06/04/[illegible]

$I \propto \frac{1}{P}$

P ↑ I ↓ [illegible]

P ↓ I ↑ [illegible] bit [illegible]

time ↓ ∴ Data [illegible] ↓.

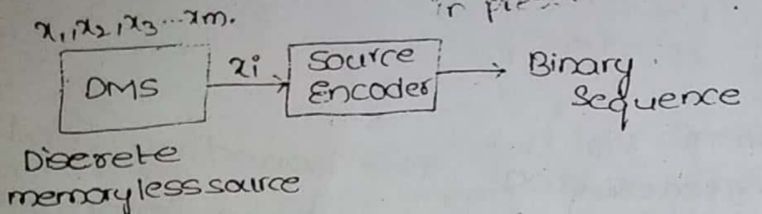
# UNIT - V

## Source coding

M. Manis

**Def: Source coding :-**

Present o/p depends only in present i/p




* A conversion of discrete memory less source which is having Symbols $x_1, x_2, \dots x_m$ with the probabilities $P(x_1), P(x_2), P(x_3) \dots p(x_m)$ is converted in to a binary code word (or) a bit sequence is called as source coding.

* The device which performs the conversion is called as Source Encoder.

**Advantages of Source coding:-**

1. The data compression is possible by the source coding operation.
2. we can minimize the average bit rate by the source encoding
3. we can reduce the data Transmission time.
4. we can reduce the storage capacity of Source Symbols by compression of that symbols.

**Few terms related to Source coding process:-**

1. **Code word length ($n_i$) :-** It is defined as how many bits consist in a Symbol (or) length of the code word. The number of binary digits in the code word.

   Ex: 100100. $\boxed{n_i = 6}$

2. **Average code word length (L) :- 'L'.** It is defined as the product in b/w probability of the Symbols and code word length, which is given as:

$$\boxed{L = \sum_{i=1}^{m} P(x_i) \times n_i}$$

3. **Code efficiency ($\eta$) :-** It is defined as the ratio b/w entropy and average code word length is called as code efficiency.

$$\boxed{\eta = \frac{H(x)}{L}}$$

4. Code Redundancy :-
which is caluclated as

$$\boxed{\gamma = 1-\eta}$$ — code efficiency.

Source coding theoram :-

This theoram states that the boundary condition in b/w average code word length and average information rate (entropy) is given as:

$$\boxed{L >= H(x)}$$

* If this boundary is satisfied then the coding is denoted as source coding.

Classification of codes :-

1). Fixed length codes :-
It is defined as the length of the each code word is fixed.

Ex:

| code word | $n_i$ |
|---|---|
| 0 0 | 2 |
| 0 1 | 2 |
| 1 0 | 2 |
| 1 1 | 2. |

2). Variable length codes :-
It is defined as the length of the each code word is varied from one code word to another code word.

Ex:

| code word | $n_i$ |
|---|---|
| 0 | 1 |
| 0 1 | 2 |
| 0 0 1 | 3 |
| 1 1 0 1 | 4. |

3). different code words :-
It is defined as one code word is different to another code word in the bit sequence.

Ex:

| code word | $n_i$ |
|---|---|
| 0 0 | 2 |
| 0 1 | 2 |
| 1 0 | 2 |
| 1 1 | 2 |

All are different

(iv) Prefix Code word :-

It is defined as no code word can be formed by adding previous code word.

Ex: code word

00

101

0111

* These codes are also called as Instantaneous codes. The Prefix codes existence can be find by using a "Craft inequality equation" and which is given as:

$$\boxed{k = \sum_{i=1}^{m} 2^{-n_i} \leq 1}$$

07/09/18

Pb) Given a discrete memory less source x with 4 possible symbols $x_1, x_2, x_3, x_4$. The symbols are having the probabilities encoded values shown below:

| $x_i$ | $P(x_i)$ | Encoded data (or) Code word | $n_i$ |
|---|---|---|---|
| $x_1$ | 0.81 | 0 | 1 |
| $x_2$ | 0.09 | 10 | 2 |
| $x_3$ | 0.09 | 110 | 3 |
| $x_4$ | 0.01 | 111 | 3 |

