MODULE – V

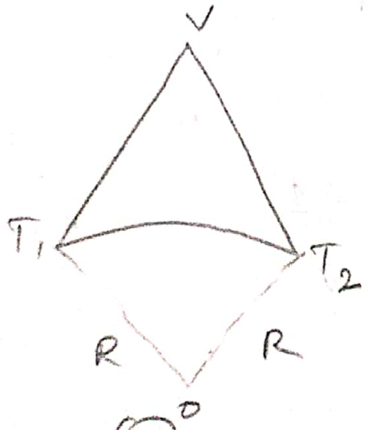
CURVES.

Curves are generally used on highways, railways where it is necessary to change the direction of motion.

A curve may be circular, parabolic, or spiral and is alway tangential to two straight directions.

Types of curves:

1. Simple circular curve
2. Compound curve
3. Reverse curve.



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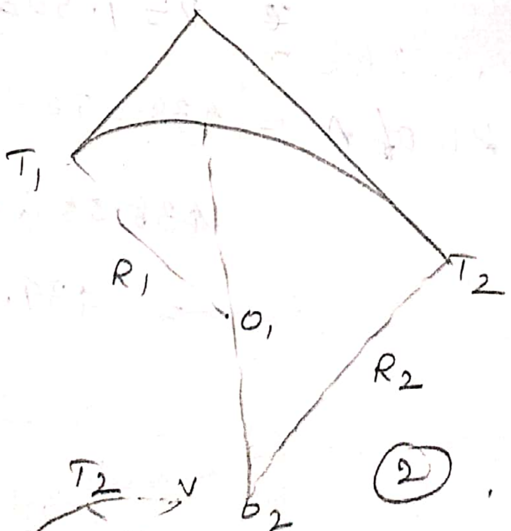

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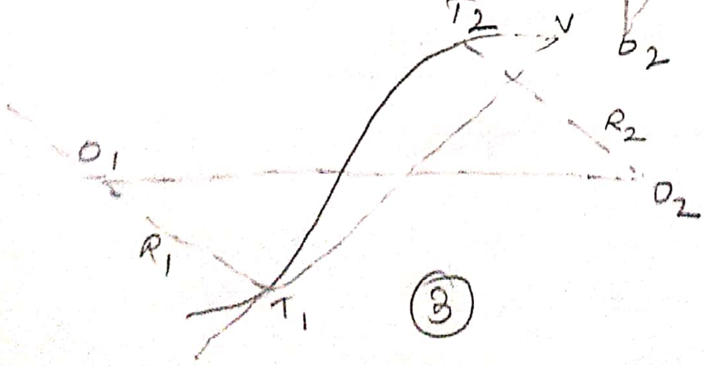

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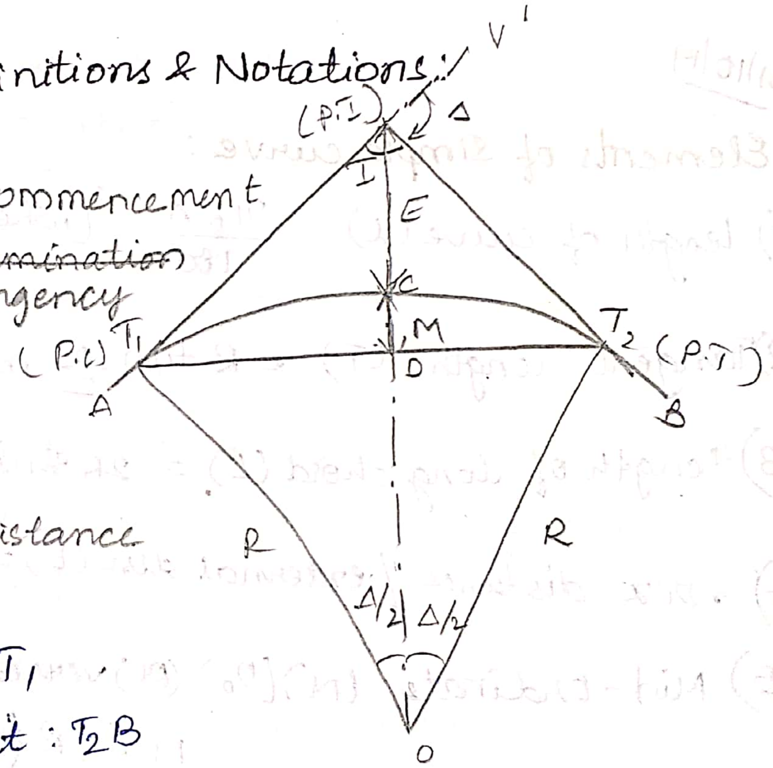
Simple curve: It is one which consists of a single arc of a circle. It is tangential to the both straight lines.

Compound curve: It consists of two or more simple arcs that run in the same direction and joins at a common tangent point.

Reverse curve: It is the one which consists of two circular arcs of same ~~arc~~ & are different Radii having their centres to the different sides of common tangent. Both the arcs thus bend in different directions with common tangent at their junction.

Simple Curves: Definitions & Notations:-



P.C – point of commencement.

P.T – Point of ~~termination~~ tangency

P.I –

M – Mid ordinate

E – ~~Ape~~ External distance

1. Back tangent: AT_1
2. Forward tangent: T_2B
3. Point of intersection: V.
4. P.C → point of commencement or point of curve
5. P.T – point of tangency i.e end of the curve
6. Intersection angle: $\underline{|I}$ = I
7. Deflection angle: Δ
9. Tangent distance: (T) VT_1 and VT_2
10. External distance: E – V to C
11. Length of l: 'l' The curved length from T_1 to T_2

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12. long chord (L): It is a chord joining PC and PT

13. Mid ordinate (M): It is the ordinate from the midpoint of long chord to the mid point of the curve

14. ~~Normal~~ Right hand curve: If the curve deflects to the right of the direction of the progress of survey, it is called as right hand curve.

