Unit - 01

Pulse Digital Modulation

Communication is the process of establishing connection or link between two points for information exchange. (or)

Communication is simply the process of conveying message at a distance or communication is the basic process of exchanging information.

→ Depending on message signal communication is classified into two types. They are

1. Analog communication.
2. Digital communication.

Analog communication :-

In Analog communication the modulating signal is Analog in nature. The modulating signal is transmitted with the help of transmitting antenna. At the receiver the signal is received and processed to recover original message signal. The block diagram of analog communication system is shown below.

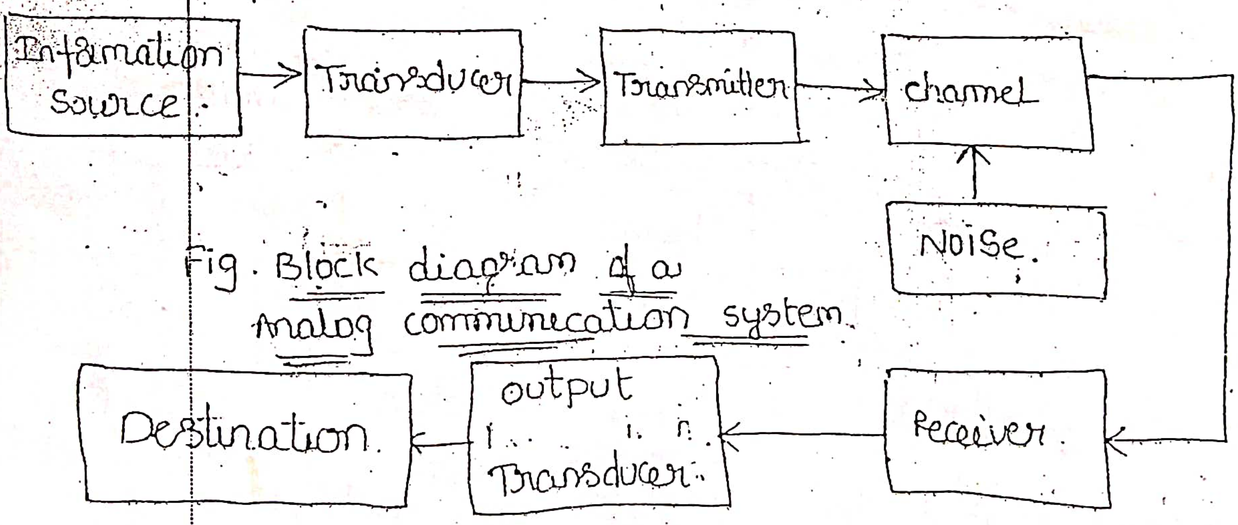


Fig. Block diagram of a Analog communication system.

Presently all the AM, FM radio transmission and TV transmissions are examples of analog communications.

Digital communications

In digital communication the modulating signal is digital in nature.

Elements of digital communication system:-

The below figure elements of digital communication system. The overall purpose of the system is to transmit the message coming out of a source to a destination point at as high rate and accuracy as possible. The communication channels accepts electrical signals and the output of the channel is usually distorted version of the input due to the non-ideal nature of the communication channel.

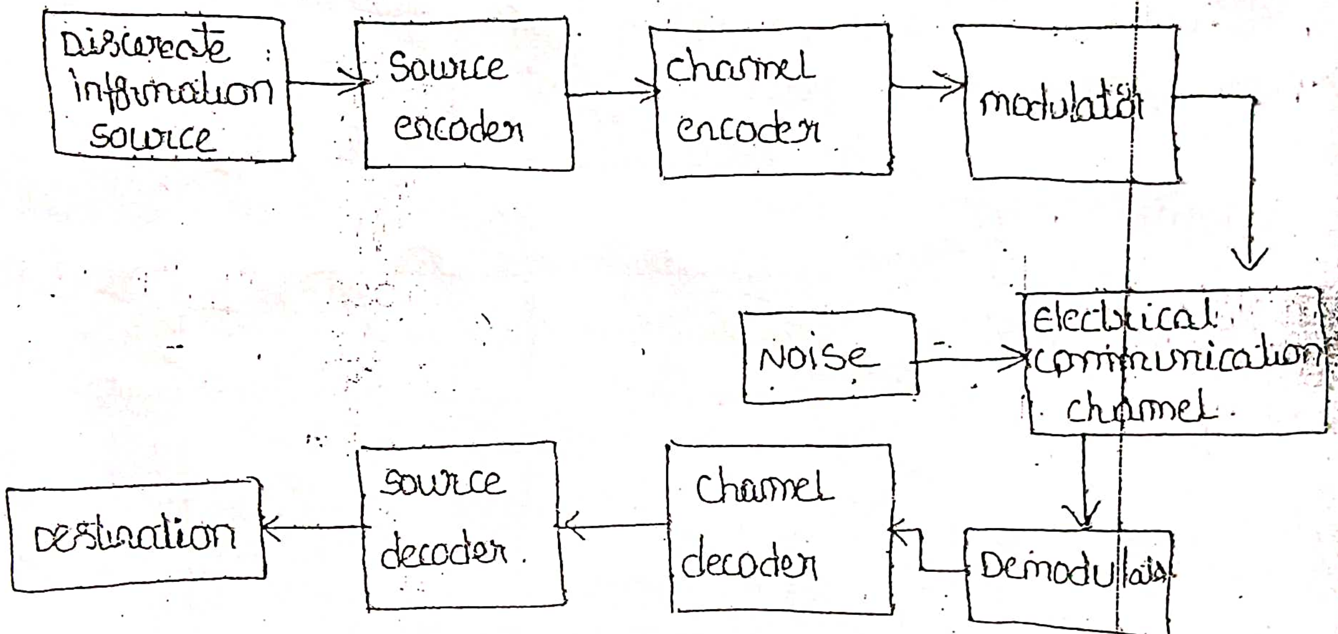


Fig. Elements of digital communication system.

In addition to this, the information bearing signals is also corrupted by unpredictable electrical signals from both manmade and natural causes. Thus, the Smearing and the noise introduce errors in the information being transmitted and limits the rate at which information can be communicated from the source to the destination.

Discrete information source :-

Information source may be classified into two categories based upon the nature of their output i.e analog information sources and discrete information sources. In case of analog communication, the information source is analog.

In case of digital communication, the information source produces a message signal which is not continuously varying with time. The output of discrete information sources such as a teletype or the numerical output of a computer consists of a sequence of discrete symbols or letters. Discrete information sources are characterized by the following parameters.

i) Source alphabet :- These are the letters, digits & special characters available from the information source.

ii) Symbol rate :- It is the rate at which the information source generates source alphabets. It is generally represented in symbols / sec (unit).

iii) Source alphabet probabilities :- Each source alphabet from the source has independent occurrence rate in the sequence. As an example, letters A, E, I, occur frequently in the sequence. Hence probability of the occurrence of each source alphabet can become one of the important property which is useful in digital communication.

iv) Probabilistic dependence of symbols in a sequence :-

The information carrying capacity of each source alphabet is different in a particular sequence. This parameter defines average information content of the symbols. This means that the source information rate is the product of symbol rate and source entropy

Information rate = Symbol rate X Source Entropy
(Bits|sec). (symbols|sec) (Bits|symbol)

Thus, the information rate represents minimum average data rate required to transmit information from source to the destination.

Source Encoder and decoder:-

The symbols produced by the information source are given to the source encoder. These symbols cannot be transmitted directly. They are first converted into digital form by the source encoder. Each binary '1' and '0' is known as a bit. The group of bits is called a codeword. The Source encoder assigns codewords to the symbols. Source Encoders must have following important parameters.

i) Block Size:-

Block size describes the maximum number of distinct codewords which can be represented by a source encoder. This depends on the number of bits in the codeword. As an example, the block size of 8-bits source encoder will be 2^8 i.e 256 code words.

ii) Average data rate:- Average data rate means the number of bits per second generated by a source encoder.

Data rate = Symbol rate x codeword length.

iii) Codeword length:-

Code word length is the no. of bits used to represent each codeword. As an example, if 8 bits are assigned to each codeword.

iv). Efficiency of the encoder :-

The efficiency of the encoder is the ratio of minimum source information rate to the actual output data rate of the source encoder.

Channel Encoder and decoder :-

After converting the message signal in the form of binary sequence by the source encoder, the signal is transmitted through the channel. The communication channel adds noise and interface to the signal being transmitted. Hence errors are introduced in the binary sequence received at the receiver end.

Channel encoder is mainly used to avoid the errors in digital communication system is done by adding redundant bits. channel encoder is characterized by certain parameters.

1) The coding rate that depends upon the redundant bits added by the channel encoder.
2) The coding method used.
3) coding efficiency which is the ratio of data rate at the input to the data rate at the output of the Encoder.
4) Error control capabilities.
5) Feasibility of the encoder and decoder.

This means that the channel encoder and decoder serve to increase the reliability of a received signal.

Digital modulators and demodulators :-

Digital continuous wave modulators are ASK, FSK, & PSK. These modulators use a continuous carrier wave, therefore they are also known as digital cw modulators. At the receiver end, the digital demodulator converts the input modulated signal into the sequence of binary bits.

A digital modulation method must have following important parameters.

1) Bandwidth needed to transmit the signal.
2) probability of symbol or bit error.
3) Synchronous or Asynchronous method of detection.
4) complexity of implementation.

Communications channel :-

The connection between transmitter and receiver is established through a communication channel. The communication can take place through wirelines, wireless or fiber optic channel. Each and every communication channel has some inherent problems. These are

i) Signal Alteration :-

The signal attenuation in channel occurs due to the internal resistance of the channel and fading of the signal.

ii) Amplitude and phase distortion :-

The transmitted signal is distorted in Amplitude and phase due to the non-linear characteristics of the communication channel.

iii) Additive noise interference :-
It is produced due to internal solid state devices and resistors etc. used to implement a communication system.

iv) multipath distortion :- It occurs mostly in wireless communication channels.

Destination :- It is the final state where the information is reached.

Advantages of digital communication system :-

→ These communication systems are simpler and cheaper.
→ only permitted receivers can receives data.
→ It having a wide dynamic range.
→ Noise can be easily tolerated.
→ In this communication, channel coding is used therfore the errors may be detected and corected at the receivers.

Disadvantages :-

→ Transmission bandwidth is required for digital communication.

