 1} = \mathbf{0.48}$$

$$\Rightarrow \gamma = 0.48$$

vii) Regulation:

The variation of V_{dc} with I_{dc} for a full-wave rectifier is obtained as follows:

$$V_{dc} = I_{dc} \times R_L$$

$$= \frac{2I_m}{\pi} R_L \qquad \because I_{dc} = \frac{2I_m}{\pi}$$

$$= \frac{2V_m R_L}{\pi (R_L + R_f)}$$

$$= \frac{2V_m}{\pi}\left(1 - \frac{R_f}{R_f + R_L}\right) \qquad = \frac{2V_m}{\pi} - I_{dc} R_f$$

$$\therefore V_{dc} = \frac{2V_m}{\pi} - I_{dc} R_f$$

The percentage regulation of the Full-wave rectifier is given by

$$\% \text{ Regulation} = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100$$

$$= \frac{\frac{2V_m}{\pi} - \left(\frac{2V_m}{\pi} - I_{dc} R_f\right)}{\frac{2V_m}{\pi} - I_{dc} R_f} \times 100 \qquad = \frac{I_{dc} R_f}{I_{dc} R_L} \times 100$$

$$\Rightarrow \% \text{ Regulation} = \frac{R_f}{R_L} \times 100$$

viii) Transformer Utilization Factor (UTF):

The average TUF in full-wave rectifying circuit is determined by considering the primary and secondary winding separately. There are two secondaries here. Each secondary is associated with one diode. This is just similar to secondary of half-wave rectifier. Each secondary has TUF as 0.287.

TUF of primary = P_{dc} / Volt-Amp rating of primary

$$\therefore (TUF)_P = \frac{I_{dc}^2 . R_L}{\frac{I_m}{\sqrt{2}} . \frac{V_m}{\sqrt{2}}} = \frac{\left(\frac{2 I_m}{\pi}\right)^2 . R_L}{\frac{V_m I_m}{2}}$$

$$= \frac{4 I_m^2}{\pi^2} . \frac{2R_L}{I_m^2 (R_f + R_L)} \qquad = \frac{8}{\pi^2} \frac{1}{1 + \frac{R_f}{R_L}}$$

If $R_L >> R_f$ then $(TUF)_p = \frac{8}{\pi^2} = 0.812.$

$$\therefore (TUF)_{av} = P_{dc} \text{ / V-A rating of transformer}$$

$$= \frac{(TUF)_p + (TUF)_s + (TUF)_s}{3}$$

$$= \frac{0.812 + 0.287 + 0.287}{3} = 0.693$$

$$\therefore (TUF) = \mathbf{0.693}$$

ix) Peak Inverse Voltage (PIV):

Peak Inverse Voltage is the maximum possible voltage across a diode when it is reverse biased. Consider that diode D_1 is in the forward biased i.e., conducting and diode D_2 is reverse biased i.e., non-conducting. In this case a voltage V_m is developed across the load resistor R_L. Now the voltage across diode D_2 is the sum of the voltages across load resistor R_L and voltage across the lower half of transformer secondary V_m. Hence PIV of diode $D_2 = V_m + V_m = 2V_m$.

Similarly PIV of diode D_1 is $2V_m$.

x) Form factor (F):

The Form Factor F is defined as F = rms value / average value

$$F = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{0.707\, I_m}{0.63\, I_m} = 1.12 \qquad \mathbf{F=1.12}$$

xi) Peak Factor (P):

The peak factor P is defined as

$$P = \text{Peak Value / rms value} = \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = 1.414 \qquad \mathbf{P = 1.414}$$

Problems from previous External Question Paper:

4) A Full-Wave rectifier circuit is fed from a transformer having a center-tapped secondary winding. The rms voltage from wither end of secondary to center tap is 30V. if the diode forward resistance is 5J and that of the secondary is 10J for a load of 900J,
Calculate:

i) Power delivered to load,
ii) % regulation at full-load,
iii) Efficiency at full-load and
iv) TUF of secondary.

Solution: Given V_{rms} = 30V, R_f =5J, Rs=10J, R_L=900J

But $V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow V_m = 30 \times \sqrt{2}$ = 42.426 V.

$$I_m = \frac{V_m}{R_f + R_S + R_L} = \frac{30\sqrt{2}}{5 + 10 + 900} = 46.36 \text{ mA.}$$

$$I_{dc} = \frac{2I_m}{\pi} = \frac{2 \times 46.36}{\pi} = 29.5\text{mA}$$

i) Power delivered to the load $= I_{dc}^2 R_L = \left(29.5 \times 10^{-3}\right)^2 \times 900 = \mathbf{0.783W}$

ii) % Regulation at full-load = $\frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100$

$V_{no-load} = \frac{2V_m}{\pi} = \frac{2 \times 42.426}{\pi}$ = **27.02 V.**

$V_{full-load} = I_{dc} R_L$ = 29.5 x 10^{-3} x 900 = 26.5 V

% Regulation = $\frac{27.02 - 26.5}{26.5} \times 100$ = **1.96 %**

iii) Efficiency of Rectification = $\frac{81.2}{1 + \frac{R_f + R_S}{R_L}} = \frac{81.2}{1 + \frac{15}{900}}$ = **79.8%**

iv) TUF of secondary = DC power output / secondary ac rating

Transformer secondary rating = $V_{rms} I_{rms}$ = $30 \times \frac{46.36}{\sqrt{2}} \times 10^{-3}$ W

$P_{dc} = I_{dc}^2 R_L$

$\therefore TUF = \frac{0.783}{30 \times \frac{46.36}{\sqrt{2}} \times 10^{-3}}$ = **0.796**

5) A Full-wave rectifier circuit uses two silicon diodes with a forward resistance of 20J each. A dc voltmeter connected across the load of 1kJ reads 55.4volts. Calculate

i) I_{RMS},
ii) Average voltage across each diode,
iii) Ripple factor, and
iv) Transformer secondary voltage rating.

Solution:

Given R_f =20J, R_L=1kJ, V_{dc} = 55.4V

For a FWR $V_{dc} = \frac{2V_m}{\pi}$ $\therefore V_m = \frac{55.4 \times \pi}{2}$ = **86.9 V**

$I_m = \frac{V_m}{R_f + R_L}$ =**0.08519A**

i) $I_{rms} = \frac{I_m}{\sqrt{2}}$ = 0.06024A

ii) V= 86.9/2 = 43.45V

iii) Ripple factor

$\gamma = \sqrt{\left(\frac{I_{rms}}{I_{dc}}\right)^2 - 1}$, $I_{dc} = \frac{2I_m}{\pi}$ =**0.05423A** $I_{rms} = \frac{I_m}{\sqrt{2}}$ =**0.06024A**

$\therefore \gamma = 0.48$

iv) Transformer secondary voltage rating: $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{86.9}{\sqrt{2}}$ = **61.49 Volts.**

6) A 230V, 60Hz voltage is applied to the primary of a 5:1 step down, center tapped transformer used in the Full-wave rectifier having a load of 900J. If the diode resistance and the secondary coil resistance together has a resistance of 100J. Determine:

i) dc voltage across the load,
ii) dc current flowing through the load,
iii) dc power delivered to the load, and
iv) ripple voltage and its frequency.

Solution: Given $V_{p(rms)}$ = 230V

$$\frac{N_2}{N_1} = \frac{2V_{S(rms)}}{V_{P(rms)}} \qquad \Rightarrow \frac{1}{5} = \frac{2V_{S(rms)}}{230}$$

$$\Rightarrow V_{S(rms)} = 23\text{V}$$

Given R_L =900J, $R_f + R_S$ =100J

$$I_m = \frac{V_{sm}}{R_f + R_S + R_L} = \frac{\sqrt{2}V_{s(rms)}}{R_f + R_S + R_L} = \frac{\sqrt{2} \times 23}{900 + 100} = 0.03252 \text{ Amp.}$$

$$\therefore I_{dc} = \frac{2I_m}{\pi} = \frac{2 \times 0.03252}{\pi} = \mathbf{0.0207\ Amp.}$$

i) $V_{DC} = I_{DC} R_L$ = 0.0207 X 100 = **18.6365 Volts.**

ii) I_{DC} = **0.0207 Amp.**

iii) $P_{dc} = I_{dc}^2 R_L$ (or) $V_{DC} I_{DC}$ = **0.3857 Watts.**

iv) PIV = $2V_{sm}$ = 2 X $\sqrt{2}$ X 23 = **65.0538 Volts**

v) Ripple factor = 0.482 = $\frac{V_{r(rms)}}{V_{DC}}$

Therefore, ripple voltage = $V_{r(rms)}$ = 0.482 x 18.6365
= **8.9827 Volts.**

Frequency of ripple = 2f = 2x60 = **120 Hz**

Bridge Rectifier

The full-wave rectifier circuit requires a center tapped transformer where only one half of the total ac voltage of the transformer secondary winding is utilized to convert into dc output. The need of the center tapped transformer in a Full-wave rectifier is eliminated in the bridge rectifier.

The bridge rectifier circuit has four diodes connected to form a bridge. The ac input voltage us applied to diagonally opposite ends of the bridge. The load resistance is connected between the other two ends of the bridge. The bridge rectifier circuits and its waveforms are shown in figure.

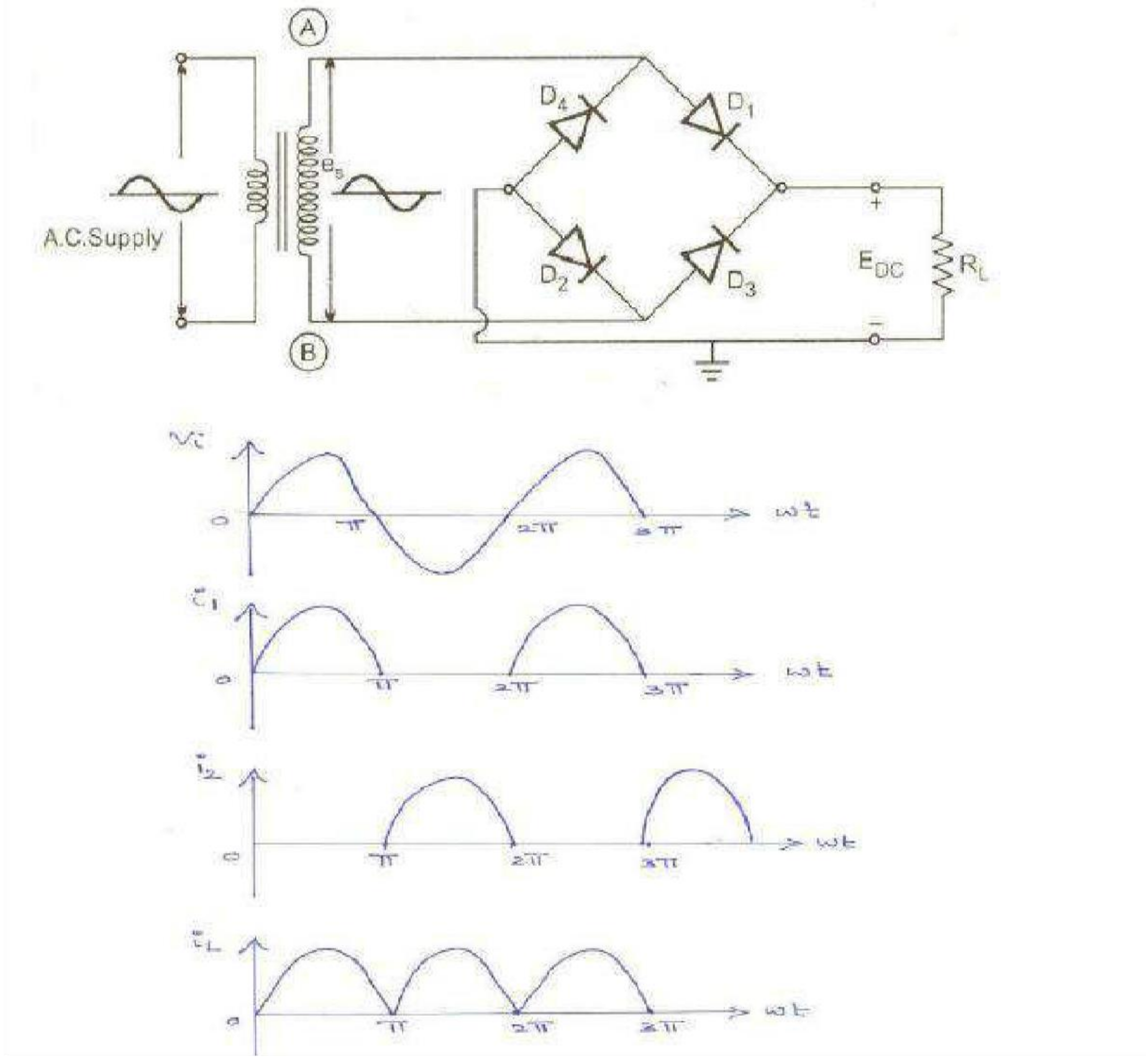


Figure 1.26: Circuit diagram and Input & Output waveforms of Bridge rectifier

Operation:

For the positive half cycle of the input ac voltage diodes D_1 and D_3 conduct, whereas diodes D_2 and D_4 do not conduct. The conducting diodes will be in series through the load resistance R_L, so the load current flows through the R_L.

During the negative half cycle of the input ac voltage diodes D_2 and D_4 conduct, whereas diodes D_1 and D_3 do not conduct.

The conducting diodes D_2 and D_4 will be in series through the load resistance R_L and the current flows through the R_L, in the same direction as in the previous half cycle. Thus a bidirectional wave is converted into a unidirectional wave.

Analysis:

The average values of output voltage and load current, the rms values of voltage and current, the ripple factor and rectifier efficiency are the same as for as center tapped full-wave rectifier.

Hence,

$$V_{dc} = \frac{2V_m}{\pi}$$

$$I_{dc} = \frac{2I_m}{\pi} \qquad I_m = \frac{V_m}{R_f + R_L}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

Since the each half cycle two diodes conduct simultaneously

$$\gamma = 0.48$$

$$\eta = \frac{81.2}{1 + \frac{2R_f}{R_L}}$$

The transformer utilization factor (TUF) of primary and secondary will be the same as there is always through primary and secondary.

TUF of secondary = P_{dc} / V-A rating of secondary

$$= \frac{I_{dc}^2 R_L}{V_{rms} I_{rms}} = \frac{\left(\frac{2I_m}{\pi}\right)^2 R_L}{\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}} = \mathbf{0.812}$$

TUF in case of secondary of primary of FWR is 0.812

$$\therefore (TUF)_{av} = \frac{(TUF)_p + (TUF)_s}{2}$$

$$= \frac{0.812 + 0.812}{2} = \mathbf{0.812}$$

$$\therefore TUF = \mathbf{0.812}$$

The reverse voltage appearing across the reverse biased diodes is $2V_m$, but two diodes are sharing it, therefore the PIV rating of the diodes is V_m.

Advantages of Bridge rectifier circuit:

1) No center-tapped transformer is required.
2) The TUF is considerably high.
3) PIV is reduced across the diode.

Disadvantages of Bridge rectifier circuit:

The only disadvantage of bridge rectifier is the use of four diodes as compared to two diodes for center-tapped FWR. This reduces the output voltage.

Problems:

7. A bridge rectifier uses four identical diodes having forward resistance of 5J and the secondary voltage of $30V_{(rms)}$. Determine the dc output voltage for I_{DC}=200mA and the value of the ripple voltage.

Solution: $V_{s(rms)}$=30V, R_S=5J, R_f=5J, I_{DC}=200mA

Now $I_{DC} = 2\frac{I_m}{\pi}$

$$\therefore I_m = \frac{200 \times 10^{-3} \times \pi}{2} = 0.3415 \text{ Amp.}$$

But $$I_m = \frac{V_{sm}}{R_S + 2R_f + R_L} = \frac{\sqrt{2}\, V_{s(rms)}}{R_S + 2R_f + R_L}$$

$$\Rightarrow 0.3415 = \frac{\sqrt{2} \times 30}{5 + (2 \times 5) + R_L}$$

$\Rightarrow R_L = 120.051\Omega \approx 120\Omega$

$V_{DC} = I_{DC} R_L = 200 \times 10^{-3} \times 120 = 24\text{Volts}$

$$\text{Ripple factor} = \frac{V_{r(rms)}}{V_{dc}}$$

For Bridge rectifier, ripple factor = 0.482

$\therefore V_{r(rms)}$ = rms value of ripple voltage

= Vdcx0.482

= 24x0.482

=**11.568 Volts**

8. In a bridge rectifier the transformer is connected to 220V, 60Hz mains and the turns ratio of the step down transformer is 11:1. Assuming the diode to be ideal, find:

i) I_{dc}

ii) voltage across the load

iii) PIV assume load resistance to be 1kΩ

Solution: $\frac{N_2}{N_1} = \frac{1}{11}$, $V_{p(rms)} = 220V$, $f = 60Hz$, $R_L = 1k\Omega$

$$\frac{N_2}{N_1} = \frac{V_{S(rms)}}{V_{P(rms)}} \Rightarrow \frac{1}{11} = \frac{V_{S(rms)}}{220} \Rightarrow V_{S(rms)} = \frac{220}{11} = 20V$$

$$V_{sm} = \sqrt{2}\, V_{s(rms)}$$

i) $$I_m = \frac{V_{sm}}{R_L} = \frac{28.2842}{1 \times 10^{-3}} = 28.2842 \text{ mA}$$

$$\therefore I_{dc} = \frac{2 I_m}{\pi} = \textbf{18 mA}$$

ii) $V_{dc} = I_{dc} R_L = 18 \times 10^{-3} X \times 10^{-3} = \textbf{18 Volts}$

iv) $PIV = V_{sm} = \textbf{28.2842 Volts}$

FILTERS

The output of a half-wave (or) full-wave rectifier circuit is not pure d.c., but it contains fluctuations (or) ripple, which are undesired. To minimize the ripple content in the output, filter circuits are used. These circuits are connected between the rectifier and load. Ideally, the output of the filter should be pure d.c. practically, the filter circuit will try to minimize the ripple at the output, as far as possible. Basically, the ripple is ac, i.e., varying with time, while dc is a constant w.r.t. time.

Hence in order to separate dc from ripple, the filter circuit should use components which have widely different impedance for ac and dc. Two such components are inductance and capacitance. Ideally, the inductance acts as a short circuit for dc, but it has large impedance for ac.

Similarly, the capacitor acts as open for dc if the value of capacitance is sufficiently lar enough. Hence, in a filter circuit, the inductance is always connected in series with the load, and t capacitance is connected in parallel to the load.

Definition of a Filter:

Filter is an electronic circuit composed of a capacitor, inductor (or) combination of both a connected between the rectifier and the load so as to convert pulsating dc to pure dc.

The different types of filters are:

1) Inductor Filter,
2) Capacitor Filter,
3) LC (or) L-Section Filter, and
4) CLC (or) ∏-section Filter.

Inductor Filter:

Half-Wave rectifier with series Inductor Filter:

The Inductor filter for half-wave rectifier is shown in figure below.

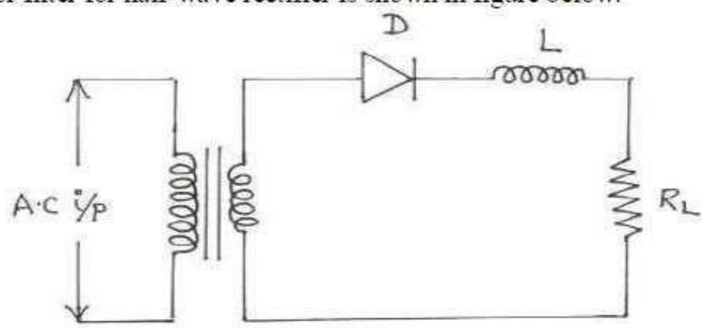


Fig. Series Inductor filter for HWR

In this filter the inductor (choke) is connected in series with the load. The operation of the inductor filter depends upon the property of the inductance to oppose any change of current that may flow through it.

Expression for ripple factor:

For a half-wave rectifier, the output current is given by,

$$i = I_m \left[\frac{1}{\pi} + \frac{1}{2}\sin\omega t - \frac{2}{\pi} \sum_{\substack{K=even \\ K\neq 0}} \frac{\cos\omega t}{(K+1)(K-1)} \right]$$

$$i = \frac{I_m}{\pi} + \frac{I_m}{2}\sin\omega t - \frac{2I_m}{\pi}\left[\frac{\cos 2\omega t}{3} + \frac{\cos 4\omega t}{15} + \dots\right] \quad \text{..............} (1)$$

Neglecting the higher order terms, we have

$$I_{dc} = \frac{I_m}{\pi} = \frac{V_m}{\pi R_L} \quad \text{..................} (2)$$

If I_1 be the rms value of fundamental component of current, then

$$I_{dc} = \frac{I_m}{2\sqrt{2}} = \frac{V_m}{2\sqrt{2}\,(R_L + j\omega L)} = \frac{V_m}{2\sqrt{2}\,(R_L^2 + j\omega^2 L^2)^{\frac{1}{2}}} \quad \text{............(3)}$$

At operating frequency, the reactance offered by inductance 'L' is very large compared to R_L (i.e., $\omega L >> R_L$) and hence R_L can be neglected.

$$\therefore I_1 = \frac{V_m}{2\sqrt{2}\,\omega L} \quad \text{.............(4)}$$

If I_2 be rms value of second harmonic,

$$\text{Then } \therefore I_2 = \frac{2I_m}{3\sqrt{2}\pi} = \frac{2V_m}{3\sqrt{2}\pi\left(R_L^2 + 4\omega^2 L^2\right)^{\frac{1}{2}}} = \frac{V_m}{3\sqrt{2}\,\pi\omega L} \quad (\because R_L << \omega L) \quad \text{....... (5)}$$

If I_{ac} be the rms value of all current components, then $I_{ac} = \sqrt{I_1^2 + I_2^2}$

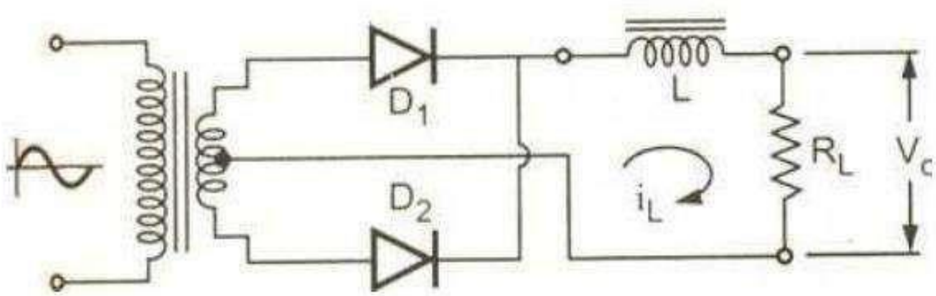
$$\text{Now, } \gamma = \frac{V}{V_{dc}} = \frac{I_{ac} R}{I_{dc} R_L} = \frac{I_{ac}}{I_{dc}} = \frac{\sqrt{\left(\frac{V}{2\sqrt{2}\omega L}\right)^2 + \left(\frac{V}{3\sqrt{2}\pi\omega L}\right)^2}}{\frac{V_m}{\pi R_L}}$$

$$= \frac{\frac{V_m}{\omega L}\sqrt{\frac{1}{8} + \frac{1}{18\pi^2}}}{\frac{V_m}{\pi R_L}} = \frac{\pi R_L}{\omega L}\sqrt{\frac{1}{8} + \frac{1}{18\pi^2}}$$

$$= \frac{1.13 R_L}{\omega L} \qquad \therefore \gamma = \frac{1.13 R_L}{\omega L} \quad \text{............(6)}$$

Full-wave rectifier with series inductor filter:

A FWR with series inductor filter is shown in figure.



The inductor offers high impedance to a.c. variations. The inductor blocks the a.c. component and allows only t
he dc component to reach the load.

To analyze the inductor filter for a FWR, the Fourier series can be written as

$$V_o = \frac{2V_m}{\pi} - \frac{4V_m}{\pi}\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 6\omega t + \ldots\ldots\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

The dc component is $\frac{2V_m}{\pi}$

Assuming the third and higher terms contribute little output voltage is

$$V_O = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For the sake of simplicity, the diode drop and diode resistance are neglected because tl introduce a little error. Thus for dc component, the current $I_m = \frac{V_m}{R_L}$. For ac component, impedance of L and R_L will be in series and is given by,

$$Z = \sqrt{R_L^2 + (2\omega L)^2}, \text{ frequency of ac component} = 2\omega$$
$$= \sqrt{R_L^2 + 4\omega^2 L^2}$$

Thus for ac component $I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}}$

The current flowing in a FWR is given by, $i = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi}\cos 2\omega t \ldots\ldots\ldots\ldots(3)$

Substituting the value of I_m for dc and ac equation (3), we get,

$$i = \frac{2V_m}{\pi R_L} - \frac{4V_m}{3\pi\sqrt{R_L^2 + 4\omega^2 L^2}}\cos(2\omega t - \phi) \quad \ldots\ldots\ldots\ldots(4)$$

Where Φ is the angle by which the load current lags behind the voltage. This is given by

$$\phi = \tan^{-1}\frac{2\omega L}{R_L}$$

Expression for Ripple Factor:

$$\gamma = \frac{I_{r,rms}}{I_{dc}}$$

From equation (4), $I_{dc} = \frac{2V_m}{\pi R_L}$, $I_{r,rms} = \frac{4V_m}{3\pi\sqrt{2}\sqrt{R_L^2 + 4\omega^2 L^2}}$

$$\therefore \gamma = \frac{\frac{4V_m}{3\pi\sqrt{2}} \frac{1}{\sqrt{R_L^2 + 4\omega^2 L^2}}}{\frac{2V_m}{\pi R_L}} \qquad \therefore \gamma = \frac{2}{3\sqrt{2}} \frac{1}{\sqrt{1 + \frac{4\omega^2 L^2}{R_L^2}}}$$

If $\frac{4\omega^2 L^2}{R_L^2} >> 1$, then $\gamma = \frac{1}{3\sqrt{2}} \frac{R_L}{\omega L} = 0.236 \frac{R_L}{\omega L}$.

$$\therefore \gamma = \frac{R_L}{3\sqrt{2}\,\omega L} \text{ .... (6)}$$

The expression shows that ripple varies inversely as the magnitude of the inductance, Also, the ripple is smaller for smaller values of R_L i.e., for high currents.

When R_L □□□ the value of □ is given by □ □ $\frac{2}{3\sqrt{2}}$ = 0.471 (close to the value 0.482 of rectifier). Thus the inductor filter should be used when R_L is consistently small.

Problems:

9. A full-wave rectifier with a load resistance of 15kJ uses an inductor filter of 15H. The peak value of the applied voltage is 250V and the frequency is 50 cycles/second. Calculate the dc load current, ripple factor and dc output voltage.

Solution: The rectified output voltage across load resistance R_L up to second harmonic is

$$V_o = \frac{2V_m}{\pi} - \frac{2V_m}{\pi} \cos\omega t$$

Therefore, DC component of output voltage is given by $V_{dc} = \frac{2V_m}{\pi}$

$$\therefore I_{dc} = \frac{V_{dc}}{R_L} = \frac{2V_m}{\pi R_L}$$

$$= \frac{2 \times 250}{\pi \times 15 \times 10^{-3}} = 10.6 \times 10^{-3} \text{ A} = 10.6 \text{ mA}$$

$V_{dc} = I_{dc} R_L = (2.12 \times 10^{-3})(15 \times 10^{3}) = 31.8$ V.

Peak value of ripple voltage = $\frac{4V_m}{3\pi}$

$$\therefore V_{ac} = \frac{1}{\sqrt{2}} \frac{4V_m}{3\pi}$$

Now $$I_{ac} = \frac{\frac{1}{\sqrt{2}} \frac{4V_m}{3\pi}}{\sqrt{R_L^2 + (2\omega L)^2}} = \frac{2\sqrt{2}V_m}{3\pi\sqrt{R_L^2 + (2\omega L)^2}}$$

$$= \frac{2 \times 1.414 \times 250}{3 \times 3.14\sqrt{(15 \times 10^3) + (4 \times 3.14 \times 50 \times 15)^2}}$$

$= 4.24 \times 10^{-3}$ A = **4.24 mA**

So, ripple factor, $\gamma = \frac{I_{ac}}{I_{dc}} = \frac{4.24mA}{10.6mA} =$ **0.4**

Capacitor Filter:

Half-wave rectifier wit capacitor filter:

The half-wave rectifier with capacitor input filter is shown in figure below:

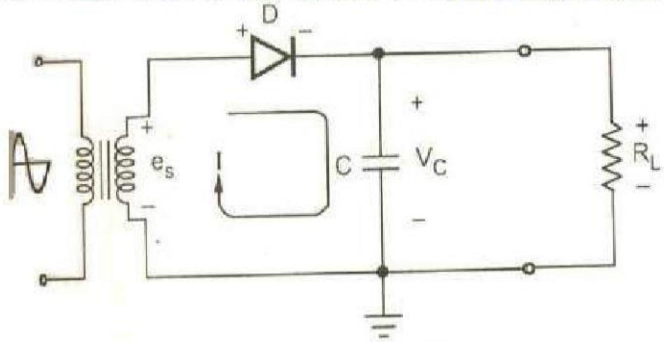


Fig. HWR with capacitor filter.

The filter uses a single capacitor connected in parallel with the load R_L. In order to minimize the ripple in the output, the capacitor C used in the filter circuit is quite large of the order of tens of microfarads.

The operation of the capacitor filter depends upon the fact that the capacitor stores energy during the conduction period and delivers this energy to the load during non-conduction period.

Operation:

During, the positive quarter cycle of the ac input signal, the diode D is forward biased and hence it conducts. This quickly charges the capacitor C to peak value of input voltage V_m. Practically the capacitor charge (V_m-$V\gamma$) due to diode forward voltage drop.

When the input starts decreasing below its peak value, the capacitor remains charged at V_m and the ideal diode gets reverse biased. This is because the capacitor voltage which is cathode voltage of diode becomes more positive than anode.

Therefore, during the entire negative half cycle and some part of the next positive half cycle, capacitor discharges through R_L. The discharging of capacitor is decided by R_LC, time constant which is very large and hence the capacitor discharge very little from V_m.

In the next positive half cycle, when the input signal becomes more than the capacitor voltage, he diode becomes forward biased and charges the capacitor C back to V_m. The output waveform is shown in figure below:

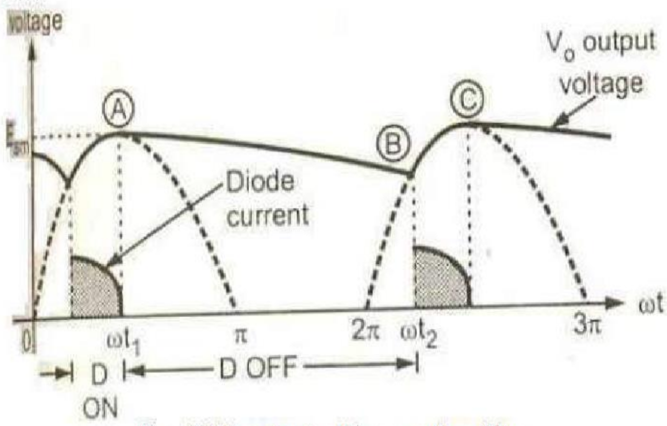
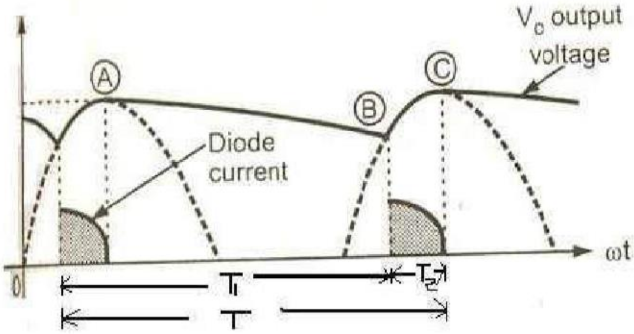


Fig. HWR output with capacitor filter.

The discharging if the capacitor is from A to B, the diode remains non-conducting. The diode conducts only from B to C and the capacitor charges.

Expression for Ripple factor:



Let, T = time period of the ac input voltage

T_1 = time for which the diode is non conducting.

T_2 = time for which diode is conducting.