15. Left hand curve: If the curve deflects to the left of the direction of progress of survey, it is called left hand curve.

16. P. I - point of Intersection

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Elements of simple curve:

1) length of curve (l) $= \frac{\pi R \Delta}{180°}$ (where Δ is indegrees)

2) Tangent length (T) $= R \tan \frac{\Delta}{2}$ m

3) length of long chord (L) $= 2R \sin \frac{\Delta}{2}$ m

4) Apex distance / external dist (E) $= R\left(\sec \frac{\Delta}{2} - 1\right)$

5) Mid-Ordinate (M) (O_o) (or) versed sine of Curve

$$M = R\left(1 - \cos \frac{\Delta}{2}\right)$$

6) $\Delta = 180 - I$

7) ~~chino~~ chainage of 1st tangent point

T_1 = chainage of intersection 'V' − tangent length (T)

8) Chainage of 2nd tangent point

T_2 = chainage of T_1 + length of curve (l)

Setting out simple curve:

This can be done by two methods

1) Linear methods
2) Angular methods

1. Linear Methods:

In this ~~linear~~ chain/tape is used.

→ These methods are used when high degree of accuracy is not required and when curve is short

2. Angular Methods:

In this method theodolite is used with or without a chain or tape.

Linear methods of setting out a curve:

a) ~~the~~ By ordinates or offsets from long chord

b) By successive bisection of arcs

c) By offsets from tangents

(i) Radial offsets

(ii) Perpendicular offsets.

d) By offsets from chords produced or by deflection distances

Angular or instrumental methods of setting out a curve:

(1) Rankine's method of tangential (or) deflection angle

(2) Two theodolite method

(3) Tacheometric method.

a) By ordinates or offsets from long chord

$$O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$O_x = \sqrt{R^2 - x^2} - (R - O_0)$$

Problem:

Calculate the ordinates at 10m distance for a circular curve having long chord of 80m and a versed ~~sign~~ sine of 4m.

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Sol:- $x = 10m$ $L = 80m$ $M = 4m$, $O_0 = 4m$

$$O_x = \sqrt{R^2 - x^2} - (R - O_0)$$

$$\Rightarrow O_0 = R - \sqrt{R^2 - \left(\frac{L}{2}\right)^2}$$

$$4 = R - \sqrt{R^2 - (40)^2}$$

$$\sqrt{R^2 - (40)^2} = R - 4$$

$$\cancel{R^2} - (40)^2 = \cancel{R^2} + 16 - 8R$$

$$8R = 16 + (40)^2$$

$$8R = 1616$$

$$\boxed{R = 202m}$$

$$\begin{array}{r} 1600 \\ 16 \\ \hline 1616 \end{array}$$

$$O_x = \sqrt{R^2 - x^2} - (R - O_0)$$

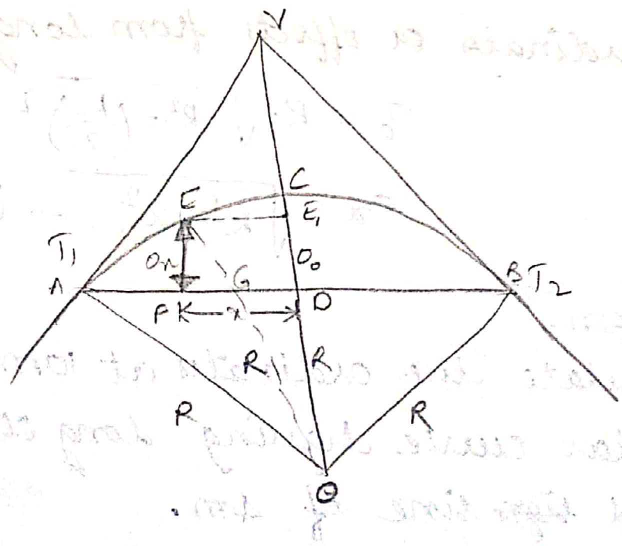
$$O_{10} = \sqrt{(202)^2 - (10)^2} - (202 - 4)$$

$$O_{10} = 3.75 m$$

$$O_{20} = \sqrt{(202)^2 - (20)^2} - (202 - 4) = 3.007 m$$

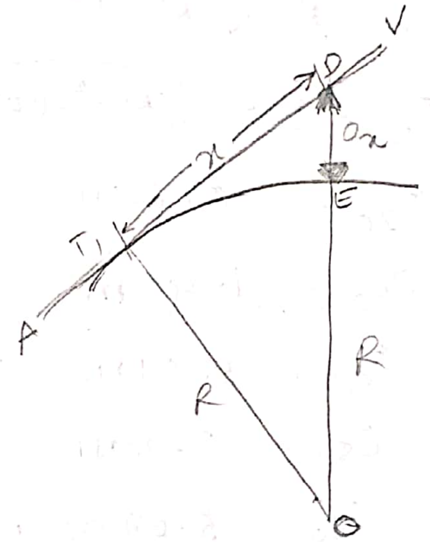
$$O_{30} = \sqrt{(202)^2 - (30)^2} - (202 - 4) = 1.75 m$$

$$O_{40} = \sqrt{(202)^2 - (40)^2} - (202 - 4) = 0$$



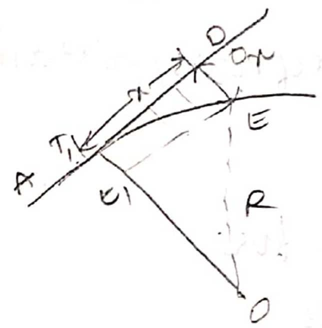
(iii) By offsets from the tangents setting out by Radial offset.

a) $O_x = \sqrt{R^2+x^2} - R$



b) Setting out by perpendicular offset:

$O_x = R - \sqrt{R^2 - x^2}$



Problem:

Determine the offsets to be set out at half chain interval along the tangents to locate a 16 chain curve, the length of each chain being 20m.