→ Digital communication needs synchronization in case of synchronous modulation.

Comparison of Analog and digital modulation :-

Analog modulation.	Digital modulation.
· Transmitted modulated signal is analog in nature.	1. Transmitted signal is digital (train of digital pulses)
· Amplitude, frequency or phase variations in the transmitted signal represent the information or message.	Amplitude, width [illegible] of the transmitted pulses is constant. The message is transmitted in the form of code words.
1. Noise immunity is poor for Am, but improved for Fm and Pm.	3. Noise immunity is Excellent.
∴ It is not possible to separate out noise and signal. Therefore repeaters cannot be used.	4. It is possible to separate signal from noise. Therefore, repeaters can be used.
5. coding is not possible.	5. coding techniques can be used to detect & correct the errors.
6. Bandwidth required is lower than that for the digital modulation methods.	6. Due to higher bit rates, higher channel bandwidth is required.
7. FDM is used for multiplexing.	7. TDM is used for multiplexing.
8. Not suitable for transmission of secret information in military applications.	8. Due to coding techniques, it is suitable for military applications.
9. Analog modulation systems are AM, FM, PM, PAM & PWM etc.	9. Digital modulation systems are PCM, DM, ADM, DPCM etc.

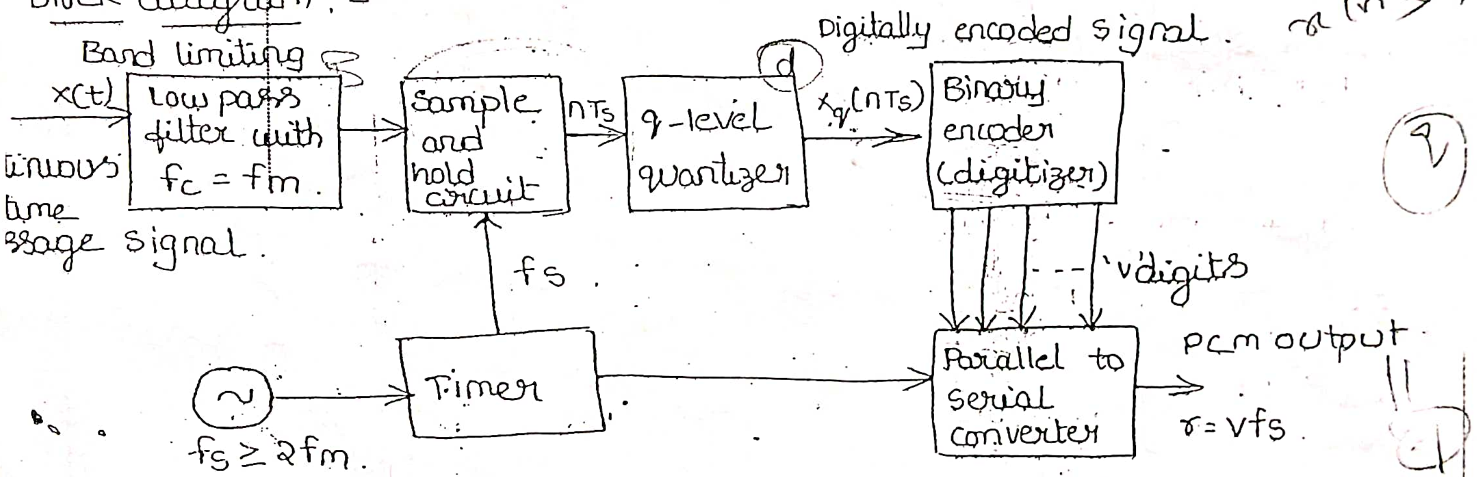
ulse code Modulation :- (PCM)

Pulse-code modulation is known as a digital pulse modulation technique. The PCM output is in the form of digital pulses of constant amplitude, width and position. The PCM system mainly consists of three main parts.

* PCM Transmitter
* Transmission path
* receiver.

* PCM Transmitter :-

Block diagram :-



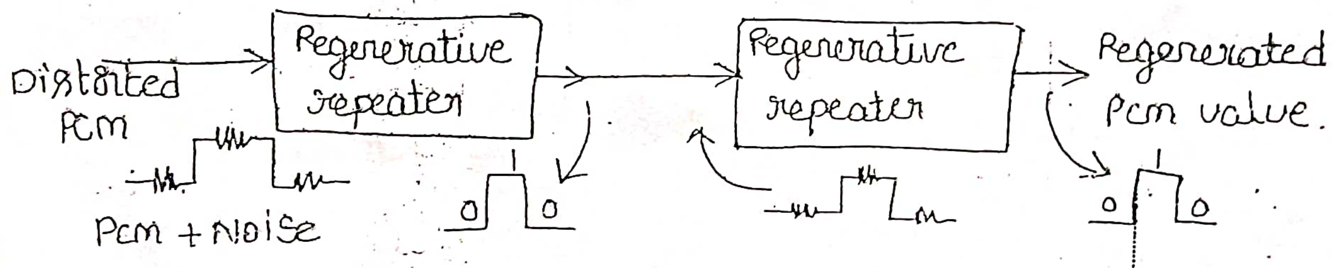
In PCM transmitter the essential operations ~~are~~ are PCM sampling, quantizing and encoding.

Here the signal x(t) is first passed through the low-pass filter of cut-off frequency f_m. This low pass filter blocks all the frequency components above the f_m. The sample and hold circuit samples this signal at the rate of f_s. The output of sample and hold circuit is denoted by $x(nT_s)$. This $x(nT_s)$ is a discrete signal. Next, the quantizer compares input $x(nT_s)$ with its fixed digital levels and assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. Now, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to v digits binary word. parallel to serial converter converts parallel form of data to serial form.

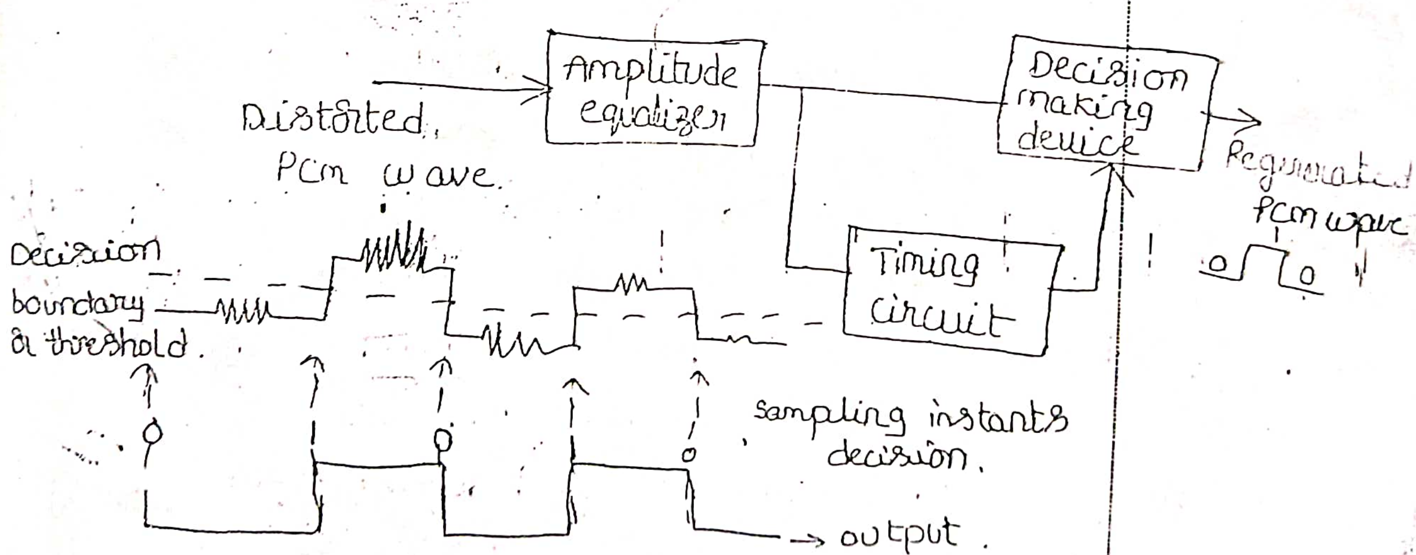
* Transmission path :-

The path between the PCM Transmitter and PCM receiver over which PCM signal travel is called as PCM Transmission path. Here we use regenerative repeaters which reduces the effect of noise and distortion. The regenerative performs 3 Basic operations namely 1. equalization
2. timing.
3. decision making.

Block diagram :-



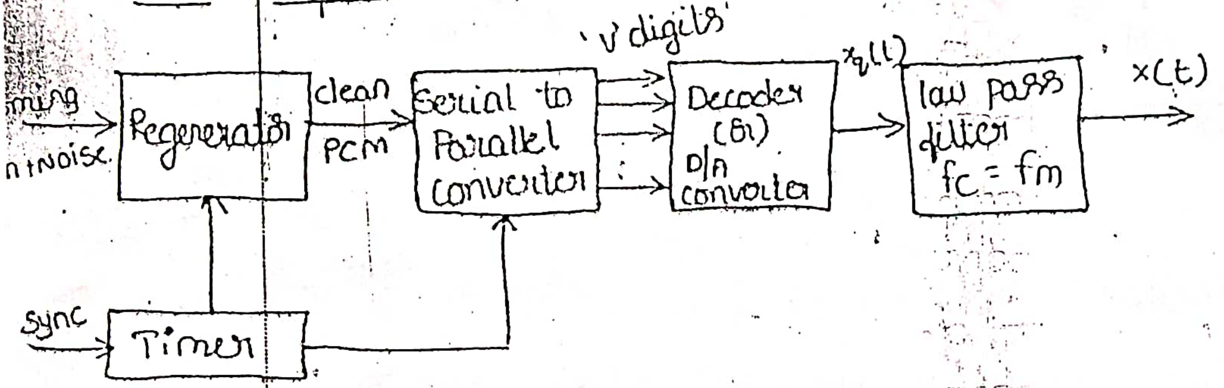
Block diagram of regenerative repeater :-



The amplitude equalizer shapes the distorted PCM wave so as to compensate for the effects of amplitude and phase distortions. The Timing circuit produces periodic pulse train. The decision making uses this pulse train for sampling the equalized PCM pulses. The decision is made by comparing equalized PCM with reference level called decision threshold.

