III - ECE AWT ①

S1 & S2. UNIT-3

A. Narasimha Reddy ANTENNA ARRAYS :- [redacted]

Antenna array :- The antenna array is a radiating system, in which the group of antennas are arranged in parallel to each other. There-fore to get the maximum radiation and the high directivity, Increased field strength.

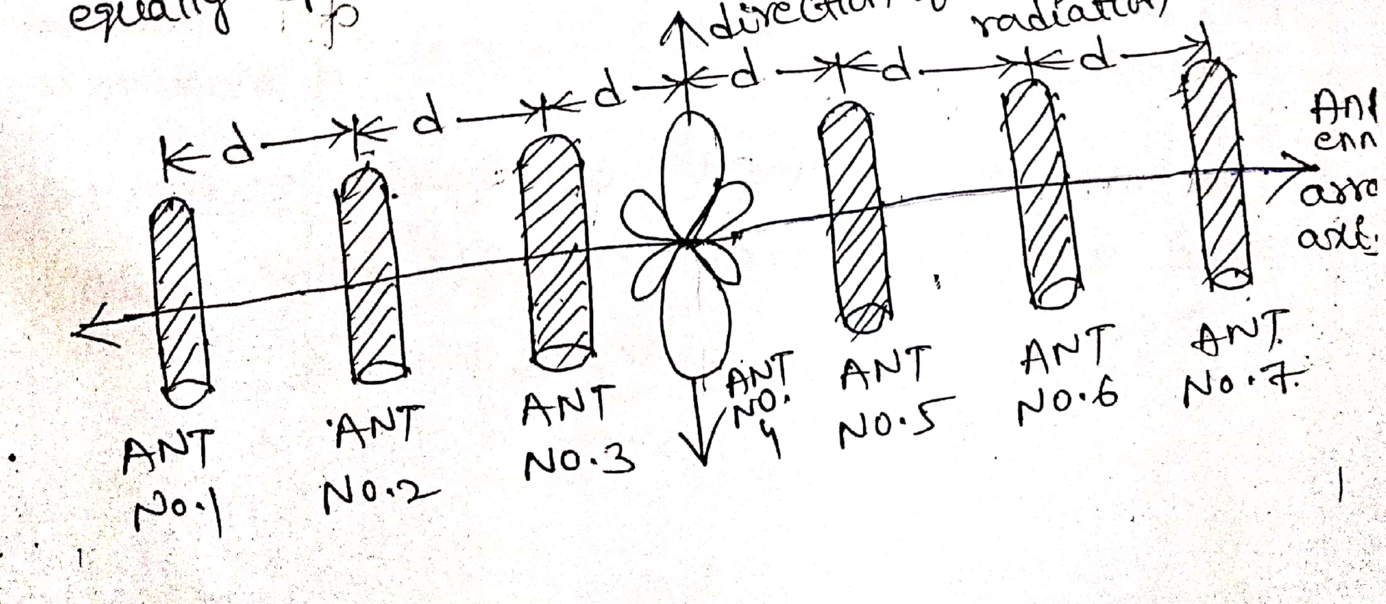
Various types of arrays :- There are different types of arrays.

(1) Broad side array
(2) End fire array
(3) collinear array
(4) parasitic array.

(1) Broad side array :-

→ Broad side array is defined as " An arrange-ment in which the principal direction of radi-ation is perpendicular to the array axis and also to the antenna plane.

→ It consists of Identical parallel antennas equally spaced along a line perpendicular to axes.



→ A horisontal radiation pattern is obtained when these arrays are vertically arranged.

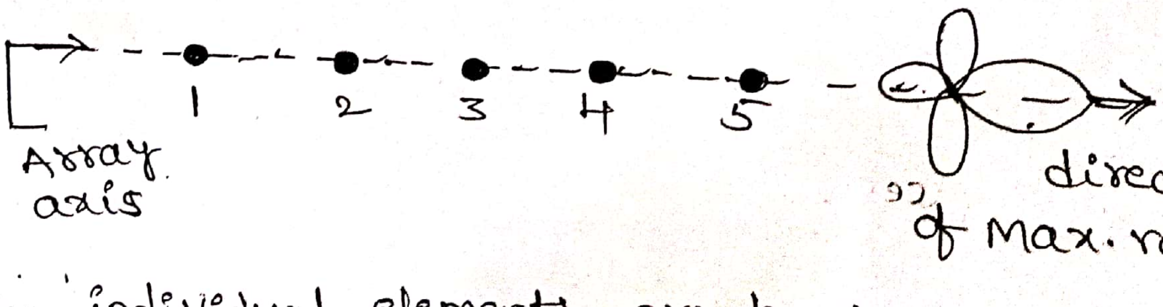
→ A vertical radiation pattern is obtained when these arrays are horizontally arranged.

→ In this broad side array, the individual elemen are having currents of equal Amplitudes and same phases.

(2) <u>End fire array</u> :-

→ An end-fire array is defined as "The arrangement in which the principal direction of maximum radiation coincides with the direction of array axis.

→ The end fire array is similar to broad-side array except that individual elements are fed in with currents out of phase 180°.



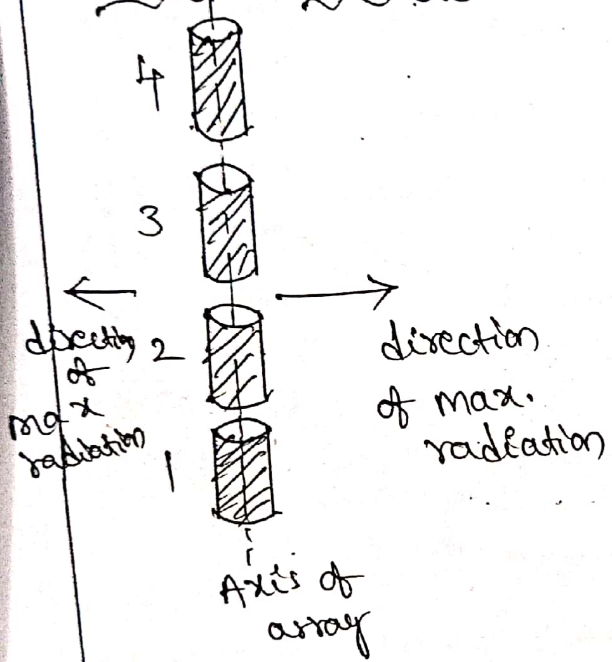
→ The individual elements are having currents of equal amplitudes and opposite phase (180°)

Collinear arrays :-

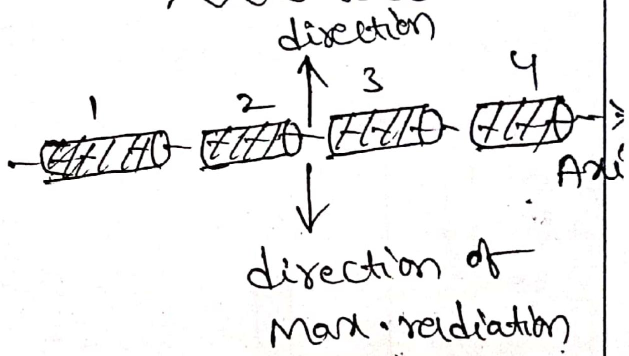
→ In collinear array, the antennas are arranged co-axially, that is antennas are mounted end to end in a single line.

→ A collinear array is a broad side radiator, in which the direction of maximum radiation is perpendicular to line of antenna.

Vertical arrangement

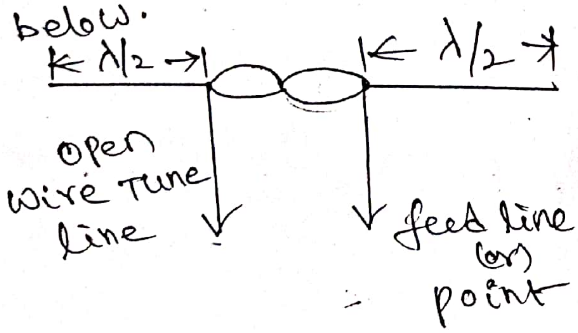


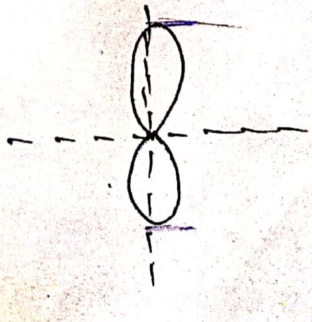
horizontal arrangement



→ The power gain of collinear array is maximum when the spacing between elements is 0.3λ to 0.5λ.

→ The two element collinear array is given below.





→ Two elements Collinear array is also Called The "Two half waves in phase".

4) parasitic arrays:-

→ Multielement arrays having number of parasitic elements are called "parasitic arrays". whether driven element is one (or) more.

→ a parasitic array with linear half wave dipole as elements is normally called as "Yagi-Uda" (or) simply "Yagi" antenna.

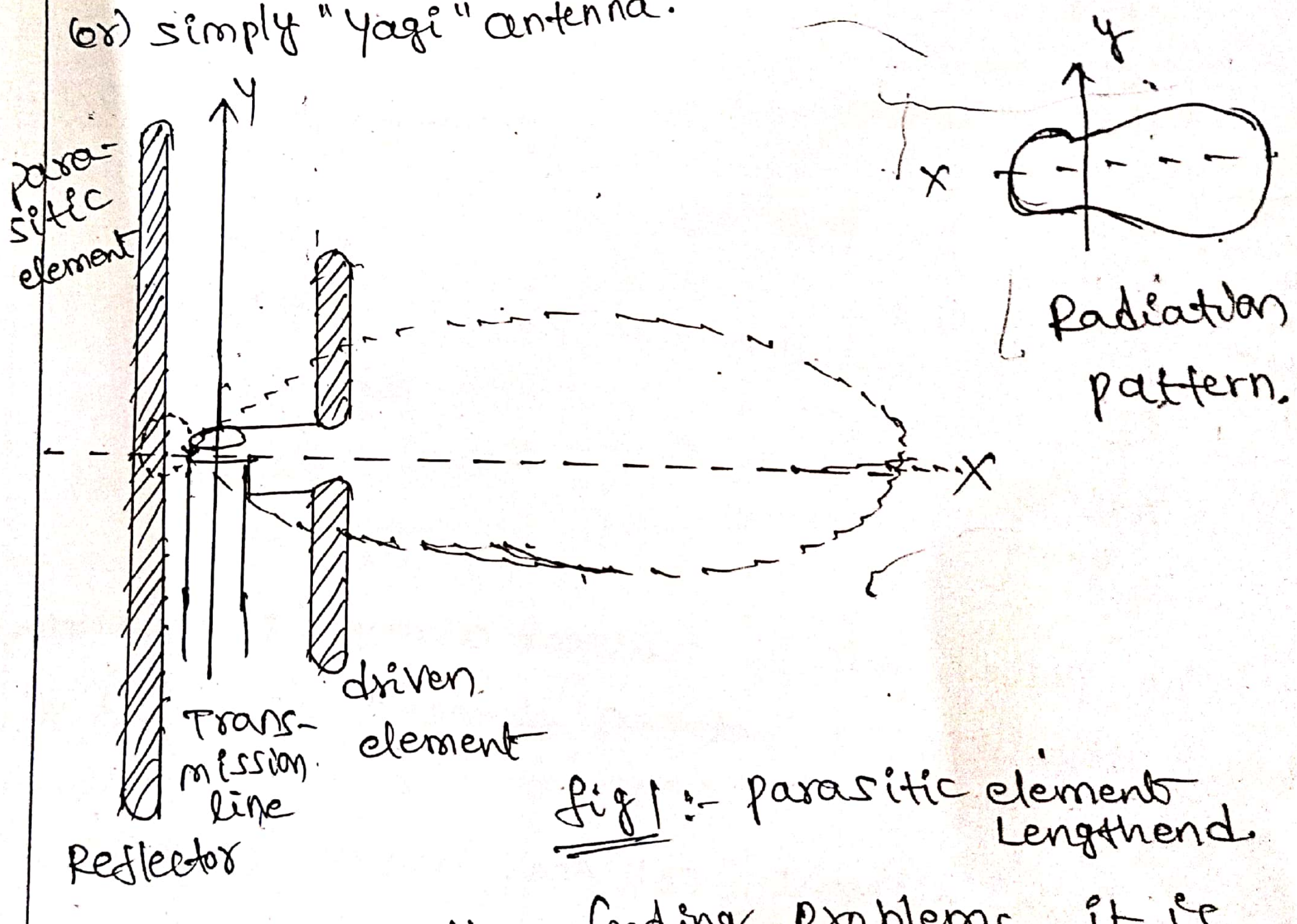
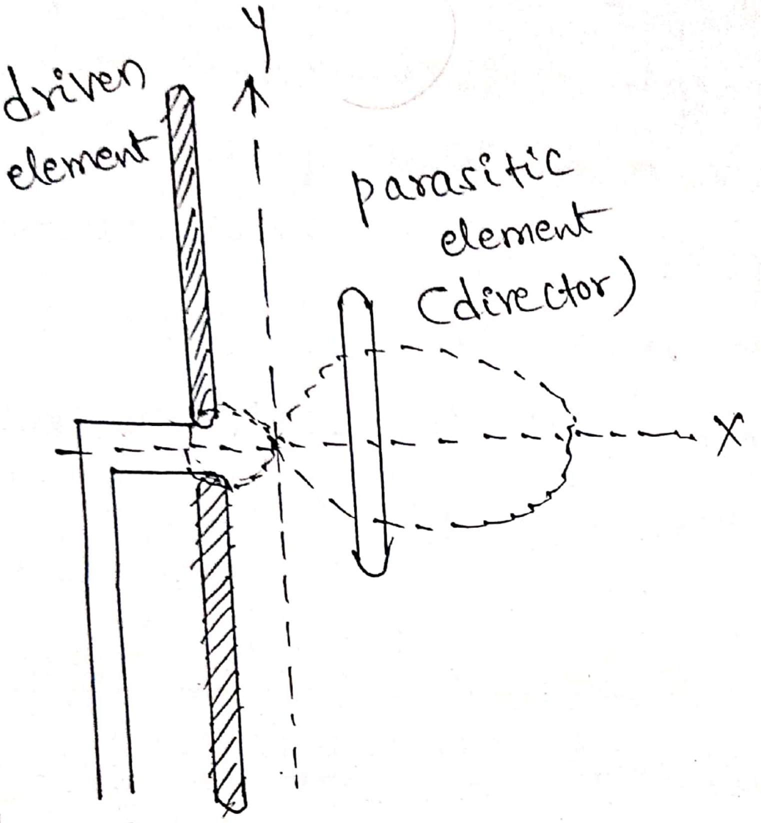