Sol:- $x = \frac{1}{2}$ Chain R = 16 chain

$= \frac{1}{2}(20) = 10\,m$ $= 16 \times 20 = 320\,m$.

~~Perpendicular offset~~ Radial offset :

$O_{10} = \sqrt{(320)^2+(10)^2} - 320 = 0.156 = 0.16\,m$

~~$O_{20} = 320 - \sqrt{(320)^2-(20)^2} =$~~

$O_{20} = \sqrt{(320)^2+20^2} - 320 = 0.624 = 0.62\,m$

$O_{30} = \sqrt{320^2+30^2} - 320 = 1.403 = 1.40\,m$

$O_{40} = \sqrt{320^2+40^2} - 320 = 2.490 = 2.49\,m$

$O_{50} = 3.88\,m$, $O_{60} = 5.58\,m$ $O_{70} = 7.57\,m$

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Ilar offsets:

$$O_{10} = R - \sqrt{R^2 - x^2}$$

$$O_{10} = 320 - \sqrt{(320)^2 - (10)^2} = 0.16m$$

$$O_{20} = 0.62m$$

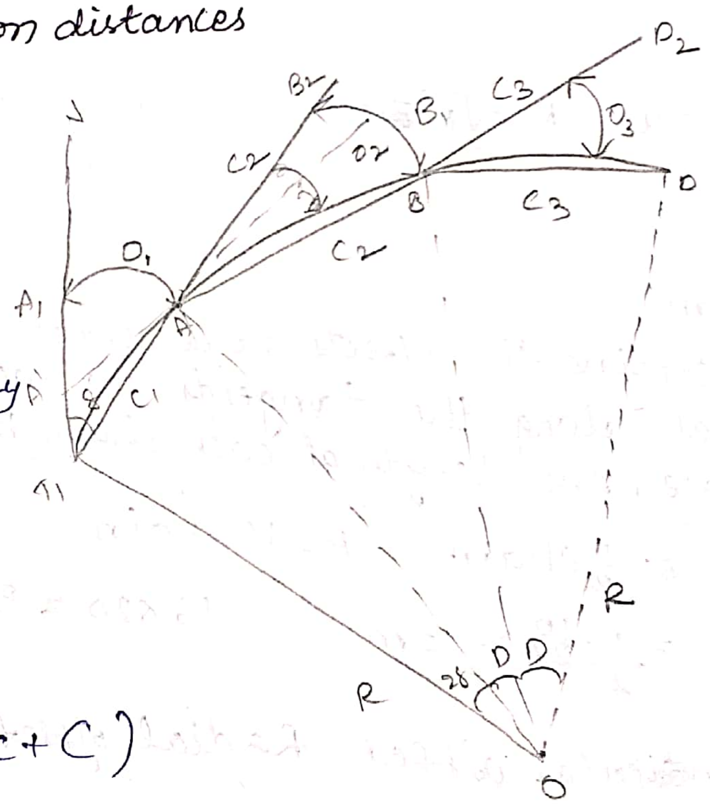
$$O_{30} = 1.40m$$

$$O_{40} = 2.51m$$

$$O_{50} = 3.93m$$

$$O_{60} = 5.67m$$

d) By ~~offset~~ deflection distances



It is useful for

→ Long curves

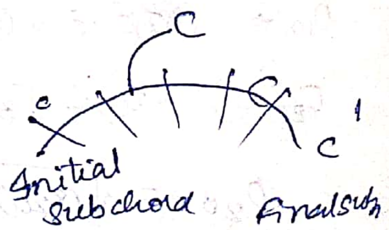
→ This method is used on highways curves where theodolite is not available.

$$O_1 = \frac{c^2}{2R}$$

$$O_2 = \frac{C}{2R}(c + C)$$

$$O_3 = O_4 = O_{n-1} = \frac{C^2}{R}$$

$$O_n = \frac{c'}{2R}(C + c')$$



problem:

Two tangents intersect at a chainage 59+60 the deflection angle being 50° 30'. Calculate necessary data for setting out a curve of 15 chains Radius to connect to connect two tangents if it is intended to set out the curve offsets from chords. Take peg interval = 100 links.

Sol:- $R = 15$ chains $\quad \Delta = 50^\circ 30'$ $\quad = 100 \times 0.2 = 20m$

$= 15 \times 20 = 300\ m$

The length of chain = 20m (100 links).

$$T = R \tan \frac{\Delta}{2}$$

$$= 300 \tan \left(\frac{50^\circ 30'}{2} \right)$$

$$= 141.48\ m$$

Length of curve $(l) = \frac{\pi R \Delta}{180^\circ} = 264.42\ m$

Chainage of (V) P.I = 59 chains + 60 links

$= 59(20) + 60(0.2)$

$= 1180 + 12$

$= 1192\ m$

Chainage of T_1 = chainage of $V - T$

$= 1192 - 141.48 = 1050.52m$

Chainage of T_2 = chainage of $T_1 + l$

$= 1050.52 + 264.42 = 1314.94\ m$

The chainage of each peg = 100 links

$= 100 \times 0.2 = 20m$

Length of first subchord : 'c' = $1060 - 1050.52$

$= 9.48\ m$

length of last subchord $c' = 1314.94 - 1300$

$= 14.94m$

$C = 20\ m$.