Let V_r be the peak to peak value of the ripple voltage which is assumed to be triangular waveform. It is known mathematically that the rms value of such a triangular waveform is

$$V_{rms} = \frac{V_r}{2\sqrt{3}}$$

During the time interval T_1, the capacitor C is discharging through the load resistance R_L. Therefore the charge lost is **$Q = C\,V_r$**

But, $i = \frac{dQ}{dt}$ $\qquad \therefore Q = \int_0^{T_1} i\,dt = I_{dc} . T_1$

As integration gives average (or) dc value,

Hence $I_{dc} . T_1 = C . V_r$

$$\therefore V_r = \frac{I_{dc} T_1}{C}$$

But $T_1 + T_2 = T$ $\qquad$ Normally, $T_1 >> T_2$,

$$\therefore T_1 + T_2 \approx T_1 \Rightarrow T_1 = T$$

$$\therefore V_r = \frac{I_{dc} \cdot T}{C} = \frac{I_{dc}}{f \cdot C} \qquad \therefore T = \frac{1}{f}$$

But $I_{dc} = \frac{V_{dc}}{R_L}$, $\qquad V_{dc} = V_m - \frac{V_r}{2}, = V_m - \frac{I_{dc}}{2fc}$

$$\therefore V_r = \frac{V_{dc}}{fCR_L}$$

Ripple factor, $\gamma = \frac{V_{rms}}{V_{dc}}$ $\qquad \Rightarrow \gamma = \frac{V_r}{2\sqrt{3} \cdot V_{dc}} = \frac{V_{dc}}{2\sqrt{3} fCR_L \cdot V_{dc}}$

$$\gamma \Rightarrow \frac{1}{2\sqrt{3}\, fCR_L}$$

The product of CR_L is the time constant of the filter circuit.

Surge current in Half-wave rectifier using capacitor filter:

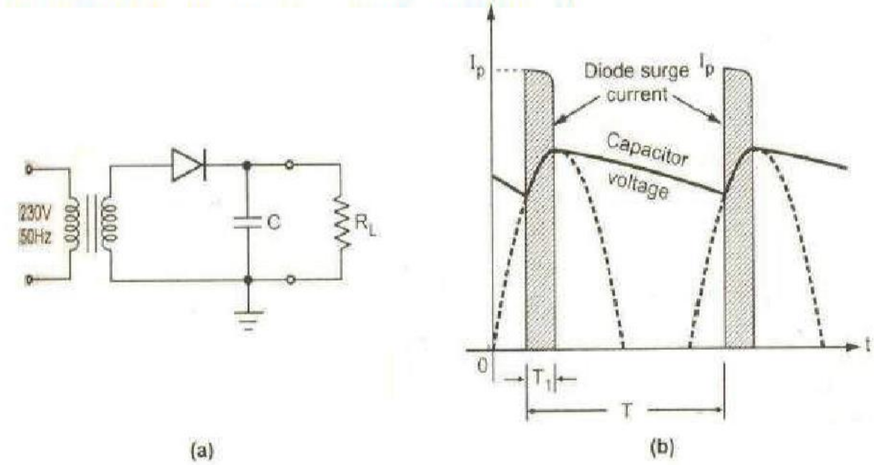


Fig. Surge current in HWR using capacitor filter

In half-wave rectifier, the diode is forward biased only for short period of time and conducts only during this time interval to charge the filter capacitance. The instant at which the diode gets forward biased, the capacitor instantaneously acts as short circuit and a surge current flow through a diode.

When the diode is non-conducting, the capacitor discharges through load resistance R_L. Thus total amount of charge that flows through conducting diode (or) diodes to recharge the capacitor must be equal to the amount of charge lost during the period when the diode (or) diodes are non-conducting and capacitor is discharging through load resistance R_L.

It can be seen that conduction period T_1 is very small compared to time period T, for the diode. Let, I_{dc} = average dc current

$I_{p(surge)}$ = peak value of the surge current.

Assume the current pulse to be rectangular assuming peak surge current flows for the entire conduction period of diode which is T_1.

Then Q (discharge) = Q (charge)

$$\therefore I_{dc} T = I_{P(surge)} T_1 \qquad \therefore I_{P(surge)} = I_{dc} \frac{T}{T_1}$$

As $T_1 << T$, it can be observed that $I_{p(surge)}$ can be many times larger than the average dc current supplied to the load.

Problem from previous External examinations:

10. A HWR circuit has filter capacitor of 1200µF and is connected to a load of 400J. The rectifier is connected to a 50Hz, 120V rms source. It takes 2msec for the capacitor to recharge during each cycle. Calculate the minimum value of the repetitive surge current for which the diode should be rated.

Solution:

Given C=1200µF, R_L=400J, f=50Hz, V_{rms}=120V

Conduction period of the diode, T_1=1ms

$$V_{sm} = \sqrt{2} \times V_{S(rms)} = \sqrt{2} \times 120 \text{ V}$$

$$V_{dc} = V_{sm} - \frac{I_{dc}}{2fC}$$

$$\Rightarrow V_{dc} = V_{sm} - \frac{V_{dc}}{2fCR_L}$$

$$\Rightarrow V_{dc} = \frac{V_{sm}}{1+\frac{1}{2fCR_L}}$$

$$= \frac{120\sqrt{2}}{1+\frac{1}{2 \times 50 \times 1200 \times 10^{-6} \times 400}} = 3.46 \text{ V}$$

$$\therefore I_{dc} = \frac{V_{dc}}{R_L} = \frac{3.46}{400} = 8.658\text{mA}$$

Now $I_{dc} = I_{p(surge)} \frac{t_1}{T}$

$$I_{P(surge)} = I_{dc} \frac{T}{t_1} = 8.658\text{mA} \times \frac{1}{50 \times 10^{-3}}$$

$$\therefore I_{P(surge)} = 0.17316 \text{ A}$$

Full-wave rectifier with capacitor filter:

The full-wave rectifier with capacitor filter is shown in the figure below:

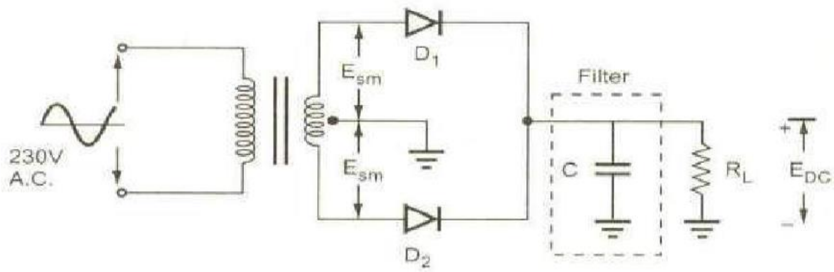


Fig. Full-wave rectifier with capacitor filter

Operation:

During the positive quarter cycle of the ac input signal, the diode D_1 is forward biased, the capacitor C gets charges through forward bias diode D_1 to the peak value of input voltage V_m.

In the next quarter cycle from $\frac{\pi}{2}$ to π the capacitor starts discharging through R_L, because once the capacitor gets charges to V_m, the diode D_1 gets reverse biased and stops conducting, so during the period from $\frac{\pi}{2}$ to π the capacitor C supplies the load current.

In the next quarter half cycle, that is, π to $\frac{3\pi}{2}$ of the rectified output voltage, if the input voltage exceeds the capacitor voltage, making D_2 forward biased, this charges the capacitor back to V_m.

In the next quarter half cycle, that is, from $\frac{3\pi}{2}$ to 2π, the diode gets reverse biased and the capacitor supplies the load current.

In FWR, as the time required by the capacitor to charge is very small and it discharges very little due to large time constant, hence ripple in the output gets reduced considerably. The output waveform is shown in figure below:

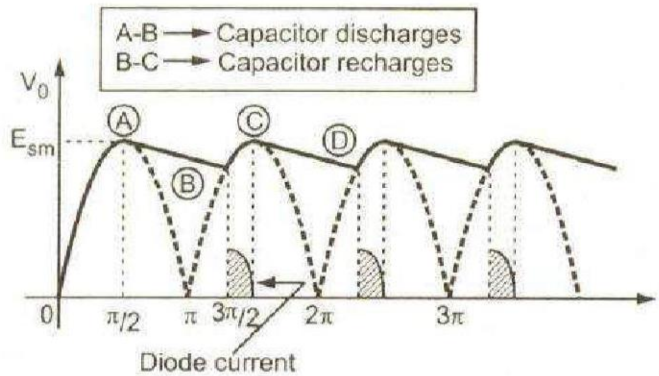
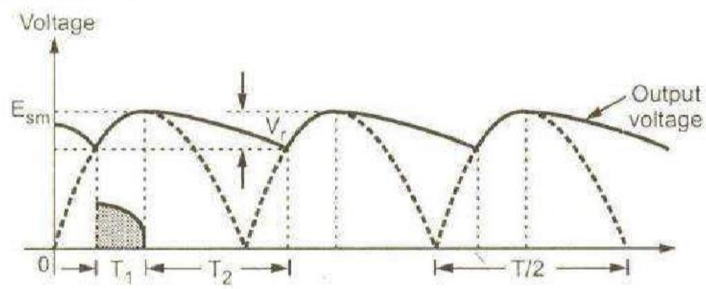


Fig. FWR output with capacitor filter.

Expression for Ripple factor:



Let, T = time period of the ac input voltage

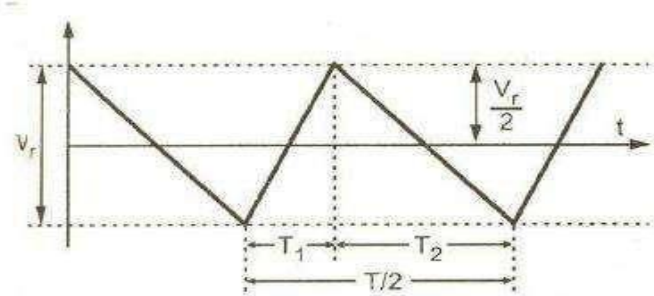
$\frac{T}{2}$ = half of the time period

T_1 = time for which diode is conducting

T_2 = time for which diode is non-conducting

During time T_1, capacitor gets charged and this process is quick. During time T_2, capacitor gets discharged through R_L. As time constant R_LC is very large, discharging process is very slow and hence $T_2 >> T_1$.

Let V_r be the peak to peak value of ripple voltage, which is assumed to be triangular as shown in the figure below:



It is known mathematically that the rms value of such a triangular waveform is,

$$V_{rms} = \frac{V_r}{2\sqrt{3}}$$

During the time interval T_2, the capacitor C is discharging through the load resistance R_L.

The charge lost is, $Q = CV_r$ But $i = \frac{dQ}{dt}$

$$\therefore Q = \int_0^{T_2} i dt = I_{DC} T_2$$

As integration gives average (or) dc value, hence $I_{dc} . T_2 = C . V_r$

$$\therefore V_r = \frac{I_{dc} T_2}{C} \qquad \text{But } T_1 + T_2 = \frac{T}{2}$$

Normally, $T_2 >> T_1$,

$$\therefore T_1 + T_2 \approx T_2 = \frac{T}{2} \quad \text{where } T = \frac{1}{f}$$

$$\therefore V_r = \frac{I_{DC}}{C} \frac{T}{2} = \frac{I_{DC} \times T}{2C} = \frac{I_{DC}}{2fC}$$

But $I_{DC} = \frac{V_{DC}}{R_L}$, $\therefore V_r = \frac{V_{DC}}{2fCR_L}$ = peak to peak ripple voltage

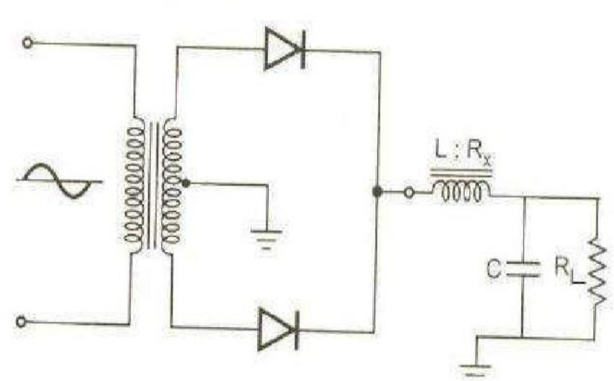
$$\text{Ripple factor, } = \frac{V_{rms}}{V_{dc}} = \frac{\frac{V_{dc}}{2fCRL}}{2\sqrt{3}} \times \frac{1}{V_{dc}} \qquad \therefore V_{rms} = \frac{V_r}{2\sqrt{3}}$$

$$\therefore \text{Ripple factor} = \frac{1}{4\sqrt{3} fCR_L}$$

L-Section Filter (or) LC Filter:

The series inductor filter and shunt capacitor filter are not much efficient to provide low ripple at all loads. The capacitor filter has low ripple at heavy loads while inductor filter at small loads. A combination of these two filters may be selected to make the ripple independent of load resistance. The resulting filter is called L-Section filter (or) LC filter (or) Choke input filter. This

name is due to the fact that the inductor and capacitor are connected as an inverted L. A full-wave rectifier with choke input filter is shown in figure below:



The action of choke input filter is like a low pass filter. The capacitor shunting the load bypasses the harmonic currents because it offers very low reactance to a.c. ripple current while it appears as an open circuit to dc current.

On the other hand the inductor offers high impedance to the harmonic terms. In this way, most of the ripple voltage is eliminated from the load voltage.

Regulation:

The output voltage of the rectifier is given by, $\upsilon = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t$

The dc voltage at no load condition is $V_{dc} = \frac{2V_m}{\pi}$

The dc voltage on load is $V_{dc} = \frac{2V_m}{\pi} - I_{dc}R$

Where $R = R_f + R_C + R_S$

R_f, R_C, R_S are resistances of diode, choke an secondary winding.

Ripple Factor:

The main aim of the filter is to suppress the harmonic components. So the reactance of the choke must be large as compared with the combined parallel impedance of capacitor and resistor.

The parallel impedance of capacitor and resistor can be made small by making the reactance of the capacitor much smaller than the resistance of the load. Now the ripple current which has passed through L will not develop much ripple voltage across R_L because the reactance of C at the ripple frequency is very small as compared with R_L.

Thus for LC filter, $X_L >> X_C$ at $2\omega = 4\Pi f$ and $R_L >> X_C$

Under these conditions, the a.c. current through L is determined primarily by $X_L = 2\omega L$ (the reactance of the inductor at second harmonic frequency). The rms value of the ripple current is

$$I_{r(rms)} = \frac{4V_m}{3\pi\sqrt{2}} \cdot \frac{1}{X_L} = \frac{2}{3\sqrt{2}X_L}\frac{2V_m}{\pi} = \frac{\sqrt{2}}{3X_L}(V_{dc})$$

Always it was stated that X_C is small as compared with R_L, but it is not zero. The a.c. voltage across the load (the ripple voltage) is the voltage across the capacitor.

Hence $V_{r(rms)} = I_{r(rms)} \times X_C$

$$= \frac{\sqrt{2}}{3X_L}V_{dc} \quad X_C$$

We know that ripple factor γ is given by

$$\gamma = \frac{V_{r(rms)}}{V_{dc}} = \frac{\sqrt{2}X_C}{3X_L}$$

But $X_C = \frac{1}{2\omega C}$ and $X_L = 2\omega L$

$$\therefore \gamma = \frac{\sqrt{2}}{3(2\omega L)} \times \frac{1}{2\omega C} = \frac{1}{6\sqrt{2}\omega^2 LC}$$

$$\therefore \gamma = \frac{1}{6\sqrt{2}\omega^2 LC}$$

This shows that ω is independent of R_L.

Multiple L-Section filters:

The number of L-sections i.e., LC circuits can be connected one after another to obtain multiple L-section filter. It gives excellent filtering and smooth dc output voltage. The figure below shows multiple L-section filter.

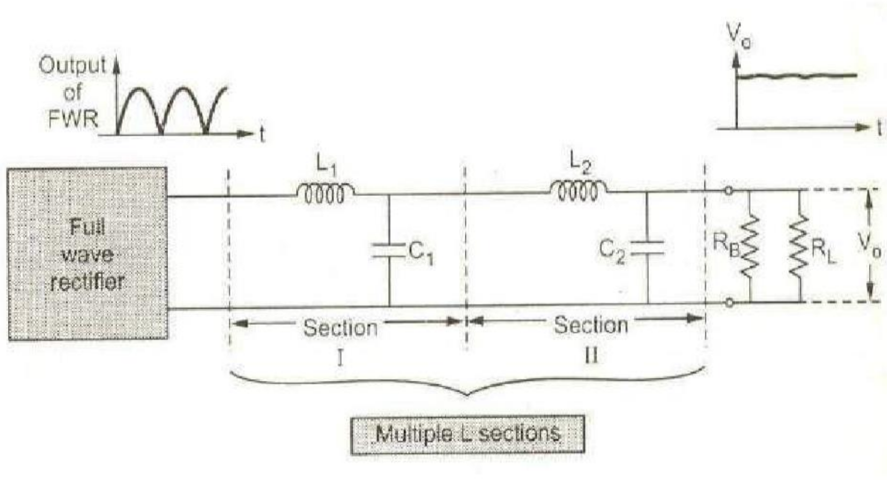


Fig. Multiple L-sections.

For two section LC filter, the ripple factor is given by $\Rightarrow \gamma = \frac{\sqrt{2}}{3} \cdot \frac{X_{C1}}{X_{L1}} \cdot \frac{X_{C}}{X_{L2}}$

CLC Filter (or) Π – section Filter:

This is capacitor input filter followed by a L-section filter. The ripple rejection capability of a Π-section filter is very good. The full-wave rectifier with Π-section filter is shown in the figure.

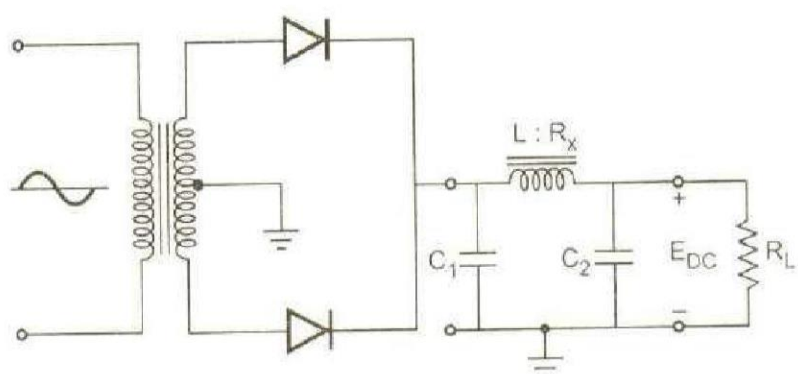


Fig. Π-section Filter.

It consists of an inductance L with a dc winding resistance as R_C and two capacitors C_1 and C_2. The filter circuit is fed from fill wave rectifier. Generally two capacitors are selected equal.

The rectifier output is given to the capacitor c_1. This capacitor offers very low reactance to the ac component but blocks dc component. Hence capacitor C_1 bypasses most of the ac component. The dc component then reaches to the choke L. The choke L offers very high reactance to dc. So it blocks ac component and does not allow it to reach to load while it allows dc component to pass through it. The capacitor C_2 now allows to pass remaining ac component and almost pure dc component reaches to the load. The circuit looks like a Π, hence called Π-Filter.

Ripple Factor:

The Fourier analysis of a triangular wave is given by

$$v = V_{dc} - \frac{V}{\pi}\left(\sin\omega t - \frac{\sin 4\omega t}{2} + \frac{\sin 6\omega t}{3} \ldots\right) \quad \ldots\ldots(1)$$

In case of full wave rectifier with capacitor filter, we have proved that

$$V_{\gamma} = \frac{I_{dc}}{2fC} = \frac{I_{dc}}{2fC_1} \quad (\therefore C = C_1 \; here) \quad \ldots\ldots(2)$$

The rms second harmonic voltage is

$$V_{r(rms)} = \frac{V_r}{\sqrt{2}} \quad \ldots\ldots(3)$$

Substituting the value of V_r from equation (2) in equation (3), we get

$$V_{r(rms)} = \frac{I_{dc}}{2\pi f C_1 \sqrt{2}} = \sqrt{2} I_{dc} XC_1 \quad \ldots\ldots(4)$$

Where $XC_1 = \frac{1}{2\omega C_1} = \frac{1}{4\pi f C_1}$ = reactance of C_1 at second harmonic frequency.

The voltage $V_{r(rms)}$ is impressed on L-section.

Now, the ripple voltage $V'_{r(rms)}$ can be obtained by multiplying $V_{r(rms)}$ by $\frac{XC_2}{X_L}$ i.e.,

$$(V'_r)_{rms} = (V_r)_{rms} \times \frac{XC_1}{X_L}$$

(or) $$(V'_r)_{rms} = \sqrt{2} I_{dc} XC_1 . \frac{XC_2}{X_L} \quad \ldots\ldots(5)$$

$$\therefore \gamma = \frac{(V'_r)_{rms}}{V_{dc}} = \frac{\sqrt{2} I_{dc} XC_1 . \frac{XC_2}{XL}}{V_{dc}}$$

$$\Rightarrow \gamma = \frac{\sqrt{2} . XC_1 . XC_2}{R_L X_L} \qquad \because \frac{I_{dc}}{V_{dc}} = \frac{1}{R_L}$$

$$\therefore \gamma = \frac{\sqrt{2} . XC_1 . XC_2}{R_L X_L}$$

Here all reactances are calculated at second harmonic frequency. Substituting the values,

we get $$\gamma = \frac{\sqrt{2}}{8\omega^3 C_1 C_2 L R_L}$$

At f= 50Hz, $$\gamma = \frac{5700}{LC_1C_2R_L}$$

Where C_1 and C_2 are in μF, L in henrys and R_L in ohms.

Multiple Π-Section Filter:

To obtain almost pure dc to the load, more Π-sections may be used one after another. Such a filter using more than one Π-section is called multiple Π-section filter. The figure shows multiple Π-section filters.

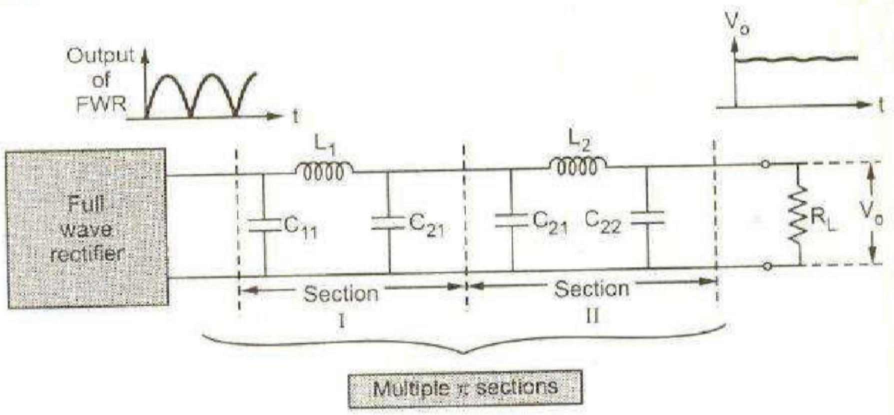


Fig. Multiple Π-section Filter.

The ripple factor of two section Π-filter is given by $\gamma = \sqrt{2}\,\frac{X_{C11}}{R_L}\cdot\frac{X_{C12}}{X_1}\cdot\frac{X_{C22}}{X_2}$

Problems:

14. Design a CLC (or) Π-section filter for V_{dc}=10V, I_L=200mA and γ=2%

Solution:

$$R_L = \frac{V_{dc}}{I_L} = \frac{10}{200\times10^{-3}} = 50\Omega$$

$$\gamma = \frac{5700}{LC_1C_2R_L} \qquad \Rightarrow 0.02 = \frac{5700}{LC_1C_2R_L} = \frac{114}{LC_1C_2}$$

If we assume L=10H and C_1=C_2=C, we have

$$\Rightarrow 0.02 = \frac{114}{LC^2} = \frac{11.4}{C^2}$$

$C^2 = 750 \Rightarrow \sqrt{570}$ = **24μF**

UNIT-II
SPECIAL PURPOSE ELECTONIC DEVICES

ZENER DIODE:

When the reverse voltage reaches breakdown voltage in normal PN junction diode, the current through the junction and the power dissipated at the junction will be high. Such an operation is destructive and the diode gets damaged. Whereas diodes can be designed with adequate power dissipation capabilities to operate in a break down
region. One such a diode is known as Zener diode.

Zener diode is heavily doped than the ordinary diode. Due to this the depletion layer will be very thin and for small applied reverse voltage(V_R) there will be sharp increase in current.

From the V-I characteristics of the Zener diode, shown in figure. It is found that the operation of Zener diode is same as that of ordinary PN diode.
Under forward-biased condition.Whereas under reverse-biased condition, breakdown of the junction occurs. The breakdown voltage depends upon the amount of doping. If the diode is heavily doped, depletion layer will be thin and consequently, breakdown occurs at lower reverse voltage and further, the breakdown voltage is sharp. Whereas a lightly doped diode has a higher breakdown voltage. Thus breakdown voltage can be selected with the amount of doping.

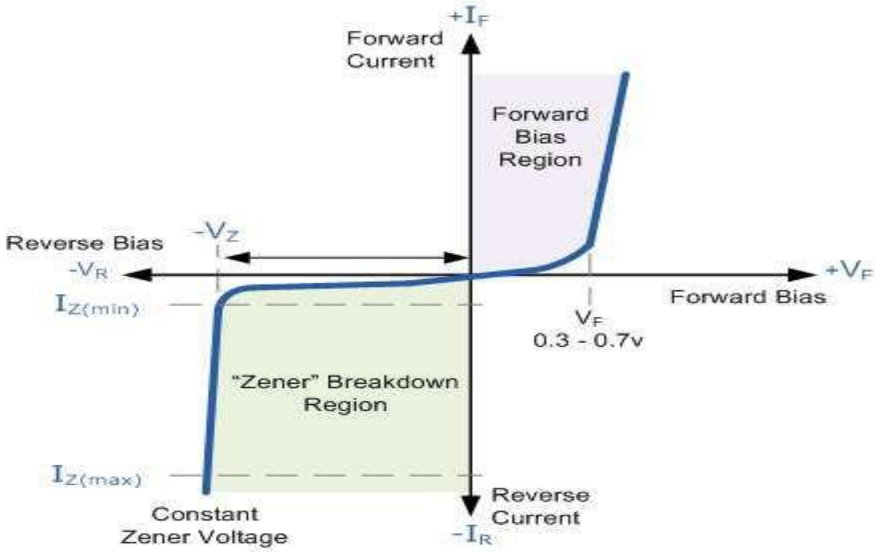


Figure 2.1: V-I Characteristics of a Zener diode

Note:

A heavily doped diode has a low Zener breakdown voltage, while a lightly doped diode has a high Zener breakdown voltage.

ZENER BREAKDOWN MECHANISM:

The sharp increasing current under breakdown conditions is due to the following two mechanisms.
(1) Avalanche breakdown
(2) Zener breakdown

The breakdown in the Zener diode at the voltage V_z may be due to any of the following mechanisms.

1. Avalanche breakdown

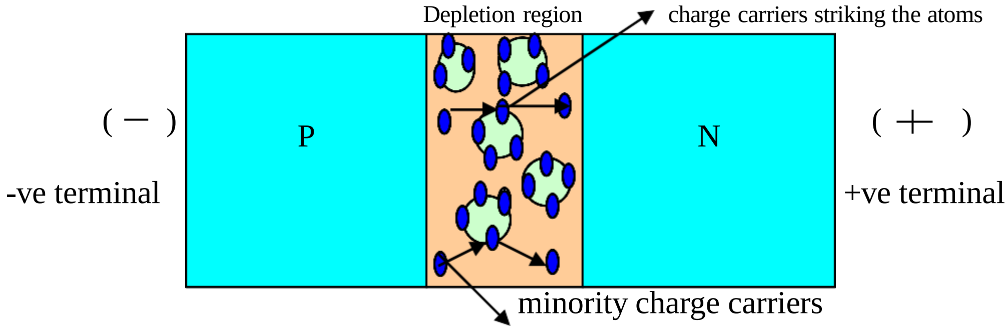
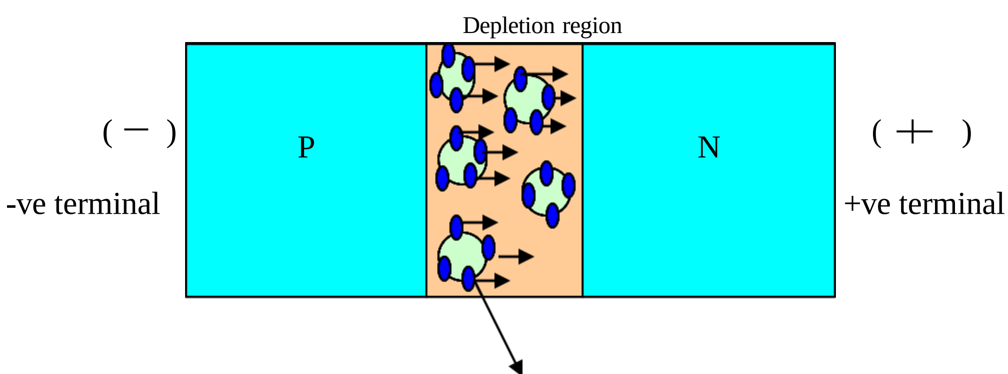


Figure 2.2: Avalanche breakdown in Zener diode

- We know that when the diode is reverse biased a small reverse saturation current I_0 flows across the junction because of the minority cariers in the depletion region.
- The velocity of the minority charge carriers is directly proportional to the applied voltage. Hence when the reverse bias voltage is increased, the velocity of minority charge carriers will also increase and consequently their energy content will also increase.
- When these high energy charge carriers strikes the atom within the depletion region they cause other charge carriers to break away from their atoms and join the flow of current across the junction as shown above. The additional charge carriers generated in this way strikes other atoms and generate new carriers by making them to break away from their atoms.
- This cumulative process is referred to as avalanche multiplication which results in the flow of large reverse current and this breakdown of the diode is called avalanche breakdown.

2. Zener breakdown

We have electric field strength = Reverse voltage/ Depletion region



electrons pulled out of their covalent bonds because of high intensity electric field

Figure 2.3: Zener breakdown in Zener diode

From the above relation we see that the reverse voltage is directly proportional to the electric field hence, a small increase in reverse voltage produces a very high intensity electric field with ina narrow Depletion region.

Therefore when the reverse voltage to a diode is increased, under the influence of high intensity electric filed large numbr of electrons within the depletion region break the covalent bonds with their atoms as shown above and thus a large reverse current flows through the diode. This breakdown is referred to as Zener breakdown.

Zener Diode as Voltage Regulator:

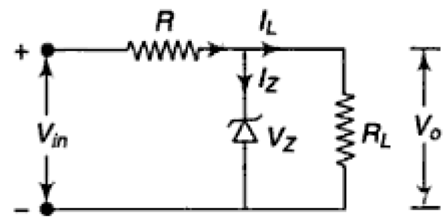


Figure 2.4: Zener diode as voltage regulator

From the Zener Characteristics shown, under reverse bias condition, the voltage across the diode remains constant although the current through the diode increases as shown. Thus the voltage across the zener diode serves as a reference voltage. Hence the diode can be used as a voltage regulator.
It is required to provide constant voltage across load resistance R_L , whereas the input voltage may be varying over a range. As shown, zener diode is reverse biased and as long as the input voltage does not fall below v_z (zener breakdown voltage), the voltage across the diode will be constant and hence the load voltage will also be constant.

Light Emitting Diode:

Light Emitting Diodes or **LED´s**, are among the most widely used of all the different types of semiconductor diodes available today and are commonly used in TV's and colour displays.
They are the most visible type of diode, that emit a fairly narrow bandwidth of either visible light at different coloured wavelengths, invisible infra-red light for remote controls or laser type light when a forward current is passed through them.

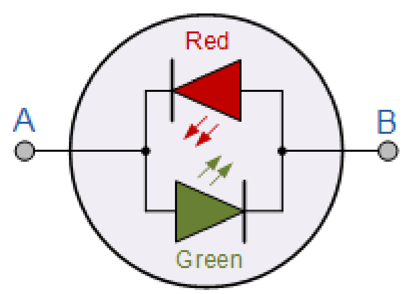


Figure 2.4: The Light Emitting Diode

The "**Light Emitting Diode**" or **LED** as it is more commonly called, is basically just a specialised type of diode as they have very similar electrical characteristics to a PN junction diode. This means that an LED will pass current in its forward direction but block the flow of current in the reverse direction.
Related Products: LEDs and LED Lighting | Optical Lenses
Light emitting diodes are made from a very thin layer of fairly heavily doped semiconductor material and depending on the semiconductor material used and the amount of doping, when forward biased an LED will emit a coloured light at a particular spectral wavelength.
When the diode is forward biased, electrons from the semiconductors conduction band recombine with holes from the valence band releasing sufficient energy to produce photons which emit a monochromatic (single colour) of light. Because of this thin layer a reasonable number of these photons can leave the junction and radiate away producing a coloured light output.

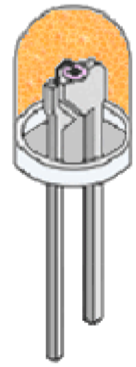


Figure 2.5: LED Construction

Then we can say that when operated in a forward biased direction **Light Emitting Diodes** are semiconductor devices that convert electrical energy into light energy.