* PCM Receiver :-

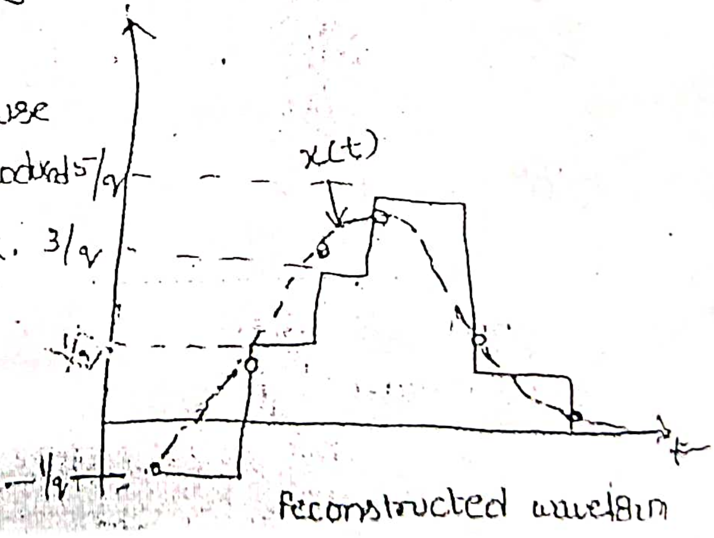
Block diagram :-



The regenerator at the start of PCM receiver reshapes the pulse and removes the noise. This signal is then converted to parallel digital words of each sample. The decoding process generating an analog signal it will denoted by $x_q(t)$. is allowed to pass through a low pass reconstruction filter to get the appropriate original message signal denoted as $x(t)$..

* It is impossible to reconstruct exact original signal x(t) because permanent quantization error introduced during quantization at the Tx. E can be reduced by increasing binary levels.

$\boxed{q\uparrow \quad v\uparrow \quad B.W\uparrow}$



Reconstructed waveform

Quantizer :-

A quantizer compares discrete time input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels which results in minimum error. This error is called quantization error.

Classification of quantization process :-

- Quantization
  - uniform quantization
    - midtread type
    - midrise type.
  - Non-uniform quantization

This classification is based on the step size.

Step Size :- The difference between two adjacent discrete values is called a "quantum" or stepsize.

i) uniform quantizer :- A uniform quantizer is that type of quantizer in which the 'step size' remains same throughout the input range.

ii) non uniform quantizer :- A Non uniform quantizer is that type of quantizer in which the 'step-size' varies according to the input signal values.

> uniform quantization is classified as.

1. mid rise type
2. midtread type

* In the input and output characteristic of midtread type the origin lies in the middle of a tread of stair case like graph.
* In the input and output characteristic of midrise type the origin lies in the middle of rising part of stair case like graph.

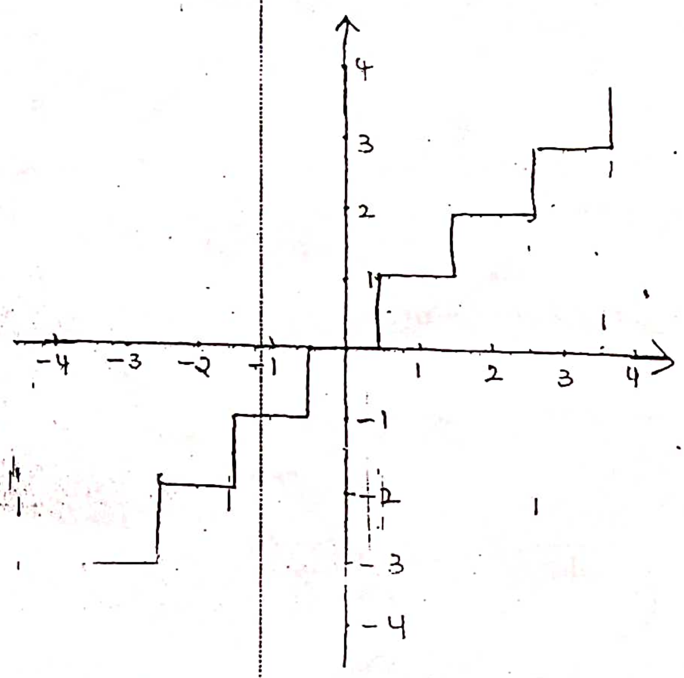


fig (a) midtread

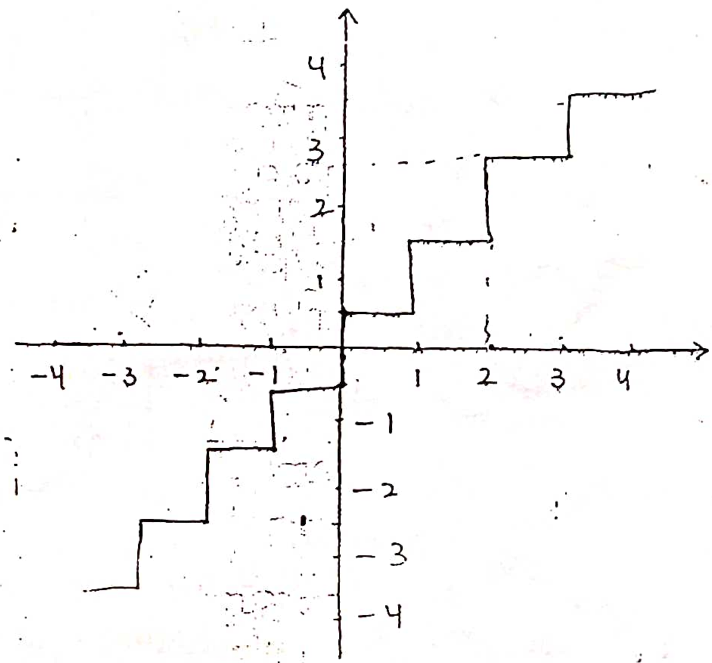


fig (b) midrise.

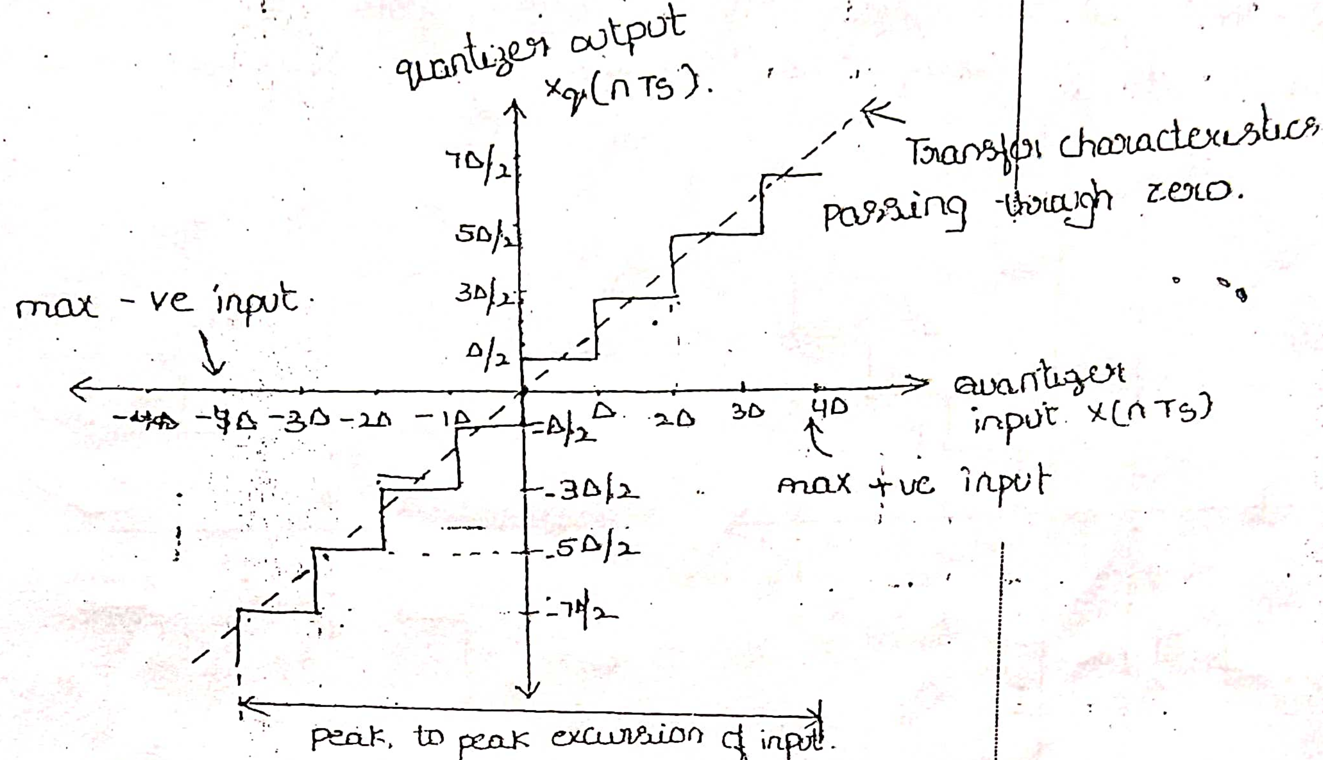
> working principle of quantizer :-

For this purpose, we shall consider uniform quantizer of midrise type. The transfer characteristics of a uniform quantizer of midrise type. let us assume that the input to the quantizer $x(nTs)$ varies from 4Δ to $+4\Delta$. This means that the peak to peak value of $x(nTs)$ will be between -4Δ to $+4\Delta$. Here Δ is the step size. The fixed digital levels are avaliable at $\pm \Delta/2$, $\pm 3/2\,\Delta$, $\pm \frac{5}{2}\Delta$, $\pm \frac{7}{2}\Delta$. These levels are avaliable at quantizer because of its characteristics.

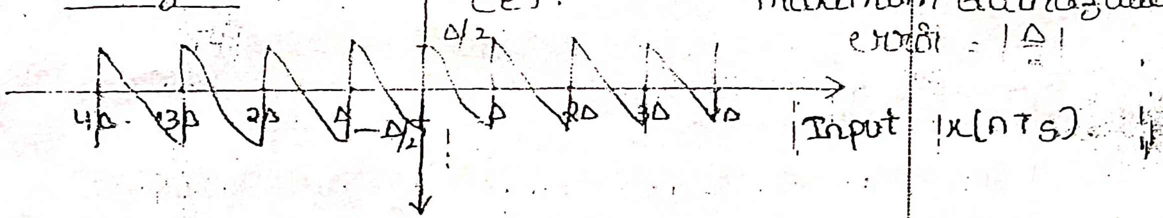
If $x(nTs) = 4\Delta$ then $x_q(nTs) = \frac{7}{2}\Delta$.