No. of full chords $= \frac{1300 - 1060}{20} = 12$ of each 20m. length

Total no. of chords $= 1 + 12 + 1$

$$= 14$$

$$O_1 = \frac{c^2}{2R}$$

$$O_1 = \frac{(9.48)^2}{2 \times 300} = 0.149\ m$$

$$O_2 = \frac{C}{2R}(c + C) = \frac{20}{2 \times 300}(9.48 + 20)$$

$$= 0.98\ m$$

$$O_3 = O_4 = O_{n-1} = \frac{c^2}{R} = \frac{(20)^2}{300} = 1.33\ m.$$

$$O_n = \frac{c'}{2R}(C + c')$$

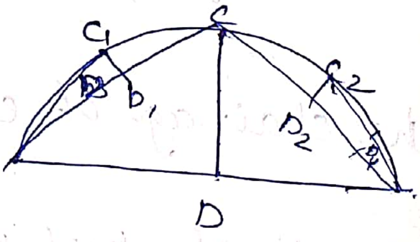
$$= \frac{14.94}{2 \times 300}(20 + 14.94) = 0.87\ m$$

b) By succesive bisection of Arcs (or) Chords.

$$CD = R\left(1 - \cos\frac{\Delta}{2}\right)$$

$$C_1D_1 = C_2D_2 = R\left(1 - \cos\frac{\Delta}{4}\right)$$

$$C_3D_3 = C_4D_4 = R\left(1 - \cos\frac{\Delta}{8}\right),$$



problem:

Q) It is required to set out a curve of radius 100m with pegs at approximately 10 m centres. $\Delta = 60°$, draw up the data necessary for pegging out the curve ~~Cant~~ chord bisection method.

Sol:- $\Delta = 60^\circ$, $R = 100m$

$CD = R\left(1 - \cos\frac{\Delta}{2}\right) = 100\left(1 - \cos\frac{60}{2}\right) = 13.40\,m$

$C_1D_1 = C_2D_2 = R\left(1 - \cos\frac{\Delta}{4}\right) = 100\left(1 - \cos\left(\frac{60}{4}\right)\right) = 3.41\,m$

$C_3D_3 = C_4D_4 = R\left(1 - \cos\frac{\Delta}{8}\right) = 100\left(1 - \cos\frac{60}{8}\right) = 0.86\,m$

Q. Two straights AB and BC are connected by a circular curve of radius 300m. Calculate elements of curve if $\Delta = 30^\circ$

Sol:- $R = 300m$, $\Delta = 30^\circ$.

1) Length of curve $l = \frac{\pi R \Delta}{180^\circ} = \frac{\pi \times 300 \times 30^\circ}{180^\circ} = 157.08\,m$

2. Tangent length $(T) = R\tan\frac{\Delta}{2} = 300\tan\frac{30^\circ}{2} = 80.38m$

3. Length of long chord $(L) = 2 \times 300 \times \sin\frac{30^\circ}{2} = 155.29m$

4. Apex distance $E = 300\left(\sec\frac{30}{2} - 1\right) = 10.58\,m$

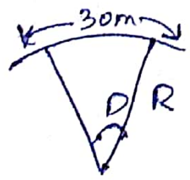
5. Mid ordinate $(M) = 300\left(1 - \cos\frac{30}{2}\right) = 10.22$

6. Ch of T_1 = Chainage of V − tangent length
$= 1192 - 80.38 = 1111.62$

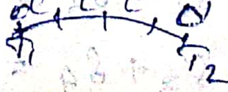
7. Ch of T_2 = ~~1192~~ $1111.62 + 157.08 = 1268.7m$

8. $I = 180 - \Delta = 180 - 30 = 150^\circ$.

1) Normal chord : (C) : A chord between two successive regular stations on a curve

2) 

$R = \frac{1719}{D}$ 3) $D = \frac{1719}{R}$

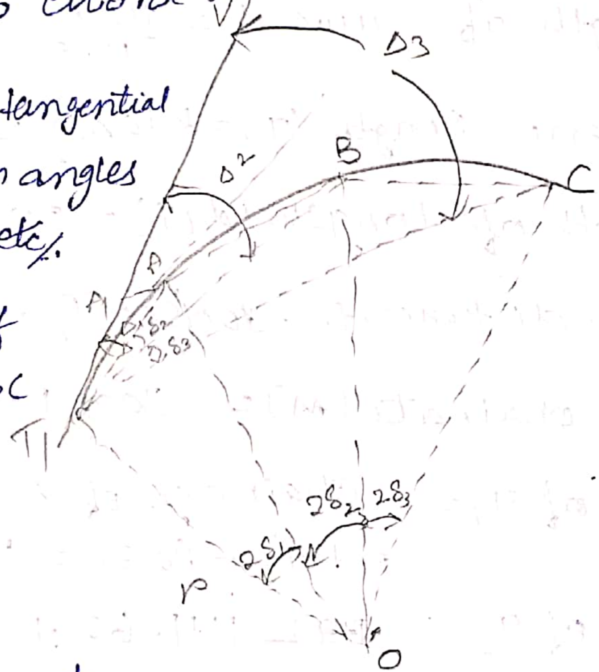
4) Subchord (C) : subchord is any chord shorter than normal chord. 

Angular methods :

1) Rankine's method of Tangential (or) deflection angles:

A deflection angle to any point on the curve is the angle at point of curve between back tangent and chord from PC to that point.

Rankine's method is based on the principle that the deflection angle to any point on a curve is measured by half the angle subtended by the arc from P.C to that point. It is assumed that length of arc is app equal to chord.