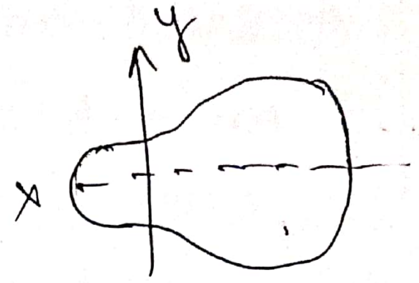
fig 1 :- parasitic elements Lengthend.

→ To overcome the feeding problems, it is desirable to use parasitic arrays.

→ The element supply power directly from source through Transmission line called driven element

The amplitude and phase of the current ③ ~~element~~ induced in a parasitic element depends on tuning and spacing between parasitic element and driven element





Radiation pattern.

fig 2:- parasitic element shortend.

→ A parasitic element lengthend by 5%. with respect to driven element acts as reflector.

→ shortend by 5% act as director.

TWO element arrays :- different cases :-

* The array of point sources nothing but the array of an isotropic radiators occupying zero volume.

* For the greater no. of point source in the array the analysis is very camplicated, and time consuming.

* The simplest condition of no. of point sou[rces]
in the array is two.

There are 3 types of two element arrays

(i) Array of two point sources with curr[ent]
of Equal amplitude and same phase.

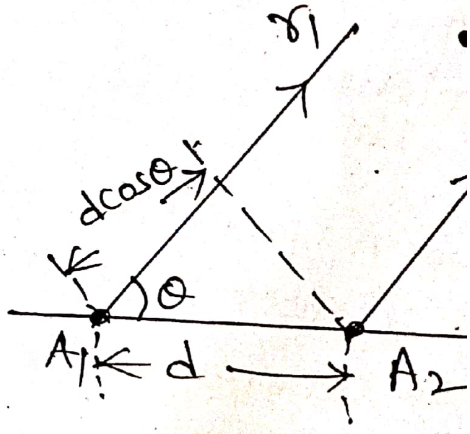
(ii) Array of two point sources with curr[ent]
of equal amplitude and opposite phase

(iii) Array of two point sources with curren[t]
of unequal amplitude and any phase.

(i) Array of two point sources with curre[nt] of equal amplitudes and same phase.

* Consider two point sources A_1 and A_2 seperated by distance 'd'.

* Let both point sources are supplied with currents equal in Amplitude (or) magnitude, same phase.



Two element array.

* The distance between point 'p' and A_1 is r_1 and distance between point 'p' and A_2 is r_2.

We can assume $r_1 = r_2 = r$.

$\therefore$ The path difference $= d\cos\theta$

In terms of wavelength, the path difference is

$$p.d = \frac{d\cos\theta}{\lambda} \rightarrow (1)$$

$\therefore$ The phase angle $\psi = 2\pi \times$ path difference

$$\Rightarrow \psi = 2\pi \times \frac{d\cos\theta}{\lambda}$$

$$\Rightarrow \psi = \frac{2\pi}{\lambda} \times d\cos\theta$$

$$\boxed{\psi = \beta d\cos\theta} \rightarrow (2)$$

Let E_1 be the far electric field at 'p' due to A_1.

$\Rightarrow$ E_2 be the far electric field at 'p' due to A_2.

The total field at point 'p' is given by

$$E = E_1 e^{-j\frac{\psi}{2}} + E_2 e^{j\psi/2}$$

assume equal amplitudes, same phase.

$$E_1 = E_2 = E_0$$

$$E = E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E = E_0 (e^{-j\psi/2} + e^{j\psi/2}) \qquad (\because e^{j\theta} + e^{-j\theta} = 2\cos\theta)$$

$$E = E_0 \left(2\cos\frac{\psi}{2}\right)$$

$$E = E_0 \left(2\cos\left(\frac{\beta d\cos\theta}{2}\right)\right)$$

$$\therefore \boxed{E = 2E_0 \cos\left(\frac{\beta d\cos\theta}{2}\right)} \rightarrow (3)$$

Where E_0 = Max. amplitude.

$$\beta = \frac{2\pi}{\lambda}, \quad d = \lambda/2$$

~~Maxima direction~~ :-

The array factor is defined as the ratio magnitude of total field to magnitude of maxi field

$$A \cdot F = \frac{|E|}{|E_{max}|} = \frac{|E|}{|2E_0|}$$

from eqn ③

$$A \cdot F = \frac{E}{2E_0} = \cos\left(\frac{\beta d \cos\theta}{2}\right)$$

maximum direction :-

For maximum direction we have to equalize ± to Array factor.

$$\therefore \cos\left(\frac{\beta d \cos\theta}{2}\right) = \pm 1$$

$$\cos\left(\frac{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta}{2}\right) = \pm 1$$

$$\cos\left(\frac{\pi}{2} \cos\theta\right) = \pm 1$$

$$\frac{\pi}{2} \cos\theta_{max} = \cos^{-1}(\pm 1)$$

$$\frac{\pi}{2} \cos\theta_{max} = \pm n\pi \quad \text{where } n = 0, 1, 2 \ldots$$

if $n = 0$ then

$$\frac{\pi}{2} \cos\theta_{max} = 0$$

$$\cos\theta_{max} = 0$$

$$\theta_{max} = \cos^{-1}(0)$$

$$\boxed{\theta_{max} = 90^\circ \text{ (or) } 270^\circ}$$

Minimum direction :-

The total field strength is minimum when $\cos\left(\frac{\pi}{2}\cos\theta\right)$ is '0'.

$$\left(\because \beta = \frac{2\pi}{\lambda},\ d = \frac{\lambda}{2}\right)$$

$$\therefore \cos\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \cos^{-1}(0)$$

$$\frac{\pi}{2}\cos\theta_{min} = \pm(2n+1)\frac{\pi}{2} \quad n=0,1,2\ldots$$

if $n=0$ then

$$\cancel{\frac{\pi}{2}}\cos\theta_{min} = \pm\cancel{\frac{\pi}{2}}$$

$$\theta_{min} = \cos^{-1}(\pm 1) \qquad \left(\because \cos 0^\circ = 1,\ \cos 180^\circ = -1\right)$$

$$\therefore \boxed{\theta_{min} = 0^\circ \text{ (or) } 180^\circ}$$

Half power point Direction :-

When the power is half the voltage (or) current is $\frac{1}{\sqrt{2}}$ times of maximum value.

$$\therefore \cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4} \quad n=0,1,2\ldots$$

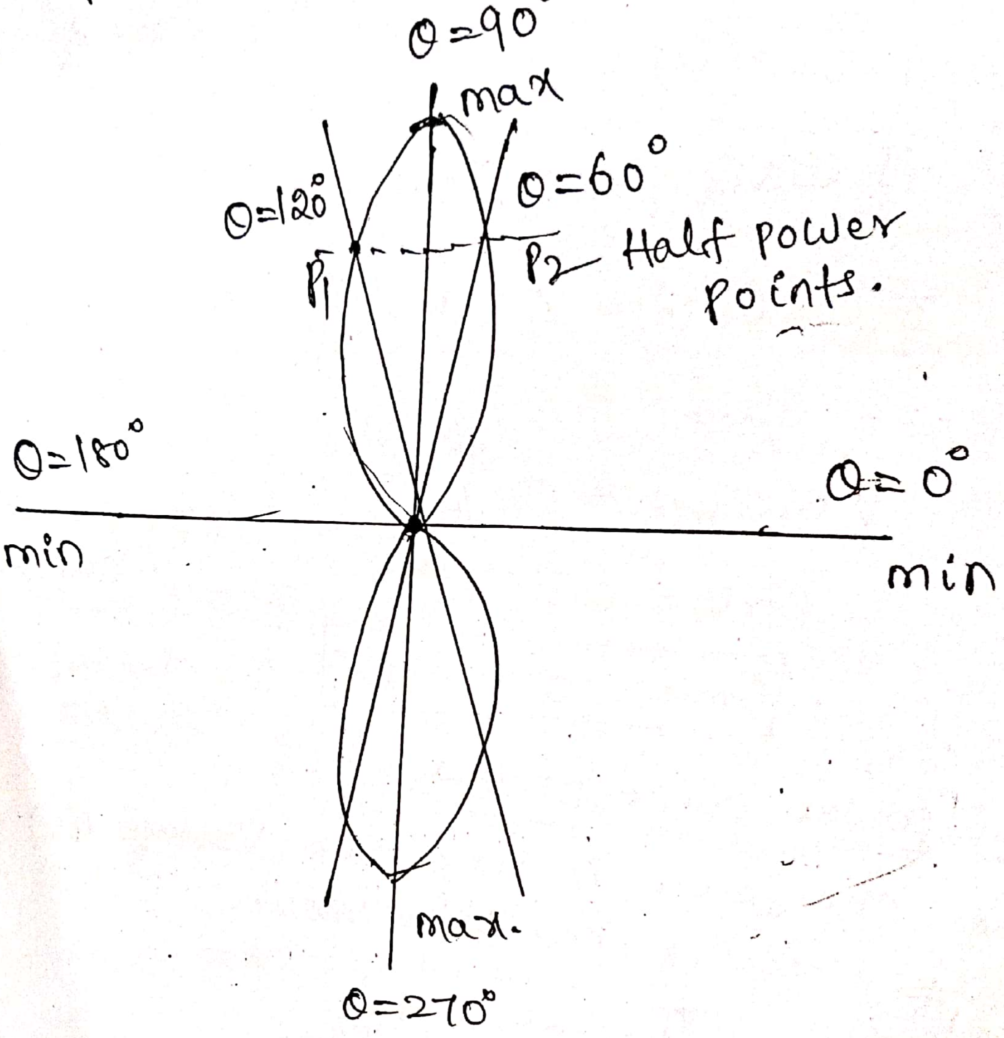
if $n=0$ then $\cancel{\frac{\pi}{2}}\cos\theta_{HPPD} = \pm\frac{\cancel{\pi}}{\cancel{4}_2}$

$$\Rightarrow \cos\theta_{HPPD} = \pm\frac{1}{2} \qquad \left(\because \cos 60^\circ = \frac{1}{2},\ \cos 120^\circ = -\frac{1}{2}\right)$$

$$\Rightarrow \theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

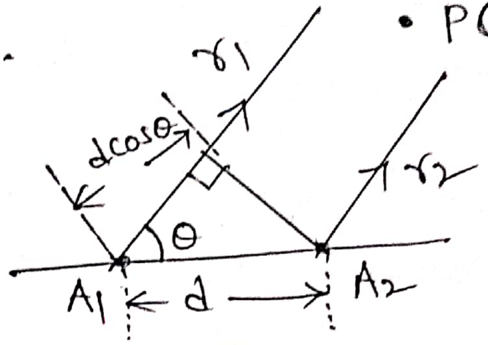
$$\therefore \boxed{\theta_{HPPD} = 60^\circ \text{ (or) } 120^\circ}$$

The field pattern for two element array with equal amplitudes and same phase is given below.



Arrays of two point sources with equal Amplitude and opposite phase:-

→ consider two point sources seperated by distance 'd' and supplied with currents equal in Amplitude but opposite phase.



'p' is distant point.

