UNIT-1

Digital Pulse Modulation

Elements of Digital Communication Systems:

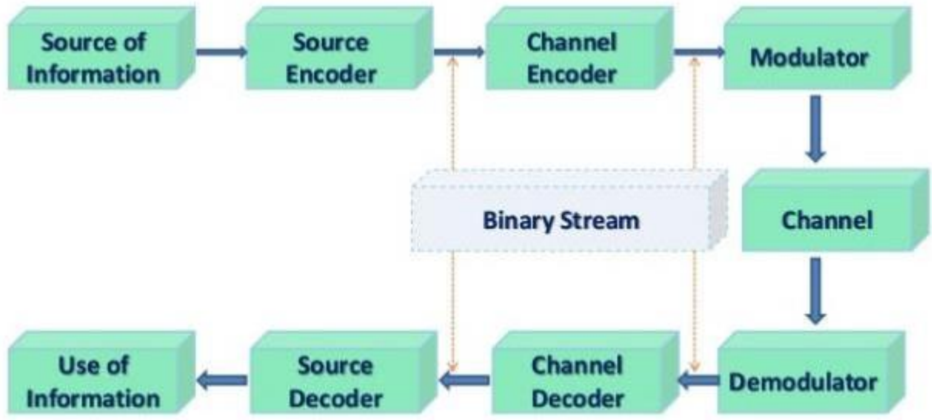


Fig. 1 Elements of Digital Communication Systems

1. **Information Source and Input Transducer:**

   The source of information can be analog or digital, e.g. analog: audio or video signal, digital: like teletype signal. In digital communication the signal produced by this source is converted into digital signal which consists of 1's and 0's. For this we need a source encoder.

2. **Source Encoder:**

   In digital communication we convert the signal from source into digital signal as mentioned above. The point to remember is we should like to use as few binary digits as possible to represent the signal. In such a way this efficient representation of the source output results in little or no redundancy. This sequence of binary digits is called ***information sequence.***

   Source Encoding or Data Compression: the process of efficiently converting the output of whether analog or digital source into a sequence of binary digits is known as source encoding.

3. **Channel Encoder:**

    The information sequence is passed through the channel encoder. The purpose of the channel encoder is to introduce, in controlled manner, some redundancy in the binary information sequence that can be used at the receiver to overcome the effects of noise and interference encountered in the transmission on the signal through the channel.

    For example take k bits of the information sequence and map that k bits to unique n bit sequence called code word. The amount of redundancy introduced is measured by the ratio *n/k* and the reciprocal of this ratio (k/n) is known as *rate of code or code rate.*

4. **Digital Modulator:**

    The binary sequence is passed to digital modulator which in turns convert the sequence into electric signals so that we can transmit them on channel (we will see channel later). The digital modulator maps the binary sequences into signal wave forms , for example if we represent 1 by sin x and 0 by cos x then we will transmit sin x for 1 and cos x for 0. (a case similar to BPSK)

5. **Channel:**

    The communication channel is the physical medium that is used for transmitting signals from transmitter to receiver. In wireless system, this channel consists of atmosphere , for traditional telephony, this channel is wired , there are optical channels, under water acoustic channels etc.We further discriminate this channels on the basis of their property and characteristics, like AWGN channel etc.

6. **Digital Demodulator:**

    The digital demodulator processes the channel corrupted transmitted waveform and reduces the waveform to the sequence of numbers that represents estimates of the transmitted data symbols.

7. **Channel Decoder:**

    This sequence of numbers then passed through the channel decoder which attempts to reconstruct the original information sequence from the knowledge of the code used by the channel encoder and the redundancy contained in the received data

Note: The average probability of a bit error at the output of the decoder is a measure of the performance of the demodulator – decoder combination.

8. **Source Decoder:**

    At the end, if an analog signal is desired then source decoder tries to decode the sequence from the knowledge of the encoding algorithm. And which results in the approximate replica of the input at the transmitter end.

9. **Output Transducer:**

Finally we get the desired signal in desired format analog or digital.

Advantages of digital communication:

- Can withstand channel noise and distortion much better as long as the noise and the distortion are within limits.
- Regenerative repeaters prevent accumulation of noise along the path.
- Digital hardware implementation is flexible.
- Digital signals can be coded to yield extremely low error rates, high fidelity and well as privacy.
- Digital communication is inherently more efficient than analog in realizing the exchange of SNR for bandwidth.
- It is easier and more efficient to multiplex several digital signals.
- Digital signal storage is relatively easy and inexpensive.
- Reproduction with digital messages is extremely reliable without deterioration.
- The cost of digital hardware continues to halve every two or three years, while performance or capacity doubles over the same time period.

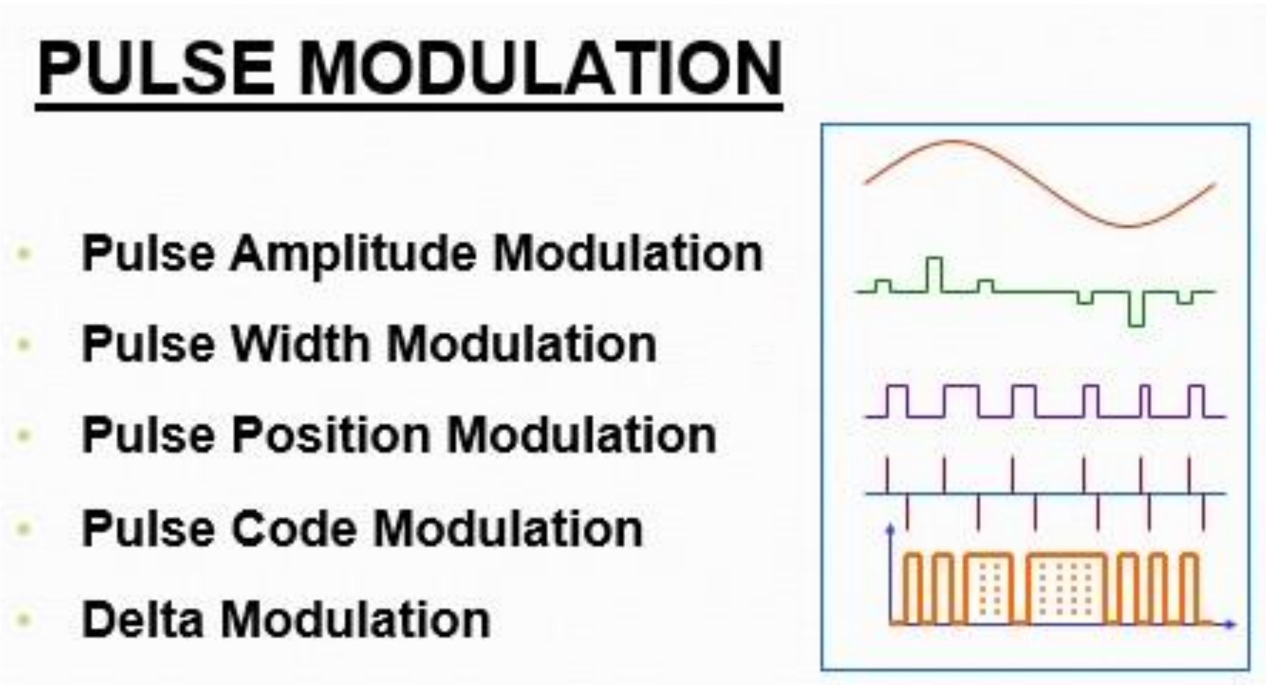
Disadvantages

- TDM digital transmission is not compatible with the FDM
- A Digital system requires large bandwidth.

Introduction to Pulse Modulation

What is the need for Pulse Modulation?

- Many Signals in Modern Communication Systems are digital
- Also, analog signals are transmitted digitally.
- Reduced distortion and improvement in signal to noise ratios.
- PAM, PWM, PPM, PCM and DM.
- In CW modulation schemes some parameter of modulated wave varies continuously with message.
- In Analog pulse modulation some parameter of each pulse is modulated by a particular sample value of the message.
- Pulse modulation is of two types
  - Analog Pulse Modulation
    - Pulse Amplitude Modulation (PAM)
    - Pulse width Modulation (PWM)
    - Pulse Position Modulation (PPM)
  - Digital Pulse Modulation
    - Pulse code Modulation (PCM)
    - Delta Modulation (DM)



Pulse Code Modulation:

Three steps involved in conversion of analog signal to digital signal

- Sampling
- Quantization
- Binary encoding

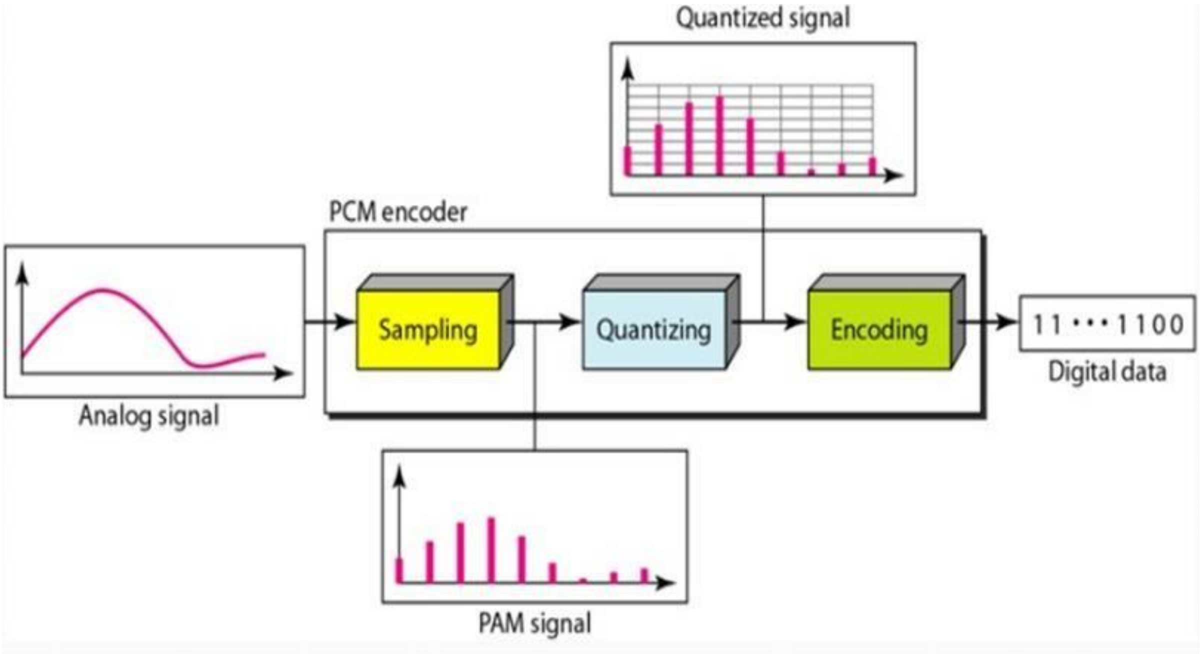


Fig. 2 Conversion of Analog Signal to Digital Signal

Note: Before sampling the signal is filtered to limit bandwidth.