$\Delta_1, \Delta_2, \Delta_3$ = total tangential angles or deflection angles to the points A, B, C etc.

capital C_1, C_2, C_3 = length of the chords T_1A, AB, BC

T_1V = Rear tangent

T_1 = P.C , $\delta_1, \delta_2, \delta_3$ = tangential angles or the angles which each of the succesive chords T_1A, AB, BC etc makes with the respective tangents to the curve at T_1, A, B

formulae :

$$a = \delta_1 = 1718.9 \frac{c \text{ (small)}}{R} \text{ minutes}$$

$$\Delta_2 = \delta_1 + \delta_2 = \Delta_1 + \delta_2$$

$$\Delta_3 = \delta_1 + \delta_2 + \delta_3 = \Delta_2 + \delta_3$$

$$\Delta_4 = \delta_1 + \delta_2 + \delta_3 + \delta_4 = \Delta_3 + \delta_4$$

$$\Delta_n = \delta_1+\delta_2+\delta_3+\cdots+\delta_n = \Delta_{n-1}+\delta_n = \frac{\Delta}{2}.$$

Hence the deflection angle for any chord is equal to deflection angle for the previous chord + tangential angle for that chord.

problem:

Calculate the necessary data for setting out the curve when two tangents intersect at a chainage (59+60), deflection angle is $50° 30'$. Radius is 15 chains $= 300 m$

Peg interval = 100 links i.e 20m. length of chain = 20m i.e 100 links. If it is intended to setout the curve by Rankine's method of tangential angles. If the theodolite has a least count of 20sec; Tabulate actual readings of deflection angles to be setout.

sol:-

$$C = 9.48 m$$

$$C' = 14.94 m \quad C = 20m$$

$$\delta_1 = 1718.9 \frac{C}{R} = 1718.9 \times \frac{9.48}{300}, = \cancel{54.31 m}$$

$$= 54.32' = 54 \quad = \cancel{54' 19' 2.06''}.$$

$$= 0° 54' 19''$$

$$\Delta_2 = \Delta_1 + \delta_2 \qquad \delta_2 = \frac{1718.9 \times 20}{R} = \frac{114.593'}{60}$$

$$= 1° 54' 35.60''$$

$$\delta_2 = \delta_3 = \cdots = \delta_{13} = 1° 54' 35.60''$$

$$\delta_{14} = 1718.9 \frac{C'}{R} = \frac{1718.9 \times 14.94}{300} = \frac{85.601'}{60}$$

$$\delta_{14} = 1° 25' 35''$$

$$\Delta_1 = \delta_1 = 0° 54' 19''$$

$$\Delta_2 = \Delta_1 + \delta_2 =$$

$$\Delta_3 = \Delta_2 + \delta_3 =$$

$2 =$ Bearing of DC $-$ Bearing of DB

$78°46'23.32'' =$ Bearing of DC $- (180° + 160°56'55.08'')$

Bearing of DC $= 59°43'18.4''$

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Tacheometric surveying :

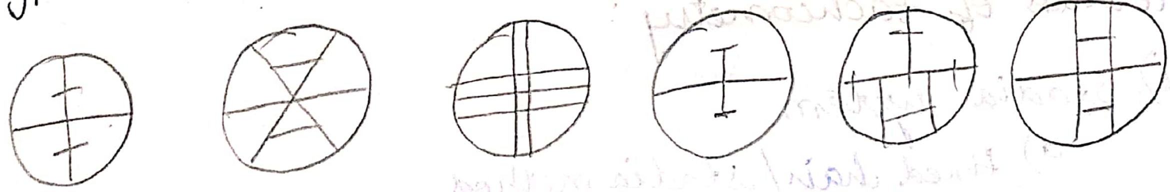
Tacheometry or telemetry is a branch of angular surveying in which the horizontal and vertical distances of points are obtained by optical means as opposed to the ordinary slower process of measurements by tape or chain.

This method is very fast and convinient. It is best adopted in obstacles such as steep and broken ground and deep ravines, streches of water (or) swamps etc where chaining is difficult or impossible.

The primary object of tacheometry is preparation of contour maps or plans requiring both horizontal and vertical control.

It provides a check on distances measured with tape.

Types of stadia diaphragms:



Features of thaceometer:

→ Multiplying const should have a value of 100 (K) and additive constant (C) should have a value of '0'

→ The axial horizontal line should be exactly midway between other two lines.

→ Telescope should be truly Anallictic lens

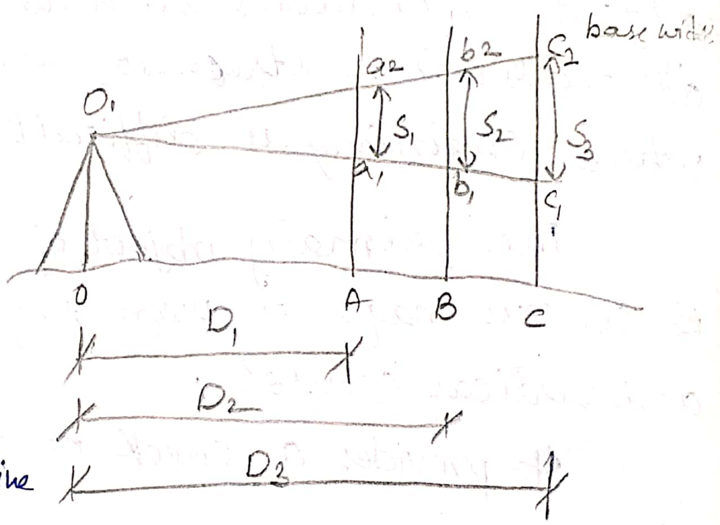
→ Telescope should be powerful having a magnification of 20-30 diameters.