The total far field at distant point 'p' is given by

$$E = -E_1 e^{-j\psi/2} + E_2 e^{j\psi/2} \rightarrow (1)$$

Let $E_1 = E_2 = E_0$

The phase of source 1 is $-\frac{\psi}{2}$, phase of source 2 is $\frac{\psi}{2}$

$$\Rightarrow E = -E_0 e^{-j\frac{\psi}{2}} + E_0 e^{j\frac{\psi}{2}}$$

$$\Rightarrow E = E_0\left[-e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right]$$

2cos θ

2j sinθ

$(\because e^{j\theta} - e^{-j\theta} = 2j\sin\theta)$

$$\Rightarrow E = E_0\left(2j\sin\frac{\psi}{2}\right)$$

$$\therefore \boxed{E = 2jE_0 \sin\left(\frac{\beta d\cos\theta}{2}\right)} \rightarrow (2) \quad (\because \psi = \beta d\cdot\cos\theta)$$

The array factor is given by

$$A.F = \frac{|E|}{|2jE_0|}$$

So eq (2) becomes

$$A.F = \frac{E}{2E_0} = \sin\left(\frac{\beta d\cos\theta}{2}\right) \quad \left(\because \beta = \frac{2\pi}{\lambda},\ d = \lambda/2\right)$$

$$A.F = \frac{E}{2E_0} = \sin\left(\frac{\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\theta}{2}\right)$$

$\therefore$ $\boxed{\text{array factor} = \sin\left(\frac{\pi}{2}\cos\theta\right)}$

<u>maximum direction</u> :- The maximum value of sine function is ± 1

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm 1$$

$$\frac{\pi}{2}\cos\theta_{max} = \sin^{-1}(\pm 1)$$

where $n = 0, 1, 2 \cdots$

$$\Rightarrow \frac{\pi}{2}\cos\theta_{max} = \pm(2n+1)\frac{\pi}{2}$$

if $n=0$ then $\cancel{\frac{\pi}{2}}\cos\theta_{max} = \pm\cancel{\frac{\pi}{2}}$

$$\therefore \theta_{max} = \cos^{-1}(\pm 1)$$

$(\because \cos 0° = 1$

$\cos 180° = -1)$

~~θ_{max} =~~

$$\boxed{\theta_{max} = 0° \text{ (or) } 180°}$$

<u>minimum direction</u> :- The minimum value of sine function is '0'

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = 0$$

$$\frac{\pi}{2}\cos\theta_{min} = \sin^{-1}(0)$$

where

$$\frac{\pi}{2}\cos\theta_{min} = \pm n\pi \quad n = 0, 1, 2 \cdots$$

if $n=0$ then $\frac{\pi}{2}\cos\theta_{min} = 0$

$$\cos\theta_{min} = 0$$

$$\theta_{min} = \cos^{-1}(0)$$

$$\therefore \boxed{\theta_{min} = 90° \text{ (or) } 270°}$$

(7)

alf power point directions :-

$$\sin\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4}$$ where $n=0,1,2\ldots$

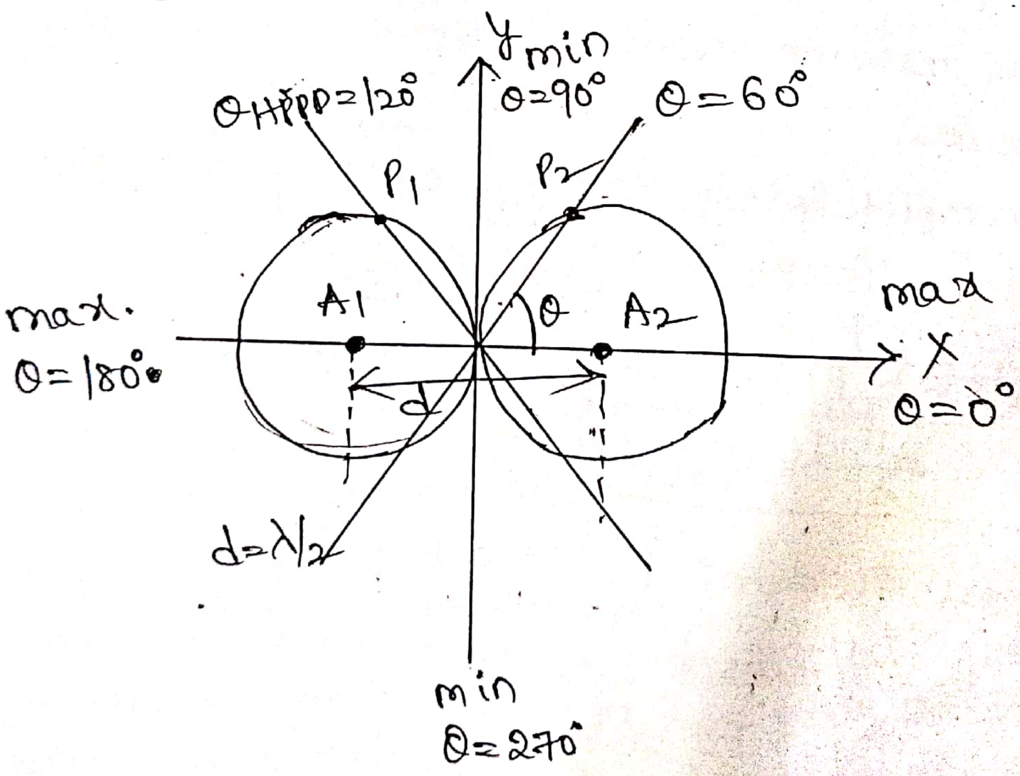
if $n=0$ then

$$\frac{\cancel{\pi}}{\cancel{2}}\cos\theta_{HPPD} = \pm\frac{\cancel{\pi}}{4_2}$$

$$\theta_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$\therefore$ $\boxed{\theta_{HPPD} = 60^\circ \text{ and } 120^\circ}$ $\left(\because \cos 60^\circ = \frac{1}{2},\ \cos 120^\circ = -\frac{1}{2}\right)$

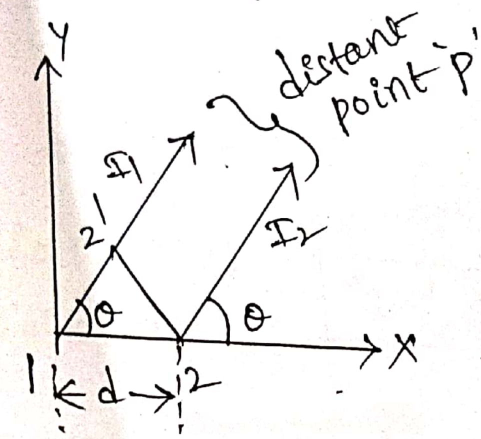
The field pattern for two point sources with spacing $d=\lambda/2$ and equal amplitudes, opposite phase (180°)



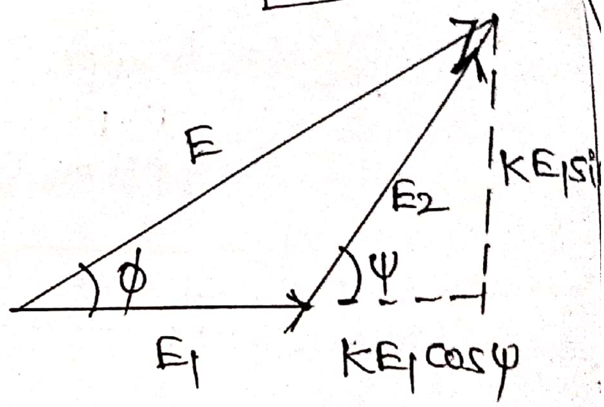
i) Array of two point sources with unequal amplitudes and any phase.

→ Let us now consider a general condition in which the amplitudes of two point sources are not equal and any phase difference say 'α'.

$$\boxed{E_2 = KE_1}$$



two point sources with unequal amplitudes & any phase



Vector diagram.

→ Let us assume source 1 is taken as reference for phase.

→ The amplitudes of source 1 and 2 at point 'p' are E_1 and E_2 ($\because E_1 > E_2$).

The total phase angle is given by

$$\boxed{\psi = \frac{2\pi}{\lambda} d\cos\theta + \alpha} \rightarrow (1)$$

The total field at 'p' is given by

$$E = E_1 e^{j \cdot 0} + E_2 e^{j\psi} = E_1 + E_2 e^{j\psi}$$

$$\Rightarrow E = E_1\left(1 + \frac{E_2}{E_1} e^{j\psi}\right)$$

$$\therefore \boxed{E = E_1(1 + K e^{j\psi})} \rightarrow (2)$$

where $K = \frac{E_2}{E_1}$

if $E_1 > E_2$ then $\frac{E_1}{E_2} > 1$

$\therefore \frac{E_2}{E_1} < 1$

$\therefore K < 1$

$$\boxed{0 \leq K \leq 1}$$

from equation ② The magnitude and phase angle can be obtained.

$$E = \left| E_1 \{ 1 + K(\cos\psi + j\sin\psi) \} \right|$$

$$\therefore E = E_1 \sqrt{(1+K\cos\psi)^2 + (K\sin\psi)^2} \angle\phi$$

ϕ = phase angle at 'p'

$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{K\sin\psi}{1+K\cos\psi}\right)}$$

principle of pattern multiplication:-

→ The pattern multiplication is a mathematical & simple method to obtain radiation patterns of arrays.

→ It is very useful in designing of arrays because it makes possible to draw the patterns of complicated arrays.

*. The total field pattern of an array of non-isotropic but similar sources is the multiplication of individual source patterns and pattern of an array of isotropic point sources each located at phase centre of individual source.

The total field pattern of an array of non isotropic but similar source is given by

$$E = \{E_i(\theta,\phi) \times E_a(\theta,\phi)\} \times \{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}$$

Where

E = Total field

$E_i(\theta,\phi)$ = field pattern of individual source

$E_a(\theta,\phi)$ = field pattern of array of isotropic source

$E_{pi}(\theta,\phi)$ = phase pattern of individual source

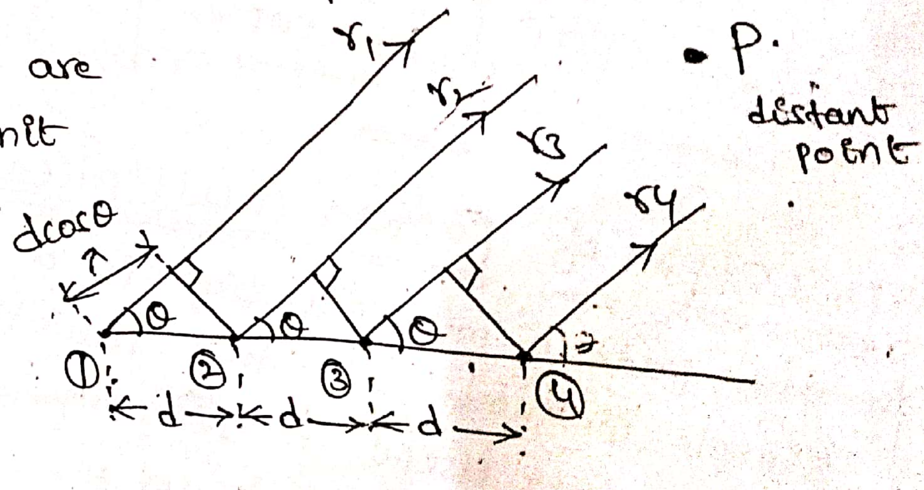
$E_{pa}(\theta,\phi)$ = phase pattern of array of isotropic point source.

3:-

Radiation pattern of 4-Isotropic elements fed in phase spaced $\frac{\lambda}{2}$ apart:-

→ Two isotropic point sources spaced $\frac{\lambda}{2}$ apart fed in phase provides a bidirectional pattern.

→ Elements ① and ② are considered as one unit and is to be placed between the middle of the elements.



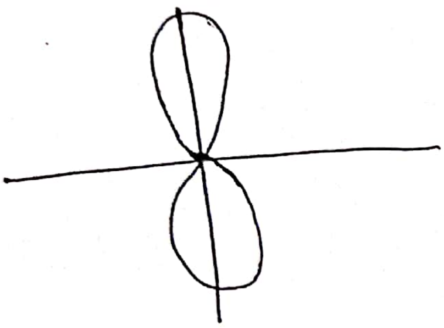
→ Also the elements ③,④ are considered as one unit assumed to be placed between the middle of the two elements.

Fig:-a four element linear array.

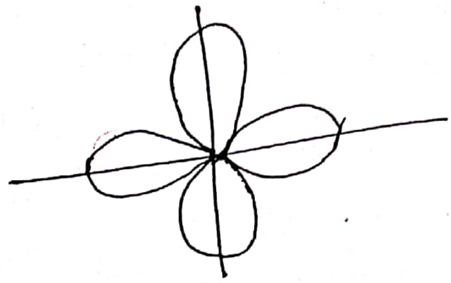
Now we can replace elements ① and ② by a single antenna located at a point midway between them by $\left(\frac{d}{2}\right)$.