Caluclate code efficiency & code redundancy

Sol: The average code word length 'L' is given as:

$\eta = \frac{H(x)}{L}$

$$\boxed{L = \sum_{i=1}^{m} P(x_i) \times n_i} \text{ bits/symbol}$$

$\gamma = 1 - \eta$

$$L = P(x_1) \times n_1 + P(x_2) \times n_2 + P(x_3) \times n_3 + P(x_4) \times n_4$$

$$= 0.81 \times 1 + 0.09 \times 2 + 0.09 \times 3 + 0.01 \times 3$$

$$= 0.81 + 0.18 + 0.27 + 0.03$$

$$\boxed{L = 1.29 \text{ bits/symbol}}$$

Average information rate (or) entropy:

$$\boxed{H(x) = \sum_{i=1}^{m=4} P(x_i) \log_2 \frac{1}{P(x_i)}}$$

$$= P(x_1) \log_2 \frac{1}{P(x_1)} + P(x_2) \log_2 \frac{1}{P(x_2)} + P(x_3) \log_2 \frac{1}{P(x_3)} + P(x_4) \log_2 \frac{1}{P(x_4)}$$

$$H(x) = 0.81 \log_2 \frac{1}{0.81} + 0.09 \log_2 \frac{1}{0.09} + 0.09 \log_2 \frac{1}{0.09} + 0$$

$$= 0.246 + 0.312 + 0.312 + 0.066$$

$$\boxed{H(x) = 0.936 \text{ bits/symbol}}$$

According to source coding theoram

$$\boxed{L >= H(x)}$$

$\therefore 1.29 >= 0.936$

* Hence we observe that source coding theoram is satisfied.

w.k.T code efficiency $\eta = \frac{H(x)}{L}$

$$\therefore \eta = \frac{0.936}{1.29}$$

$$\eta = 0.725 \times 100$$

$$\boxed{\eta = 72.5\%}$$

w.k.T code Redundancy $r = 1 - \eta$

$$r = 1 - 0.725 = 0.275$$

$$\boxed{r = 0.275}$$

(or)

$$r = 100 - 72.5 = 27.5\%$$

2) Consider a discrete memory less source X with four possible symbols $x_1, x_2, x_3$ & $x_4$. Each symbol is coded as different, which is shown in below table:

| $x_i$ | Code-A | $n_i$ | Code-B | $n_i$ | code-C | $n_i$ | Code-D | $n_i$ |
|---|---|---|---|---|---|---|---|---|
| $x_1$ | 00 | 2 | 0 | 1 | 0 | 1 | 0 | 1 |
| $x_2$ | 01 | 2 | 10 | 2 | 11 | 2 | 100 | 3 |
| $x_3$ | 10 | 2 | 11 | 2 | 100 | 3 | 110 | 3 |
| $x_4$ | 11 | 2 | 110 | 3 | 110 | 3 | 111 | 3 |

Verify the craft inequality function and identify the prefix codes in the given code word.

Sol: w.k.T craft inequality function is given as:

$$\boxed{K = \sum_{i=1}^{m} 2^{-n_i} \leq 1} \quad \therefore m = 4.$$

For code-A:

$$K = 2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} \leq 1$$

$$= \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} \leq 1$$

the craft inequality function is satisfied hence A is a prefix code.

code-B:

W.K.T ~~Craf~~ from craft inequality function

$$K = \sum_{i=1}^{m} 2^{-n_i} \le 1 \quad \because m=4$$

$$K = 2^{-1} + 2^{-2} + 2^{-2} + 2^{-3} \le 1$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} \le 1$$

$$\boxed{K = 1.125 \not\le 1} \qquad \boxed{K = 1.125 \not\le 1}$$

Hence code B is not a prefix code.

For code-C:

From craft inequality function:

$$\boxed{K = \sum_{i=1}^{m} 2^{-n_i} \le 1} \quad \because m=4$$

$$K = 2^{-1} + 2^{-2} + 2^{-3} + 2^{-3} \le 1$$

$$K = \cancel{\ } 1 \le 1$$

$\therefore$ Hence code C is a prefix code.