* For small distances up to 100m ordinary leveling staff is used.

* For greater distances stadia rod is used.

Principle of tacheometry:

This is based on isosceles $\triangle$le. ~~Inan~~ In an isosceles $\triangle$le, the ratio of $\perp$lar distance from apex to base and base width is always constant.



$$\frac{D_1}{S_1} = \frac{D_2}{S_2} = \frac{D_3}{S_3} = \frac{f}{i}$$

where $\frac{f}{i} = k$ is known as Multiplying constant

f = focal length of objective

i = Stadia intercept

Methods of tacheometry:

1) Stadia system

a) Fixed hair / stadia method

b) movable hair method / subtense method

2) Tangential system

3) Measurements by means of special instruments.

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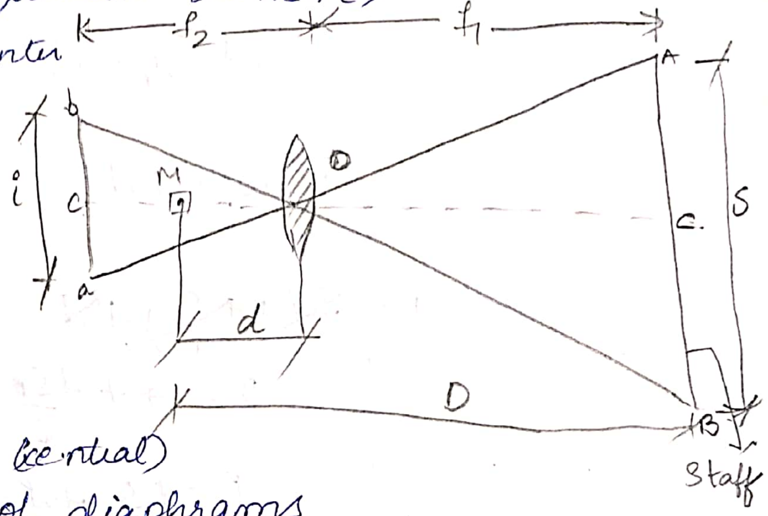
Fixed hair / stadia Method :

* principle of Stadia Method :

(Distance equation $D = KS + C$)

where 'O' is optical center of objective glass



ACB = points cut by three lines of ~~side~~ sight corresponding to three wires.

a, b, & c = top, axial (central) and bottom hairs of diaphrams

ab = i = interval between stadia hairs (stadia interval or stadia intercept)

AB = S = staff intercept

f_1 = horizontal distance of staff from optical center of objective

f_2 = horizontal distance between cross wires from 'O'

M = center of instrument corresponding to vertical axis.

d = distance of vertical axis of instrument from O

D = distance between instrument center to staff

By law of similar Δles,

$$\frac{f_1}{f_2} = \frac{S}{i}$$

$\because$ The rays ~~bOf~~ BOb & AOa pass through ~~the~~ 'O', they are straight so that Δ AOB and Δ boa are similar Δles.

$\because$ f_1, f_2 are conjugate focal distances, from lens formula

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Multiplying numerator with ff_1 on both sides

$$\frac{ff_1}{f} = \frac{ff_1}{f_1} + \frac{ff_1}{f_2}$$

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$$f_1 = f + \frac{f f_1}{f_2}$$

$$\cancel{f_1} \quad f_1 = f + f\left(\frac{s}{i}\right) \qquad \boxed{\because \frac{f_1}{f_2} = \frac{s}{i}}$$

$$\boxed{D = f_1 + d}$$

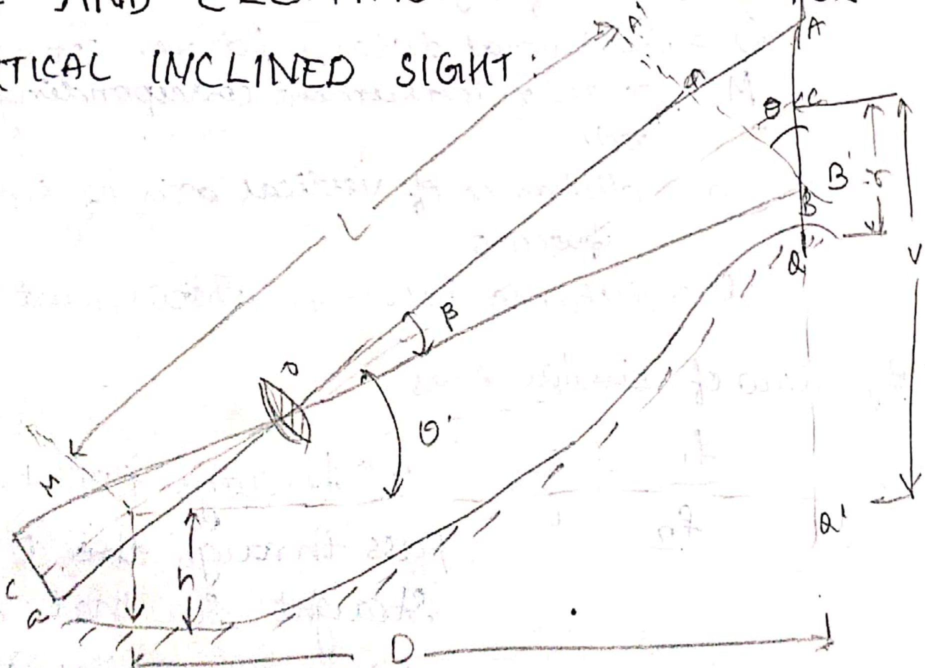
$$D = \cancel{f\left(1+\frac{s}{i}\right)+d}$$

