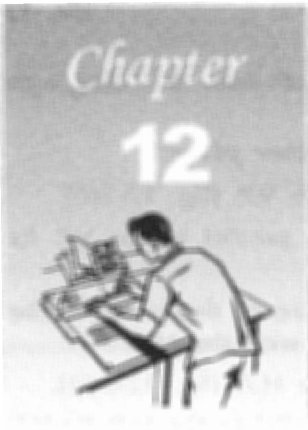


Chapter 12 PROJECTIONS OF PLANES

12-0. INTRODUCTION

Plane figures or surfaces have only two dimensions, viz. length and breadth. They do not have thickness. A plane figure may be assumed to be contained by a plane, and its projections can be drawn, if the position of that plane with respect to the principal planes of projection is known.

In this chapter, we shall discuss the following topics:

1. Types of planes and their projections.
2. Traces of planes.

12-1. TYPES OF PLANES

Planes may be divided into two main types:

(1) Perpendicular planes.

(2) Oblique planes.

(1) Perpendicular planes: These planes can be divided into the following sub-types:

(i) Perpendicular to both the reference planes.

(ii) Perpendicular to one plane and parallel to the other.

(iii) Perpendicular to one plane and inclined to the other.

(i) ***Perpendicular to both the reference planes*** (fig. 12-1): A square *ABCD* is perpendicular to both the planes. Its H.T. and V.T. are in a straight line perpendicular to *xy*.

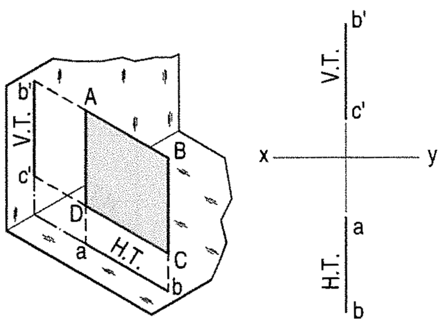


FIG. 12-1

The front view *b'c'* and the top view *ab* of the square are both lines coinciding with the V.T. and the H.T. respectively.

(ii) *Perpendicular to one plane and parallel to the other plane:*

(a) Plane, perpendicular to the H.P. and parallel to the V.P. [fig. 12-2(i)].

A triangle *PQR* is perpendicular to the H.P. and is parallel to the V.P. Its H.T. is parallel to *xy*. It has no V.T.

The front view *p'q'r'* shows the exact shape and size of the triangle. The top view *pqr* is a line parallel to *xy*. It coincides with the H.T.

(b) Plane, perpendicular to the V.P. and parallel to the H.P. [fig. 12-2(ii)].

A square *ABCD* is perpendicular to the V.P. and parallel to the H.P. Its V.T. is parallel to *xy*. It has no H.T.

The top view *abcd* shows the true shape and true size of the square. The front view *a'b'* is a line, parallel to *xy*. It coincides with the V.T.

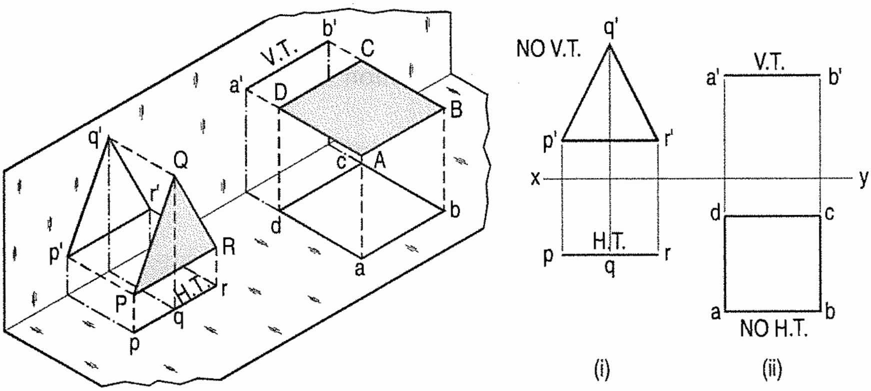


FIG. 12-2

(iii) *Perpendicular to one plane and inclined to the other plane:*

(a) Plane, perpendicular to the H.P. and inclined to the V.P. (fig. 12-3).

A square *ABCD* is perpendicular to the H.P. and inclined at an angle ø to the V.P. Its V.T. is perpendicular to *xy*. Its H.T. is inclined at ø to xy.

Its top view *ab* is a line linclined at ø to *xy*. The front view *a'b'c'd'* is smaller than *ABCD*.

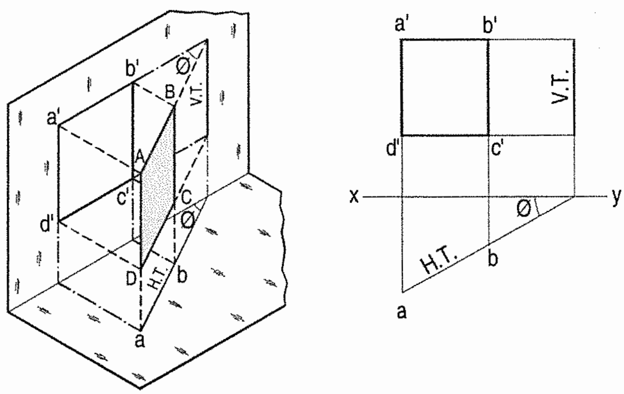


FIG. 12-3

(b) Plane, perpendicular to the V.P. and inclined to the H.P. (fig. 12-4).

A square *ABCD* is perpendicular to the V.P. and inclined at an angle θ to the H.P. Its H.T. is perpendicular to *xy*. Its V.T. makes the angle θ with *xy*. Its front view *a'b'* is a line inclined at θ to *xy*. The top view *abcd* is a rectangle which is smaller than the square *ABCD*.

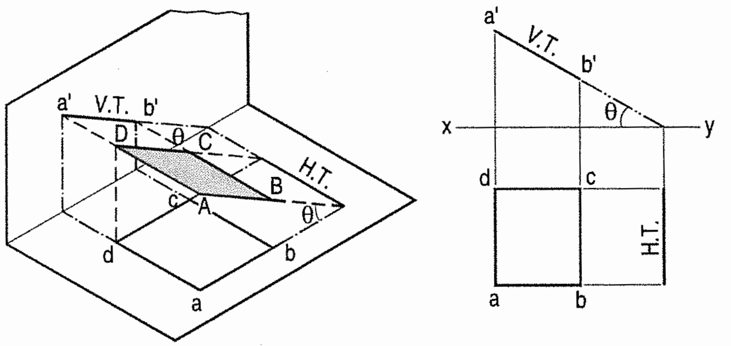


FIG. 12-4

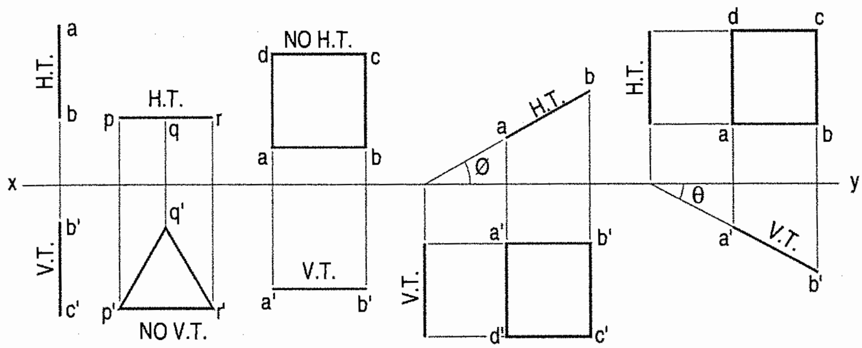


FIG. 12-5

Fig. 12-5 shows the projections and the traces of all these perpendicular planes by third-angle projection method.

(2) Oblique planes: Planes which are inclined to both the reference planes are called *oblique planes*. Representation of oblique planes by their traces is too advanced to be included in this book.

A few problems on the projections of plane figures inclined to both the reference planes are however, illustrated at the end of the chapter. They will prove to be of great use in dealing with the projections of solids.

12-2. TRACES OF PLANES

A plane, extended if necessary, will meet the reference planes in lines, unless it is parallel to any one of them.

These lines are called the *traces* of the plane. The line in which the plane meets the H.P. is called the *horizontal trace* or the H.T. of the plane. The line in which it meets the V.P. is called its *vertical trace* or the V.T. A plane is usually represented by its traces.

12-3. GENERAL CONCLUSIONS

(1) Traces:

(a) When a plane is perpendicular to both the reference planes, its traces lie on a straight line perpendicular to *xy*.

(b) When a plane is perpendicular to one of the reference planes, its trace upon the other plane is perpendicular to *xy* (except when it is parallel to the other plane).

(c) When a plane is parallel to a reference plane, it has no trace on that plane. Its trace on the other reference plane, to which it is perpendicular, is parallel to *xy*.

(d) When a plane is inclined to the H.P. and perpendicular to the V.P., its inclination is shown by the angle which its V.T. makes with *xy*. When it is inclined to the V.P. and perpendicular to the H.P., its inclination is shown by the angle which its H.T. makes with *xy*.

(e) When a plane has two traces, they, produced if necessary, intersect in *xy* (except when both are parallel to *xy* as in case of some oblique planes).

(2) Projections:

(a) When a plane is perpendicular to a reference plane, its projection on that plane is a straight line.

(b) When a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

(c) When a plane is perpendicular to one of the reference planes and inclined to the other, its inclination is shown by the angle which its projection on the plane to which it is perpendicular, makes with *xy*. Its projection on the plane to which it is inclined, is smaller than the plane itself.

Problem 12-1. *Show by means of traces, each of the following planes:*

(a) *Perpendicular to the H.P. and the V.P.*

(b) *Perpendicular to the H.P. and inclined at 30° to the V.P.*

(c) *Parallel to and 40 mm away from the V.P.*

(d) *Inclined at 45° to the H.P. and perpendicular to the V.P.*

(e) *Parallel to the H.P. and 25 mm away from it.*

Fig. 12-6 and fig. 12-7 show the various traces.

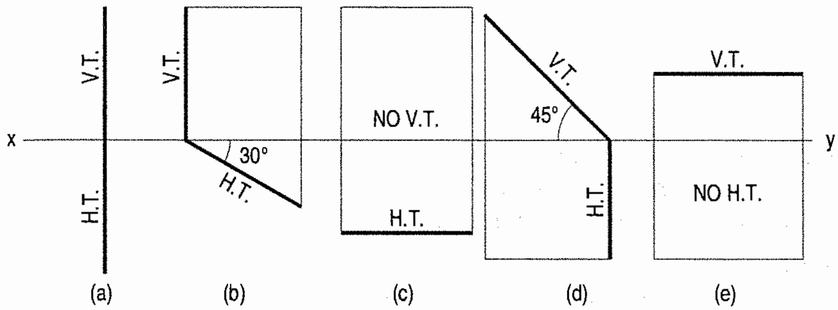
(a) The H.T. and the V.T. are in a line perpendicular to xy.

(b) The H.T. is inclined at 30° to *xy*; the V.T. is normal to *xy*; both the traces intersect in *xy*.

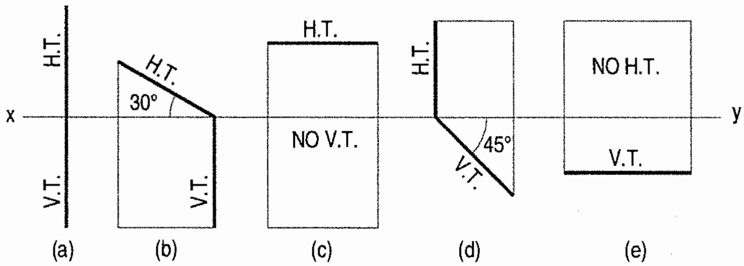
(c) The H.T. is parallel to and 40 mm away from *xy*. It has no V.T.

(d) The H.T. is perpendicular to *xy*; the V.T. makes 45° angle with *xy*; both intersect in *xy*.

(e) The V.T. is parallel to and 25 mm away from *xy*. It has no H.T.



(First-angle projection)
FIG. 12-6



(Third-angle projection)
FIG. 12-7

12-4. PROJECTIONS OF PLANES PARALLEL TO ONE OF THE REFERENCE PLANES

The projection of a plane on the reference plane parallel to it will show its true shape. Hence, beginning should be made by drawing that view. The other view which will be a line, should then be projected from it.

(1) When the plane is parallel to the H.P.: The top view should be drawn first and the front view projected from it.

Problem 12-2. (fig. 12-8): *An equilateral triangle of 50 mm side has its V.T. parallel to and 25 mm above xy. It has no H.T. Draw its projections when one of its sides is inclined at 45° to the V.P.*

As the V.T. is parallel to *xy* and as there is no H.T. the triangle is parallel to the H.P. Therefore, begin with the top view.

(i) Draw an equilateral triangle *abc* of 50 mm side, keeping one side, say *ac*, inclined at 45° to *xy*.

(ii) Project the front view, parallel to and 25 mm *above* *xy*, as shown.

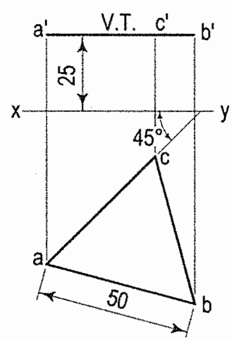


FIG. 12-8

(2) When the plane is parallel to the V.P.: Beginning should be made with the front view and the top view projected from it.

Problem 12-3. (fig. 12-9): *A square ABCD of 40 mm side has a corner on the H.P. and 20 mm in front of the V.P. All the sides of the square are equally inclined to the H.P. and parallel to the V.P. Draw its projections and show its traces.*

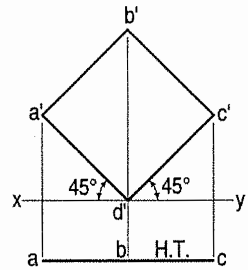


FIG. 12-9

As all the sides are parallel to the V.P., the surface of the square also is parallel to it. The front view will show the true shape and position of the square.

(i) Draw a square *a'b'c'd'* in the front view with one corner in *xy* and all its sides inclined at 45° to *xy*.

(ii) Project the top view keeping the line *ac* parallel to *xy* and 30 mm below it. The top view is its H.T. It has no V.T.

12-5. PROJECTIONS OF PLANES INCLINED TO ONE REFERENCE PLANE AND PERPENDICULAR TO THE OTHER

When a plane is inclined to a reference plane, its projections may be obtained in two stages. In the initial stage, the plane is assumed to be parallel to that reference plane to which it has to be made inclined. It is then tilted to the required inclination in the second stage.

(1) Plane, inclined to the H.P. and perpendicular to the V.P.: When the plane is inclined to the H.P. and perpendicular to the V.P., in the initial stage, it is assumed to be parallel to the H.P. Its top view will show the true shape. The front view will be a line parallel to *xy*. The plane is then tilted so that it is inclined to the H.P. The new front view will be inclined to *xy* at the true inclination. In the top view the corners will move along their respective paths (parallel to *xy*).

Problem 12-4. (fig. 12-10): *A regular pentagon of 25 mm side has one side on the ground. Its plane is inclined at 45° to the H.P. and perpendicular to the V.P. Draw its projections and show its traces.*

Assuming it to be parallel to the H.P.

(i) Draw the pentagon in the top view with one side perpendicular to *xy* [fig. 12-10(i)]. Project the front view. It will be the line *a'c'* contained by *xy*.

(ii) Tilt the front view about the point *a'*, so that it makes 45° angle with *xy*.

(iii) Project the new top view $ab_1c_1d_1e$ upwards from this front view and horizontally from the first top view. It will be more convenient if the front view is reproduced in the new position separately and the top view projected from it, as shown in fig. 12-10(ii). The V.T. coincides with the front view and the H.T. is perpendicular to *xy*, through the point of intersection between *xy* and the front view-produced.

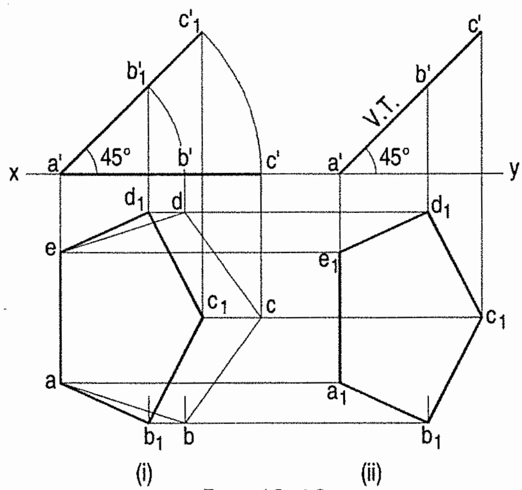


FIG. 12-10

(2) Plane, inclined to the V.P. and perpendicular to the H.P.: In the initial stage, the plane may be assumed to be parallel to the V.P. and then tilted to the required position in the next stage. The projections are drawn as illustrated in the next problem.

Problem 12-5. (fig. 12-11): *Draw the projections of a circle of 50 mm diameter, having its plane vertical and inclined at 30° to the V.P. Its centre is 30 mm above the H.P. and 20 mm in front of the V.P. Show also its traces.*

A circle has no corners to project one view from another. However, a number of points, say twelve, equal distances apart, may be marked on its circumference.

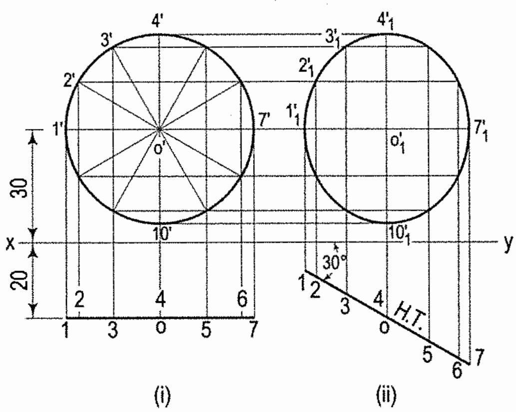


FIG. 12-11

(i) Assuming the circle to be parallel to the V.P., draw its projections. The front view will be a circle [fig. 12-11(i)], having its centre 30 mm above *xy*. The top view will be a line, parallel to and 20 mm below *xy*.

(ii) Divide the circumference into twelve equal parts (with a 30°-60° set-square) and mark the points as shown. Project these points in the top view. The centre *O* will coincide with the point 4.

(iii) When the circle is tilted, so as to make 30° angle with the V.P., its top view will become inclined at 30° to *xy*. In the front view all the points will move along their respective paths (parallel to *xy*). Reproduce the top view keeping the centre *o* at the same distance, viz. 20 mm from *xy* and inclined at 30° to *xy* [fig. 12-11(ii)].

(iv) For the final front view, project all the points upwards from this top view and horizontally from the first front view. Draw a freehand curve through the twelve points $1'_1$, $2'_1$ etc. This curve will be an ellipse.

12-6. PROJECTIONS OF OBLIQUE PLANES

When a plane has its surface inclined to one plane and an edge or a diameter or a diagonal parallel to that plane and inclined to the other plane, its projections are drawn in three stages.

(1) If the surface of the plane is inclined to the H.P. and an edge (or a diameter or a diagonal) is parallel to the H.P. and inclined to the V.P.,

(i) in the initial position the plane is assumed to be parallel to the H.P. and an edge perpendicular to the V.P.

(ii) It is then tilted so as to make the required angle with the H.P. As already explained, its front view in this position will be a line, while its top view will be smaller in size.

(iii) In the final position, when the plane is turned to the required inclination with the V.P., only the position of the top view will change. Its shape and size will not be affected. In the final front view, the corresponding distances of all the corners from *xy* will remain the same as in the second front view.

If an edge is in the H.P. or on the ground, in the initial position, the plane is assumed to be lying in the H.P. or on the ground, with the edge perpendicular to the V.P. *If a corner is in the H.P. or on the ground, the line joining that corner with the centre of the plane is kept parallel to the V.P.*

(2) Similarly, if the surface of the plane is inclined to the V.P. and an edge (or a diameter or a diagonal) is parallel to the V.P. and inclined to the H.P.,

(i) in the initial position, the plane is assumed to be parallel to the V.P. and an edge perpendicular to the H.P.

(ii) It is then tilted so as to make the required angle with the V.P. Its top view in this position will be a line, while its front view will be smaller in size.

(iii) When the plane is turned to the required inclination with the H.P., only the position of the front view will change. Its shape and size will not be affected. In the final top view, the corresponding distances of all the corners from *xy* will remain the same as in the second top view.

If an edge is in the V.P., in the initial position, the plane is assumed to be lying in the V.P. with an edge perpendicular to the H.P. *If a corner is in the V.P., the line joining that corner with centre of the plane is kept parallel to the H.P.*

Problem 12-6. (fig. 12-12): *A square ABCD of 50 mm side has its corner A in the H.P., its diagonal AC inclined at 30° to the H.P. and the diagonal BD inclined at 45° to the V.P. and parallel to the H.P. Draw its projections.*

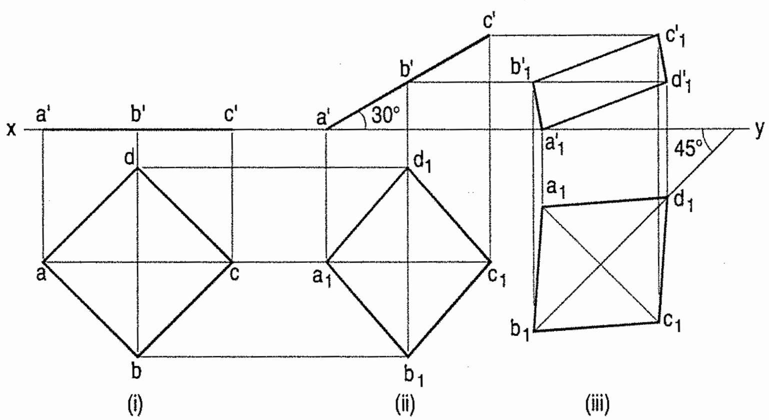


FIG. 12-12

In the initial stage, assume the square to be lying in the H.P. with *AC* parallel to the V.P.

(i) Draw the top view and the front view. When the square is tilted about the corner *A* so that *AC* makes 30° angle with the H.P., *BD* remains perpendicular to the V.P. and parallel to the H.P.

(ii) Draw the second front view with *a'c'* inclined at 30° to *xy*, keeping *a'* or *c'* in *xy*. Project the second top view. The square may now be turned so that *BD* makes 45° angle with the V.P. and remains parallel to the H.P. Only the position of the top view will change. Its shape and size will remain the same.

(iii) Reproduce the top view so that b_1d_1 is inclined at 45° to *xy*. Project the final front view upwards from this top view and horizontally from the second front view.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 27 for the following problem.