Elements of PCM System:

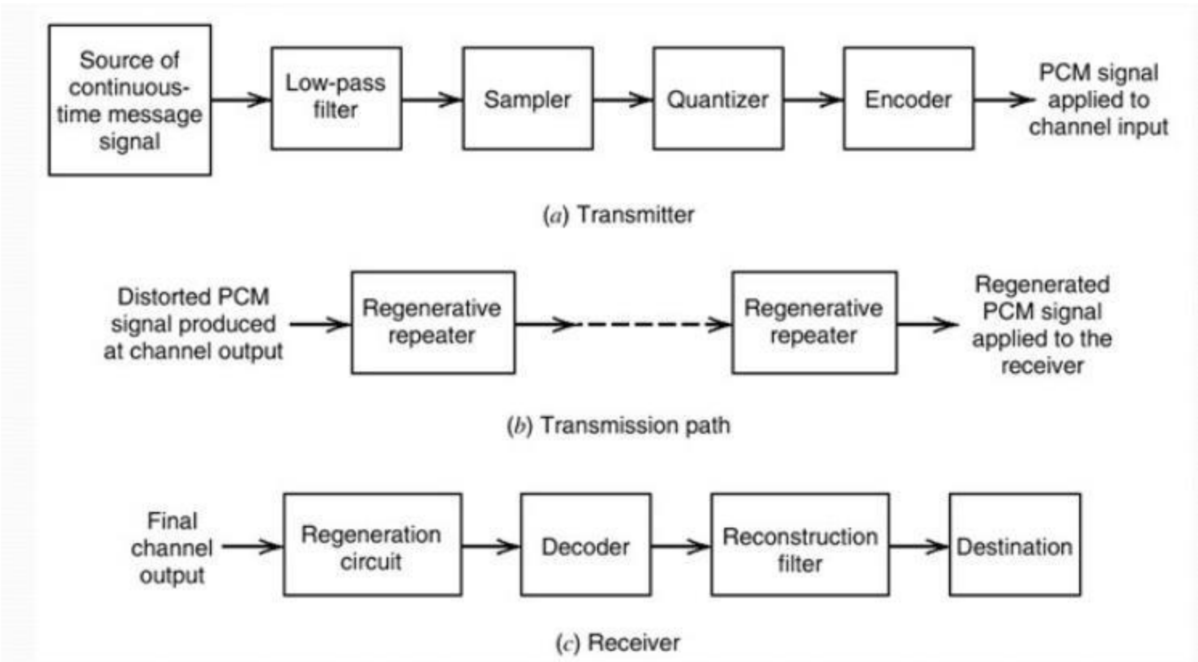


Fig. 3 Elements of PCM System

Sampling:

- Process of converting analog signal into discrete signal.
- Sampling is common in all pulse modulation techniques

- The signal is sampled at regular intervals such that each sample is proportional to amplitude of signal at that instant
- Analog signal is sampled every $T_s Secs$, called sampling interval. f_s=1/T_S is called sampling rate or sampling frequency.
- f_s=2f_m is Min. sampling rate called **Nyquist rate.** Sampled spectrum (ω) is repeating periodically without overlapping.
- Original spectrum is centered at ω=0 and having bandwidth of ω_m. Spectrum can be recovered by passing through low pass filter with cut-off ω_m.
- For f_s<2f_m sampled spectrum will overlap and cannot be recovered back. This is called **aliasing.**

Sampling methods:

- Ideal – An impulse at each sampling instant.
- Natural – A pulse of Short width with varying amplitude.
- Flat Top – Uses sample and hold, like natural but with single amplitude value.

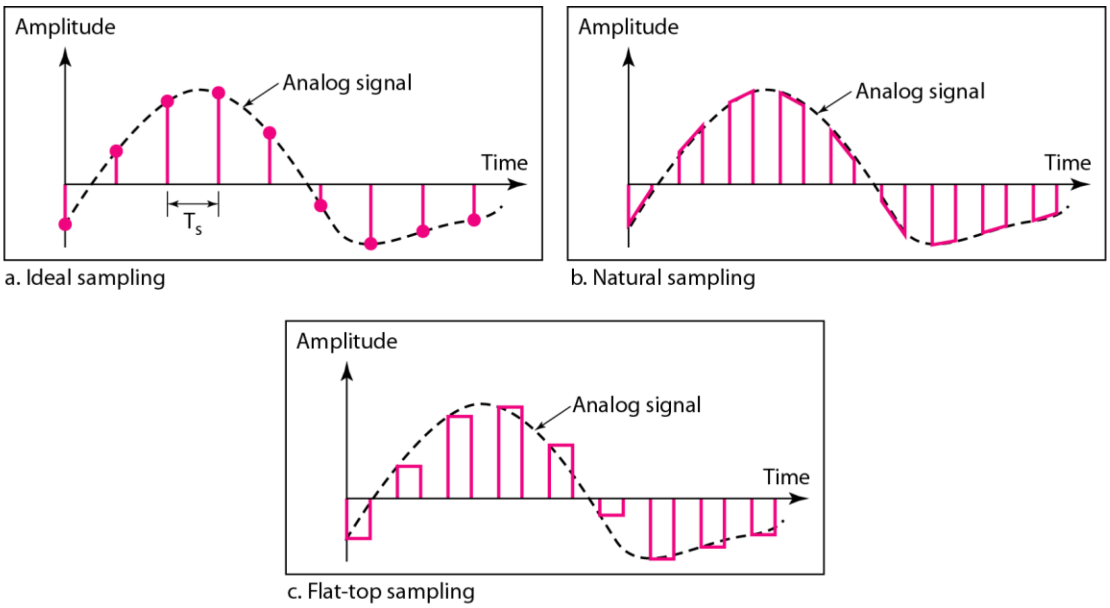


Fig. 4 Types of Sampling

Sampling of band-pass Signals:

- A band-pass signal of bandwidth 2f_m can be completely recovered from its samples.

  Min. sampling rate =2×$Bandwidth$

  =2×2f_m=4f_m

- Range of minimum sampling frequencies is in the range of 2×BW to 4×BW

Instantaneous Sampling or Impulse Sampling:

- Sampling function is train of spectrum remains constant impulses throughout frequency range. It is not practical.

Natural sampling:

- The spectrum is weighted by a **sinc** function.
- Amplitude of high frequency components reduces.

Flat top sampling:

- Here top of the samples remains constant.
- In the spectrum high frequency components are attenuated due sinc pulse roll off. This is known as **Aperture effect.**
- If pulse width increases aperture effect is more i.e. more attenuation of high frequency components.

Sampling Theorem:

Statement of sampling theorem

> 1) ***A band limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2W}$ seconds and***
>
> 2) ***A band limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.***

The first part of above statement tells about sampling of the signal and second part tells about reconstruction of the signal. Above statement can be combined and stated alternately as follows :

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal. i.e.,

$$f_s \geq 2W$$

Here f_s is the sampling frequency and

W is the higher frequency content

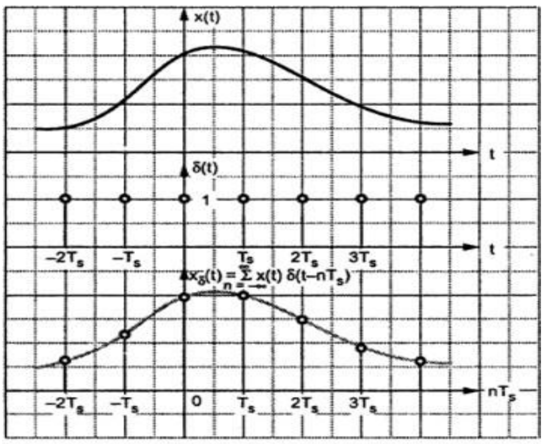


Fig. 5 CT and its DT signal

Proof of sampling theorem

There are two parts : (I) Representation of $x(t)$ in terms of its samples

(II) Reconstruction of $x(t)$ from its samples.

Part I : Representation of $x(t)$ in its samples $x(nT_s)$

Step 1 : Define $x_\delta(t)$

Step 2 : Fourier transform of $x_\delta(t)$ i.e. $X_\delta(f)$

Step 3 : Relation between $X(f)$ and $X_\delta(f)$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

Step 1 : Define $x_\delta(t)$

The sampled signal $x_\delta(t)$ is given as,

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s) \qquad \ldots\ 1$$

Here observe that $x_\delta(t)$ is the product of x_δ and impulse train $\delta(t)$ as shown in above fig In the above equation $\delta(t-nT_s)$ indicates the samples placed at $\pm T_s$, $\pm 2T_s$, $\pm 3T_s$ ... and so on.

Step 2 : FT of $x_\delta(t)$ i.e. $X_\delta(f)$

Taking FT of equation (1.3.1).

$$X_\delta(f) = \text{FT}\left\{\sum_{n=-\infty}^{\infty} x(t)\,\delta(t-nT_s)\right\}$$

$$= \text{FT \{Product of } x(t) \text{ and impulse train\}}$$

We know that FT of product in time domain becomes convolution in frequency domain. i.e.,

$$X_\delta(f) = \text{FT}\{x(t)\} * FT\{\delta(t-nT_s)\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

By definitions, $x(t) \xleftrightarrow{FT} X(f)$ and

$$\delta(t-nT_s) \xleftrightarrow{FT} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

Hence equation (1.3.2) becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$$

Since convolution is linear,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f-nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \qquad \textit{By shifting property of impulse function}$$

$$= \cdots f_s X(f-2f_s) + f_s X(f-f_s) + f_s X(f) + f_s X(f-f_s) + f_s X(f-2f_s) + \cdots$$

Comments

(i) The RHS of above equation shows that X(f) is placed at $\pm f_s, \pm 2f_s, \pm 3f_s, \cdots$

(ii) This means X(f) is periodic in f_s.

(iii) If sampling frequency is $f_s = 2W$, then the spectrums $X(f)$ just touch each other.