$$D = f + \frac{fs}{i} + d$$

$$D = \underbrace{\frac{f}{i}}_{K} s + \underbrace{f + d}_{C}$$

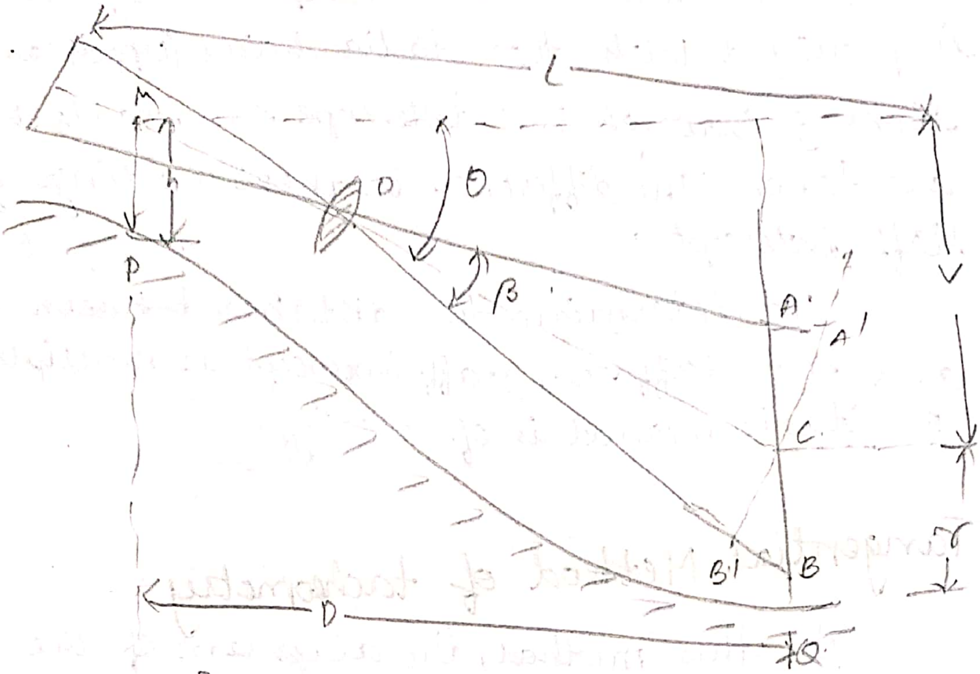
$\boxed{D = Ks + C}$ is known as distance equation

DISTANCE AND ELEVATION FORMULAE FOR STAFF VERTICAL INCLINED SIGHT:



Elevated sights vertical holding

Elevation of staff station (Q) = Elevation of instrument station $+ h + V - r$



Depressed sight - vertical holding

$$\boxed{D = KS\cos^2\theta + C\cos\theta}$$

$$\boxed{V = KS\frac{\sin 2\theta}{2} + C\sin\theta}$$

Elevation of staff station Q = Elevation of P + h − V − γ

Problem:

Two distances of 20m and 100m were accurately measured out and the intercepts on the staff b/w the outer stadia web were 0.196 m and the former distance & 0.996m at the later. Calculate the tacheometric constants.

sol:- $D = KS + C$.

$S_1 = 0.196\,m$ $D_1 = 20m$

$S_2 = 0.996\,m$ $D_2 = 100m$

$D_1 = KS_1 + C$

$20 = K(0.196) + C \rightarrow (1)$

$K = 100$

$C = 0.4$

$D_2 = KS_2 + C$

$100 = K(0.996) + C \rightarrow (2)$

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Stadia method:

In this method, the diaphram of the tachometer is provided with two stadia hairs (upper and lower) looking through the telescope the stadia hair readings are taken. The difference in these readings gives the staff intercept.

To determine the distance between the station and the staff. The staff intercept is multiplied by k i.e 100

The stadia method is of two types:

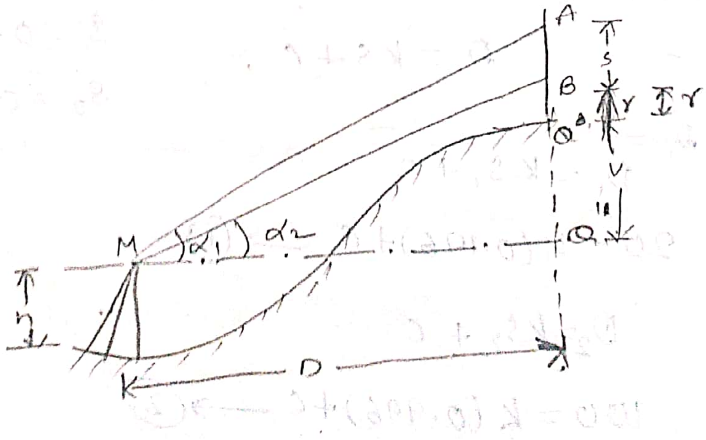
Tangential Method of tacheometry:

In this method, the diaphram of the tachometer is not provided with stadia hair. The readings are taken by single horizontal hair.

The staff consists of two vanes or targets at a known distance apart. To measure the staff intercept, two pointings are required. The angles of elevation or depression are measured and their tangents are used for finding the horizontal distances and elevations.

The stadia method requires only one observation but tangential method requires two pointings of telescope.