Problem 12-7. (Fig. 12-13): *A rectangular plane surface of size L × W is positioned in the first quadrant and is inclined at an angle of 60° with the H.P. and 30° with the V.P. Draw its projections.*

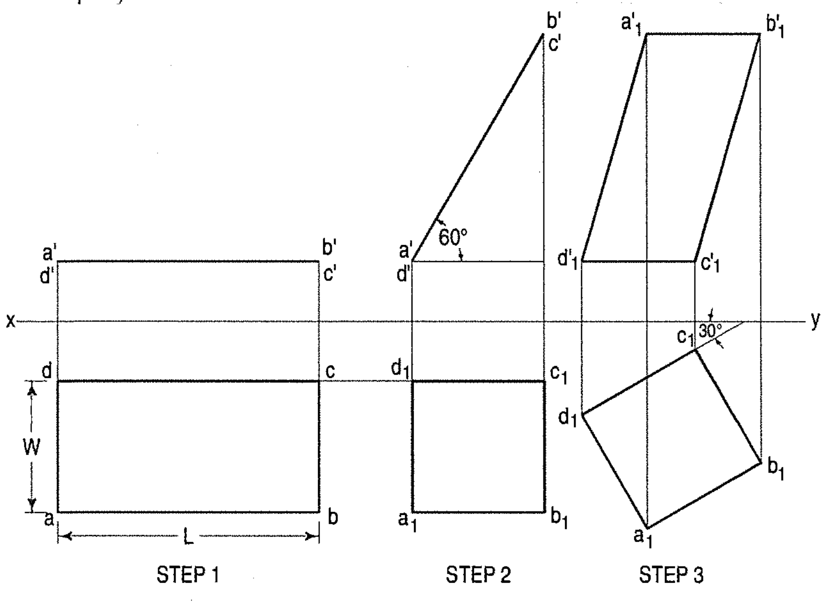


FIG. 12-13

(i) The plane is first assumed to be parallel to H.P. with its shorter edge perpendicular to V.P. In this position, true shape and size of the plane is given by its projection on H.P. The front view will be a true line parallel to the reference line *xy*.

(ii) Rotate the front view projection by 60° (the angle of inclination of plane with H.P.) as shown in Step 2 of fig. 12-13(ii). Draw vertical lines from the ends of line *a'd'* and *b'c'* to intersect horizontal lines drawn from the top view *abcd* (step 1) at points b_1, c_1, d_1 and a_1. Join $a_1b_1c_1d_1$ to obtain the top view of the plane in this inclined position.

(iii) Now rotate the edge d_1c_1 of the top view (step 2) by 30° (the angle of inclination of plane with V.P.) and reproduce it as shown in step 3 of the fig. 12-13(iii). Draw projections from a_1, b_1, c_1 and d_1 to intersect the horizontal projections from *a'd'* and *b'c'* to get the points a'_1, b'_1, c'_1 and d'_1. Join the lines $a'_1b'_1c'_1d'_1$ to obtain the final front view of the given plane surface.

Problem 12-8. (fig. 12-14): *Draw the projections of a regular hexagon of 25 mm side, having one of its sides in the H.P. and inclined at 60° to the V.P., and its surface making an angle of 45° with the H.P.*

(i) Draw the hexagon in the top view with one side perpendicular to *xy*. Project the front view *a'c'* in *xy*.

(ii) Draw *a'c'* inclined at 45° to *xy* keeping *a'* or *c'* in *xy* and project the second top view.

(iii) Reproduce this top view making a_1f_1 inclined at 60° to *xy* and project the final front view.

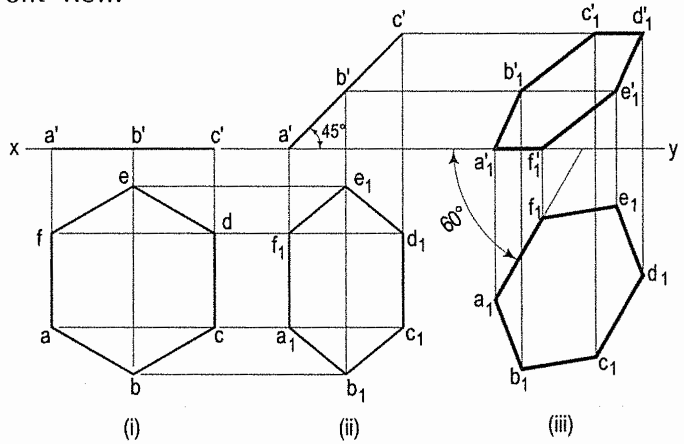


FIG. 12-14

Problem 12-9. (fig. 12-15)**:** *Draw the projections of a circle of 50 mm diameter resting in the H.P. on a point A on the circumference, its plane inclined at 45° to the H.P. and*

(a) the top view of the diameter AB making 30° angle with the V.P.;

(b) the diameter AB making 30° angle with the V.P.

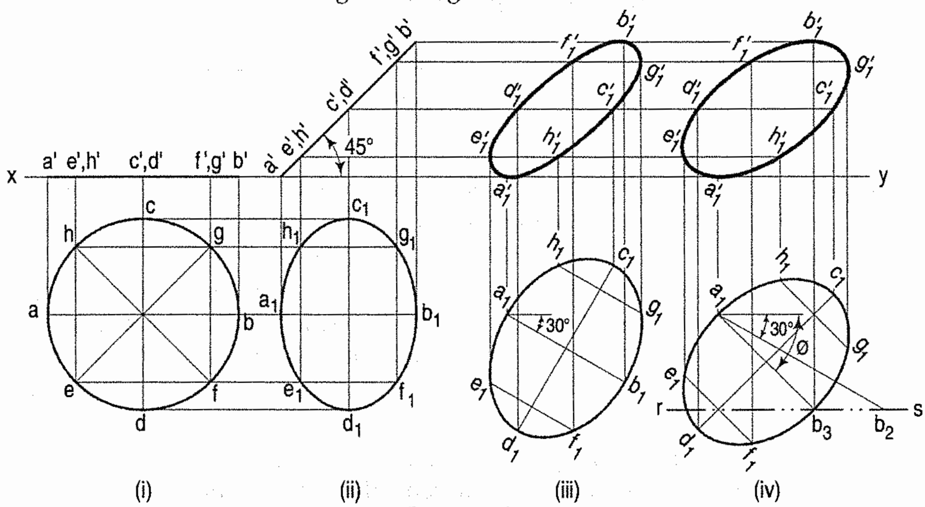


FIG. 12-15

Draw the projections of the circle with *A* in the H.P. and its plane inclined at 45° to the H.P. and perpendicular to the V.P. [fig. 12-15(i) and fig. 12-15(ii)].

(a) In the second top view, the line a_1b_1 is the top view of the diameter *AB*. Reproduce this top view so that a_1b_1 makes 30° angle with *xy* [fig. 12-15(iii)]. Project the required front view.

(b) If the diameter AB, which makes 45° angle with the H.P., is inclined at 30° to the V.P. also, its top view a_1b_1 will make an angle greater than 30° with xy. This apparent angle of inclination is determined as described below.

Draw any line a_1b_2 equal to AB and inclined at 30° to xy [fig. 12-15(iv)]. With a_1 as centre and radius equal to the top view of AB, viz. a_1b_1, draw an arc cutting rs (the path of B in the top view) at b_3. Draw the line joining a_1 with b_3, and around it, reproduce the second top view. Project the final front view. It is evident that a_1b_3 is inclined to xy at an angle ø which is greater than 30°.

Problem 12-10. (fig. 12-16): *A thin 30°-60° set-square has its longest edge in the V.P. and inclined at 30° to the H.P. Its surface makes an angle of 45° with the V.P. Draw its projections.*

In the initial stage, assume the set-square to be in the V.P. with its hypotenuse perpendicular to the H.P.

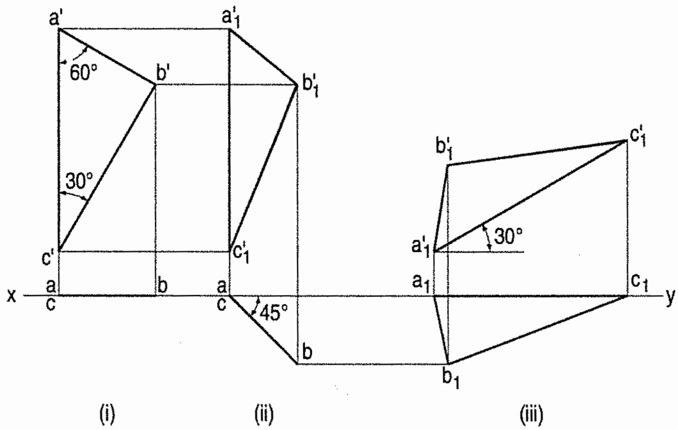


FIG. 12-16

(i) Draw the front view $a'b'c'$ and project the top view ac in xy.

(ii) Tilt ac around the end a so that it makes 45° angle with xy and project the front view $a'_1b'_1c'_1$.

(iii) Reproduce the second front view $a'_1b'_1c'_1$ so that $a'_1b'_1$ makes an angle of 30° with xy. Project the final top view $a_1b_1c_1$.

Problem 12-11. (fig. 12-17): *A thin rectangular plate of sides 60 mm × 30 mm has its shorter side in the V.P. and inclined at 30° to the H.P. Project its top view if its front view is a square of 30 mm long sides.*

As the front view of the plate is a square, its surface must be inclined to the V.P. Hence, assume the plate to be in the V.P. with its shorter edge perpendicular to the H.P.

(i) Draw the front view $a'b'c'd'$ and project the top view ab in xy [fig. 12-17(i)].

(ii) The line ab should be so inclined to xy that the front view becomes a square. Therefore, draw the square $a'_1b'_1c'_1d'_1$ of side equal to $a'd'$. With a as centre and radius equal to ab draw an arc cutting the projector through b'_1 at b. Then ab is the new top view.

(iii) Reproduce the second front view in such a way that $a'_1d'_1$ makes 30° angle with xy. Project the final top view as shown.

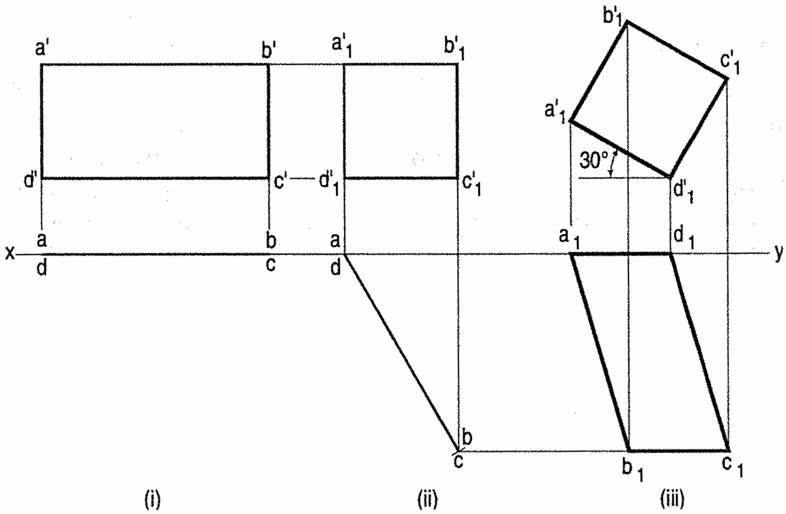
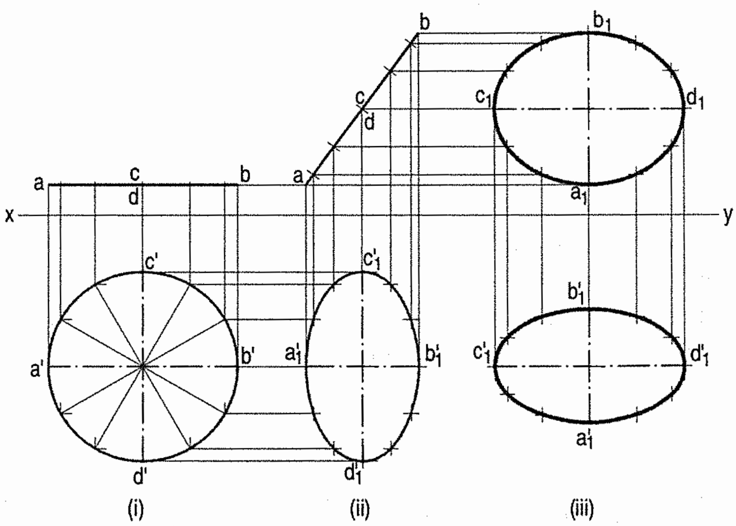


FIG. 12-17

Problem 12-12. (fig. 12-18): *A circular plate of negligible thickness and 50 mm diameter appears as an ellipse in the front view, having its major axis 50 mm long and minor axis 30 mm long. Draw its top view when the major axis of the ellipse is horizontal.*

As the plate is seen as an ellipse in the front view, its surface must be inclined to the V.P.



(Third-angle projection)

FIG. 12-18

(i) Therefore, assume it to be parallel to the V.P. and draw its front view and the top view.

(ii) Turn the line *ab* so that its length in the front view becomes 30 mm, and project the front view. It will be an ellipse whose major axis is vertical.

(iii) Reproduce this view so that the major axis $c'_1d'_1$ is horizontal, and project the required top view.

Problem 12-13. *Fig. 12-19 shows a thin plate of negligible thickness. It rests on its PQ edge with its plane perpendicular to V.P. and inclined 40° to the H.P. Draw its projections.*

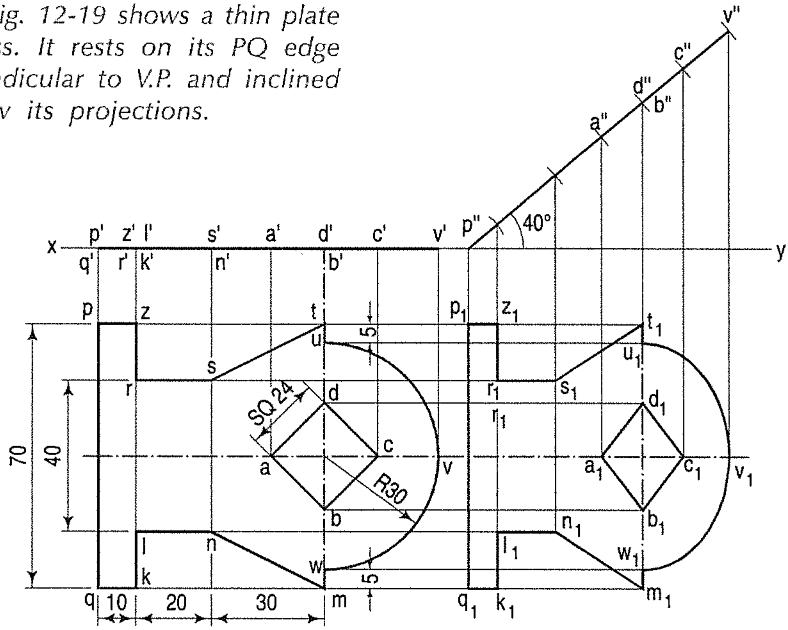


FIG. 12-19

(i) Keep the plane of plate in the H.P. and draw the projections as shown.

(ii) Tilt front view at *p*" making an angle of 40°.

(iii) Project *p*", *a*", *d*", *c*", *v*" etc. in the top view. Draw horizontal projectors intersecting previously drawn projectors from the front view. Join by smooth curve to complete the top view.

Problem 12-14. (fig. 12-20): *A pentagonal plate of 45 mm side has a circular hole of 40 mm diameter in its centre. The plane stands on one of its sides on the H.P. with its plane perpendicular to V.P. and 45° inclined to the H.P. Draw the projections.*

(i) Keep the plane of plate in the horizontal plane.

(ii) Draw top view and front view as shown.

(iii) Tilt the front view *a*" .....*d*" at *a*" making an angle of 45°. Draw the projectors from various points *a*" ....*d*".

(iv) Draw horizontal projectors from the top view *abcd* as shown. Join the intersection points and complete new top view a_1, b_1, c_1, d_1, e_1, as shown.

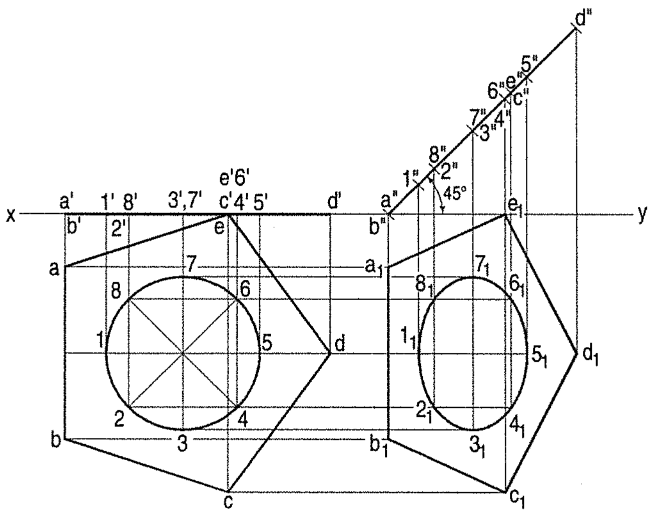


FIG. 12-20

Problem 12-15. (fig. 12-21): *A thin circular plate of 70 mm diameter is resting on its circumference such that its plane is inclined 60° to the H.P. and 30° to the V.P. Draw the projections of the plate.*

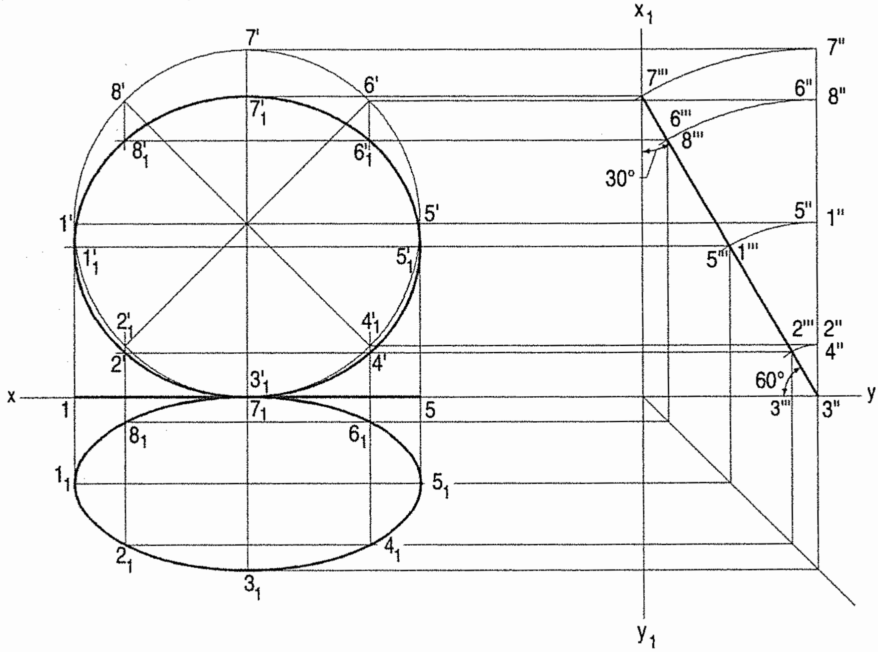


FIG. 12-21

(i) Draw the projection of the plate keeping its plane parallel to the V.P. as shown in fig. 12-21.

(ii) Mark a reference line x_1y_1 perpendicular to xy line to represent the auxiliary plane which is at right angle to both the H.P. and the V.P.

(iii) Divide the front view in eight parts and mark the points 1', 2' ....8'. Project these points on the side view as 1", 2" ....8".

(iv) Tilt the side view 3" 7" such that it touches the x_1y_1 line and also makes 60° with the xy line.

(v) Complete the projection as shown in fig. 12-21.

Problem 12-16. (fig. 12-22): *PQRS is a rhombus having diagonal PR = 60 mm and QS = 40 mm and they are perpendicular to each other. The plane of the rhombus is inclined with H.P. such that its top view appears to be square. The top view of PR makes 30° with the V.P. Draw its projections and determine inclination of the plane with the H.P.*

(i) Assume that the rhombus is lying in H.P. with its longest diagonal parallel to xy line.

(ii) Draw the plans of diagonals PR = 60 mm and QS = 40 mm (true length) perpendicular each other as shown.

(iii) Join points p, q, r and s. It is top view of the rhombus. Project the points p, q, r and s in the xy line. It is front view of the rhombus points p', q', r' and s' in the xy line as the plane of rhombus is perpendicular to the V.P.

(iv) *PR* and *QS* are lying in H.P. *pr* and *qs* are true length. As the plane of the rhombus is inclined to H.P., the top view of the rhombus is going to be a square. But diagonal *qs* does not change in the length as it is perpendicular to V.P.

(v) Draw the projectors from the points *p*, *q* and *r*, *s* parallel to the *xy*. From q_1 and s_1 draw square $p_1\ q_1\ r_1\ s_1$ such that $s_1\ q_1 = p_1\ r_1$ as shown in fig. 12-22.

(vi) Draw vertical projectors from p_1, q_1, r_1 and s_1.

(vii) Projector of p_1 intersects at p'' in xy line. Taking p'' as centre and the radius equal to 60 mm ($p'r'$), draw the arc to intersect the vertical projectors of r_1 at r''. Join $p''r''$. Measure angle of $p''r''$ with *xy* line.

(viii) Tilt diagonal p_1r_1 at 30° with xy and reproduce square $p_2q_2r_2s_2$. Draw vertical projectors from p_2, q_2, r_2 and s_2 to intersect the horizontal projectors from p'', q'', r'' and s'' at p''', q''', r''' and s'''. Join the points p''', q''', r''' and s''' as shown.

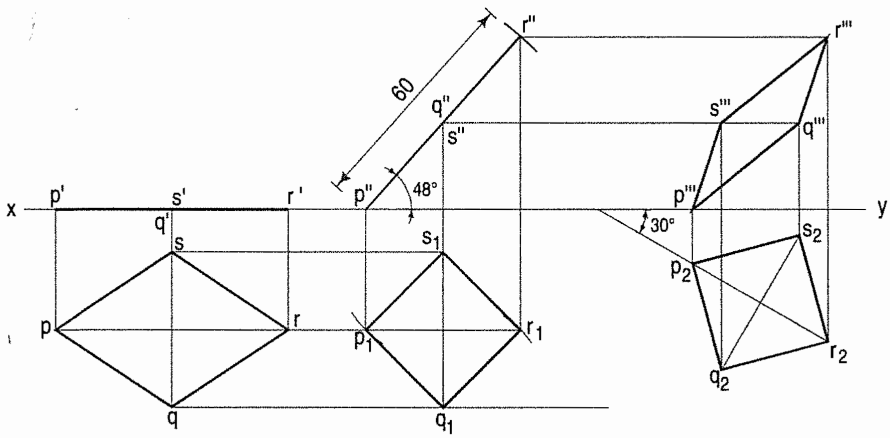


FIG. 12-22

EXERCISES 12

1. Draw an equilateral triangle of 75 mm side and inscribe a circle in it. Draw the projections of the figure, when its plane is vertical and inclined at 30° to the V.P. and one of the sides of the triangle is inclined at 45° to the H.P.