$x(nTs) = -4\Delta$ then $x_q(nTs) = -\frac{7}{2}\Delta$

Transfer characteristic



Variation of quantization, quantization error (ε).



Quantization error is defined as the error that arise when digital levels are assigned to the input levels.

quantization error $\varepsilon = x_q(nTs) - x'(nTs)$.

In above figure, it may also be observed that

for $\Delta < x(nTs) < 2\Delta$, $x_q(nTs) = \frac{3}{2}\Delta$

$-\Delta < x(nTs) < -2\Delta$, $x_q(nTs) = \frac{-3}{2}\Delta$.

This means that the maximum quantization error will be $\pm\frac{\Delta}{2}$

$$\boxed{\varepsilon_{max} = \left|\frac{\Delta}{2}\right|}$$

Transmission bandwidth in PCM system :-

In PCM system the quantizer use 'v' number of binary digits to represent each level. Then, the number of levels that may be represented by 'v' digits will be $q = 2^v$.

For example, if v = 4 bits, the total number of levels will be

$$q = 2^v = 2^4 = 16 \text{ levels.}$$

- number of bits per sample = v

number of samples per second = f_s.

no. of bits per second = no. of bits per samples × number of samples per second.

= 'v' bits per sample × f_s samples per second

signaling rate in PCM; $r = v f_s$.

$$f_s \geq 2f_m.$$

The bandwidth required for PCM system must be greater than half of signalling rate

$$BW \geq \frac{1}{2} r$$

$$BW \geq \frac{1}{2} v f_s \Rightarrow B.W \geq \frac{1}{2} v \cdot 2 f_m$$

$$\therefore BW \geq v f_m.$$

> Quantization Noise / Error in PCM :-

Because of quantization, inherent errors are introduced in the signal. This error is called quantization error. The quantization error is given as

$$\epsilon = x_q(nT_s) - x(nT_s)$$

let us consider amplitude ranges from $-X_{max}$ to $+X_{max}$ in uniform quantization. Then Total amplitude range is

$$= X_{max} - (-X_{max})$$

$$= 2X_{max}.$$

If total amplitude range is divided into 'q' levels. then step size Δ is

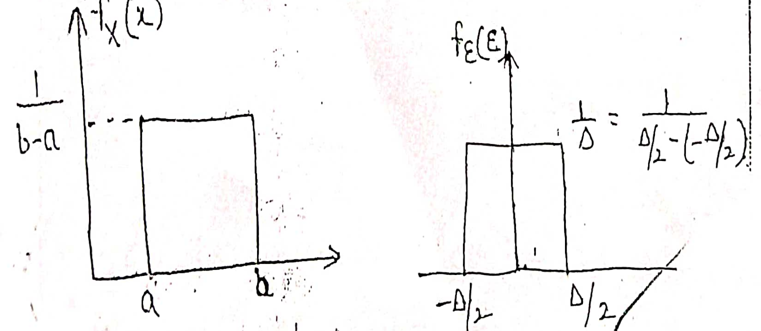
$$\Delta = \frac{2.X_{max}}{q} \quad (\because X_{max} = 1, \; -X_{max} = -1$$

Therefore, step size would be $\Delta = \frac{2}{q}$.

Now, if step size 'Δ' is considered as sufficiently small, then it may be assumed that the quantization error 'ε' will be an uniformly distributed random variable. The maximum quantization error is

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right| \qquad -\frac{\Delta}{2} \le \varepsilon_{max} \le \frac{\Delta}{2}.$$

Hence, over the interval $(-\frac{\Delta}{2}, \frac{\Delta}{2})$ quantization error may be assumed as an uniformly distributed random variable.



a uniform distribution

A uniform distribution for quantization error.

An uniformly distributed random variable 'x' over an interval (a, b), the probability density function of uniformaly distributed random variable 'x' is given as.

$$f_X(x) = \begin{cases} 0 & \text{for } x \leq a \\ \frac{1}{b-a} & \text{for } a < x \leq b \\ 0 & \text{for } x > b \end{cases}$$

Thus the help of equation, the probability density function (PDF) for quantization error ε may be defined as.

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \leq \frac{\Delta}{2} \\ \frac{\Delta}{2} & \text{for } -\frac{\Delta}{2} < \varepsilon < \frac{\Delta}{2} \\ 0 & \text{for } \varepsilon > \frac{\Delta}{2} \end{cases}$$

The noise power is expressed as.

$$\text{noise power} = \frac{v^2_{noise}}{R}.$$

v^2_{noise} is mean square value of noise (ε).

we know that $E(x) = \int_{-\infty}^{\infty} x\, f_X(x)\, dx$

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

Similarly

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\Delta/2}^{\Delta/2} \varepsilon^2 \cdot \frac{1}{\Delta}\, d\varepsilon = \frac{1}{\Delta}\left[\frac{\varepsilon^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\cancel{\Delta}} \times \left[\frac{\Delta^3}{4}\right] = \frac{\Delta^2}{12}.$$

$\therefore$ Quantization noise or error $= \frac{\Delta^2}{12}$

Signal to noise ratio of PCM system :-

$$\frac{S}{N} = \frac{\text{signal power (Normalized)}}{\text{Noise power (Normalized)}} = \frac{P}{\Delta^2/12}$$

quantization noise : $\frac{\Delta^2}{12}$ $\left(\Delta = \frac{2x_{max}}{2^v}\right)$

$$\Delta^2 = \left(\frac{2x_{max}}{2^v}\right)^2 = \frac{4 \cdot x_{max}^2}{2^{2v}} \qquad (x_{max} = 1)$$

quantization = $\frac{\cancel{4}}{2^{2v}} \cdot \frac{1}{\cancel{12}_3} = \frac{1}{3}$

Then quantization noise is = $\frac{4x_{max}^2}{2^{2v}} \cdot \frac{1}{12}$

Then $$\frac{S}{N} = \frac{P}{\frac{\cancel{4}x_{max}^2}{2^{2v}} \cdot \frac{1}{\cancel{12}\,3}} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$$

$\therefore \frac{S}{N} \leq 3 \times 2^{2v}$. $(X_{max} = 1)$

$(P \leq 1)$

The signal to noise ratio in db.

$$\left(\frac{S}{N}\right)_{dB} \leq 10 \log \left(3 \times 2^{2v}\right)$$

$$\leq 10 \log_{10} 3 + 20v \log_{10} 2$$

$$\left(\frac{S}{N}\right)_{dB} \leq 4.8 + 6v.$$

Signal to noise ratio for sinusoidal input

we know $\frac{S}{N} = 3P2^{2v}$

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consider modulating signal is a sinusoidal voltage having peak amplitude X_{max}.

$$P = \frac{V^2}{R} \quad \text{let } (R = 1)$$

$$V_{rms} = \frac{A_m}{\sqrt{2}} = \frac{x_{max}}{\sqrt{2}}$$

$$P = \frac{x_{max}^2}{2} = \frac{1}{2} \quad (\because x_{max} = 1)$$

$$\left(\frac{S}{N}\right)_{dB} = 10 \log \left(3 \times P \times 2^{2V}\right)$$

$$= 10 \log \left(\frac{3}{2} \times 2^{2V}\right)$$

$$= 1.8 + 6V.$$

Non uniform quantization :-

If the quantizer characteristics is non-linear and the step size is not constant. in stead if it is variable, dependent on the amplitude of input signal then the quantization is known as non-uniform quantization. In non-uniform quantization, the step size is reduced with the reduction in signal level. for weak signals ($P<<1$), the step size is small, therefore the quantization noise reduces, to improve the signal to quantization noise ratio for weak signals. The step size is thus varied according to the signal level to keep the signal to noise ratio adequately high. This is non-uniform quantization. The non-uniform quantization is practically achieved through a process called "companding".

· Companding :-

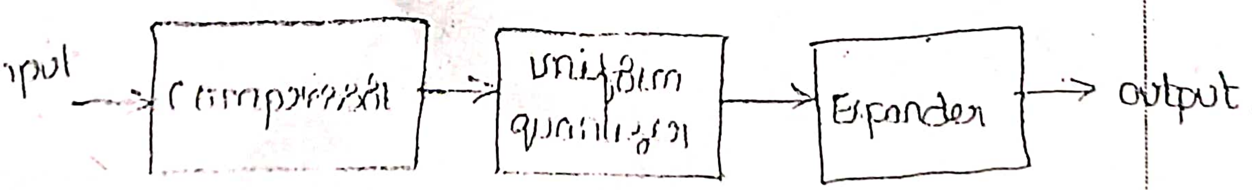
It is used to improve the signal to quantization noise ratio of weak signals.

$$N = \frac{\Delta^2}{12}$$

In uniform quantization, once the step size is fixed, the quantization noise power remains constant and signal power is not constant. It is proportional to the square of signal amplitude. Hence signal power will be small for weak signals, but quantization noise power is constant. Therefore, the signal to quantization noise for the weak signals is very poor. Companding is a term derived from two words.

Companding : Compressing + Expanding.

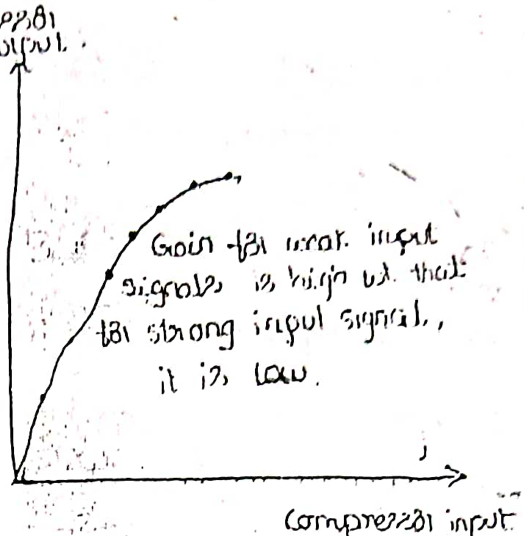
In practice, it is difficult to implement the non-uniform quantization because it is not known in advance about the changes in the signal levels. Therefore, a particular method is used. Therefore the weak signals are amplified and strong signals are attenuated. This process is called compression and the block is called compressor.