The construction of a Light Emitting Diode is very different from that of a normal signal diode. The PN junction of an LED is surrounded by a transparent, hard plastic epoxy resin hemispherical shaped shell or body which protects the LED from both vibration and shock.

Surprisingly, an LED junction does not actually emit that much light so the epoxy resin body is constructed in such a way that the photons of light emitted by the junction are reflected away from the surrounding substrate base to which the diode is attached and are focused upwards through the domed top of the LED, which itself acts like a lens concentrating the amount of light. This is why the emitted light appears to be brightest at the top of the LED.

However, not all LEDs are made with a hemispherical shaped dome for their epoxy shell. Some indication LEDs have a rectangular or cylindrical shaped construction that has a flat surface on top or their body is shaped into a bar or arrow. Generally, all LED's are manufactured with two legs protruding from the bottom of the body.

Also, nearly all modern light emitting diodes have their cathode, (–) terminal identified by either a notch or flat spot on the body or by the cathode lead being shorter than the other as the anode (+) lead is longer than the cathode (k).

Unlike normal incandescent lamps and bulbs which generate large amounts of heat when illuminated, the light emitting diode produces a "cold" generation of light which leads to high efficiencies than the normal "light bulb" because most of the generated energy radiates away within the visible spectrum. Because LEDs are solid-state devices, they can be extremely small and durable and provide much longer lamp life than normal light sources.

Light Emitting Diode Colours:

So how does a light emitting diode get its colour. Unlike normal signal diodes which are made for detection or power rectification, and which are made from either Germanium or Silicon semiconductor materials, **Light Emitting Diodes** are made from exotic semiconductor compounds such as Gallium Arsenide (GaAs), Gallium Phosphide (GaP), Gallium Arsenide Phosphide (GaAsP), Silicon Carbide (SiC) or Gallium Indium Nitride (GaInN) all mixed together at different ratios to produce a distinct wavelength of colour.

Different LED compounds emit light in specific regions of the visible light spectrum and therefore produce different intensity levels. The exact choice of the semiconductor material used will determine the overall wavelength of the photon light emissions and therefore the resulting colour of the light emitted.

Thus, the actual colour of a light emitting diode is determined by the wavelength of the light emitted, which in turn is determined by the actual semiconductor compound used in forming the PN junction during manufacture.

Therefore the colour of the light emitted by an LED is NOT determined by the colouring of the LED's plastic body although these are slightly coloured to both enhance the light output and to indicate its colour when its not being illuminated by an electrical supply.

Light emitting diodes are available in a wide range of colours with the most common being RED, AMBER, YELLOW and GREEN and are thus widely used as visual indicators and as moving light displays.

Recently developed blue and white coloured LEDs are also available but these tend to be much more expensive than the normal standard colours due to the production costs of mixing together two or more complementary colours at an exact ratio within the semiconductor compound and also by injecting nitrogen atoms into the crystal structure during the doping process.

Light Emitting Diode Colours:

From the table shown below we can see that the main P-type dopant used in the manufacture of **Light Emitting Diodes** is Gallium (Ga, atomic number 31) and that the main N-type dopant used is Arsenic (As, atomic number 33) giving the resulting compound of Gallium Arsenide (GaAs) crystalline structure.

The problem with using Gallium Arsenide on its own as the semiconductor compound is that it radiates large amounts of low brightness infra-red radiation (850nm-940nm approx.) from its junction when a forward current is flowing through it.

The amount of infra-red light it produces is okay for television remote controls but not very useful if we want to use the LED as an indicating light. But by adding Phosphorus (P, atomic number 15), as a third dopant the overall wavelength of the emitted radiation is reduced to below 680nm giving visible red light to the human eye. Further refinements in the doping process of the PN junction have resulted in a range of colours spanning the spectrum of visible light as we have seen above as well as infra-red and ultra-violet wavelengths.

Typical LED Characteristics			
Semiconductor Material	Wavelength	Colour	V_F @ 20mA
GaAs	850-940nm	Infra-Red	1.2v
GaAsP	630-660nm	Red	1.8v
GaAsP	605-620nm	Amber	2.0v
GaAsP:N	585-595nm	Yellow	2.2v
AlGaP	550-570nm	Green	3.5v
SiC	430-505nm	Blue	3.6v
GaInN	450nm	White	4.0v

By mixing together a variety of semiconductor, metal and gas compounds the following list of LEDs can be produced.

Liquid crystal display:

We always use devices made up of Liquid Crystal Displays (LCDs) like computers, digital watches and also DVD and CD players. They have become very common and have taken a giant leap in the screen industry by clearly replacing the use of Cathode Ray Tubes (CRT). CRT draws more power than LCD and are also bigger and heavier. All of us have seen an LCD, but no one knows the exact working of it. Let us take a look at the working of an LCD.

Figure 2.6: LCD

The article below is developed as two sections:-

1. Basics of LCD Displays 2. Working Principle of LCD

Note:- If you are looking for a note on technical specifications of LCD Displays for Interfacing it with micro controllers:- here we have a great article on the same:- A Note on Character LCD Display.If you want to know about the invention history of LCD go through the article:- Invention History of Liquid Crystal Display (LCD).

We get the definition of LCD from the name "Liquid Crystal" itself. It is actually a combination of two states of matter – the solid and the liquid. They have both the properties of solids and liquids and maintain their respective states with respect to another. Solids usually maintain their state unlike liquids who change their orientation and move everywhere in the particular liquid. Further studies have showed that liquid crystal materials show more of a liquid state than that of a solid. It must also be noted that liquid crystals are more heat sensitive than usual liquids. A little amount of heat can easily turn the liquid crystal into a liquid. This is the reason why they are also used to make thermometers.

Basics of LCD Displays:

The liquid-crystal display has the distinct advantage of having a low power consumption than the LED. It is typically of the order of microwatts for the display in comparison to the some order of milliwatts for LEDs. Low power consumption requirement has made it compatible with MOS integrated logic circuit. Its other advantages are its low cost, and good contrast. The main drawbacks of LCDs are additional requirement of light source, a limited temperature range of operation (between 0 and 60° C), low reliability, short operating life, poor visibility in low ambient lighting, slow speed and the need for an ac drive.

Basic structure of an LCD

A liquid crystal cell consists of a thin layer (about 10 u m) of a liquid crystal sandwiched between two glass sheets with transparent electrodes deposited on their inside faces. With both glass sheets transparent, the cell is known as transmittive type cell. When one glass is transparent and the other has a reflective coating, the cell is called reflective type. The LCD does not produce any illumination of its own. It, in fact, depends entirely on illumination falling on it from an external source for its visual effect

Types of LCD/Liquid Crystal Displays:

Two types of display available are dynamic scattering display and field effect display.

When dynamic scattering display is energized, the molecules of energized area of the display become turbulent and scatter light in all directions. Consequently, the activated areas take on a frosted glass appearance resulting in a silver display. Of course, the unenergized areas remain translucent.

Field effect LCD contains front and back polarizers at right angles to each other. Without electrical excitation, the light coming through the front polarizer is rotated 90° in the fluid.
Now, let us take a look at the different varieties of liquid crystals that are available for industrial purposes. The most usable liquid crystal among all the others is the nematic phase liquid crystals.

Nematic Phase LCD:

The greatest advantage of a nematic phase liquid crystal substance is that it can bring about predictable controlled changes according to the electric current passed through them. All the liquid crystals are according to their reaction on temperature difference and also the nature of the substance.
Twisted Nematics, a particular nematic substance is twisted naturally. When a known voltage is applied to the substance, it gets untwisted in varying degrees according to our requirement. This in turn is useful in controlling the passage of light. A nematic phase liquid crystal can be again classified on the basis in which the molecules orient themselves in respect to each other. This change in orientation mainly depends on the director, which can be anything ranging from a magnetic field to a surface with microscopic grooves. Classification includes Smectic and also cholesteric. Smectic can be again classified as smectic C, in which the molecules in each layer tilt at an angle from the previous layer. Cholesteric, on the other hand has molecules that twist slightly from one layer to the next, causing a spiral like design. There are also combinations of these two called Ferro-electric liquid crystals (FLC), which include cholesteric molecules in a smectic C type molecule so that the spiral nature of these molecules allows the microsecond switching response time. This makes FLCs to be of good use in advanced displays.
Liquid crystal molecules are further classified into thermotropic and lyotropic crystals. The former changes proportionally with respect to changes in pressure and temperature. They are further divided into nematic and isotropic. Nematic liquid crystals have a fixed order of pattern while isotropic liquid crystals are distributed randomly. The lyotropic crystal depends on the type of solvent they are mixed with. They are therefore useful in making detergents and soaps.

Making of LCD:

- Though the making of LCD is rather simple there are certain facts that should be noted while making it.
- The basic structure of an LCD should be controllably changed with respect to the applied electric current.
- The light that is used on the LCD can be polarized.
- Liquid crystals should be able to both transmit and change polarized light.
- There are transparent substances that can conduct electricity.

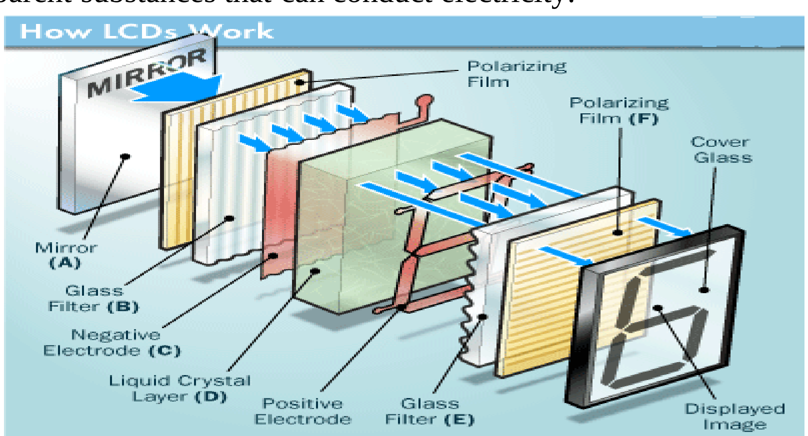


Figure 2.6: Working of LCD

To make an LCD, you need to take two polarized glass pieces. The glass which does not have a polarized film on it must be rubbed with a special polymer which creates microscopic grooves in the surface. It must also be noted that the grooves are on the same direction as the polarizing film. Then, all you need to do is to add a coating of nematic liquid crystals to one of the filters. The grooves will cause the first layer of molecules to align with the filter's orientation. At right angle to the first piece, you must then add a second piece of glass along with the polarizing film. Till the uppermost layer is at a 90-degree angle to the bottom, each successive layer of TN molecules will keep on twisting. The first filter will naturally be polarized as the light strikes it at the beginning. Thus the light passes through each layer and is guided on to the next with the help of molecules. When this happens, the molecules tend to change the plane of vibration of the light to match their own angle. When the light reaches the far side of the liquid crystal substance, it vibrates at the same angle as the final layer of molecules. The light is only allowed an entrance if the second polarized glass filter is same as the final layer. Take a look at the figure above.

Working of LCD:

The main principle behind liquid crystal molecules is that when an electric current is applied to them, they tend to untwist. This causes a change in the light angle passing through them. This causes a change in the angle of the top polarizing filter with respect to it. So little light is allowed to pass through that particular area of LCD. Thus that area becomes darker comparing to others.

For making an LCD screen, a reflective mirror has to be setup in the back. An electrode plane made of indium-tin oxide is kept on top and a glass with a polarizing film is also added on the bottom side. The entire area of the LCD has to be covered by a common electrode and above it should be the liquid crystal substance. Next comes another piece of glass with an electrode in the shape of the rectangle on the bottom and, on top, another polarizing film. It must be noted that both of them are kept at right angles. When there is no current, the light passes through the front of the LCD it will be reflected by the mirror and bounced back. As the electrode is connected to a temporary battery the current from it will cause the liquid crystals between the common-plane electrode and the electrode shaped like a rectangle to untwist. Thus the light is blocked from passing through. Thus that particular rectangular area appears blank.

Photodiode Theory of Operation:

A silicon photodiode is a solid-state device which converts incident light into an electric current. It consists of a shallow diffused p-n junction, normally a p-on-n configuration although "P-type" devices (n-on-p) are available for enhanced responsivity in the 1µm region. Modern day silicon photodiodes are generally made by planar diffusion or ion-implantation methods.

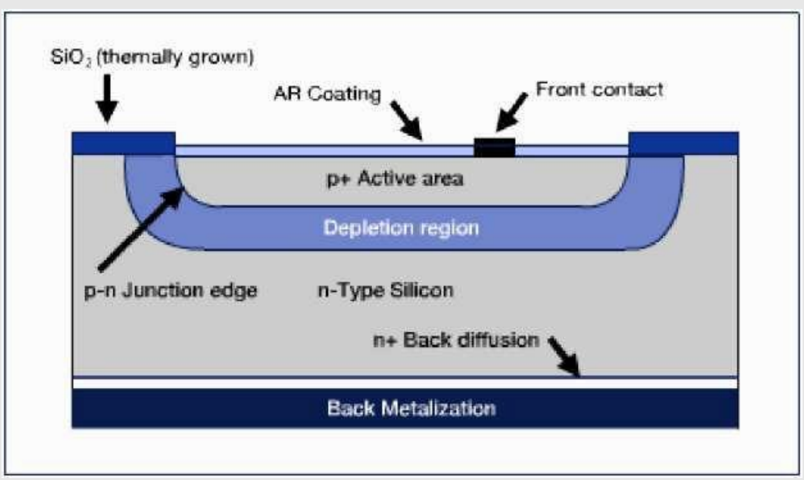


Figure 2.7: Operation of Photodiode

In the p-on-n planar diffused configuration, shown in the figure, the junction edge emerges on the top surface of the silicon chip, where it is passivated by a thermally grown oxide layer.

Basic photodiode physics:

The p-n junction and the depletion region are of major importance to the operation of a photodiode. These photodiode regions are created when the p-type dopant with acceptor impurities (excess holes), comes into contact with the n-type silicon, doped with donor impurities (excess electrons). The holes and the electrons, each experiencing a lower potential on the opposite side of the junction, begin to flow across the junction into their respective lower potential areas. This charge movement establishes a depletion region, which has an electric field opposite and equal to the low potential field, and hence no more current flows.

When photons of energy greater than 1.1eV (the bandgap of silicon) fall on the device, they are absorbed and electron-hole pairs are created. The depth at which the photons are absorbed depends upon their energy; the lower the energy of the photons, the deeper they are absorbed. The electron-hole pairs drift apart, and when the minority carriers reach the junction, they are swept across by the electric field. If the two sides are electrically connected, an external current flows through the connection. If the created minority carriers of that region recombine with the bulk carriers of that region before reaching the junction field, the carriers are lost and no external current flows.

The equivalent circuit of a photodiode is shown in the figure below. The photodiode behaves as a current source when illuminated. When operated without bias, this current is distributed between the internal shunt resistance and external load resistor. In this mode, a voltage develops which creates a forward bias, thus reducing its ability to remain a constant current source. When operated with a reverse voltage bias, the photodiode becomes an ideal current source.

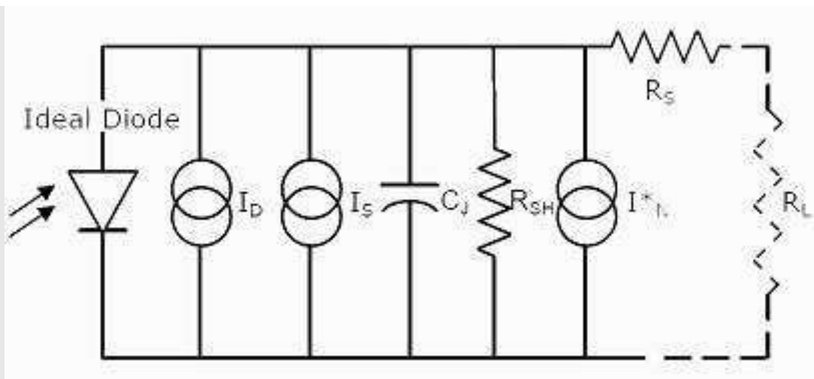


Figure 2.8: Equivalent Circuit of Photodiode

I_D = Dark current, Amps

I_S = Light Signal Current (I_S=RP_O)

R = Photodiode responsivity at wavelength of irradiance, Amps/Watt

P_O = Light power incident on photodiode active area, Watts

R_{SH} = Shunt Resistance, Ohms

I^*_N = Noise Current, Amps rms

C = Junction Capacitance, Farads

R_S = Series Resistance, Ohms

R_L = Load Resistance, Ohms

Silicon photodiodes are typically sensitive to light in the spectral range from about 200 nm (near UV) to about 1100 nm (near IR). Photosensor responsivity (R) is measured in Amperes (A) of photocurrent generated per Watt (W) of incident light power. Actual light levels in most applications typically range from picoWatts to milliWatts, which generate photocurrents from pico-Amps to milli-Amps. Responsivity in Amps/Watt varies with the wavelength of the incident light, with peak values from 0.4 to 0.7 A/W. The silicon photodiode response is well matched to light sources emitting in the UV to near infrared spectrum, such as HeNe lasers; GaAlAs and GaAs LEDs and laser diodes; and Nd:YAG lasers. Select a detector from the IR, Blue/Visible or UV series for a spectral response curve best matched to the spectral irradiance of your light source.

The silicon photodiode response is usually linear within a few tenths of a percent from the minimum detectable incident light power up to several milliWatts. Response linearity improves with increasing applied reverse bias and decreasing effective load resistance.

Heating the silicon photodiode shifts its spectral response curve (including the peak) toward longer wavelengths. Conversely, cooling shifts the response toward shorter wavelengths. The following values are typical for the temperature dependence of responsivity for different wavelength regions:-

UV to 500nm:	-0.1%/°C to -2%/°C
500 to 700nm:	~0%/°C
~900nm:	0.1 %/°C
1064 nm:	0.75%/°C to 0.9%/°C

Modes of operation:

A silicon photodiode can be operated in either the *photovoltaic* or *photoconductive* mode. In the photovoltaic mode, the photodiode is unbiased; while for the photoconductive mode, an external reverse bias is applied. Mode selection depends upon the speed requirements of the application, and the amount of dark current that is tolerable. In the photovoltaic mode, dark current is at a minimum. Photodiodes exhibit their fastest switching speeds when operated in the photoconductive mode.

Photodiodes and Op-Amps can be coupled such that the photodiode operates in a short circuit current mode. The op-amp functions as a simple current to voltage converter.

Varactor Diode:

Varactor Diode is a reverse biased p-n junction diode, whose capacitance can be varied electrically. As a result these diodes are also referred to as varicaps, tuning diodes, voltage variable capacitor diodes, parametric diodes and variable capacitor diodes. It is well known that the operation of the p-n junction depends on the bias applied which can be either forward or reverse in characteristic. It is also observed that the span of the depletion region in the p-n junction decreases as the voltage increases in case of forward bias. On the other hand, the width of the depletion region is seen to increase with an increase in the applied voltage for the reverse bias scenario. Under such condition, the p-n junction can be considered to be analogous to a capacitor (Figure 1) where the p and n layers represent the two plates of the capacitor while the depletion region acts as a

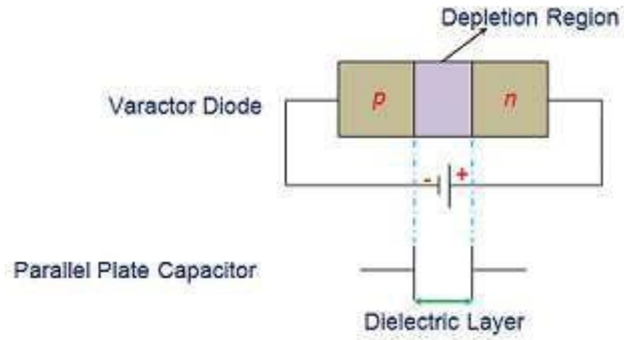


Figure 2.9: Varactor diode as Parallel plate

Dielectric separating them. Thus one can apply the formula used to compute the capacitance of a parallel plate capacitor even to the varactor diode.

Hence, mathematical expression for the capacitance of varactor diode is given by $C_j = \frac{\varepsilon A}{d}$

Where, C_j is the total capacitance of the junction. ε is the permittivity of the semiconductor material. A is the cross-sectional area of the junction. d is the width of the depletion region. Further the relationship between the capacitance and the reverse bias
voltage is given as

$$C_j = \frac{CK}{(V_b - V_R)^m}$$

Where, C_j is the capacitance of the varactor diode. C is the capacitance of the varactor diode when unbiased. K is the constant, often considered to be 1. V_b is the barrier potential. V_R is the applied reverse voltage. m is the material dependent constant. In addition, the electrical circuit equivalent of a **varactor diode** and its symbol are shown by Figure 2. This indicates that the maximum operating frequency of the circuit is dependent on the series resistance (R_s) and the diode capacitance, which can be mathematically given as

$$F = \frac{1}{2\pi R_s C_j}$$

In addition, the quality factor of the varactor diode is given by the equation

$$Q = \frac{F}{f}$$

Where, F and f represent the cut-off frequency and the operating frequency, respectively.

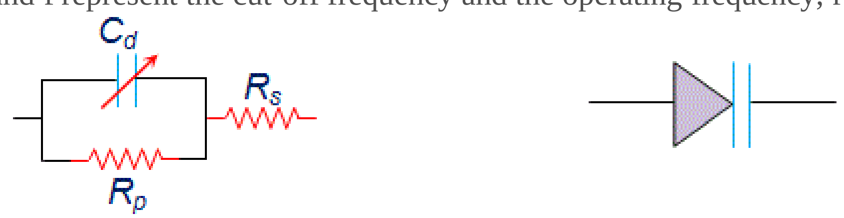


Figure 2.10; (a) Equivalent circuit of varactor diode (b) Symbolic representation

As a result, one can conclude that the capacitance of the varactor diode can be varied by varying the magnitude of the reverse bias voltage as it varies the width of the depletion region, d. Also it is evident from the capacitance equation that d is inversely proportional to C. This means that the junction capacitance of the **varactor diode** decreases with an increase in the depletion region width caused to due to an increase in the reverse bias voltage (V_R), as shown by the graph in Figure 3. Meanwhile it is important to note that although all the diodes exhibit the similar property, varactor diodes are specially manufactured to achieve the objective.

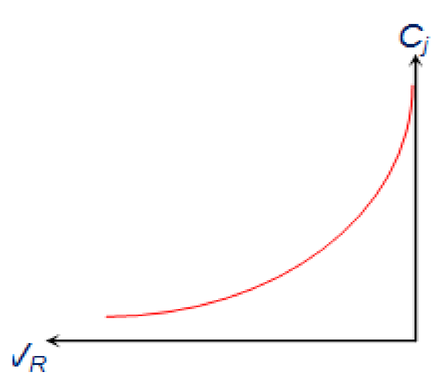


Figure 2.11: Characteristics of Varactor diode

In other words varactor diodes are manufactured with an intention to obtain a definite C-V curve which can be accomplished by controlling the level of doping during the process of manufacture. Depending on this, varactor diodes can be classified into two types viz., abrupt varactor diodes and hyper-abrupt varactor diodes, depending on whether the p-n junction diode is linearly or non-linearly doped (respectively).

These varactor diodes are advantageous as they are compact in size, economical, reliable and less prone to noise when compared to other diodes. Hence, they are used in

1. Tuning circuits to replace the old style variable capacitor tuning of FM radio
2. Small remote control circuits
3. Tank circuits of receiver or transmitter for auto-tuning as in case of TV
4. Signal modulation and demodulation.
5. Microwave frequency multipliers as a component of LC resonant circuit
6. Very low noise microwave parametric amplifiers
7. AFC circuits
8. Adjusting bridge circuits
9. Adjustable bandpass filters
10. Voltage Controlled Oscillators (VCOs)
11. RF phase shifters
12. Frequency multipliers

Tunnel Diode:

The application of transistors is very high in frequency range are hampered due to the transit time and other effects. Many devices use the negative conductance property of semiconductors for high frequency applications. Tunnel diode is one of the most commonly used negative conductance devices. It is also known as Esaki diode after L. Esaki for his work on this effect. This diode is a two terminal device. The concentration of dopants in both p and n region is very high. It is about 10^{24} - 10^{25} m^{-3} the p-n junction is also abrupt. For this reasons, the depletion layer width is very small. In the current voltage characteristics of tunnel diode, we can find a negative slope region when forward bias is applied. Quantum mechanical tunneling is responsible for the phenomenon and thus this device is named as tunnel diode.

The doping is very high so at absolute zero temperature the Fermi levels lies within the bias of the semiconductors. When no bias is applied any current flows through the junction.

Characteristics of Tunnel Diode

When reverse bias is applied the Fermi level of p - side becomes higher than the Fermi level of n-side. Hence, the tunneling of electrons from the balance band of p-side to the conduction band of n-side takes place. With the interments of the reverse bias the tunnel current also increases. When forward junction is a applied the Fermi level of n - side becomes higher that the Fermi level of p - side thus the tunneling of electrons from the n - side to p - side takes place. The amount of the tunnel current is very large than the normal junction current. When the forward bias is increased, the tunnel current is increased up to certain limit. When the band edge of n - side is same with the Fermi level in p - side the tunnel current is maximum with the further increment in the forward bias the tunnel current decreases and we get the desired negative conduction region. When the forward bias is raised further, normal p-n junction current is obtained which is exponentially proportional to the applied voltage. The V - I characteristics of the tunnel diode is given,

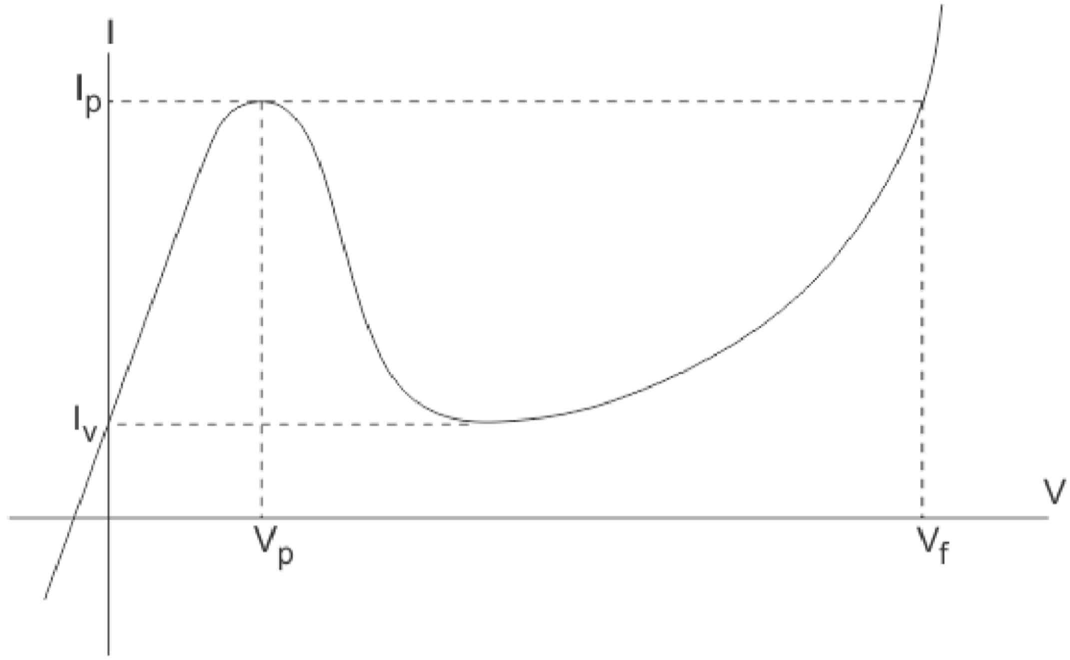


Figure 2.12: Characteristics of Tunnel diode

The negative resistance is used to achieve oscillation and often Ck+ function is of very high frequency frequencies.

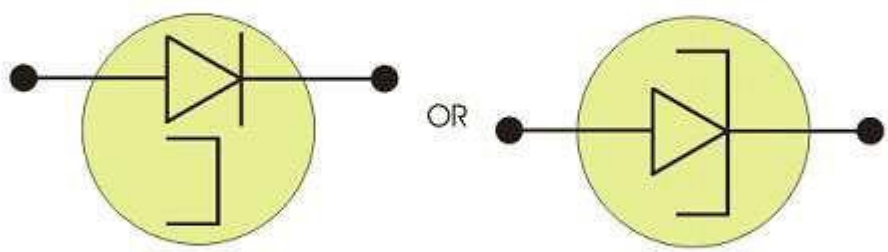


Figure 2.13: Symbol of Tunnel diode

Applications of Tunnel Diode:

Tunnel diode is a type of sc diode which is capable of very fast and in microwave frequency range. It was the quantum mechanical effect which is known as tunneling. It is ideal for fast oscillators and receivers for its negative slope characteristics. But it cannot be used in large integrated circuits – that's why it's an applications are limited.

SCR (Silicon Controlled Rectifier):

It is a four layered PNPN device and is a prominent member of thyristor family. It consists of three diodes connected back to back with gate connection or two complementary transistors connected back to back. It is widely used as switching device in power control applications. It can switch ON for variable length of time and delivers selected amount of power to load. It can control loads, by switching the current OFF and ON up to many thousand times a second. Hence it posses advantages of RHEOSTAT and a switch with none of their disadvantages.

Construction:

As shown in figure it is a four layered three terminal device. Layers being alternately P-type and N-type silicon. Junctions are marked $J_1J_2J_3$. Whereas terminals are anode (A), cathode (C) and gate (G). The gate terminal is connected to inner P-type layer and it controls the firing or switching of SCR.

Biasing:

The biasing of SCR is shown in figure . The junction J_1 and J_3 become forward biased while J_2 is reverse biased. In figure polarity is reversed. It is seen that now junction J_1 and J_3 become reverse biased and only J_2 is forward biased

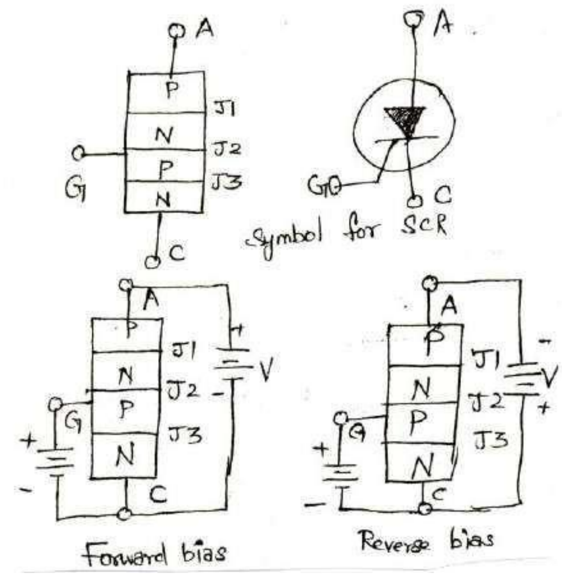


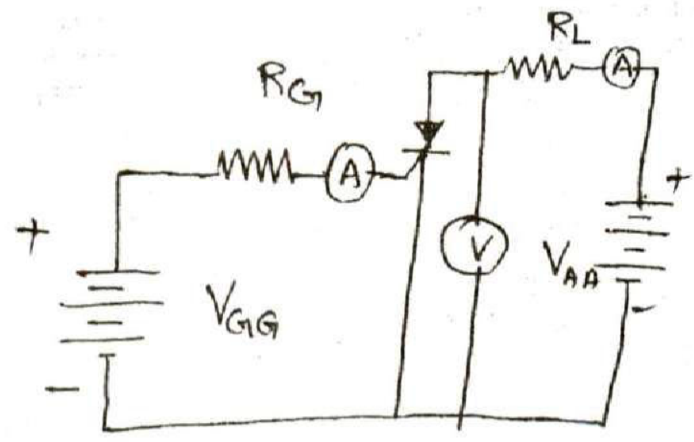
Figure 2.14: Circuit and Symbol of SCR

Operation of SCR:

- In SCR a load is connected in series with anode and is kept at positive potential with respect to cathode when the gate is open ile., no voltage is applied at the gate. Under this condition, junctions J_1 and J_3 are forward biased and junction J_2 is reverse biased. Due to this, no current flows through R_L and hence the SCR is cut off.
- However when the anode voltage is increased gradually to break over voltage, then breakdown occurs at junction J_2 due to this charge carriers are able to flow from cathode to anode easily, hence SCR starts conducting and is said to be in **ON state.** The SCR offers very small forward resistance so that it allow infinitely high current. The current flowing thorough the SCR is limited only by the anode voltage and external resistance.
- If the battery connections of the applied voltage are reversed as shown in figure the junction J_1 and J_3 are reverse biased. J_2 is forward biased. If the applied reverse voltage is small the **SCR is OFF** and hence no current flows through the device. If the reverse voltage is increased to reverse breakdown voltage, the junction J_1 and J_3 will breakdown due to avalanche effect. This causes current to flow through the SCR.
- From the above discussion we conclude that the SCR can be used to conduct only in forward direction. Therefore SCR is called as **"unidirectional device".**

V-I **Characteristics of SCR:**

The "**forward characteristics'** of SCR may be obtained using the figure. The volt-ampere characteristics of a SCR for I_G =0 is shown in figure.



