SOURCE COODING UNIT-V

Coding offers the most significant application of the information theory. The main purpose of coding is to improve the efficiency of the communication system in some sense.

Coding is a procedure for mapping a given set of messages $[m_1, m_2 \cdots m_N]$ into a new set of encoded messages $[c_1 c_2 \cdots c_N]$ in such a way that the transformation is one-to-one. i.e for each message, there is only one encoded message. This is "source coding".

In this, the code words of a code dictionary are of different lengths and the efforts are directed towards getting minimum average length of code, resulting in increased efficiency of transmission.

It is possible to devise codes for a special purpose without relevance to the efficiency of transmission. This is "channel coding"

It is also possible to resort to codes that do not have a one-to-one association. However, in this chapter we will study only the one-to-one codes, which

(i) improve some sort of transmission efficiency
(ii) reduce probability of error by detecting and correcting errors.

The following terminology is associated with coding:

(a) Letter, symbol, or character: Any individual member of the alphabet set.

(b) Message (or) word: ~~The numbers of letters in a message~~ A finite sequence of letters of an alphabet.

(c) length of the word: The number of letters in a message.

(d) coding, encoding or enciphering: A procedure for associating words constructed from a finite alphabet of a language with the given words of another language in a one-to-one manner.

(e) decoding or deciphering: The inverse operation of assigning words of second language corresponding to the given words of the first language.

(f) uniquely decipherable, or separable encoding and decoding:
In this operation, the correspondence of all possible sequence of words between the two languages in one-to-one when there is no space between the words.

(g) Irreducibility or prefix property: when no encoded words can be obtained from each other by the addition of more letters, the code is said to be irreducible or of a prefix property.

* when a code is irreducible, it is also uniquely decipherable; but the vice versa is not true.

an addition of a 0, or a '1' to any of the codes does not produce other code words.

0110101000101101010 it is uniquely decoded as

$c_1 c_3 c_2 c_2 c_1 c_1 c_2 c_3 c_3 c_2$

→ An efficient source coding technique should satisfy two functional requirements

(1) The code word must be in binary form

(2) The code word is uniquely decodable

→ There are two types of coding techniques

(1) fixed length coding

(2) variable length coding.

In fixed length coding the codewords of a code dictionary are of same length.

In variable length coding the code words of a code dictionary are of different length. In variable length coding, if the probability of occurence of a message is high then it will be assign with a code word which is having less number of bits. If the probability of occurence of a message is low then it will assign with a code word which is having more number of bits.

There are some parameters of source coding

- Average codeword length
- Average data rate
- efficiency

Channel capacity:-

The Mutual information $I[X:Y]$ indicates a measure of the average information per symbol transmitted in the system.

A suitable measure for efficiency of transmission of information may be introduced by comparing the actual rate and upper bound of the rate of information transmission for a given c

According to Shannon concept, channel capacity is d as the maximum of mutual information.

∴ The channel capacity 'c' is given by

$$c = \max[I[X:Y]]$$

$$c = \max[H(X) - H(X/Y)].$$

* The transmission efficiency or channel efficiency is defined as

$$\eta = \frac{\text{Actual transinformation}}{\text{maximum transinformation}}$$

$$\boxed{\eta = \frac{I[X:Y]}{\max[I[X:Y]]} = \frac{I[X:Y]}{c}}$$

The redundancy of the channel is 'R'

$$\boxed{R = 1 - \eta}$$

* sometimes the unit of $I[X:Y]$ and 'c' is taken as "bits/sec".

Let M be the number of symbols in an encoding alphabet. Let there be N messages $m_1, m_2 --- m_N$ with the probabilities $P(m_1), P(m_2) --- P(m_N)$.

let n_i be the number of symbols in the i^{th} message. The average length of the message & the average length per code word is given by.

$$\bar{L} = \sum_{i=1}^{N} P(n_i) \cdot n_i \quad \text{letts/message.}$$

$\bar{L}$ should be minimum to have an efficient transmission.

∴ coding efficiency is given by

$$\eta = \frac{\bar{L}_{min}}{\bar{L}}$$

Let $H(x)$ be the entropy of the source in bits/message.

$\log M$ be the maximum average information associated with each letter in bits/letter.