At the receiver we use expansion. The circuit used for providing expansion is known as expander. The compression of signal at the transmitter and expansion at the receiver is called companding.

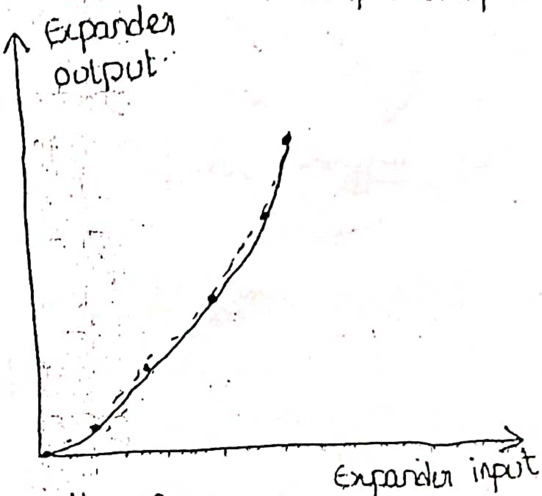
Compressor characteristic :-

The compressor provides a higher gain to the weak signals and smaller gain to the strong input signals. Thus, weak signals are artifically boosted to improve the signal to quantization noise ratio.

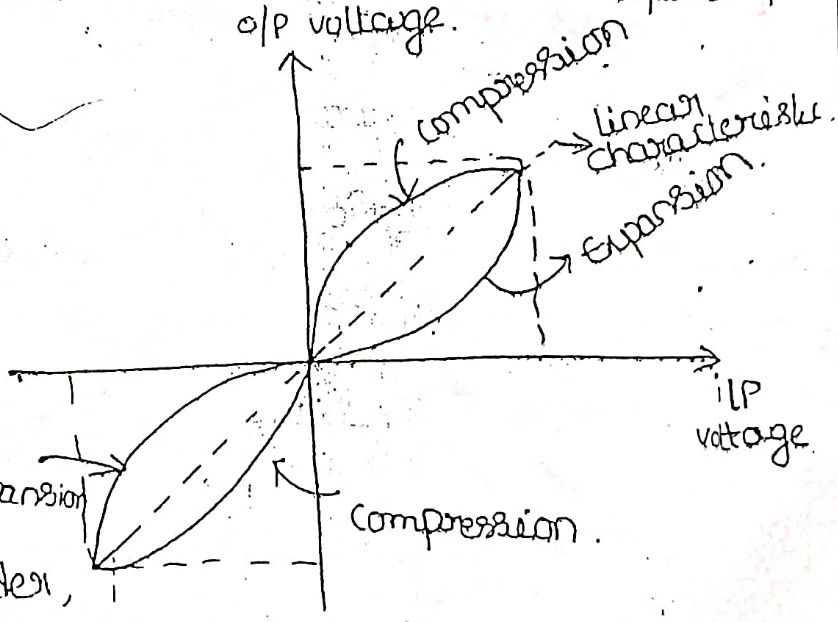


Expander characteristics :-

This is exactly inverse of compressor characteristics. The artificially boosted signals by the compressor are brought back to their original amplitudes at the receiver end.



Compander characteristics :-



It is the combination of both compressor and Expander characteristics. Due to inverse nature of compressor and Expander, the overall characteristics of the compander is a straight line.

Different types of compressor :-

There are two types of companding in PCM system.

1) μ - law companding.
2) A - Law companding.

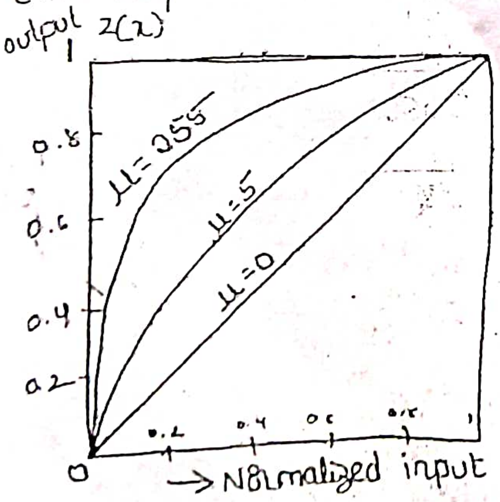
μ - law companding :-

In the μ - law companding, the compressor characteristic is continuous. It is approximately linear for smaller values of input levels and logarithmic for high input levels.

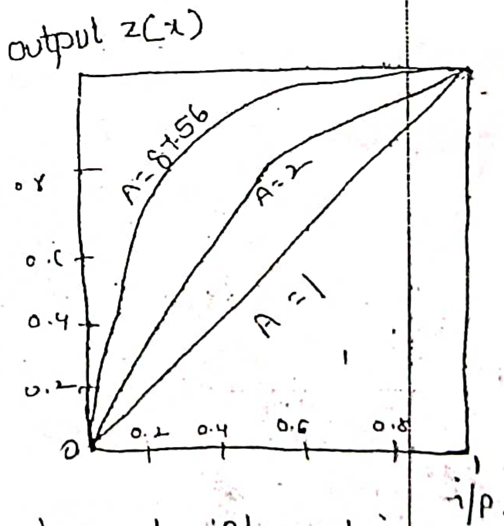
$$Z(x) = (\text{Sgn } x) \frac{\ln(1+\mu|x|/x_{max})}{\ln(1+\mu)} \qquad 0 \leq |x|/x_{max} \leq 1.$$

Here, $Z(x)$ represents the output and x is the input to the compressor. Also $|x|/x_{max}$ represents the normalized value of input with respect to the maximum value x_{max}. Sgn represents ± 1. i.e positive and negative values of input and output. ($\mu = 255$).

> The μ - law companding is used for speech and music signals. It is used for telephone systems in US, Canada and Japan. (.



characteristic of a μ-law compressor.



characteristic of a A - law compressor.

A-law companding :-

In the A-law companding, the compressor characteristic is piecewise, made up of a linear segment for low levels inputs and a logarithmic segment for high level inputs. The A-law companding is used for PCM telephone system in europe. practically A = 87.56.

$$\frac{z(x)}{x_{max}} = \begin{cases} \dfrac{A|x|/x_{max}}{1+\log_e A} & \text{for } 0 \le \dfrac{|x|}{x_{max}} \le \dfrac{1}{A} \\ \dfrac{1+\log_e[A|x|/x_{max}]}{1+\log_e A} & \text{for } \dfrac{1}{A} \le \dfrac{|x|}{x_{max}} \le 1 \end{cases}$$

Applications of PCM :-

1. PCM is used in telephony.
2. PCM is used in space communication.

Advantages :-

1. PCM Provides high noise immunity.
2. We can store the PCM signal due to its digital nature.
3. We can use various coding techniques so that only the desired person can decode the received signal.
4. Due to digital nature of the signal, we can place repeaters between the transmitter and the receivers. Repeaters further reduces the effect of noise.

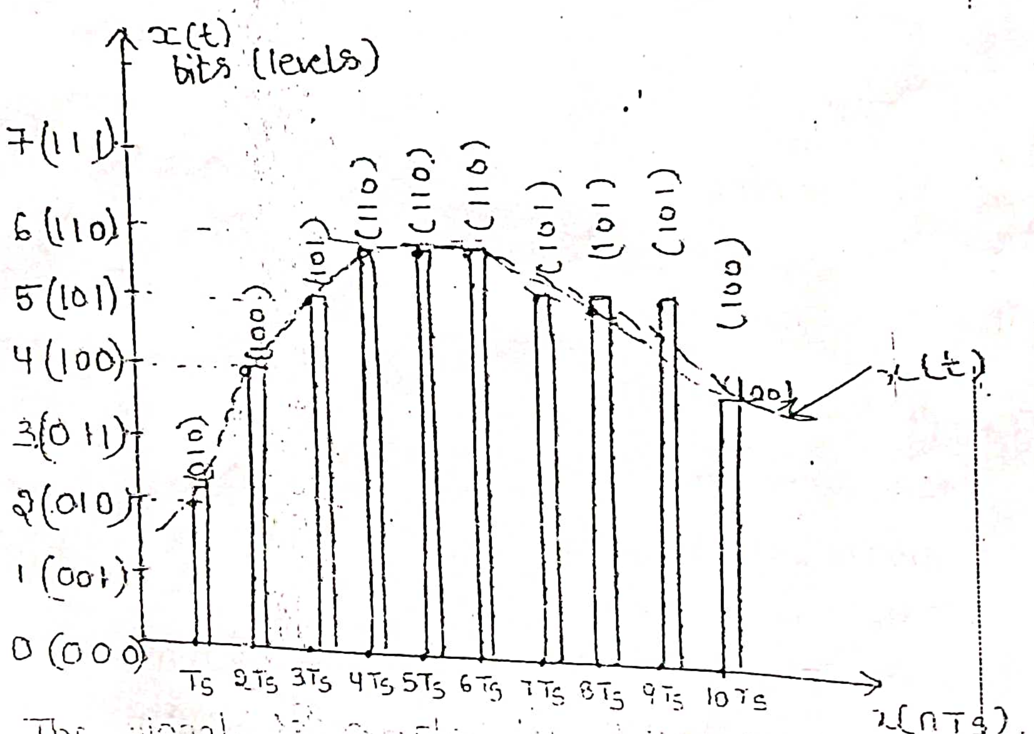
Drawbacks of PCM :-

1. The encoding, decoding and quantizing circuitry of PCM is complex.
2. PCM requires a large bandwidth as compared to the other systems.