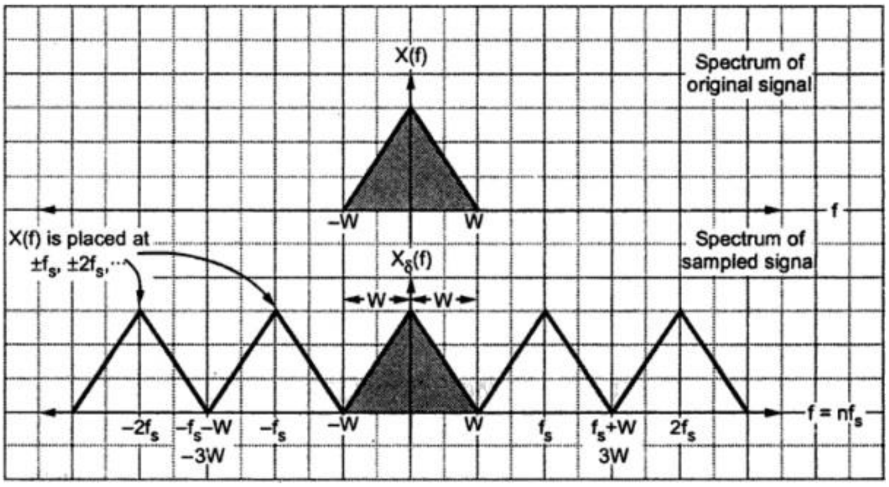


Fig. 6 Spectrum of original signal and sampled signal

Step 3 : Relation between $X(f)$ and $X_\delta(f)$

Important assumption : Let us assume that $f_s = 2W$, then as per above diagram.

$$X_\delta(f) = f_s X(f) \qquad \text{for } -W \le f \le W \text{ and } f_s = 2W$$

or
$$X(f) = \frac{1}{f_s} X_\delta(f) \qquad \dots \quad 3$$

Step 4 : Relation between $x(t)$ and $x(nT_s)$

DTFT is,
$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$\therefore$
$$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n} \qquad \dots \quad 4$$

In above equation 'f' is the frequency of DT signal. If we replace $X(f)$ by $X_\delta(f)$, then 'f' becomes frequency of CT signal. i.e.,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi \frac{f}{f_s} n}$$

In above equation 'f' is frequency of CT signal. And $\frac{f}{f_s}$ = Frequency of DT signal in equation 4 Since $x(n) = x(nT_s)$, i.e. samples of $x(t)$, then we have,

$$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} \text{ since } \frac{1}{f_s} = T_s$$

Putting above expression in equation 3 ,

$$X(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}$$

Inverse Fourier Transform (IFT) of above equation gives $x(t)$ i.e.,

$$x(t) = IFT\left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}\right\} \quad \ldots \quad 5$$

Comments :

i) Here $x(t)$ is represented completely in terms of $x(nT_s)$.

ii) Above equation holds for $f_s = 2W$. This means if the samples are taken at the rate of 2W or higher, $x(t)$ is completely represented by its samples.

iii) First part of the sampling theorem is proved by above two comments.

Part II : Reconstruction of $x(t)$ from its samples

Step 1 : Take inverse Fourier transform of $X(f)$ which is in terms of $X_\delta(f)$.

Step 2 : Show that $x(t)$ is obtained back with the help of interpolation function.

Step 1 : The IFT of equation 5 becomes,

$$x(t) = \int_{-\infty}^{\infty} \left\{\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}\right\} e^{j2\pi f t}\, df$$

Here the integration can be taken from $-W \le f \le W$. Since $X(f) = \frac{1}{f_s} X_\delta(f)$ for $-W \le f \le W$. (See Fig. 6).

$$\therefore \qquad x(t) = \int_{-W}^{W} \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s} \cdot e^{j2\pi f t}\, df$$

Interchanging the order of summation and integration,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{1}{f_s} \int_{-W}^{W} e^{j2\pi f(t-nT_s)}\, df$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \cdot \left[\frac{e^{j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)} \right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \left\{ \frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)} \right\}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \cdot \frac{1}{f_s} \cdot \frac{\sin 2\pi W(t-nT_s)}{\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2Wt - 2WnT_s)}{\pi(f_s t - f_s nT_s)}$$

Here $f_s = 2W$, hence $T_s = \frac{1}{f_s} = \frac{1}{2W}$. Simplifying above equation,

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\sin \pi(2Wt-n)}{\pi(2Wt-n)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \mathrm{sinc}(2Wt-n) \qquad \text{since } \frac{\sin \pi\theta}{\pi\theta} = \mathrm{sinc}\,\theta \qquad \dots \qquad 6$$

Step 2 : Let us interpret the above equation. Expanding we get,

$$x(t) = \cdots + x(-2T_s)\mathrm{sinc}(2Wt+2) + x(-T_s)\mathrm{sinc}(2Wt+1) + x(0)\mathrm{sinc}(2Wt) + x(T_s)\mathrm{sinc}(2Wt-1) + \cdots$$

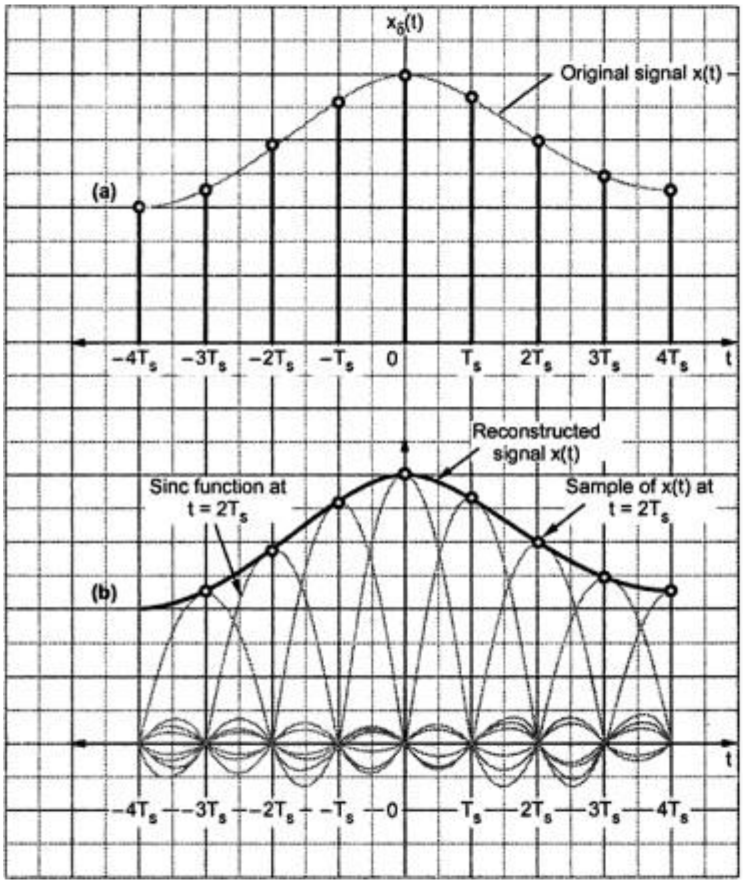


Fig. 7 (a) Sampled version of signal x(t)
(b) Reconstruction of x(t) from its samples

Comments :

i) The samples $x(nT_s)$ are weighted by sinc functions.

ii) The sinc function is the interpolating function. Fig. 7 shows, how $x(t)$ is interpolated.

Step 3 : Reconstruction of $x(t)$ by lowpass filter

When the interpolated signal of equation 6 is passed through the lowpass filter of bandwidth $-W \leq f \leq W$, then the reconstructed waveform shown in above Fig. 7(b) is obtained. The individual sinc functions are interpolated to get smooth $x(t)$.

PCM Generator:

The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Fig. 8 shows the PCM generator.

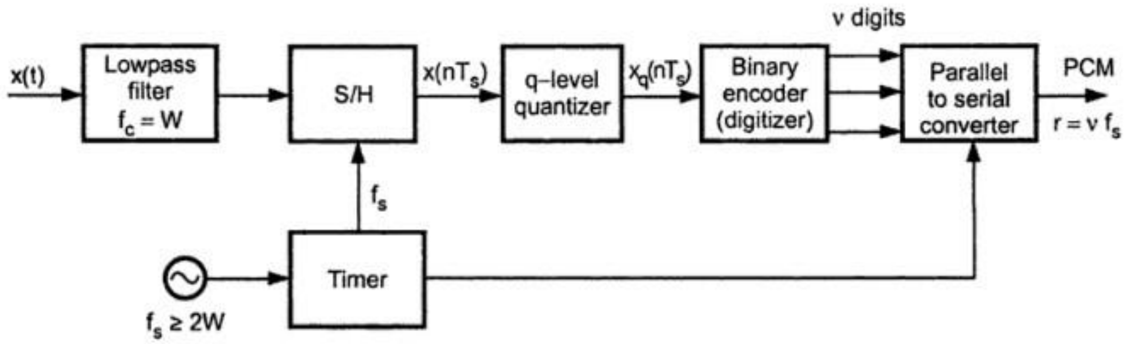


Fig. 8 PCM generator

In the PCM generator of above figure, the signal $x(t)$ is first passed through the lowpass filter of cutoff frequency 'W' Hz. This lowpass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 8 output of sample and hold is called $x(nT_s)$. This $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It assigns any one of the digital level to $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

Now coming back to our discussion of PCM generation, the quantized signal level $x_q(nT_s)$ is given to binary encoder. This encoder converts input signal to 'v' digits binary word. Thus $x_q(nT_s)$ is converted to 'V' binary bits. The encoder is also called digitizer.

It is not possible to transmit each bit of the binary word separately on transmission line. Therefore '*v*' binary digits are converted to serial bit stream to generate single baseband signal. In a parallel to serial converter, normally a shift register does this job. The output of PCM generator is thus a single baseband signal of binary bits.

An oscillator generates the clocks for sample and hold an parallel to serial converter. In the pulse code modulation generator discussed above ; sample and hold, quantizer and encoder combinely form an analog to digital converter.

Transmission BW in PCM:

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v \quad \dots \quad 1$$

Here 'q' represents total number of digital levels of q-level quantizer.

For example if $v = 3$ bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to 'v' binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per second = f_s

$\therefore$ Number of bits per second is given by,

$$\begin{aligned}\text{(Number of bits per second)} &= \text{(Number of bits per samples)} \\ &\quad \times \text{(Number of samples per second)} \\ &= v \text{ bits per sample} \times f_s \text{ samples per second} \quad \dots \quad 2\end{aligned}$$

The number of bits per second is also called signaling rate of PCM and is denoted by 'r' i.e.,

$$\boxed{\textbf{Signaling rate in PCM : } r = v f_s} \quad \dots \quad 3$$

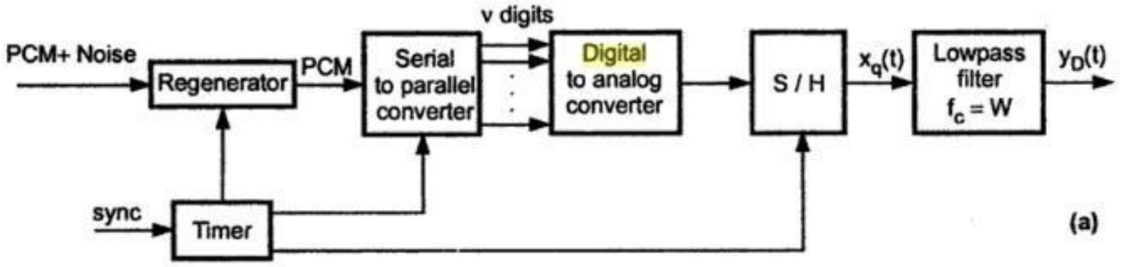
Here $f_s \geq 2W$.

Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

$$\text{Transmission Bandwidth of PCM : } \begin{cases} B_T \geq \frac{1}{2} r & \dots \quad 4 \\ B_T \geq \frac{1}{2} v f_s \quad \text{Since } f_s \geq 2W & \dots \quad 5 \\ B_T \geq v W & \dots \quad 6 \end{cases}$$

PCM Receiver:

Fig. 9 (a) shows the block diagram of PCM receiver and Fig. 9 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.



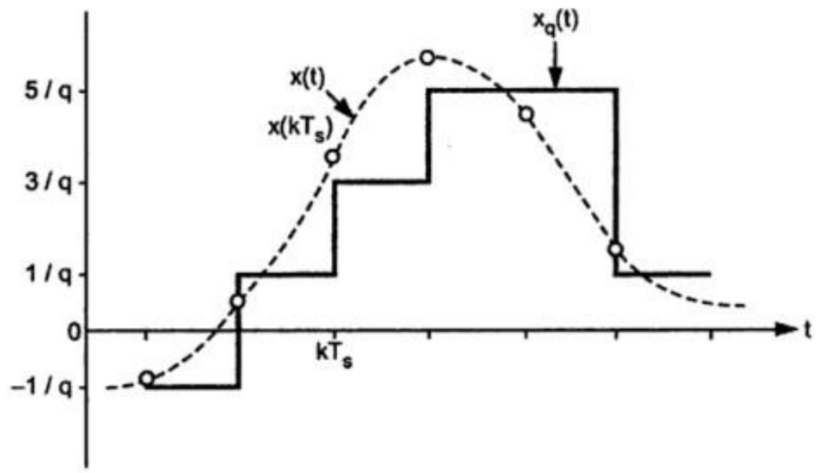


Fig. 9 (a) PCM receiver
(b) Reconstructed waveform

The digital word is converted to its analog value $x_q(t)$ along with sample and hold. This signal, at the output of S/H is passed through lowpass reconstruction filter to get $y_D(t)$. As shown in reconstructed signal of Fig. 9 (b), it is impossible to reconstruct exact original signal $x(t)$ because of permanent quantization error introduced during quantization at the transmitter. This quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits 'v' increases the signaling rate as well as transmission bandwidth as we have seen in equation 3 and equation 6. Therefore the choice of these parameters is made, such that noise due to quantization error (called as quantization noise) is in tolerable limits.

Quantization

- The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels.

- **Quantization** is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal
- Both sampling and quantization result in the loss of information.
- The quality of a Quantizer output depends upon the number of quantization levels used.
- The discrete amplitudes of the quantized output are called as **representation levels** or **reconstruction levels**.
- The spacing between the two adjacent representation levels is called a **quantum** or **step-size**.
- There are two types of Quantization
    - Uniform Quantization
    - Non-uniform Quantization.
- The type of quantization in which the quantization levels are uniformly spaced is termed as a **Uniform Quantization**.
- The type of quantization in which the quantization levels are unequal and mostly the relation between them is logarithmic, is termed as a **Non-uniform Quantization.**

<u>Uniform Quantization:</u>

- There are two types of uniform quantization.
    - Mid-Rise type
    - Mid-Tread type.
- The following figures represent the two types of uniform quantization.

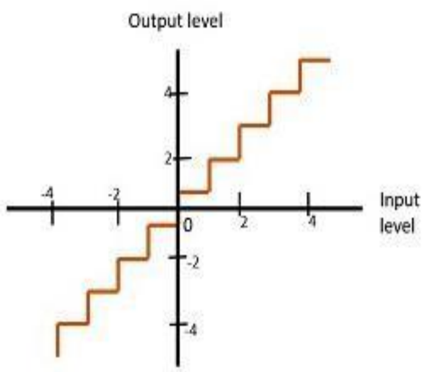


Fig 1 : Mid-Rise type Uniform Quantization

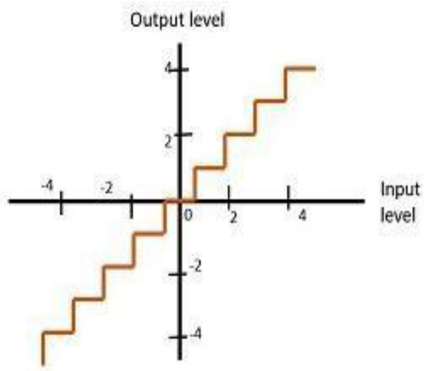


Fig 2 : Mid-Tread type Uniform Quantization

- The **Mid-Rise** type is so called because the origin lies in the middle of a raising part of the stair-case like graph. The quantization levels in this type are even in number.
- The **Mid-tread** type is so called because the origin lies in the middle of a tread of the stair-case like graph. The quantization levels in this type are odd in number.
- Both the mid-rise and mid-tread type of uniform quantizer is symmetric about the origin.

Quantization Noise and Signal to Noise ratio in PCM System:

Derivation of Quantization Error/Noise or Noise Power for Uniform (Linear) Quantization

Step 1 : Quantization Error

Because of quantization, inherent errors are introduced in the signal. This error is called *quantization error*. We have defined quantization error as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \quad \text{.........................} (1)$$

Step 2 : Step size

Let an input $x(nT_s)$ be of continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

Therefore the total amplitude range becomes,

$$\text{Total amplitude range} = x_{max} - (-x_{max}) = 2x_{max} \quad \text{.........................} (2)$$

If this amplitude range is divided into 'q' levels of quantizer, then the step size 'δ' is given as,

$$\delta = \frac{x_{max} - (-x_{max})}{q} = \frac{2x_{max}}{q} \quad \text{....................} (3)$$

If signal $x(t)$ is normalized to minimum and maximum values equal to 1, then

$$x_{max} = 1, \quad -x_{max} = -1 \quad \text{...............................} (4)$$

Therefore step size will be,

$$\delta = \frac{2}{q} \quad \text{(for normalized signal)} \quad \text{...............................} (5)$$

Step 3 : Pdf of Quantization error

If step size 'δ' is sufficiently small, then it is reasonable to assume that the quantization error 'ε' will be uniformly distributed random variable. The maximum quantization error is given by

$$\varepsilon_{max} = \left|\frac{\delta}{2}\right| \quad \text{............................} (6)$$

i.e.

$$-\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2} \quad \text{..............................} (7)$$

Thus over the interval $\left(-\frac{\delta}{2}, \frac{\delta}{2}\right)$ quantization error is uniformly distributed random variable.

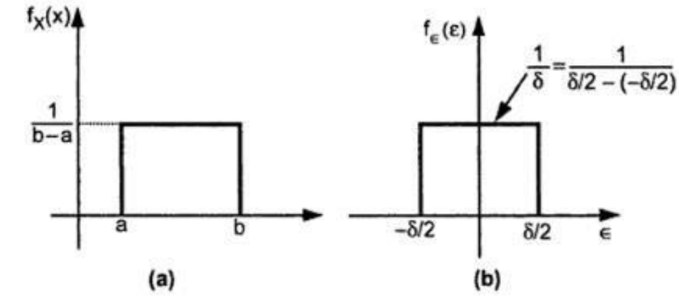


Fig. 10 (a) Uniform distribution
(b) Uniform distribution for quantization error

In above figure, a random variable is said to be uniformly distributed over an interval (a, b). Then PDF of 'X' is given by, (from equation of Uniform PDF).

$$f_X(x) = \begin{cases} 0 & \text{for} \quad x \le a \\ \frac{1}{b-a} & \text{for} \quad a < x \le b \\ 0 & \text{for} \quad x > b \end{cases} \qquad \text{..........................(8)}$$

Thus with the help of above equation we can define the probability density function for quantization error 'ε' as,

$$f_\varepsilon(\varepsilon) = \begin{cases} 0 & \text{for} \quad \varepsilon \le \frac{\delta}{2} \\ \frac{1}{\delta} & \text{for} \quad -\frac{\delta}{2} < \varepsilon \le \frac{\delta}{2} \\ 0 & \text{for} \quad \varepsilon > \frac{\delta}{2} \end{cases} \qquad \text{...................................(9)}$$

Step 4 : Noise Power

quantization error 'ε' has zero average value. That is mean 'm_ε' of the quantization error is zero.

The signal to quantization noise ratio of the quantizer is defined as,

$$\frac{S}{N} = \frac{\text{Signal power (normalized)}}{\text{Noise power (normalized)}} \quad \cdots 10$$

If type of signal at input i.e., $x(t)$ is known, then it is possible to calculate signal power.

The noise power is given as,

$$\text{Noise power} = \frac{V_{noise}^2}{R} \quad \cdots (11)$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\varepsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \bar{\varepsilon}^2 \quad \cdots (12)$$

The mean square value of a random variable 'X' is given as,

$$\overline{X}^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx \quad \text{By definition} \quad \cdots (13)$$

Here $$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)\, d\varepsilon \quad \cdots (14)$$

From equation 9 we can write above equation as,

$$E[\varepsilon^2] = \int_{-\delta/2}^{\delta/2} \varepsilon^2 \times \frac{1}{\delta}\, d\varepsilon$$

$$= \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta/2}^{\delta/2} = \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12} \quad \cdots (15)$$

∴ From equation 1.8.25, the mean square value of noise voltage is,

$$V_{noise}^2 = \text{mean square value} = \frac{\delta^2}{12}$$

When load resistance, $R = 1$ ohm, then the noise power is normalized i.e.,

$$\text{Noise power (normalized)} = \frac{V_{noise}^2}{1} \qquad \text{[with } R = 1 \text{ in equation 11]}$$

$$= \frac{\delta^2 / 12}{1} = \frac{\delta^2}{12}$$

Thus we have,

Normalized noise power

or Quantization noise power $= \frac{\delta^2}{12}$ **; For linear quantization.**

or Quantization error (in terms of power) ... (16)

Derivation of Maximum Signal to Quantization Noise Ratio for Linear Quantization:

signal to quantization noise ratio is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

$$= \frac{\text{Normalized signal power}}{(\delta^2 / 12)} \qquad \text{... (17)}$$

The number of bits 'v' and quantization levels 'q' are related as,

$$q = 2^v \qquad \text{... (18)}$$

Putting this value in equation (3) we have,

$$\delta = \frac{2\,x_{max}}{2^v} \qquad \text{... (19)}$$

Putting this value in equation 1.8.30 we get,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\left(\frac{2\,x_{max}}{2^v}\right)^2 + 12}$$

Let normalized signal power be denoted as 'P'.

$$\frac{S}{N} = \frac{P}{\frac{4\,x_{max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{max}^2} \cdot 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus,

$$\textbf{Maximum signal to quantization noise ratio : } \frac{S}{N} = \frac{3P}{x_{max}^2} \cdot 2^{2v} \quad \cdots (20)$$

This equation shows that signal to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input $x(t)$ is normalized, i.e.,

$$x_{max} = 1 \quad \cdots (21)$$

Then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P \quad \cdots (22)$$

If the destination signal power 'P' is normalized, i.e.,

$$P \leq 1 \quad \cdots (23)$$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v} \quad \cdots (24)$$

Since $x_{max} = 1$ and $P \leq 1$, the signal to noise ratio given by above equation is normalized.