∴ The minimum average number of letters per message is given by the ratio

$$\bar{L}_{min} = \frac{H(x)}{\log M}.$$

$$\therefore \text{coding efficiency } \eta = \frac{\bar{L}_{min}}{\bar{L}} = \frac{H(x)}{\bar{L} \log M}.$$

Redundancy $R = 1 - \eta$.

Example:-

Let us use a binary code for coding. Let the code letters be '0' and '1'. Thus M is '2'.

Message	code1	length of code 1	code2	length of code 2	probability
m_1	00	$n_1 = 2$	0	$n_1 = 1$	$P_1 = 1/2$
m_2	01	$n_2 = 2$	10	$n_2 = 2$	$P_2 = 1/4$
m_3	10	$n_3 = 2$	110	$n_3 = 3$	$P_3 = 1/8$
m_4	11	$n_4 = 2$	111	$n_4 = 3$.	$P_4 = 1/8$.

for code 1:-

$$\bar{L} = \sum_{i=1}^{N} n_i \, p(n_i)$$

$$= \sum_{i=1}^{4} n_i \, p(n_i) = n_1 p(n_1) + n_2 p(n_2) + n_3 p(n_3) + n_4 p(n_4)$$

$$= 2(1/2) + 2(1/4) + 2(1/8) + 2(1/8)$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1 + \frac{1}{2} + \frac{1}{2} = 1+1 = 2 \text{ letters/message.}$$

$$H(X) = -\sum_{k=1}^{4} P_k \log_2 P_k$$

$$= -\left[P_1 \log P_1 + P_2 \log P_2 + P_3 \log P_3 + P_4 \log P_4\right]$$

$$= -\left[1/2 \log 1/2 + 1/4 \log 1/4 + 1/8 \log 1/8 + 1/8 \log 1/8\right].$$

$$= 1/2 + 1/2 + 3/4 = 7/4 \text{ bits/message.}$$

$$\eta = \frac{H(X)}{\bar{L} \log M} = \frac{7/4}{2 \log 2} = 7/8 = 87.5\ \%$$

$$\bar{L} = \sum_{i=1}^{N} n_i p(n_i) = n_1 p(n_1) + n_2 p(n_2) + n_3 p(n_3) + n_4 p(n_4)$$

$$= 1(1/2) + 2(1/4) + 3(1/8) + 3(1/8)$$

$$= 1/2 + 1/2 + \frac{3}{8} + \frac{3}{8} = 1 + \frac{6}{8} = \frac{14}{8} = 7/4 \text{ letters/message.}$$

$$\therefore \eta = \frac{H(X)}{\bar{L}\log_2 M} = \frac{7/4}{7/4 \log_2 2} = 100\%.$$

$\therefore$ we can say that the second coding technique is better as compared to the first.

Shannon - Fano Coding:-

An efficient coding technique can be obtained by the following procedure is known as shannon-fano algorithm.

- → List out the source symbols in order of decreasing probability.
- → partition the symbols set into two sets that are as close to equiprobable as possible (two equal probability sets). and assign "zero" to the upper set and "one" to the lower set.
- → continue this process, each time, partition the sets with as nearly equal probabilities as possible until further partitioning is not possible.

① obtain the shannon-fano coding for the discrete memoryless source 'x' it has 4 symbols x_1, x_2, x_3 and x_4 with probabilities $1/2, 1/4, 1/8$ and $1/8$ respectively and show that the coding efficiency is 100% and find Redundancy.

Sol.

symbol	probability	code	length of the code
x_1	1/2	0	1
x_2	1/4	1 0	2
x_3	1/8	1 1 0	3
x_4	1/8	1 1 1	3

$$\bar{L} = \sum_{i=1}^{N} n_i p(n_i) = 1(1/2) + 2(1/4) + 2(3)(1/8)$$

$$= 1.75 \text{ letters/symbol.}$$

$$H = -\sum_{K=1}^{4} P_K \log_2 P_K = -\left[1/2 \log 1/2 + 1/4 \log 1/4 + 2(1/8) \log 1/8\right]$$

$$= 1.75 \text{ bits/message.}$$

$$\therefore \eta = \frac{H(x)}{\bar{L} \log M} = \frac{1.75}{1.75 \log_2 2} = 100\%.$$

$\Rightarrow$ redundancy $= 1 - \eta = 1 - 1 = 0$.

Source x it has 8 symbols x_1 to x_8 with probabilities 1/2, 1/8, 1/8, 1/16, 1/16, 1/16, 1/32, 1/32 respectively. find efficiency of the code.