For code-D:

$$K = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-3}$$

$$K = 0.875 \le 1$$

Hence code D is a prefix code.

Pb) 3). A DMS source having 5 symbols with probability values and encoded data are given in below table;

| $x_i$ | $p(x_i)$ | encoded data | $n_i$ |
|---|---|---|---|
| $x_1$ | 1/2 | 0 | 1 |
| $x_2$ | 1/6 | 00 | 2 |
| $x_3$ | 1/6 | 010 | 3 |
| $x_4$ | 1/12 | 110 | 3 |
| $x_5$ | 1/12 | 111 | 3. |

Calculate efficiency and Redundancy.

Sol: W.K.T Average code word length

$$\boxed{L = \sum_{i=1}^{m} p(x_i) \times n_i} \quad \because m=5$$

$$L = P(x_1)\times n_1 + P(x_2)\times n_2 + P(x_3)\times n_3 + P(x_4)\times n_4 + P(x_5)\times n_5$$

$$= 1/2 \times 1 + 1/6 \times 2 + 1/6 \times 3 + 1/12 \times 3 + 1/12 \times 3$$

$$= 0.5 + 0.33 + 0.5 + 0.25 + 0.25$$

$$\boxed{L = 1.83 \text{ bits/symbol}}$$

W.K.T Entropy $H = \sum_{i=1}^{m} P(x_i) \log_2 \frac{1}{P(x_i)}$ $\quad \boxed{\because m=5}$

$$= 1/2 \cdot \log_2 2 + 1/6 \log_2 6 + 1/6 \log_2 6 + 1/12 \log_2 12 + 1/12 \log_2 12$$

$$= 1/2 + 0.430 + 0.43 + 0.298 + 0.298$$

$$\boxed{H = 1.956 \text{ bits/symbol}} \qquad L \neq H.$$

* Hence $L \neq H$ so that source coding theoram is not satisfied. The caluclation of efficiency and redundancy are not possible.

Source coding techniques (or) entropy coding:-

This coding is used to compress the binary data by taking the values of each symbol probability.

* If the symbol probability is more, then that symbol will be encoded with less no. of binary digits & if the symbol probability is less than that symbol will be encoded with more no. of binary digits.

* By this way the compression operation is possible by source coding techniques (or) entropy coding techniques.

* There are two types of source coding techniques:
    1. Shannon fano coding.
    2. Huffman coding.

* The Shannon fano coding have less efficiency as compared with Huffman coding.

## Shannon fano coding :-

This coding operation can be easily implemented with the help of Shannon fano ~~code~~ algorithm.

## Shannon fano algorithm :-

**Step 1:-** List all the source symbols probabilities and in the order of decreasing.

**Step 2:-** Divide the probablities in to two sets such a way first set is approximately equals to second set.

**Step 3:-** Assign 0's for upper set elements & assign 1's for lower set elements.

**Step 4:-** Continue this process upto the end of symbol probabilities.

1). A DMS source having 6 symbols with 6 possible probabilities given in below table. obtain the code for each symbol using shonnon fano coding & caluclate the efficiency & redundancy of the codes.

| $x_i$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ |
|---|---|---|---|---|---|---|
| $P(x_i)$ | 0.30 | 0.20 | 0.12 | 0.08 | 0.25 | 0.05 |

**Sol:-** Shannon fano coding :-

| $x_i$ | $P(x_i)$ | Step-1 | Step-2 | Step-3 | Step-4 | encoded o/p | Code word length $n_i$ |
|---|---|---|---|---|---|---|---|
| $x_1$ | 0.30 | 0 | 0 | | | 00 | 2 |
| $x_5$ | 0.25 | 0 | 1 | | | 01 | 2 |
| $x_2$ | 0.20 | 1 | 0 | | | 10 | 2 |
| $x_5$ | 0.12 | 1 | 1 | 0 | | 110 | 3 |
| $x_4$ | 0.08 | 1 | 1 | 1 | 0 | 1110 | 4 |
| $x_6$ | 0.05 | 1 | 1 | 1 | 1 | 1111 | 4 |