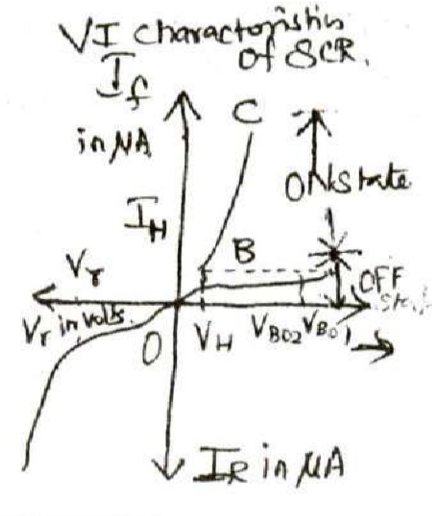


Figure 2.15: Circuit for V-I characteristics of SCR

V_r in volts, V_{B01} for $I_0 = 0$, V_{B02} for I_G=1ma

1. As the applied anode to cathode voltage is increased above zero, a very small current flows through the device, under this condition the SCR is off. It will be continued until, the applied voltage the forward break over voltage (point A).
2. If the anode- cathode (applied) voltage exceeds the break over voltage it conducts heavily the SCR turn ON and anode to cathode voltage decreases quickly to a point B because, under this condition the SCR offers very low resistance hence it drops very low voltage across it.
3. At this stage is SCR allows more current to flow through it. The amplitude of the current is depending upon the supply voltage and load resistance connected in the circuit.
4. The current corresponding to the point ‘B’ is called the **“holding current (I_H)”.** It can be defined as the minimum value of anode current required to keep the SCR in ON State. If the SCR falls below this holding current the SCR turns OFF.
5. If the value of the gate current I_G is increased above zero. (I_G >0) the SCR turns ON even at lower break over voltage as shown in figure.
6. The region lying between the points OA is called **forward blocking region.** In this region SCR is ON.
7. Once the SCR is switched ON then the gate looses all the control. So SCR cannot be turned OFF by varying the gate voltage. It is possible only by reducing the applied voltage.

To obtain the **“reverse characteristics”** the following points are followed.

1. In this case the SCR is reverse biased. If the applied reverse voltage is increased above zero, hence a very small current flows through the SCR. Under this condition the SCR is OFF, it continues till the applied reverse voltage reaches breakdown voltage.
2. As the applied reverse voltage is increased above the breakdown voltage, the avalanche breakdown occurs hence SCR starts conducting in the reverse direction. It is shown in curve DE. Suppose the applied voltage is increased to a very high value, the device may get damaged.

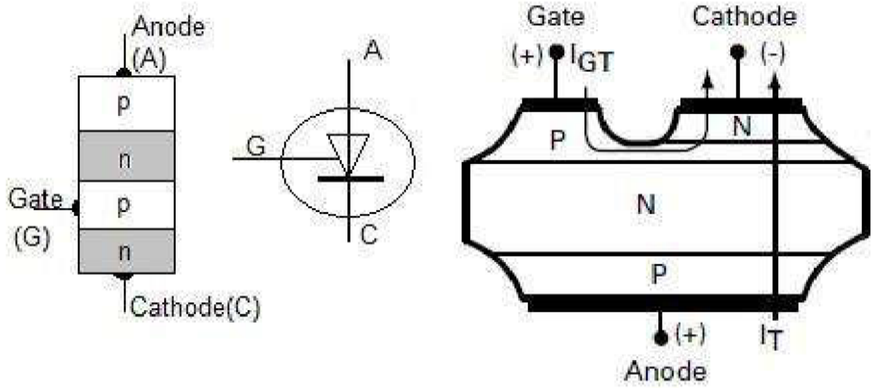


Figure 2.16: Structure and symbol of SCR

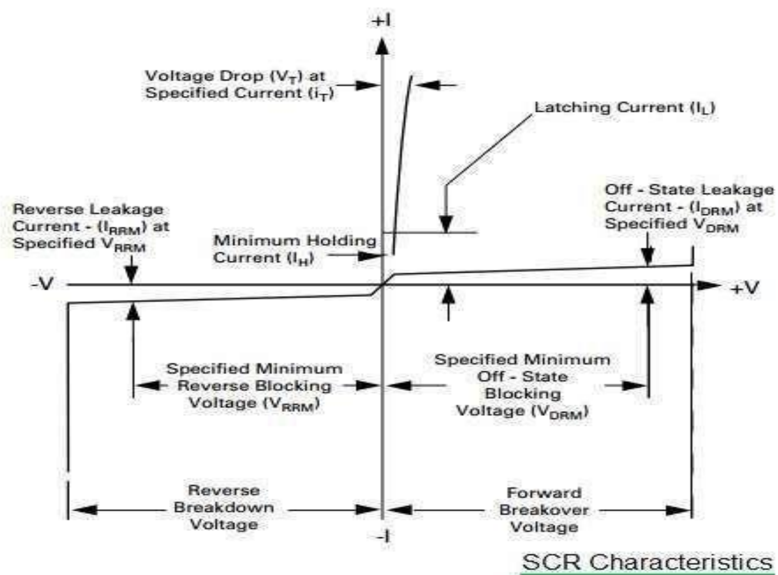
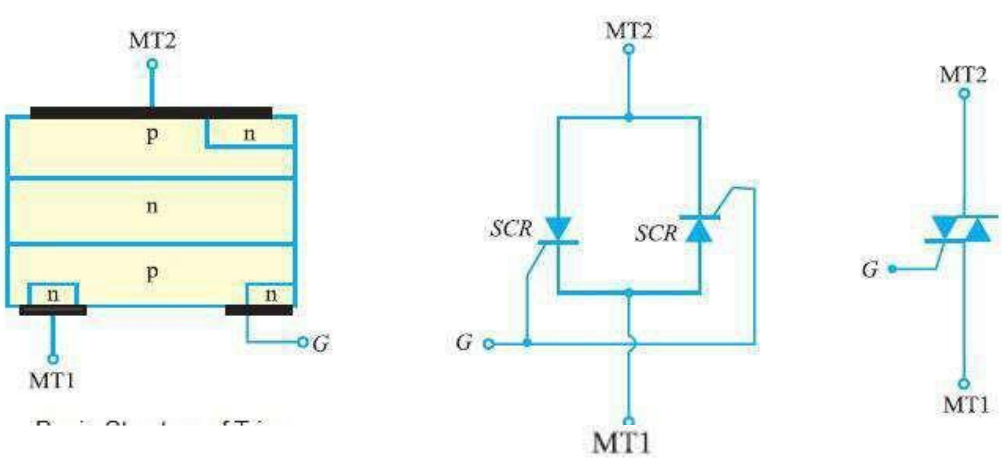


Figure 2.17: Characteristics of SCR

The full form of SCR is Silicon Controlled Rectifier.

- It is a three terminal device.
- It has 4 layers of semiconductor.
- It is a unidirectional switch. It conducts current only in one direction. Hence it can control DC power only OR it can control forward biased half cycle of AC input in the load.
- Basically SCR can only control either positive or negative half cycle of AC input.



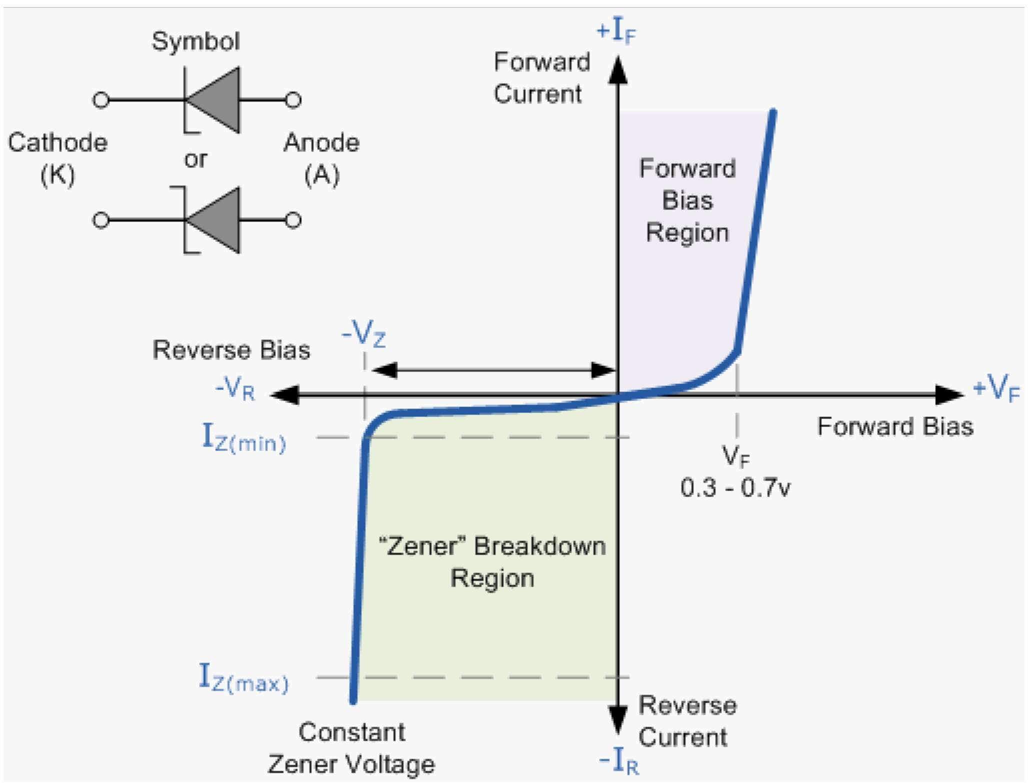
ZENER DIODES

The **Zener diode** is like a general-purpose signal diode consisting of a silicon PN junction. When biased in the forward direction it behaves just like a normal signal diode passing the rated current, but as soon as a reverse voltage applied across the zener diode exceeds the rated voltage of the device, the diodes breakdown voltage $V_B$is reached at which point a process called *Avalanche Breakdown* occurs in the semiconductor depletion layer and a current starts to flow through the diode to limit this increase in voltage.

The current now flowing through the zener diode increases dramatically to the maximum circuit value (which is usually limited by a series resistor) and once achived this reverse saturation current remains fairly constant over a wide range of applied voltages. This breakdown voltage point, $V_B$is called the "zener voltage" for zener diodes and can range from less than one volt to hundreds of volts.

The point at which the zener voltage triggers the current to flow through the diode can be very accurately controlled (to less than 1% tolerance) in the doping stage of the diodes semiconductor construction giving the diode a specific *zener breakdown voltage*, (V_Z) for example, 4.3V or 7.5V. This zener breakdown voltage on the I-V curve is almost a vertical straight line.

Zener Diode I-V Characteristics



The **Zener Diode** is used in its "reverse bias" or reverse breakdown mode, i.e. the diodes anode connects to the negative supply. From the I-V characteristics curve above, we can see that the zener diode has a region in its reverse bias characteristics of almost a constant negative voltage regardless of the value of the current flowing through the diode and remains nearly constant even with large changes in current as long as the zener diodes current remains between the breakdown current $I_{Z(min)}$and the maximum current rating$I_{Z(max)}$.

This ability to control itself can be used to great effect to regulate or stabilise a voltage source against supply or load variations. The fact that the voltage across the diode in the breakdown region is almost constant turns out to be an important application of the zener diode as a voltage regulator. The function of a regulator is to provide a constant output voltage to a load connected in parallel with it in spite of the ripples in the supply voltage or the variation in the load current and the zener diode will continue to regulate the voltage until the diodes current falls below the minimum $I_{Z(min)}$value in the reverse breakdown region.

TUNNELDIODE:

A **tunnel diode** or **Esaki diode** is a type of semiconductor diode which is capable of very fast operation, well into the microwave frequency region, by using quantum mechanical effects.

It was invented in August 1957 by Leo Esaki when he was with Tokyo Tsushin Kogyo, now known as Sony. In 1973 he received the Nobel Prize in Physics, jointly with Brian Josephson, for discovering the electron tunneling effect used in these diodes. Robert Noyce independently came up with the idea of a tunnel diode while working for William Shockley, but was discouraged from pursuing it.

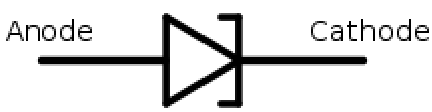


Fig:Tunnel diode schematic symbol

These diodes have a heavily dopedp–n junction only some 10 nm (100 Å) wide. The heavy doping results in a broken bandgap, where conduction band electron states on the n-side are more or less aligned with valence band hole states on the p-side.

Tunnel diodes were manufactured by Sony for the first time in 1957 followed by GeneralElectric and other companies from about 1960, and are still made in low volume today. Tunnel diodes are usually made from germanium, but can also be made in gallium arsenide and silicon materials. They can be used as oscillators, amplifiers, frequency converters anddetectors.

Tunnelling Phenomenon:

In a conventional semiconductor diode, conduction takes place while the p–n junction is forward biased and blocks current flow when the junction is reverse biased. This occurs up to a point known as the “reverse breakdown voltage” when conduction begins (often accompanied by destruction of the device). In the tunnel diode, the dopant concentration in the p and n layers are increased to the point where the **reverse breakdown voltage** becomes **zero** and the diode conducts in the reverse direction.

However, when forward-biased, an odd effect occurs called "quantum mechanical tunnelling" which gives rise to a region where an *increase* in forward voltage is accompanied by a *decrease* in forward current. This negative resistance region can be exploited in a solid state version of the dynatron oscillator which normally uses a tetrodethermionic valve (or tube).

Forward bias operation

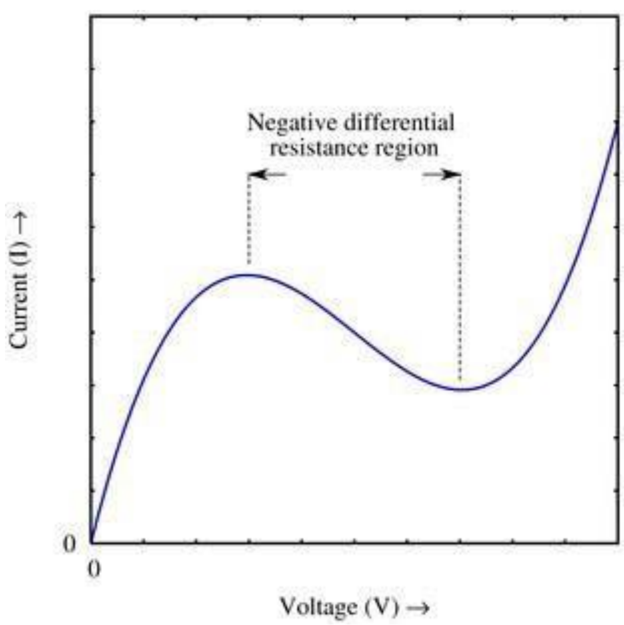
Under normal forward bias operation, as voltage begins to increase, electrons at first tunnel through the very narrow p–n junction barrier because filled electron states in the conduction band on the n-side become aligned with empty valence band hole states on the p-side of the p-n junction. As voltage increases further these states become more misaligned and the current drops – this is called *negative resistance* because current decreases with increasing voltage. As voltage increases yet further, the diode begins to operate as a normal diode, where electrons travel by conduction across the p–n junction, and no longer by tunneling through the p–n junction barrier. Thus the most important operating region for a tunnel diode is the negative resistance region.

Reverse bias operation

When used in the reverse direction they are called **back diodes** and can act as fast rectifiers with zero offset voltage and extreme linearity for power signals (they have an accurate square law characteristic in the reverse direction).

Under reverse bias filled states on the p-side become increasingly aligned with empty states on the n-side and electrons now tunnel through the pn junction barrier in reverse direction – this is the Zener effect that also occurs in zener diodes.

Technical comparisons



A rough approximation of the VI curve for a tunnel diode, showing the negative differential resistance region.The Japanese physicist Leo Esaki invented the tunnel diode in 1958.It consists of a p-n junction with highly doped regions. Because of the thinness of the junction, the electrons can pass through the potential barrier of the dam layer at a suitable polarization, reaching the energy states on the other sides of the junction. The current-voltage characteristic of the diode is represented in Figure 1. In this sketch *i p* and *Up* are the peak, and *iv* and *Uv* are the valley values for the current and voltage respectively. The form of this dependence can be qualitatively explained by considering the tunneling processes that take place in a thin p-njunction.

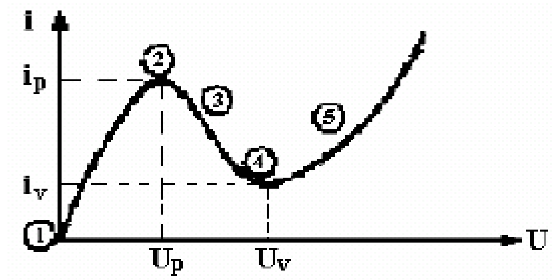


Figure 1.

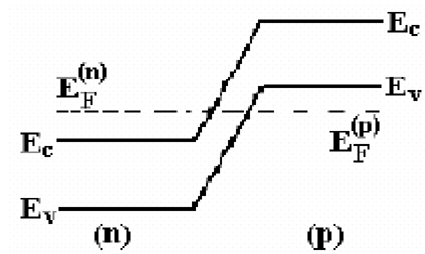


Figure 2.

Energy band structure of tunnel diode:

For the degenerated semiconductors, the energy band diagram at thermal equilibrium is presented in Figure2.

In Figure 3 the tunneling processes in different points of the current voltage characteristic for the tunnel diode are presented.

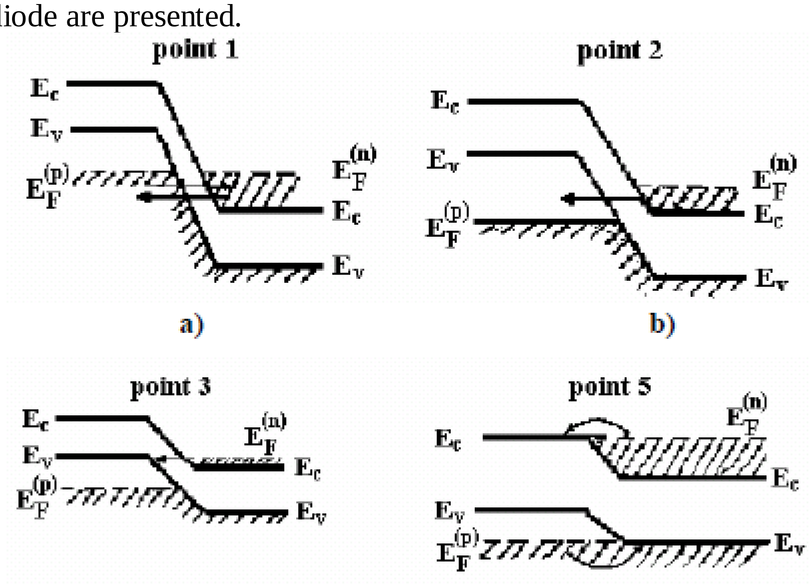


Figure 3.

In Fig. 3a, the thermal equilibrium situation corresponding to point 1

from the Fig. 1 diagram presented; in this case the electrons will uniformly tunnel in both directions, so the current will be null. At a direct polarization, a non-zero electron flow will tunnel from the occupied states of the conduction band of the n region to the emptystates of the valence band from the p region. The current attains a maximum when the overlap of the empty and occupied states reaches the maximum value; a minimum value is reached when there are no states for tunneling on the sides of the barrier. In this case, the tunnel current should drop to zero.

Advantages of tunnel diodes:

- Environmental immunity i.e peak point is not a function oftemperature.
- lowcost.
- low noise.
- low powerconsumption.
- High speed i.e tunneling takes place very fast at the speed of light in the order of nanoseconds
- simplicity i.e a tunnel diode can be used along with a d.c supply and a few passive elements to obtain various applicationcircuits.

Applications for tunnel diodes:

- local oscillators for UHF televisiontuners
- Trigger circuits inoscilloscopes
- High speed counter circuits and very fast-rise time pulse generatorcircuits
- The tunnel diode can also be used as low-noise microwaveamplifier.

VARACTORDIODE:

Varactor diode is a special type of diode which uses transition capacitance property i.e voltage variable capacitance .These are also called as varicap,VVC(voltage variable capacitance) or tuning diodes.

The varactor diode symbol is shown below with a diagram representation.

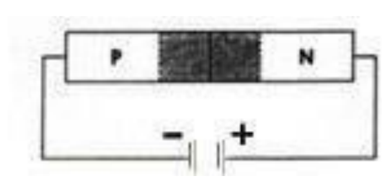


Fig:symbol of varactor diode

When a reverse voltage is applied to a PN junction , the holes in the p-region are attracted to the anode terminal and electrons in the n-region are attracted to the cathode terminal creating a region where there is little current.This region ,the depletion region, is essentially devoid of carriers and behaves as the dielectric of a capacitor.

The depletion region increases as reverse voltage across it increases; and since capacitance varies inversely as dielectric thickness, the junction capacitance will decrease as the voltage across the PN junction increases. So by varying the reverse voltage across a PN junction the junction capacitance can be varied .This is shown in the typical varactor voltage-capacitance curve below.

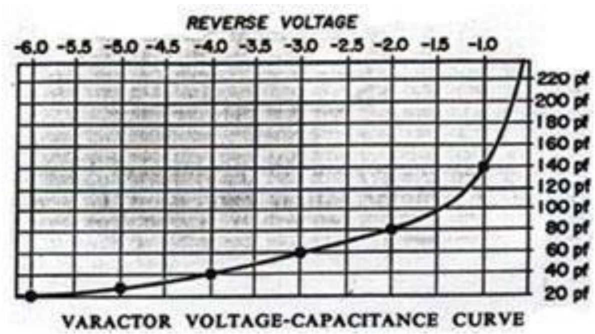


Fig:voltage- capacitance curve

Notice the nonlinear increase in capacitance as the reverse voltage is decreased. This nonlinearity allows the varactor to be used also as a harmonic generator.

Major varactor considerations are:
(a) Capacitancevalue
(b) Voltage
(c) Variation in capacitance withvoltage.
(d) Maximum workingvoltage
(e) Leakage current

Applications:

- Tunedcircuits.
- FMmodulators
- Automatic frequency controldevices
- Adjustable bandpassfilters
- Parametric amplifiers
- Televisionreceivers.

PRINCIPLE OF OPERATION OFSCR

A **silicon-controlled rectifier** (or **semiconductor-controlled rectifier**) is a four-layer solid state device that controls current. The name "silicon controlled rectifier" or **SCR** is General Electric's trade name for a type of thyristor. The SCR was developed by a team of power engineers led by Gordon Hall and commercialized by Frank W. "Bill" Gutzwiller in 1957.symbol of SCR is given below:

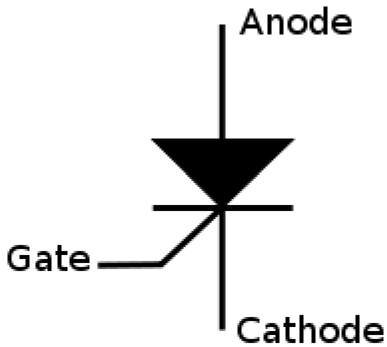


Fig : symbol of SCR

Construction of SCR

An SCR consists of four layers of alternating P and N type semiconductor materials. Silicon is used as the intrinsic semiconductor, to which the proper dopants are added. The junctions are either diffused or alloyed. The planar construction is used for low power SCRs (and all the junctions are diffused). The mesa type construction is used for high power SCRs. In this case, junction J2 is obtained by the diffusion method and then the outer two layers are alloyed to it, since the PNPN pellet is required to handle large currents. It is properly braced with tungsten or molybdenum plates to provide greater mechanical strength. One of these plates is hard soldered to a copper stud, which is threaded for attachment of heat sink. The doping of PNPN will depend ontheapplicationofSCR,sinceitscharacteristicsaresimilartothoseofthethyratron.Today, the term thyristor applies to the larger family of multilayer devices that exhibit bistable state-change behaviour, that is, switching either ON or OFF.

The operation of a SCR and other thyristors can be understood in terms of a pair of tightly coupled bipolar junction transistors, arranged to cause the self-latching action.The following figures are construction of SCR,its two transistor model and symbol respectively

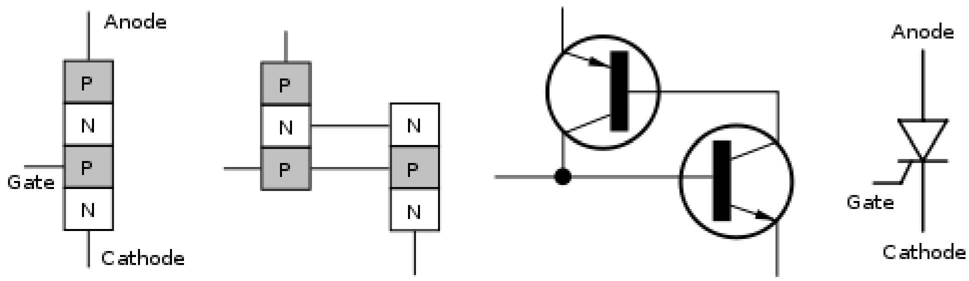
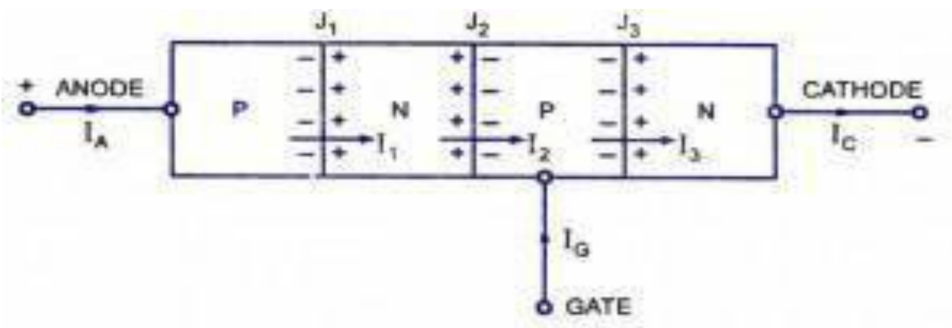


Fig: Construction,Two transistor model of SCR and symbol of SCR

. SCR Working Principle



Diagrammatic Representation Showing Current Flow and Voltage Bias in An SCR

The **SCR** is a four-layer, three-junction and a three-terminal device and is shown in fig.a. The end P-region is the anode, the end N-region is the cathode and the inner P-region is the gate. The anode to cathode is connected in series with the load circuit. Essentially the device is a switch. Ideally it remains off (voltage blocking state), or appears to have an infinite impedance until both the anode and gate terminals have suitable positive voltages with respect to the cathode terminal. The thyristor then switches on and current flows and continues to conduct without further gate signals. Ideally the thyristor has zero impedance in conduction state. For switching off or reverting to the blocking state, there must be no gate signal and the anode current must be reduced to zero. Current can flow only in onedirection.

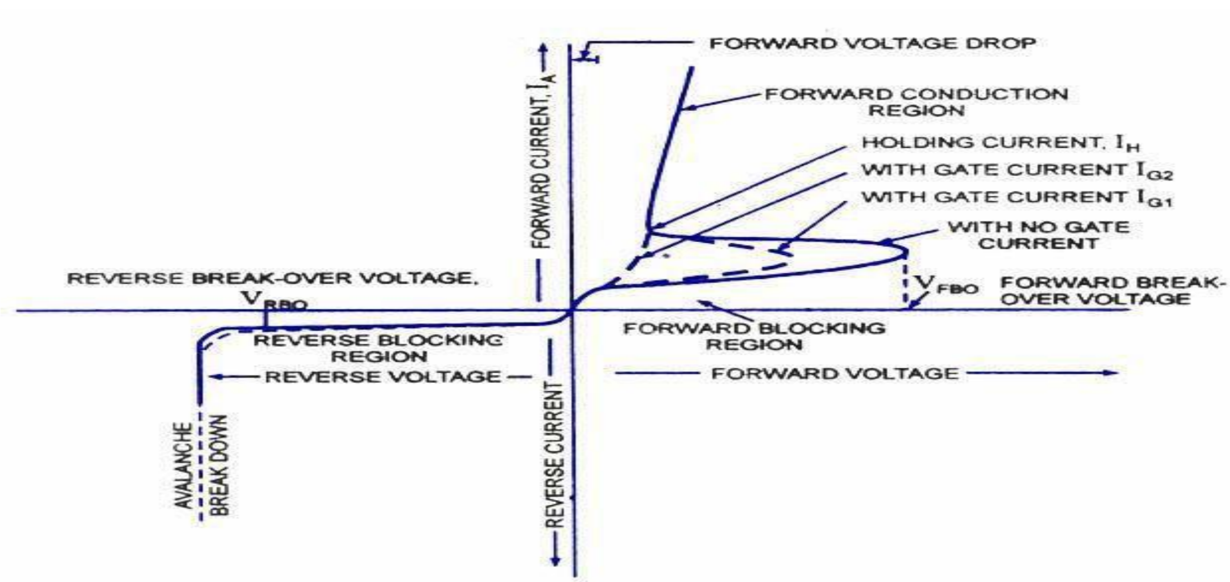
In absence of external bias voltages, the majority carrier in each layer diffuses until there is a built-in voltage that retards further diffusion. Some majority carriers have enough energy to cross the barrier caused by the retarding electric field at each junction. These carriers then become minority carriers and can recombine with majority carriers. Minority carriers in each layer can be accelerated across each junction by the fixed field, but because of absence of external circuit in this case the sum of majority and minority carrier currents must be zero.

A voltage bias, as shown in figure, and an external circuit to carry current allow internal currents which include the following terms:

The current I_x is due to

- Majority carriers (holes) crossing junctionJ_1
- Minority carriers crossing junctionJ_1
- Holes injected at junction J_2 diffusing through the N-region and crossing junction $J_1$and
- Minority carriers from junction J_2 diffusing through the N-region and crossing junction J_1.

V I characteristics of SCR:



V-I Characteristics of SCR

As already mentioned, the **SCR** is a four-layer device with three terminals, namely, the anode, the cathode and the gate. When the anode is made positive with respect to the cathode, junctions J_1 and J_3 are forward biased and junction $J_2$is reverse-biased and only the leakage current will flow through the device. The SCR is then said to be in the forward blocking state or in the forwardmodeoroffstate.Butwhenthecathodeismadepositivewithrespecttotheanode, junctions J_1 and J_3 are reverse-biased, a small reverse leakage current will flow through the SCR and the SGR is said to be in the reverse blocking state or in reverse mode.

When the anode is positive with respect to cathode i.e. when the SCR is in forward mode, the SCR does not conduct unless the forward voltage exceeds certain value, called the forward breakover voltage, V_{FB0}. In non-conducting state, the current through the SCR is the leakage current which is very small and is negligible. If a positive gate current is supplied, the SCR can become conducting at a voltage much lesser than forward break-over voltage. The larger the gate current, lower the break-over voltage. With sufficiently large gate current, the SCR behaves identical to PN rectifier. Once the SCR is switched on, the forward voltage drop across it is suddenly reduced to very small value, say about 1 volt. In the conducting or on-state, the current through the SCR is limited by the external impedance.

When the anode is negative with respect to cathode, that is when the SCR is in reverse mode or in blocking state no current flows through the SCR except very small leakage current of the order of few micro-amperes. But if the reverse voltage is increased beyond a certain value, called the reverse break-over voltage, V_{RB0}avalanche break down takes place. Forward break-over voltage $V_{FB0}$is usually higher than reverse breakovervoltage,$\mathbf{V_{RBO}}$.

From the foregoing discussion, it can be seen that the SCR has two stable and reversible operating states. The change over from off-state to on-state, called turn-on, can be achieved by increasing the forward voltage beyond $\mathbf{V_{FB0}}$. A more convenient and useful method of turn-on the device employs the gate drive. If the forward voltage is less than the forward break-over voltage, V_{FB0}, it can be turned-on by applying a positive voltage between the gate and the cathode. This method is called the gate control. Another very important feature of the gate is that once the SCR is triggered to on-state the gate loses itscontrol.

The switching action of gate takes place only when

(i) SCR is forward biased i.e. anode is positive with respect to cathode,and

(ii) Suitable positive voltage is applied between the gate and thecathode.

Once the SCR has been switched on, it has no control on the amount of current flowing through it. The current through the SCR is entirely controlled by the external impedance connected in the circuit and the applied voltage. There is, however, a very small, about 1 V, potential drop across the SCR. The forward current through the SCR can be reduced by reducing the applied voltage or by increasing the circuit impedance. There is, however, a minimum forward current that must be maintained to keep the SCR in conducting state. This is called the holding current rating of SCR. If the current through the SCR is reduced below the level of holding current, the device returns to off-state or blockingstate.