Differential pulse code modulation (DPCM) :-

i) **Reason to use DPCM :-** The samples of a signals are highly correlated with each other. This is due to the fact any signal does not change fast. This means that its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with a little difference. When these samples are encoded by a standard PCM system, the resulting encoded signal contains some redundant information.

ii) **Redundant information in PCM :-**

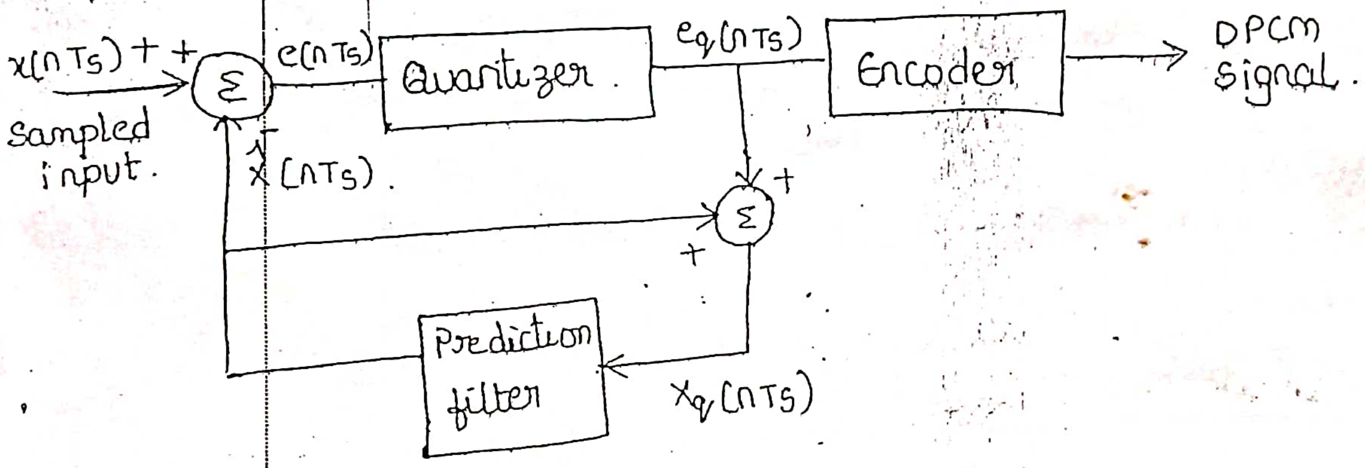


The signal is sampled [illegible] sampling at Ts, 2Ts, 3Ts ... nTs. If we observe, 4Ts, 5Ts and 6Ts are encoded to same value of (110). This information can be carried by one sample. But three samples are carrying the information. This is redundant. If this redundancy is reduced, then overall bitrate will decrease and no. of bits to be transmitted in one sample will also reduce.

If the redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation Scheme is known as differential pulse code modulation (DPCM). (13)

DPCM transmitter :-

It works on the principle of prediction. The value of present sample is predicated from past sample. The sampled signal is denoted by $x(nT_S)$ and predicated signal by $\hat{x}(nT_S)$. The predicted value is produced by prediction filter. The quantizer output is added with previous prediction and given as input to prediction filter $x_q(nT_S)$. we can observe that $e_q(nT_S)$ is small and can be encoded by using small no of bits. Thus bits per sample is reduced.



prediction error is

$$e(nT_S) = x(nT_S) - \hat{x}(nT_S).$$

quantizer output

$$e_q(nT_S) = e(nT_S) + q(nT_S)$$

$q(nT_S)$ is the quantization error.

input to ~~quan~~ prediction filter.

$$x_q(nT_S) = \hat{x}(nT_S) + e_q(nT_S).$$

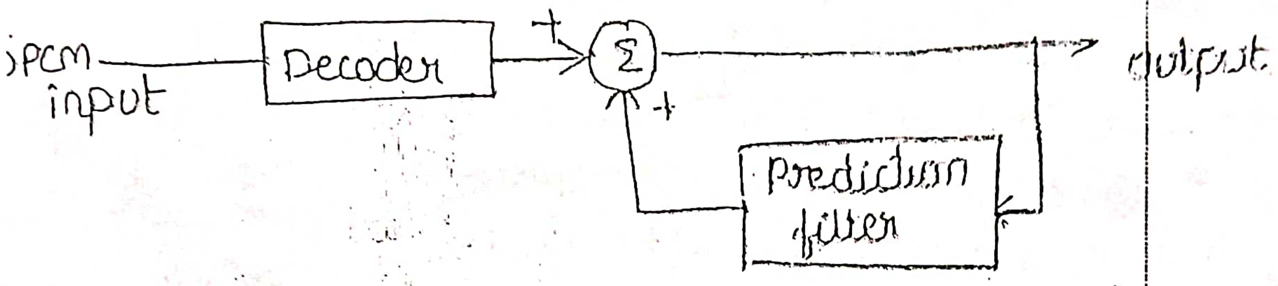
$$= \hat{x}(nT_S) + e(nT_S) + q(nT_S)$$

$$= \hat{x}(nT_s) + x(nT_s) - \hat{x}(nT_s) + q(nT_s)$$

$$= x(nT_s) + q(nT_s)$$

Hence the prediction filter input does not depends on prediction filter characteristics.

DPCM Receiver :-



The decoder first reconstruct the quantized error signal from incoming binary signal. The prediction filter o/p and quantized error signals are added to give quantized version of original signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$ which is introduced permanently in the reconstructed signal.

Advantage of DPCM : Salient features :-

1. As the difference between $x(nT_s)$ and $\hat{x}(nT_s)$ is being encoded and transmitted by the DPCM technique, A small difference voltage is to be quantized and encoded.
2. This will requires less number of quantization levels and hence less number of bits to represent them.
3. Thus signaling rate and bandwidth of a DPCM system will be less than that of PCM.

Delta modulation :-

i) Reason to use Delta modulation :-

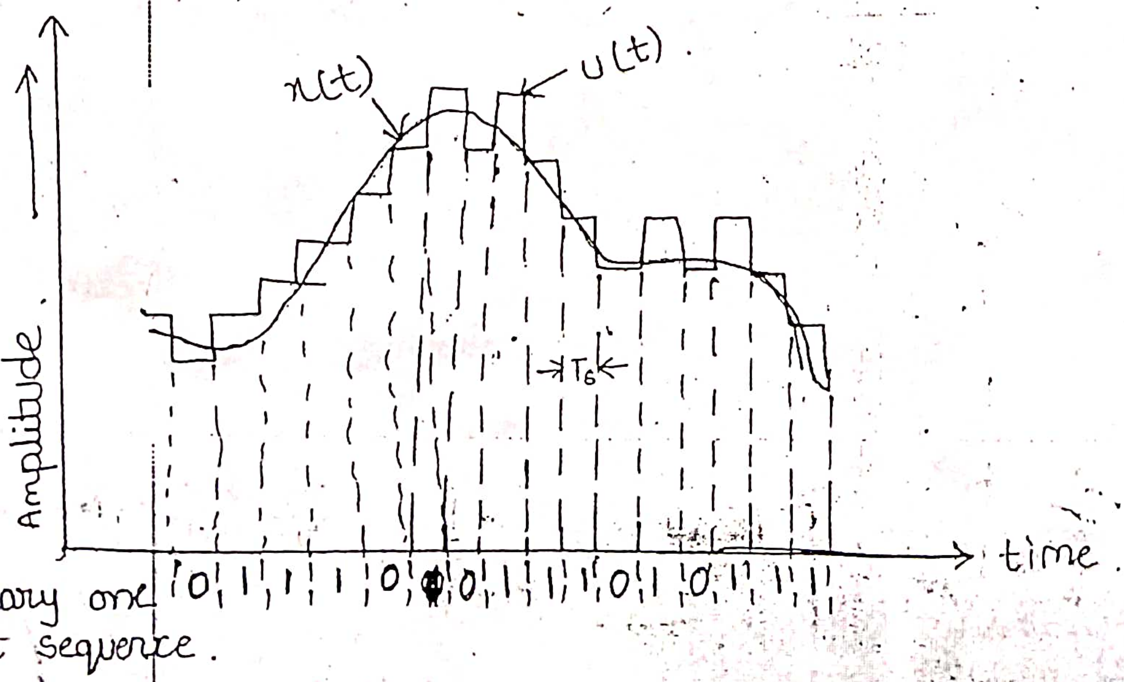
we have observed in PCM that it transmits all the bits which are used to code a sample. Hence, signaling rate and transmission channel bandwidth are quite large in PCM. To overcome this problem. Delta modulation is used.

ii) working principle :-

Delta modulation transmits only one bit per sample. Hence the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted. Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is kept fixed. The difference between input signal $x(t)$ and staircase approximated signal is confined to two levels i.e $+\Delta$ and $-\Delta$. If difference is positive, then signal is increased by one step. If difference is negative, then signal is reduced by one step.

If step is increased, '1' is transmitted.

If step is decreased, '0' is transmitted.



Mathematical Expressions :-

Thus, the principle of delta modulation can be explained with the help of few equations as shown.

The error is given as,

$$e(nT_S) = x(nT_S) - \hat{x}(nT_S).$$

where $e(nT_S)$ = error at present sample

$x(nT_S)$ = sampled signal of $x(t)$

$\hat{x}(nT_S)$ = last sample approximation of the staircase waveform.

If we assume $u(nT_S)$ as the present sample approximation of staircase output, then

$$u[(n-1)T_S] = \hat{x}(nT_S).$$

Now, let us define a quantity $b(nT_S)$ in such a way that

$$b(nT_S) = \Delta \, \text{sgn}\,[e(nT_S)].$$

T_S = Sampling interval.

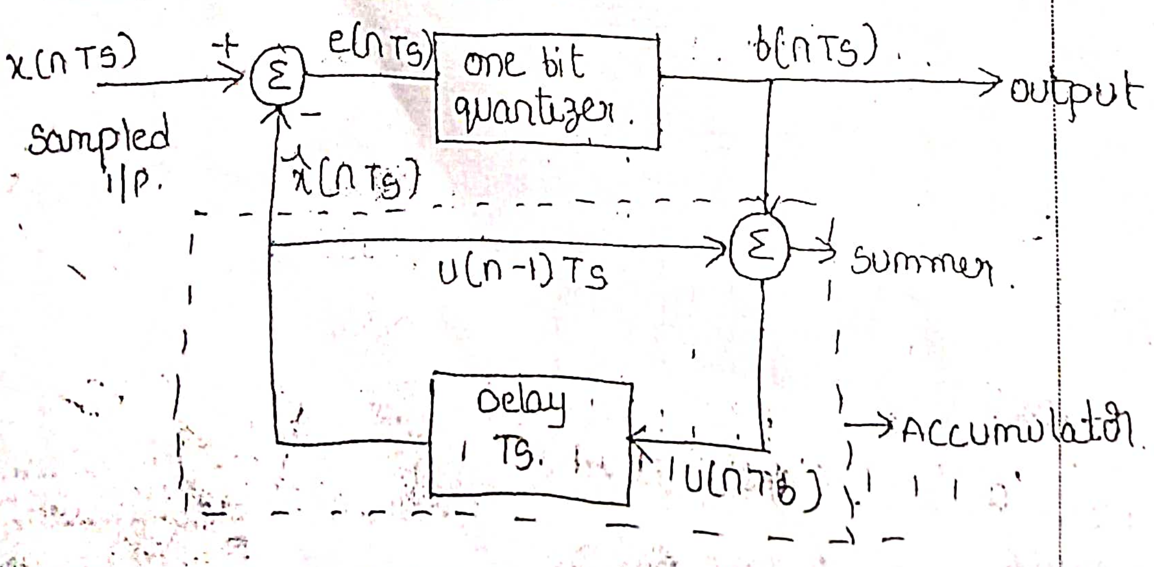
In other words.

$$b(nT_S) = \begin{cases} +\Delta & \text{if } x(nT_S) \geq \hat{x}(nT_S) \\ -\Delta & \text{if } x(nT_S) < \hat{x}(nT_S). \end{cases}$$

Also, if $b(nT_S) = +\Delta$ then '1' is transmitted and

$b(nT_S) = -\Delta$ then '0' is transmitted.