Expressing the signal to noise ratio in decibels,

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB \quad \text{since power ratio.}$$

$$\leq 10 \log_{10} [3 \times 2^{2v}]$$

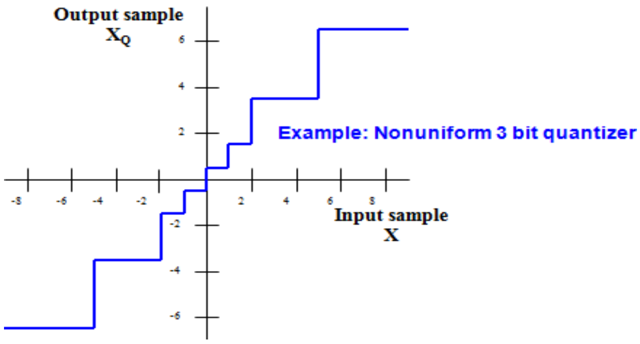
$$\leq (4.8 + 6v)\, dB$$

Thus,

Signal to Quantization noise ratio for normalized values of power 'P' and amplitude of input $x(t)$: $\left(\frac{S}{N}\right) dB \leq (4.8 + 6v)\, dB$ ⋯ (25)

Non-Uniform Quantization:

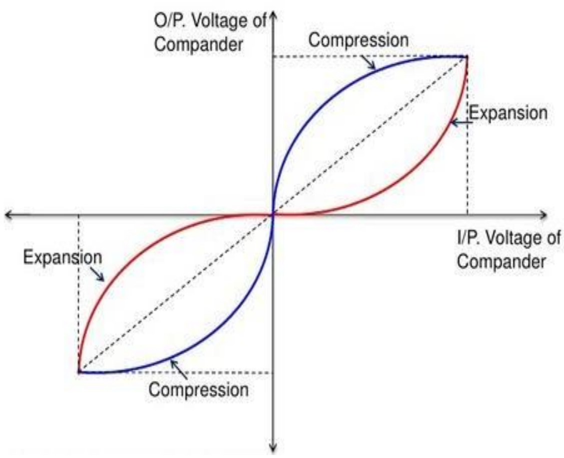
In non-uniform quantization, the step size is not fixed. It varies according to certain law or as per input signal amplitude. The following fig shows the characteristics of Non uniform quantizer.

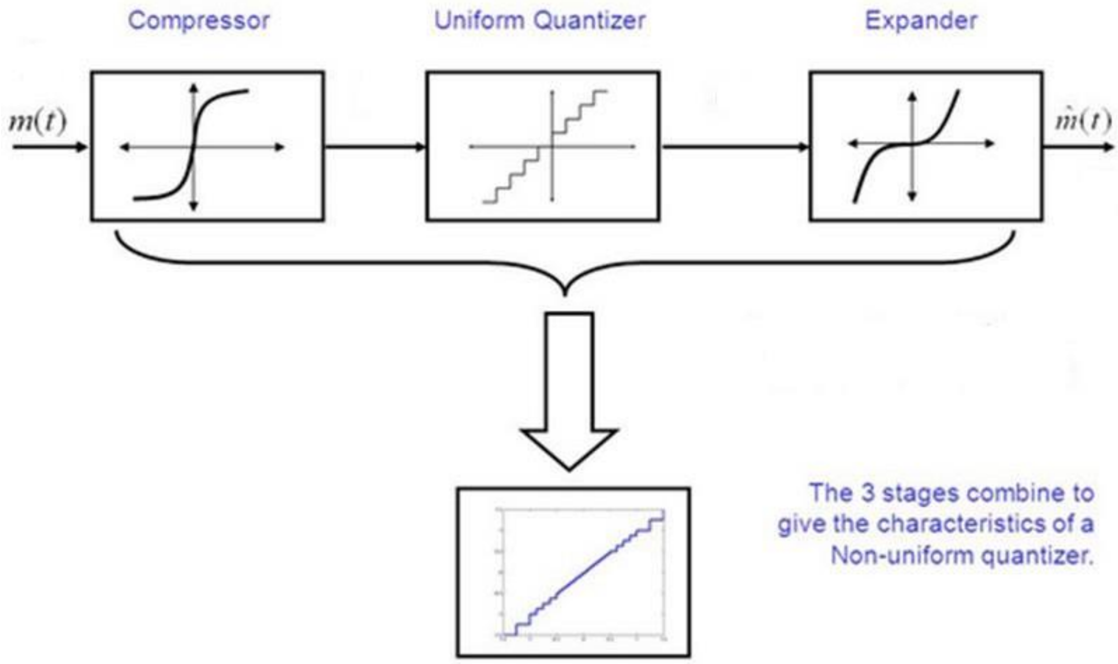


In this figure observe that step size is small at low input signal levels. Hence quantization error is also small at these inputs. Therefore signal to quantization noise power ratio is improved at low signal levels. Stepsize is higher at high input levels. Hence signal to noise power ratio remains almost same throughout the dynamic range of quantizer.

Companding PCM System:

- Non-uniform quantizers are difficult to make and expensive.
- An alternative is to first pass the speech signal through nonlinearity before quantizing with a uniform quantizer.
- The nonlinearity causes the signal amplitude to be ***compressed***.
  - The input to the quantizer will have a more uniform distribution.
- At the receiver, the signal is ***expanded*** by an inverse to the nonlinearity.
- The process of compressing and expanding is called ***Companding***.





μ - Law Companding for Speech Signals

Normally for speech and music signals a μ - law compression is used. This compression is defined by the following equation,

$$Z(x) = (\text{Sgn } x)\frac{ln(1+\mu|x|)}{ln(1+\mu)} \quad |x| \leq 1 \qquad \cdots (1)$$

Below Fig shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

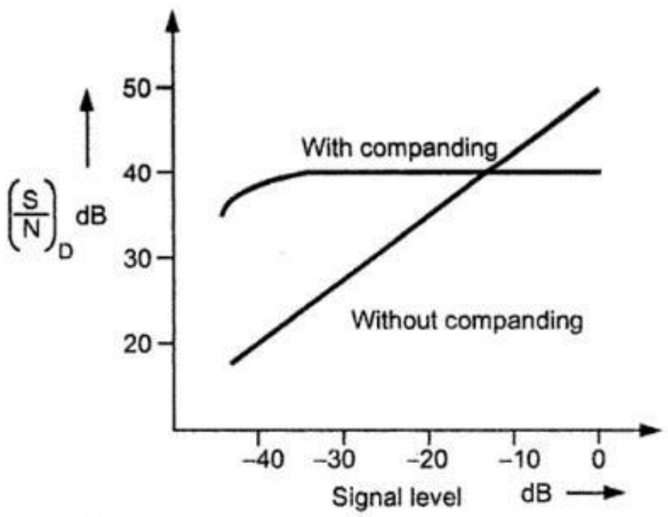


Fig. 11 PCM performance with μ - law companding

It can be observed from above figure that signal to noise ratio of PCM remains almost constant with companding.

A-Law for Companding

The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \quad \cdots (2)$$

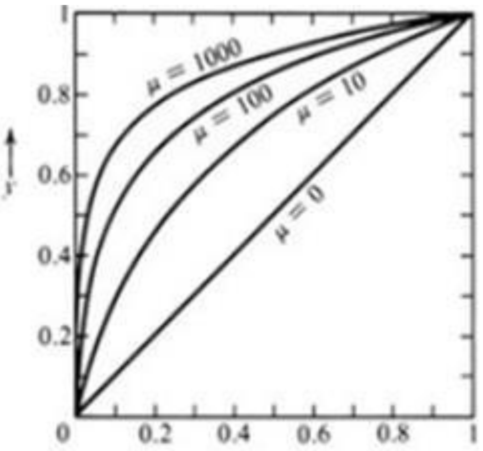
When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

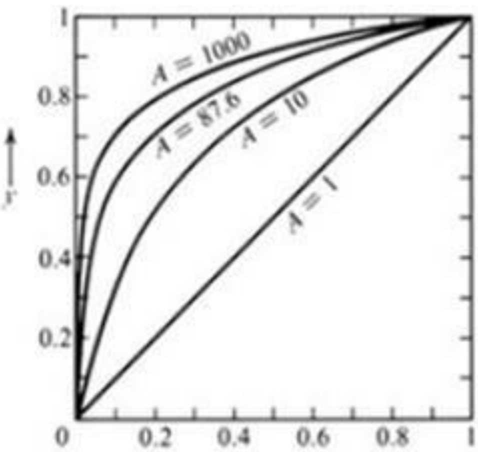
Signal to Noise Ratio of Companded PCM

The signal to noise ratio of companded PCM is given as,

$$\frac{S}{N} = \frac{3q^2}{[\ln(1+\mu)]^2} \quad \cdots (3)$$

Here $q = 2^v$ is number of quantization levels.





<u>Differential Pulse Code Modulation (DPCM):</u>

Redundant Information in PCM:

The samples of a signal are highly corrected with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. When these samples are encoded by standard PCM system, the resulting encoded signal contains redundant information.

Fig. shows a continuous time signal $x(t)$ by dotted line. This signal is sampled by flat top sampling at intervals $T_s, 2T_s, 3T_s \dots nT_s$. The sampling frequency is selected to be higher than nyquist rate. The samples are encoded by using 3 bit (7 levels) PCM. The sample is quantized to the nearest digital level as shown by small

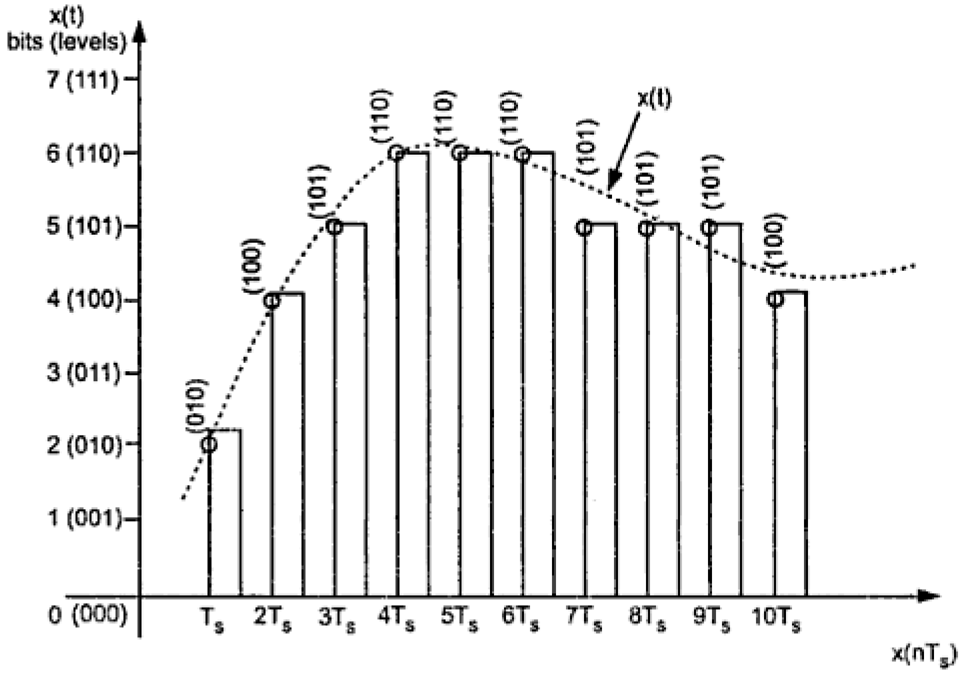


Fig. Redundant information in PCM

circles in the diagram. The encoded binary value of each sample is written on the top of the samples. We can see from Fig. that the samples taken at $4T_s, 5T_s$ and $6T_s$ are encoded to same value of (110). This information can be carried only by one sample. But three samples are carrying the same information means it is redundant. Consider another example of samples taken at $9T_s$ and $10T_s$. The difference between these samples is only due to last bit and first two bits are redundant, since they do not change.