The SCR can be switched off by reducing the forward current below the level of holding current which may be done either by reducing the applied voltage or by increasing the circuitimpedance.

Note : The gate can only trigger or switch-on the SCR, it cannot switch off.

Alternatively the SCR can be switched off by applying negative voltage to the anode (reverse mode), the SCR naturally will be switched off.

Here one point is worth mentioning, the SCR takes certain time to switch off. The time, called the turn-off time, must be allowed before forward voltage may be applied again otherwise the device will switch-on with forward voltage without any gate pulse. The turn-off time is about 15 micro-seconds, which is immaterial when dealing with power frequency, but this becomes important in the inverter circuits, which are to operate at highfrequency.

Merits of SCR

1.Very small amount of gate drive isrequired.
2.SCRs with high voltage and current ratings areavailable.
3.On state losses of SCR areless.

Demerits of SCR

1.Gate has no control,once SCR is turned on.
2.External circuits are required for turning it off.
3.Operationg frequencies are low.
4.Additional protection circuits are required.

Application of SCRs

SCRs are mainly used in devices where the control of high power, possibly coupled with high voltage, is demanded. Their operation makes them suitable for use in medium to high-voltage AC power control applications, such as lamp dimming, regulators and motorcontrol.

SCRs and similar devices are used for rectification of high power AC in high-voltage directcurrent power transmission

PHOTODIODE:

The photo diode is a semiconductor p-n junction device whose region of operation is limited to the reverse biased region.The figure below shows the symbol of photodiode

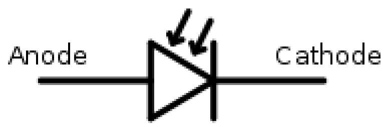
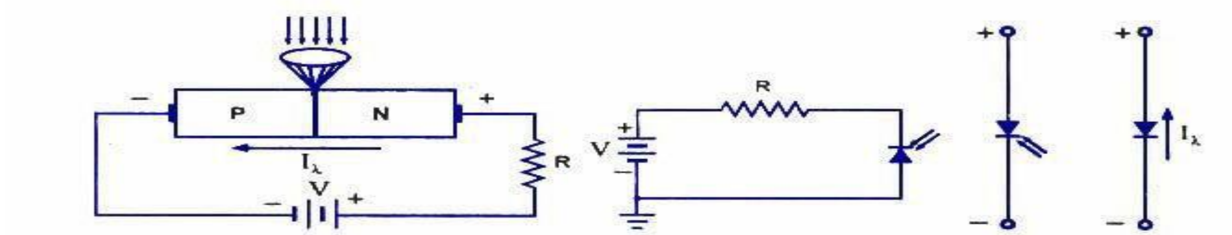


Fig:Symbol for photodiode.

Principle of operation:

A photodiode is a type of photo detector capable of converting light into either current or voltage, depending upon the mode of operation. The common, traditional solar cell used to generate electric solar power is a large area photodiode. A photodiode is designed to operate in reverse bias. The deletion region width is large.Under normal conditions it carries small reverse current due to minority charge carriers.When light is incident through glass window on the p-n junction,photons in the light bombard the p-n junction and some energy s imparted to the valence electrons.So valence electrons break covalent bonds and become free electrons.Thus more electron-hole pairs are generated.Thus total number of minority charge carriers increases and hence reverse current increases.This is the basic principle of operation of photodiode.



Basic Biasing Arrangement and Construction Photodiode

Characteristics of photodiode:

When the P-N junction is reverse-biased, a reverse saturation current flows due to thermally generated holes and electrons being swept across the junction as the minority carriers. With the increase in temperature of the junction more and more hole-electron pairs are created and so the reverse saturation current I_0 increases. The same effect can be had by illuminating the junction. When light energy bombards a P-N junction, it dislodges valence electrons. The more light striking the junction the larger the reverse current in a diode. It is due to generation of more and more charge carriers with the increase in level of illumination. This is clearly shown in ‘ figure for different intensity levels. The dark current is the current that exists when no light is incident. It is to be noted here that current becomes zero only with a positive applied bias equals to V_Q. The almost equal spacing between the curves for the same increment in luminous flux reveals that the reverse saturation current I_0increases linearly with the luminous flux as shown in figure. Increase in reverse voltage does not increase the reverse current significantly, because all available charge carriers are already being swept across the junction. For reducing the reverse saturation current I_0 to zero, it is necessary to forward bias the junction by an amount equal to barrier potential. Thus the photodiode can be used as a photoconductivedevice.

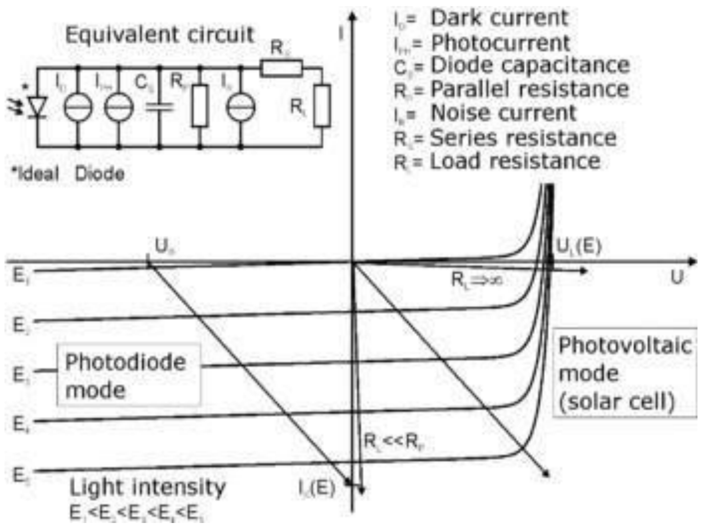


Fig: characteristics of photodiode

On removal of reverse bias applied across the photodiode, minority charge carriers continue to be swept across the junction while the diode is illuminated. This has the effect of increasing the concentration of holes in the P-side and that of electrons in the N-side But the barrier potential is negative on the P-side and positive on the N-side, and was created by holes flowing from P to N-side and electrons from N to P-side during fabrication of junction. Thus the flow of minority carriers tends to reduce the barrier potential.

When an external circuit is connected across the diode terminals, the minority carrier; return to the original side via the external circuit. The electrons which crossed the junction from P to N-side now flow out through the N-terminal and into the P-terminal This means that the device is behaving as a voltage cell with the N-side being the negative terminal and the P-side the positive terminal. Thus, the photodiode is & photovoltaic device as well as photoconductive device.

Advantages:

The advantages of photodiode are:

1.It can be used as variable resistancedevice.

2.Highly sensitive to the light.
3.The speed of operation is veryhigh.

Disadvantages:

1.Temperature dependent dark current.
2.poor temperature stability.
3.Current needs amplification for driving other circuits.

Applications:

1.Alarm system.
2.counting system.

Problems:

1.In a particular application single phase half wave rectifier using SCR is used.The average load voltage is 80V.If supply voltage is 230V,50Hz a.c.find the firing angle of the SCR.

2.In a particular application single phase half wave rectifier using SCR s used.The supply voltage is 325sin ωt where ω=100πrad/sec.Find the time for which SCR remains OFF ifforward breakover voltage is125V.

UNIT III
RECTIFIERS AND FILTERS

7.1 Circuit Model of a Diode

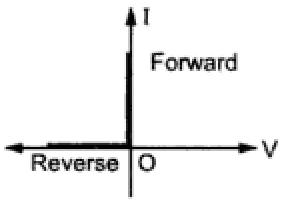


Fig. 7.1 V-I characteristics of an ideal diode

We have seen that a diode has a very important property that it permits only unidirectional conduction. It conducts very well in forward direction when forward biased and conducts very poorly in reverse direction when reverse biased. An ideal diode should conduct instantaneously when forward biased and should not conduct at all when reverse biased. The V-I characteristics of such an ideal diode would be as shown in the Fig. 7.1. **An ideal diode acts like a switch.**

But in a practical diode, there is a voltage drop across a diode and it starts conducting heavily after its cut-in voltage V_γ is exceeded by the forward biased voltage applied. Similarly it is not a perfect conductor with zero resistance in forward biased condition. It has some finite resistance in forward biased condition.

So voltage drop across forward biased diode is made up of,

1. The drop equal to cut in voltage which is to be maintained across the diode to keep it forward biased.
2. The drop due to its forward resistance due to forward current ($I_f r_f$).

Hence in analyzing various devices involving diodes it is necessary to replace diode by its circuit model which cosiders its cut-in voltage and forward resistance. **So a diode is replaced by a model with a battery equal to cut-in voltage of diode, the forward resistance of a diode in series with an ideal diode as shown in Fig. 7.2.**

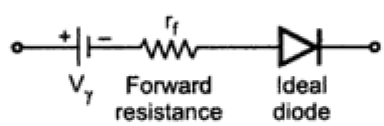


Fig. 7.2 Circuit model of a diode

Key Point: *Hence the total voltage drop across the diode is sum of the cut-in voltage and the drop across its forward resistance due to the current flowing through it.*

If the forward resistance is not given, a diode can be replaced by a battery having voltage equa to cut in voltage of diode.

Key Point: *A reverse biased diode can be replaced by an open circuit from the circuit analysis point of view.*

Such a circuit model of diode helps to understand the working of various electronic circuits and applications. Let us study now the various p-n junction applications such as half wave, full wave and bridge rectifier.

Block Diagram of a typical DC Power Supply:

A d.c. power supply is an important element of any type of an electronic circuit. In everyday life, we are using number of electronic devices such as transistors, deck, T.V., V.C.R. which operate fully or partly on d.c. power supply in the range of 0 to 24 V. The successful operation of the device depends on the proper functioning of the d.c. power supply. The power supply tries to provide a smooth, constant d.c. voltage, as required by an electronic device. As we know, M.S.E.B. (Maharashtra State Electricity Board) provides a power supply of an alternating type (a.c.) of 230 V, 50 Hz. Then the question is, how the various electronic circuits and devices get a d.c. power supply ?

A typical d.c. power supply consists of various stages. The Fig.3.1 shows the block diagram of a typical d.c. power supply consisting of various circuits. The nature of voltages at various points is also shown in the Fig. 3.1.

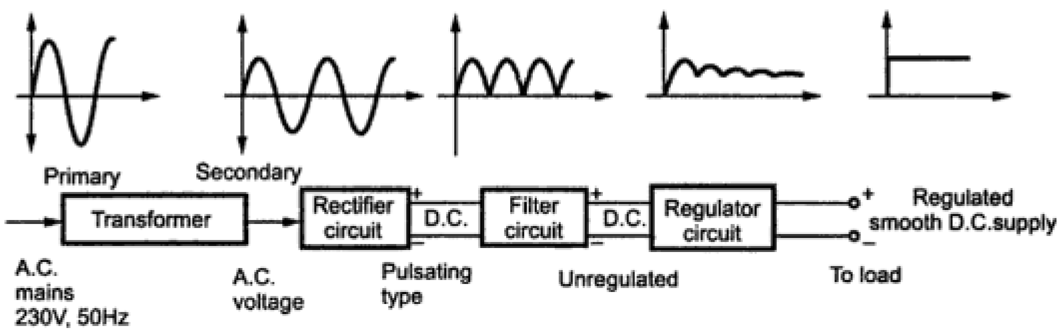


Fig. 3.1 Block diagram of a typical d.c. power supply

The a.c. voltage (230 V, 50 Hz) is connected to the primary of the transformer. The transformer steps down the a.c.voltage, to the level required for the desired d.c. output. Thus, with suitable turns ratio we get desired a.c. secondary voltage. The rectifier circuit converts this a.c. voltage into a pulsating d.c. voltage. A pulsating d.c. voltage means a unidirectional voltage containing large varying component called ripple in it. The filter circuit is used after a rectifier circuit, which reduces the ripple content in the pulsating d.c. and tries to make it smoother. Still then the filter output contains some ripple. This voltage is called unregulated d.c.voltage. A circuit used after the filter is a regulator circuit which not only makes the d.c. voltage smooth and almost ripple free but it also keeps the d.c. output voltage constant though input d.c. voltage varies under certain conditions. It keeps the output voltage constant under variable load conditions, as well. The output of a regulator is called d.c. supply, to which the load can be connected. Now a days, complete regulator circuits are available in the integrated circuit (IC) form.

In this chapter, we shall study the various rectifier circuits and filter circuits.

3.2 Rectifiers

A **rectifier** is a device which converts a.c. voltage to pulsating d.c. voltage, using one or more p-n junction diode.

The p-n junction diode conducts only in one direction. It conducts when forward biased while practically it does not conduct when reverse biased. Thus if an alternating voltage is applied across a p-n junction diode, during positive half cycle the diode will be forward biased and will conduct successfully. While during the negative half cycle it will be reversed biased and will not conduct at all. Thus the conduction occurs only during positive half cycle. If the resistance is connected in series with the diode, the output voltage across the resistance will be unidirectional i.e. d.c. Thus p-n junction diode subjected to an a.c. voltage acts as a rectifier converting alternating voltage to a pulsating d.c. voltage.

3.2.1 The Important Characteristics of a Rectifier Circuit

The important points to be studied while analysing the various rectifier circuits are,

a) **Waveform of the load current :** As rectifier converts a.c. to pulsating d.c., it is important to analyze the nature of the current through load which ultimately determines the waveform of the load voltage.

b) **Regulation of the output voltage :** As the load current changes, load voltage changes. Practically load voltage should remain constant. So concept of regulation is to study the effect of change in load current on the load voltage.

c) **Rectifier efficiency :** It signifies, how efficiently the rectifier circuit converts a.c. power into d.c. power.

d) **Peak value of current in the rectifier circuit :** The peak value is the maximum value of an alternating current in the rectifier circuit. This decides the rating of the rectifier circuit element which is diode.

e) **Peak value of voltage across the rectifier element in the reverse direction (PIV) :** When the diode is not conducting, the reverse voltage gets applied across the diode. The peak value of such voltage decides the peak inverse voltage i.e. PIV rating of a diode.

f) **Ripple factor :** The output of the rectifier is of pulsating d.c. type. The amount of a.c. content in the output can be mathematically expressed by a factor called ripple factor.

Using one or more diodes following rectifier circuits can be designed.

1. Half wave rectifier
2. Full wave rectifier
3. Bridge rectifier

Let us discuss the various rectifier circuits in detail.

3.3 Half Wave Rectifier

In half wave rectifier, rectifying element conducts only during positive half cycle of input a.c. supply. The negative half cycles of a.c. supply are eliminated from the output.

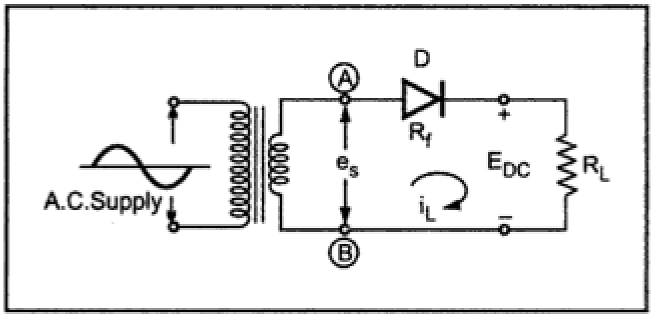


Fig. 3.2 Halfwave rectifier

This rectifier circuit consists of resistive load, rectifying element, i.e. p-n junction diode, and the source of a.c. voltage, all connected in series. The circuit diagram is shown in Fig. 3.2. Usually, the rectifier circuits are operated from ac mains supply. To obtain the desired d.c.voltage across the load, the a.c. voltage is applied to rectifier circuit using suitable step-up or step-down transformer, mostly a step-down one, with necessary turns ratio.

The input voltage to the half-wave rectifier circuit shown in Fig. 3.2 is a sinusoidal a.c. voltage, having a frequency which is the supply frequency, 50 Hz.

The transformer decides the peak value of the secondary voltage. If the N_1 are primary number of turns and N_2 are secondary number of turns and E_{pm} is the peak value of the primary voltage then,

$$\frac{N_2}{N_1} = \frac{E_{sm}}{E_{pm}}$$

where E_{sm} is the peak value of the secondary a.c. voltage.

As the nature of E_{sm} is sinusoidal the instantaneous value will be,

$$e_s = E_{sm} \sin \omega t$$

$$\omega = 2\pi f$$

$$f = \text{supply frequency}$$

Let R_f represents the forward resistance of the diode. Assume that, under reverse biased condition, the diode acts almost as open circuit, conducting no current.

3.3.1 Operation of the Circuit

During the positive half cycle of secondary a.c voltage, terminal (A) becomes positive with respect to terminal (B). The diode is forward biased and the current flows in the circuit in the clockwise direction, as shown in Fig. 3.2. The current will flow for almost full positive half cycle. This current is also flowing through load resistance R_L hence denoted as i_L, the load current.

During negative half cycle when terminal (A) is negative with respect to terminal (B), diode becomes reverse biased. Hence no current flows in the circuit. Thus the circuit current, which is also the load current, is in the form of half sinusoidal pulses.

The load voltage, being the product of load current and load resistance, will also be in the form of half sinusoidal pulses. The different waveforms are illustrated in Fig. 3.3.

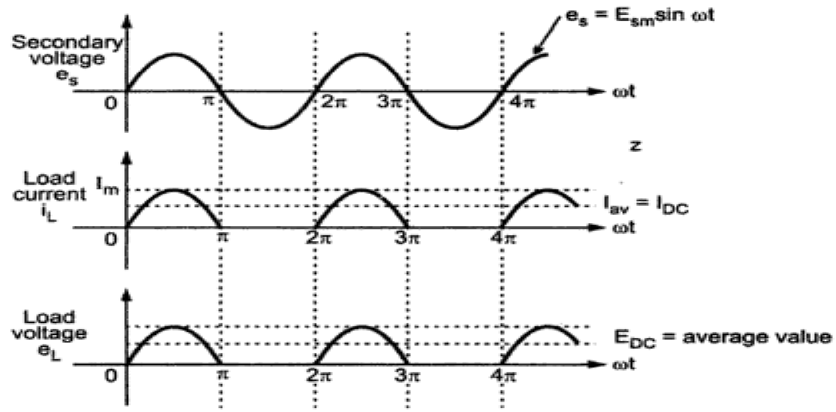


Fig. 3.3 Load current and load voltage waveforms for halfwave rectifier

The d.c. output waveform is expected to be a straight line but the half wave rectifier **gives** output in the form of positive sinusoidal pulses. Hence the output is called **pulsating d.c**. It is discontinuous in nature. Hence it is necessary to calculate the average value of load current and average value of output voltage.

3.3.2 Average DC Load Current (I_{DC})

The average or dc value of alternating current is obtained by integration.

For finding out the average value of an alternating waveform, we have to determine the area under the curve over one complete cycle i.e. from 0 to 2π and then dividing it by the base i.e. 2π.

Mathematically, current waveform can be described as,

$$i_L = I_m \sin \omega t \qquad \text{for } 0 \le \omega t \le \pi$$

$$i_L = 0 \qquad \text{for } \pi \le \omega t \le 2\pi$$

where I_m = peak value of load current

$$\therefore \quad I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i_L \, d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} I_m \sin(\omega t)\, d(\omega t)$$

As no current flows during negative half cycle of ac input voltage, i.e. between $\omega t = \pi$ to $\omega t = 2\pi$, we change the limits of integration.

$$\therefore \quad I_{DC} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{I_m}{2\pi}\left[-\cos(\omega t)\right]_0^{\pi} = -\frac{I_m}{2\pi}\left[\cos(\pi) - \cos(0)\right]$$

$$= -\frac{I_m}{2\pi}[-1-1] = \frac{I_m}{\pi}$$

$$\therefore \quad I_{DC} = \frac{I_m}{\pi} = \text{average value}$$

Applying Kirchhoff's voltage law we can write,

$$I_m = \frac{E_{sm}}{R_f + R_L + R_s}$$

where R_s = resistance of secondary winding of transformer. If R_s is not given it should be neglected while calculating I_m.

3.3.3 Average DC Load Voltage (E_{DC})

It is the product of average D.C. load current and the load resistance R_L.

$$E_{DC} = I_{DC} R_L$$

Substituting value of I_{DC},

$$E_{DC} = \frac{I_m}{\pi} R_L = \frac{E_{sm}}{(R_f + R_L + R_s)\pi} R_L$$

The winding resistance R_s and forward diode resistance R_f are practically very small compared to R_L.

$$\therefore \quad E_{DC} = \frac{E_{sm}}{\pi\left[\frac{R_f + R_s}{R_L} + 1\right]}$$

But as R_f and R_s are small compared to R_L, $(R_f + R_s)/R_L$ is negligibly small compared to 1. So neglecting it we get,

$$\therefore \quad E_{DC} \approx \frac{E_{sm}}{\pi}$$

3.3.4 R.M.S. Value of Load Current (I_{RMS})

The R.M.S means squaring, finding mean and then finding square root. Hence R.M.S. value of load current can be obtained as,

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} (I_m \sin \omega t)^2 \, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} \left(I_m{}^2 \sin^2 \omega t \, d(\omega t)\right)}$$

$$= I_m \sqrt{\frac{1}{2\pi}\int_0^{\pi} \frac{[1 - \cos(2\omega t)] d(\omega t)}{2}}$$

$$= I_m \sqrt{\frac{1}{2\pi}\left\{\frac{\omega t}{2} - \frac{\sin(2\omega t)}{4}\right\}_0^{\pi}}$$

$$= I_m \sqrt{\frac{1}{2\pi}\left(\frac{\pi}{2}\right)} \qquad \text{as } \sin(2\pi) = \sin(0) = 0$$

$$= \frac{I_m}{2}$$

$$\therefore \quad I_{RMS} = \frac{I_m}{2}$$

Note : Students must remember that this R.M.S. value is for **half wave** rectified waveform hence it is $I_m/2$. For full sine wave it is $I_m/\sqrt{2}$.

3.3.5 D.C. Power Output (P_{DC})

The d.c. power output can be obtained as,

$$P_{DC} = E_{DC} I_{DC} = I_{DC}^2 R_L$$

$$\text{D.C. Power output} = I_{DC}^2 R_L = \left[\frac{I_m}{\pi}\right]^2 R_L = \frac{I_m{}^2}{\pi^2} R_L$$

$$\therefore \quad P_{DC} = \frac{I_m{}^2}{\pi^2} R_L$$

where

$$I_m = \frac{E_{sm}}{R_f + R_L + R_s}$$

$$\therefore \quad P_{DC} = \frac{E_{sm}{}^2 R_L}{\pi^2 [R_f + R_L + R_s]^2}$$

3.3.6 A.C. Power Input (P_{AC})

The power input taken from the secondary of transformer is the power supplied to three resistances namely load resistance R_L , the diode resistance R_f and winding resistance R_s. The a.c. power is given by,

$$P_{AC} = I_{RMS}^2 [R_L + R_f + R_s]$$

but

$$I_{RMS} = \frac{I_m}{2} \qquad \text{for half wave,}$$

$$\therefore \quad P_{AC} = \frac{I_m^2}{4} [R_L + R_f + R_s]$$

3.3.7 Rectifier Efficiency (η)

The rectifier efficiency is defined as the ratio of output d.c. power to input a.c. power.

$$\therefore \quad \eta = \frac{\text{D.C. output power}}{\text{A.C. input power}} = \frac{P_{DC}}{P_{AC}}$$

$$\therefore \quad \eta = \frac{\frac{I_m^2}{\pi^2} R_L}{\frac{I_m^2}{4} [R_f + R_L + R_s]} = \frac{(4/\pi^2) R_L}{(R_f + R_L + R_s)}$$

$$\therefore \quad \eta = \frac{0.406}{1 + \left(\frac{R_f + R_s}{R_L}\right)}$$

If $(R_f + R_s) << R_L$ as mentioned earlier, we get the maximum theoretical efficiency of half wave rectifier as,

$$\%\eta_{max} = 0.406 \times 100 = 40.6\ \%$$

Thus in half wave rectifier, maximum 40.6% a.c. power gets converted to d.c. power in the load. If the efficiency of rectifier is 40% then what happens to the remaining 60% power. It is present interms of ripples in the output which is fluctuating component present in the output. Thus more the rectifier efficiency, less are the ripple contents in the output.

3.3.8 Ripple Factor (γ)

It is seen that the output of half wave rectifier is not pure d.c. but a pulsating d.c. The output contain pulsating components called **ripples.** Ideally there should not be any ripples in the rectifier output. The measure of such ripples present in the output is with the help of a factor called **ripple factor** denoted by γ. It tells how smooth is the output. Smaller the ripple factor closer is the output to a pure d.c. The ripple factor expresses how much successful the circuit is in obtaining pure d.c. from a.c. input.

Mathematically ripple factor is defined as the ratio of R.M.S. value of the a.c. component to the average or d.c.component.

$$\text{Ripple factor} \quad \gamma = \frac{\text{R.M.S. value of a.c. component}}{\text{Average or d.c.component}}$$

Now the output current is composed of a.c. component as well as d.c. component.

Let I_{ac} = r.m.s. value of a. c. component present in output

I_{DC} = d.c. component present in output

I_{RMS} = R.M.S. value of total output current

$$\therefore \quad I_{RMS} = \sqrt{I_{ac}^2 + I_{DC}^2}$$

$$\therefore \quad I_{ac} = \sqrt{I_{RMS}^2 - I_{DC}^2}$$

Now Ripple factor $= \dfrac{I_{ac}}{I_{DC}}$ as per definition

$$\therefore \quad \gamma = \frac{\sqrt{I_{RMS}^2 - I_{DC}^2}}{I_{DC}}$$

$$\therefore \quad \gamma = \sqrt{\left(\frac{I_{RMS}}{I_{DC}}\right)^2 - 1}$$

This is the general expression for ripple factor and can be used for any rectifier circuit.

Now for a half wave circuit,

$$I_{RMS} = \frac{I_m}{2} \quad \text{while} \quad I_{DC} = \frac{I_m}{\pi}$$

$$\therefore \quad \gamma = \sqrt{\left[\frac{\left(\frac{I_m}{2}\right)}{\left(\frac{I_m}{\pi}\right)}\right]^2 - 1} = \sqrt{\frac{\pi^2}{4} - 1} = \sqrt{1.4674}$$

$$\therefore \quad \gamma = 1.211$$

This indicates that the ripple contents in the output are 1.211 times the d.c. component i.e. 121.1 % of d.c. component. The ripple factor for half wave is very high which indicates that the half wave circuit is a poor converter of a.c. to d.c. The ripple factor is minimised using filter circuits along with rectifiers.

3.3.9 Load Current

The load current i_L which is composed of a.c. and d.c. components can be expressed using Fourier series as,

$$i_L = I_m\left[\frac{1}{\pi}+\frac{1}{2}\sin\omega t-\frac{2}{3\pi}\cos 2\omega t-\frac{2}{15\pi}\cos 4\omega t \ ...\right]$$

This expression shows that the current may be considered to be the sum of an infinite number of current components, according to Fourier series.

The first term of the series is the average or d.c. value of the load current. The second term is a varying component having frequency same as that of a.c. supply voltage. This is called fundamental component of the current having frequency same as the supply. The third term is again a varying component having frequency twice the frequency of supply voltage. This is called second harmonic component. Similarly all the other terms represent the a.c. components and are called harmonics.

Thus ripple in the output is due to the fundamental component alongwith the various harmonic components. And the average value of the total pulsating d.c. is the d.c. value of the load current, given by the constant term in the series, I_m / π

3.3.10 Peak Inverse Voltage (PIV)

The Peak Inverse Voltage is the peak voltage across the diode in the reverse direction i.e. when the diode is reverse biased. In half wave rectifier, the load current is ideally zero when the diode is reverse biased and hence the maximum value of the voltage that can exist across the diode is nothing but E_{sm}.

$$\therefore \quad \text{PIV of diode} = E_{sm} = \text{Maximum value of secondary voltage}$$

$$= \pi E_{DC}|_{I_{DC}=0}$$

This is called PIV rating of a diode. So diode must be selected based on this PIV rating and the circuit specifications.

3.3.11 Transformer Utilization Factor (T.U.F.)

The factor which indicates how much is the utilization of the transformer in the circuit is called Transformer Utilization Factor (T.U.F.)

The T.U.F. is defined as the ratio of d.c. power delivered to the load to the a.c power rating of the transformer. While calculating the a.c. power rating, it is necessary to consider r.m.s. value of a.c. voltage and current.

The T.U.F. for half wave rectifier can be obtained as,

$$\text{A.C. power rating of transformer} = E_{RMS}\, I_{RMS}$$

$$= \frac{E_{sm}}{\sqrt{2}}\cdot\frac{I_m}{2} = \frac{E_{sm}\, I_m}{2\sqrt{2}}$$

Remember that the secondary voltage is purely sinusoidal hence its r.m.s. value is $1/\sqrt{2}$ times maximum while the current is half sinusoidal hence its r.m.s. value is 1/2 of the maximum, as derived earlier.

$$\text{D.C. power delivered to the load} = I_{DC}^2\, R_L$$

$$= \left(\frac{I_m}{\pi}\right)^2 R_L$$

$$\therefore \quad \text{T.U.F.} = \frac{\text{D.C. Power delivered to the load}}{\text{A.C. Power rating of the transformer}}$$

$$= \frac{\left(\frac{I_m}{\pi}\right)^2 R_L}{\left(\frac{E_{sm} I_m}{2\sqrt{2}}\right)}$$

Neglecting the drop across R_f and R_s we can write,

$$E_{sm} = I_m R_L$$

$$\therefore \quad \text{T.U.F} = \frac{I_m^2}{\pi^2} \cdot \frac{R_L . 2\sqrt{2}}{I_m^2 R_L}$$

$$= \frac{2\sqrt{2}}{\pi^2}$$

$$= 0.287$$

The value of T.U.F. is low which shows that in half wave circuit, the transformer is not fully utilized.

3.3.12 Disadvantages of Half Wave Rectifier Circuit

1. The ripple factor of half wave rectifier circuit is 1.21, which is quite high. The output contains lot of varying components.
2. The maximum theoretical rectification efficiency is found to be 40%. The practical value will be less than this. This indicates that half wave rectifier circuit is quite inefficient.
3. The circuit has low transformer utilization factor, showing that the transformer is not fully utilized.
4. The dc current is flowing through the secondary winding of the transformer which may cause dc saturation of the core of the transformer. To minimize the saturation, transformer size have to be increased accordingly. This increases the cost.

Because of all these disadvantages, the half-wave rectifier circuit is normally not used as a power rectifier circuit.

Ex. 3.1 : *A half wave rectifier circuit is supplied from a 230 V, 50 Hz supply with a step down ratio of 3 :1 to a resistive load of 10 kΩ. The diode forward resistance is 75 Ω while transformer secondary resistance is 10 Ω. Calculate maximum, average, RMS values of current, D.C. output voltage, efficiency of rectification and ripple factor.*

Sol. : The circuit is shown in the Fig. 3.4

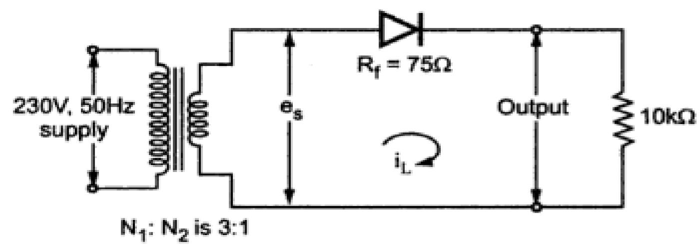


Fig. 3.4

The given values are,

$R_f = 75\,\Omega$, $R_L = 10\,k\Omega$, $R_s = 10\,\Omega$

The given supply voltages are always r.m.s. values.

$$E_p(RMS) = 230\text{ V},\ \frac{N_1}{N_2} = \frac{3}{1} \text{ i.e. } \frac{N_2}{N_1} = \frac{1}{3}$$

$$\frac{N_2}{N_1} = \frac{E_s(RMS)}{E_p(RMS)}$$

$$\therefore \quad \frac{1}{3} = \frac{E_s(RMS)}{230}$$

$$\therefore \quad E_s(RMS) = 76.667\text{ V}$$

This is r.m.s. value of the transformer secondary voltage.

$$\therefore \quad E_{sm} = \sqrt{2}\,E_s(RMS) = \sqrt{2} \times 76.667$$

$$= 108.423\text{ V}$$

$$\therefore \quad I_{sm} = \frac{E_{sm}}{R_s + R_f + R_L} = I_m$$

$$\therefore \quad I_m = \frac{108.423}{10 + 75 + 10 \times 10^3}$$

$$= 10.75\text{ mA}$$

$$\therefore \quad I_{av} = I_{DC} = \frac{I_m}{\pi} = \frac{10.75}{\pi}$$

$$= 3.422\text{ mA}$$

$$I_{RMS} = \frac{I_m}{2} \text{ for half wave}$$

$$= \frac{10.75}{2} = 5.375\text{ mA}$$

$$E_{DC} = \text{d.c output voltage} = I_{DC} R_L$$

$$= 3.422 \times 10^{-3} \times 10 \times 10^3$$

$$= 34.22\text{ V}$$

$$P_{DC} = \text{d.c. output power} = E_{DC} I_{DC}$$
$$= 34.22 \times 3.422 \times 10^{-3}$$
$$= 0.1171 \text{ W}$$

This also can be obtained as,

$$P_{DC} = \frac{I_m^2}{\pi^2} R_L$$
$$= \frac{(10.75 \times 10^{-3})^2}{\pi^2} \times 10 \times 10^3$$
$$= 0.1171 \text{ W}$$
$$P_{AC} = \text{a.c. input power}$$
$$= I_{RMS}^2 [R_s + R_f + R_L]$$
$$= (5.375 \times 10^{-3})^2 [10 + 75 + 10 \times 10^3]$$
$$= 0.2913 \text{ W}$$
$$\therefore \quad \% \eta = \frac{P_{DC}}{P_{AC}} \times 100 = \frac{0.1171}{0.2913} \times 100$$
$$= 40.19\%$$

The ripple factor is constant for half wave rectifier and is 1.21.

$$\therefore \quad \gamma = 1.21$$

3.4 Full Wave Rectifier

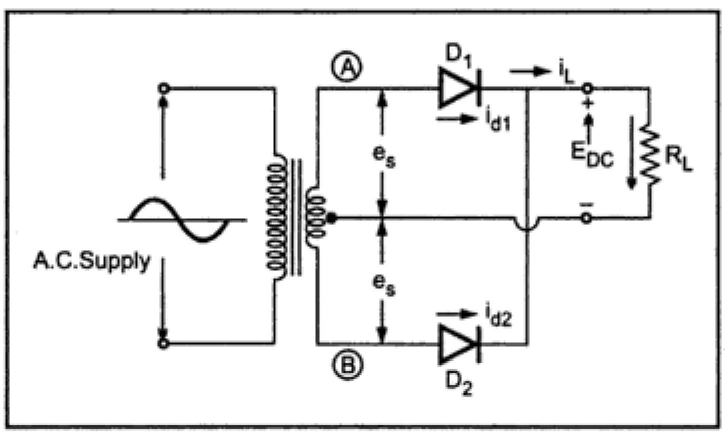


Fig. 3.6 Full wave rectifier

The full wave rectifier conducts during both positive and negative half cycles of input a.c. supply. In order to rectify both the half cycles of a.c. input, two diodes are used in this circuit. The diodes feed a common load R_L with the help of a center tap transformer. The a.c. voltage is applied through a suitable power transformer with proper turns ratio.