Delta modulation transmitter :-



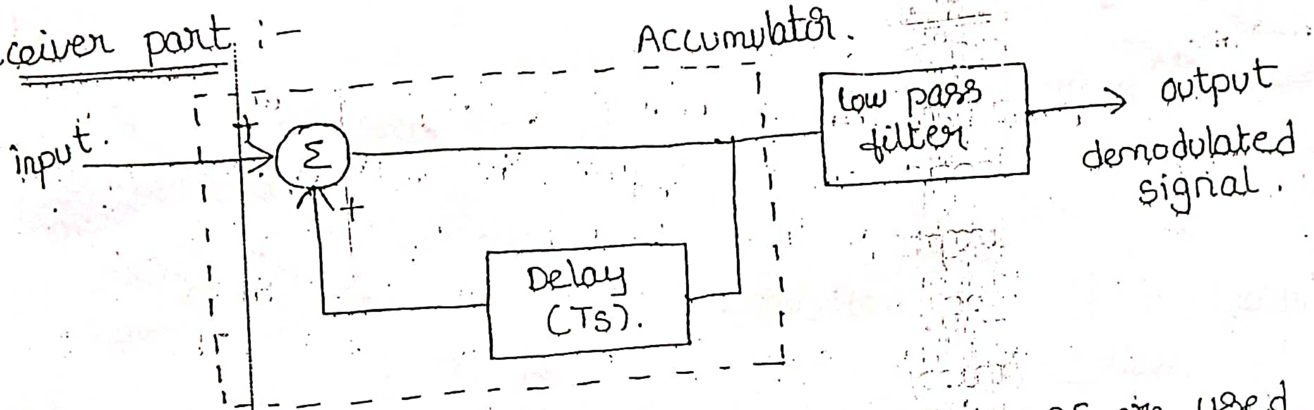
The summer in accumulator adds quantizer o/p with previous sample approximation. This gives the present sample approximation i.e.

$$u(nT_s) = u[(n-1)T_s] + [\pm\Delta]$$

(or) $$u(nT_s) = u[(n-1)T_s] + b(nT_s).$$

Thus, depending on the sign of $e(nT_s)$. one bit quantizer generates an output of $+\Delta$ or $-\Delta$. If stepsize is $+\Delta$, then binary '1' is transmitted. If stepsize is $-\Delta$ then binary 0 is transmitted.

Receiver part :-



At receiver end, the accumulator and LPF are used. The accumulator generates the stair case approximated signal output and is delayed by one sampling period T_s. It is added to the input signal. If input is '1' then it adds $+\Delta$ step to the previous output. If input is '0' then one step 'Δ' is subtracted from the delayed signal. The LPF smoothens the staircase signal to reconstruct original message signal $x(t)$.

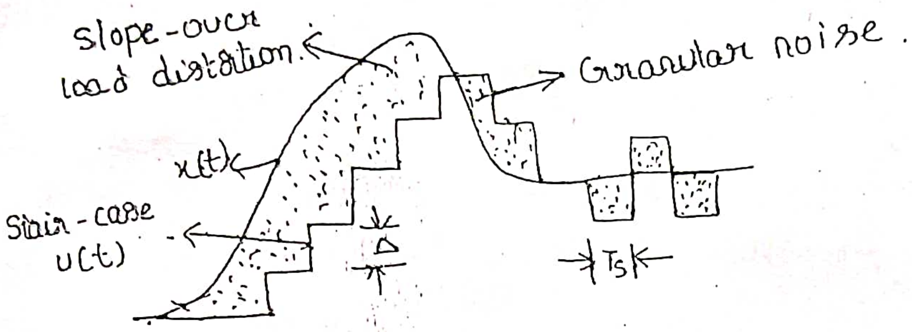
Advantages of DM :-

1. Since, DM transmits only one bit for one sample, the signaling rate and transmission channel bandwidth is quite small.
2. The Tx and Rx implementation for DM is very simple.

Drawbacks of DM :-

The DM has two major drawbacks as under

1. Slope overload distortion
2. Granular or idle noise.



i) Slope overload distortion :-

The distortion arises because of large dynamic range of the input signal. From the figure, the rate of rise of input signal x(t) is so high that staircase signal cannot approximate it. Hence, there is a large error between the staircase approximated and the original input signal, x(t). This error (or) noise is called as "slope-over load distortion".

To minimize this error, step size (Δ) must be increased when slope of signal x(t) is high. Since step size of DM remains fixed. its maximum or minimum slope occurs along straight lines. Therefore this modulator is known as linear delta modulator.

ii) Granular (or) Idle noise :-

This noise occurs when step size is too large compared to small variations in the input signal. It means for very small variations in the input signal, the staircase signal is changed by large amount (Δ). From the figure, when the input signal is almost flat. u(t) keeps on oscillating by ±Δ around the signal..

The error between input and approximated signal is called as granular noise. To overcome this problem, step size should be maintained low.

Bit Rate (sampling rate) of DM :-

Delta modulation bit rate (r) = No. of bits transmitted / sec.

= No. of samples / sec × No. of bits / sample

$= f_s \times 1$

$= f_s$.

The DM bit rate is $(1/N)$ times the bit rate of a PCM system, where 'N' is the number of bits per transmitted PCM codeword. From this, we can say that channel bandwidth required for DM is reduced to great extent when compared to that of a PCM system.

Example :- Given a sinewave of frequency f_m and amplitude A_m. applied to a DM with step size 'Δ'. show that slope overload distortion will occur if, $A_m > \frac{\Delta}{2\pi f_m T_s}$, here T_s is sampling period.

Solution :- let us consider a sinewave is given as

$$x(t) = A_m \sin(2\pi f_m t).$$

The maximum slope of delta modulator may be given as.

$$\text{max. slope} = \frac{\text{step size}}{\text{sampling period}} = \frac{\Delta}{T_s}.$$

we know that, slope over load distortion will take place if slope of $x(t)$ is greater than slope of DM.

$$\max \left| \frac{d}{dt} x(t) \right| > \frac{\Delta}{T_s}.$$

$$\max \left| \frac{d}{dt} A_m \sin(2\pi f_m t) \right| > \frac{\Delta}{T_s}$$

$$\max \left| A_m \cos(2\pi f_m t)\, 2\pi f_m \right| > \frac{\Delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\Delta}{T_s} \quad [\because \max.\ \text{of } \cos\theta = 1]$$

$$\boxed{A_m > \frac{\Delta}{T_s (2\pi f_m)}}$$

Hence proved.

Evaluation of maximum output signal to noise ratio :-

we know that the condition to avoid the slope over load distortion is expressed as.

$$A_m \leq \frac{\Delta}{2\pi f_m T_s}.$$

where A_m is peak amplitude of the sinusoidal signal

Δ is step size, f_m is maximum frequency of signal.

T_s is sampling period.

$$\therefore A_m = \frac{\Delta}{2\pi f_m T_s}.$$

Therefore, the maximum value of the output signal power is expressed as

$$\text{signal power} = \frac{V_{rms}^2}{R} \quad [\because R = 1]$$

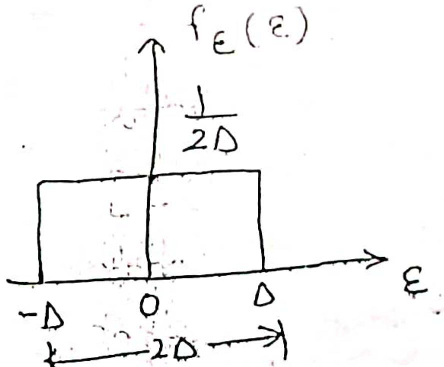
$$V_{rms} = \frac{A_m}{\sqrt{2}}$$

$$P_{max} = \frac{A_m^2}{2}$$

$$P_{max} = \frac{\Delta^2}{4\pi^2 f_m^2 T_s^2} \cdot \frac{1}{2} = \frac{\Delta^2}{8\pi^2 f_m^2 T_s^2}$$

Now, we require to obtain the expression for quantization noise power. The quantization error in DM lie in the intervals $(-\Delta, \Delta)$. The quantization nois error is uniformly distributed as shown in the figure.

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & t < -\Delta \\ \frac{1}{2\Delta} & -\Delta < t < \Delta \\ 0 & t > \Delta \end{cases}$$



$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon$$

$$= \int_{-\Delta}^{\Delta} \varepsilon^2 \cdot \frac{1}{2\Delta}\, d\varepsilon = \frac{1}{2\Delta} \int_{-\Delta}^{\Delta} \varepsilon^2 d\varepsilon$$

$$= \frac{1}{2\Delta} \left[\frac{\varepsilon^3}{3}\right]_{-\Delta}^{\Delta} = \frac{1}{2\Delta}\left[\Delta^3 - (-\Delta^3)\right]$$

$$= \frac{1}{6\cancel{\Delta}} \times 2\cancel{\Delta}^3 = \frac{\Delta^2}{3}$$

The DM signal is passed through a reconstruction LPF at output of a DM receiver. The bandwidth of LPF is w such that $F_m \geq f_m$ and $F_m << f_s$ normalized power at the filter output,

$$N = \frac{f_m}{f_s} \times \frac{\Delta^2}{3} = \frac{f_m T_s \Delta^2}{3} \quad \left(\because T_s = \frac{1}{f_s}\right)$$

Hence signal to noise ratio is given as,

$$\frac{S}{N} = \frac{\text{signal power (normalized)}}{\text{Noise power (normalized)}}$$

$$= \frac{\Delta^2}{8\pi^2 f_m^2 T_S^2} \times \frac{3}{\Delta^2 F_M T_S}$$

$$\boxed{\frac{S}{N} = \frac{3}{8\pi^2 f_m^2 F_M T_S^3}}$$

This is the desired expression for the output, signal to quantization noise ratio.