Principle of DPCM

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced. This type of digital pulse modulation scheme is called Differential Pulse Code Modulation.

DPCM Transmitter

The differential pulse code modulation works on the principle of prediction. The value of the present sample is predicted from the past samples. The prediction may not be exact but it is very close to the actual sample value. Fig. shows the transmitter of Differential Pulse Code Modulation (DPCM) system. The sampled signal is denoted by $x(nT_s)$ and the predicted signal is denoted by $\hat{x}(nT_s)$. The comparator finds out the difference between the actual sample value $x(nT_s)$ and predicted sample value $\hat{x}(nT_s)$. This is called error and it is denoted by $e(nT_s)$. It can be defined as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \qquad \text{..................(1)}$$

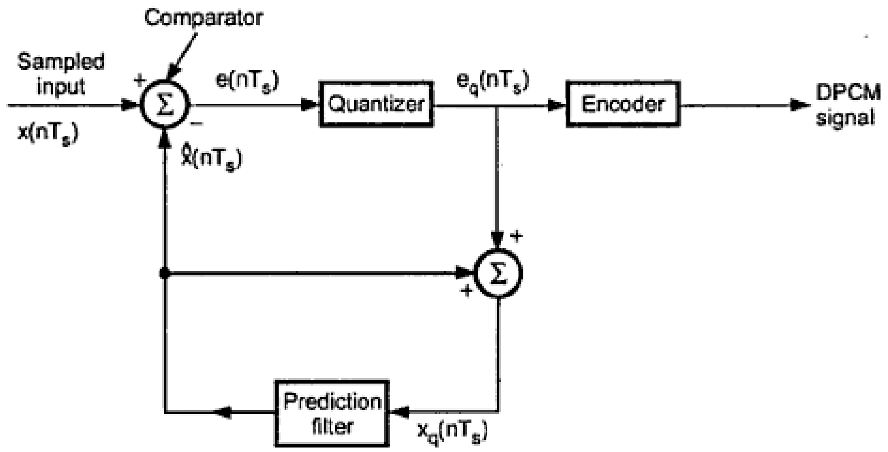


Fig. Differential pulse code modulation transmitter

Thus error is the difference between unquantized input sample $x(nT_s)$ and prediction of it $\hat{x}(nT_s)$. The predicted value is produced by using a prediction filter. The quantizer output signal $e_q(nT_s)$ and previous prediction is added and given as input to the prediction filter. This signal is called $x_q(nT_s)$. This makes the prediction more and more close to the actual sampled signal. We can see that the quantized error signal $e_q(nT_s)$ is very small and can be encoded by using small number of bits. Thus number of bits per sample are reduced in DPCM.

The quantizer output can be written as,

$$e_q(nT_s) = e(nT_s) + q(nT_s) \qquad \text{.................(2)}$$

Here $q(nT_s)$ is the quantization error. As shown in Fig. the prediction filter input $x_q(nT_s)$ is obtained by sum $\hat{x}(nT_s)$ and quantizer output i.e.,

$$x_q(nT_s) = \hat{x}(nT_s) + e_q(nT_s) \qquad \text{..................(3)}$$

Putting the value of $e_q(nT_s)$ from equation 2 in the above equation we get,

$$x_q(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s) \qquad \text{................(4)}$$

Equation 1 is written as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s)$$

$$\therefore \quad e(nT_s) + \hat{x}(nT_s) = x(nT_s) \qquad \text{...............(5)}$$

$\therefore$ Putting the value of $e(nT_s) + \hat{x}(nT_s)$ from above equation into equation 4 we get,

$$x_q(nT_s) = x(nT_s) + q(nT_s) \qquad \text{..................(6)}$$

Thus the quantized version of the signal $x_q(nT_s)$ is the sum of original sample value and quantization error $q(nT_s)$. The quantization error can be positive or negative. Thus equation 6 does not depend on the prediction filter characteristics.

Reconstruction of DPCM Signal

Fig. shows the block diagram of DPCM receiver.

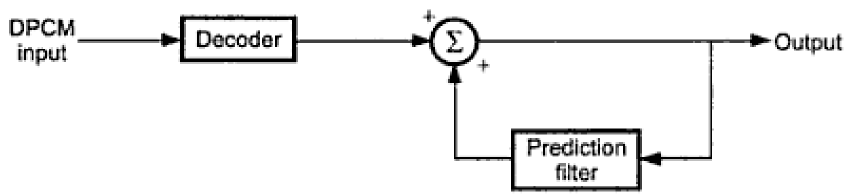


Fig. DPCM receiver

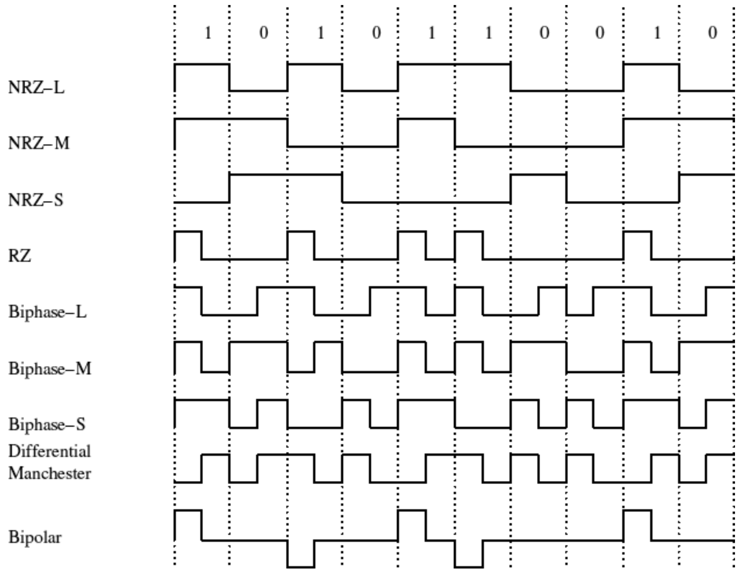
The decoder first reconstructs the quantized error signal from incoming binary signal. The prediction filter output and quantized error signals are summed up to give the quantized version of the original signal. Thus the signal at the receiver differs from actual signal by quantization error $q(nT_s)$, which is introduced permanently in the reconstructed signal.

Line Coding:

In telecommunication, a line code is a code chosen for use within a communications system for transmitting a digital signal down a transmission line. Line coding is often used for digital data transport.

The waveform pattern of voltage or current used to represent the 1s and 0s of a digital signal on a transmission link is called **line** encoding. The common types of

line encoding are unipolar, polar, bipolar and Manchester encoding. **Line codes** are used commonly in computer communication networks over short distances.



Signal	Comments
NRZ–L	Non-return to zero level. This is the standard positive logic signal format used in digital circuits. 1 forces a high level 0 forces a low level
NRZ–M	Non return to zero mark 1 forces a transition 0 does nothing
NRZ–S	Non return to zero space 1 does nothing 0 forces a transition
RZ	Return to zero 1 goes high for half the bit period 0 does nothing
Biphase–L	Manchester. Two consecutive bits of the same type force a transition at the beginning of a bit period. 1 forces a negative transition in the middle of the bit 0 forces a positive transition in the middle of the bit
Biphase–M	There is always a transition at the beginning of a bit period. 1 forces a transition in the middle of the bit 0 does nothing
Biphase–S	There is always a transition at the beginning of a bit period. 1 does nothing 0 forces a transition in the middle of the bit
Differential Manchester	There is always a transition in the middle of a bit period. 1 does nothing 0 forces a transition at the beginning of the bit
Bipolar	The positive and negative pulses alternate. 1 forces a positive or negative pulse for half the bit period 0 does nothing

Time Division Multiplexing:

The sampling theorem provides the basis for transmitting the information contained in a band-limited message signal $m(t)$ as a sequence of samples of $m(t)$ taken uniformly at a rate that is usually slightly higher than the Nyquist rate. An important feature of the sampling process is a *conservation of time*. That is, the transmission of the message samples engages the communication channel for only a fraction of the sampling interval on a periodic basis, and in this way some of the time interval between adjacent samples is cleared for use by other independent message sources on a time-shared basis. We thereby obtain a *time-division multiplex* (TDM) *system*, which enables the joint utilization of a common communication channel by a plurality of independent message sources without mutual interference among them.

The concept of TDM is illustrated by the block diagram shown in Figure Each input message signal is first restricted in bandwidth by a low-pass anti-aliasing filter to remove the frequencies that are nonessential to an adequate signal representation. The low-pass filter outputs are then applied to a *commutator*, which is usually implemented using electronic switching circuitry. The function of the commutator is twofold: (1) to take a narrow sample of each of the N input messages at a rate f_s that is slightly higher than $2W$, where W is the cutoff frequency of the anti-aliasing filter, and (2) to sequentially interleave these N samples inside the sampling interval T_s. Indeed, this latter function is the essence of the time-division multiplexing operation. Following the commutation process, the multiplexed signal is applied to a *pulse modulator*, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel. It is clear that the use of time-division multiplexing introduces a bandwidth expansion factor N, because the scheme must squeeze N samples derived from N independent message sources into a time slot equal to one sampling interval. At the receiving end of the system, the received signal is applied to a *pulse demodulator*, which performs the reverse operation of the pulse modulator. The narrow samples produced at the pulse demodulator output are distributed to the appropriate low-pass reconstruction filters by means of a *decommutator*, which operates in *synchronism* with the commutator in the transmitter. This synchronization is essential for a satisfactory operation of the system.

The way this synchronization is implemented depends naturally on the method of pulse modulation used to transmit the multiplexed sequence of samples.