The full wave rectifier circuit is shown in the Fig.3.6.

For the proper operation of the circuit, a center-tap on the secondary winding of the transformer is essential.

3.4.1 Operation of the Circuit

Consider the positive half cycle of ac input voltage in which terminal (A) is positive and terminal (B) negative. The diode D_1 will be forward biased and hence will conduct;

while diode D_2 will be reverse biased and will act as open circuit and will not conduct. This is illustrated in Fig.3.7.

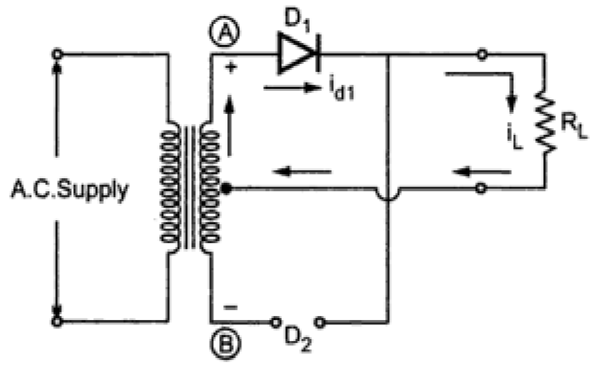


Fig. 3.7 Current flow during positive half cycle

The diode D_1 supplies the load current, i.e. $i_L = i_{d1}$ .This current is flowing through upper half of secondary winding while the lower half of secondary winding of the transformer carries no current since diode D_2 is reverse biased and acts as open circuit.

In the next half cycle of ac voltage, polarity reverses and terminal (A) becomes negative and (B) positive. The diode D_2 conducts, being forward biased, while D_1 does not, being reverse biased. This is shown in Fig. 3.8.

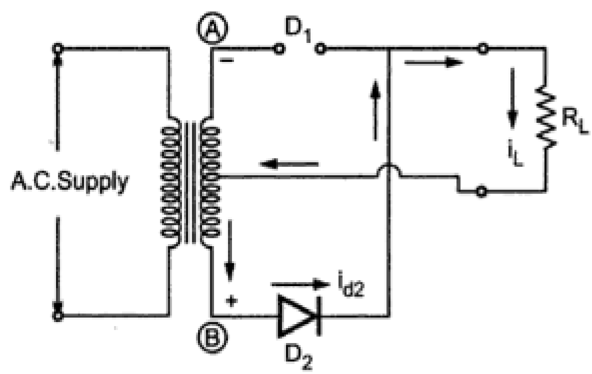


Fig. 3.8 Current flow during negative half cycle

The diode D_2 supplies the load current, i.e. $i_L = i_{d2}$. Now the lower half of the secondary winding carries the current but the upper half does not.

It is noted that the **load current flows in both half cycles of ac voltage and in the same direction through the load resistance**. Hence we get rectified output across the load. The load current is sum of individual diode currents flowing in corresponding half cycles. It is also noted that the two diodes do not conduct simultaneously but in alternate half cycles.The individual diode currents and the load current are shown in Fig. 3.9

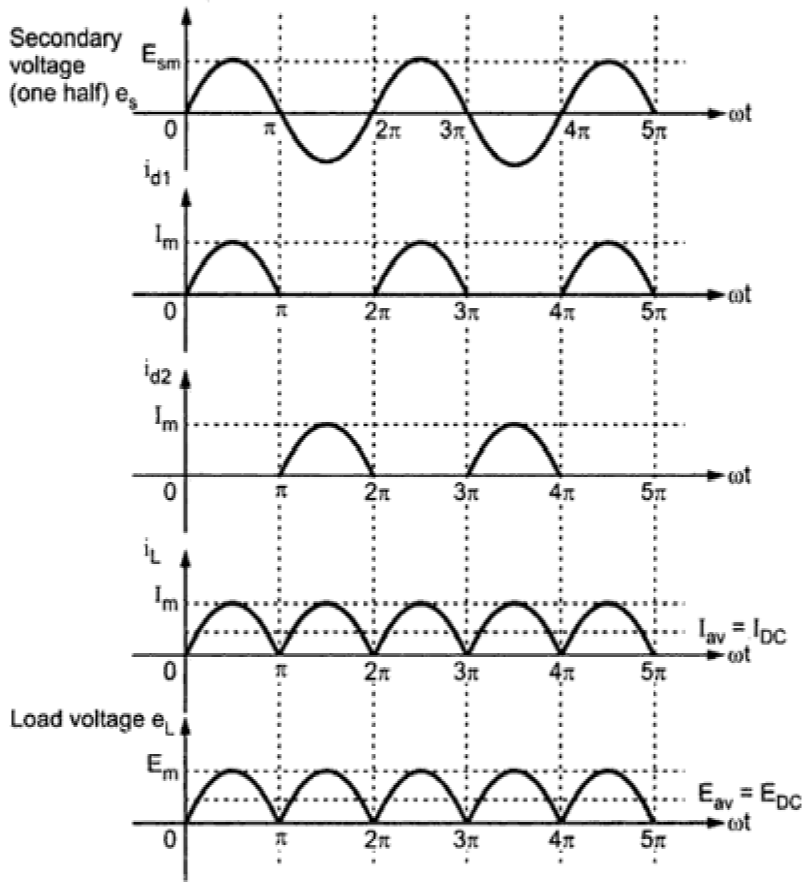


Fig. 3.9 Load current and voltage waveforms full wave rectifier

Thus the full wave rectifier circuit essentially consists of two half-wave rectifier circuits working independently (working in alternate half cycles of a c) of each other but feeding a common load. The output load current is still pulsating d.c. and not pure d.c.

3.4.2 Maximum Load Current

Let R_f = forward resistance of diodes

R_s = winding resistance of each half of secondary

R_L = load resistance

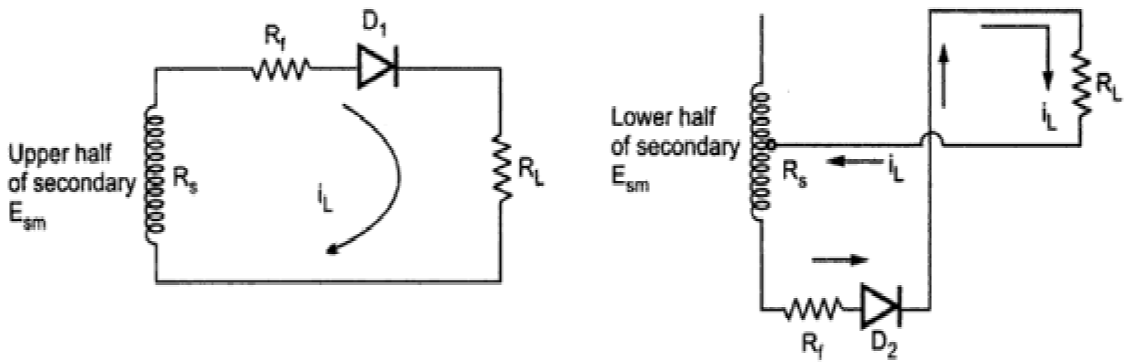
e_s = instantaneous a.c. voltage across each half of secondary

$\therefore$ $e_s = E_{sm} \sin \omega t$

$\omega = 2 \pi f$

E_{sm} = maximum value of a.c. input voltage across each half of secondary winding

Hence we can write the expression for the maximum value of load current, looking at equivalent circuit shown in Fig 3.10.



(a) Fig. 3.10 (b)

$$\therefore \quad I_m = \frac{E_{sm}}{R_s + R_f + R_L}$$

where I_m = maximum value of load current i_L

3.4.3 Average DC Load Current (I_{DC})

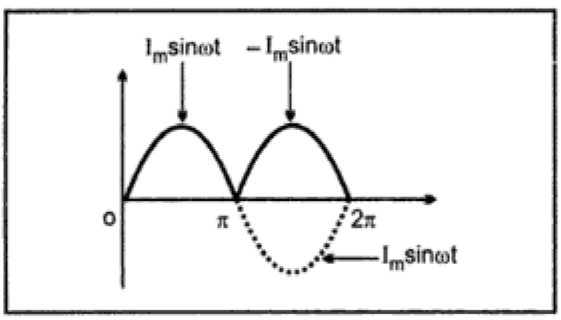


Fig. 3.11

Consider one cycle of load current i_L from 0 to 2π to obtain the average value which is d.c. value of load current.

$$i_L = I_m \sin \omega t \qquad 0 \le \omega t \le \pi$$

But for π to 2π, the current i_L is again positive while sin ωt term is negative during π to 2π. Hence in the region π to 2π the positive i_L can be represented as negative of $I_m \sin(\omega t)$.

$$\therefore \quad i_L = -I_m \sin \omega t \qquad \pi \le \omega t \le 2\pi$$

$$\therefore \quad I_{av} = I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} i_L \, d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin \omega t \, d(\omega t) + \int_{\pi}^{2\pi} -I_m \sin \omega t \, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}\left[\int_0^{\pi} \sin \omega t \, d(\omega t) - \int_{\pi}^{2\pi} \sin \omega t \, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}\left[(-\cos\omega t)_0^{\pi} - (-\cos\omega t)_{\pi}^{2\pi}\right]$$

$$= \frac{I_m}{2\pi}[-\cos\pi + \cos 0 + \cos 2\pi - \cos\pi]$$

but $\cos\pi = -1$

$$= \frac{I_m}{2\pi}[-(-1)+1+1-(-1)] = \frac{4I_m}{2\pi}$$

$\therefore$ $I_{DC} = \frac{2I_m}{\pi}$ for full wave rectifier

For half wave it is I_m/π and full wave rectifier is the combination of two half wave circuits acting alternately in two half cycles of input. Hence obviously the d.c. value for full wave circuit is $2\,I_m/\pi$.

3.4.4 Average DC Load Voltage (E_{DC})

The d.c. load voltage is,

$$E_{DC} = I_{DC}R_L = \frac{2I_m R_L}{\pi}$$

Substituting value of I_m,

$$E_{DC} = \frac{2\,E_{sm}\ R_L}{\pi\,[R_f + R_s + R_L]}$$

$$= \frac{2\,E_{sm}}{\pi\left[1 + \frac{R_f + R_s}{R_L}\right]}$$

But as R_f and $R_s << R_L$ hence $\frac{R_f + R_s}{R_L} << 1$

$$\therefore \quad E_{DC} = \frac{2\,E_{sm}}{\pi}$$

3.4.5 RMS Load Current (I_{RMS})

The R.M.S. value of current, I_{RMS}, is obtained as follows :

$$I_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_L^2\,d(\omega t)}$$

Since two half wave rectifier are similar in operation we can write,

$$I_{RMS} = \sqrt{\frac{2}{2\pi}\int_0^{\pi}[I_m\ \sin\omega t]^2\,d(\omega t)}$$

$$= I_m\sqrt{\frac{1}{\pi}\int_0^{\pi}\left[\frac{1-\cos 2\omega t}{2}\right]d(\omega t)} \text{ as } \sin^2\omega t$$

$$= \frac{1-\cos 2\omega t}{2}$$

$$\therefore \quad I_{RMS} = I_m \sqrt{\frac{1}{2\pi}\left[[\omega t]_0^{\pi} - \left(\frac{\sin 2\omega t}{2}\right)_0^{\pi}\right]}$$

$$= I_m \sqrt{\frac{1}{2\pi}[\pi - 0]}$$

$$= I_m \sqrt{\frac{1}{2\pi}(\pi)} \qquad \text{as } \sin(2\pi) = \sin(0) = 0$$

$$\therefore \quad I_{RMS} = \frac{I_m}{\sqrt{2}}$$

3.4.6 DC Power Output (P_{DC})

$$\text{D.C. Power output} = E_{DC} I_{DC} = I_{DC}^2 R_L$$

$$\therefore \quad P_{DC} = I_{DC}^2 R_L = \left(\frac{2I_m}{\pi}\right)^2 R_L$$

$$\therefore \quad P_{DC} = \frac{4}{\pi^2} I_m^2 R_L$$

Substituting value of I_m we get,

$$P_{DC} = \frac{4}{\pi^2} \frac{E_{sm}^2}{(R_s + R_f + R_L)^2} \times R_L$$

Note : Instead of remembering this formula students can use the expression $E_{DC}I_{DC}$ or $I_{DC}^2 R_L$ to calculate P_{DC} while solving the problems.

3.4.7 AC Power Input (P_{AC})

The a.c. power input is given by,

$$P_{AC} = I_{RMS}^2 (R_f + R_s + R_L)$$

$$= \left(\frac{I_m}{\sqrt{2}}\right)^2 (R_f + R_s + R_L)$$

$$\therefore \quad P_{AC} = \frac{I_m^2 (R_f + R_s + R_L)}{2}$$

Substituting value of I_m we get,

$$\therefore \quad P_{AC} = \frac{E_{sm}^2}{(R_f + R_s + R_L)^2} \times \frac{1}{2} \times (R_f + R_s + R_L)$$

$$\therefore \quad P_{AC} = \frac{E_{sm}^2}{2(R_f + R_s + R_L)}$$

3.4.8 Rectifier Efficiency (η)

$$\eta = \frac{P_{DC} \text{ output}}{P_{AC} \text{ input}}$$

$$\therefore \quad \eta = \frac{\frac{4}{\pi^2} I_m^2 R_L}{\frac{I_m^2 (R_f + R_s + R_L)}{2}}$$

$$\therefore \quad \eta = \frac{8 R_L}{\pi^2 (R_f + R_s + R_L)}$$

But if $R_f + R_s << R_L$, neglecting it from denominator

$$\eta = \frac{8 R_L}{\pi^2 (R_L)} = \frac{8}{\pi^2}$$

$$\therefore \quad \% \eta_{max} = \frac{8}{\pi^2} \times 100 = 81.2 \%$$

This is the maximum theoretical efficiency of full wave rectifier.

3.4.9 Ripple Factor (γ)

As derived earlier in case of half wave rectifier the ripple factor is given by a general expression,

$$\text{Ripple factor} = \sqrt{\left[\frac{I_{RMS}}{I_{DC}}\right]^2 - 1}$$

For full wave $I_{RMS} = I_m / \sqrt{2}$ and $I_{DC} = 2I_m / \pi$ so,

substituting in the above equation.

$$\text{Ripple factor} = \sqrt{\left[\frac{I_m / \sqrt{2}}{2I_m / \pi}\right]^2 - 1}$$

$$= \sqrt{\frac{\pi^2}{8} - 1}$$

$$\therefore \quad \text{Ripple factor} = \gamma = 0.48$$

This indicates that the ripple contents in the output are 48 % of the d.c. component which is much less than that for half wave circuit.

3.4.10 Load Current (i_L)

The fourier series for the load current is obtained by taking the sum of the series for the individual rectifier current. The two diodes conduct in alternate half cycles, i.e. there is a phase difference of π radians between two diode currents. Hence,

$$i_{d_1} = I_m \left[\frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{3\pi} \cos 2\omega t - \frac{2}{15\pi} \cos 4\omega t \ldots\right]$$

and $i_{d_2} = i_{d_1}$ with ωt replaced by $(\omega t + \pi)$

$$\therefore \quad i_{d_2} = I_m \left[\frac{1}{\pi} + \frac{1}{2} \sin(\omega t + \pi) - \frac{2}{3\pi} \cos 2(\omega t + \pi) - \frac{2}{15\pi} \cos 4(\omega t + \pi) \ldots\right]$$

$$= I_m\left[\frac{1}{\pi}-\frac{1}{2}\sin\omega t-\frac{2}{3\pi}\cos(2\omega t+2\pi)-\frac{2}{15\pi}\cos(4\omega t+4\pi)...\right]$$

$$= I_m\left[\frac{1}{\pi}-\frac{1}{2}\sin\omega t-\frac{2}{3\pi}\cos 2\omega t-\frac{2}{15\pi}\cos 4\omega t...\right]$$

Then the fourier series for the load current is,

$$i_L = i_{d_1}+i_{d_2}$$

$$= I_m\left[\frac{2}{\pi}-\frac{4}{3\pi}\cos 2\omega t-\frac{4}{15\pi}\cos 4\omega t\cdots\right]$$

The first term in the above series represents the average or dc value, while the remaining terms "ripple". It is seen that the lowest frequency of the ripple is 2f, i.e. twice the supply frequency of ac supply. The lowest ripple frequency in the load current of the full-wave connection, is double than that in the half-wave connection.

As seen from Fig. 3.7 and Fig 3.8 the individual diode currents are flowing in opposite directions through the two halves of the secondary winding. Hence the net secondary current will be difference of individual diode currents.

Thus, $$i_{sec} = i_{d_1}-i_{d_2}$$

The fourier series of i_{scc} is obtained by the difference between the series of individual diode currents. Using above relations we can write,

$$i_{sec} = I_m \sin\omega t$$

Hence under ideal conditions, the secondary current is purely sinusoidal. No d.c. component flows through the secondary hence there is no danger of saturation. This reduces the transformer losses and overall size and cost of the circuit. Thus the transformer gets utilised effectively.

3.4.11 Peak Inverse Voltage (PIV)

It can be observed from the circuit diagram that when the diode is reversed biased then full transformer secondary voltage gets impressed across it. The drop across conducting diode is assumed zero. Thus the peak value of the inverse voltage to which diode gets subjected is voltage across both the parts of the transformer secondary.

$$\therefore \quad \text{PIV of diode} = 2E_{sm}$$

$$= \pi E_{DC}|_{I_{DC}=0}$$

where E_{sm}= maximum value of a.c. voltage across half the transformer secondary.

3.4.12 Transformer Utilization Factor (T.U.F.)

In full wave rectifier, the secondary current flows through each half separately in every half cycle. While the primary of transformer carries current continuously. Hence T.U.F is calculated for primary and secondary windings separately and then the average T.U.F. is determined.

$$\text{Secondary T.U.F} = \frac{\text{DC power to load}}{\text{AC power rating of secondary}}$$

$$= \frac{I_{DC}^2 R_L}{E_{RMS}\, I_{rms}} = \frac{\left(\frac{2}{\pi}I_m\right)^2 R_L}{\frac{E_{sm}}{\sqrt{2}}\times\frac{I_m}{\sqrt{2}}}$$

Neglecting forward resistance R_f of diode, $E_{sm} \approx I_m R_L$.

$$\therefore \quad \text{Secondary T.U.F.} = \frac{\frac{4}{\pi^2} \times I_m^2 R_L}{\frac{I_m^2 R_L}{2}} = \frac{8}{\pi^2}$$

$$= 0.812$$

The primary of the transformer is feeding two half-wave rectifiers separately. These two half-wave rectifiers work independently of each other but feed a common load. We have already derived the T.U.F. for half wave circuit to be equal to 0.287. Hence

$$\text{T.U.F. for primary winding} = 2 \times \text{T.U.F. of half wave circuit}$$
$$= 2 \times 0.287$$
$$= 0.574.$$

The average T.U.F for fullwave circuit will be

$$\begin{array}{l}\text{Average T.U.F. for} \\ \text{full wave rectifier circuit}\end{array} = \frac{\text{T.U.F of primary} + \text{T.U.F of secondary}}{2}$$

$$= \frac{0.574 + 0.812}{2}$$

$$= 0.693$$

$\therefore$ Average T.U.F. for full-wave rectifier = 0.693

Thus in full-wave circuit transformer gets utilized more than the half wave rectifier circuit.

3.4.13 Voltage Regulation

The secondary voltage should not change with respect to the load current. The voltage regulation is the factor which tells us about the change in d.c. output voltage as load changes from no load to full load condition.

If $(V_{dc})_{NL}$ = D.C. voltage on no load

$(V_{dc})_{FL}$ = D.C. voltage on full load

then voltage regulation is defined as

$$\text{Voltage regulation} = \frac{(V_{dc})_{NL} - (V_{dc})_{FL}}{(V_{dc})_{FL}} \qquad \text{... (3.1)}$$

Less the value of voltage regulation, better is the performance of rectifier circuit.

For a full wave circuit,

$$(V_{dc})_{NL} = \frac{2\,E_{sm}}{\pi} \qquad \text{... (3.2)}$$

and

$$(V_{dc})_{FL} = I_{DC}\,R_L \qquad \text{... (3.3)}$$

The regulation can be expressed as,

$$\% R = \frac{\frac{2E_{sm}}{\pi} - I_{DC}R_L}{I_{DC}R_L} \times 100$$

Now $$I_m = \frac{E_{sm}}{R_f + R_L + R_s}$$

$\therefore$ $$E_{sm} = I_m(R_f + R_L + R_s)$$

and $$I_{DC} = \frac{2I_m}{\pi}$$

$\therefore$ $$\%R = \frac{\frac{2I_m}{\pi}[R_f + R_L + R_s] - \frac{2I_m}{\pi}R_L}{\frac{2I_m}{\pi}R_L} \times 100$$

$$= \frac{R_f + R_L + R_s - R_L}{R_L} \times 100$$

$$= \frac{R_f + R_s}{R_L} \times 100$$

Neglecting winding resistance R_s, the regulation can be expressed as,

$$\% R \approx \frac{R_f}{R_L} \times 100$$

where R_f = forward resistance of the diode.

3.4.14 Comparison of Full Wave and Half Wave Circuit

For comparison, we assume that the full-wave and half-wave circuits use identical diodes, identical load resistances and the voltage across half the secondary winding of transformer used in full-wave circuit is the same as the voltage across the secondary winding of the transformer used in half-wave circuit.

1. The d.c. load current in case of full wave circuit is twice to that in half wave circuit; similarly the D.C. load voltage in full wave circuit is twice that in half wave circuit.
2. The lowest ripple frequency in full wave circuit is twice that in half wave circuit. Now to remove ripple the additional circuits called filter circuits are used along with rectifier circuits. But as the frequency is more in full-wave, the capacitor values required in capacitance filter are much less hence smaller elements are sufficient in filter circuits used with full wave circuit to reduce ripple.
3. Because there is no net d.c. current through windings of the transformer used in full wave circuit, the losses are less as compared to losses in transformer used in half wave circuit.
4. The full wave connection gives dc power output four times as large, when compared with half wave connection.
5. The efficiency of rectification in a full wave connection is twice that for half wave connection.
6. The ripple factor is less for full-wave, i.e. rectification is more nearly complete for full wave as compared to half-wave.

Ex. 3.3: *A full-wave rectifier circuit is fed from a transformer having a center-tapped secondary winding. The rms voltage from either end of secondary to center tap is 30V. If the diode forward resistance is 2 Ω and that of the half secondary is 8 Ω, for a load of 1 kΩ, calculate*

a) Power delivered to load,
b) % Regulation at full load,
c) Efficiency of rectification,
d) TUF of secondary.

Sol.: Given : $E_s = 30\text{ V},\ R_f = 2\Omega,\ R_s = 8\Omega,\ R_L = 1\text{k}\Omega$

$$E_s = E_{RMS} = 30\text{ V}$$

$$E_{sm} = E_s\sqrt{2} = 30\sqrt{2}\ \text{volt} = 42.426\text{ V}$$

$$I_m = \frac{E_{sm}}{R_f + R_L + R_s} = \frac{30\sqrt{2}}{2+1000+8}\ \text{A}$$

$$= 42\text{ mA}$$

$$I_{DC} = \frac{2}{\pi} I_m = 26.74\text{ mA}$$

a) Power delivered to load

$$= I_{DC}^2\ R_L = (26.74 \times 10^{-3})^2\ (1\text{k}\Omega)$$

$$= 0.715\text{ W}$$

b) V_{DC}, no load $= \frac{2}{\pi} E_{sm} = \frac{2}{\pi} \times 30\sqrt{2}$

$= 27 \text{ V}$

V_{DC}, full load $= I_{DC} R_L = (26.74 \text{ mA}) (1 \text{ k}\Omega)$

$= 26.74 \text{ V}$

$$\% \text{ Regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

$$= \frac{27 - 26.74}{26.74} \times 100$$

$$= 0.97 \%$$

c) Efficiency of rectification $= \frac{\text{D.C. output}}{\text{A.C. input}}$

$$= \frac{8}{\pi^2} \times \frac{1}{1 + \frac{R_f + R_s}{R_L}} = \frac{8}{\pi^2} \times \frac{1}{1 + \frac{(2+8)}{1000}}$$

$$= 0.802 \text{ i.e. } 80.2\%$$

d) Transformer secondary rating

$$= E_{RMS} \; I_{RMS}$$

$$= [30 \text{ V}] \left[\frac{42 \text{ mA}}{\sqrt{2}}\right]$$

$$= 0.89 \text{ W}$$

$\therefore$ T.U.F. $= \frac{\text{D.C. power output}}{\text{A.C. rating}}$

$$= \frac{0.715}{0.89}$$

$$= 0.802$$

3.5 Bridge Rectifier

The bridge rectifier circuits are mainly used as,

a) a power rectifier circuit for converting ac power to dc power, and

b) a rectifying system in rectifier type ac meters, such as ac voltmeter, in which the ac voltage under measurement is first converted into dc and measured with conventional meter. In this system, the rectifying elements are either copper oxide type or selenium type.

The basic bridge rectifier circuit is shown in Fig. 3.13.

The bridge rectifier circuit is essentially a full-wave rectifier circuit, using four diodes, forming the four arms of an electrical bridge. To one diagonal of the bridge, the ac voltage is applied through a transformer if necessary, and the rectified dc voltage is taken from the other diagonal of the bridge. The main advantage of this circuit is that it does not require a center tap on the secondary winding of the transformer. Hence wherever possible, ac voltage can be directly applied to the bridge.

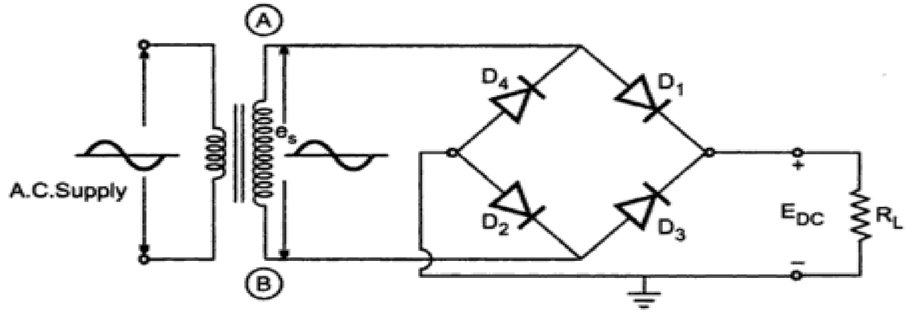


Fig. 3.13 Bridge rectifier circuit

3.5.1 Operation of the Circuit

Consider the positive half of ac input voltage. The point A of secondary becomes positive. The diodes D_1 and D_2 will be forward biased, while D_3 and D_4 reverse biased. The two diodes D_1 and D_2 conduct in series with the load and the current flows as shown in Fig. 3.14.

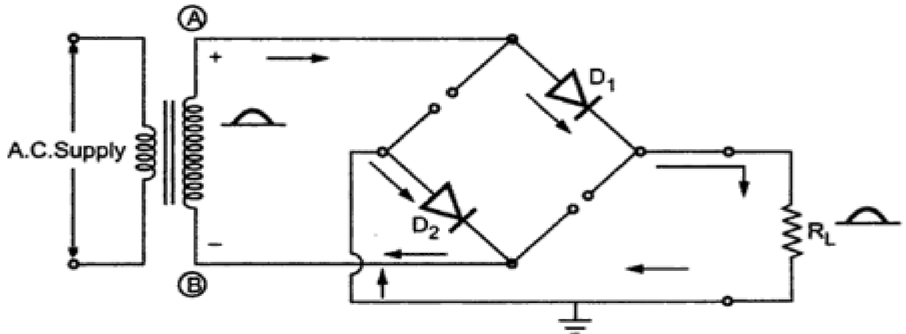


Fig. 3.14 Current flow during positive half cycle

In the next half cycle, when the polarity of ac voltage reverses hence point B becomes positive diodes D_3 and D_4 are forward biased, while D_1 and D_2 reverse biased. Now the diodes D_3 and D_4 conduct is series with the load and the current flows as shown in Fig. 3.14.

It is seen that in both cycles of ac, the load current is flowing in the same direction hence, we get a full-wave rectified output.

The waveforms of load current and voltage remain exactly same as shown before for full- wave rectifier.

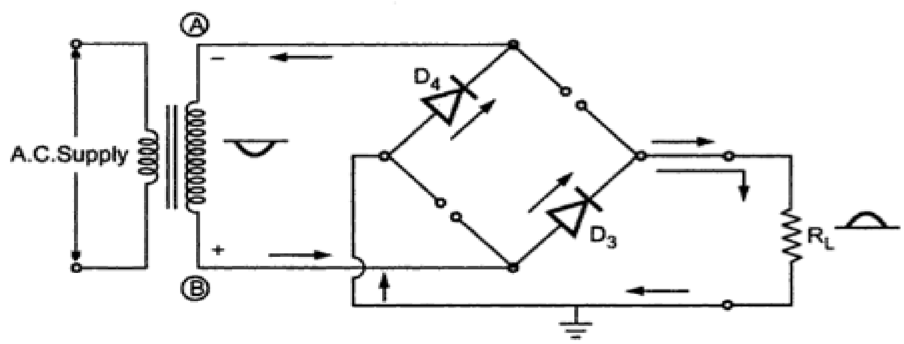


Fig. 3.15 Current flow during negative half cycle

3.5.2 Expressions for Various Parameters

The bridge rectifier circuit, being basically a full-wave rectifier circuit; all the characteristic discussed previously for a full-wave circuit using two diodes, are the characteristic of a bridge rectifier circuit.

The relation between I_m the maximum value of load current and I_{DC} , I_{RMS} remains same as derived earlier for the full wave rectifier circuit.

$$I_{DC} = \frac{2I_m}{\pi} \text{ and } I_{RMS} = \frac{I_m}{\sqrt{2}}$$

The expression for I_m will change slightly. This will be clear from the equivalent circuit shown in the Fig. 3.16.

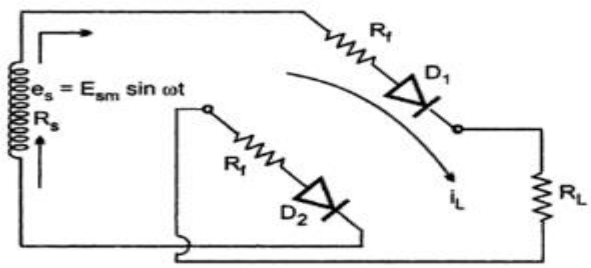


Fig. 3.16

In each half cycle two diodes conduct simultaneously. Hence maximum value of load current is,

$$I_m = \frac{E_{sm}}{R_s + 2R_f + R_L}$$

So the only modification is that instead of R_f , which is forward resistance of each diode, the term 2 R_f appears in the denominator.