2. A regular hexagon of 40 mm side has a corner in the H.P. Its surface is inclined at 45° to the H.P. and the top view of the diagonal through the corner which is in the H.P. makes an angle of 60° with the V.P. Draw its projections.

3. Draw the projections of a regular pentagon of 40 mm side, having its surface inclined at 30° to the H.P. and a side parallel to the H.P. and inclined at an angle of 60° to the V.P.

4. Draw the projections of a rhombus having diagonals 125 mm and 50 mm long, the smaller diagonal of which is parallel to both the principal planes, while the other is inclined at 30° to the H.P.

5. Draw a regular hexagon of 40 mm side, with its two sides vertical. Draw a circle of 40 mm diameter in its centre. The figure represents a hexagonal plate with a hole in it and having its surface parallel to the V.P. Draw its projections when the surface is vertical and inclined at 30° to the V.P. Assume the thickness of the plate to be equal to that of a line.

6. Draw the projections of a circle of 75 mm diameter having the end *A* of the diameter *AB* in the H.P., the end *B* in the V.P., and the surface inclined at 30° to the H.P. and at 60° to the V.P.

7. A semi-circular plate of 80 mm diameter has its straight edge in the V.P. and inclined at 45° to the H.P. The surface of the plate makes an angle of 30° with the V.P. Draw its projections.

8. The top view of a plate, the surface of which is perpendicular to the V.P. and inclined at 60° to the H.P. is a circle of 60 mm diameter. Draw its three views.

9. A plate having shape of an isosceles triangle has base 50 mm long and altitude 70 mm. It is so placed that in the front view it is seen as an equilateral triangle of 50 mm sides and one side inclined at 45° to *xy*. Draw its top view.

10. Draw a rhombus of diagonals 100 mm and 60 mm long, with the longer diagonal horizontal. The figure is the top view of a square of 100 mm long diagonals, with a corner on the ground. Draw its front view and determine the angle which its surface makes with the ground.

11. A composite plate of negligible thickness is made-up of a rectangle 60 mm × 40 mm, and a semi-circle on its longer side. Draw its projections when the longer side is parallel to the H.P. and inclined at 45° to the V.P., the surface of the plate making 30° angle with the H.P.

12. A 60° set-square of 125 mm longest side is so kept that the longest side is in the H.P. making an angle of 30° with the V.P. and the set-square itself inclined at 45° to the H.P. Draw the projections of the set-square.

13. A plane figure is composed of an equilateral triangle *ABC* and a semi-circle on *AC* as diameter. The length of the side *AB* is 50 mm and is parallel to the V.P. The corner *B* is 20 mm behind the V.P. and 15 mm below the H.P. The plane of the figure is inclined at 45° to the H.P. Draw the projections of the plane figure.

14. An equilateral triangle *ABC* having side length as 50 mm is suspended from a point *O* on the side *AB* 15 mm from *A* in such a way that the plane of the triangle makes an angle of 60° with the V.P. The point *O* is 20 mm below the H.P. and 40 mm behind the V.P. Draw the projections of the triangle.

15. *PQRS* and *ABCD* are two square thin plates with their diagonals measuring 30 mm and 60 mm. They are touching the H.P. with their corners *P* and *A* respectively, and touching each other with their corresponding opposite corners *R* and *C*. If the plates are perpendicular to each other and perpendicular to V.P. also, draw their projections and determine the length of their sides.

Chapter 13

PROJECTIONS OF SOLIDS

13-0. INTRODUCTION

A solid has three dimensions, viz. length, breadth and thickness. To represent a solid on a flat surface having only length and breadth, at least two orthographic views are necessary. Sometimes, additional views projected on auxiliary planes become necessary to make the description of a solid complete.

This chapter deals with the following topics:

1. Types of solids.
2. Projections of solids in simple positions.
   (a) Axis perpendicular to the H.P.
   (b) Axis perpendicular to the V.P.
   (c) Axis parallel to both the H.P. and the V.P.
3. Projections of solids with axes inclined to one of the reference planes and parallel to the other.
   (a) Axis inclined to the V.P. and parallel to the H.P.
   (b) Axis inclined to the H.P. and parallel to the V.P.
4. Projections of solids with axes inclined to both the H.P. and the V.P.
5. Projections of spheres.

13-1. TYPES OF SOLIDS

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 28 for the types of solids.

Solids may be divided into *two* main groups:

(1) Polyhedra

(2) Solids of revolution.

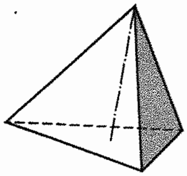
(1) Polyhedra: A polyhedron is defined as a solid bounded by planes called *faces*. When all the faces are equal and regular, the polyhedron is said to be regular.

There are *seven* regular polyhedra which may be defined as stated below:

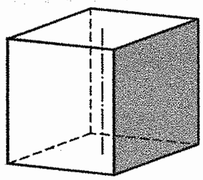
(i) *Tetrahedron* (fig. 13-1): It has four equal faces, each an equilateral triangle.

(ii) *Cube or hexahedron* (fig. 13-2): It has six faces, all equal squares.

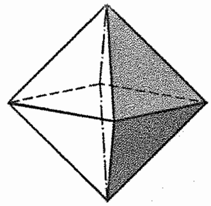
(iii) *Octahedron* (fig. 13-3): It has eight equal equilateral triangles as faces.



Tetrahedron
FIG. 13-1



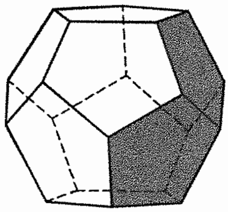
Cube
FIG. 13-2



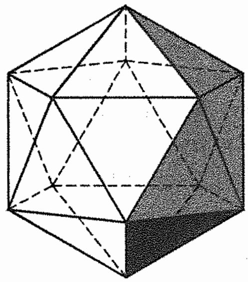
Octahedron
FIG. 13-3

(iv) *Dodecahedron* (fig. 13-4): It has twelve equal and regular pentagons as faces.

(v) *Icosahedron* (fig. 13-5): It has twenty faces, all equal equilateral triangles.



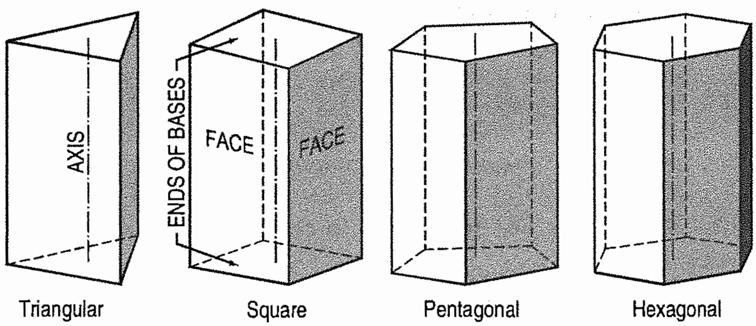
Dodecahedron
FIG. 13-4



Icosahedron
FIG. 13-5

(vi) *Prism:* This is a polyhedron having two equal and similar faces called its ends or bases, parallel to each other and joined by other faces which are parallelograms. The imaginary line joining the centres of the bases is called the axis.

A right and regular prism (fig. 13-6) has its axis perpendicular to the bases. All its faces are equal rectangles.



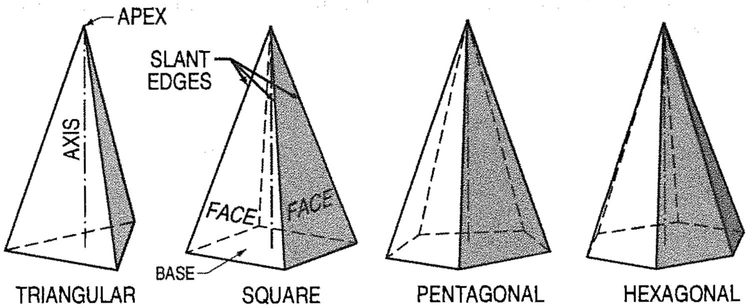
Prisms
FIG. 13-6

(vii) *Pyramid:* This is a polyhedron having a plane figure as a base and a number of triangular faces meeting at a point called the vertex or apex. The imaginary line joining the apex with the centre of the base is its axis.

A right and regular pyramid (fig. 13-7) has its axis perpendicular to the base which is a regular plane figure. Its faces are all equal isosceles triangles.

Oblique prisms and pyramids have their axes inclined to their bases.

Prisms and pyramids are named according to the shape of their bases, as triangular, square, pentagonal, hexagonal etc.



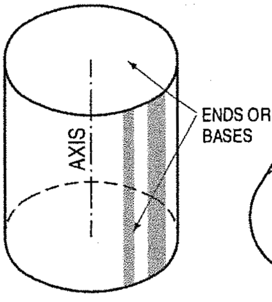
Pyramids
FIG. 13-7

(2) Solids of revolution:

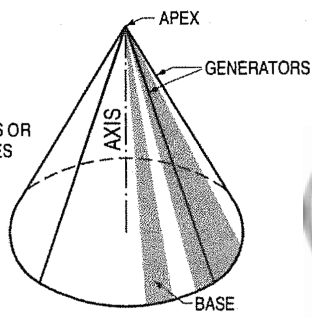
(i) *Cylinder* (fig. 13-8): A *right circular cylinder* is a solid generated by the revolution of a rectangle about one of its sides which remains fixed. It has two equal circular bases. The line joining the centres of the bases is the axis. It is perpendicular to the bases.

(ii) *Cone* (fig. 13-9): A *right circular cone* is a solid generated by the revolution of a right-angled triangle about one of its perpendicular sides which is fixed.

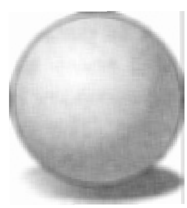
It has one circular base. Its axis joins the apex with the centre of the base to which it is perpendicular. Straight lines drawn from the apex to the circumference of the base-circle are all equal and are called *generators* of the cone. The length of the generator is the slant height of the cone.



Cylinder
FIG. 13-8



Cone
FIG. 13-9



Sphere
FIG. 13-10

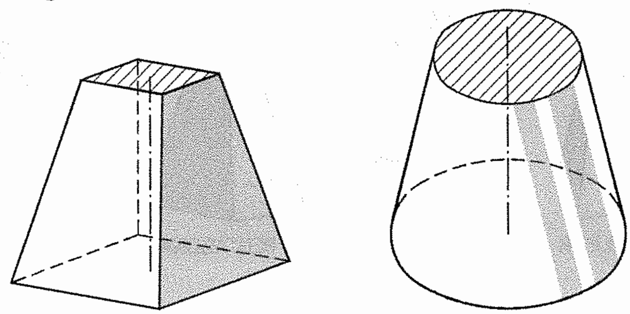
(iii) *Sphere* (fig. 13-10): A *sphere* is a solid generated by the revolution of a semi-circle about its diameter as the axis. The mid-point of the diameter is the centre of the sphere. All points on the surface of the sphere are equidistant from its centre.

Oblique cylinders and cones have their axes inclined to their bases.

(iv) *Frustum*: When a pyramid or a cone is cut by a plane parallel to its base, thus removing the top portion, the remaining portion is called its *frustum* (fig. 13-11).

(v) *Truncated*: When a solid is cut by a plane inclined to the base it is said to be *truncated.*

In this book mostly right and regular solids are dealt with. Hence, when a solid is named without any qualification, it should be understood as being right and regular.



Frustums
FIG. 13-11

13-2. PROJECTIONS OF SOLIDS IN SIMPLE POSITIONS

A solid in simple position may have its axis perpendicular to one reference plane or parallel to both. When the axis is perpendicular to one reference plane, it is parallel to the other. Also, when the axis of a solid is perpendicular to a plane, its base will be parallel to that plane. We have already seen that when a plane is parallel to a reference plane, its projection on that plane shows its true shape and size.

Therefore, the projection of a solid on the plane to which its axis is perpendicular, will show the true shape and size of its base.

Hence, when the axis is perpendicular to the ground, i.e. to the H.P., the top view should be drawn first and the front view projected from it.

When the axis is perpendicular to the V.P., beginning should be made with the front view. The top view should then be projected from it.

When the axis is parallel to both the H.P. and the V.P., neither the top view nor the front view will show the actual shape of the base. In this case, the projection of the solid on an auxiliary plane perpendicular to both the planes, viz. the side view must be drawn first. The front view and the top view are then projected from the side view. The projections in such cases may also be drawn in two stages.

(1) Axis perpendicular to the H.P.:

Problem 13-1. (fig. 13-12): *Draw the projections of a triangular prism, base 40 mm side and axis 50 mm long, resting on one of its bases on the H.P. with a vertical face perpendicular to the V.P.*

(i) As the axis is perpendicular to the ground i.e. the H.P. begin with the top view. It will be an equilateral triangle of sides 40 mm long, with one of its

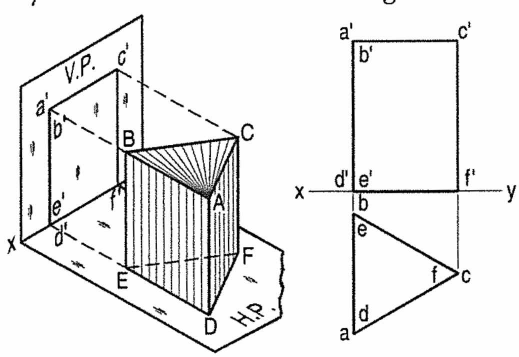


FIG. 13-12

sides perpendicular to xy. Name the corners as shown, thus completing the top view. The corners d, e and f are hidden and coincide with the top corners a, b and c respectively.

(ii) Project the front view, which will be a rectangle. Name the corners. The line $b'e'$ coincides with $a'd'$.

Problem 13-2. (fig. 13-13): *Draw the projections of a pentagonal pyramid, base 30 mm edge and axis 50 mm long, having its base on the H.P. and an edge of the base parallel to the V.P. Also draw its side view.*

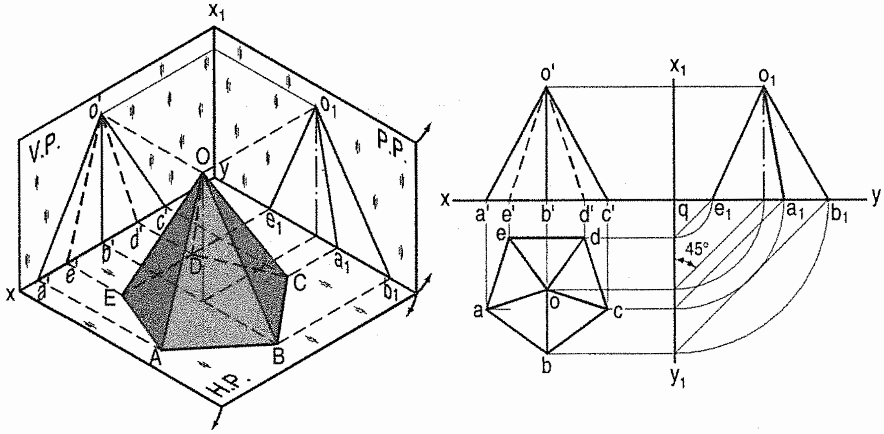


FIG. 13-13

(i) Assume the side DE which is nearer the V.P., to be parallel to the V.P. as shown in the pictorial view.

(ii) In the top view, draw a regular pentagon $abcde$ with ed parallel to and nearer xy. Locate its centre o and join it with the corners to indicate the slant edges.

(iii) Through o, project the axis in the front view and mark the apex o', 50 mm above xy. Project all the corners of the base on xy. Draw lines $o'a'$, $o'b'$ and $o'c'$ to show the visible edges. Show $o'd'$ and $o'e'$ for the hidden edges as dashed lines.

(iv) For the side view looking from the left, draw a new reference line x_1y_1 perpendicular to xy and to the right of the front view. Project the side view on it, horizontally from the front view as shown. The respective distances of all the points in the side view from x_1y_1, should be equal to their distances in the top view from xy. This is done systematically as explained below:

(v) From each point in the top view, draw horizontal lines upto x_1y_1. Then draw lines inclined at 45° to x_1y_1 (or xy) as shown. Or, with q, the point of intersection between xy and x_1y_1 as centre, draw quarter circles. Project up all the points to intersect the corresponding horizontal lines from the front view and complete the side view as shown in the figure. Lines o_1d_1 and o_1c_1 coincide with o_1e_1 and o_1a_1 respectively.

Problem 13-3. (fig. 13-14): *Draw the projections of (i) a cylinder, base 40 mm diameter and axis 50 mm long, and (ii) a cone, base 40 mm diameter and axis 50 mm long, resting on the H.P. on their respective bases.*

(i) Draw a circle of 40 mm diameter in the top view and project the front view which will be a rectangle [fig. 13-14(ii)].

(ii) Draw the top view [fig. 13-14(iii)]. Through the centre o, project the apex o', 50 mm above *xy*. Complete the triangle in the front view as shown.

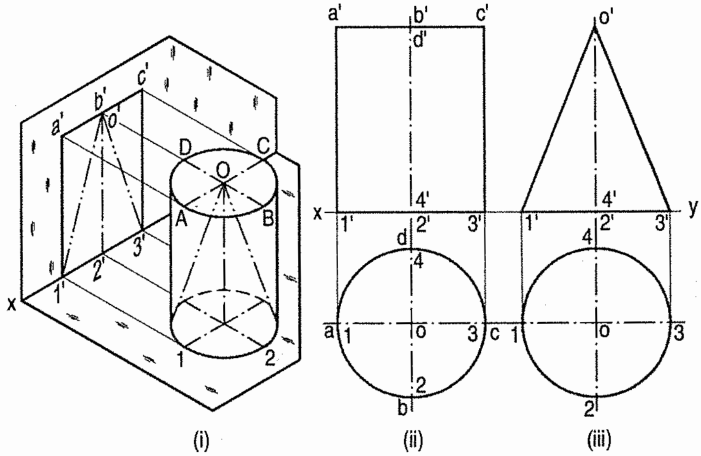


FIG. 13-14

In the pictorial view [fig. 13-14(i)], the cone is shown as contained by the cylinder.

Problem 13-4. (fig. 13-15): *A cube of 50 mm long edges is resting on the H.P. with its vertical faces equally inclined to the V.P. Draw its projections.*

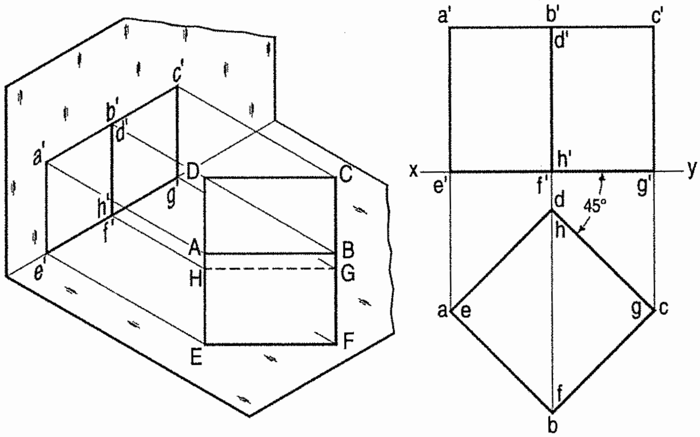


FIG. 13-15

Begin with the top view.

(i) Draw a square *abcd* with a side making 45° angle with *xy*.

(ii) Project up the front view. The line *d' h'* will coincide with *b' f'*.

Problem 13-5. (fig. 13-16): *Draw the projections of a hexagonal pyramid, base 30 mm side and axis 60 mm long, having its base on the H.P. and one of the edges of the base inclined at 45° to the V.P.*

(i) In the top view, draw a line *af* 30 mm long and inclined at 45° to *xy*. Construct a regular hexagon on *af*. Mark its centre *o* and complete the top view by drawing lines joining it with the corners.

(ii) Project up the front view as described in problem 13-2, showing the line *o'e'* and *o'f'* for hidden edges as dashed lines.

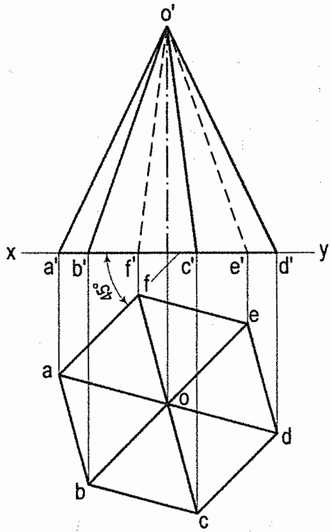


FIG. 13-16

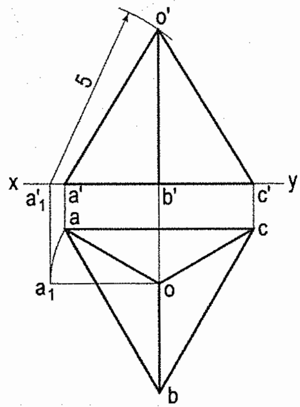


FIG. 13-17

Problem 13-6. (fig. 13-17): *A tetrahedron of 5 cm long edges is resting on the H.P. on one of its faces, with an edge of that face parallel to the V.P. Draw its projections and measure the distance of its apex from the ground.*

All the four faces of the tetrahedron are equal equilateral triangles of 5 cm side.

(i) Draw an equilateral triangle *abc* in the top view with one side, say *ac*, parallel to *xy*. Locate its centre *o* and join it with the corners.

(ii) In the front view, the corners *a'*, *b'* and *c'* will be in *xy*. The apex *o'* will lie on the projector through *o* so that its true distance from the corners of the base is equal to 5 cm.

(iii) To locate *o'*, make *oa* (or *ob* or *oc*) parallel to *xy*. Project a_1 to a'_1 on *xy*. With a'_1 as centre and radius equal to 5 cm cut the projector through *o* in *o'*. Draw lines *o'a'*, *o'b'* and *o'c'* to complete the front view. *o'b'* will be the distance of the apex from the ground.

(2) Axis perpendicular to the V.P.:

Problem 13-7. (fig. 13-18): *A hexagonal prism has one of its rectangular faces parallel to the H.P. Its axis is perpendicular to the V.P. and 3.5 cm above the ground.*

Draw its projections when the nearer end is 2 cm in front of the V.P. Side of base 2.5 cm long; axis 5 cm long.

(i) Begin with the front view. Construct a regular hexagon of 2.5 cm long sides with its centre 3.5 cm above *xy* and one side parallel to it.