Similarly replace elements ③ and ④ by single antenna having same pattern

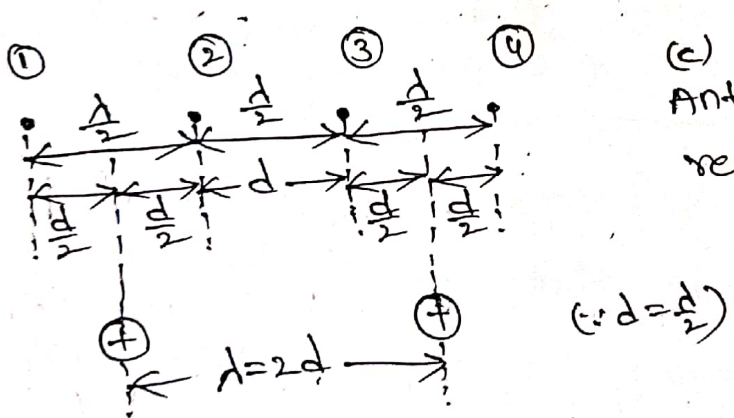
The resultant radiation pattern of four elements array can be obtained as multiplication of patterns.



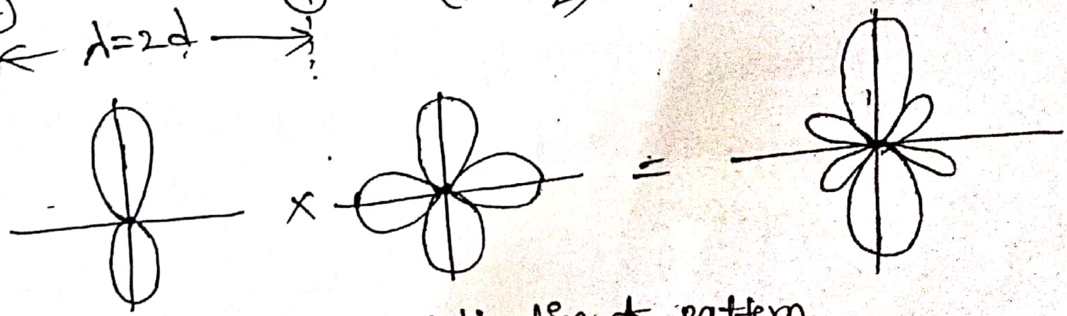
(a) radiation pattern of two antennas spaced at distance $\frac{\lambda}{2}$ and with equal currents, same phase.



(b) radiation pattern of two antennas spaced at distance λ with equal currents, same phase.



(c) Antennas ①, ② and ③, ④ replaced by single antenna seperately



(d) multiplication of pattern.

n element uniform linear array:-

→ An array is said to be linear, if the individual elements of the array are spaced equally along a line and it is uniform if the array are fed with currents of equal amplitude & uniform progressive phase shift.

→ Now we shall calculate pattern of linear array of 'n' isotropic point sources which are spaced equally.

→ The total far field at distant point 'P' is

$$E_t = E_0 e^{j\cdot 0} + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} + \cdots + E_0 e^{j(n-1)\psi}$$

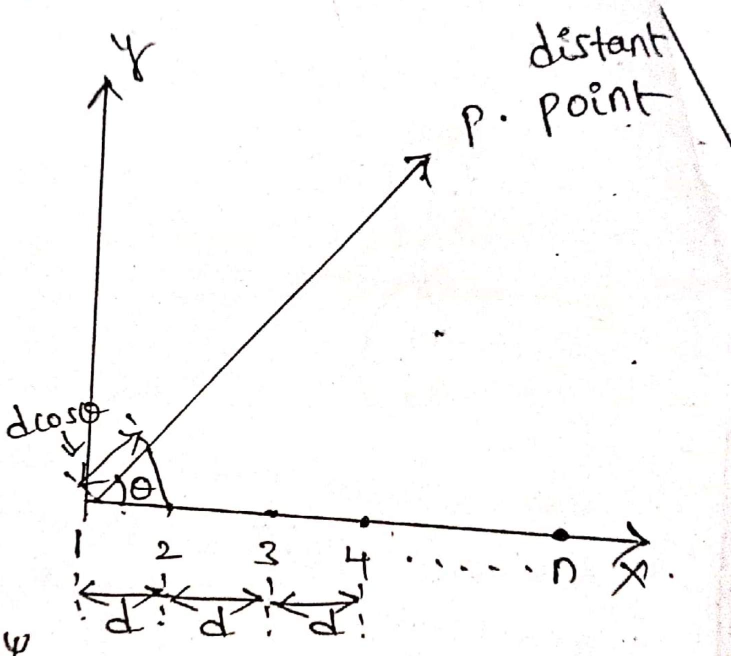


fig:- Linear array with 'n' isotropic point sources.

$$\Rightarrow \boxed{E_t = E_0\left[1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + e^{j4\psi} + \cdots e^{j(n-1)\psi}\right]} \rightarrow (1)$$

where $\psi = (\beta d \cos\theta + \alpha)$ radian

= Total phase difference of fields at 'P'

α = phase difference in adjacent sources (or) progressive phase shift b/w two point sources.

Multiplying eq ① by $e^{j\psi}$

$$E_t e^{j\psi} = E_0\left(e^{j\psi} + e^{2j\psi} + e^{j3\psi} + e^{j4\psi} + \cdots + e^{jn\psi}\right) \rightarrow ②$$

Subtracting eq ② from ①

$$① - ② \Rightarrow E_t - E_t e^{j\psi} = \left\{E_0(1 + \cancel{e^{j\psi}} + \cancel{e^{2j\psi}} + \cancel{e^{j3\psi}} + \cdots + \cancel{e^{j(n-1)\psi}}\right\}$$

$$- \left\{E_0(\cancel{e^{j\psi}} + \cancel{e^{j2\psi}} + \cancel{e^{j3\psi}} + \cdots + e^{jn\psi}\right\}.$$

$$\therefore E_t(1 - e^{j\psi}) = E_0(1 - e^{jn\psi})$$

$$\Rightarrow E_t = E_0\left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right] = E_0\left[\frac{1 - e^{jn\frac{\psi}{2}} \cdot e^{jn\frac{\psi}{2}}}{1 - e^{j\frac{\psi}{2}} \cdot e^{j\frac{\psi}{2}}}\right] \rightarrow ③$$

$$= E_0\left[\frac{e^{jn\frac{\psi}{2}} \cdot e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}} \cdot e^{jn\frac{\psi}{2}}}{e^{j\frac{\psi}{2}} \cdot e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}} \cdot e^{j\frac{\psi}{2}}}\right]$$

$$= E_0\left[\frac{e^{jn\frac{\psi}{2}}\left(e^{-jn\frac{\psi}{2}} - e^{jn\frac{\psi}{2}}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}\right]$$

$$= E_0\left[\frac{-\cancel{2j} \times \sin n\frac{\psi}{2}}{-\cancel{2j} \times \sin\frac{\psi}{2}}\right] \cdot e^{j(n-1)\frac{\psi}{2}}$$

$$\therefore E_t = E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] e^{j\phi}$$

Where $\phi = \frac{(n-1)}{2}\psi \rightarrow ④$

$$|E_t| = \left|E_0\left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]\left[\cos\phi + j\sin\phi\right]\right|$$

$$\therefore \boxed{E_t = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right] \angle\phi}$$ Where $\phi = \left(\frac{n-1}{2}\right)\psi$

The total far field pattern of Linear array of n-isotropic point source is

$$\boxed{E_t = E_0 \left[\frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}\right]}$$

There are 3 different cases under the n-element uniform linear arrays.

(1). broad side array

(2) End fire array

(3) End fire array with increased directivity

broad side array :-

→ An array is said to be broad side array, if the maximum direction of radiation perpendicular to the line of array (ie) 90° and 270°. Broad side sources are in phase. $\alpha = 0°$, $\psi = 0$ for maximum.

$$\therefore \psi = \beta d\cos\theta + \alpha = \beta d\cos\theta + 0$$

$$\Rightarrow 0 = \beta d\cos\theta$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0) = 90° \text{ (or) } 270°$$

$$\therefore \boxed{\theta = 90° \text{ (or) } 270°}$$

Directions of pattern ~~[illegible]~~ maxima:- (minor lobe) (11)

For array of n-isotropic point sources of equal amplitude & spacing we are using S.A Schelkunoff procedure.

$$E_t = E_0 \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

this is maximum when numerator is maximum

$\therefore \sin n\frac{\psi}{2} = 1$

$n\frac{\psi}{2} = \sin^{-1}(1) = \pm(2N+1)\frac{\pi}{2}$

$N = 1, 2, 3, 4 \ldots\ldots$

$N = 0$ for major lobe maxima.

$\Rightarrow \frac{\psi}{\cancel{2}} = \pm(2N+1)\frac{\pi}{\cancel{2}} \times \frac{1}{n}$

$\psi = \pm\frac{(2N+1)\pi}{n}$

$\beta d\cos(\theta_{max})_{minor} + \alpha = \pm\frac{(2N+1)\pi}{n}$

$\beta d\cos(\theta_{max})_{minor} = \pm\frac{(2N+1)\pi}{n} - \alpha$

$\cos(\theta_{max})_{minor} = \frac{1}{\beta d}\left[\pm\frac{(2N+1)\pi}{n} - \alpha\right]$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{(2N+1)\pi}{n} - \alpha\right\}\right]}$$

where $(\theta_{max})_{minor}$ = minor lobe maxima

For broad side array $\alpha = 0$.

$(\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{(2N+1)\pi}{n}\right\}\right]$

$\Rightarrow (\theta_{max})_{minor} = \cos^{-1}\left[\frac{1}{\frac{2\pi}{\lambda}\cdot d}\left\{\pm\frac{(2N+1)\cancel{\pi}}{n}\right\}\right]$

$$\boxed{\therefore (\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\lambda}{2nd}\right]}$$

For example

Let $n=4$, $d=\lambda/2$: $\alpha=0$

$$(\theta_{max})_{minor} = \cos^{-1}\left[\frac{(2N+1)\cancel{\lambda}}{\cancel{2}\cdot 4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}\right] = \cos^{-1}\left(\pm\frac{(2N+1)}{4}\right)$$

for $N=1$ $(\theta_{max})_{minor} = \cos^{-1}\left(\pm\frac{3}{4}\right)$

$= \pm 0.7\cancel{22}$ radians

$= \pm 41.4°$ degrees.. $(\because 1\,rad = 57.3°)$

(or)

$= \pm 138.6°$ degrees.

$$\therefore \boxed{(\theta_{max})_{minor} = \pm 41.4° \text{ (or) } \pm 138.6°}$$

Direction of pattern minima :-

According to S.A schelkunoff procedure

$$E_t = E_0 \frac{\sin n\frac{\psi}{2}}{\sin\frac{\psi}{2}}$$

$(\because \sin\frac{\psi}{2} \neq 0)$

$\Rightarrow \sin n\frac{\psi}{2} = 0$

$n\frac{\psi}{2} = \sin^{-1}(0) = \pm N\pi$, $N = 1,2,3,\ldots$

$\psi = \pm\frac{2N\pi}{n}$

$\beta d\cos(\theta_{min})_{minor} + \alpha = \pm\frac{2N\pi}{n}$

$\cos(\theta_{min})_{minor} = \frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}$

$$\boxed{(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right\}\right]}$$

For broad side $\alpha = 0$, $\beta = \frac{2\pi}{\lambda}$

$$\therefore (\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\frac{2\pi}{\lambda}\cdot d}\left(\pm\frac{2N\pi}{n}\right)\right]$$

$$= \cos^{-1}\left[\pm\frac{\cancel{2}N\lambda}{\cancel{2}nd}\right]$$

$$\therefore \boxed{(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]}$$

For example if $n=4$, $d=\lambda/2$

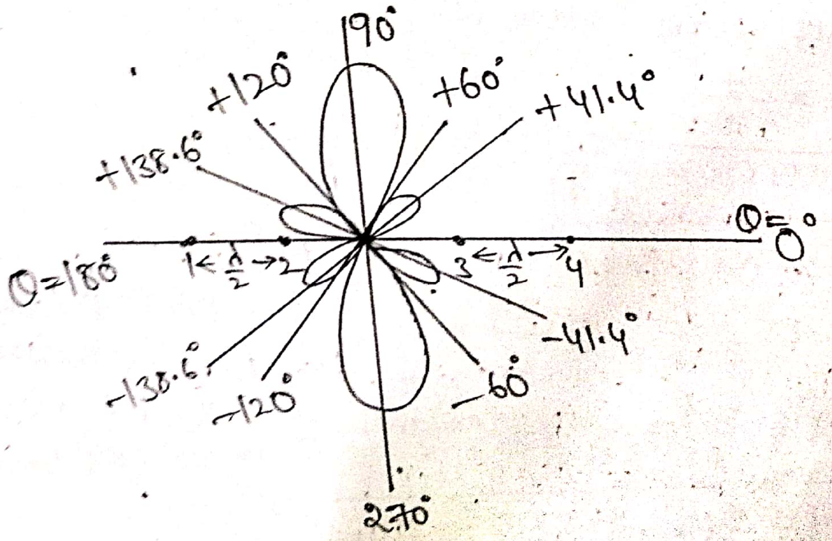
$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{1\cdot\cancel{\lambda}}{\cancel{2}4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}\right] = \cos^{-1}\left(\pm\frac{1}{2}\right) \quad \text{for } N=1$$

$$\therefore \boxed{(\theta_{min})_{minor} = \pm 60^\circ, \pm 120^\circ}$$

for $N=2$

$$(\theta_{min})_{minor} = \cos^{-1}\left[\pm\frac{\cancel{2}\cdot\cancel{\lambda}}{\cancel{2}4\cdot\frac{\cancel{\lambda}}{\cancel{2}}}\right] = \cos^{-1}(\pm 1)$$

$$\boxed{\therefore (\theta_{min})_{minor} = 0^\circ, 180^\circ}$$



Beam width of major lobe:-

It is defined as

(i) The angle between first nulls (or)

(ii) The double angle between first null and major lobe maximum directions.