The remaining expressions are identical to those derived for two diode full wave rectifier and reproduced for the convenience of the reader.

$$E_{DC} = I_{DC} R_L = \frac{2E_{sm}}{\pi}$$

$$P_{DC} = I_{DC}^2 R_L = \frac{4}{\pi^2} I_m^2 R_L$$

$$P_{AC} = I_{RMS}^2 (R_s + 2R_f + R_L)$$

$$= \frac{I_m^2 (2R_f + R_s + R_L)}{2}$$

$$\eta = \frac{8R_L}{\pi^2 (R_s + 2R_f + R_L)}$$

$$\%\eta_{max} = 81.2\%$$

$$\gamma = 0.48$$

The E_{sm} is the maximum value of a.c. voltage across full secondary winding of the transformer used.

As the current flows through the entire secondary of the transformer for all the time, the transformer utilization factor is 0.812. This is more than the T.U.F for full wave rectifier circuit.

The basic voltage regulation expression remains same as,

$$\% R = \frac{(V_{dc})_{NL} - (V_{dc})_{FL}}{(V_{dc})_{FL}} \times 100$$

Approximately it can be expressed as,

$$\% R \approx \frac{2R_f}{R_L} \times 100$$

3.5.3 Advantages of Bridge Rectifier Circuit

1) The current in both the primary and secondary of the power transformer flows for the entire cycle and hence for a given power output, power transformer of a small size and less cost may be used.
2) No center tap is required in the transformer secondary. Hence, wherever possible, ac voltage can directly be applied to the bridge.
3) The current in the secondary of the transformer is in opposite direction in two half cycles. Hence net d.c. component flowing is zero which reduces the losses and danger of saturation.
4) Due to pure alternating current in secondary of transformer, the transformer gets utilised effectively and hence the circuit is suitable for applications where large powers are required.
5) As two diodes conduct in series in each half cycle, inverse voltage appearing across diodes get shared. Hence the circuit can be used for high voltage applications. Such a peak reverse voltage appearing across diode is called peak inverse voltage rating (PIV) of diode.

3.5.4 Disadvantages of Bridge Rectifier

The only disadvantage of bridge rectifier is the use of four diodes as compared to two diodes in normal full wave rectifier. This causes additional voltage drop as indicated by term $2R_f$ present in expression of I_m instead of R_f . This reduces the output voltage.

Ex. 3.5 : *A 5 kΩ load is fed from a bridge rectifier connected across a tranformer secondary whose primary is connected to 460 V, 50 Hz supply. The ratio of number of primary turns to secondary turns is 2:1.*
Calculate d.c. load current, d.c. load voltage, ripple voltage and P.I.V. rating of diode.

Sol. : $R_L = 5\ k\Omega = 5 \times 10^3\ \Omega,\ N_1 : N_2$ is 2:1

$$E_p = 460\ V_{R.M.S}\ \text{value}$$

$$\therefore \quad \frac{E_s}{E_p} = \frac{N_2}{N_1} = \frac{1}{2}$$

$$\therefore \quad E_s = \frac{1}{2} \times E_p = 230\ V$$

$$\therefore \quad E_{sm} = \sqrt{2} \times E_s = 230 \times \sqrt{2} = 325.269\ V.$$

$$\text{now} \quad I_{DC} = \frac{2 I_m}{\pi} \quad \text{where } I_m = \frac{E_{sm}}{R_L} \text{ neglecting } R_f.$$

$$\therefore \quad I_{DC} = \frac{2 E_{sm}}{\pi R_L} = \frac{2 \times 325.269}{\pi \times 5 \times 10^{13}}$$

$$= 41.41\ mA$$

$$\text{D.C. load voltage } E_{DC} = I_{DC} \times R_L = 41.41 \times 10^{-3} \times 5 \times 10^3$$

$$= 207.072\ V$$

$$\text{Ripple voltage} = \text{Ripple factor} \times V_{DC}$$

Ripple factor for bridge rectifier is 0.482.

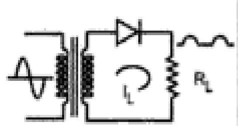
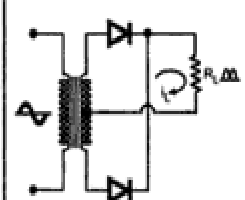
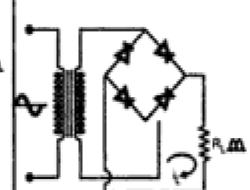
$$\therefore \quad \text{Ripple voltage} = 0.482 \times 207.072$$

$$= 99.8\ V$$

$$\text{P.I.V. rating of each diode} = E_{sm} \text{ for bridge rectifier}$$

$$= 325.27\ V$$

3.6 Comparison of Rectifier Circuits

		Half Wave	Full Wave	Bridge
1.	Circuit diagram with input and output waveform			
2.	Number of diodes	1	2	4
3.	Average DC current (I_{DC})	$\frac{I_m}{\pi}$	$\frac{2I_m}{\pi}$	$\frac{2I_m}{\pi}$
4.	Average DC voltage (E_{DC})	$\frac{E_{sm}}{\pi}$	$\frac{2E_{sm}}{\pi}$	$\frac{2E_{sm}}{\pi}$

5.	RMS current (I_{RMS})	$\frac{I_m}{2}$	$\frac{I_m}{\sqrt{2}}$	$\frac{I_m}{\sqrt{2}}$
6.	DC power output (P_{DC})	$\frac{I_m^2 R_L}{\pi^2}$	$\frac{4}{\pi^2} I_m^2 R_L$	$\frac{4}{\pi^2} I_m^2 R_L$
7.	AC power input (P_{AC})	$\frac{I_m^2 (R_L + R_f + R_s)}{4}$	$\frac{I_m^2 (R_f + R_s + R_L)}{2}$	$\frac{I_m^2 (2R_f + R_s + R_L)}{2}$
8.	Maximum rectifier efficiency (η)	40.6 %	81.2 %	81.2 %
9.	Ripple factor (γ)	1.21	0.482	0.482
10	Maximum load current (I_m)	$\frac{E_{sm}}{R_s + R_f + R_L}$	$\frac{E_{sm}}{R_s + R_f + R_L}$	$\frac{E_{sm}}{R_s + 2R_f + R_L}$

Table 3.1

Module 1 Questions:

1. Draw the circuit diagram of half wave rectifier and explain its operation with the help of waveforms.
2. Derive the expressions for Ripple Factor and Efficiency of Half Wave Rectifier.
3. Derive the expressions for Average DC current, Average DC Voltage, RMS value of current, DC Power Output and AC Power Input of a Half Wave Rectifier.
4. Draw the circuit diagram of Full wave rectifier and explain its operation with the help of waveforms.
5. Derive the expressions for Ripple Factor and Efficiency of Full Wave Rectifier.
6. Derive the expressions for Average DC current, Average DC Voltage, RMS Value of Current, DC Power Output and AC Power Input of a Full Wave Rectifier.
7. A Half wave rectifier has a load of 3.5kΩ. If the diode resistance and the secondary coil Resistance together have resistance of 800Ω and the input voltage of 240V, Calculate (i) Peak, Average and RMS value of the current flowing, (ii) DC power output, (iii) AC Power input and (iv) efficiency of the rectifier.
8. With neat diagram, explain Bridge Rectifier.
9. A bridge rectifier uses four identical diodes having forward resistance of 5Ω each. Transformer secondary resistance is 5Ω and the secondary voltage of 30V (rms). Determine the DC output voltage for I_{DC} = 200mA and the value of the ripple voltage.

3.7 Filter Circuits

It is seen that the output a half-wave or full-wave rectifier circuit is not pure d.c.; but it contains fluctuations or ripple, which is undesired . To minimize the ripple in the output, filter circuits are used. These circuits are connected between the rectifier and load, as shown in Fig. 3.18.

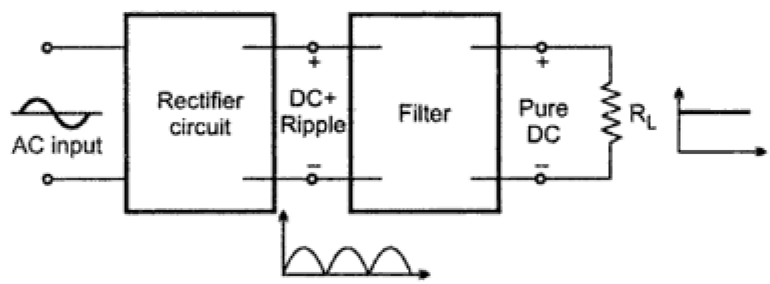


Fig. 3.18 Power supply using rectifier and filter

An ac input is applied to the rectifier. At the output of the rectifier, there will be DC and ripple voltage present, which is the input to the filter. Ideally the output of the filter should be pure DC. Practically, the filter circuit will try to minimize the ripple at the output, as far as possible.

Basically the ripple is ac, i.e. varying with time, while DC is a constant w.r.t. time. Hence in order to separate DC from ripple, the filter circuit should use components which have widely different impedance for a.c. and d.c. Two such components are inductance and capacitance. Ideally, the inductance acts as a short circuit for d.c., but it has a large impedance for a.c.. Similarly, the capacitor acts as open for d.c. and almost short for a.c. if the value of capacitance is sufficiently large enough.

Since ideally, inductance acts as short circuit for d.c., it cannot be placed in shunt arm across the load, otherwise the d.c. will be shorted. Hence, in a filter circuit, the inductance is always connected in series with the load. The inductance used in filter circuits is also called "choke".

Similarly, since the capacitance is open for d.c., i.e. it blocks d.c.; hence it cannot be connected in series with the load. It is always connected in shunt arm, parallel to the load.

Thus filter is an electronic circuit composed of capacitor, inductor or combination of both and connected between the rectifier and the load so as to convert pulsating d.c. to pure d.c.

There are basically two types of filter circuits,

- Capacitor input filter
- Choke input filter

Looking from the rectifier side, if the first element, in the filter circuit is capacitor then the filter circuit is called capacitor input filter.While if the first element is an inductor, it is called choke input filter. The choke input filter is not in use now a days as inductors are bulky, expensive and consume more power. Let us discuss the operation of a capacitor input filter.

3.8 Capacitor Input Filter

The block schematic of capacitor input filter is shown in the Fig. 3.19, looking from the rectifier side the first element in filter is a capacitor.

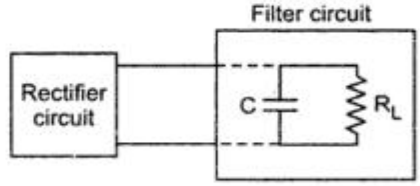


Fig. 3.19

The Fig. 3.20 shows a full -wave rectifier circuit, followed by a capacitor input filter. The filter uses a single capacitor connected in shunt arm i.e. in parallel with the load resistance R_L.

In order to minimize the ripple in the output, the capacitor C used in the filter circuit is quite large, of the order of tens of microfarads.

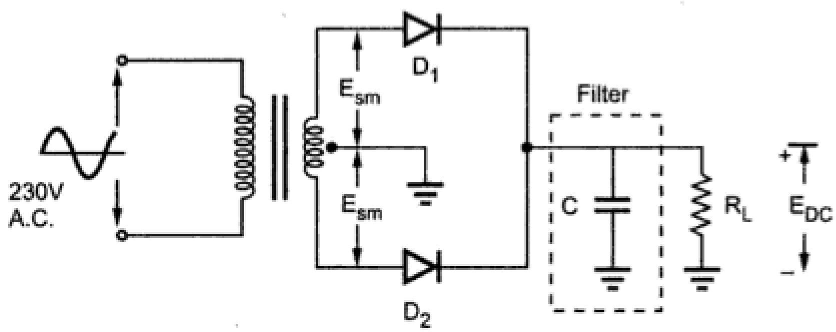


Fig. 3.20 Capacitor input filter

3.8.1 Operation of the Circuit

During the first quarter cycle of the rectified output voltage, obtained from the rectifier circuit, the capacitor C gets charged to peak value E_{sm}. This is shown in the Fig.3.21.

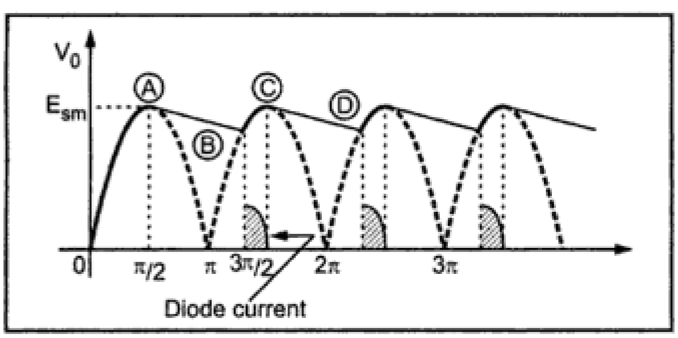


Fig. 3.21

Now in the next quarter cycle from $\frac{\pi}{2}$ to π, the rectifier output voltage, shown dotted starts decreasing. But as capacitor C is charged upto the maximum value E_{sm}, it makes the conducting diode reverse biased and the diode stops conducting. The current through diode reduces to zero when capacitor charges upto E_{sm} which is shown by a shaded portion in the Fig. 3.21.

Now the capacitor C starts discharging through load resistance R_L. As the capacitor C is large, the time constant CR_L is large and capacitor discharges to less extent, from point A to B as shown in Fig. 3.21. At this point, it can be seen that the rectifier output voltage, in the quarter π to $\frac{3\pi}{2}$ exceeds the capacitor voltage, at point B. And the another diode gets forward biased and starts conducting. The capacitor C, again starts charging and quickly gets charged through the forward biased diode having very small forward resistance. The time required by the capacitor to charge to the peak value is quite small and diode current again reduces to zero.

In this circuit, the two diodes are conducting in alternate half cycles of the output of the rectifier circuit. The diodes are not conducting for the entire half cycle but only for a part of the half cycle, during which the capacitor is getting charged. When the capacitor is discharging through the load resistance R_L, both the diodes are non-conducting. The capacitor supplies the load current. As the time required by capacitor to charge is very small and it discharges very little due to large time constant, hence ripple in the output gets reduced considerably. Though the diodes conduct partly, the load current gets maintained due to the capacitor. This filter is very popularly used in practice.

3.8.2 Exact Analysis of Capacitor Input Filter

For the exact analysis of capacitor input filter, consider a basic half wave rectifier circuit using capacitor input filter as shown in the Fig. 3.22.

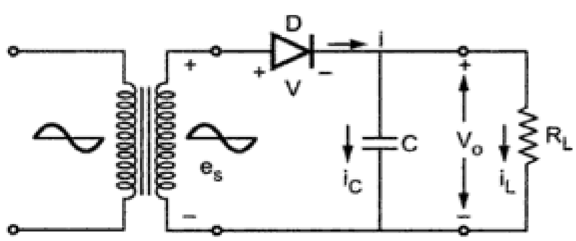


Fig. 3.22

The a.c. input voltage to the rectifier circuit is,

$$e_s = E_{sm} \sin \omega t$$

Let V be the voltage drop across the diode and V_o be the output voltage.

The diode current i has two components, one from the capacitor i_C and other through load resistance R_L which is i_L.

The waveforms of various voltages and currents are shown in the Fig. 3.23.

The waveforms of various voltages and currents are shown in the Fig. 3.23.

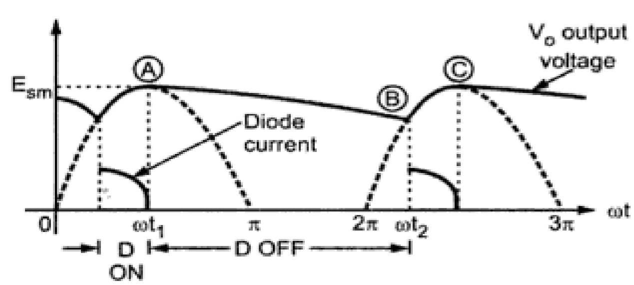


Fig. 3.23

As already seen, the diode conducts only for the period of the half cycle when the capacitor C is charging. The diode current flows only for part of the half cycle shown shaded in the Fig. 3.23. From point A to B the diode is nonconducting, while from B to C it is conducting. The point or time at which the diode starts conducting is called **'cut-in' time** while the time at which it stops conducting is called **'cut-out' time**.

The cut-out time is denoted as ωt_1 while the cut-in time is denoted as ωt_2.

Let us analyse what happens when diode is conducting and diode is not conducting.

I) **Diode conducting :** Neglecting the drop across the diode and drop across the secondary winding of the transformer, the entire transformer voltage is impressed directly across the parallel combination of C and R_L .

$$\text{The diode current } i = \frac{e_s}{Z} = e_s Y \qquad \text{... (3.4)}$$

where $\frac{1}{Z} = Y =$ admittance of C and R_L in parallel.

$$\therefore \quad Y = \sqrt{\frac{1}{R_L^2} + \omega^2 C^2} \angle \phi$$

where $\phi = \tan^{-1}(\omega C R_L)$

$$\therefore \quad i = e_s Y = E_{sm} \sin \omega t . [Y]$$

$$i = E_{sm}\left[\sqrt{\frac{1}{R_L^2} + \omega^2 C^2}\right] \sin(\omega t + \phi) \qquad \text{... (3.5)}$$

At the cutout time t_1, the diode current becomes zero.

$$\therefore \quad 0 = E_{sm}\left[\sqrt{\frac{1}{R_L^2} + \omega^2 C^2}\right] \sin(\omega t_1 + \phi) \qquad \text{... (3.6)}$$

To satisfy the above equation, $\sin(\omega t_1 + \phi) = 0$

$\therefore$ Trignometrically $\sin(\omega t_1 + \phi) = n\pi$

For n = 1, $\pi = (\omega t_1 + \phi)$

$$\therefore \quad \omega t_1 = \pi - \phi = \pi - \tan^{-1}(\omega C R_L) \qquad \text{... (3.7)}$$

$$\text{and at} \quad t = t_1, V_o = E_{sm} \sin(\omega t_1) \qquad \text{... (3.8)}$$

II) **Diode Non-Conducting :** During the time interval between t_1 and t_2, transformer voltage is less than the voltage across the capacitor C. Hence the diode is reverse biased and does not conduct. The capacitor C discharges exponentially through R_L with a time constant CR_L. The capacitor voltage, which is also the output voltage, is given by,

$$V_o = Ke^{-t/CR_L} \qquad \text{... (3.9)}$$

where K depends on initial conditions.

But at $t = t_1$, $V_o = E_{sm} \sin \omega t_1$

Hence, substituting in above equation, we get

$$E_{sm} \sin \omega t_1 = K e^{-t_1/CR_L}$$

$$\therefore \quad K = \frac{E_{sm} \sin \omega t_1}{e^{-t_1/CR_L}} = E_{sm} \sin \omega t_1 \times e^{t_1/CR_L}$$

Therefore, $$V_o = \left[E_{sm} \sin \omega t_1 \times e^{+t_1/CR_L} \right] \times e^{-t_1/CR_L}$$

$$\therefore \quad V_o = [E_{sm} \sin \omega t_1] \, e^{\frac{-(t-t_1)}{CR_L}} \quad \text{... (3.10)}$$

where this exponentially decaying V_o, intersects the sinusoidal curve $E_{sm} \sin \omega t$ in the following cycle, at the cut-in point t_2. This is point B in the Fig. 3.23.

At this point B, transformer voltage is more than the voltage across C and it is increasing. Hence the diode becomes forward biased and starts conducting. The same cycle repeats again.

Thus we have seen that,

$$V_o = E_{sm} \sin \omega t \qquad \text{for } \omega t_2 < \omega t < \omega t_1$$

and $$V_o = [E_{sm} \sin \omega t_1] \, e^{-(t-t_1)/CR_L}$$

$$\text{for } \omega t_1 < \omega t < (2\pi + \omega t_2)$$

Here ωt_1 and ωt_2 represent the cut-out and cut-in angles in the first half cycle, respectively.

And $$\omega t_1 = \pi - \tan^{-1}(\omega C R_L) \quad \text{... (3.11)}$$

The analysis of a full wave rectifier circuit with capacitor input filter follows in the same fashion. The cut-out point is the same as found for half wave rectifier, however, the cut-in point lies between π and 2π, as shown in the Fig. 3.24.

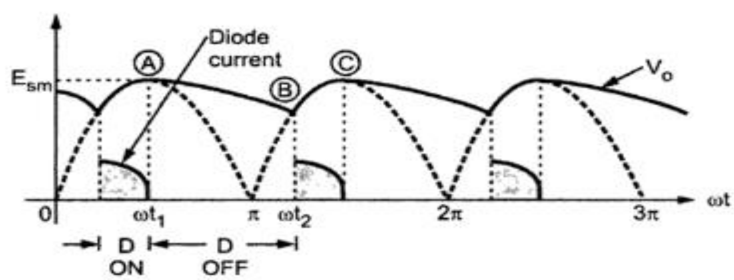


Fig. 3.24

3.8.3 Derivation of Expression of Ripple Factor for Capacitor Input Filter

Consider a full wave rectifier circuit using a capacitor input filter.

The basic circuit diagram and various waveforms for the circuit are shown in the Fig. 3.25 (a) and 3.25 (b).

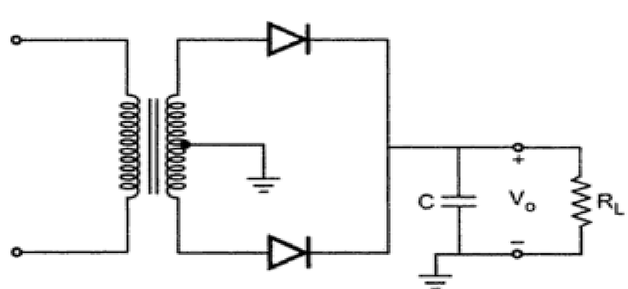


Fig. 3.25 (a)

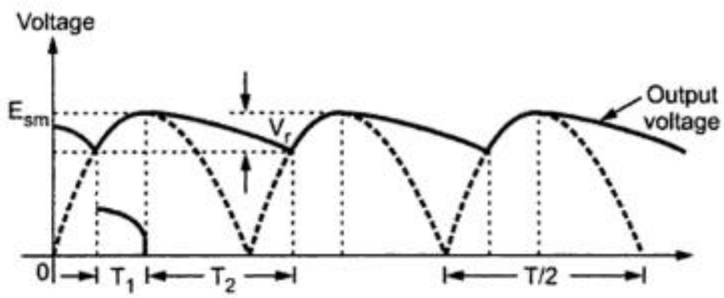


Fig. 3.25 (b)

Let $\frac{T}{2}$ be the time for half cycle of a.c. input voltage.

During the time interval T_1 the diode is conducting and the capacitor C is getting charged.

While in the time interval T_2 , the diode is reverse biased and the capacitor discharges through the load resistance R_L .

Let V_r be the peak to peak value of ripple voltage, which is assumed to be triangular as shown in the Fig. 3.26.

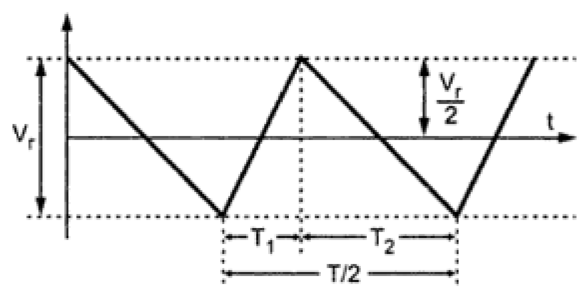


Fig. 3.26 Triangular approximation of ripple voltage

It can be shown mathematically that the r.m.s. value of such a triangular waveform is

$$V_{rms} = \frac{V_r}{2\sqrt{3}} \quad \text{... (3.12)}$$

During the time interval T_2, the capacitor C is discharging through the load resistance R_L . The charge lost is,

$$Q = CV_r \quad \text{... (3.13)}$$

But $$i = \frac{dQ}{dt}$$

$\therefore$ $$Q = \int_0^{T_2} i\,dt = I_{DC}\,T_2$$

As **integration gives average or dc value**

Hence $$I_{DC} T_2 = CV_r \quad \text{... (3.14)}$$

$$\therefore \quad V_r = \frac{I_{DC} T_2}{C}$$

Now, $$T_1 + T_2 = \frac{T}{2} \qquad \text{Normally, } T_2 >> T_1$$

$$\therefore \quad T_1 + T_2 \approx T_2 = \frac{T}{2} \qquad \text{where } T = \frac{1}{f}$$

$$\therefore \quad V_r = \frac{I_{DC}}{C}\left[\frac{T}{2}\right] = \frac{I_{DC} \times T}{2C} = \frac{I_{DC}}{2\,fC}$$

But $$I_{DC} = \frac{E_{DC}}{R_L}$$

$$\therefore \quad V_r = \frac{E_{DC}}{2\,f\,C\,R_L} \quad \text{... (3.15)}$$

$$\text{Ripple factor} = \frac{V_{rms}}{E_{DC}} = \frac{\frac{E_{DC}}{2\,f\,C\,R_L}}{2\sqrt{3}} \times \frac{1}{E_{DC}}, \text{ Since } V_{rms} = \frac{V_r}{2\sqrt{3}}$$

$$\therefore \quad \text{Ripple factor} = \frac{1}{4\sqrt{3}\,f\,C\,R_L} \quad \text{for full wave} \quad \text{... (3.16)}$$

For **half wave rectifier** with capacitor input filter the ripple factor is

$$\text{Ripple factor} = \frac{1}{2\sqrt{3}\,f\,C\,R_L} \quad \text{for half wave} \quad \text{... (3.17)}$$

The product CR_L is the time constant of the filter circuit.

From the expression of the ripple factor, it is clear that increasing the value of capacitor C, the ripple factor gets decreased. Thus the output can be made smoother, reducing the ripple content by selecting large value of capacitor. However very large value of capacitor cannot be used because larger the value of capacitor, larger the initial charging surge current. This may exceed the current rating of the diodes in the rectifier. Otherwise the diodes used must be of higher current rating which increases the cost.

Similarly, the another factor controlling the ripple factor is load resistance R_L. As the load current drawn increases, for the same d.c. output voltage, the load resistance decreases. This increases the ripple contents in the output. Hence the filter is not suitable for the variable loads. The desirable feature of the filter is high voltage and less ripple at the output for small load currents.

The d.c. output voltage from a **capacitor filter** fed from a **full wave rectifier** is given by,

$$E_{DC} = E_{sm} - I_{DC}\left[\frac{1}{4\,fC}\right]$$

While the d.c. output voltage from a **capacitor filter** fed from a **half wave rectifier** is given by,

$$E_{DC} = E_{sm} - I_{DC}\left[\frac{1}{2\,fC}\right]$$

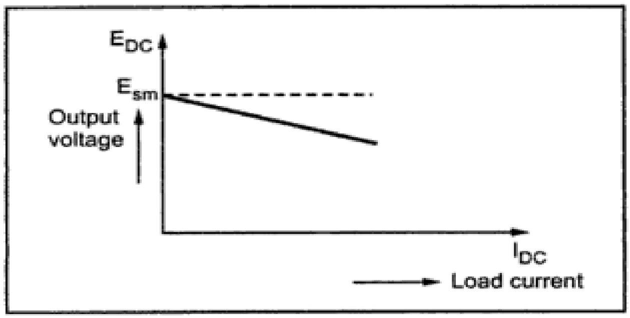


Fig. 3.27 Load regulation with capacitor input filter

From the above expressions, it can be seen that as the current drawn by the load increases, the d.c. output voltage decreases. Hence this filter circuit is having poor regulation. The load regulation graph i.e. regulation characteristics for the capacitor input filter is shown in the Fig. 3.27.

Ex. 3.9 : *A 100 μF capacitor, when used as filtering element, has 12 V, d.c. across it with a terminal load resistance of 2 k Ω . If the rectifier is full-wave and supply frequency is 50 Hz, what is the percentage of ripple in the output ? Draw the neat circuit diagram.*

Sol. : The circuit diagram is shown in the Fig. 3.28.

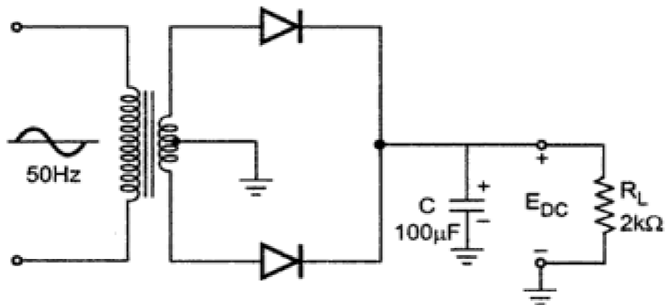


Fig. 3.28

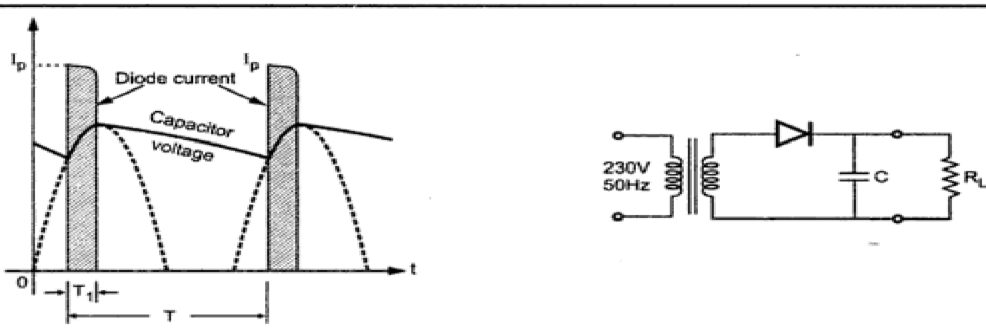
Given : $R_L = 2k\,\Omega$, $C = 100\,\mu F$, $E_{DC} = 12$ V, Supply frequency = 50 Hz

$$\text{Ripple factor} = \frac{1}{4\sqrt{3}\, f\, C\, R_L} = \frac{1}{4\sqrt{3}\,[50][100\times10^{-6}\,][2\times10^{3}\,]}$$

$$= \frac{1}{4\sqrt{3}\,(50)\,(2\times10^{-1}\,)} = 0.01443$$

$\therefore$ % of ripple in the output $= 0.01443 \times 100 = 1.443$ %

3.8.4 Surge Current in Rectifier using Capacitor-Input Filter



(a) (b)

Fig. 3.29 Surge current in rectifier using capacitor input filter

We have seen that in a rectifier circuit using capacitor - input filter, the diode is forward biased for only a short period of half cycle and it conducts only during this time interval to charge the filter capacitor. When the diode is not conducting, the capacitor discharges through the load resistor R_L. Hence the total amount of charge that flows through the forward biased diode(s) to recharge the capacitor must equal the amount of charge lost during the period the filter capacitor discharges into R_L. This is shown in Fig. 3.29 (a) and (b). (See Fig. on previous page) When the load current is small, i.e. R_L is large, the capacitor voltage does not decay significantly, so the conduction interval T_1 is small compared to the period T between rectifier pulses.

Let I_{DC} be the average or dc current and I_p the peak current. Assume the current pulse to be rectangular.

Then

$$Q\,(\text{discharge}) = Q(\text{charge}) \qquad \text{... (3.18)}$$

$$\therefore \quad I_{DC}\,T = I_p\,T_1$$

$$\therefore \quad I_p = I_{DC}\,\frac{T}{T_1} \qquad \text{... (3.19)}$$

Since $T >> T_1$, the peak current through the diode may be many times larger than the average current supplied to the load. The diode(s) must be capable of supplying this repetitive surge current.

Increasing the value of filter capacitance C decreases the ripple and increases the DC voltage. However, increased capacitance shortens the conduction period T_1 as the capacitor voltage decays less when $R_L C$ time constant is large. Hence the large capacitance increases the peak diode current. To limit surge current, sometimes a small resistance is connected between the rectifier and the filter capacitor, as shown in Fig.3.30.

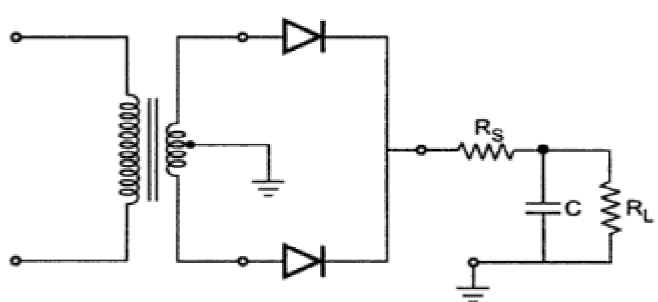


Fig. 3.30 Prevention of surge current

However, this resistance R_s will reduce the DC output voltage.