(ii) Project down the top view, keeping the line for nearer end, viz. 1-4, 2 cm below *xy*.

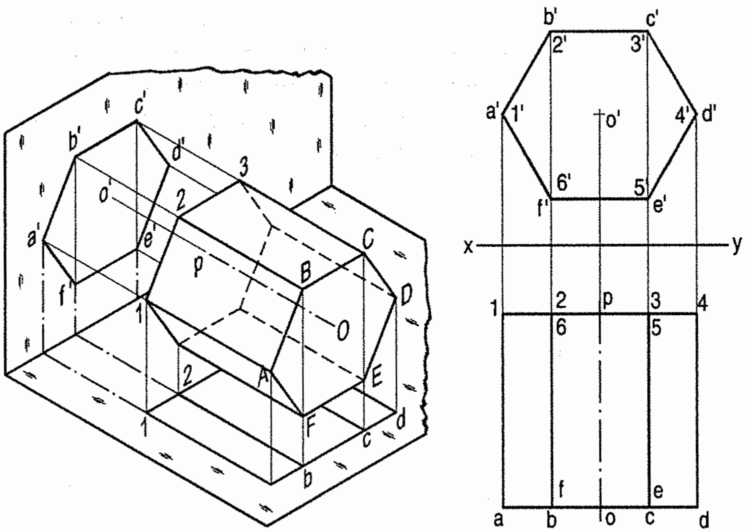


FIG. 13-18

Problem 13-8. (fig. 13-19): *A square pyramid, base 40 mm side and axis 65 mm long, has its base in the V.P. One edge of the base is inclined at 30° to the H.P. and a corner contained by that edge is on the H.P. Draw its projections.*

(i) Draw a square in the front view with the corner *d'* in *xy* and the side *d'c'* inclined at 30° to it. Locate the centre *o'* and join it with the corners of the square.

(ii) Project down all the corners in *xy* (because the base is in the V.P.). Mark the apex *o* on a projector through *o'*. Draw lines for the slant edges and complete the top view.

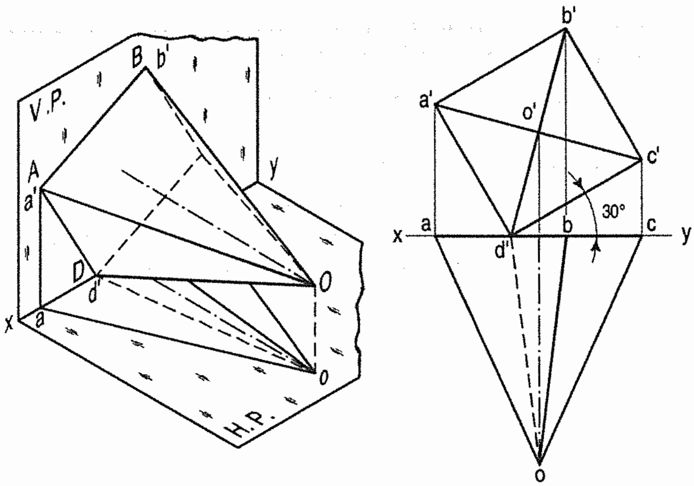


FIG. 13-19

(3) Axis parallel to both the H.P. and the V.P.:

Problem 13-9. (fig. 13-20): *A triangular prism, base 40 mm side and height 65 mm is resting on the H.P. on one of its rectangular faces with the axis parallel to the V.P. Draw its projections.*

As the axis is parallel to both the planes, begin with the side view.

(i) Draw an equilateral triangle representing the side view, with one side in *xy*.

(ii) Project the front view horizontally from this triangle.

(iii) Project down the top view from the front view and the side view, as shown.

This problem can also be solved in two stages as explained in the next article.

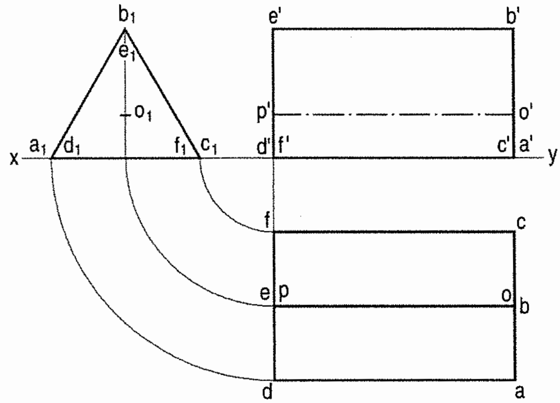


FIG. 13-20

EXERCISES 13(a)

Draw the projections of the following solids, situated in their respective positions, taking a side of the base 40 mm long or the diameter of the base 50 mm long and the axis 65 mm long.

1. A hexagonal pyramid, base on the H.P. and a side of the base parallel to and 25 mm in front of the V.P.
2. A square prism, base on the H.P., a side of the base inclined at 30° to the V.P. and the axis 50 mm in front of the V.P.
3. A triangular pyramid, base on the H.P. and an edge of the base inclined at 45° to the V.P.; the apex 40 mm in front of the V.P.
4. A cylinder, axis perpendicular to the V.P. and 40 mm above the H.P., one end 20 mm in front of the V.P.
5. A pentagonal prism, a rectangular face parallel to and 10 mm above the H.P., axis perpendicular to the V.P. and one base in the V.P.
6. A square pyramid, all edges of the base equally inclined to the H.P. and the axis parallel to and 50 mm away from both the H.P. and the V.P.
7. A cone, apex in the H.P. axis vertical and 40 mm in front of the V.P.
8. A pentagonal pyramid, base in the V.P. and an edge of the base in the H.P.

13-3. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO ONE OF THE REFERENCE PLANES AND PARALLEL TO THE OTHER

When a solid has its axis inclined to one plane and parallel to the other, its projections are drawn in two stages.

(1) In the initial stage, the solid is assumed to be in simple position, i.e. *its axis perpendicular to one of the planes.*

If the axis is to be inclined to the ground, i.e. the H.P., it is assumed to be perpendicular to the H.P. in the initial stage. Similarly, if the axis is to be inclined to the V.P., it is kept perpendicular to the V.P. in the initial stage.

Moreover

(i) if the solid has an edge of its base parallel to the H.P. or in the H.P. or on the ground, that edge should be kept perpendicular to the V.P.; if the edge of the base is parallel to the V.P. or in the V.P., it should be kept perpendicular to the H.P.

(ii) If the solid has a corner of its base in the H.P. or on the ground, the sides of the base containing that corner should be kept equally inclined to the V.P.; if the corner is in the V.P., they should be kept equally inclined to the H.P.

(2) Having drawn the projections of the solid in its simple position, the final projections may be obtained by one of the following *two* methods:

(i) **Alteration of position:** The position of one of the views is altered as required and the other view projected from it.

(ii) **Alteration of reference line or auxiliary plane:** A new reference line is drawn according to the required conditions, to represent an auxiliary plane and the final view projected on it.

In the first method, the reproduction of a view accurately in the altered position is likely to take considerable time, specially, when the solid has curved surfaces or too many edges and corners. In such cases, it is easier and more convenient to adopt the second method. Sufficient care must however be taken in transferring the distances of various points from their respective reference lines.

After determining the positions of all the points for the corners in the final view, difficulty is often felt in completing the view correctly. The following sequence for joining the corners may be adopted:

(a) Draw the lines for the edges of the visible base. The base, which (compared to the other base) is further away from *xy* in one view, will be fully visible in the other view.

(b) Draw the lines for the longer edges. The lines which pass through the figure of the visible base should be dashed lines.

(c) Draw the lines for the edges of the other base.

It should always be remembered that, when two lines representing the edges cross each other, one of them must be hidden and should therefore be drawn as a dashed line.

13-3-1. AXIS INCLINED TO THE V.P. AND PARALLEL TO THE H.P.

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 29 for the following problem.

Problem 13-10. (fig. 13-21): *Draw the projections of a pentagonal prism, base 25 mm side and axis 50 mm long, resting on one of its rectangular faces on the H.P., with the axis inclined at 45° to the V.P.*

In the simple position, assume the prism to be on one of its faces on the ground with the axis perpendicular to the V.P.

Draw the pentagon in the front view with one side in *xy* and project the top view [fig. 13-21(i)].

The shape and size of the figure in the top view will not change, so long as the prism has its face on the H.P. The respective distances of all the corners in the front view from *xy* will also remain constant.

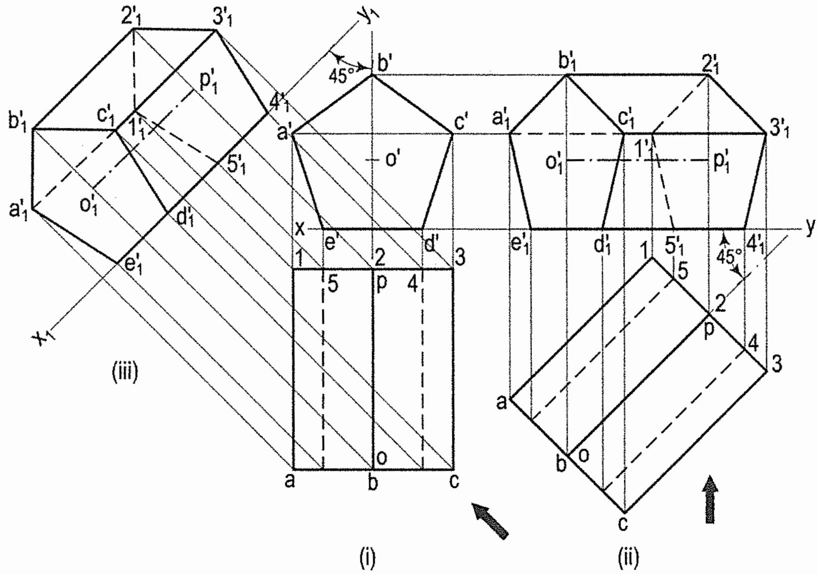


FIG. 13-21

Method I: [fig. 13-21(ii)]:

(i) Alter the position of the top view, i.e. reproduce it so that the axis is inclined at 45° to *xy*. Project all the points upwards from this top view and horizontally from the first front view, e.g. a vertical from *a* intersecting *a* horizontal from *a'* at a point a'_1.

(ii) Complete the pentagon $a'_1b'_1c'_1d'_1e'_1$ for the fully visible end of the prism. Next, draw the lines for the longer edges and finally, draw the lines for the edges of the other end. Note carefully that the lines $a'_1 1'_1$, $1'_1 2'_1$ and $1'_1 5'_1$ are dashed lines. $e'_1 5'_1$ is also hidden but it coincides with other visible lines.

Method II: [fig. 13-21(iii)]:

(i) Draw a new reference line x_1y_1, making 45° angle with the top view of the axis, to represent an auxiliary vertical plane.

(ii) Draw projectors from all the points in the top view perpendicular to x_1y_1 and on them, mark points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the front view. Join the points as already explained. The auxiliary front view and the top view are the required projections.

Problem 13-11. (fig. 13-22): *Draw the projections of a cylinder 75 mm diameter and 100 mm long, lying on the ground with its axis inclined at 30° to the V.P. and parallel to the ground.*

Adopt the same methods as in the previous problem. The ellipses for the ends should be joined by common tangents. Note that half of the ellipse for the hidden base will be drawn as dashed line.

Fig. 13-22(iii) shows the front view obtained by the method II.

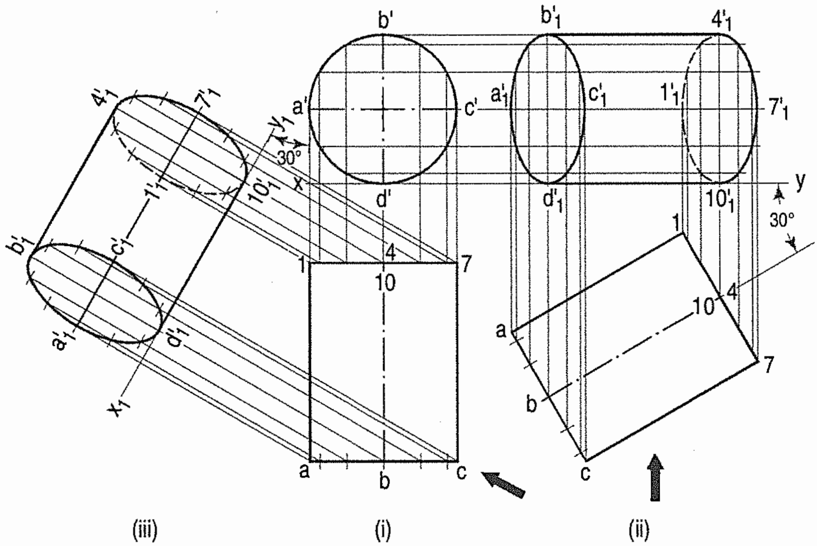


FIG. 13-22

13-3-2. AXIS INCLINED TO THE H.P. AND PARALLEL TO THE V.P.

Problem 13-12. (fig. 13-23): *A hexagonal pyramid, base 25 mm side and axis 50 mm long, has an edge of its base on the ground. Its axis is inclined at 30° to the ground and parallel to the V.P. Draw its projections.*

In the initial position assume the axis to be perpendicular to the H.P.

Draw the projections with the base in *xy* and its one edge perpendicular to the V.P. [fig. 13-23(i)].

If the pyramid is now tilted about the edge *AF* (or *CD*) the axis will become inclined to the H.P. but will remain parallel to the V.P. The distances of all the corners from the V.P. will remain constant.

The front view will not be affected except in its position in relation to *xy*. The new top view will have its corners at same distances from *xy*, as before.

Method I: [fig. 13-23(ii)]:

(i) Reproduce the front view so that the axis makes 30° angle with *xy* and the point *a*' remains in *xy*.

(ii) Project all the points vertically from this front view and horizontally from the first top view. Complete the new top view by drawing (a) lines joining the apex o'_1 with the corners of the base and (b) lines for the edges of the base.

The base will be partly hidden as shown by dashed line a_1b_1, e_1f_1 and f_1a_1. Similarly o_1f_1 and o_1a_1 are also dashed lines.

Method II: [fig. 13-23(iii)]:

(i) Through *a′* draw a new reference line x_1y_1 inclined at 30° to the axis, to represent an auxiliary inclined plane.

(ii) From the front view project the required top view on x_1y_1, keeping the distance of each point from x_1y_1 equal to the distance of its first top view from *xy*, viz. e_1q = *eb*' etc.

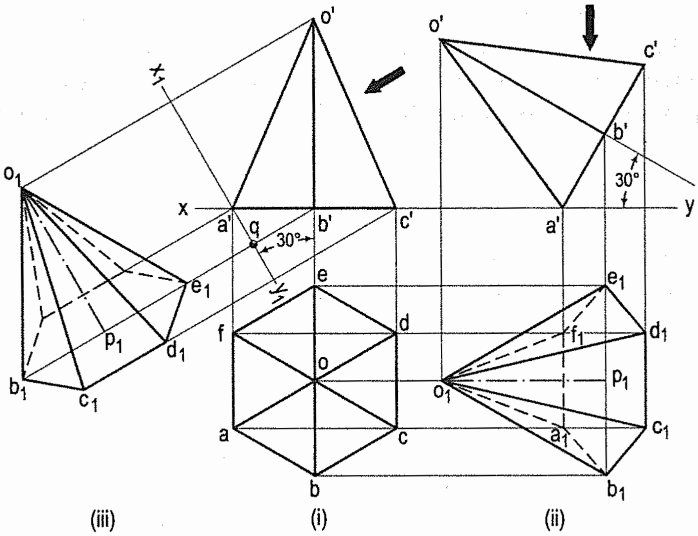


FIG. 13-23

Problem 13-13. (fig. 13-24): *Draw the projections of a cone, base 75 mm diameter and axis 100 mm long, lying on the H.P. on one of its generators with the axis parallel to the V.P.*

(i) Assuming the cone to be resting on its base on the ground, draw its projections.

(ii) Re-draw the front view so that the line *o*'7' (or *o*'1') is in *xy*. Project the required top view as shown. The lines from o_1 should be tangents to the ellipse.

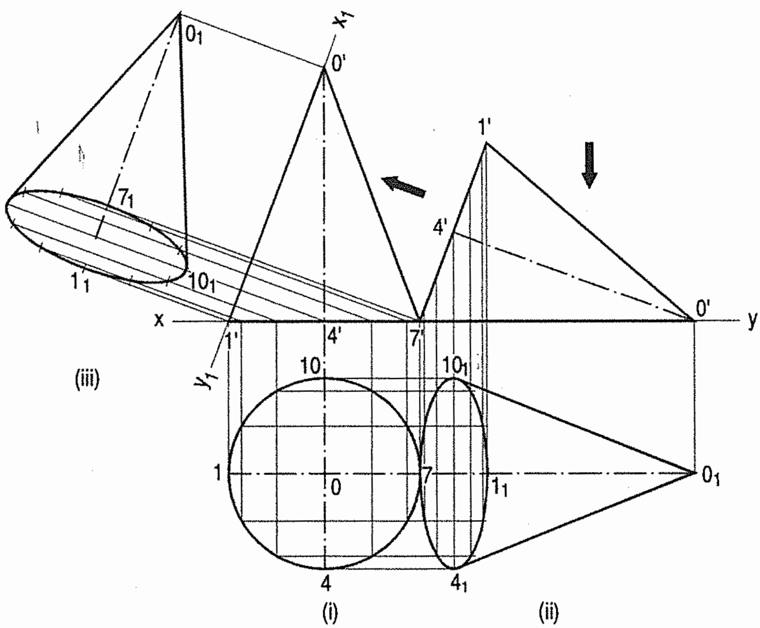
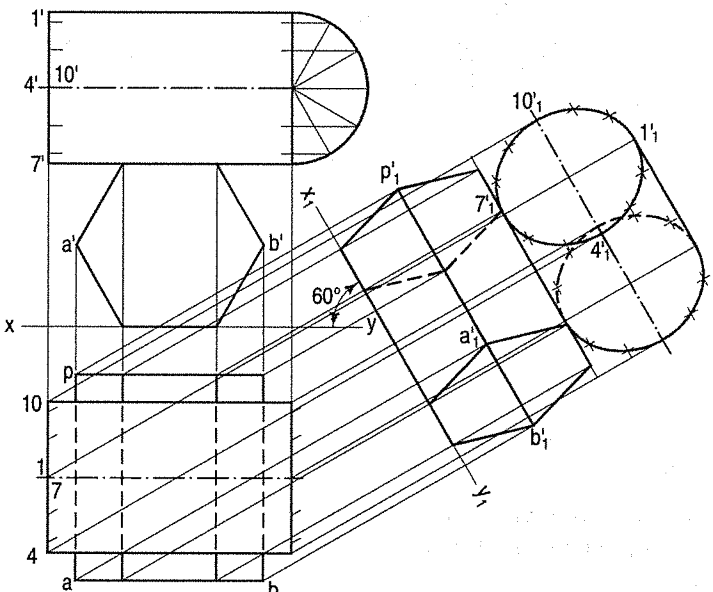


FIG. 13-24

The top view obtained by auxiliary-plane method is shown in fig. 13-24(iii). The new reference line x_1y_1 is so drawn as to contain the generator o'1' instead of o'7' (for sake of convenience). The cone is thus lying on the generator o'1'. Note that $1'1_1$ = 1'1, $o'o_1$ = 4'o etc. Also note that the base is fully visible in both the methods.

Problem 13-14. *The projections of a cylinder resting centrally on a hexagonal prism are given in fig. 13-25(i). Draw its auxiliary front view on a reference line inclined at 60° to xy.*

See fig. 13-25(ii) which is self-explanatory.



(i) (ii)

FIG. 13-25

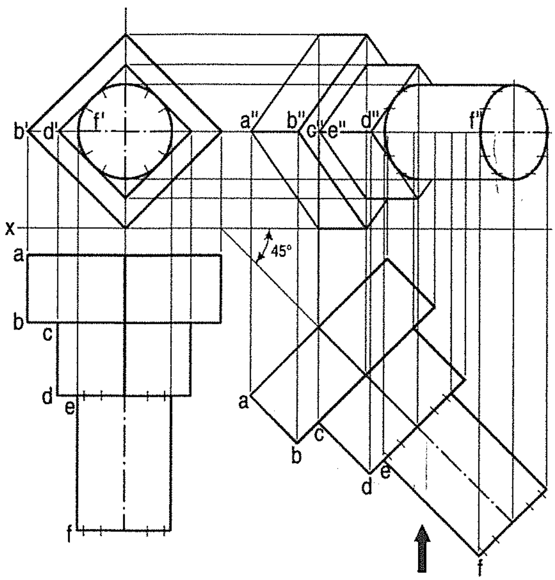


FIG. 13-26

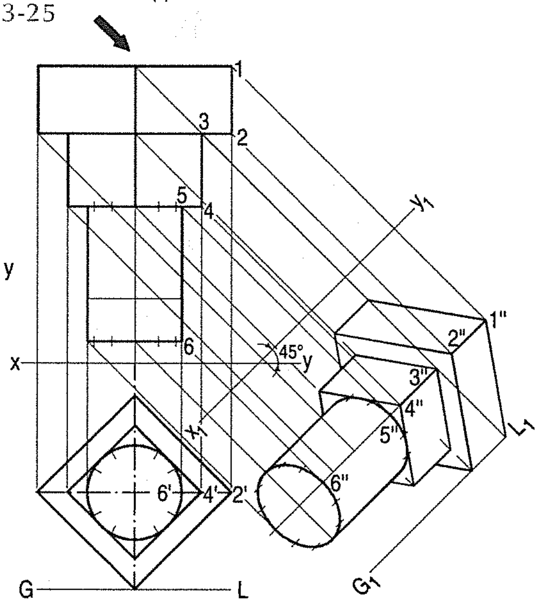


FIG. 13-27

Problem 13-15. (fig. 13-26 and fig. 13-27): *A square-headed bolt 25 mm diameter, 125 mm long and having a square neck has its axis parallel to the H.P. and inclined at 45° to the V.P.*

All the faces of the square head are equally inclined to the H.P. Draw its projections neglecting the threads and chamfer.

See fig. 13-26. The projections are obtained by the change-of-position method. The length of the bolt is taken shorter.