W.K.T

$$H(x) = \sum_{i=1}^{m=6} P(x_i) \cdot \log_2 \frac{1}{P(x_i)}$$

$$= P(x_1)\log_2\frac{1}{P(x_1)} + P(x_2)\log_2\frac{1}{P(x_2)} + P(x_3)\cdot\log_2\frac{1}{P(x_3)} + P(x_5)\log_2\frac{1}{P(x_5)} + P(x_6)\cdot\log_2\frac{1}{P(x_6)} + P(x_4)\cdot\log_2\frac{1}{P(x_4)}$$

$= 0.30\log_2\frac{1}{0.3} + 0.25\log_2\frac{1}{0.25} + 0.2\log_2\frac{1}{0.2} + 0.12\log_2\frac{1}{0.12} + 0.08\log_2\frac{1}{0.08} + 0.05\log_2\frac{1}{0.05}$

$= 0.52 + 0.15 + 0.46 + 0.36 + 0.29 + 0.21$

$\boxed{H(x) = 2.34 \text{ bits/symbol.}}$

$L = P(x_1)\cdot n_1 + P(x_2)\cdot n_2 + P(x_3)\cdot n_3 + P(x_4)\cdot n_4 + P(x_5)\cdot n_5 + P(x_6)\cdot n_6$

$= 0.3\times 2 + 0.25\times 2 + 0.20\times 2 + 0.12\times 3 + 0.08\times 4 + 0.05\times 4$

$= 0.6 + 0.5 + 0.4 + 0.36 + 0.32 + 0.2$

$\boxed{L = 2.38 \text{ bits/symbol}}$

Here $L \geq H(x)$ so that source coding theoram is Satisfied.

∴ Code efficiency $\eta = \frac{H(x)}{L}$

$\eta = \frac{2.34}{2.38} = 0.983 \simeq 98.3\%$

∴ code redundancy $r = 1-\eta$ (or) $100-\eta\%$.

$r = 1 - 0.983$

$= 0.017$

$\boxed{r = 17.6\%}$

2). Given that the symbols of probabilities $x_i$ & $P(x_i)$

| $x_i$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|---|---|---|---|---|---|---|---|
| $P(x_i)$ | 0.4 | 0.12 | 0.08 | 0.08 | 0.2 | 0.08 | 0.04. |

Sol:-

| $x_i$ | $P(x_i)$ | step-1 | step-2 | step-3 | step-4 | encoded o/p | Code word length $n_i$ |
|---|---|---|---|---|---|---|---|
| $x_1$ | 0.4 | 0 | 0 | | | 00 | 2 |
| $x_5$ | 0.2 | 0 | 1 | | | 01 | 2 |
| $x_2$ | 0.12 | 1 | 0 | 0 | | 100 | 3 |
| $x_3$ | 0.08 | 1 | 0 | 1 | | 101 | 3 |
| $x_4$ | 0.08 | 1 | 1 | 0 | | 110 | 3 |
| $x_6$ | 0.08 | 1 | 1 | 1 | 0 | 1110 | 4 |
| $x_7$ | 0.04 | 1 | 1 | 1 | 1 | 1111 | 4 |

W.K.T $H(x) = \sum_{k=1}^{m=7} P(x_i)\log_2\frac{1}{P(x_i)}$

$= P(x_1)\log_2\frac{1}{P(x_1)} + P(x_2)\log_2\frac{1}{P(x_2)} + P(x_3)\cdot\log_2\frac{1}{P(x_3)} + P(x_4)\log_2\frac{1}{P(x_4)} + \ldots P(x_7)\log_2\frac{1}{P(x_7)}$

$= 0.4\log_2\frac{1}{0.4} + 0.2\log_2\frac{1}{0.2} + 0.12\log_2\frac{1}{0.12} + 3\left[0.08\log_2\frac{1}{0.08}\right] + 0.04\log_2\frac{1}{0.04}$