Let the Complementary angle. $\gamma = 90° - \theta_{min}$.

$\Rightarrow \theta_{min} = 90° - \gamma$

Beam width of major lobe = 2x angle between first null and major lobe maximum.

$\Rightarrow BWFN = 2 \times \gamma$

But $\theta_{min} = \cos^{-1}\left(\pm \frac{N\lambda}{nd}\right)$

$90° - \gamma = \cos^{-1}\left(\pm \frac{N\lambda}{nd}\right)$

$\cos(90° - \gamma) = \pm \frac{N\lambda}{nd}$

$\sin\gamma = \pm \frac{N\lambda}{nd}$ (∵ γ is very small $\sin\gamma \simeq \gamma$)

$$\boxed{\gamma = \pm \frac{N\lambda}{nd}}$$

first null occurs when $N=1$

$\gamma = \frac{\pm\lambda}{nd}$

$$\therefore \boxed{BWFN = \frac{2\lambda}{nd}}$$

if $N\lambda >> nd$ then

$2\gamma = \frac{2\lambda}{nd} = \frac{2\lambda}{L}$ (∵ $L = (n-1)d$ = Total Length of array)

$\therefore 2\gamma = \frac{2\lambda}{L} = \frac{2}{(L/\lambda)}$ radians. $L \simeq nd$ (if n is very large)

$$\boxed{BWFN = 2\gamma = \frac{2 \times 57.3°}{(L/\lambda)} = \frac{114.6°}{(L/\lambda)}}$$

the half power beam width is

$$HPBW = \frac{BWFN}{2} = \frac{1}{(L/\lambda)} \text{ radians.}$$

$$\boxed{HPBW = \frac{57.3}{(L/\lambda)} \text{ degrees.}}$$

<u>End fire array</u> :-

→ An array is said to be end fire, if the maximum direction of radiation coincides with the array axis (or) line (ie) $\theta = 0°$ (or) $180°$.

$\because \psi = 0$, and $\theta = 0°$ (or) $180°$

$$\psi = \beta d \cos\theta + \alpha$$

$$0 = \beta d \cos 0° + \alpha$$

$$0 = \beta d + \alpha$$

$$\boxed{\alpha = -\beta d}$$

$\left(\because \beta = \frac{2\pi}{\lambda},\ d = \frac{\lambda}{2}\right.$

$\left.\alpha = -\frac{\cancel{2}\pi}{\cancel{\lambda}} \cdot \frac{\cancel{\lambda}}{\cancel{2}} = -180°\right)$

<u>direction of pattern maxima</u>:-

According to S.A Schelkunoff procedure

$$\sin n\frac{\psi}{2} = 1$$

$$n\frac{\psi}{\cancel{2}} = \sin^{-1}(1) = \pm(2N+1)\frac{\pi}{\cancel{2}}$$

$$\Rightarrow n\psi = \pm(2N+1)\pi$$

$$\psi = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d \cos(\theta_{max})_{minor} + \alpha = \pm\frac{(2N+1)\pi}{n}$$

$$\beta d \cos(\theta_{max}) - \beta d = \pm\frac{(2N+1)\pi}{n}$$

$\rightarrow \beta d\left(\cos(\theta_{max})_{minor} - 1\right) = \pm \frac{(2N+1)\pi}{n}$

$$\cos(\theta_{max})_{minor} - 1 = \pm \frac{(2N+1)\pi}{\beta n d}$$

$$\cos(\theta_{max})_{minor} = \pm \frac{(2N+1)\pi}{\beta n d} + 1$$

$$\boxed{(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2N+1)\pi}{\beta n d} + 1\right]}$$

if $n=4$, $d=\lambda/2$, $\alpha = -\pi$

for $N=1$

$$(\theta_{max})_{minor} = \cos^{-1}\left[\pm \frac{(2\cdot1+1)\cancel{\pi}}{\frac{2\cancel{\pi}}{\cancel{\lambda}}\cdot 4 \cdot \frac{\cancel{\lambda}}{2}} + 1\right]$$

$$= \cos^{-1}\left[\pm \frac{3}{4} + 1\right] = \cos^{-1}\left[\frac{7}{4}, \frac{1}{4}\right]$$

But $\cos^{-1}\left(\frac{7}{4}\right)$ doesn't exist

$\therefore (\theta_{max})_{minor} = \cos^{-1}\left(\frac{1}{4}\right) = 75.5^\circ$

$$\boxed{\therefore (\theta_{max})_{minor} = 75.5^\circ}$$

for $N=2$

$$(\theta_{max})_{2\,minor} = \cos^{-1}\left[\pm \frac{(2\cdot2+1)\cancel{\pi}}{\frac{2\cancel{\pi}}{\cancel{\lambda}}\cdot 4 \cdot \frac{\cancel{\lambda}}{2}} + 1\right] = \cos^{-1}\left(\pm \frac{5}{4} + 1\right)$$

$$= \cos^{-1}\left(\frac{5}{4}+1, -\frac{5}{4}+1\right)$$

$$= \cos^{-1}\left(\frac{-1}{4}\right) = -75.5^\circ$$

$$\boxed{(\theta_{max})_{2}\,minor = -75.5^\circ}$$

Directions of pattern minima:-

According to S.A. Schelkunoff procedure

$\sin \frac{n\psi}{2} = 0$

$\frac{n\psi}{2} = \sin^{-1}(0) = \pm N\pi \quad N = 1, 2, 3 \ldots$

$\psi = \pm \frac{2N\pi}{n}$

$\beta d \cos(\theta_{min})_{minor} + \alpha = \pm \frac{2N\pi}{n}$

$\beta d \cos(\theta_{min})_{minor} - \beta d = \pm \frac{2N\pi}{n} \quad (\because \alpha = -\beta d)$

$\beta d \{(\cos \theta_{min})_{minor} - 1\} = \pm \frac{2N\pi}{n}$

$\cos(\theta_{min})_{minor} - 1 = \pm \frac{2N\pi}{\beta n d} = \frac{\pm \cancel{2} N \cancel{\pi}}{\frac{\cancel{2\pi}}{\lambda} \cdot n \cdot d}$

$\therefore (\cos \theta_{min} - 1) = \pm \frac{N\lambda}{nd}$

$\left(\cos 2 \cdot \frac{\theta_{min}}{2} - 1\right) = \pm \frac{N\lambda}{nd}$

$\cancel{1} - 2\sin^2 \frac{\theta_{min}}{2} - \cancel{1} = \pm \frac{N\lambda}{nd}$

$2\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{nd}$

$\sin^2 \frac{\theta_{min}}{2} = \pm \frac{N\lambda}{2nd}$

$\sin \frac{\theta_{min}}{2} = \pm \sqrt{\frac{N\lambda}{2nd}}$

$\frac{\theta_{min}}{2} = \sin^{-1}\left(\pm \sqrt{\frac{N\lambda}{2nd}}\right)$

$$\boxed{(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm \sqrt{\frac{N\lambda}{2nd}}\right)}$$

For example $n=4$, $d=\frac{\lambda}{2}$

$N=1$: $(\theta_{min})_1 = 2\sin^{-1}\left(\pm\sqrt{\frac{1\cdot\lambda}{2\cdot 4\cdot \frac{\lambda}{2}}}\right)$

$= 2\sin^{-1}\left(\pm\frac{1}{2}\right) = 2\times(\pm 30) = \pm 60^\circ$

$N=2$: $(\theta_{min})_2 = 2\sin^{-1}\left(\pm\sqrt{\frac{2\cdot\lambda}{2\cdot 4\cdot \frac{\lambda}{2}}}\right) = 2\sin^{-1}\left(\pm\sqrt{\frac{1}{2}}\right)$

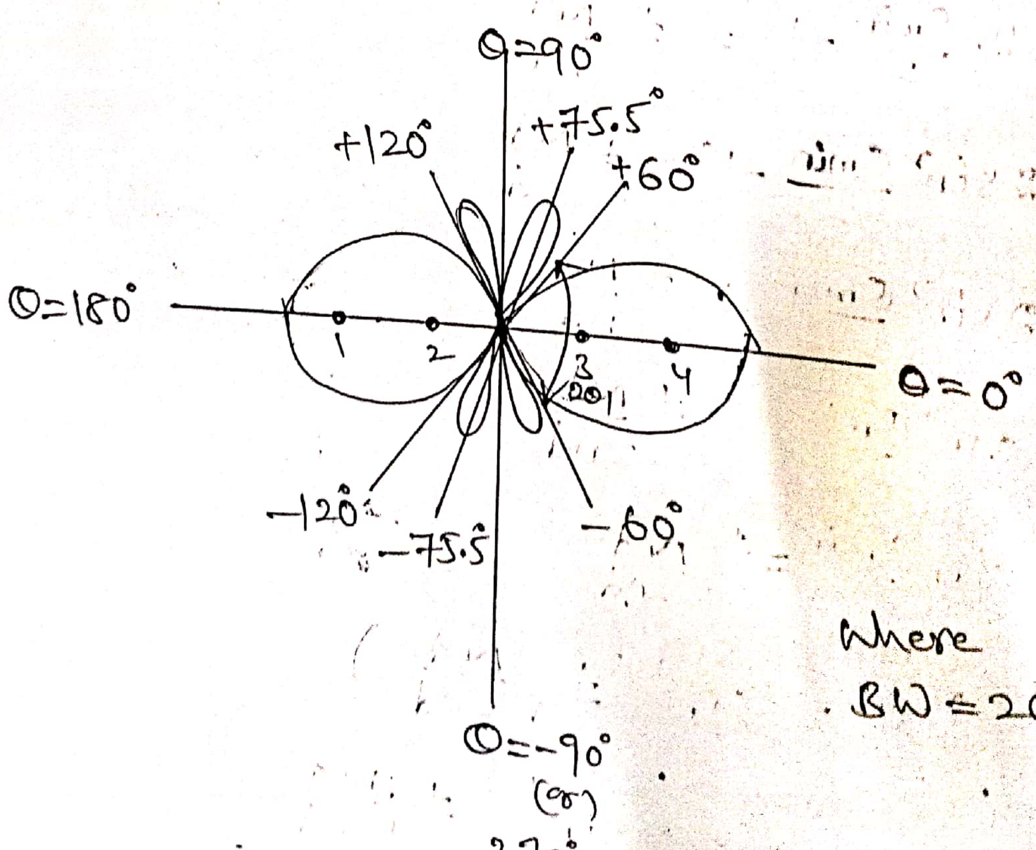
$= 2\sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right) = 2\times(\pm 45^\circ) = \pm 90^\circ$

$N=3$: $(\theta_{min})_3 = 2\sin^{-1}\left(\pm\sqrt{\frac{3\cdot\lambda}{2\cdot 4\cdot \frac{\lambda}{2}}}\right) = 2\sin^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$

$= 2\times(\pm 60^\circ) = \pm 120^\circ$

$N=4$: $(\theta_{min})_4 = 2\sin^{-1}\left(\pm\sqrt{\frac{4\cdot\lambda}{2\cdot 4\cdot \frac{\lambda}{2}}}\right) = 2\sin^{-1}(\pm 1)$

$= 2\times(\pm 90^\circ) = \pm 180^\circ$.



Where $BW = 2\theta_1$.

Beam width of major lobes:-

The complementary angle r is not required in this End fire array case. because the beam width of end fire array is larger than broad side.