Sol:-

Symbol	probability	code	length of the code
x_1	1/2	0	1
x_2	1/8	1 0 0	3
x_3	1/8	1 0 1	3
x_4	1/16	1 1 0 0	4
x_5	1/16	1 1 0 1	4
x_6	1/16	1 1 1 0	4
x_7	1/32	1 1 1 1 0	5
x_8	1/32	1 1 1 1 1	5

$$\bar{L} = \sum_{i=1}^{N} n_i p(n_i) = \sum_{i=1}^{8} n_i p(n_i)$$

$$= 1(1/2) + 3(1/8) + 3(1/8) + 4(1/16) + 4(1/16) + 4(1/16) + 5(1/32) + 5(1/32)$$

$$= \frac{1}{2} + \frac{3}{4} + \frac{3}{4} + \frac{5}{16} = \frac{1}{2} + \frac{3}{2} + \frac{5}{16} = 2.312 \text{ letters/message}$$

$$H = -\sum_{k=1}^{8} P_k \log_2 P_k$$

$$= -\left[1/2 \log 1/2 + 1/8 \log 1/8 + 1/8 \log 1/8 + 1/16 \log 1/16 + 1/16 \log 1/16 + 1/16 \log 1/16 + 1/32 \log 1/32 + 1/32 \log 1/32 \right]$$

$$= 2.312 \text{ bits/message.}$$

$$\therefore \eta = \frac{H(x)}{\bar{L} \log_2 M} = \frac{2.312}{2.312 (\log_2 2)} = 100\%.$$

③ Apply the shannon-Fano coding procedure for the following message ensemble:

$$[X] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8]$$

$$[P] = [1/4 \quad 1/8 \quad 1/16 \quad 1/16 \quad 1/16 \quad 1/4 \quad 1/16 \quad 1/8]$$

Take M=2.

sol:-

Message	probability	Encoded message	length (n_i)
x_1	1/4	0 0	2
x_6	1/4	0 1	2
x_2	1/8	1 0 0	3
x_8	1/8	1 0 1	3
x_3	1/16	1 1 0 0	4
x_4	1/16	1 1 0 1	4
x_5	1/16	1 1 1 0	4
x_7	1/16	1 1 1 1	4

$$\bar{L} = \sum_{i=1}^{8} n_i P_i = n_1P_1 + n_2P_2 + n_3P_3 + n_4P_4 + n_5P_5 + n_6P_6 + n_7P_7 + n_8P_8$$

$$= \left(2\times 1/4\right) + \left[3\times 1/8\right] + \left[4\times\tfrac{1}{16}\right] + \left[4\times 1/16\right] + \left[4\times 1/16\right] + \left[2\times 1/4\right]$$

$$+ \left(4\times\tfrac{1}{16}\right) + \left[3\times\tfrac{1}{8}\right]$$

$$= \frac{1}{2} + \frac{3}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{2} + \frac{1}{4} + \frac{3}{8}$$

$$\bar{L} = 2 + \frac{3}{4} = 2.75 \text{ letters/message}$$

$$H(X) = -\sum_{k=1}^{8} P_k \log_2 P_k = 2.75 \text{ letters/message.}$$

ensemble

$[X] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7]$

$[P] = [0.4 \quad 0.2 \quad 0.12 \quad 0.08 \quad 0.08 \quad 0.08 \quad 0.04]$.

Take $M=2$.

sol. case i

Message	probability	Encoded message	Length
x_1	0.4	0 0	2
x_2	0.2	0 1	2
x_3	0.12	1 0 0	3
x_4	0.08	1 0 1	3
x_5	0.08	1 1 0	3
x_6	0.08	1 1 1 0	4
x_7	0.04	1 1 1 1	4

$$\bar{L} = \sum_{i=1}^{7} n_i P_i = (2\times0.4) + (2\times0.2) + (3\times0.12) + (3\times0.08) + (3\times0.08) + 4(0.08) + 4(0.04)$$

$= 2.52$ letters/message

case ii:-

message	probability	Encoded message	length
x_1	0.4	0	1
x_2	0.2	1 0 0	3
x_3	0.12	1 0 1	3
x_4	0.08	1 1 0 0	4
x_5	0.08	1 1 0 1	4
x_6	0.08	1 1 1 0	4
x_7	0.04	1 1 1 1	4

$\bar{L} = 2.48$ letters/message.