$= 0.528 + 0.464 + 0.367 + 0.874 + 0.185$

$$\boxed{H(x) = 2.418 \text{ bits/symbol}}$$

w.k.T

$$L = \sum_{i=1}^{m} P(x_i) \times n_i$$

$$= P(x_1) \cdot n_1 + P(x_2) \times n_2 + P(x_3) \times n_3 + P(x_4) \times n_4 + P(x_5) \times n_5 + P(x_6) \times n_6 + P(x_7) \times n_7$$

$$= 0.4 \times 2 + 0.2 \times 2 + 0.12 \times 3 + 0.08 \times 3 + 0.08 \times 3 + 0.08 \times 4 + 0.04 \times 4$$

$$\boxed{L = 2.52 \text{ bits/symbol}}$$

∴ From above we can say that $L \geq H(x)$ Hence the Source code theoram is satisfied.

w.k.T efficiency: $\eta = \frac{H(x)}{L}$

$$\eta = \frac{2.418}{2.52}$$

$$\eta = 0.95$$

$$\therefore \boxed{\eta = 95\%}$$

Code redundancy $r = 1 - \eta = 1 - 0.95$

$$r = 0.05$$

$$\boxed{r = 5\%}$$

## Huffman coding :-

* This coding is very efficient technique as compared to shannon fano coding and we can obtain high efficiency in the encoding process.

* To obtain Huffman coding, there is a simple procedure is exist, which is known as Huffman algorithm.

### Huffman algorithm :-

1. List the source symbols in the order of decreasing probability.
2. Combine the probabilities of two symbols, which are having lowest probability values and reorder the resultant probabilities in the decreasing process, this step is called as Reduction. The same procedure is repeated until there are two probabilities remaining.
3. Start the encoding with last Reduction, which consists of exactly two ordered probabilities. Assign '0' for upper probability and assign '1' for lower probability.
4. Now, go back and assign '0' and assign '1' w.r.t base probabilities.
5. Keep progressing this way until the first column is reached.

Pb) :- i). The DMS source emits five symbols with the following probabilities. listed in below table. Encode the symbols using Huffman coding and calculate efficiency & Redundancy of the code.

| $x_i$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|---|---|---|---|---|---|
| $P(x_i)$ | 0.4 | 0.15 | 0.16 | 0.19 | 0.10 |

Sol: Huffman coding:-

| $x_i$ | $P(x_i)$ | Encoded | Step-1 | | Step-3 | | Step 4 | | Encoded | $n_i$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_1$ | 0.4 | 1 | 0.4 | 1 | 0.4 | 1 | 0.6 | 0 | 1 | 1 |
| $x_4$ | 0.19 | 000 | 0.25 | 01 | 0.35 | 00 | 0.4 | 1 | 000 | 3 |
| $x_3$ | 0.16 | 001 | 0.19 | 000 | 0.25 | 01 | | | 001 | 3 |
| $x_2$ | 0.15 | 010 | 0.16 | 001 | | | | | 010 | 3 |
| $x_5$ | 0.10 | 011 | | | | | | | 011 | 3 |

$$\log_2\left(1/p_k\right) = \frac{\log_{10}(1/p_k)}{\log_{10} 2} = \frac{0.397}{0.301}$$

w.k.T

$$H(x) = \sum_{i=1}^{m} P(x_i)\log_2\frac{1}{P(x_i)}$$

$$H(x) = (0.4)\log_2\left(\frac{1}{0.4}\right) + (0.19)\log_2\left(\frac{1}{0.19}\right) + (0.16)\log_2\left(\frac{1}{0.16}\right) + 0.15\log_2\left(\frac{1}{0.15}\right) + 0.10\log_2\left(\frac{1}{0.10}\right)$$

$$= 0.528 + 0.455 + 0.423 + 0.410 + 0.332$$

$$\boxed{H(x) = 2.148} \text{ bits/symbol}$$

w.k.T

$$L = \sum_{i=1}^{m} P(x_i) \times n_i$$

$$= P(x_1)\times n_1 + P(x_2)\times n_2 + P(x_3)\times n_3 + P(x_4)\times n_4 + P(x_5)\times n_5$$

$$= 0.4\times 1 + 0.19\times 3 + 0.16\times 3 + 0.15\times 3 + 0.1\times 3$$

$$\boxed{L = 2.2} \text{ bits/symbol}$$

∴ Hence $L \geq H(x)$ source code theoram is

verified.