$\therefore$ Beam width $= 2 \times$ angle between first nulls & maximum of major lobe

$$\boxed{BW = 2 \times \theta_1}$$

$$\theta_{min} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$$

$$\sin\left(\frac{\theta_{min}}{2}\right) = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\Rightarrow \sin\left(\frac{\theta_{min}}{2}\right) \cong \frac{\theta_{min}}{2} = \pm\sqrt{\frac{N\lambda}{2nd}}$$

$$\therefore \boxed{\theta_{min} = \pm 2\sqrt{\frac{N\lambda}{2nd}} = \pm\sqrt{\frac{2N\lambda}{nd}}}$$

if L is Total length of array

$$L = (n-1)d \Rightarrow \boxed{L \cong nd}$$

$$\boxed{\theta_{min} = \pm\sqrt{\frac{2N\lambda}{nd}} = \pm\sqrt{\frac{2N\lambda}{L}}}$$

Beam width between first nulls (BWFN) $= 2 \times \theta_{min}$

$$\therefore BWFN = 2\times\left(\pm\sqrt{\frac{2N\lambda}{nd}}\right) = \pm 2\sqrt{\frac{2N\lambda}{nd}}$$

$$\Rightarrow 2\theta_1 = \pm 2\sqrt{\frac{2N}{(L/\lambda)}} = \pm 2\sqrt{\frac{2\times 1}{(L/\lambda)}} \text{ for } N = 1$$

$$\therefore BWFN = \pm 2\sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}} \text{ rad} = \pm 2\times 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}} \text{ degree}$$

$$\boxed{BWFN = \pm 114.6\sqrt{\frac{2}{(L/\lambda)}}}$$

$$\boxed{HPBW = \frac{BWFN}{2} = \pm 57.3\sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}}}$$

③ **End fire array with increased directivity:-**

→ The maximum radiation can be obtained along the axis of the uniform array by allowing progressive phase shift 'α' between elements equal to $\pm\beta d$.

$\alpha = -\beta d$ for $\theta = 0°$, $\quad$ ($\because \psi = \beta d \cos\theta + \alpha$)

$\alpha = +\beta d$ for $\theta = 180°$ $\quad$ and $\psi = 0$ for max.

$\Rightarrow 0 = \beta d \cos\theta + \alpha$

$\alpha = -\beta d \cos\theta$)

→ This produces a maximum field in the direction $\theta = 0°$ but does not give maximum directivity.

→ To improve the directivity of an end fire array without destroying other characteristics:

→ In 1938 Hansen and Woodyard proposed, the required phase shift between closely spaced elements of a very long array should be

$$\alpha = -\left(\beta d + \frac{\pi}{n}\right) \cong -\left(\beta d + \frac{2.94}{n}\right) \text{ for maximum } \theta = 0 \rightarrow (1)$$

$$\alpha = +\left(\beta d + \frac{\pi}{n}\right) \cong +\left(\beta d + \frac{2.94}{n}\right) \text{ for maximum } \theta = 180° \rightarrow (2)$$

These conditions are referred to as "Hansen Woodyard conditions for Increased directivity".

→ The above conditions also cannot achieve maximum directivity at $\theta = 0°$ and $\theta = 180°$

The magnitude of maximum value is not be unity and side lobe level is not -13.46 db.

→ To increase the directivity due to "Hansen-Woodyard conditions from eqns ①, ② with assumptions of $|\psi|$ values.

(i) For maximum radiation along $\theta = 0$:-

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=0} \simeq \frac{\pi}{n} \rightarrow ③$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^\circ} \simeq \pi \rightarrow ④$$

(ii) For maximum radiation along $\theta = 180^\circ$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=180^\circ} \simeq \frac{\pi}{n} \rightarrow ⑤$$

$$|\psi| = |\beta d \cos\theta + \alpha|_{\theta=0^\circ} \simeq \pi \rightarrow ⑥$$

→ The main requirement is to fulfil the condition $|\psi| = \pi$ for each array

→ For array of n-elements the condition $|\psi| = \pi$ is satisfied by using eqns ①, ② for $\theta = 0^\circ$ and $\theta = 180^\circ$

The spacing between two elements is

$$d = \left(\frac{n-1}{n}\right)\frac{\lambda}{4}$$

if the no. of elements considered is large then

$$d = \frac{\cancel{n}}{\cancel{n}} \cdot \frac{\lambda}{4} \Rightarrow \boxed{d = \frac{\lambda}{4}}$$

→ Hence for large uniform array the spacing is $\frac{\lambda}{4}$ to increase the directivity.

Comparison of characteristics :-

S.No	Type of array	Directions of minor lobe maxima
1.	General case	$(\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\pi}{n} - \alpha\right.$
2.	Broad side ($\alpha = 0$)	$(\theta_{max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm\frac{(2N+1)\pi}{n}\right]\right.$
3.	Ordinary End fire $\alpha = \pm\beta d$	$(\theta_{max})_{minor} = \cos^{-1}\left[\pm\frac{(2N+1)\pi}{\beta n d}+1\right]$ $= \cos^{-1}\left[\pm\frac{(2N+1)\lambda}{2nd}+1\right]$

S.No	Type of array	Directions of minor lobe minima
1.	General case	$(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n} - \alpha\right.\right.$
2.	Broad side ($\alpha = 0$)	$(\theta_{min})_{minor} = \cos^{-1}\left[\frac{1}{\beta d}\left\{\pm\frac{2N\pi}{n}\right\}\right]$ $= \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]$
3.	Ordinary End fire $\alpha = \pm\beta d$	$(\theta_{min})_{minor} = 2\sin^{-1}\left(\pm\sqrt{\frac{N\lambda}{2nd}}\right)$

S.No	Type of array	Beam width between first nulls
1.	Broad side array	$BWFN = \frac{2\lambda}{nd} = \frac{114.6^\circ}{(L/\lambda)}$
2.	Ordinary End fire array	$BWFN = 2\sqrt{\frac{2N\lambda}{nd}}$ rad $= 114.6^\circ\sqrt{\frac{2}{(L/\lambda)}}$

Type of array	HPBW (Half power Beam width)
Broad side array	$HPBW = \frac{57.3^\circ}{\left(\frac{L}{\lambda}\right)} = \frac{\lambda}{nd}$ rad.
End fire array	$HPBW = 57.3^\circ\sqrt{\frac{2}{(L/\lambda)}} = \sqrt{\frac{2}{\left(\frac{L}{\lambda}\right)}}$ rad ($\because$ 1 rad $= 57.3^\circ$)
End fire array with Increased directivity.	$HPBW = \frac{52^\circ}{\sqrt{L/\lambda}}$

Directivity relations:-

For a broad side array

$$\boxed{D = 2n\left(\frac{d}{\lambda}\right)}$$

$$\Rightarrow D = 2\left(\frac{nd}{\lambda}\right)$$

$$\Rightarrow \boxed{D = 2\left(\frac{L}{\lambda}\right)}$$

$L = (n-1)d$. if n is large

($\because nd \cong L$ = Total length of array)

For an end fire array

$$\boxed{D = 4n\left(\frac{d}{\lambda}\right)}$$

$$\Rightarrow D = 4\left(\frac{nd}{\lambda}\right)$$

$$\therefore \boxed{D = 4\left(\frac{L}{\lambda}\right)}$$

($\because L = (n-1)d$
$\Rightarrow L \cong nd$)

For an end fire array with incre...

$$\boxed{D = 1.789\left[4n\left(\frac{d}{\lambda}\right)\right]}$$

$$\Rightarrow D = 1.789\left[4\left(\frac{nd}{\lambda}\right)\right] \qquad (\because L = (n-1)d$$

$\therefore L \cong nd$) if n is very large.

$$\therefore \boxed{D = 1.789\left[4\left(\frac{L}{\lambda}\right)\right]}$$

Concept of scanning arrays (or) phased arrays:

→ An array which gives maximum radiation in any direction by controlling phase excitation in each element. such an array is commonly called "phased array".

→ The array in which the phase and the amplitude of most of the elements is variable.

→ We get the direction of maximum radiation and pattern shape along with side lobes is controlled is called "phased array".

Let the array gives maximum radiation in $\theta = \theta_0$ direction.

$$\therefore \psi = \beta d \cos\theta + \alpha$$

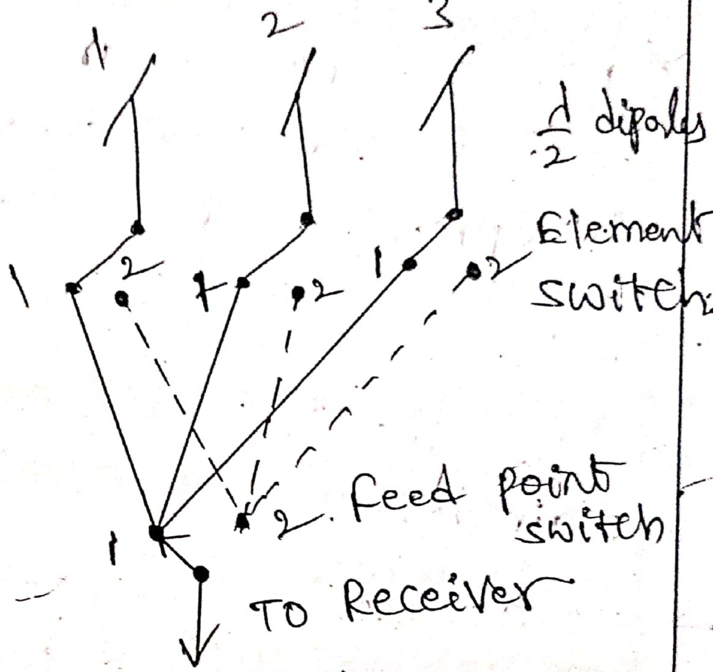
at $\psi = 0$, the radiation is maximum

$$0 = \beta d \cos\theta_0 + \alpha \qquad \text{where } 0 \le \theta_0 \le \pi$$

$$\therefore \boxed{\alpha = -\beta d \cos\theta_0}$$

From above equation, the maximum radiation can be achieved in any direction if the progressive phase difference between the elements is controlled.

→ Let us consider a three element array. The elements of array is considered as $\frac{\lambda}{2}$ dipole.



→ All the cables are of same length.

→ All the cables are taken together at common feed point

→ The mechanical switches are used, one switch at each antenna, and one switch at a common feed point.

→ By operating switch, the beam can be shifted to any phase shift.

Binomial arrays :-

→ In the binomial arrays, the amplitudes of radiating sources are arranged according to coefficients of binomial series.

$$(a+b)^{n-1} = a^{n-1} + \frac{(n-1)}{1!} a^{n-2} b^1 + \frac{(n-1)(n-2)}{2!} a^{n-3} b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} b^3 + \cdots$$

 Scanned by CamScanner

Where n = no. of radiating sources in this array.

→ Binomial array can be defined as it is an array in which the amplitudes of the antenna elements are arranged according to the coefficients of the binomial series.

→ For uniform linear array, the array length is increased to increased the directivity. ($\because$ spacing is $\frac{\lambda}{4}$)

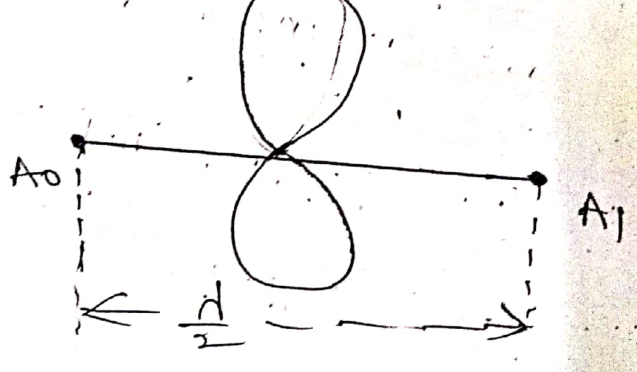
→ But for some applications the secondary lobes should be eliminated, with respect to main lobes.

→ To achieve such a pattern the array is arranged in such a way that broad side array radiate more strongly at the centre.

Let us consider array of two identical point sources spaced $\frac{\lambda}{2}$ apart.

The far field pattern is given by

$$\boxed{E = \cos\left(\frac{\pi}{2}\cos\theta\right)}$$



Dis advantages of binomial array :-

HPBW increases and hence the directivity decreases.

* For design of a large array, larger amplitude ratio of sources is required.

Effect of uniform and Non-uniform amplitude distributions :-

- In the design of linear inphase antenna arrays of non-uniform amplitudes C.L Dolph used the Tchebyscheff polynomial, the name is "Dolph-Tchebyscheff arrays"
- It is also called as "Chebyshev arrays" (or) "Dolph-chebyshev arrays"
- C.L Dolph proposed that for a linear broad-side arrays, it is possible to minimize the beam width of main lobe for a specified side lobe level, vice versa
- That means If the beam width between first nulls is specified then the side lobe level is minimized.
- The current distribution that produce such a pattern is called "Dolph-Tchebyscheff distribution"!

* Therefore Dolph-Tchebyshev distribution provides compromise optimum value between two conflicting properties.