Adaptive delta modulation :-

To overcome the quantization errors due to slope overload and granular noise, the step size (Δ) is made adaptive to variations in input signal $x(t)$. If the i/p is varying slowly, the step size is reduced. This method is known as Adaptive delta modulation (ADM).

Transmitter part :-

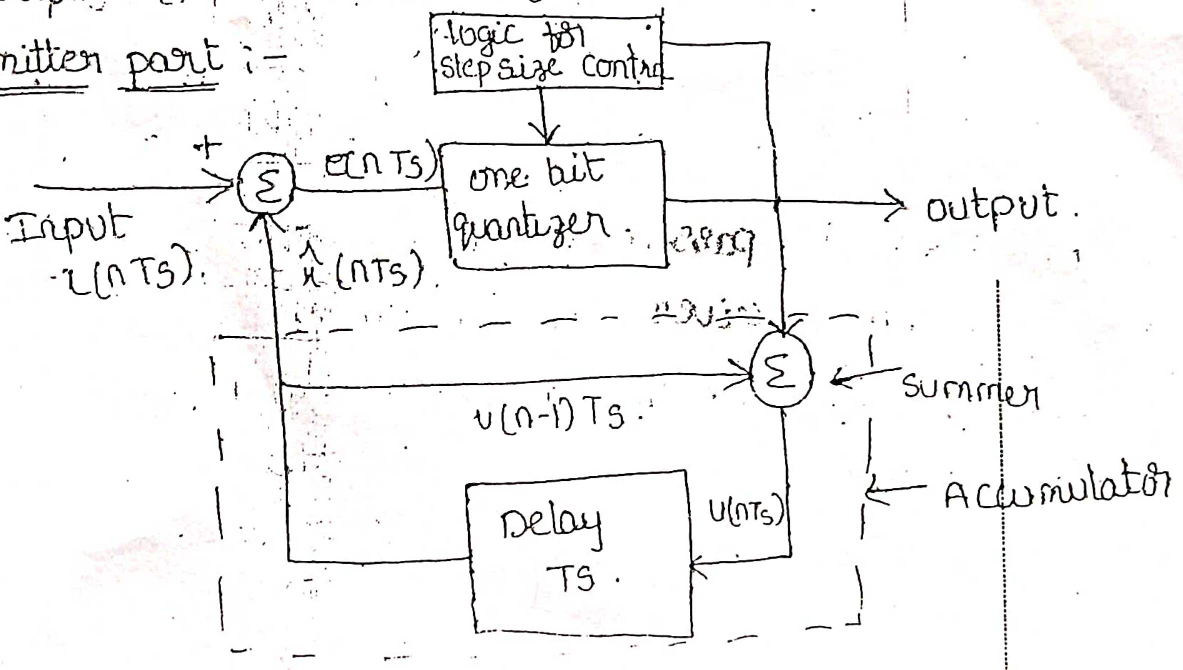


Fig. shows the transmitter part of ADM. The logic for step size control is added to diagram.

The step size may increase or decrease according to specified rule depending on one bit quantizer output. If quantizer output is high, then step size may be doubled for next sample. If quantizer output is low, then step size may be reduced by one step.

Receiver part :-

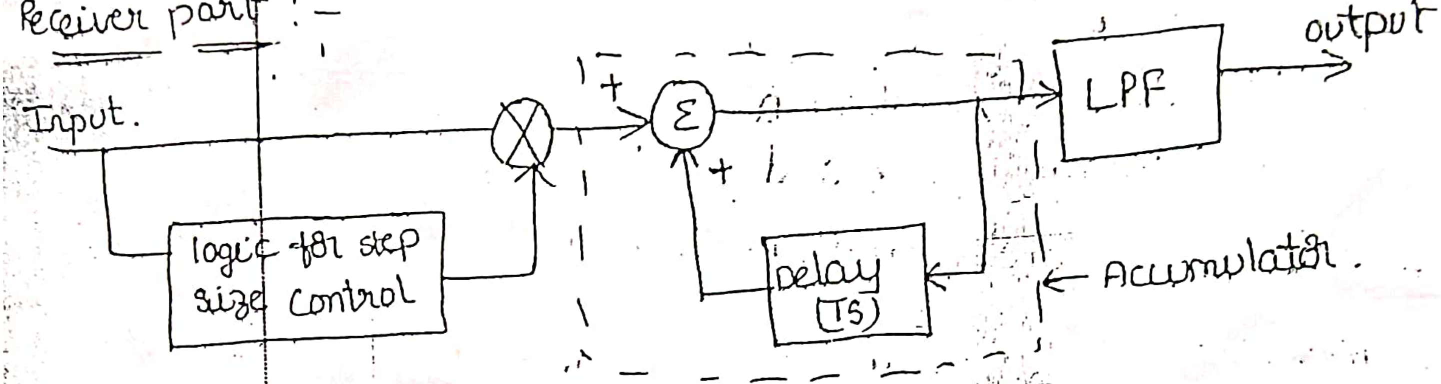
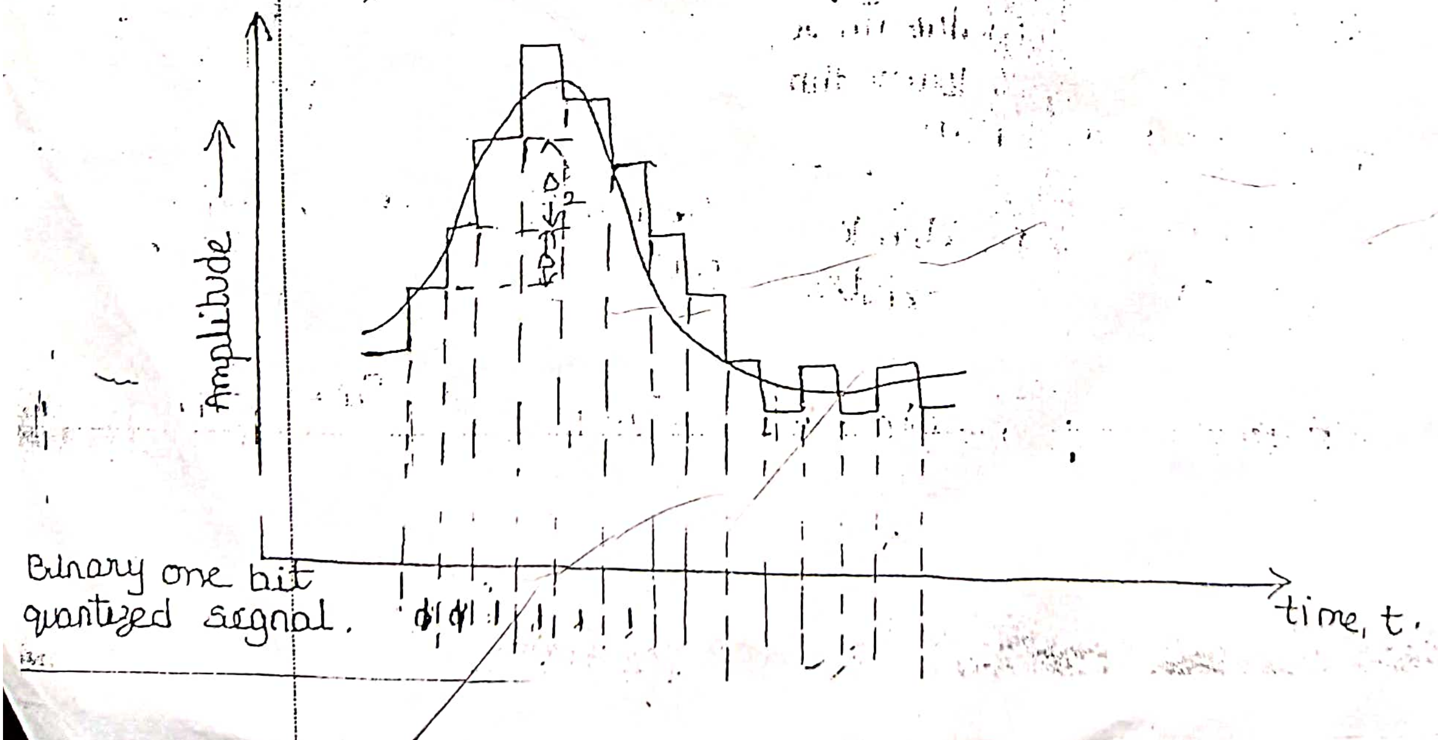


Fig. shows the receiver part of ADM. There are two portions. The first portion produces the step size from each incoming bit. Exactly, the same process is followed as that in transmitter. The previous input and present input decides stepsize. It is then applied to an accumulator which forms stair case waveform. Then LPF smoothens out the staircase waveform to reconstruct the original signal.



dvantages of ADM :-

- The signal to noise ratio becomes better than ordinary DM because of reduction in slope overload distortion and idle noise.
- Because of variable step size, dynamic range of ADM is wider than simple DM.
3. Bandwidth is better than Delta modulation.
4. Implementation of Tx and Rx is very simple

fference between PCM, DPCM, DM & ADM :-

PCM	DPCM	DM	ADM
It can use 4,8,16 bits / sample.	Bits can be more than one but less than PCM.	It uses only one bit per sample	only one bit is used to encode one sample.
No. of levels depends on no. of bits level size is fixed.	Here fixed no. of levels are used.	step size is fixed and cannot be varied.	According to signal variation, step size varies.
Error depends on no. of levels used.	slope over load distortion and quantization noise are present.	Slope over load & granular noise are present	quantization noise is present but other errors are absent
highest B.W is required since no. of bits are high.	Bandwidth required is lower than PCM.	Lowest B.W is required.	Lowest B.W is required.
No. feedback in Tx and Rx part.	Feed back exists.	Feed back present in Tx.	Feed exists.
system complex.	simple	simple	simple.