Thus, it can be seen that the second method is better as it gives a lower value for $\bar{L}$.

$$H(X) = -\sum_{K=1}^{7} P_K \log P_K = 2.42 \text{ bits/message.}$$

$$\therefore \eta = \frac{H(X)}{\bar{L} \log_2 M} = \frac{2.42}{2.48 \log_2 2} = 97.6\%.$$

The above example shows that sometimes shannon-Fano metho
is ambiguous. The ambiguity arises due to the availability of more th
one equally valid schemes of partitioning resulting in ambiguity.
Moreover, as M is increased, this method is not suitable and the
formation of M approximately equiprobable groups is rather difficult.

⑤ Apply the shannon-Fano coding procedure for the following message ensemble

$$[X] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7]$$

$$[P] = [0.4 \quad 0.2 \quad 0.12 \quad 0.08 \quad 0.08 \quad 0.08 \quad 0.04] \qquad \text{Take } M = 3.$$

sol.

Message	probability	Encoded message	length.
x_1	0.4	0	1
x_2	0.2	1 0	2
x_3	0.12	1 1	2
x_4	0.08	2 0	2
x_5	0.08	2 1	2
x_6	0.08	2 2 0	3
x_7	0.04	2 2 1	3.

$$\bar{L} = \sum_{K=1}^{7} P_K n_K = (0.4\times1) + (0.2\times2) + (0.12\times2) + (0.08\times2) + (0.08\times2) + (0.08\times3) + (0.04\times3))$$

$$= 1.72 \text{ letters/message.}$$

$$\eta = \frac{H(X)}{\bar{L} \log_2 M} = \frac{2.42}{1.72 \times \log_2 3} = 88.7\%.$$

Another type of source code is Huffman code which leads to the lowest possible value of $\bar{L}$ for a given M, resulting in a "maximum efficiency" or "minimum redundancy". Hence it is also known as the "minimum redundancy" or "optimum code".

The procedure to find Huffman code is as follows.

→ Arrange the given messages in an order of decreasing probability.

→ Combine the last two probabilities which results in new probability, then again arrange all the probabilities in decreasing order. again repeat the same step until we get only two probabilities.

→ Assign '1' to the first probability and '0' to the second probability for the last two probabilities. and move back and apply the same process to get the code words for each message.

) Solve the following example by Huffman method

$$[X] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7]$$

$$[P] = [0.4 \quad 0.2 \quad 0.12 \quad 0.08 \quad 0.08 \quad 0.08 \quad 0.04]$$

Sol. Take $M=2$.

Message	probability	first reduction	second reduction	Third reduction	fourth reduction	fifth reduction.
x_1	0.4 (0)	0.4 (0)	0.4 (0)	0.4 (0)	0.4 (0)	0.6 (1)
x_2	0.2 (111)	0.2 (111)	0.2 (111)	0.24 (10)	0.36 11	0.4 (0)
x_3	0.12 (101)	0.12 (101)	0.16 (110)	0.20 111	0.24 10	
x_4	0.08 1101	0.12 (100)	0.12 101	0.16 110		
x_5	0.08 1100		0.12 100			
x_6	0.08 1001	0.08 1101				
x_7	0.04 1000	0.08 1100				

code length	length code
$c_1 = 1$	0
$c_2 = 3$	111
$c_3 = 3$	101
$c_4 = 4$	1101
$c_5 = 4$	1100
$c_6 = 4$	1001
$c_7 = 4$	1000

$$\bar{L} = \sum_{i=1}^{N} n_i P_i \text{ letters/message}$$

$$= \sum_{i=1}^{7} n_i p_i = 1(0.4) + 3(0.2) + 3(0.12) + 4(0.08) + 4(0.08) + 4(0.08) + 4(0.04) = 2.48 \text{ letters/message}$$

... letters/message.