3.9 Inductor Filter or Choke Filter

In this type of filter, an inductor (choke) is connected in series with the load. It is known that the inductor opposes change in the current. So the ripple which is change in the current is opposed by the inductor and it tries to smoothen the output. Consider a full wave rectifier with inductor filter which is also called choke filter. Fig. 3.31 (a) shows the circuit diagram while Fig. 3.31 (b) shows the current waveform obtained by using choke filter with full wave rectifier.

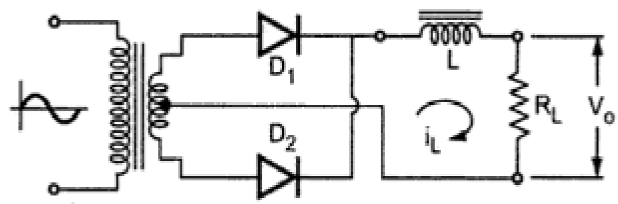


Fig. 3.31 (a) Circuit diagram of choke filter

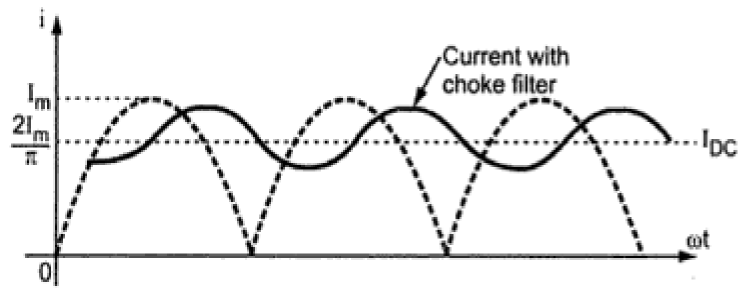


Fig. 3.31 (b) Current waveform of choke filter

3.9.1 Operation of the Circuit

In the positive half cycle of the secondary voltage of the transformer, the diode D_1 is forward biased. Hence the current flows through D_1, L and R_L. While in the negative half cycle, the diode D_1 is reverse biased while diode D_2 is forward biased. Hence the current flows through D_2, L and R_L. Hence we get unidirectional current through R_L. Due to inductor L which opposes change in current, it tries to make the output smooth by opposing the ripple content in the output.

We know that the fourier series for the load current for full wave rectifier as,

$$i_L = I_m \left[\frac{2}{\pi} - \frac{4}{3\pi} \cos 2\omega t - \frac{4}{15\pi} \cos 4\omega t \right]$$

Neglecting higher order harmonics we get,

$$i_L = \frac{2I_m}{\pi} - \frac{4I_m}{3\pi} \cos 2\omega t \quad \text{... (3.20)}$$

Neglecting diode forward resistances and the resistance of choke and transformer secondary we can write the d.c. component of current as

$$\frac{2I_m}{\pi} = \frac{2V_m}{\pi R_L} \quad \text{... (3.21)}$$

as $$I_m = \frac{V_m}{R_L}$$

While the second harmonic component represents a.c. component or ripple present and can be written as,

$$I_m = \frac{V_m}{Z} \text{ for a.c. component} \quad \text{... (3.22)}$$

Now

$$Z = R_L + j2X_L = \sqrt{R_L^2 + 4\omega^2 L^2} \angle \phi \quad \text{... (3.23)}$$

where

$$\phi = \tan^{-1}\frac{2\omega L}{R_L} \quad \text{... (3.24)}$$

$$\therefore \quad I_m = \frac{V_m}{\sqrt{R_L^2 + 4\omega^2 L^2}} \quad \text{... (3.25)}$$

The ripple present is the second harmonic component having frequency 2ω. Hence while calculations the effective inductive reactance must be calculated at 2ω hence represented as $2X_L$ in the above expression.

Hence equation (3.20) modifies as,

$$i_L = \frac{2V_m}{\pi R_L} - \frac{V_m}{3\pi\sqrt{R_L^2 + 4\omega^2 L^2}} \cos(2\omega t - \phi) \quad \text{... (3.26)}$$

3.9.2 Expression for the Ripple Factor

Ripple factor is given by,

$$\text{Ripple factor} = \frac{I_{rms}}{I_{DC}} \quad \text{... (3.27)}$$

where

$$I_{rms} = \frac{I_m}{\sqrt{2}} \text{ of a.c. component}$$

$$I_{rms} = \frac{4V_m}{3\sqrt{2}\,\pi\sqrt{R_L^2 + 4\omega^2 L^2}} \quad \text{... (3.28)}$$

while

$$I_{DC} = \frac{2V_m}{\pi R_L} \quad \text{... (3.29)}$$

$$\therefore \quad \text{Ripple factor} = \frac{\dfrac{4V_m}{3\sqrt{2}\,\pi\sqrt{R_L^2 + 4\omega^2 L^2}}}{\dfrac{2V_m}{\pi R_L}} \quad \text{... (3.30)}$$

$$= \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{1 + \dfrac{4\omega^2 L^2}{R_L^2}}} \quad \text{... (3.31)}$$

Initially on no load condition, $R_L \to \infty$ and hence $\frac{4\omega^2 L^2}{R_L^2} \to 0$.

$$\therefore \quad \text{Ripple factor} = \frac{2}{3\sqrt{2}} = 0.472 \quad \text{... (3.32)}$$

This is very close to normal full wave rectifier without filtering.

But as load increases, R_L decreases hence $\frac{4\omega^2 L^2}{R_L^2} >> 1$. So neglecting 1 we get,

$$\text{Ripple factor} = \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{\frac{4\omega^2 L^2}{R_L^2}}} \quad \text{... (3.33)}$$

$$= \frac{R_L}{3\sqrt{2}.\omega L} \quad \text{... (3.34)}$$

So as load changes, ripple charges which is inversely proportional to the value of the inductor. Smaller the value of R_L, smaller is the ripple hence the filter is suitable for low load resistances i.e. for high load current applications.

3.10 L-Section Filter

This is also called **choke input filter** as the filter element looking from the rectifier side is an inductance L. The d.c. winding resistance of the choke is R_x. The circuit is also called L-type filter or LC filter. The circuit is shown in the Fig. 3.32.

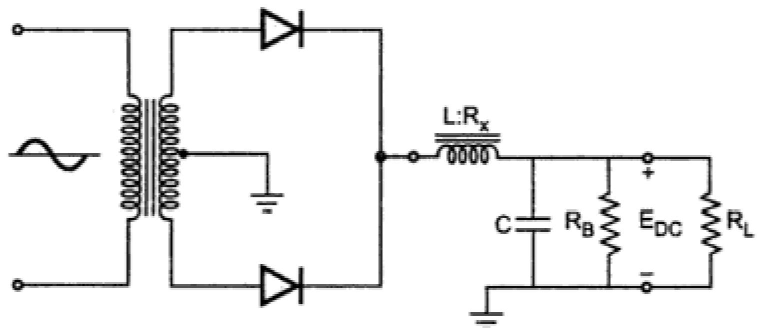


Fig. 3.32 Choke input filter

The basic requirement of this filter circuit is that the current through the choke must be continuous and not interrupted. An interrupted current through the choke may develop a large back e.m.f. which may be in excess of PIV rating of the diodes and /or maximum voltage rating of the capacitor C. Thus this back e.m.f. is harmful to the diodes and capacitor. To eliminate the back e.m.f. developed across the choke, the current through it must be maintained continuous.

This is assured by connecting a bleeder resistance, R_B across the output terminals.

We have seen that the lowest ripple frequency for a full wave rectifier circuit is twice the supply frequency of a.c. input voltage to the rectifier. Let f, in Hz, be the supply frequency. Then angular supply frequency will be ω rad/s, where $\omega = 2\pi f$. Then the lowest ripple angular frequency will be "2ω" rad/s.

3.10.1 Derivation of Ripple Factor

The analysis of the choke input filter circuit is based on the following assumptions :

Since the filter elements, L and C, are having reasonably large values, the reactance X_L of the inductance of L at 2ω i.e. $X_L = 2\omega L$ is much larger than R_x. Also the reactance X_L is much larger than the reactance of C, X_C at 2ω as,

$$X_C = \frac{1}{2\omega C} \quad \text{... (3.35)}$$

Let R be the equivalent resistance of bleeder resistance R_B and the load resistance R_L connected in parallel. Then ,

$$R = R_B || R_L = \frac{R_B R_L}{R_B + R_L} \quad \text{... (3.36)}$$

We will assume that reactance of C at '2ω' is much less than R, i.e. $X_C << R$.

The capacitance C is in parallel with R. Hence the equivalent impedance of X_C and R will be nearly equal to X_C, as per our assumption.

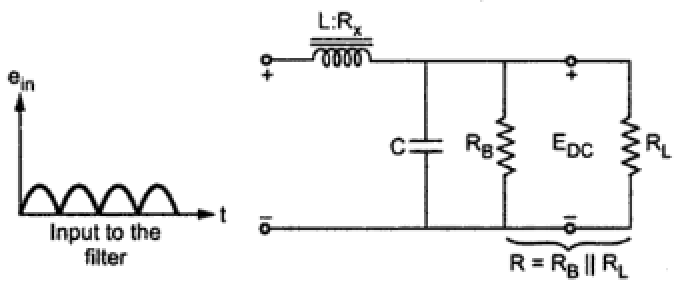


Fig. 3.33

The input voltage e_{in}, to the choke-input filter is the output voltage of the full wave rectifier, having the waveform as illustrated in Fig. 3.33. Using Fourier series, the input voltage "e_{in}" can be written as,

$$e_{in} = E_{sm}\left[\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t - \frac{4}{15\pi}\cos 4\omega t \ldots\right] \quad \text{... (3.37)}$$

where E_{sm} = the maximum value of half secondary voltage of the transformer.

The first term $\frac{2}{\pi} E_{sm}$, in the Fourier series indicates the d.c. output voltage of the rectifier, while the remaining terms ripple. The amplitude of the lowest ripple component, which is the second harmonic component of the supply frequency, is $\frac{4}{3\pi}$ while the amplitude of the fourth harmonic component, 5ω, is $\frac{4}{15\pi}$. The amplitude of the fourth harmonic is just one-fifth or 20% of the amplitude of second harmonic component. The higher harmonics will have still less amplitudes compared to the amplitude of the second harmonic component. Hence all harmonics, except the second harmonic, can be neglected. The equation for "e_{in}" can now be approximately written as,

$$e_{in} \approx E_{sm}\left[\frac{2}{\pi} - \frac{4}{3\pi}\cos 2\omega t\right] \quad \text{... (3.38)}$$

The d.c. current in the circuit will be,

$$I_{DC} = \frac{\frac{2}{\pi} E_{sm}}{R_x + R} \qquad \text{... (3.39)}$$

$$R = R_B || R_L$$

$$\therefore \quad E_{DC} \text{ across the load} = I_{DC} R = \frac{2}{\pi} \frac{E_{sm}}{R_x + R} \times R$$

$$\therefore \quad E_{DC} = \frac{2}{\pi} \frac{E_{sm}}{1 + \frac{R_x}{R}} \qquad \text{... (3.40)}$$

Normally, R_x is much less than R, i.e. $R_x << R$

Then, $$E_{DC} \approx \frac{2}{\pi} E_{sm} \qquad \text{... (3.41)}$$

Thus the choke input filter circuit gives approximately constant d.c. voltage across the load. In other words, this filter circuit is having better load regulation compared to that of capacitor input filter in which case the d.c. load voltage depends upon the d.c. load current drawn. Let us calculate the ripple factor for choke input filter, based on the assumptions already made.

The impedance Z_2 of the filter circuit for second harmonic component of input, i.e. at 2ω, will be,

$$Z_2 = (R_x) + (j\, 2\omega L) + \left[\frac{1}{j 2\omega C} || R \right] \qquad \text{... (3.42)}$$

But, $\frac{1}{2\omega C} << R$, and $2\omega L >> R_x$, as per assumptions.

Hence, $$|Z_2| \approx 2\omega L \qquad \text{... (3.43)}$$

Second harmonic component of the current in the filter circuit, will be

$$I_{2m} = \frac{\frac{4}{3\pi} E_{sm}}{Z_2} \approx \frac{\frac{4}{3\pi} E_{sm}}{2\omega L} \qquad \text{... (3.44)}$$

The second harmonic voltage across the load is

$$E_{2m} = I_{2m} \times \left[\frac{1}{2\omega C} || R \right] \approx I_{2m} \times \frac{1}{2\omega C} \qquad \text{... (3.45)}$$

Since , $$\frac{1}{2\omega C} << R$$

$$\therefore \quad E_{2m} = I_{2m} \times \frac{1}{2\omega C} = \frac{\frac{4}{3\pi} E_{sm}}{2\omega L} \times \frac{1}{2\omega C} \qquad \text{... (3.46)}$$

$$\therefore \quad E_{2m} = \frac{4}{3\pi} \frac{E_{sm}}{4\omega^2 LC} = \frac{E_{sm}}{3\pi\omega^2 LC} \qquad \text{... (3.47)}$$

$$\therefore \quad E_{2rms} = \frac{E_{2m}}{\sqrt{2}} = \frac{E_{sm}}{3\sqrt{2}\,\pi\,\omega^2 LC} \qquad \text{... (3.48)}$$

Hence the ripple factor is given by,

$$\text{Ripple factor} = \frac{E_{2rms}}{E_{DC}} \quad \text{... (3.49)}$$

$$= \frac{E_{sm}}{3\sqrt{2}\,\pi\,\omega^2\,LC} \times \frac{1}{\frac{2}{\pi}\frac{E_{sm}}{1+\frac{R_x}{R}}}$$

$$= \frac{1}{6\omega^2\,LC\sqrt{2}}\left(1+\frac{R_x}{R}\right) \quad \text{but } R_x << R$$

$$\therefore \quad \text{Ripple factor} \approx \frac{1}{6\sqrt{2}\,\omega^2\,LC} \quad \text{... (3.50)}$$

It is seen that the ripple factor for choke-input filter does not depend upon the load resistance unlike the capacitor input filter.

3.10.2 The Necessity of Bleeder Resistance R_B

We have seen,

$$I_{DC} = \frac{2}{\pi}\frac{E_{sm}}{R_x+R}$$

$$I_{2m} = \frac{4}{3\pi}\frac{E_{sm}}{2\omega L}$$

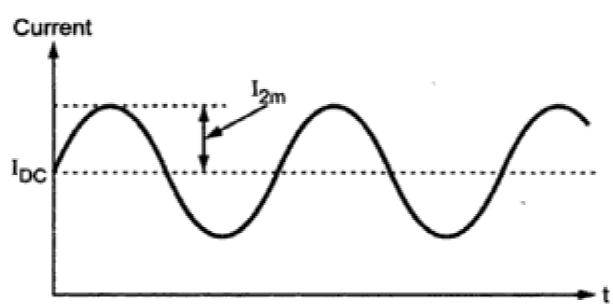


Fig. 3.34 (a)

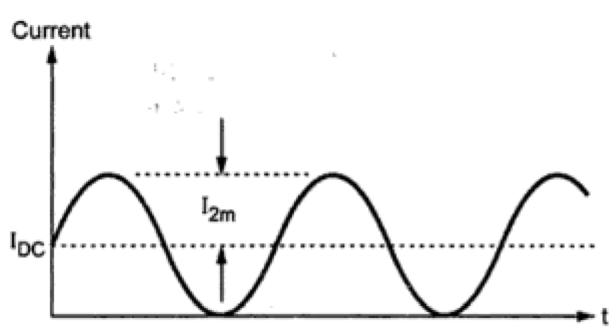


Fig. 3.34 (b)

Thus I_{DC} is seen to depend on load resistance $[R = R_B \| R_L]$ while I_{2m} does not. I_{2m} is constant, independent of R_L. The second harmonic current I_{2m} is superimposed on I_{DC}, as shown in Fig. 3.34 (a).

If the load resistance is increased, I_{DC} will decrease, but I_{2m} will not. If the load resistance is still further increased, a stage may come where I_{DC} may become less than I_{2m}. In such situation, for a certain period of time in each cycle, the net current in the circuit will be zero. In other words, the current will be interrupted and not continuous. As already, explained, this interruption of current, producing large back e.m.f. is harmful to both the diodes and filter capacitor C. To avoid such situation, certain minimum load current has to be drawn. For this purpose, the bleeder resistance R_B is connected. The bleeder resistance R_B is so selected that it draws, a minimum current through choke, the condition for which is,

$$I_{DC} = I_{2m} \quad \text{... (3.51)}$$

$$\therefore \quad \frac{2}{\pi}\frac{E_{sm}}{R_x + R} = \frac{4}{3\pi}\frac{E_{sm}}{2\omega L} \quad \text{... (3.52)}$$

$$\therefore \quad R + R_x = 3\omega L \quad \text{... (3.53)}$$

Usually $R_x << R$, then $R \approx 3\omega L$

Considering the worst case that the load resistance R_L is not connected, then $R = R_B$

$$\text{Since} \quad R = R_B \| R_L$$

$$\therefore \quad R_B = 3\omega L = 3\,(2\,\pi f)\,L \quad \text{... (3.54)}$$

$$f = 50 \text{ Hz}$$

$$\therefore \quad R_B = 3 \times 2 \times \pi \times 50\,L = 943\,L \quad \text{... (3.55)}$$

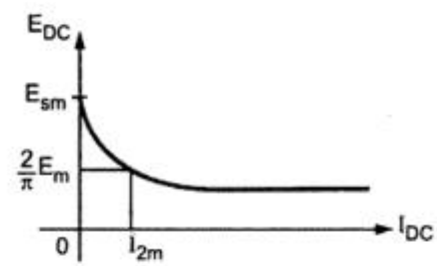


Fig. 3.35

Practically, R_B is selected to be equal to 900 L. The values of L and C are selected depending on the ripple factor desired.

The regulation characteristic of the choke-input filter is shown in Fig. 3.34.

So long as the value of $[R + R_x]$ is less than $3\omega L$, the d.c. load voltage is approximately constant at $\frac{2}{\pi}E_{sm}$, since I_{DC} is then greater than I_{2m}. However, as $[R + R_x]$ becomes larger than $3\omega L$, i.e. for light loads, the d.c. voltage increases and approaches E_{sm} as $R \rightarrow \infty$, since I_{DC} is smaller than I_{2m}.

3.11 Comparison Between Full Wave Choke Input and Capacitor Input Filters

For capacitor input filter, the dc output voltage is high as the capacitor always charges to peak value, but the regulation is poor as compared to choke input type. For capacitor input type, ripple voltage increases with the load current increase, unlike the choke input where the ripple is independent of the load.

The capacitor input filters are suitable for fixed loads, whereas choke input is always used when good regulation of the d.c. load voltage is important and especially when a variable load is to be fed. The choke-input filters are normally employed in polyphase rectifier systems employing mercury-arc rectifiers where the arc is not to be allowed to extinguish.

3.12 CLC Filter or π Filter

This is a capacitor filter followed by a L section filter. The ripple rejection capability of π filter is very good. It is shown in the Fig. 3.36.

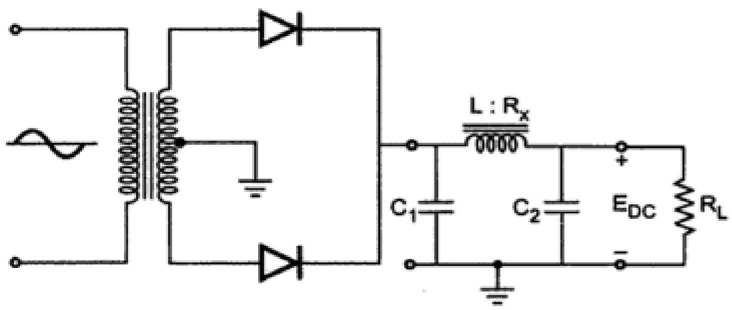


Fig. 3.36 π type filter

It consists of an inductance L with a d.c. winding resistance as R_x and two capacitors C_1 and C_2. The filter circuit is fed from full wave rectifier. Generally two capacitors are selected equal. This circuit is basically a capacitor input filter since the first element looking from the rectifier side is a capacitor. All the features, advantages, disadvantages discussed previously for the capacitor input filter using single capacitor are applicable equally to the π filter.

The rectifier output is given to the capacitor C_1. This capacitor offers very low reactance to the a.c. component but blocks d.c. component. Hence capacitor C_1 bypasses most of the a.c. component. The d.c. component then reaches to the choke L. The choke L offers very high reactance to a.c. component and low reactance to d.c. So it blocks a.c. component and does not allow it to reach to load while it allows d.c. component to pass through it. The capacitor C_2 now allows to pass remaining a.c. component and almost pure d.c. component reaches to the load. The circuit looks like a π hence called π filter. To obtain almost pure d.c. to the load, more such π sections may be used one after another.

The output voltage is given by,

$$E_{DC} = E_{sm} - \frac{V_r}{2} - I_{DC} R_x$$

where V_r = Peak to peak ripple voltage

$$R_x = \text{D.C. resistance of choke}$$

$$\text{Now} \quad V_r = \frac{I_{DC}}{2fC} \text{ for full wave}$$

$$\text{and} \quad = \frac{I_{DC}}{fC} \text{ for half wave}$$

3.12.1 Ripple Factor

The ripple factor for this filter is given by ,

$$\text{Ripple factor} = \frac{\sqrt{2} \times X_{C_1} \times X_{C_2}}{X_L \; R_L}$$

The various reactances X_{C_1}, X_{C_2}, X_L are to be calculated at twice the supply frequency since the circuit is fed from a full wave rectifier circuit.

Hence,

$$X_{C_1} = \frac{1}{2\omega C_1}$$

$$X_{C_2} = \frac{1}{2\omega C_2}$$

$$X_L = 2\omega L$$

$$\therefore \quad \text{Ripple factor} = \frac{\sqrt{2}\left(\frac{1}{2\omega C_1}\right)\left(\frac{1}{2\omega C_2}\right)}{(2\omega L)\,(R_L)}$$

$$\therefore \quad \gamma = \frac{\sqrt{2}}{8\omega^3 \, LC_1 \, C_2 \, R_L}$$

Since this π type filter employes three fitering elements, the riple is reduced to the great extent.

If C_1 and C_2 are expressed in microfarads and frequency f is assumed to be 50 Hz then we get,

$$\gamma = \frac{\sqrt{2}}{8(2\pi \times 50)^3 \times \left(C_1 \times 10^{-6} \times C_2 \times 10^{-6} \times L \times R_L\right)}$$

$$\therefore \quad \gamma \approx \frac{5700}{L\, C_1 C_2 R_L}$$

where C_1 and C_2 are in µF, L in henries and R_L in ohms.

Ex. 3.11 : *Calculate the ripple factor for a π type filter, employing 10 H choke and two equal capacitors 16 µF each and fed from a full wave rectifier and 50 Hz mains. The load resistance is 4 kΩ.*

Sol. : The given values are,

$C_1 = C_2 = 16\ \mu F$, $L = 10\ H$, $R_L = 4\ k\Omega = 4000\ \Omega$, $f = 50\ Hz$

$$\therefore \quad \gamma = \frac{\sqrt{2}}{8\omega^3 LC_1C_2R_L}$$

$$= \frac{\sqrt{2}}{8(2\pi f)^3 LC_1C_2R_L}$$

$$= \frac{\sqrt{2}}{8(2\pi \times 50)^3 \times 10 \times 16 \times 10^{-6} \times 16 \times 10^{-6} \times 4000}$$

$$= 5.56 \times 10^{-4}$$

Thus the ripple is 0.055% which shows that with π filter the output is almost pure d.c.

Students can use another expression for γ which will give same result but while using it, remember that C_1, C_2 are in μF, f is 50 Hz, L in henries and R_L in ohms.

$$\therefore \quad \gamma = \frac{5700}{10 \times 16 \times 16 \times 4000} = 5.56 \times 10^{-4}$$

This is same as calculated above.

Ex. 3.25 : *For a capacitor input filter connected to full wave rectifier circuit, the output voltage is 20 V. This was obtained for a load of $R_L = 200\ \Omega$ and $C = 50\ \mu F$. Calculate the ripple factor. Assume $f = 50$ Hz.*

Sol. : The ripple factor is given by

$$\text{Ripple factor} = \frac{1}{4\sqrt{3} f C R_L}$$

$$= \frac{1}{4\sqrt{3} \times 50 \times 50 \times 10^{-6} \times 200}$$

$$= 0.2886$$

$$\therefore \quad \%\ \text{Ripple in the output} = 0.2886 \times 100 = 28.88\%$$

Comparison of Filters

Table . shows the comparison of various types of filters, when used with full-wave circuits. In all these filters, the resistances of diodes, transformer and filter elements are considered negligible and a 60 Hz power line is assumed.

Table Comparison of Various Types of Filters

	Type of Filter				
	None	*L*	*C*	*L-Section*	*π-Section*
V_{dc} at no load	$0.636\ V_m$	$0.636\ V_m$	V_m	V_m	V_m
V_{dc} at load I_{dc}	$0.636\ V_m$	$0.636\ V_m$	$V_m - \frac{4170\ I_{dc}}{C}$	$0.636\ V_m$	$V_m - \frac{4170\ I_{dc}}{C}$
Ripple factor Γ	0.48	$\frac{R_L}{16000\ L}$	$\frac{2410}{CR_L}$	$\frac{0.83}{LC}$	$\frac{3330}{LC_1C_2R_L}$
Peak inverse voltage (PIV)	$2V_m$	$2V_m$	$2V_m$	$2V_m$	$2V_m$

5.11 Multiple L- Section Filter

The number of L-sections i.e. LC circuits can be connected one after another to obtain multiple L-section filter. It gives excellent filtering and smooth d.c. output voltage.

The Fig. 5.48 shows multiple L-section filter.

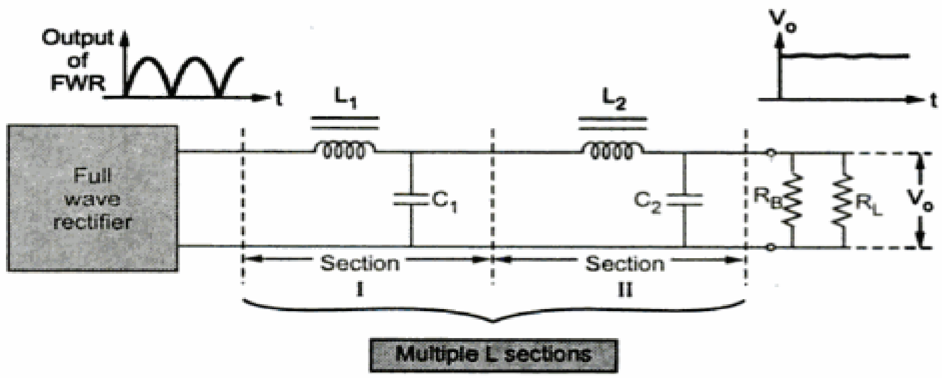


Fig. 5.48 Multiple L-section filter

5.11.1 Derivation of the Ripple Factor

The following assumptions are made in the analysis of multiple L section filter :

1. Reactances of all the inductors are much greater than the reactances of the capacitors.
2. Reactance of the last capacitor is small compared with the resistance of the load.

Thus, X_{C2} = Reactance of capacitor $C_2 = \frac{1}{2\omega C_2}$

X_{C1} = Reactance of capacitor $C_1 = \frac{1}{2\omega C_1}$

X_{L1} = Reactance of inductor $L_1 = 2\omega L_1$

The second harmonic current component flowing through the capacitor C_1 is given by,

$$I'_{2m} = \frac{\frac{4}{3\pi}E_{sm}}{2\omega L_1} \quad \text{... Refer equation (10) of section 5.10}$$

Now $2\omega L_1$ is X_{L1} hence r.m.s. value of this current is given by,

$$I'_2(\text{r.m.s.}) = \frac{I'_{2m}}{\sqrt{2}} = \frac{4}{3\pi}\times\frac{1}{\sqrt{2}}\times\frac{E_{sm}}{X_{L1}}$$

But $$E_{DC} = \frac{2E_{sm}}{\pi} \quad \text{... for full wave}$$

$$\therefore \quad I'_2(\text{r.m.s.}) = \frac{4}{3\pi}\times\frac{1}{\sqrt{2}}\times\frac{\pi}{2}\frac{E_{DC}}{X_{L1}}$$

$$\therefore \quad \boxed{I'_2(\text{r.m.s.}) = \frac{\sqrt{2}}{3}\frac{E_{DC}}{X_{L1}}} \quad \text{...(1)}$$

Thus the voltage across C_1 is = $I'_2(\text{r.m.s.})X_{C1}$.

The current through second filtering section is I_2 and neglecting reactance of the last capacitor, it is given by,

$$I_2(\text{r.m.s.}) = \frac{\text{Voltage across } C_1}{X_{L2}} = \frac{I'_2(\text{r.m.s.})X_{C1}}{X_{L2}} \quad ...(2)$$

And the ripple voltage across the load is the voltage across capacitor C_2,

$$\therefore \quad V_r(\text{r.m.s.}) = I_2(\text{r.m.s.}) \times X_{C2} = \frac{I'_2(\text{r.m.s.})X_{C1}}{X_{L2}} \times X_{C2}$$

Using (1),

$$V_r(\text{r.m.s.}) = \frac{\sqrt{2}}{3} \times \frac{E_{DC} \times [X_{C1}]\,[X_{C2}]}{[X_{L1}]\,[X_{L2}]} \quad ...(3)$$

The ripple factor is given by,

$$\therefore \quad \gamma = \frac{V_r(\text{r.m.s.})}{E_{DC}} = \frac{\sqrt{2}}{3}\frac{X_{C1}}{X_{L1}}\frac{X_{C2}}{X_{L2}} \quad ...(4)$$

Using the expressions for the reactances,

$$\gamma = \frac{\sqrt{2}}{3} \times \frac{\frac{1}{2\omega C_1} \times \frac{1}{2\omega C_2}}{2\omega L_1 \times 2\omega L_2} = \frac{\sqrt{2}}{3} \times \frac{1}{16\omega^4 L_1 L_2 C_1 C_2} \quad ...(5)$$

For multiple LC filter having 'n' similar setions with $L_1 = L_2 \; ... \; L_n = L$ and $C_1 = C_2 = \; ... \; C_n = C$ then the ripple factor is given by,

$$\therefore \quad \gamma = \frac{\sqrt{2}}{3}\frac{1}{(4\omega^2 LC)^n}$$

The value of critical inductance is given by,

$$L_{critical} = \frac{R_L}{3\omega}$$

The value of bleeder resistance R_B is given by,

$$R_B \leq 3\omega L$$

5.12.2 Multiple π Section Filter

To obtain almost pure d.c. to the load, more π sections may be used one after another. Such a filter using more than one π sections is called **multiple π section filter.** The Fig. 5.48 shows multiple π section filter.

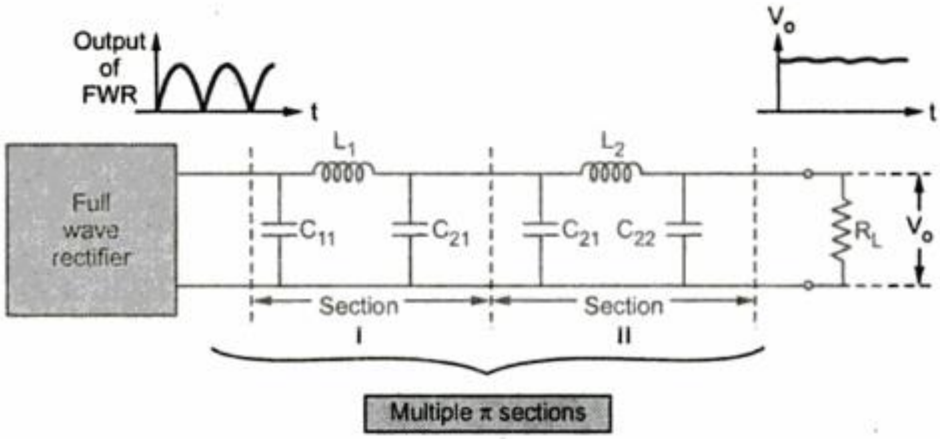


Fig. 5.51 Multiple π section filter

The ripple factor for multiple π section filter is given by,

$$\therefore \qquad \gamma = \sqrt{2}\,\frac{X_{C11}}{R_L}\,\frac{X_{C21}}{X_{L1}}\,\frac{X_{C22}}{X_{L2}} \ldots$$

This expression can be extended to include n number of such π sections.

Module 2 Questions:

1. Draw the circuit of capacitor filter and explain its operation.
2. Derive the expression for ripple factor of HWR and FWR with capacitor filter.
3. Draw the circuit of inductor filter and explain its operation.
4. Derive the expression for ripple factor of inductor filter. Mention the need of Bleeder resistor.
5. Discuss the L Section Filter with neat diagram.
6. Derive the Ripple Factor For L Section Filter.
7. Derive the expression for Ripple Factor of CLC Filter.
8. Compare the different types of filter circuits in terms of ripple factors.