$$\eta = \frac{H(x)}{L} = \frac{2.148}{2.2} = 0.976$$

$$\boxed{\eta = 97.6\%}$$

w.k.T

$$\gamma = 1-\eta = 1-0.976$$

$$\gamma = 0.024$$

$$\boxed{\gamma = 2.4\%}$$

Code variance: $(\sigma^2)$

$$\sigma^2 = \sum_{i=1}^{m} p_i(n_i - L)^2$$

$$\sigma^2 = 0.4(1-2.2)^2 + 0.19(3-2.2)^2 + 0.16(3-2.2)^2 + 0.15(3-2.2)^2 + 0.1(3-2.2)^2$$

$$\sigma^2 = 0.576 + 0.121 + 0.102 + 0.096 + 0.064$$

$$\boxed{\sigma^2 = 0.959}$$

The theoram states that channel capacity is always greater than (or) equal to Information rate i.e "$C \geq R$" to obtain a loss less Transmission line.

* The units for channel capacity is "bits per second" (or) "bps".
* Before taking the relation b/w 'R' & 'c' we have to caluclate channel capacity. The channel capacity can be calculated with the help of bandwidth and signal to noise ratio.

which is expressed as:

$$\boxed{C = B\log_2\left[1+\frac{S}{N}\right]}$$

B: Bandwidth.
S/N: Signal to noise ratio

* The theoram states the expression b/w 'c' and 'R', which is given as:

$$C \geq R.$$

* The relation trade of b/w bandwidth and signal to noise ratio is always directly proportional to each other:

$$C \propto B$$

$$C \propto S/N.$$

$$\boxed{B \propto S/N}$$

Pb).1) A voice channel of a telephone N/w has a B.w of 3.4KHz. Caluclate the channel capacity of the telephone N/w for a S/N of 30 dB.

Sol: Given: B.w = 3.4KHz = $3.4 \times 10^3$ Hz.

Channelcapacity = ? ; $(S/N)_{in\,dB}$ = 30dB.

* The S/N ratio in dB can be converted to normal value as

$$\left(\frac{S}{N}\right)_{indB} = 10\log_{10}(S/N)$$

$$30^3 = 10\log_{10}(S/N)$$

$$\boxed{S/N = 10^3 = 1000}$$

w.k.T

$$C = B\log_2\left[1+\frac{S}{N}\right] = 3.4\times10^3 \cdot \left[\log_2\left[1+1000\right]\right]$$

$$C = 33888.5 \qquad 1.9$$

$$\boxed{C = 33.88\text{ kbps}}$$

$\frac{3.0103}{0.3010}$ $\log_2(1001)$ $\frac{\log_{10} 2}{0.3010}$

Give an Additive white Gaussian noise (AWGN) channel with 4KHz Band width and noise power spectral density of $\eta/2 = 10^{-12}$ watts/Hz. The Signal power required at the reciever is 0.1 milliwatt. Calculate the channel capacity.

Sol: Given: $B.W\ (B) = 4KHz = 4\times10^3\ Hz$

Signal power $(S) = 0.1 \times 10^{-3}$ watt

Note: The noise power of the Additive white Gaussian noise channel (AWGN) is related with power spectral density and Band width of the channel which is given as:

$\boxed{N = \eta B}$ . $\boxed{N = \eta \times B}$ $S = 0.1\times10^{-3}$

Given $\eta/2 = 10^{-12}$ watts/Hz

$\eta = (10^{-12}\times2)$ watts/Hz.

$\therefore$ Noise power $(N) = 2\times10^{-12} \times 4\times10^{3}$

$N = 8\times10^{-9}$.

$\therefore \frac{S}{N} = \frac{0.1\times10^{-3}}{8\times10^{-9}} = 12500$

$\boxed{\frac{S}{N} = 12500}$

$\therefore$ w.k.T channel capacity $\boxed{C = B\log_2\left(1+\frac{S}{N}\right)}$

$C = 4\times10^3 \log_2[1+12500]$

$C = 54.439$.