Message	probability	first reduction	second reduction
x_1	0.4 (2)	0.4 (2)	0.4 (2)
x_2	0.2 (0)	0.2 (0)	0.4 (1)
x_3	0.12 (11)	0.2 12	0.2 (0)
x_4	0.08 (10)	0.12 11	
x_5	0.08 122	0.08 10	
x_6	0.08 121		
x_7	0.04 120		

code word	codeword length
$c_1 = 2$	$n_1 = 1$
$c_2 = 0$	$n_2 = 1$
$c_3 = 11$	$n_3 = 2$
$c_4 = 10$	$n_4 = 2$
$c_5 = 122$	$n_5 = 3$
$c_6 = 121$	$n_6 = 3$
$c_7 = 120$	$n_7 = 3$.

$$\bar{L} = \sum_{i=1}^{7} p_i n_i = 1.6 \text{ letters/message.}$$

$$\eta = \frac{H(X)}{\bar{L}\log_2 M} = \frac{2.42}{1.6 \log_2 3} = 95.4\%.$$

⑧ Encode ~~the~~ following for $M=4$.

$$[X] = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8]$$

$$[P(x)] = [0.2 \quad 0.2 \quad 0.15 \quad 0.15 \quad 0.1 \quad 0.1 \quad 0.05 \quad 0.05]$$

Sol:- case i:-

message	probability	first reduction	second reduction
x_1	0.2 13	0.3 (0)	0.7 (1)
x_2	0.2 12	0.2 13	0.3 (0)
x_3	0.15 11	0.2 12	
x_4	0.15 10	0.15 11	
x_5	0.1 03	0.15 10	
x_6	0.1 02		
x_7	0.05 01		
x_8	0.05 00		

codeword	length
13	2
12	2
11	2
10	2
03	2
02	2
01	2
00	2

$$\bar{L} = \sum_{i=1}^{8} P_i n_i = 2 \text{ letters/message}$$

$$H(X) = -\sum_{K=1}^{8} P_K \log_2 P_K =$$

$$\eta = \frac{H(X)}{\bar{L} \log_2 M} \quad \text{where } M=4.$$

Message	I	II reduction		reduction
x_1	0.2 (1)	0.2 (1)		0.45 (0)
x_2	0.2 (2)	0.2 (2)		0.2 (1)
x_3	0.15 (3)	0.15 (3)		0.2 (2)
x_4	0.15 (00)	0.15	00	0.15 (3)
x_5	0.1 (01)	0.1	01	
x_6	0.1 (02)	0.1	02	
x_7	0.05 (030)	0.1	03	
x_8	0.05 (031)			

codeword	length of codeword
$c_1 = 1$	$n_1 = 1$
$c_2 = 2$	$n_2 = 1$
$c_3 = 3$	$n_3 = 1$
$c_4 = 00$	$n_4 = 2$
$c_5 = 01$	$n_5 = 2$
$c_6 = 02$	$n_6 = 2$
$c_7 = 030$	$n_7 = 3$
$c_8 = 031$	$n_8 = 3$

$$\bar{L} = \sum_{i=1}^{8} n_i P_i = (0.2\times1) + (0.2\times1) + (0.15\times1) + (0.15\times2) + (0.1\times2) + (0.1\times2) + (0.05\times3) + (0.05\times3).$$

$$= 1.55 \text{ letters/message.}$$

$\bar{L}$ in first case is greater than $\bar{L}$ in second case

$\therefore$ case ii is advantageous.

Thus, it should be ensured that the number of terms to be combined in the last reduction should be equal to M. The number of terms to be combined in the first reduction shoo be decided to ensure M terms in the last reduction which comes out to be equal to [N-K[M-1]] where K is the highest integer that gives a value greater than 1 to the expression [N-K[M-1)]. It may be noted that all intermediate reductions combine exactly M terms.

Shannon's theorem:-

Shannon's theorem is fundamental to the theory of communications. It is concerned with the rate of transmission of information over a communication channel.

Shannon's theorem says that it is possible, in principle, to derive a means where by a communications system will transmit information with an arbitrarily small probability of error provided that the information rate 'R' is less than or equal to a rate "c" called the channel capacity.

" Given a source of M equally likely messages with M>>1, which is generating information at a rate R'. Given a channel with the channel capacity c" Then if

$$\boxed{R \leq C}$$

there exists a coding technique such that the output of the source may be transmitted over the channel with a probability of error in the received message which may be made arbitrarily small.

A Number of communication signals use continuous sources and use the channel continuously.

Ex:- AM, FM.

The information theory concept of discrete channels can be extended to continuous channel. The definitions of different entropies in the discrete case were based on the concept of different averages.

In a similar way we may define different entropies in the case of continuous distributions.