Case(i): Both Angles are angles of elevation



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Take Δle BMQ'

$$\tan\alpha_2 = \frac{V}{D}$$

$$V = D\tan\alpha_2 \rightarrow (1)$$

Δ AMQ'

$$\tan\alpha_1 = \frac{(V+S)}{D}$$

$$V+S = D\tan\alpha_1 \rightarrow (2)$$

(2) − (1)

$$S = D\tan\alpha_1 - D\tan\alpha_2$$

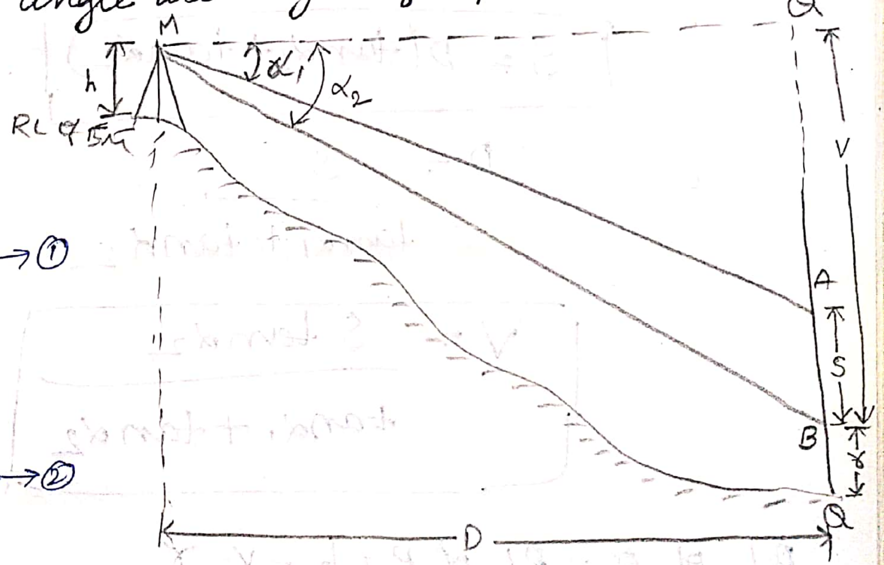
$$S = D(\tan\alpha_1 - \tan\alpha_2)$$

$$D = \frac{S}{\tan\alpha_1 - \tan\alpha_2}$$

$$V = D\tan\alpha_2$$

$$\boxed{V = \frac{S\tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}}$$

Case (ii): Both angle are angles of depression



Δ AMQ'

$$\tan\alpha_1 = \frac{V-S}{D}$$

$$V-S = D\tan\alpha_1 \rightarrow (1)$$

Δ BMQ'

$$\tan\alpha_2 = \frac{V}{D}$$

$$V = D\tan\alpha_2 \rightarrow (2)$$

~~(2) − (1)~~

~~(1) − (2)~~ (2) − (1).

$$S = D(\tan\alpha_2 - \tan\alpha_1)$$

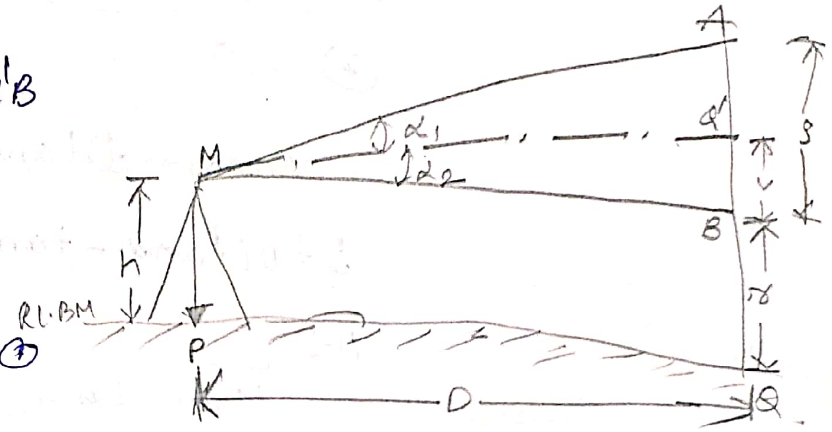
$$\boxed{D = \frac{S}{\tan\alpha_2 - \tan\alpha_1}}$$

$$V = \frac{S\tan\alpha_2}{\tan\alpha_2 - \tan\alpha_1}$$

RL of Q' = RL ~~of~~ BM + h − V − γ

Case (iii):

One angle elevation and other depression



Consider Δ MQ'B

$$\tan\alpha_2 = \frac{V}{D}$$

$$\boxed{V = D\tan\alpha_2} \rightarrow (1)$$

Δ MAQ'

$$\tan\alpha_{\cancel{2}} = \frac{S-V}{D}$$

$$S - V = \cancel{to}\ D\tan\alpha_1 \rightarrow (2)$$

(1) + (2)

$$\boxed{S = D(\tan\alpha_1 + \tan\alpha_2)}$$

$$D = \frac{S}{\tan\alpha_1 + \tan\alpha_2}$$

$$\boxed{V = \frac{S\tan\alpha_2}{\tan\alpha_1 + \tan\alpha_2}}$$

RL of Q = RL of P + h − V − γ

problem:

The vertical angles to vanes fixed at ~~one A~~ 1m and 3m above the foot of the staff vertically at a station A were $+2^\circ 30'$ and $+5^\circ 48'$ respectively. Find horizontal distance and RL of A if height of the instrument determined from observation on to a BM is 438.556 m above datum.

Sol:-

$$\alpha_2 = 2^\circ 30'$$

$$\alpha_1 = 5^\circ 48'$$

case(i)

$$D = \frac{S}{\tan\alpha_1 - \tan\alpha_2} \qquad V = D \tan\alpha_2$$

$$D = \frac{3-1}{-\tan(2^\circ 30') + \tan(5^\circ 48')} = 34.53\,m$$

$$V = (34.53)\tan(5^\circ 48')$$

$$V = 1.508\,m.$$

$$RL \text{ of } A = 438.556 + V - r$$

$$= 438.556 + 1.508 - 1$$

$$= 439.064\,m$$