$\boxed{C = 54.4\ kbps}$

3). An anlog signal having 4 KHz Bandwidth is sampled at 1.25 times of the Nyquist rate and each sample is quantized in to 256 of the equally likely levels. Assume that the successful samples are statistically independent.

What is i) the information rate of source.

ii). Can the o/p of the source be transmitted without error over an AWGN with the B.W of 10 KHz and S/N ratio is 20 dB

iii) Find the S/N ratio required for error free transmission free transmission of the o/p of the source, if S/N is 20dB

Sol: Given:

$B.W = 4kHz$. is can be taken from B.W

∴ The maximum frequency $(f_m) = 4kHz$.

$$B.W = f_m = 4kHz = 4 \times 10^3 \text{ Hz}.$$

Nyquist rate $(f_s) = 2f_m$

$$f_s = 2(4 \times 10^3) = 8 \times 10^3 \text{ Hz}.$$

Sampled rate $(r) = 1.25 f_s$

$$= 1.25 \times 8 \times 10^3$$

$$\boxed{r = 10^4 \text{ sym/sec.}}$$

$$\boxed{r = 10{,}000 \text{ symbol/sec.}}$$

The Quantized levels $M = 256. = 2^8$.

W.K.T $M = 2^n$

Entropy $H = \log_2 M$

$$H = \log_2 2^8$$

$$\boxed{H = 8 \text{ bits/msg}}$$

i). W.K.T Information rate (R)

$$R = r \times H$$

$$= 10{,}000 \times 8$$

$$= 80{,}000$$

$$\boxed{R = 80 \text{ kbps}}$$

ii) Given: $B.W = 10kHz$

$$\left(\frac{S}{N}\right) = 20dB$$

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}(S/N)$$

$$20 = 10 \log_{10}(S/N)$$

$$\frac{S}{N} = 10^2$$

$$\boxed{\frac{S}{N} = 100}$$

W.K.T channel capacity

$$C = B \log_2 \left[ 1 + \frac{S}{N} \right]$$

$$= 10^4 \log_2 [1 + 100]$$

$$C = 10^4 \log_2 101 = 66.582 \text{ kbps}$$

According to channel capacity theoram $C \geq R$

$$\Rightarrow \boxed{C \ngeq R}$$

Here the condition is not satisfied so that error free transmission is not possible.

iii) According to, Shannon-hartley channel capacity theoram

$$\boxed{C \geq R.}$$

W.K.T $C = B \log_2 \left[ 1 + \frac{S}{N} \right]$

$$\Rightarrow B \log_2 \left[ 1 + \frac{S}{N} \right] \geq R$$

$$10^4 \log_2 \left[ 1 + \frac{S}{N} \right] \geq 80000$$

$$\log_2 \left[ 1 + \frac{S}{N} \right] \geq 8$$

$$1 + \frac{S}{N} \geq 2^8$$

$$1 + \frac{S}{N} \geq 256$$

$$\boxed{\frac{S}{N} \geq 255.}$$

In dB :-

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10} \left(\frac{S}{N}\right)$$

$$= 10 \log_{10} (255)$$

$$\boxed{\left(\frac{S}{N}\right)_{dB} = 24.06 \text{ dB}}$$

iv) B.W = ? ; $\left(\frac{S}{N}\right)_{dB} = 20 \text{ dB} \Rightarrow \frac{S}{N} = 100.$

$$C = B \log_2 \left[ 1 + \frac{S}{N} \right] \qquad C \geq R$$

$$\Rightarrow B \log_2 [1 + S/N] \geq R$$

$$B \log_2 [1 + 100] \geq 80 \times 10^3$$

$B \log_2 101 \geq 80 \times 10^3$.

$B \geq \frac{80 \times 10^4}{6.9188}$.

$\boxed{B \geq 12 \text{ kHz}}$

Code Varience :- $(\sigma^2)$ :-

It is defined as the difference b/w one code word to another code word in the generated binary sequence of source coding methods.

* which is caluclated by using:

$$\boxed{\sigma^2 = \sum_{i=1}^{m} P(x_i) [n_i - L]^2}$$