If $p(x)$ is the probability density function associated with the signal $x(t)$ Then the entropy of source is

$$H(x) = -\int_{-\infty}^{\infty} p(x) \log_2 p(x)\, dx$$

In a similar way the different entropies associated with two dimensional random variables with a joint probability density $P(xy)$ & marginal densities $P_1(x)$ & $P_2(y)$ may be defined as

$$H(X) = -\int_{-\infty}^{\infty} p(x) \log_2 p(x)\, dx$$

$$H(y) = -\int_{-\infty}^{\infty} p(y) \log_2 p(y)\, dy$$

$$H(x\,y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\,y) \log_2 p(x\,y)\, dx\,dy.$$

$$H(x/y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\,y) \log_2 p(x/y)\, dx\,dy.$$

$$H(y/x) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\,y) \log_2 p(y/x)\, dx\,dy.$$

$$\int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\ y)\,dx\,dy = 1.$$

<u>Mutual Information:-</u>

$$I[x:y] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x;y) \log_2\left[\frac{p(x|y)}{p(x)}\right] dx\,dy$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\ y) \log_2\left[\frac{p(x)}{p(x|y)}\right] dx\,dy$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\ y) \log_2^{z}\,dx\,dy.$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\ y)\ \frac{\log z}{\log 2}\cdot\frac{\log e}{\log e}\,dx\,dy.$$

$$I(x:y) = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x\ \ y)\ \log_e^{z}\ \log_2^{e}\,dx\,dy.$$

$$\left[\log_e^{z} = -1 + \frac{z}{1!} + \frac{z^2}{2!} + ----\right]$$

$$I[x:y] = -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x;y)\ (z-1)\ \log_2^{e}\,dx\,dy$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\left[\frac{p(x)}{p(x|y)}-1\right]\log_2^{e}\,dx\,dy$$

$$= -\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{p(x;y)\,p(x)}{p(x|y)}\log_2^{e}\,dx\,dy + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p(x,y)\log_2^{e}\,dx\,d$$

$-\infty \quad -\infty$

$$I[x;y] \geq -\log_2^e + \log_2^e$$

$$\boxed{I[x;y] \geq 0}$$

Hence the transinformation of a continuous system is non-negative.

Capacity of Gaussian channel:-

A Theorem which is complementary to shannon's theorem and applies to a channel in which the noise is gaussian is known as the shannon-Hartely theorem.

The channel capacity of a white, bandlimited gaussian channel

$$C = B \log_2 \left[1 + \frac{S}{N}\right] \text{ bits/sec.} \rightarrow (1)$$

Where B = Channel Bandwidth

S = Signal power

N = total Noise within the channel Bandwidth

$N = \eta B$ with $\eta/2$ power spectral density (two-sided).

A particular encoder-decoder is used with a gaussian channel and an error probability Pe results, then with a non-gaussian channel another encoder-decoder can be designed so that the pe will be smaller. channel capacity equations can be derived for a number of non-gaussian channels, as follows.

For the purpose of transmission over the channel, the messages are represented by fixed voltage levels. source generates one message after another in sequence, the transmitted signal s(t) takes a waveform as shown in below figure.

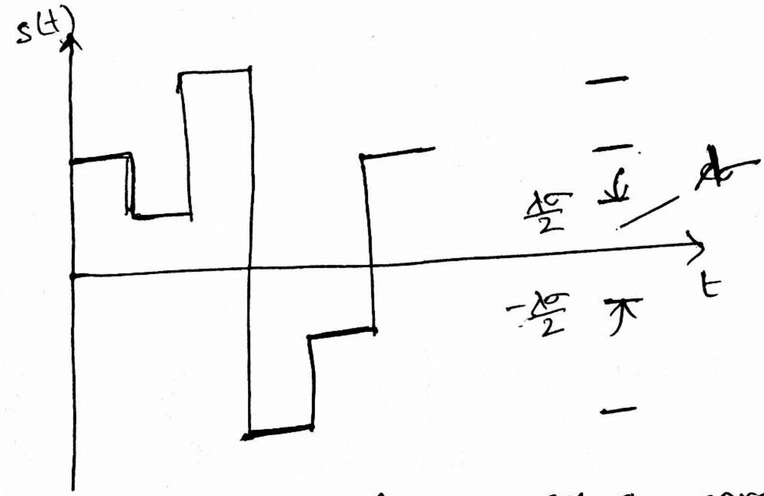

Fig:- A sequence of messages is represented by waveform s(t) which assumes voltage levels corresponding to the messages.