Fig. 13-27 shows the views in third-angle projection, obtained by the auxiliary-plane *method.*

Problem 13-16. (fig. 13-28): *A hexagonal prism, base 40 mm side and height 40 mm has a hole of 40 mm diameter drilled centrally through its ends. Draw its projections when it is resting on one of its corners on the H.P. with its axis inclined at 60° to the H.P. and two of its faces parallel to the V.P.*

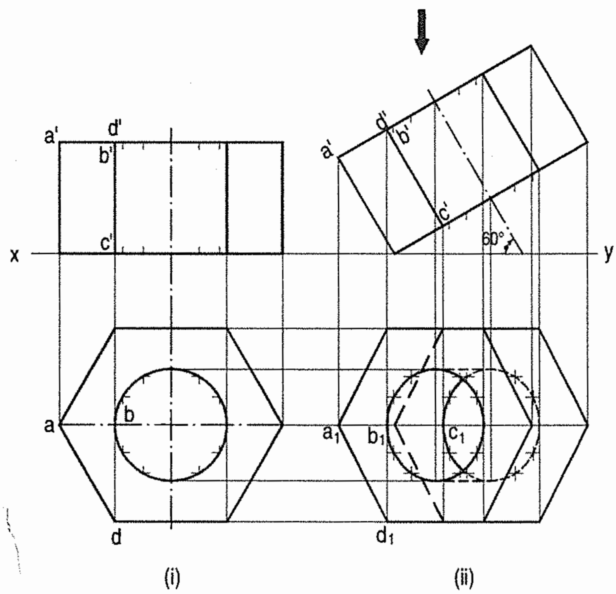


FIG. 13-28

(i) Begin with the top view and project up the front view assuming the axis to be vertical.

(ii) Tilt the front view, and project the required top view. Note that a part of the ellipse for the lower end of the hole will be visible.

Problem 13-17. (fig. 13-29): *The projections of a hopper made of tin sheet are given. Project another top view on an auxiliary inclined plane making 45° angle with the H.P.*

(i) Draw a new reference line x_1y_1 inclined at 45° to xy and project the required top view on it, from the front view.

(ii) Show carefully, the visible ellipses for the outer as well as the inner parts of the hopper rings.

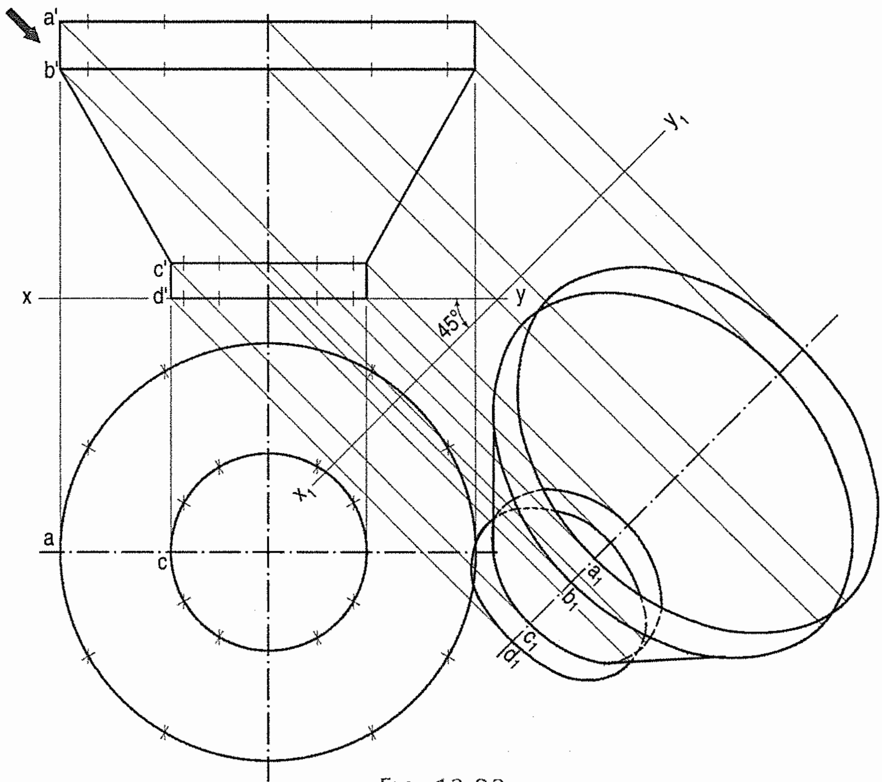


FIG. 13-29

13-4. PROJECTIONS OF SOLIDS WITH AXES INCLINED TO BOTH THE H.P. AND THE V.P.

The projections of a solid with its axis inclined to both the planes are drawn in three stages:

(i) Simple position

(ii) Axis inclined to one plane and parallel to the other

(iii) Final position.

The second and final positions may be obtained either by the alteration of the positions of the solid, i.e. the views, or by the alteration of reference lines.

Problem 13-18. *A square prism, base 40 mm side and height 65 mm, has its axis inclined at 45° to the H.P. and has an edge of its base, on the H.P and inclined at 30° to the V.P. Draw its projections.*

Method I: (fig. 13-30):

(i) Assuming the prism to be resting on its base on the ground with an edge of the base perpendicular to the V.P., draw its projections.

Assume the prism to be tilted about the edge which is perpendicular to the V.P., so that the axis makes 45° angle with the H.P.

(ii) Hence, change the position of the front view so that the axis is inclined at 45° to *xy* and *f'* (or *e'*) is in *xy*. Project the second top view.

Again, assume the prism to be turned so that the edge on which it rests, makes an angle of 30° with the V.P., keeping the inclination of the axis with the ground constant. The shape and size of the second top view will remain the same; only its position will change. In the front view, the distances of all the corners from xy will remain the same as in the second front view.

(iii) Therefore, reproduce the second top view making f_1g_1 inclined at 30° to xy. Project the final front view upwards from this top view and horizontally from the second front view, e.g. a vertical from a_1 and a horizontal from a' intersecting at a'_1. As the top end is further away from xy in the top view it will be fully visible in the front view. Complete the front view showing the hidden edges by dashed lines.

(iv) The second top view may be turned in the opposite direction as shown. In this position, the lower end of the prism, viz. $e'_1f'_1g'_1h'_1$ will be fully visible in the front view.

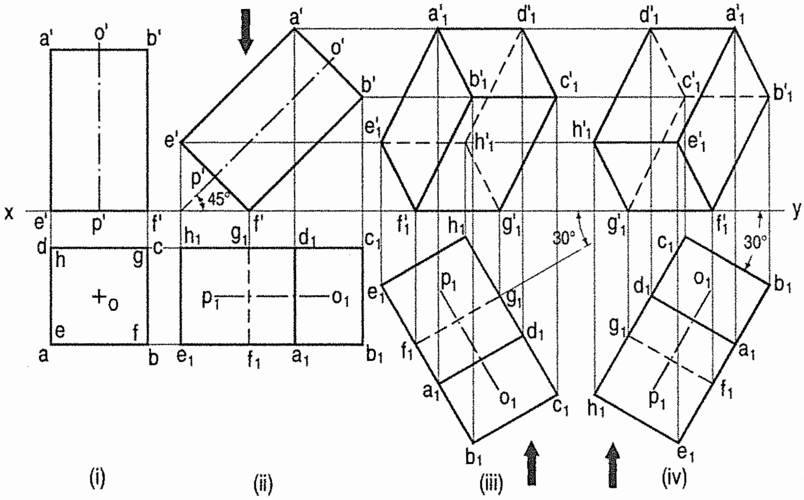


FIG. 13-30

Method II: (fig. 13-31):

(i) Draw the top view and the front view in simple position.

(ii) Through f', draw a new reference line x_1y_1 making 45° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 30° to the line f_1g_1. From the auxiliary top view, project the required front view, keeping the distance of each point from x_2y_2, equal to its distance (in the first front view) from x_1y_1 i.e. $a'_1q_1 = a'q$ etc. The problem is thus solved by change-of-reference line method only.

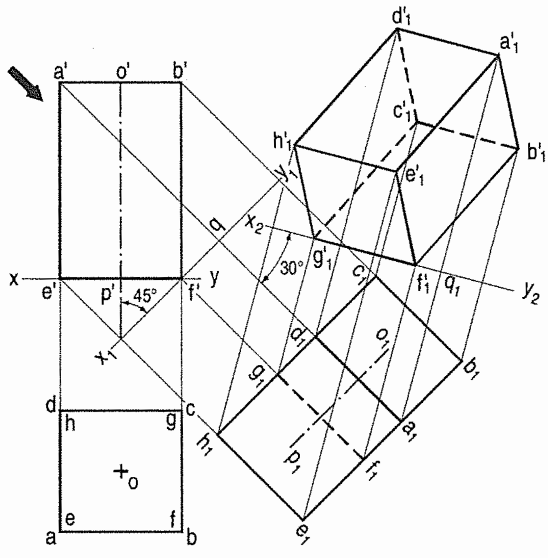


FIG. 13-31

Note: The new reference line satisfying the required conditions may be drawn in various positions, as explained in chapter 11.

Problem 13-19. (fig. 13-32): *Draw the projections of a cone, base 45 mm diameter and axis 50 mm long, when it is resting on the ground on a point on its base circle with (a) the axis making an angle of 30° with the H.P. and 45° with the V.P.; (b) the axis making an angle of 30° with the H.P. and its top view making 45° with the V.P.*

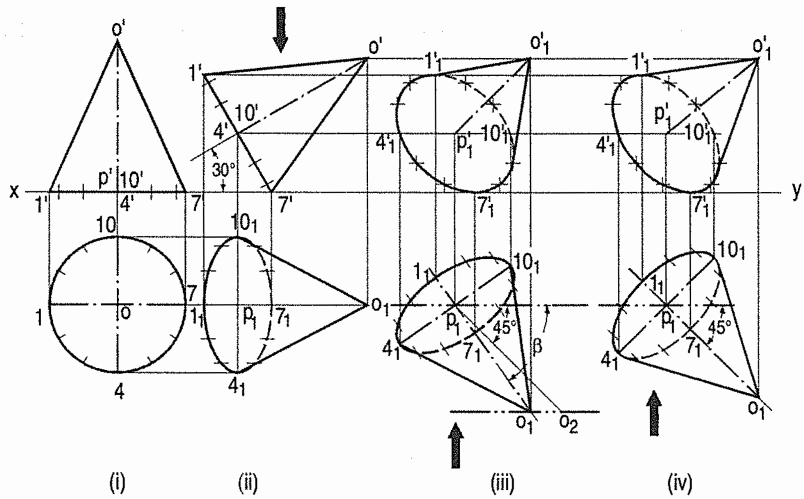


FIG. 13-32

(i) Draw the top view and the front view of the cone with the base on the ground.

(ii) Tilt the front view so that the axis makes 30° angle with *xy*. Project the second top view.

(a) In order that the axis may make an angle of 45° with the V.P., let us determine the apparent angle of inclination which the top view of the axis, viz. o_1p_1 should make with *xy* and which will be greater than 45°.

(iii) Mark any point p_1 below *xy*. Draw a line p_1o_2 equal to the true length of the axis, viz, *o'p'*, and inclined at 45° to *xy*. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the apparent angle of inclination and is greater than 45°. Around p_1o_1 as axis, reproduce the second top view and project the final front view as shown.

Note that the base of the cone is not visible in the front view because it is nearer *xy* in the top view.

(b) When the top view of the axis is to make 45° angle with the V.P., it is evident that p_1o_1 should be inclined at 45° to *xy*. Hence, reproduce the top view accordingly and project the required front view [fig. 13-32(iv)].

Problem 13-20. (fig. 13-33): *A pentagonal pyramid, base 25 mm side and axis 50 mm long has one of its triangular faces in the V.P. and the edge of the base contained by that face makes an angle of 30° with the H.P. Draw its projections.*

(i) In the initial position, assume the pyramid as having its base in the V.P. and an edge of the base perpendicular to the H.P. The front view will have to be drawn first and the top view projected from it.

(ii) Change the position of the top view so that the line o1 (for the face o1 5) is in *xy*. Project the second front view.

(iii) Tilt this front view so that the line $1'_1 5'_1$ makes 30° angle with *xy*. Project the final top view. Note that the base is not visible in the top view as it is nearer *xy* in the front view.

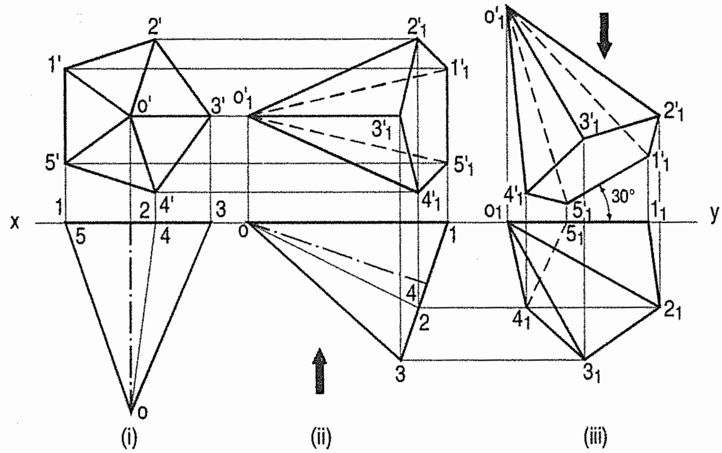


FIG. 13-33

Problem 13-21. (fig. 13-34): *A square pyramid, base 38 mm side and axis 50 mm long, is freely suspended from one of the corners of its base. Draw its projections, when the axis as a vertical plane makes an angle of 45° with the V.P. When a pyramid is suspended freely from a corner of its base, the imaginary line joining that corner with the centre of gravity of the pyramid will be vertical.*

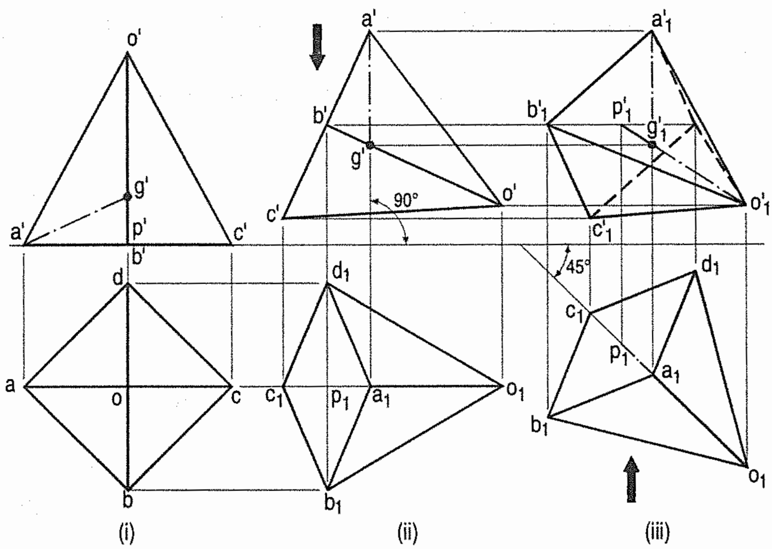


FIG. 13-34

The centre of gravity of a pyramid lies on its axis and at a distance equal to $\frac{1}{4}$ of the length of the axis from the base.

Assume the pyramid to be suspended from the corner A of the base.

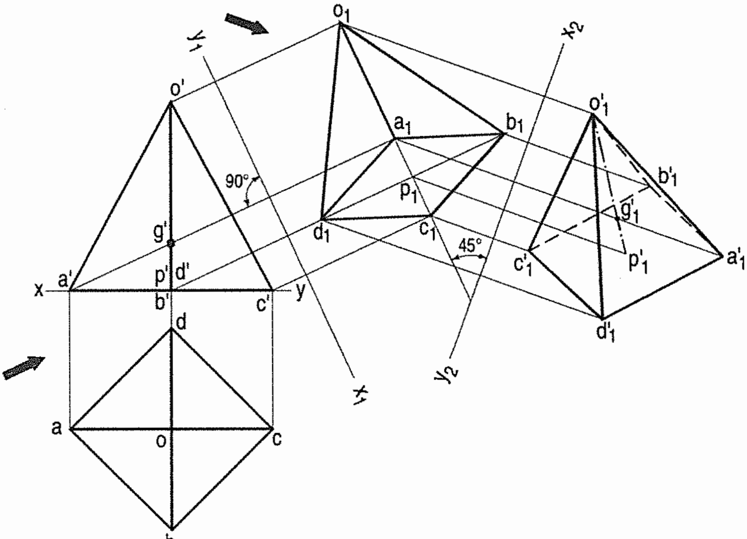


FIG. 13-35

In the initial position, the pyramid should be kept with its base on the ground and the line joining A with the centre of gravity G, parallel to the V.P. In the top view, g will coincide with o the top view of the axis.

(i) Draw a square $abcd$ (in the top view) with ag, i.e. ao parallel to xy. Project the front view. Making g' at a distance equal to $\frac{1}{4}$ of the axis from xy. Join a' with g'.

(ii) Tilt the front view so that $a'g'$ is perpendicular to xy and project the top view. The axis will still remain parallel to the V.P.

(iii) Reproduce this top view so that o_1p_1 (the top view of the axis) is inclined at 45° to xy. The axis as a vertical plane will thus be making 45° angle with the V.P. Project the final front view.

Fig. 13-35 shows the projections obtained by the change-of-reference-line method.

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 30 for the following problem.

Problem 13-22. (fig. 13-36): *A hexagonal pyramid, base 25 mm side and axis 55 mm long, has one of its slant edges on the ground. A plane containing that edge and the axis is perpendicular to the H.P. and inclined at 45° to the V.P. Draw its projections when the apex is nearer the V.P. than the base.*

Assume the pyramid to be resting on the ground on its base with a slant edge parallel to the V.P.

(i) Draw the top view of the pyramid with a side of the hexagon parallel to xy. The lines ao and do for the slant edges will also be parallel to xy. Project the front view.

(ii) Tilt this front view so that $a'o'$ or $d'o'$ is in xy. Project the second top view.

(iii) Draw a new reference line x_1y_1 making 45° angle with o_1d_1 (the top view of the axis) and project the final front view.

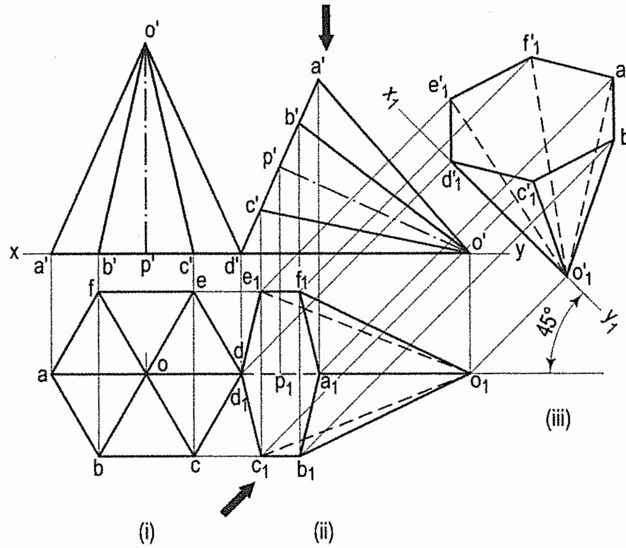


FIG. 13-36

The problem is thus solved by combination of the change-of-position and change-of-reference-line methods.

Problem 13-23. (fig. 13-37): *Draw the projections of a cube of 25 mm long edges resting on the H.P on one of its corners with a solid diagonal perpendicular to the V.P.*

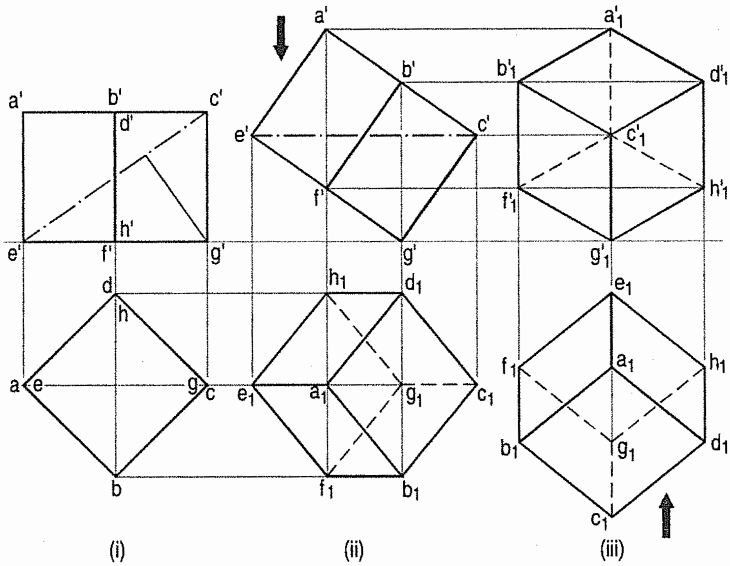


FIG. 13-37

Assume the cube to be resting on one of its faces on the H.P. with a solid diagonal parallel to the V.P.

(i) Draw a square *abcd* in the top view with its sides inclined at 45° to *xy*. The line *ac* representing the solid diagonals *AG* and *CE* is parallel to *xy*. Project the front view.

(ii) Tilt the front view about the corner *g'* so that the line *e'c'* becomes parallel to *xy*. Project the second top view. The solid diagonal *CE* is now parallel to both the H.P. and the V.P.

(iii) Reproduce the second top view so that the top view of the solid diagonal, viz. e_1c_1 is perpendicular to *xy*. Project the required front view.

Problem 13-24. (fig. 13-38): *A triangular prism, base 40 mm side and axis 50 mm long, is lying on the H.P. on one of its rectangular faces with the axis perpendicular to the V.P. A cone, base 40 mm diameter and axis 50 mm long, is resting on the H.P. and is leaning centrally on a face of the prism, with its axis parallel to the V.P. Draw the projections of the solids and project another front view on a reference line making 60° angle with xy.*

It will first be necessary to draw the cone with its base on the H.P. to determine the length of its generator and to project the top view.

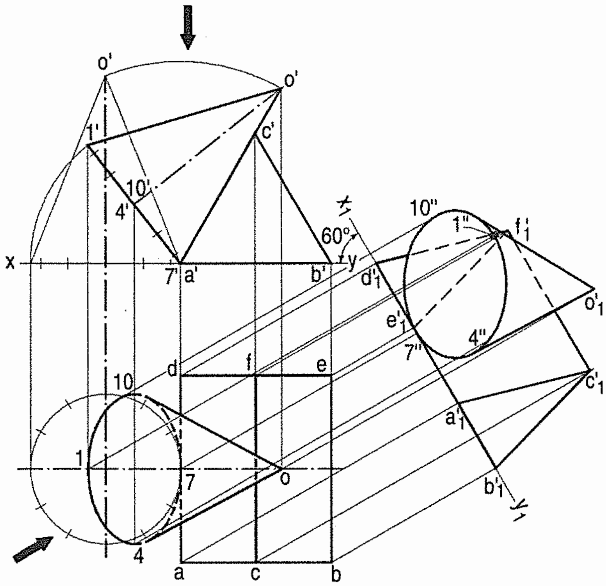


FIG. 13-38

Next, draw a triangle *a'b'c'* for the prism and a triangle *o'1'7'* for the cone as shown by the construction lines. Project the top view. Draw a reference line x_1y_1 and project the required front view as shown.