The TDM system is highly sensitive to dispersion in the common channel, that is, to variations of amplitude with frequency or lack of proportionality of phase with frequency. Accordingly, accurate equalization of both magnitude and phase responses of the channel is necessary to ensure a satisfactory operation of the system;

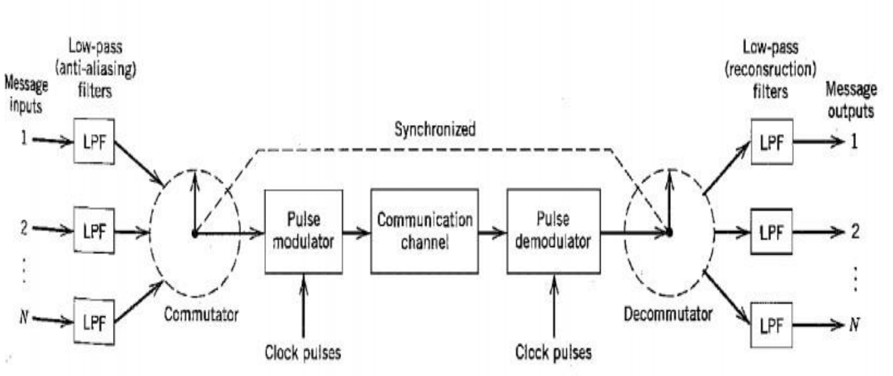


FIGURE Block diagram of TDM system.

TDM is immune to nonlinearities in the channel as a source of crosstalk. The reason for this behaviour is that different message signals are not simultaneously applied to the channel.

Introduction to Delta Modulation

PCM transmits all the bits which are used to code the sample. Hence signaling rate and transmission channel bandwidth are large in PCM. To overcome this problem Delta Modulation is used.

Delta Modulation

Operating Principle of DM

Delta modulation transmits only one bit per sample. That is the present sample value is compared with the previous sample value and the indication,whether the amplitude is increased or decreased is sent. Input signal $x(t)$ is approximated to step signal by the delta modulator. This step size is fixed . The difference between the input signal $x(t)$ and staircase approximated signal confined to two levels, i.e. $+\delta$ and $-\delta$. If the difference is positive, then approximated signal is increased by one step i.e. 'δ'. If the difference is negative, then approximated signal is reduced by 'δ'. When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted. Thus for each sample, only one binary bit is transmitted. Fig. shows the analog signal $x(t)$ and its staircase approximated signal by the delta modulator.

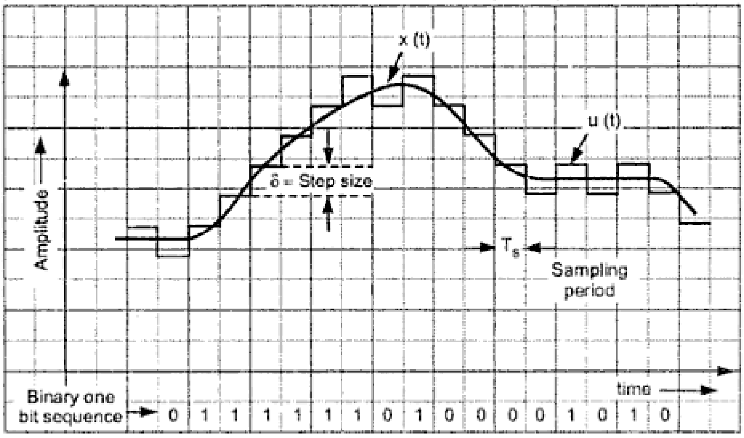


Fig. Delta modulation waveform

The principle of delta modulation can be explained by the following set of equations. The error between the sampled value of $x(t)$ and last approximated sample is given as,

$$e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{... (1)}$$

Here, $e(nT_s)$ = Error at present sample

$x(nT_s)$ = Sampled signal of $x(t)$

$\hat{x}(nT_s)$ = Last sample approximation of the staircase waveform.

We can call $u(nT_s)$ as the present sample approximation of staircase output.

Then, $u[(n-1)T_s] = \hat{x}(nT_s)$ ... (2)

= Last sample approximation of staircase waveform.

Let the quantity $b(nT_s)$ be defined as,

$$b(nT_s) = \delta\, sgn[e(nT_s)] \quad \text{... (3)}$$

That is depending on the sign of error $e(nT_s)$ the sign of step size δ will be decided. In other words,

$$b(nT_s) = +\delta \quad \text{if} \quad x(nT_s) \geq \hat{x}(nT_s)$$

$$= -\delta \quad \text{if} \quad x(nT_s) < \hat{x}(nT_s) \quad \text{... (4)}$$

If $b(nT_s) = +\delta$; binary '1' is transmitted

and if $b(nT_s) = -\delta$; binary '0' is transmitted.

T_s = Sampling interval.

DM Transmitter

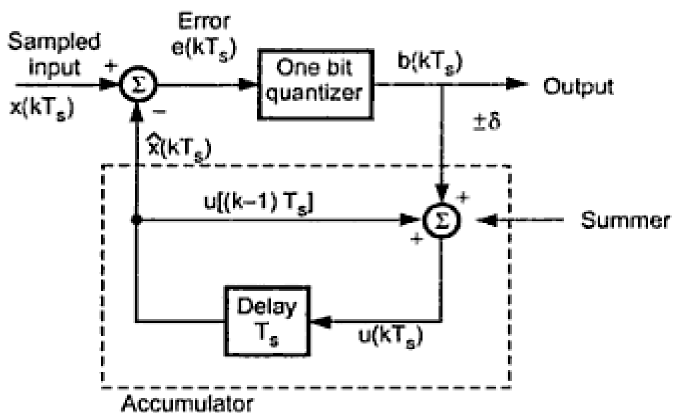
Fig. (a) shows the transmitter based on equations 3 to 5.

The summer in the accumulator adds quantizer output ($\pm\delta$) with the previous sample approximation. This gives present sample approximation. i.e.,

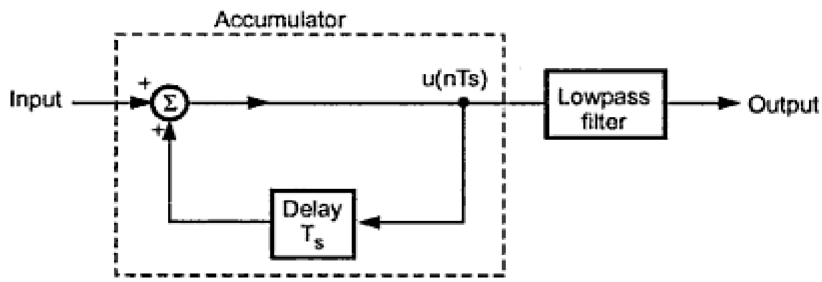
$$u(nT_s) = u(nT_s - T_s) + [\pm\delta] \quad \text{or}$$

$$= u[(n-1)T_s] + b(nT_s) \quad \ldots (5)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s. The sampled input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.



(a)



(b)

Fig. (a) Delta modulation transmitter and
(b) Delta modulation receiver

Depending on the sign of $e(nT_s)$ one bit quantizer produces an output step of $+\delta$ or $-\delta$. If the step size is $+\delta$, then binary '1' is transmitted and if it is $-\delta$, then binary '0' is transmitted.

DM Receiver

At the receiver shown in Fig. (b), the accumulator and low-pass filter are used. The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s. It is then added to the input signal. If input is binary '1' then it adds $+\delta$ step to the previous output (which is delayed). If input is binary '0' then one step 'δ' is subtracted from the delayed signal. The low-pass filter has the cutoff frequency equal to highest frequency in $x(t)$. This filter smoothen the staircase signal to reconstruct $x(t)$.

Advantages and Disadvantages of Delta Modulation

Advantages of Delta Modulation

The delta modulation has following advantages over PCM,

1. Delta modulation transmits only one bit for one sample. Thus the signaling rate and transmission channel bandwidth is quite small for delta modulation.
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter involved in delta modulation.

Disadvantages of Delta Modulation

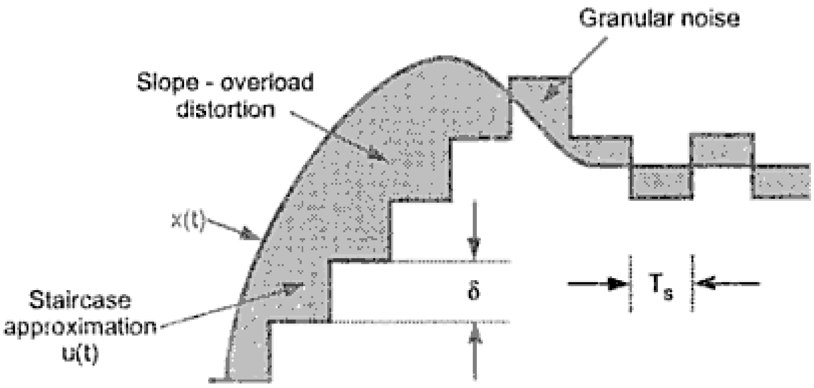


Fig. Quantization errors in delta modulation

The delta modulation has two drawbacks -

Slope Overload Distortion (Startup Error)

This distortion arises because of the large dynamic range of the input signal.

As can be seen from Fig. the rate of rise of input signal $x(t)$ is so high that the staircase signal cannot approximate it, the step size 'δ' becomes too small for staircase signal $u(t)$ to follow the steep segment of $x(t)$. Thus there is a large error between the staircase approximated signal and the original input signal $x(t)$. This error is called *slope overload distortion*. To reduce this error, the step size should be increased when slope of signal of $x(t)$ is high.

Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore this modulator is also called Linear Delta Modulator (LDM).

Granular Noise (Hunting)

Granular noise occurs when the step size is too large compared to small variations in the input signal. That is for very small variations in the input signal, the staircase signal is changed by large amount (δ) because of large step size. Fig shows that when the input signal is almost flat, the staircase signal $u(t)$ keeps on oscillating by $\pm\delta$ around the signal. The error between the input and approximated signal is called *granular noise*. The solution to this problem is to make step size small.

Thus large step size is required to accommodate wide dynamic range of the input signal (to reduce slope overload distortion) and small steps are required to reduce granular noise. Adaptive delta modulation is the modification to overcome these errors.

Adaptive Delta Modulation

Operating Principle

To overcome the quantization errors due to slope overload and granular noise, the step size (δ) is made adaptive to variations in the input signal $x(t)$. Particularly in the steep segment of the signal $x(t)$, the step size is increased. When the input is varying slowly, the step size is reduced. Then the method is called *Adaptive Delta Modulation (ADM)*.

The adaptive delta modulators can take continuous changes in step size or discrete changes in step size.

Transmitter and Receiver

Fig. (a) shows the transmitter and (b) shows receiver of adaptive delta modulator. The logic for step size control is added in the diagram. The step size increases or decreases according to certain rule depending on one bit quantizer output.