The received signal is accompanied by noise whose root-mean-square voltage is "σ". The levels have been separated by an interval $\lambda\sigma$.

Where λ is a number assumed large enough to allow recognition of individual levels with an acceptable probability of error.

Assuming an even number of levels, the levels are located at voltages $\pm\lambda\sigma/2$, $\pm 3\lambda\sigma/2$, etc;

If there are M possible messages, then there must be M levels.

The average signal power is

$$S = \frac{2}{M}\left\{\left(\frac{\lambda\sigma}{2}\right)^2 + \left(\frac{3\lambda\sigma}{2}\right)^2 + \cdots + \left[\frac{(M-1)\lambda\sigma}{2}\right]^2\right\}$$

$$S = \frac{M^2-1}{12}(\lambda\sigma)^2$$

The number of levels for a given average signal power is

$$M = \left[1 + \frac{12S}{\lambda^2\sigma^2}\right]^{1/2}$$

average amount of information.

$$H = \log_2 M = \log_2 \left[1 + \frac{12}{d^2}\frac{S}{N}\right]^{1/2}$$

$$H = 1/2 \log_2 \left[1 + \frac{12}{d^2}\cdot\frac{S}{N}\right] \text{ bits/message.}$$

and $T = \gamma = 0.5/B$

$\therefore \gamma = 1/T = 2B$ messages/sec.

$\therefore$ The information rate R is given by

$$R = \gamma H$$

$$= 2B \times \frac{1}{2} \log_2 \left[1 + \frac{12}{\lambda^2}\cdot\frac{S}{N}\right]$$

$$R = B\log_2 \left[1 + \frac{12}{\lambda^2}\cdot\frac{S}{N}\right] \rightarrow ②$$

comparing equation ② with equation ① of shannon-Hartley theorem, we observe that the result would be identical if we set

$\frac{12}{d^2} = 1$ that is $d = 3.5$.

Bandwidth, S/N Tradeoff:-

The shannon-Hartley theorem indicates that a Noiseless Gaussian channel ($S/N = \infty$) has an infinite capacity.

for a fixed signal power and in the presence of white gaussian noise the channel capacity approaches an upper limit with increasing bandwidth. Now calculate that limit

W.K.T $C = B\log_2 \left[1 + \frac{S}{N}\right]$

$$C = \frac{S}{\eta}\cdot\frac{\eta B}{S}\log_2\left(1 + \frac{S}{N}\right) = \frac{S}{\eta}\log_2\left[1 + \frac{S}{\eta B}\right]^{\frac{\eta B}{S}}$$

$(\because N = \eta B)$.

$$\left[\lim_{x\to 0}(1+x)^{1/x} = e\right]$$

$$\therefore C_\infty = \underset{B\to\infty}{Lt}\, C = \frac{S}{\eta}\log_2^e = 1.44\, S/\eta .$$

calculate the capacity of the low pass channel with the usable B.W of 3KHz & S/N is 10^3 at the channel o/p assume that the channel noise to Gaussian white.

Given $B.W = 3KHz$.

$$\frac{S}{N} = 10^3$$

$$C = B\log_2\left[1+\frac{S}{N}\right]$$

$$= 3\times10^3 \log_2\left[1+10^3\right]$$

$$C = 29.901\times10^3 \text{ bits/sec}.$$

→ Consider a source emitting 8 symbols with probabilities 0.25, 0.125, 0.0625, 0.0625, 0.0625, 0.25, 0.0625, 0.125 design a code using shannon-fano algorithm and find efficiency.

sol:

symbol	probability	code	length
x_1	0.25	0 0	2
x_2	0.25	0 1	2
x_3	0.125	1 0 0	3
x_4	0.125	1 0 1	3
x_5	0.0625	1 1 0 0	4
x_6	0.0625	1 1 0 1	4
x_7	0.0625	1 1 1 0	4
x_8	0.0625	1 1 1 1	4

$$\bar{L} = \sum_{K=1}^{8} P_K n_K = 2.75 \text{ letters}$$

$$H = -\sum_{K=1}^{8} P_K \log P_K$$

$$= 2.75 \text{ bits/message}$$

$$\therefore \eta = \frac{H(x)}{\bar{L}\log_2^2}$$

$$= 100\%$$