Problem 13-25. *A pentagonal prism is resting on one of the corners of its base on the H.P. The longer edge containing that corner is inclined at 45° to the H.P. The axis of the prism makes an angle of 30° to the V.P. Draw the projections of the solid.*

Also, draw the projections of the solid when the top view of axis is inclined at 30° to xy. Take the side of base 45 mm and height 70 mm.

(i) Assuming the prism to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of its base perpendicular to *xy*.

(ii) Redraw the front view so that the edge *c'3'* is inclined at 45° to *xy*. Project the required top view as shown in fig. 13-39(i).

(iii) Determine the apparent angle of inclination which the top view of the axis should make with *xy* when the axis makes an angle of 30° with the V.P.

(iv) Mark any point p_1 below *xy*. Draw a line p_1o_2 equal to the true length of the axis (70 mm) and inclined at 30° to *xy*. With p_1 as centre and radius equal to p_1o_1 (the length of the top view of the axis) draw an arc cutting the locus of o_2 at o_1. Then β is the required apparent angle of inclination. Considering p_1o_1 as axis, reproduce the second top view and project the final front view as shown in fig. 13-39(i).

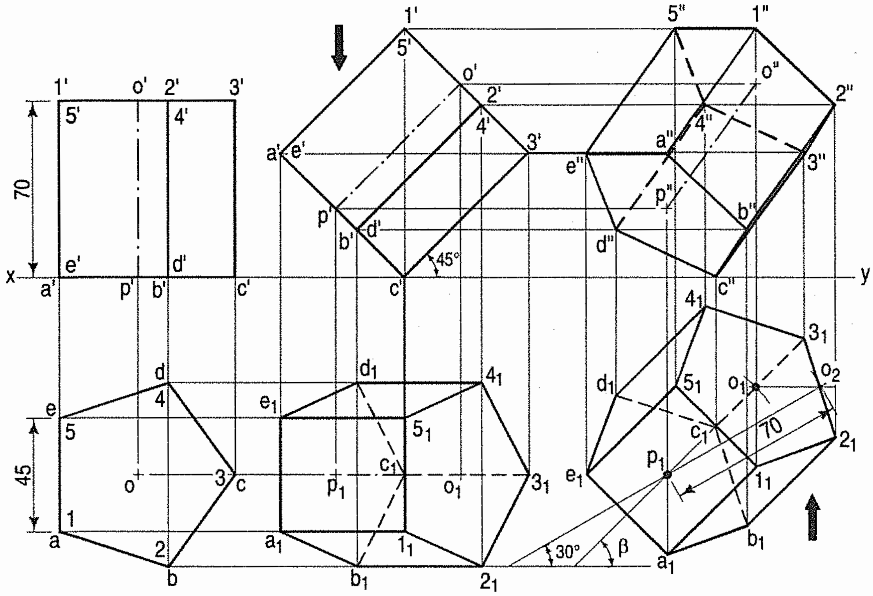


FIG. 13-39(i)

(v) When the top view of axis makes an angle of 30° with the V.P., it is evident that p_1o_1 is inclined at 30° to xy. Hence, reproduce the top view and the front view as shown in fig. 13-39(ii).

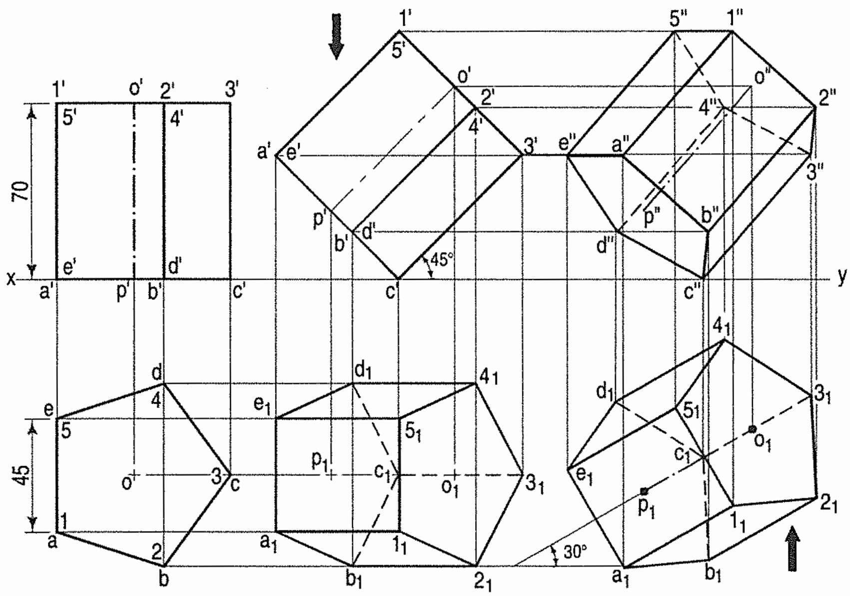


FIG. 13-39(ii)

Problem 13-26 (fig 13-40)**:** *A square prism, with the side of its base 40 mm and axis 70 mm long is lying on one of its base edges on the H.P. in such a way that this base edge makes an angle of 45° with the V.P. and the axis is inclined at 30° to the H.P. Draw the projections of the solid using the 'auxiliary plane method'.*

(i) In the initial position assume the axis of the prism to be perpendicular to the H.P. Draw the projections as shown.

(ii) Draw a new reference line x_1y_1 making an angle of 30° with the front view of the axis, to represent an auxiliary horizontal plane. Draw projectors from a', b' c', d' and 1', 2', 3', 4' perpendicular to x_1y_1 and on them, mark these points keeping the distance of each point from x_1y_1 equal to its distance from *xy* in the top view. Join the points as shown.

(iii) Draw another reference line x_2y_2 inclined at 45° to the line *a*1 (or *b*2). From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1, i.e. *q*'1" = *q*1' etc. Join the points as shown. Note that the view is obtained by observing the auxiliary top view from the top, along the projectors.

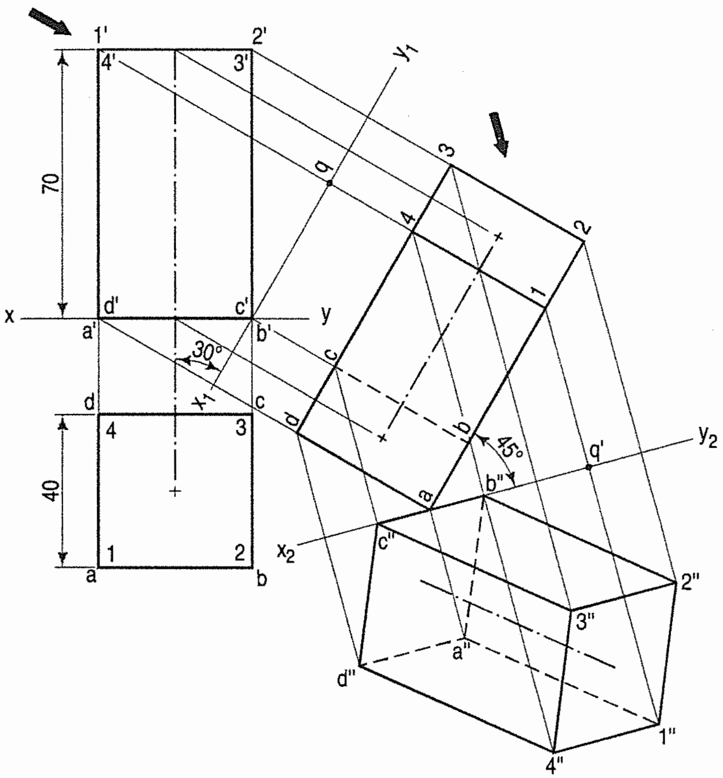


FIG. 13-40

Problem 13-27. *A hexagonal prism, with the side of the hexagon 30 mm and height of 70 mm is resting on the H.P. on one of the edges of its hexagonal base in such a way that, the edge is at 60° to the V.P. and the base is at 30° to the H.P. Draw to scale 1:1, the view from the front and the view from the top.*

Refer to fig. 13-41.

(i) Draw the top view and the front view in simple position keeping the axis perpendicular to the H.P.

(ii) Draw a new reference line x_1y_1 making 60° angle with the axis. On it, project the auxiliary top view.

(iii) Draw another reference line x_2y_2 inclined at 60° to the edge of base c_1d_1. From the auxiliary top view, project the required new front view, keeping the distance of each point from x_2y_2, equal to its distance from x_1y_1 i.e. $q3' = q'3''$ etc. Join the points as shown. It should be noted that the edge of base away from x_2y_2 will be observed as full lines and nearest lines from x_2y_2 will be dotted lines. i.e. $c''d''$, $d''e''$ and $e''f''$ are full lines while $f''a''$, $a''b''$ and $b''c''$ are dotted lines. Note that the view is drawn by observing the auxiliary top view from the top along the projectors.

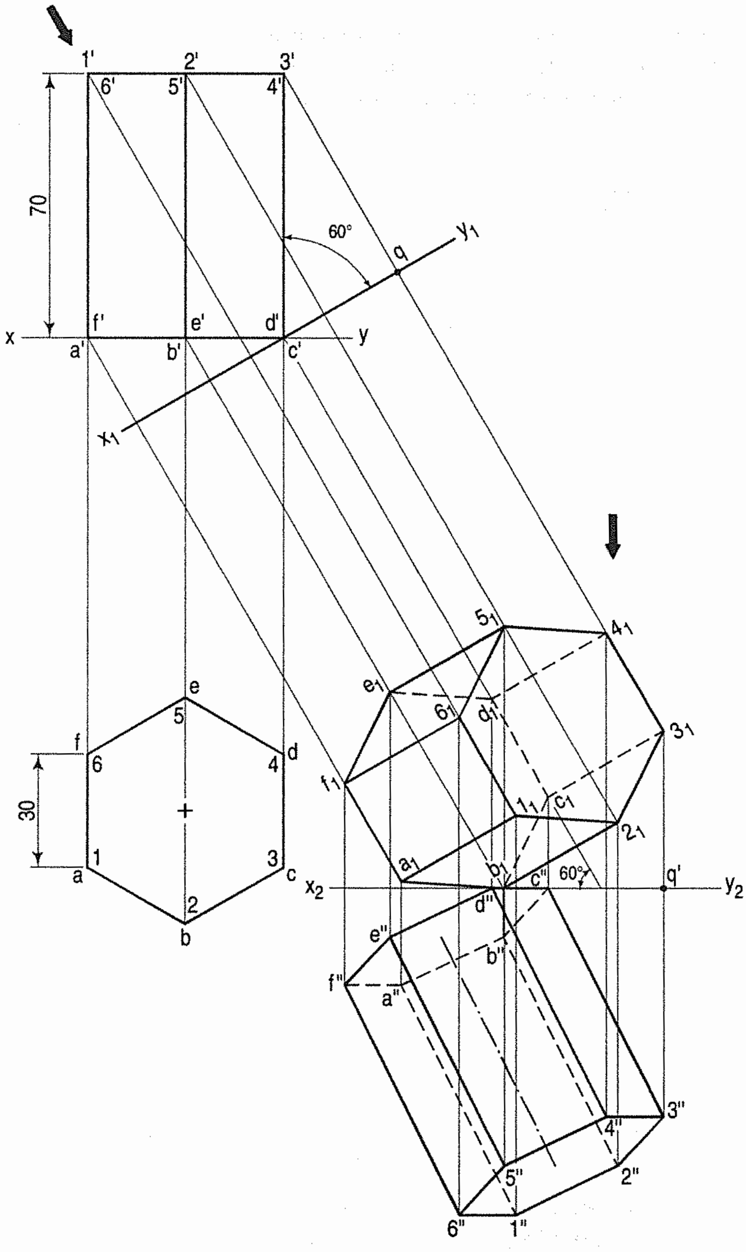


FIG. 13-41

Problem 13-28. *A regular pentagonal prism lies with its axis inclined at 60° to the H.P. and 30° to the V.P. The prism is 60 mm long and has a face width of 25 mm. The nearest corner is 10 mm away from the V.P. and the farthest shorter edge is 100 mm from the H.P. Draw the projections of the solid.*

(i) Draw initial position of the prism as shown in fig. 13-42.

(ii) With 4' as centre and radius equal to 100 mm, draw an arc. Mark tangent to the arc making 60° with the axis as shown. This is a new reference line x_1y_1. Project the required new top view.

(iii) Draw another reference line x_2y_2 inclined at 30° angle to the axis of new top view. Project the various points to obtain new front view as shown in fig. 13-42. Observe the auxiliary top view from the top along the projectors.

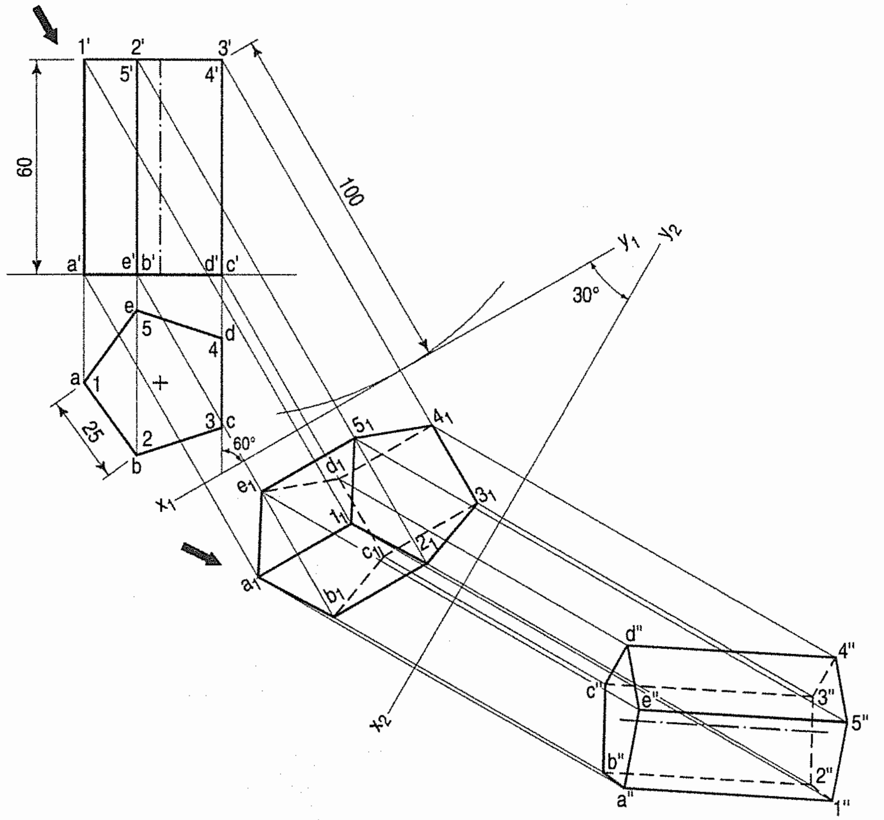


FIG. 13-42

Problem 13-29. *A square pyramid of 50 mm side of base and 50 mm length of axis is resting on one of its triangular faces on the H.P. having a slant edge containing that face parallel to the V.P. Draw the projections of the pyramid.*

(i) Assuming the axis of pyramid perpendicular to the H.P., draw the front view and the top view as shown in fig. 13-43.

(ii) Draw new reference line x_1y_1 coinciding with o'c' in the front view. Project new top view, keeping the distance of a_1, b_1...o_1 from x_1y_1 equal to the distance of a, b,...o from xy. Join these points.

(iii) Draw another reference line x_2y_2 parallel to the slant edge o_1c_1 or o_1b_1. Project new front view as shown. Observe auxiliary top view from the base *a, b, c, d, o* along projectors.

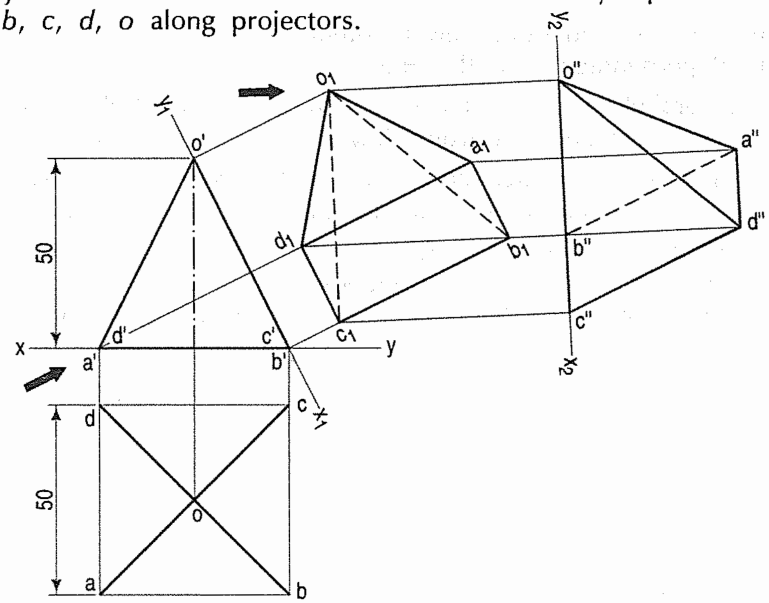


FIG. 13-43

Problem 13-30. *A regular pentagonal pyramid, base 30 mm side and height 80 mm rests on one edge of its base on the ground so that the highest point in the base is 30 mm above the ground. Draw its projection when the axis is parallel to the V.P.*

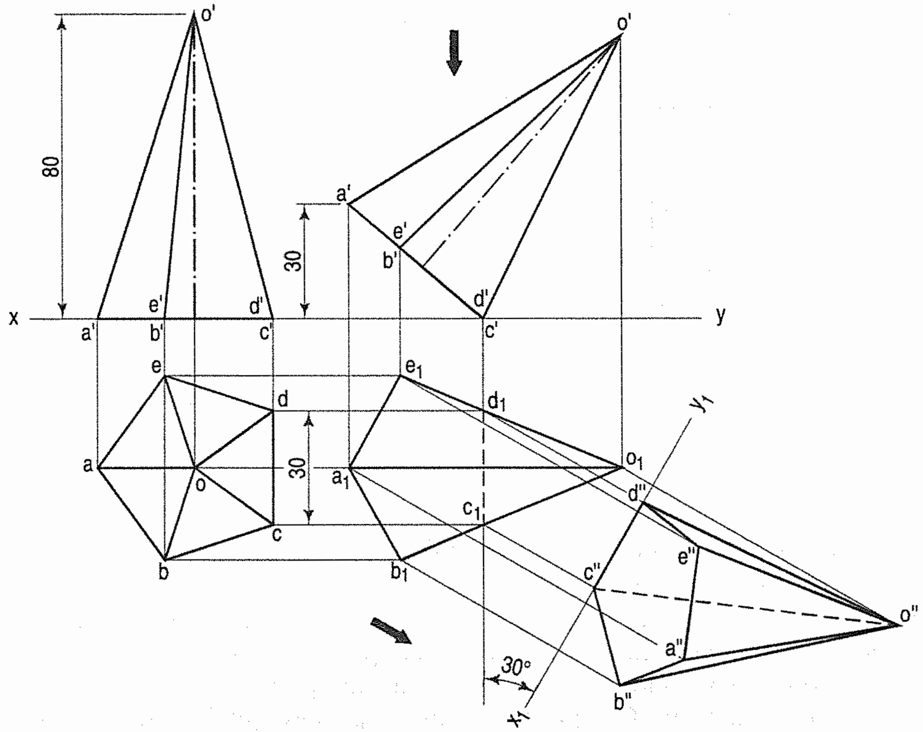


FIG. 13-44

Draw another front view on a reference line inclined at 30° to the edge on which it is resting so that the base is visible.

(i) Draw top view and front view in simple position assuming the axis of the pyramid perpendicular to the H.P.

(ii) Draw a parallel line at a distance of 30 mm from *xy*. Mark the point c' on the line *xy* and reproduce the front view as shown fig. 13-44.

(iii) Project points *a*', *b*', *c*' etc. and obtain new top view keeping distance of points a_1, b_1, c_1 etc. from *xy* equal to distance of *a*, *b*, *c*, etc. from the line *xy*.

(iv) Draw another reference line x_1y_1 making an angle of 30° with the side of base c_1d_1 and obtain a new front view as shown. Note that the base is visible. Observe from the base *a*, *b*, *c*, *d*, *e* along the projectors.

Problem 13-31. *A regular pentagonal pyramid with the sides of its base 30 mm and height 80 mm rests on an edge of the base. The base is tilted until its apex is 50 mm above the level of the edge of the base on which it rests. Draw the projection of the pyramid when the edge on which it rests, is parallel to the V.P. and the apex of the pyramid points towards V.P.*

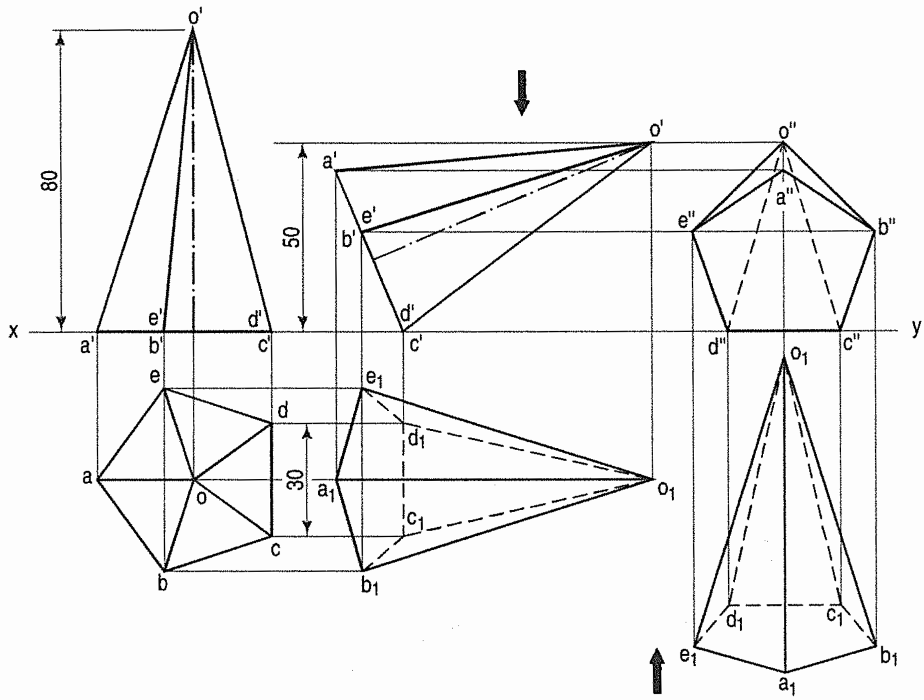


FIG. 13-45

(i) Draw top view and front view assuming the axis of the pyramid perpendicular to the H.P. as shown in fig. 13-45.