* According to C.L Dolph, the current distribution is optimum provided that distance between two array elements $d \leq \frac{\lambda}{2}$

* For practical design of array, the narrow beamwidth for side lobe levels upto 20-30 dB in UHF and VHF bands.

* 20 dB level is considered for good, 30 dB level considered for excellent. But very difficult to 40 dB level and not exist.

Tchebyscheff polynomials :- (Chebyshev arrays)

The Tchebyscheff polynomial with variable x is denoted by $T_m(x)$.

It is given by

$$T_m(x) = \cos(m \cos^{-1} x), \quad -1 < x < 1 \rightarrow 1(a)$$

$$T_m(x) = \cosh(m \cosh^{-1} x), \quad |x| > 1 \rightarrow 1(b)$$

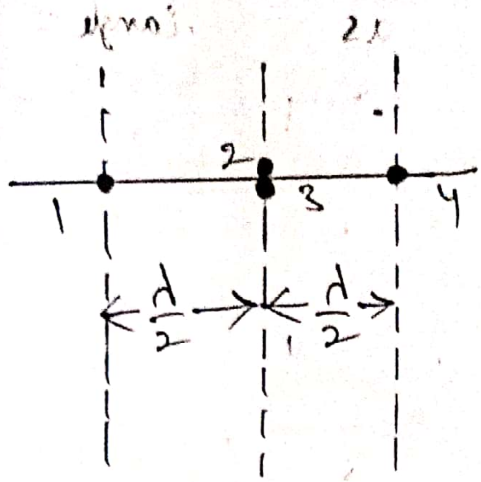
Where 'm' is an integer constant from '0' to ∞.

Let m=0. Then eqn 1(a) becomes

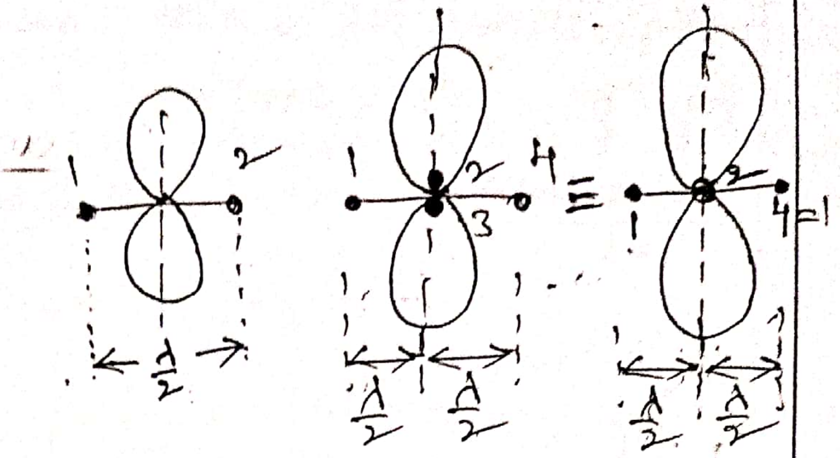
$$T_0(x) = \cos(0 \cdot \cos^{-1} x)$$

$$\boxed{\begin{aligned} &\text{Let } \delta = \cos^{-1} x \\ &\Rightarrow x = \cos \delta \end{aligned}}$$

The arrangement of 4-elements with $\frac{\lambda}{2}$ Spacing :-



fig(a)
arrangement of 4-elements with $\frac{\lambda}{2}$ spacing

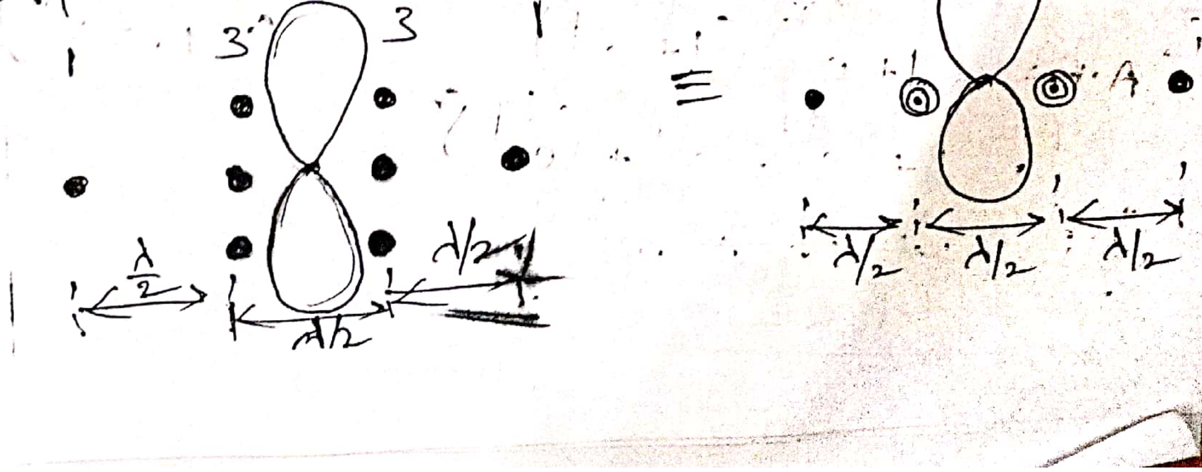


fig(b) :- pattern for 2-element array and 4-element array

→ In the uniform 4-element array, the resultant pattern shows four side lobes. the secondary lobes appear in resultant pattern, because the pattern have a spacing ~~between~~ greater than $\frac{\lambda}{2}$.

• So if we reduce the spacing between the two elements to $\frac{\lambda}{2}$ wavelength then the side lobes are reduced, and only main lobes are obtained.

→ 4-sources with amplitude ratio is 1:3:3:1



The binomial series coefficients are ob[tained] by pascal's triangle.

pascal's triangle

no. of sources	Relative amplitude
n=1	1
n=2	1 1
n=3	1 2 1
n=4	1 3 3 1
n=5	1 4 6 4 1
n=6	1 5 10 10 5 1
n=7	1 6 15 20 15 6 1
n=8	1 7 21 35 35 21 7 1
n=9	1 8 28 56 70 56 28 8 1

∴ The pattern for binomial array is given by

$$\boxed{E = \cos^{n-1}\left[\frac{\pi}{2}\cos\theta\right]}$$

The array factor is given by

$$A.F = (1+e^{j\psi})^{N-1}$$

The array factor for multiplication pattern is

$$A.F = (1+e^{j\psi})(1+e^{j\psi}) \quad (\because N=3)$$

$$= 1+e^{j\psi}+e^{j\psi}+(e^{j\psi})^2$$

$$\boxed{AF = 1+2e^{j\psi}+e^{j2\psi}}$$

$T_0(x) = \cos(0 \cdot \delta) = \cos 0$

$\therefore \boxed{T_0(x) = 1}$

Let $m=1$:-

$T_1(x) = \cos(1 \cdot \cos^{-1}x) = \cos\delta$

$\therefore \boxed{T_1(x) = x}$ $\quad \left(\because \delta = \cos^{-1}x \quad x = \cos\delta\right)$

Let $m=2$

$T_2(x) = \cos(2 \cdot \cos^{-1}x) = \cos 2\delta$

$= 2\cos^2\delta - 1.$ $\quad (\because \cos 2\theta = 2\cos^2\theta - 1)$

$\boxed{T_2(x) = 2x^2 - 1.}$

Let $m=3$:-

$T_3(x) = \cos(3 \cdot \cos^{-1}x) = \cos 3\delta$

$= 4\cos^3\delta - 3\cos\delta$ $\quad (\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta)$

$\boxed{T_3(x) = 4x^3 - 3x}$

Let $m=4$:-

$T_4(x) = \cos(4 \cdot \cos^{-1}x) = \cos 4\delta$

$= \cos 2(2\delta)$

$= 2\cos^2(2\delta) - 1$

$= 2\left[2\cos^2\delta - 1\right]^2 - 1$ $\quad (\because x = \cos\delta)$

$= 2\left[4\cos^4\delta + 1 - 4\cos^2\delta\right] - 1$

$= 8x^4 + 2 - 8x^2 - 1$

$\therefore \boxed{T_4(x) = 8x^4 - 8x^2 + 1}$

For higher values of 'm' can be calculated by using recursive formula

$$\boxed{T_{m+1}(x) = 2x\,T_m(x) - T_{m-1}(x)}$$

To obtain $T_5(x)$ put $m=4$ in above expression

$\therefore T_{4+1}(x) = T_5(x) = 2x\,T_4(x) - T_3(x)$

$\Rightarrow T_5(x) = 2x[8x^4 - 8x^2 + 1] - [4x^3 - 3x]$

$= 16x^5 - 16x^3 + 2x - 4x^3 + 3x$

$\therefore \boxed{T_5(x) = 16x^5 - 20x^3 + 5x}$

To obtain $T_6(x)$ put $m=5$ in Recursive formula

$T_6(x) = 2x\,T_5(x) - T_4(x)$

$= 2x[16x^5 - 20x^3 + 5x] - [8x^4 - 8x^2 + 1]$

$= 32x^6 - 40x^4 + 10x^2 - 8x^4 + 8x^2 - 1$

$\therefore \boxed{T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1}$

To obtain $T_7(x)$ put $m=6$ in Recursive formula

$T_7(x) = 2x\,T_6(x) - T_5(x)$

$= 2x[32x^6 - 48x^4 + 18x^2 - 1] - [16x^5 - 20x^3 + 5x]$

$= 64x^7 - 96x^5 + 36x^3 - 2x - 16x^5 + 20x^3 - 5x$

$\therefore \boxed{T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x}$

∴ The polynomials are given below

$T_0(X) = 1$ $m = 0$

$T_1(X) = X$ $m = 1$

$T_2(X) = 2X^2 - 1$ $m = 2$

$T_3(X) = 4X^3 - 3X$ $m = 3$

$T_4(X) = 8X^4 - 8X^2 + 1$ $m = 4$

$T_5(X) = 16X^5 - 20X^3 + 5X$ $m = 5$

$T_6(X) = 32X^6 - 48X^4 + 18X^2 - 1$ $m = 6$

$T_7(X) = 64X^7 - 112X^5 + 56X^3 - 7X$ $m = 7$

$T_8(X) = 128X^8 - 256X^6 + 160X^4 - 32X^2 + 1$; $m = 8$

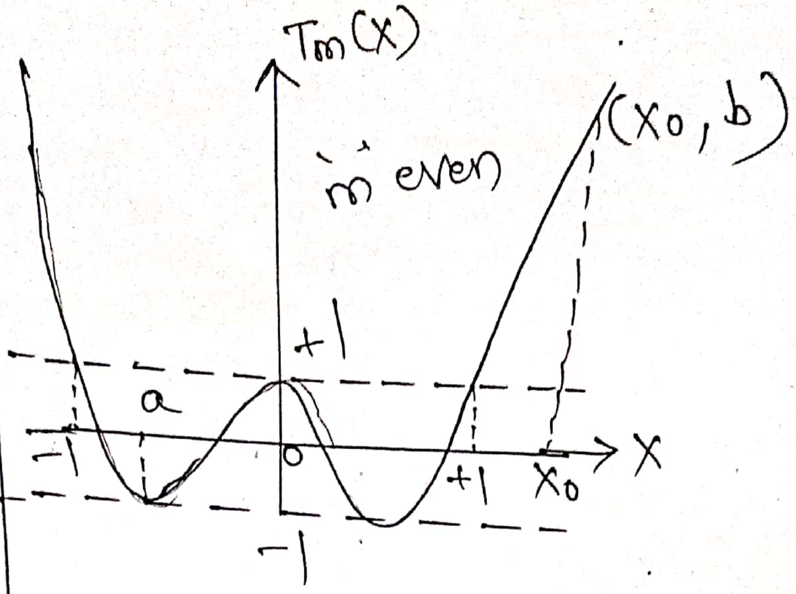
$T_9(X) = 256X^9 - 576X^7 + 432X^5 - 120X^3 + 9X$; $m = 9$.

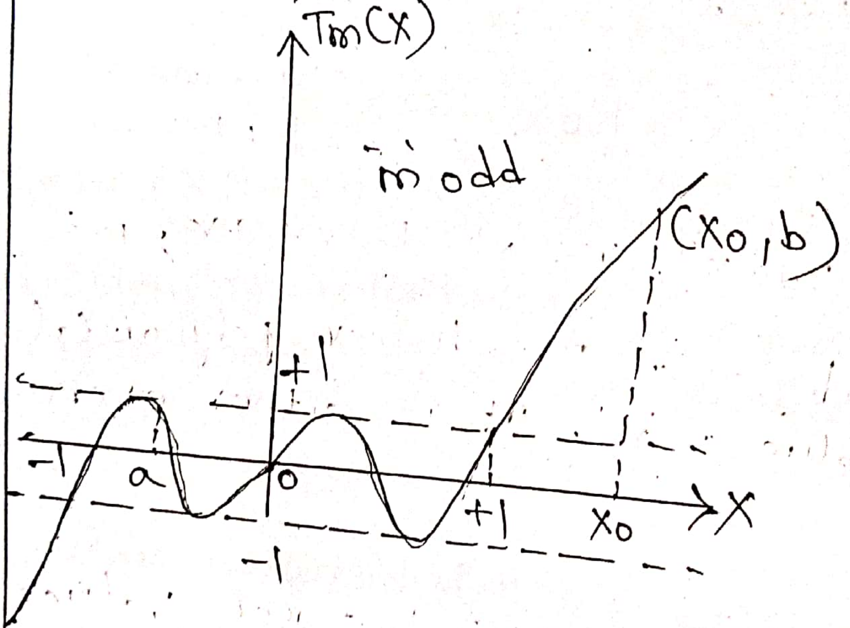
Therefore the degree of Tchebyshev polynomial is same as value of 'm'. It is either even or odd.