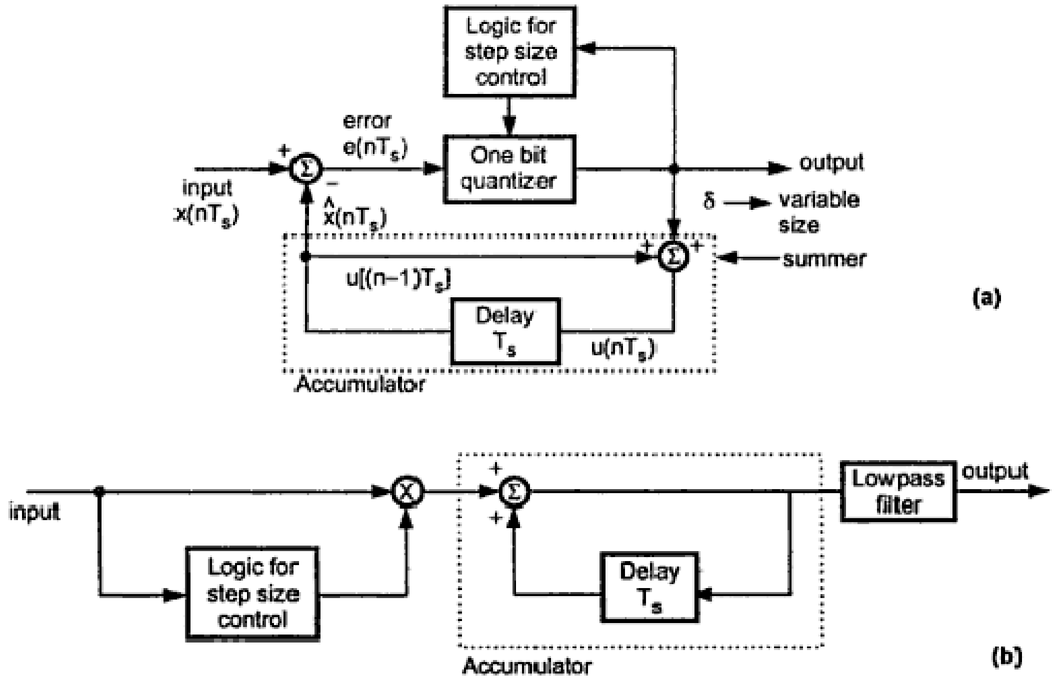


Fig. Adaptive delta modulator (a) Transmitter (b) Receiver

For example if one bit quantizer output is high (1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step. Fig. shows the waveforms of adaptive delta modulator and sequence of bits transmitted.

In the receiver of adaptive delta modulator shown in Fig. (b) the first part generates the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decides the step size. It is then given to an accumulator which builds up staircase waveform. The low-pass filter then smoothens out the staircase waveform to reconstruct the smooth signal.

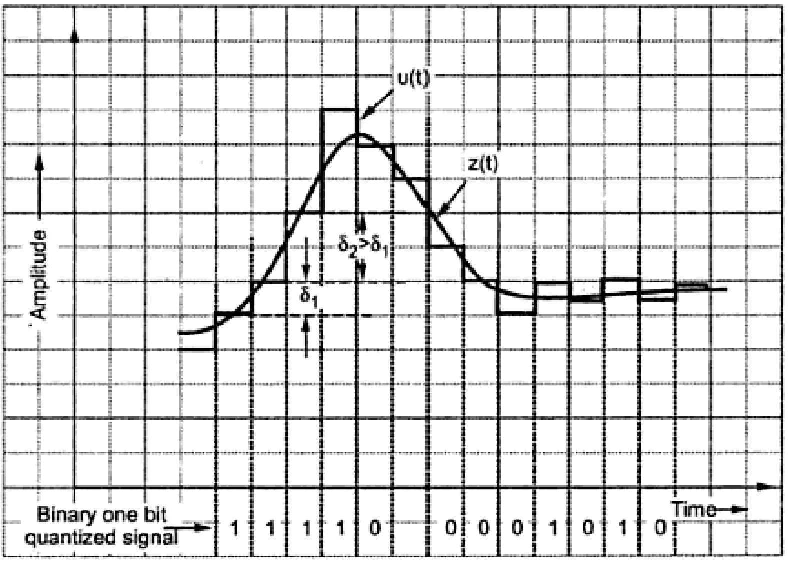


Fig. Waveforms of adaptive delta modulation

Advantages of Adaptive Delta Modulation

Adaptive delta modulation has certain advantages over delta modulation. i.e.,

1. The signal to noise ratio is better than ordinary delta modulation because of the reduction in slope overload distortion and granular noise.
2. Because of the variable step size, the dynamic range of ADM is wide.
3. Utilization of bandwidth is better than delta modulation.

Plus other advantages of delta modulation are, only one bit per sample is required and simplicity of implementation of transmitter and receiver.

Condition for Slope overload distortion occurrence:

Slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

where T_s is the sampling period.

Let the sine wave be represented as,

$$x(t) = A_m \sin(2\pi f_m t)$$

Slope of $x(t)$ will be maximum when derivative of $x(t)$ with respect to 't' will be maximum. The maximum slope of delta modulator is given

$$\text{Max. slope} = \frac{\text{Step size}}{\text{Sampling period}}$$

$$= \frac{\delta}{T_s} \qquad \text{.............(1)}$$

Slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e.

$$\max\left|\frac{d}{dt}x(t)\right| > \frac{\delta}{T_s}$$

$$\max\left|\frac{d}{dt}A_m \sin(2\pi f_m t)\right| > \frac{\delta}{T_s}$$

$$\max|A_m\, 2\pi f_m \cos(2\pi f_m t)| > \frac{\delta}{T_s}$$

$$A_m\, 2\pi f_m > \frac{\delta}{T_s}$$

or

$$\boxed{A_m > \frac{\delta}{2\pi f_m T_s}} \qquad \text{...............(2)}$$

Expression for Signal to Quantization Noise power ratio for Delta Modulation:

To obtain signal power :

slope overload distortion will not occur if

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

Here A_m is peak amplitude of sinusoided signal

δ is the step size

f_m is the signal frequency and

T_s is the sampling period.

From above equation, the maximum signal amplitude will be,

$$A_m = \frac{\delta}{2\pi f_m T_s} \qquad \text{..................(1)}$$

Signal power is given as,

$$P = \frac{V^2}{R}$$

Here V is the rms value of the signal. Here $V = \frac{A_m}{\sqrt{2}}$. Hence above equation becomes,

$$P = \left(\frac{A_m}{\sqrt{2}}\right)^2 / R$$

Normalized signal power is obtained by taking R = 1. Hence,

$$P = \frac{A_m^2}{2}$$

Putting for A_m from equation 1

$$P = \frac{\delta^2}{8\pi^2 f_m^2 T_s^2} \qquad \text{.................(2)}$$

This is an expression for signal power in delta modulation.

(ii) To obtain noise power

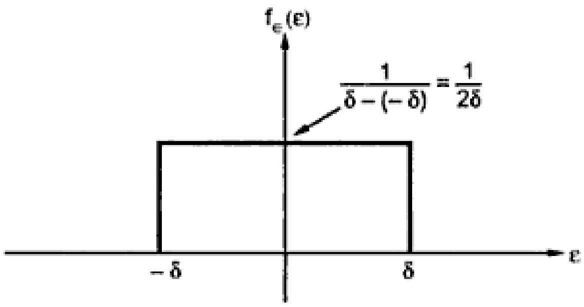


Fig. Uniform distribution of quantization error

We know that the maximum quantization error in delta modulation is equal to step size 'δ'. Let the quantization error be uniformly distributed over an interval $[-\delta, \delta]$. This is shown in Fig. From this figure the PDF of quantization error can be expressed as,

$$f_\epsilon(\varepsilon) = \begin{cases} 0 & for \quad \varepsilon < \delta \\ \frac{1}{2\delta} & for \quad -\delta < \varepsilon < \delta \\ 0 & for \quad \varepsilon > \delta \end{cases} \qquad ....(3)$$

The noise power is given as,

$$\text{Noise power} = \frac{V_{noise}^2}{R}$$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF $f_\epsilon(\varepsilon)$, its mean square value is given as,

$$\text{mean square value} = E[\varepsilon^2] = \overline{\varepsilon^2}$$

mean square value is given as,

$$E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\epsilon(\varepsilon)\, d\varepsilon$$

From equation 3

$$E[\varepsilon^2] = \int_{-\delta}^{\delta} \varepsilon^2 \cdot \frac{1}{2\delta}\, d\varepsilon$$

$$= \frac{1}{2\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\delta}^{\delta}$$

$$= \frac{1}{2\delta}\left[\frac{\delta^3}{3} + \frac{\delta^3}{3}\right] = \frac{\delta^2}{3} \qquad ....(4)$$

Hence noise power will be,

$$\text{noise power} = \left(\frac{\delta^2}{3}\right) / R$$

Normalized noise power can be obtained with R = 1. Hence,

$$\text{noise power} = \frac{\delta^2}{3} \qquad ....(5)$$

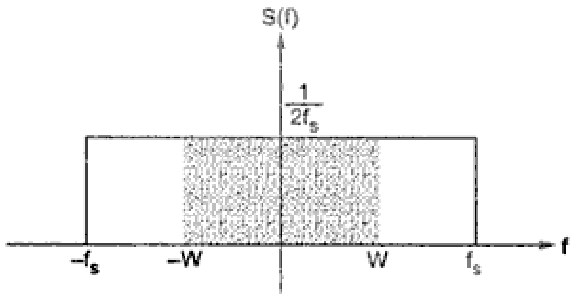


Fig. PSD of noise

This noise power is uniformly distributed over $-f_s$ to f_s range. This is illustrated in Fig. At the output of delta modulator receiver there is lowpass reconstruction filter whose cutoff frequency is 'W'. This cutoff frequency is equal to highest signal frequency. The reconstruction filter passes part of the noise power at the output as Fig. From the geometry of Fig. output noise power will be,

$$\text{Output noise power} = \frac{W}{f_s} \times \text{noise power} = \frac{W}{f_s} \times \frac{\delta^2}{3}$$

We know that $f_s = \frac{1}{T_s}$, hence above equation becomes,

$$\text{Output noise power} = \frac{W T_s \delta^2}{3} \qquad \text{.................(6)}$$

(iii) To obtain signal to noise power ratio

Signal to noise power ratio at the output of delta modulation receiver is given as,

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}$$

From equation 2. and equation 6

$$\frac{S}{N} = \frac{\dfrac{\delta^2}{8\pi^2 f_m^2 T_s^2}}{\dfrac{W T_s \delta^2}{3}}$$

$$\boxed{\frac{S}{N} = \frac{3}{8\pi^2 W f_m^2 T_s^3}} \qquad \text{..............(7)}$$

This is an expression for signal to noise power ratio in delta modulation.