(ii) Draw a parallel line at a distance of 50 mm from *xy*. Reproduce the front view as shown. Draw projectors from points *a*', *b*', *c*' etc. vertically from the front view and horizontally from the points *a*, *b*, *c* etc. from the previous top view. Complete the new top view, joining the intersection of the projectors in the correct sequence as shown.

(iii) Redraw the top view keeping c_1d_1 parallel to xy. Project the points a_1, b_1, c_1 etc. vertically from the new top view and horizontal projectors from the points a', b', c' etc. of the front view. Join the intersection points of both the projectors in the correct sequence as shown.

Problem 13-32. *A right regular pentagonal pyramid, with the sides of the base 30 mm and height 65 mm rests on the edge of its base on the horizontal plane, the base being tilted until the vertex is at 60 mm above the H.P. Draw the projections of the pyramid when the edge on which it rests, is made parallel to FRP. Assuming the pyramid to be resting on its base on the horizontal plane, draw its projections keeping one of the sides of the base perpendicular to xy.*

Method I: Changing position of reference line [fig. 13-46(i)]:

(i) With o' as centre and radius equal to 60 mm, draw an arc. Draw the tangent to the arc passing through c' or d'. This is a new reference line x_1y_1. Project the required top view.

(ii) Draw another reference line x_2y_2 parallel to c_1d_1. Project new front view as shown. Observe auxiliary top view from the base a, b, c, d, e along the projectors.

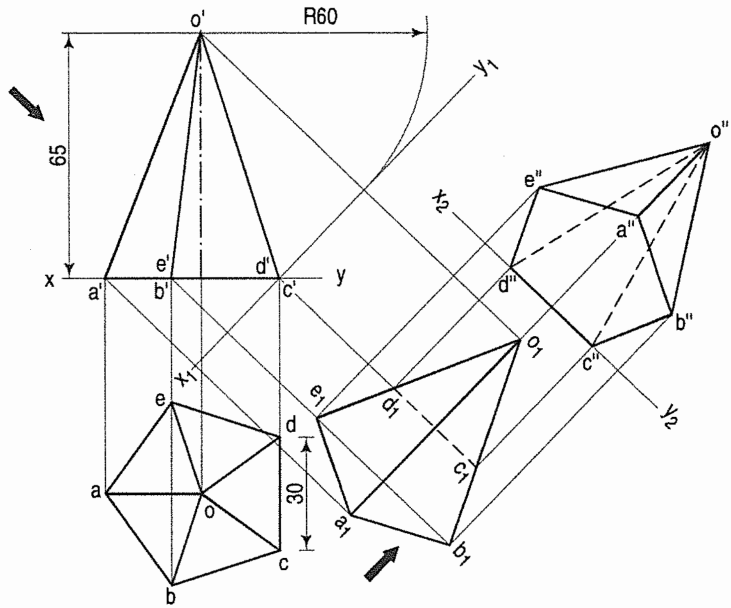


FIG. 13-46(i)

Method II: Changing positions of solid [(fig. 13-46(ii)]:

(i) Draw a line 60 mm parallel to xy. Mark point c' or d' on xy. With c' as centre and the radius equal to o'c', draw an arc cutting the above line at o'. With o' and c' as centre and radius equal to o'c' and a'c' draw an arc cutting each other at the point a'. Join a', o' and c' as shown. Project the required top view as shown.

(ii) Redraw the top view keeping side of base c'd' parallel to xy. Project new front view as shown.

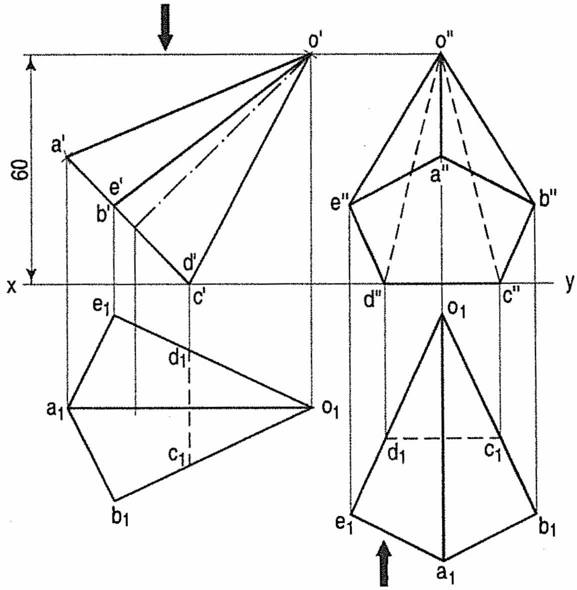


FIG. 13-46(ii)

Problem 13-33. *The front view, part top view and part auxiliary view of a casting are given in fig. 13-47(i). Project its side view.*

See fig. 13-47(ii).

The construction for the ellipse for 38 mm diameter circle has been shown in detail. Horizontal distances are taken from the auxiliary view. Other ellipses are drawn in the same manner.

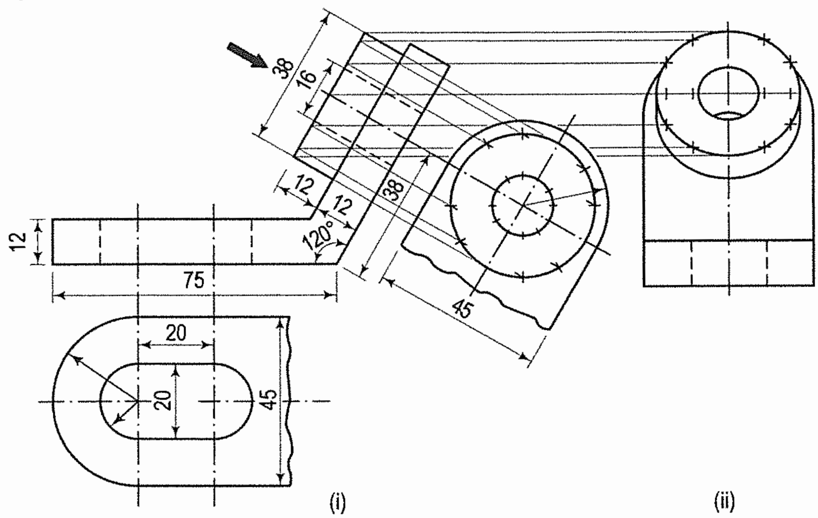


FIG. 13-47

13-5. PROJECTIONS OF SPHERES

The projection of a sphere in any position on any plane is always a circle whose diameter is equal to the diameter of the sphere (fig. 13-48). This circle represents the contour of the sphere.

A flat circular surface is formed when a sphere is cut by a plane. A hemisphere (i.e. a sphere cut by a plane passing through its centre) has a flat circular face of diameter equal to that of the sphere.

When it is placed on the ground on its flat face, its front view is a semi-circle, while its top view is a circle [fig. 13-49(i)].

When the flat face is inclined to the H.P. or the ground and is perpendicular to the V.P. it is seen as an ellipse (partly hidden) in the top view [fig. 13-49(ii)], while the contour of the hemisphere is shown by the arc of the circle drawn with radius equal to that of the sphere.

Fig. 13-50 shows the projections of a sphere, a small portion of which is cut off by a plane. Its flat face is perpendicular to the H.P. and inclined to the V.P. An ellipse is seen in the front view within the circle for the sphere.

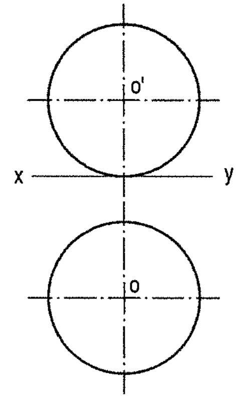


FIG. 13-48

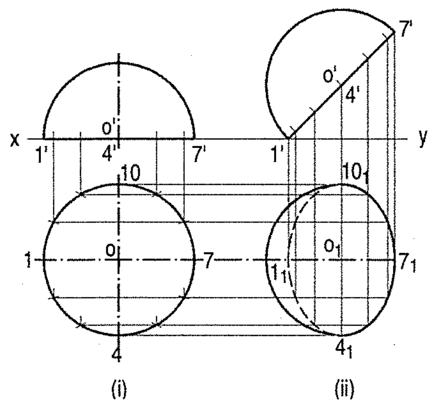


FIG. 13-49

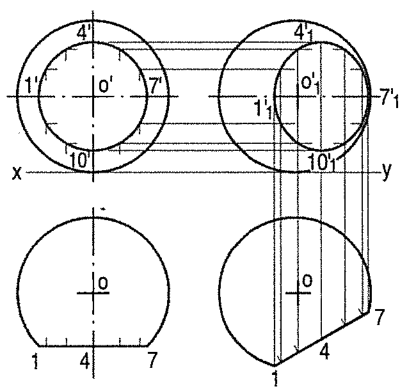


FIG. 13-50

When the flat face of a cut sphere is perpendicular to the V.P. and inclined to the H.P., its projections can be drawn as described in problem 13-34.

Problem 13-34. (fig. 13-51)**:** *A brass flower-vase is spherical in shape with flat, circular top 35 cm diameter and bottom 25 cm diameter and parallel to each other. The greatest diameter is 40 cm. Draw the projections of the vase when its axis is parallel to the V.P. and makes an angle of 60° with the ground.*

(i) Draw the front view of the vase resting on its bottom with its axis vertical. Project the top view.

(ii) Tilt the front view so that the axis makes 60° angle with *xy* and project the top view. Note that a part of the ellipse for the bottom is also visible.

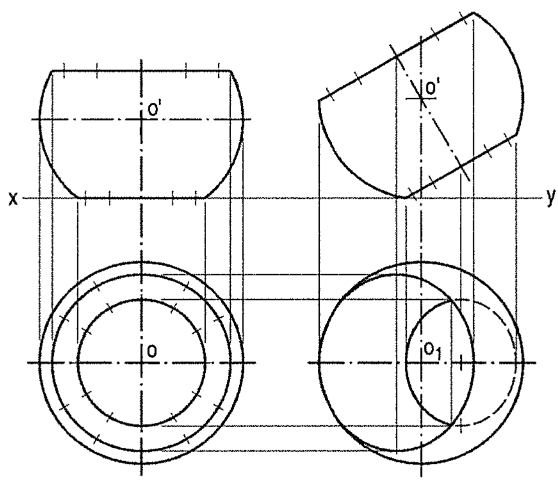


FIG. 13-51

(1) Spheres in contact with each other: Projections of two equal spheres resting on the ground and in contact with each other, with the line joining their centres parallel to the V.P., are shown in fig. 13-52.

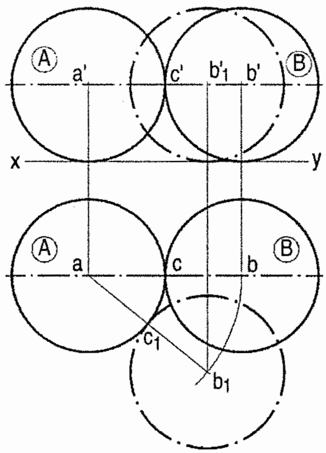


FIG. 13-52

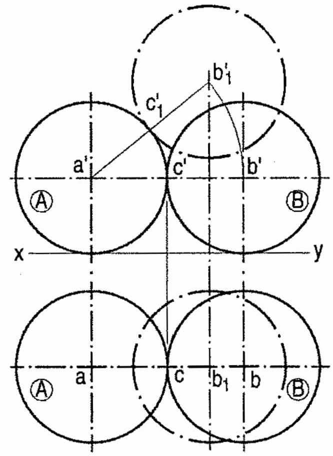


FIG. 13-53

As the spheres are equal in size, the line joining their centres is parallel to the ground also. Hence, both ab and *a'b'* show the true length of that line (i.e. equal to the sum of the two radii or the diameter of the spheres). The point of contact between the two spheres is also visible in each view.

If the position of one of the spheres, say sphere *B*, is changed so that the line joining their centres is inclined to the V.P., in the front view, the centre *b'* will move along the line *a'b'* to b'_1. The true length of the line joining the centres and the point of contact are now seen in the top view only.

When the sphere *B* is so moved that it remains in contact with the sphere *A* and the line joining their centres is parallel to the V.P., but inclined to the ground (fig. 13-53), the true length of that line and the point of contact are visible in the front view only.

Problem 13-35. (fig. 13-54): *Three equal spheres of 38 mm diameter are resting on the ground so that each touches the other two and the line joining the centres of two of them is parallel to the V.P.*

A fourth sphere of 50 mm diameter is placed on top of the three spheres so as to form a pile. Draw three views of the arrangement and find the distance of the centre of the fourth sphere above the ground.

As the spheres are resting on the ground and are equal in size, the lines joining their centres will be parallel to the ground. In the top view, the centres will lie at the corners of an equilateral triangle of sides equal to the sum of the two radii, i.e. 40 mm.

Draw (in the top view) an equilateral triangle *abc* of 40 mm long sides with one side, say *ab*, parallel to *xy*. At its corners, draw three circles of 40 mm diameter. Project the front view. The centres will lie on a line parallel to and 20 mm above *xy*.

When the fourth sphere is placed on top, its centre *d* in the top view will be in the centre of the triangle. In the front view, it will lie on a projector through *d*.

The true distance between the centre of the top sphere and that of any one of the bottom spheres will be equal to the sum of the two radii, viz. 20 mm + 25 mm, i.e. 45 mm.

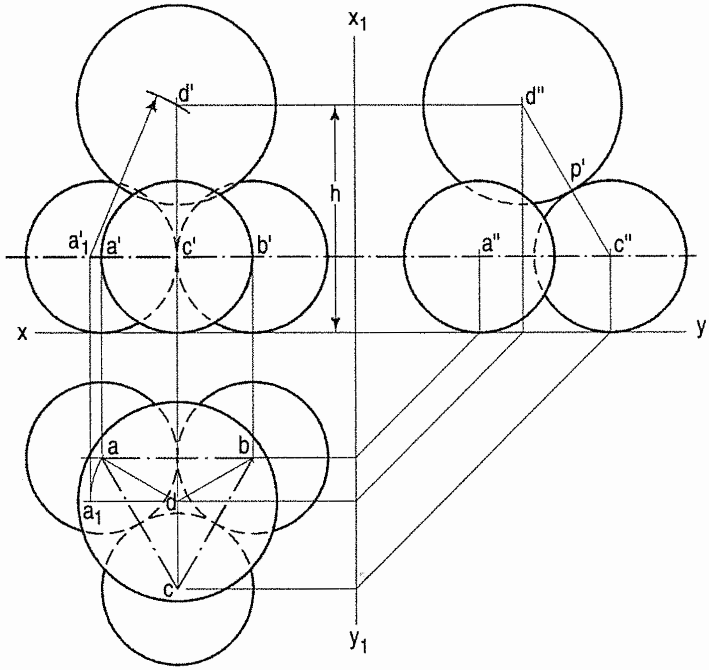


FIG. 13-54

But as none of the lines *da, db* or *dc* is parallel to *xy*, their front views will not show their true lengths. Therefore, to locate the position of the centre of the top sphere in the front view,

(i) make one of the lines, say *da*, parallel to *xy*;

(ii) project a_1 to a'_1 on the path of *a'* and

(iii) with a'_1 as centre and radius equal to 45 mm, draw an arc cutting the projector through *d* at the required point *d'*. With *d'* as centre and radius equal to 25 mm, draw the required circle which will be partly hidden as shown. *h* is the distance of the centre of the sphere from the ground.

(iv) Project the side view. As *c'd'* is parallel to the new reference line, *c''d''* will be equal to 45 mm and the point of contact *p'* between the spheres having centres *c* and *d* will be visible.

(2) Unequal spheres: When two unequal spheres are on the ground and are in contact with each other, their point of contact and the true length of the line joining their centres will be seen in the front view if that line is parallel to the V.P. In the top view, the length of the line will be shorter but will remain constant even when it is inclined to the V.P.

Problem 13-36. (fig. 13-55): *Three spheres A, B and C of 75 mm, 50 mm and 30 mm diameters respectively, rest on the ground each touching the other two. Draw their projections and show the three points of contact when the line joining the centres of the spheres A and B is parallel to the V.P.*

(i) With centre *a'* and radius equal to 37.5 mm, draw a circle of sphere *A*, mark *a'* at 37.5 mm above *xy* in front view. With *a'b'* equal to 62.5 mm, mark point *b'* 25 mm above *xy*. With *b'* as centre and radius equal to 25 mm, draw a circle of sphere *B* in front view.

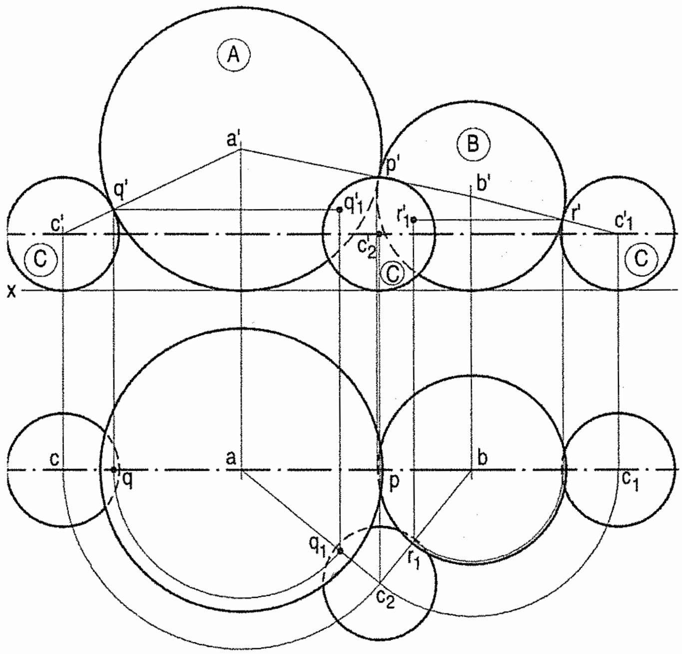


FIG. 13-55

(ii) Project the centres and obtain points *a* and *b* on a line parallel to *xy* in top view. With *a* as centre and radius equal to 37.5 mm of sphere *A*, and with *b* as centre and radius equal to 25 mm of sphere *B*, draw circles in the top view.

(iii) Similarly, draw the views of sphere *C* in contact with spheres *A* and *B*.

(iv) With *a* as centre and radius equal to *ac*, and with *b* as centre and radius equal to bc_1, draw arcs intersecting each other at c_2. With c_2 centre draw top view of the sphere *C*.

(v) Draw the projector through c_2 to cut the path of c' at c'_2. Then c'_2 is the required centre of the sphere *C* in the front view. *p*, q_1 and r_1, and *p'*, q'_1 and r'_1 are the points of contact in the top view and the front view respectively.

Problem 13-37. (fig. 13-56): *A square prism, base 20 mm side and axis 50 mm long, is resting on its base on the ground with two faces perpendicular to the V.P. Determine the radius of four equal spheres resting on the ground, each touching a face of the prism and other two spheres. Draw the projections of the arrangement.*

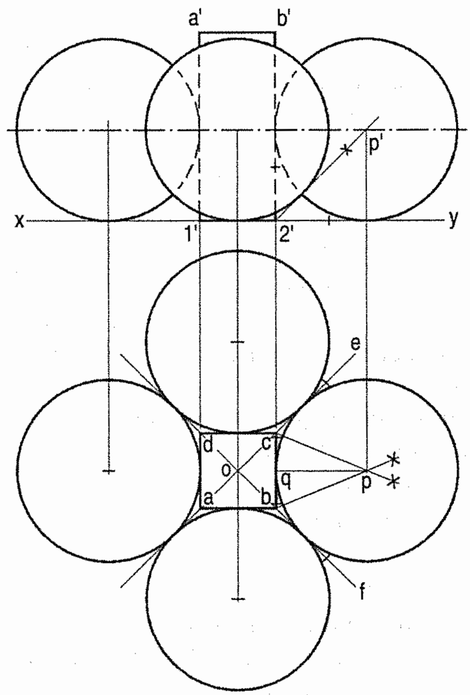


FIG. 13-56

(i) Draw the front and top views of the prism. In the top view, draw diagonals of the square (intersecting each other at *o*) and produce them on both sides.

(ii) Draw the bisectors of angles *bce* and *cbf* intersecting each other at *p*. From *p*, draw a perpendicular *pq* to *bc*. Then *pq* is the required radius of the sphere and *p* is the centre of the circle for the sphere.

(iii) Obtain the other three centres in the same manner. Or, with *o* as centre and radius equal to *op*, draw a circle to cut the centre lines through *o* at the required centres. Draw the four circles.

(iv) Draw a bisector of angle *b'2'y* intersecting the projector through *p* at *p'*. Then *p'* is the centre of the sphere in the front view. The centres for the other circles will lie on the horizontal line through *p'*. Project their exact positions from the top view and draw the circles.

Problem 13-38. (fig. 13-57): *Six equal spheres are resting on the ground, each touching other two spheres and a triangular face of a hexagonal pyramid resting on its base on the ground.*

Draw the projections of the solids when a side of the base of the pyramid is perpendicular to the V.P.

Determine the diameter of each sphere. Base of the pyramid 20 mm side; axis 50 mm long.

(i) Draw the projections of the pyramid in the required position. Assuming the solid to be a prism, locate the positions of the centre of one sphere (viz. *p* and *p'*) in the two views.

(ii) Draw a line joining *p'* with *a'* (the centre of the base) which coincides with 2'. The centre of the required sphere will lie on this line. Draw a bisector of angle o'3'y cutting *a'p'* at *c'*. Draw a line *c'q'* perpendicular to *xy*.

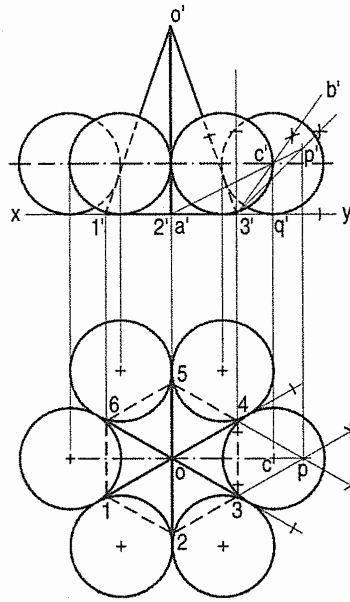


FIG. 13-57

(iii) With c' as centre and radius *c'q'*, draw one of the required circles. Project *c'* to *c* on *op* in the top view. Then *c* is the centre of the circle in the top view. Other centres may be located in the top view as shown and projected down in the front view.

(iv) Draw the six circles in the top view and four in the front view as shown in the fig. 13-57.

Problem 13-39. *The projections of a paper-weight with a spherical knob are given in fig. 13-58(i). Draw the two views and project another top view when its flat base makes an angle of 60° with the H.P.*

See fig. 13-58(ii) which is self-explanatory.