Properties:-

- All the polynomials oscillate between values -1 and 1
- In the region $|x| < 1$; the m^{th} order polynomial crosses the axis 'm' times.
- In the region $|x| > 1$, the Tchebyshev polynomial go on increasing.

The Tchebyshev polynomial waveforms are given by





The mth order polynomial is $T_m(x) = \cos(m\cos^{-1}x)$

The nulls are given by the roots.

$$\cos(m\cos^{-1}x) = 0$$

$$\Rightarrow \cos(m\delta) = 0$$

$$m\delta = \cos^{-1}(0)$$

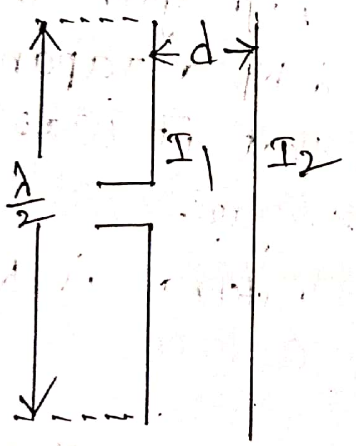
$$m\delta = (2k-1)\frac{\pi}{2}, \quad k = 1, 2, 3 \ldots m$$

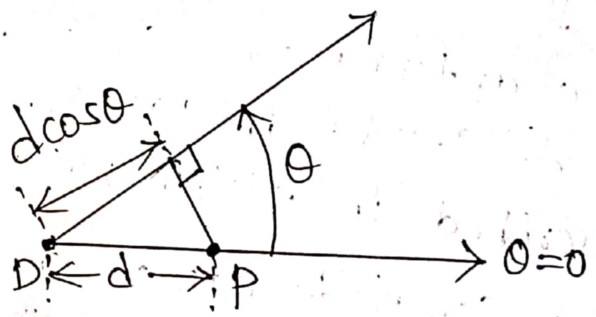
$$\boxed{\delta = \frac{(2k-1)\pi}{2m}} \quad k = 1, 2, 3 \ldots m$$

Arrays with parasitic elements:-

The element in which current is induced due to the field in other elements is called as "parasitic element".

* one (or) more parasitic elements coupled magnetically with the driven element forms an "array of parasitic elements".
* It is also called as parasitic antenna.
* The effect of parasitic element on the directional pattern of the antenna depends on the magnitude and the phase of the induced current in parasitic element.



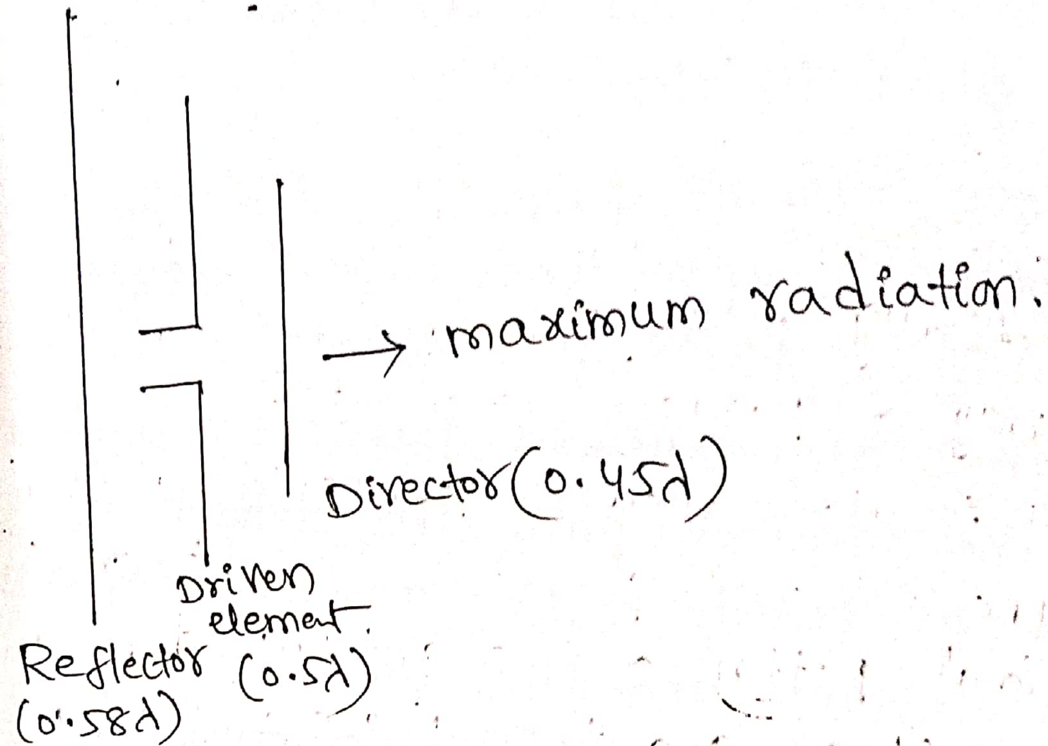


D = Driven element

P = parasitic element.

* When the parasitic element is larger than its resonant ($\frac{\lambda}{2}$) length, it is inductive in nature. Then such element is acts as "reflector".
* When the parasitic element is shorter than its resonant ($\frac{\lambda}{2}$) length. it is capacitive in nature then such element is acts as "Director".

The 3-element parasitic array is given below.

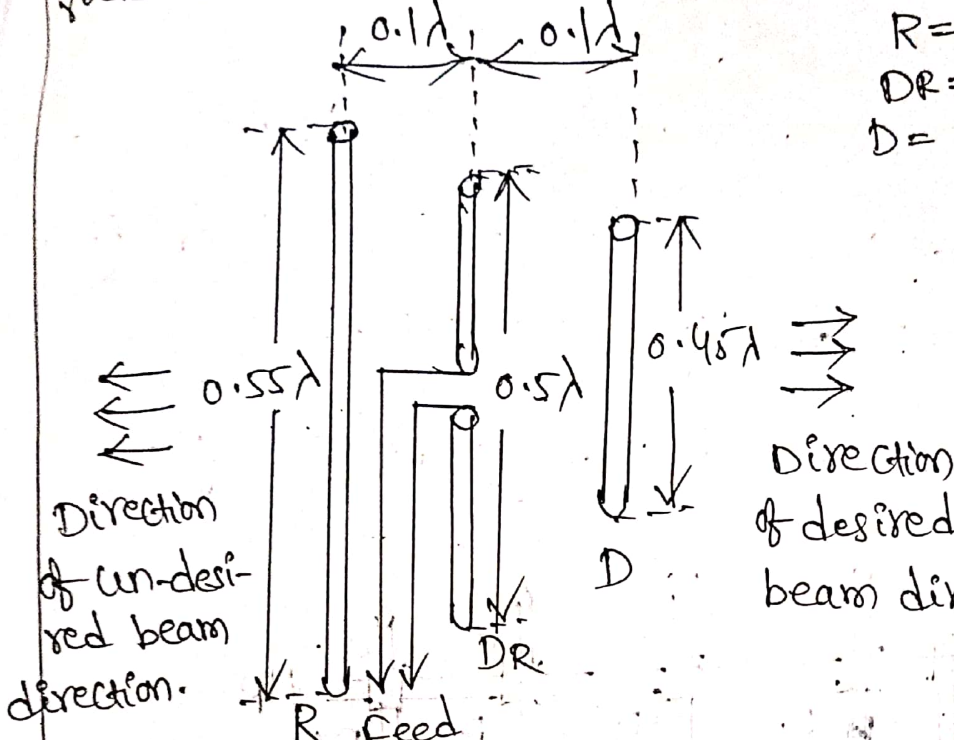


Yagi-Uda Array :- (or) Yagi-Uda antenna

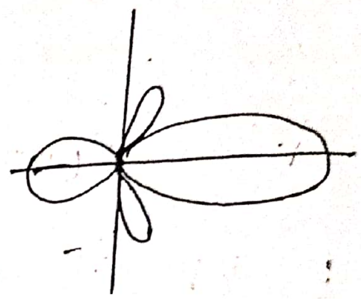
* Yagi-uda antennas are most high gain antenna
* The antenna was first invented by a Japanese prof. S. Uda, in 1940's.. after that, it was described in english by prof. H-Yagi.
* The complete name of this antenna is known as "Yagi-Uda antenna".
* It consists of a driven element, a reflector, and one (or) more directors.
* That is Yagi-uda antenna is an array of a driven element and one (or) more parasitic elements

) Desp. The driven element is a resonant half wave dipole usually of metallic rod at frequency of operation.

the parasitic elements are continuous metallic rods arranged in parallel to driven element.



fig(a) :- Yagi-Uda array



fig(b) Radiation pattern.

* The parasitic element Receive their excitation from voltages induced in these elements by the current flow in the driven element.

* Generally the spacing between driven element and parasitic elements is 0.1 λ to 0.15 λ.

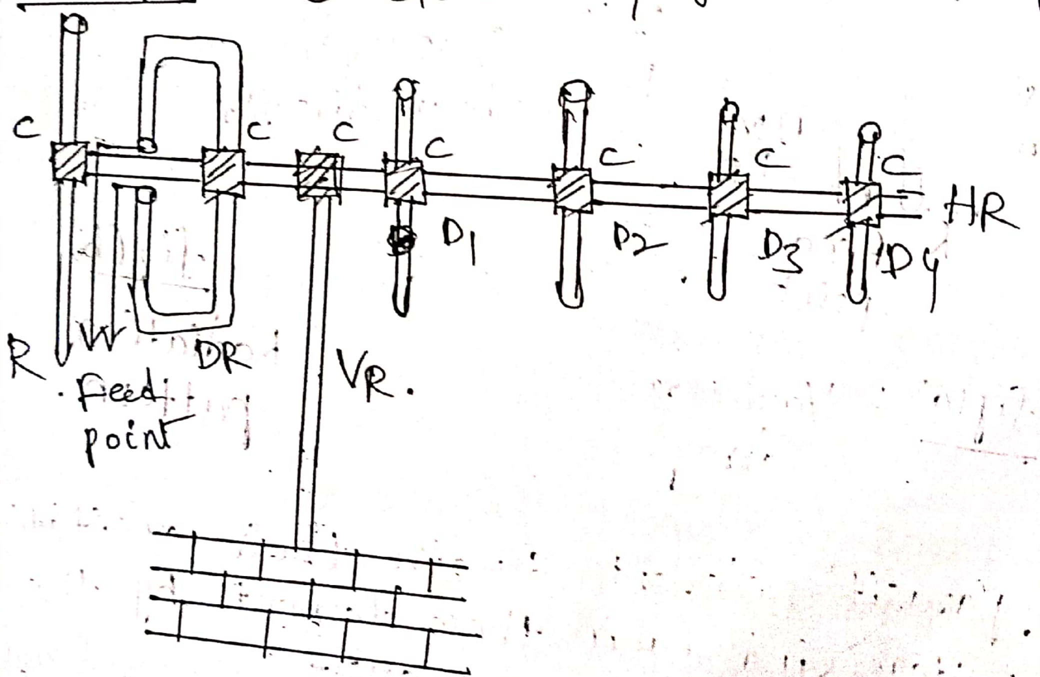
$$\text{Reflector length} = \frac{152}{f(MHz)} \text{ meter} \quad (\because 1 \text{ meter} = 3.3 \text{ ft})$$

$$\text{Driven element length} = \frac{143}{f(MHz)} \text{ meter}$$

$$\text{Director length} = \frac{137}{f(MHz)} \text{ meter}$$

* The spacing between elements and lengths of the parasitic elements determines the phases of the currents.
* A parasitic element of length greater than $\frac{\lambda}{2}$ then it is Inductive and is called as "Reflector"
* A parasitic element of length less than $\frac{\lambda}{2}$ then it is capacitive and is called as "Director".
* The element of length is equal to $\frac{\lambda}{2}$ then it is driven element (or) dipole element.

Example :- 6-element Yagi-Uda array.



Where R = Reflector

DR = driven element = folded dipole

D_1, D_2, D_3, D_4 = directors

VR = Vertical rod to support horizontal rod

HR = horizontal rod to support elements

C = clamps.

 Scanned by CamScanner