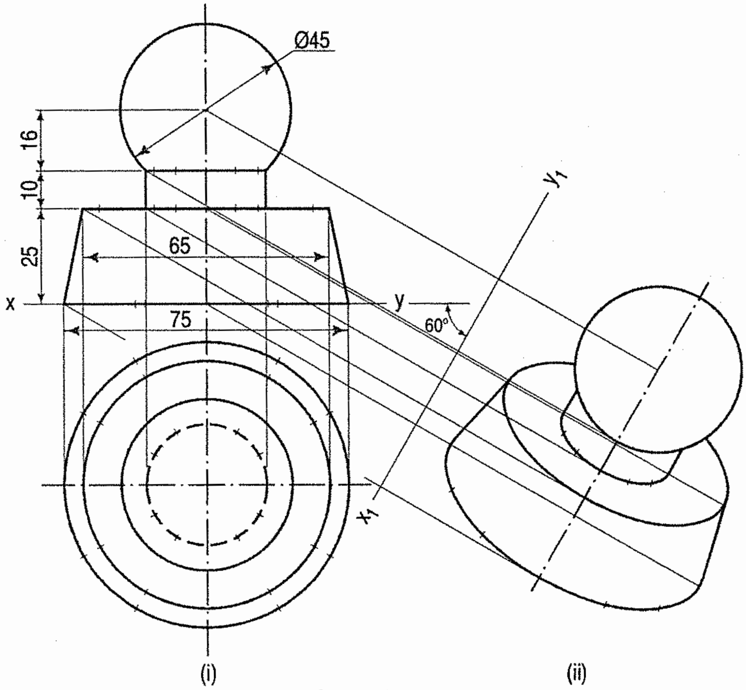


FIG. 13-58

Problem 13-40. (fig. 13-59): *A vertical hexagonal prism of base side 20 mm and thickness 15 mm has one side of hexagon perpendicular to the V.P. A right cone of 34 mm diameter and height 40 mm is placed on the top face of prism such that the base of cone touches top surface of prism while the axes of both coincide. Draw the front view and top view of the combined object. Draw also projections when axes of combined solid is inclined at 35° with auxiliary plane.*

(i) Draw the top view of hexagonal prism keeping one of sides perpendicular to *xy*. (i.e. *ab* or *ed*). Project above *xy* line and draw the front view of prism of height 15 mm.

(ii) Inscribe circle in the top view touching sides of the prism. Project it in the front view and mark the height of the cone as shown.

(iii) Draw auxiliary x_1y_1 inclined at 35° with the axes of the combined solids.

(iv) Draw the projectors from the various points of combined solids in the front view.

(v) Taking distance of various points from the top view of combined solids from *xy* and mark same distances along the respective projectors.

(vi) Complete auxiliary top view as shown.

Problem 13-41. (fig. 13-60): *A vertical cylindrical disc of thickness 10 mm and diameter 50 mm is resting on the ground. A vertical frustum of pentagonal pyramid, having bottom of 20 mm sides, top face of 40 mm sides with 60 mm height is resting on the top surface of the disc so that axes of the both solids coincide. Take one of sides of the base of pentagon is perpendicular to V.P. Draw the projections of combined solid when the axis of combined solids is inclined to 30° with the H.P.*

(i) Draw the top view of frustum of pyramid (pentagon) keeping one of the sides perpendicular to *xy* as shown.

(ii) Project the front view marking height of cylindrical disc and frustum of the pyramid 10 mm and 60 mm respectively.

(iii) Draw a line at angle of 30° with *xy*. (As axis is inclined with H.P., its inclination observed in the front view).

(iv) Reproduce the front view considering inclined line as axes of the combined solids.

(v) Draw the vertical projectors from various points of the front view.

(vi) Draw horizontal projectors to intersect respective vertical projectors. Obtain the auxiliary top view as shown.

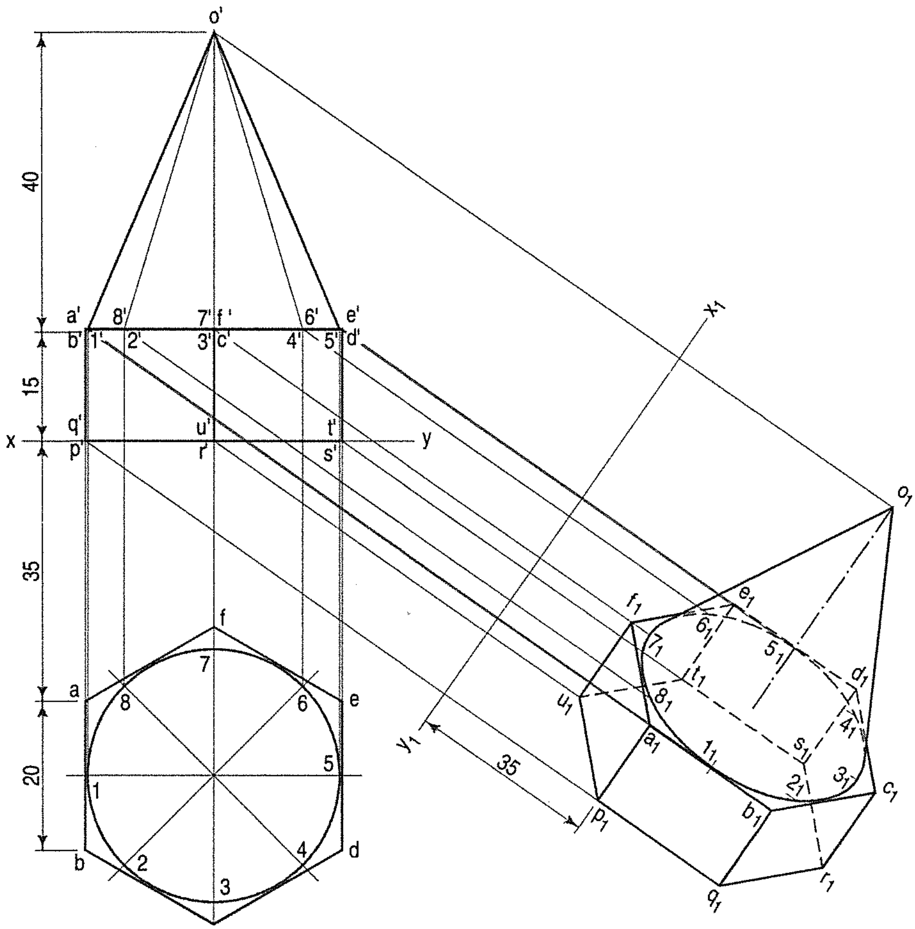


FIG. 13-59

Problem 13-42. (fig. 13-61): *A right hexagonal prism of side 25 mm and 20 mm thick with one side of the base is perpendicular to the V.P. resting on the ground. A vertical frustum of square pyramid of base 20 mm sides and top face side 30 mm and height 50 mm is resting on the prism such that one side of square makes 45° with the V.P. Assume that axes of both solids are coinciding. Draw the projections of the combined solids when top corner of the square pyramid is 70 mm above the ground (H.P.). Determine angle of combined solids with the H.P.*

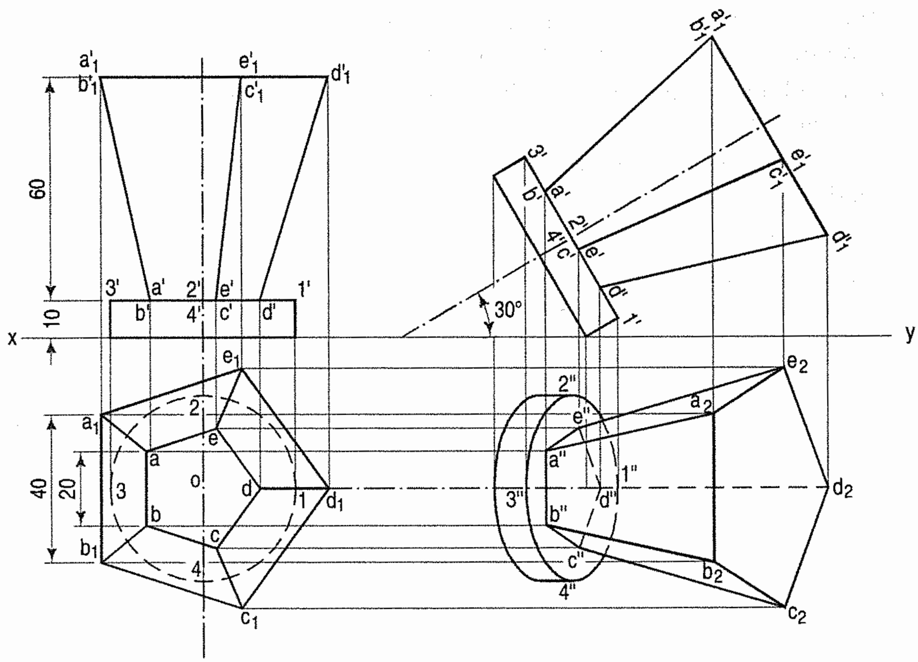


FIG. 13-60

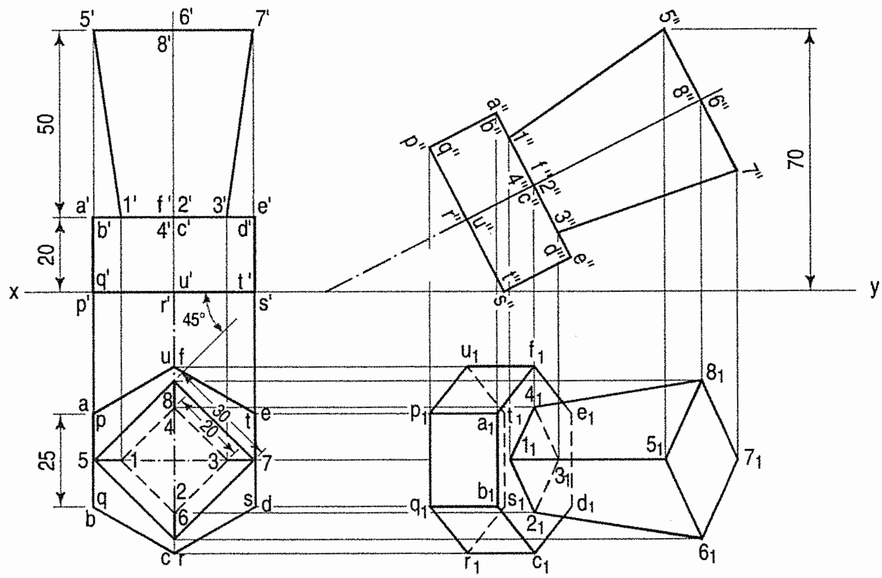


FIG. 13-61

(i) Draw the top view and front view as shown in figure. Keep one side of the hexagonal perpendicular to the *xy*.

(ii) Project the front view as shown.

(iii) Draw a parallel line at distance 70 mm away from *xy*. Reproduce the front view of the combined solids as shown.

(iv) Draw the projectors from the new front view.

(v) Draw from the top view horizontal projectors to intersect respective projectors drawn from the new front view.

(vi) Complete the top view as shown.

EXERCISES 13(b)

1. A rectangular block 75 mm × 50 mm × 25 mm thick has a hole of 30 mm diameter drilled centrally through its largest faces. Draw the projections when the block has its 50 mm long edge parallel to the H.P. and perpendicular to the V.P. and has the axis of the hole inclined at 60° to the H.P.
2. Draw the projections of a square pyramid having one of its triangular faces in the V.P. and the axis parallel to and 40 mm above the H.P. Base 30 mm side; axis 75 mm long.
3. A cylindrical block, 75 mm diameter and 25 mm thick, has a hexagonal hole of 25 mm side, cut centrally through its flat faces. Draw three views of the block when it has its flat faces vertical and inclined at 30° to the V.P. and two faces of the hole parallel to the H.P.
4. Draw three views of an earthen flower pot, 25 cm diameter at the top, 15 cm diameter at the bottom, 30 cm high and 2.5 cm thick, when its axis makes an angle of 30° with the vertical.
5. A tetrahedron of 75 mm long edges has one edge parallel to the H.P. and inclined at 45° to the V.P. while a face containing that edge is vertical. Draw its projections.
6. A hexagonal prism, base 30 mm side and axis 75 mm long, has an edge of the base parallel to the H.P. and inclined at 45° to the V.P. Its axis makes an angle of 60° with the H.P. Draw its projections.
7. A pentagonal prism is resting on a corner of its base on the ground with a longer edge containing that corner inclined at 45° to the H.P. and the vertical plane containing that edge and the axis inclined at 30° to the V.P. Draw its projections. Base 40 mm side; height 65 mm.
8. Draw three views of a cone, base 50 mm diameter and axis 75 mm long, having one of its generators in the V.P. and inclined at 30° to the H.P., the apex being in the H.P.
9. A square pyramid, base 40 mm side and axis 90 mm long, has a triangular face on the ground and the vertical plane containing the axis makes an angle of 45° with the V.P. Draw its projections.
10. A frustum of a pentagonal pyramid, base 50 mm side, top 25 mm side and axis 75 mm long, is placed on its base on the ground with an edge of the base perpendicular to the V.P. Draw its projections. Project another top view on a reference line parallel to the line which shows the true length of the slant edge. From this top view, project a front view on an auxiliary vertical plane inclined at 45° to the top view of the axis.
11. Draw the projections of a cone, base 50 mm diameter and axis 75 mm long, lying on a generator on the ground with the top view of the axis making an angle of 45° with the V.P.
12. The front view, incomplete top view and incomplete auxiliary top view of a casting are given in fig. 13-47. Draw all the three views completely in the third-angle projection.

13. A line sketch (in two views) of a shed with a curved roof is given in fig. 13-62. Draw its front view on an auxiliary vertical plane inclined at 60° to the V.P. All dimensions are in metres. Scale, 10 mm = 0.5 m.

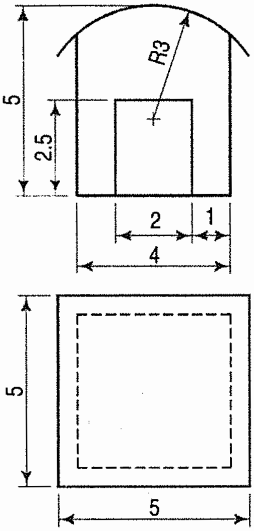


FIG. 13-62

14. The front view of a hexagonal pyramid [base 25 mm side] having one of its triangular faces resting centrally on a triangular face of a square pyramid [base 50 mm side and axis 50 mm long] is given in fig. 13-63. The plane containing the two axes is parallel to the V.P. Draw the top view of the solids. From this top view, project a front view on a reference line x_1y_1 inclined at 30° to xy; (ii) from the given front view, project another top view on a reference line x_2y_2 inclined at 45° to xy.

15. A cube of 50 mm long edges is resting on the ground with its vertical faces equally inclined to the V.P. A hexagonal pyramid, base 25 mm side and axis 50 mm long, is placed centrally on top of the cube so that their axes are in a straight line and two edges of its base parallel to the V.P. Draw the front and top views of the solids. Project another top view on an A.I.P. making an angle of 45° with the H.P. From this top view project another front view on an auxiliary vertical plane inclined at 30° to the top view of the combined axis.

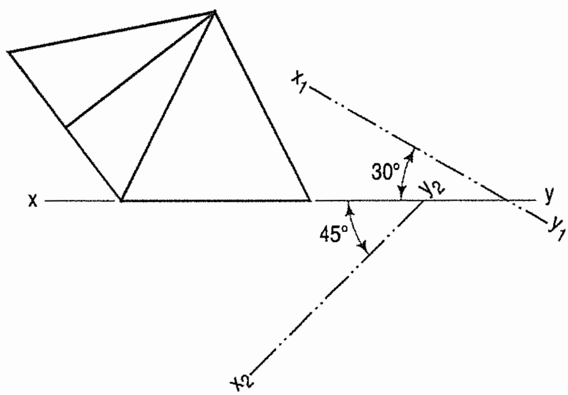


FIG. 13-63

16. Four equal spheres of 25 mm diameter are resting on the ground, each touching the other two spheres, so that a line joining the centres of two touching spheres is inclined at 30° to the V.P. A fifth sphere of 30 mm diameter is placed centrally on top of the four spheres, thus forming a pile. Draw the projections of the spheres and measure the height of the centre of the top sphere above the ground.

17. Three spheres of 25 mm, 50 mm and 75 mm diameter respectively are resting on the ground so that each touches the other two. Draw their projections when the top view of the line joining centres of any two of them is perpendicular to the V.P.

18. Three equal cones, base 50 mm diameter and axis 75 mm long, are placed on the ground on their bases, each touching the other two. A sphere of 40 mm diameter is placed centrally between them. Draw three views of the arrangement and determine the height of the centre of the sphere above the ground.

19. Five equal spheres are resting on the ground each touching the other two spheres and a vertical face of a pentagonal prism of 25 mm side. Determine the diameter of the spheres and draw the projections when a side of the base of the prism is perpendicular to the V.P.

20. Four equal spheres are resting on the ground, each touching the other two spheres and a triangular face of a square pyramid, having base 25 mm side and axis 50 mm long. Draw their projections and find the diameter of the spheres.
21. One of the body diagonals of a cube of 45 mm edge is parallel to the H.P. and inclined at 45° to the V.P. Draw the front view and the top view of the cube.
22. A pentagonal pyramid, base 40 mm side and height 75 mm rests on one edge of its base on the ground so that the highest point in the base is 25 mm above the ground. Draw its projections when the axis is parallel to the V.P. Draw another front view on a reference line inclined at 30° to the edge on which it is resting, and so that the base is visible.
23. A thin lamp shade in the form of a frustum of a cone has its larger end 200 mm diameter, smaller end 75 mm diameter and height 150 mm. Draw its three views when it is lying on its side on the ground and the axis parallel to the V.P.
24. A bucket made of tin sheet has its top 200 mm diameter and bottom 125 mm diameter with a circular ring 40 mm wide attached at the bottom. The total height of the bucket is 250 mm. Draw its projections when its axis makes an angle of 60° with the vertical.
25. A hexagonal pyramid, side of the base 25 mm long and height 70 mm, has one of its triangular faces perpendicular to the H.P. and inclined at 45° to the V.P. The base-side of this triangular face is parallel to the H.P. Draw its projections.
26. A pentagonal pyramid has an edge of the base in the V.P. and inclined at 30° to the H.P., while the triangular face containing that edge makes an angle of 45° with the V.P. Draw three views of the pyramid. Length of the side of the base is 30 mm, while that of the axis is 80 mm.
27. A square pyramid, base 40 mm side and axis 75 mm long is placed on the ground on one of its slant edges, so that the vertical plane passing through that edge and the axis makes an angle of 30° with the V.P. Draw its three views.
28. A hexagonal prism, side of base 40 mm and height 50 mm is lying on the ground on one of its bases with a vertical face perpendicular to the V.P. A tetrahedron is placed on the prism so that the corners of one of its faces coincide with the alternate corners of the top surface of the prism. Draw the projections of the solids. Project another top view on an auxiliary inclined plane making 45° with the H.P.
29. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 150 mm and 100 mm respectively and its length is 150 mm. It is situated in such a way that its axis is parallel to the H.P. and lies in a plane inclined at 60° to the V.P. Draw the projections of the duct, assuming the thickness of the duct-sheet to be negligible.
30. A pentagonal pyramid, base 30 mm edge and axis 75 mm long, stands upon a circular block, 75 mm diameter and 25 mm thick, so that their axes are in a straight line. Draw the projections of the solids when the base of the block is inclined at 30° to the ground, an edge of the base of the pyramid being parallel to the V.P.
31. The body diagonal of a cube is 75 mm long. The cube has a central 25 mm square hole. The faces of the hole make 45° with the side faces of the cube. Draw the projections of the cube when a body diagonal is perpendicular to the H.P.

32. A bucket, 300 mm diameter at the top and 225 mm diameter at the bottom has a circular ring 225 mm diameter and 50 mm wide attached at the bottom. The total height of the bucket is 300 mm. Draw the projections of the bucket when its axis is inclined at 60° to the H.P. and as a vertical plane makes an angle of 45° with the V.P. Assume the thickness of the plate of the bucket to be equal to that of a line.

33. The vertex-angle of the cone just touching the edges of a vertical hexagonal pyramid 125 mm in height is 45°. Draw the projections of the pyramid on a 45° inclined plane when the former is truncated by a plane making 45° with the axis and bisecting the axis.

34. A knob of a machine handle consists of 15 mm diameter × 150 mm long cylindrical portion and 40 mm diameter spherical portion. The centre of the sphere lies on the axis of the cylindrical portion. Draw the projections if its axis is inclined at 45° to the horizontal plane.

35. Six equal spheres rest on the ground in contact with each other and also with the slanting faces of a regular upright hexagonal pyramid, 25 mm edge of base and 125 mm length of axis. Draw the projections and find the diameter of the sphere.

36. A cylinder, 100 mm diameter and 150 mm long, has a rectangular slot 50 mm × 30 mm cut through it. The axis of the slot bisects the axis of the cylinder at right angles and the 50 mm side of the slot makes an angle of 60° with the base of the cylinder. Draw three views of the cylinder.

37. A very thin glass shade for a table lamp is the portion of a sphere 125 mm diameter included between two parallel planes at 15 mm and 55 mm from the centre, making the height 70 mm. If the axis of the shade is inclined at 30° to the vertical, obtain the projections of the shade.

38. A cone frustum, base 75 mm diameter, top 35 mm diameter and height 65 mm has a hole of 30 mm diameter drilled through it so that the axis of the hole coincides with that of the cone. It is resting on its base on the ground and is cut by a section plane perpendicular to the V.P., parallel to an end generator and passing through the top end of the axis. Draw sectional top view and sectional side view of the frustum.

39. Three vertical poles *AB*, *CD* and *EF* are respectively 5, 8 and 12 metres long. Their ends *B*, *D* and *F* are on the ground and lie at the corners of an equilateral triangle of 10 metres long sides. Determine graphically the distance between the top ends of the poles, viz. *AC*, *CE* and *EA*.

40. Two cylinders of 80 mm diameter each meet each other at right angles. The axis of one of the cylinders is parallel to both the reference planes and is 40 mm in front of the axis of the other cylinder. Draw three views of the cylinders showing lines of intersection in them. Take any suitable lengths of the cylinders.

41. A tetrahedron of side 40 mm rests on the top face of a hexagonal prism of base and height 25 mm such that their apex coincide. Draw the projections when the combination rests with one of the sides of the prism on the H.P., is perpendicular to the V.P., and the axis is inclined at 30° to the H.P.

42. A pentagonal pyramid, base 30 mm side and axis 70 mm long, has one of its slant edges in the H.P. and inclined at 30° to the V.P. Draw the projections of the solid when the apex is towards the observer.