

LEVELLING & CONTOURING

Leveling:

Leveling: Basic definitions, types of levels and ~~staves~~ leveling staves, Temporary and permanent adjustments - method of leveling. Booking and determination of level - HI method - Rise & fall method, effect of curvature of earth and refraction.

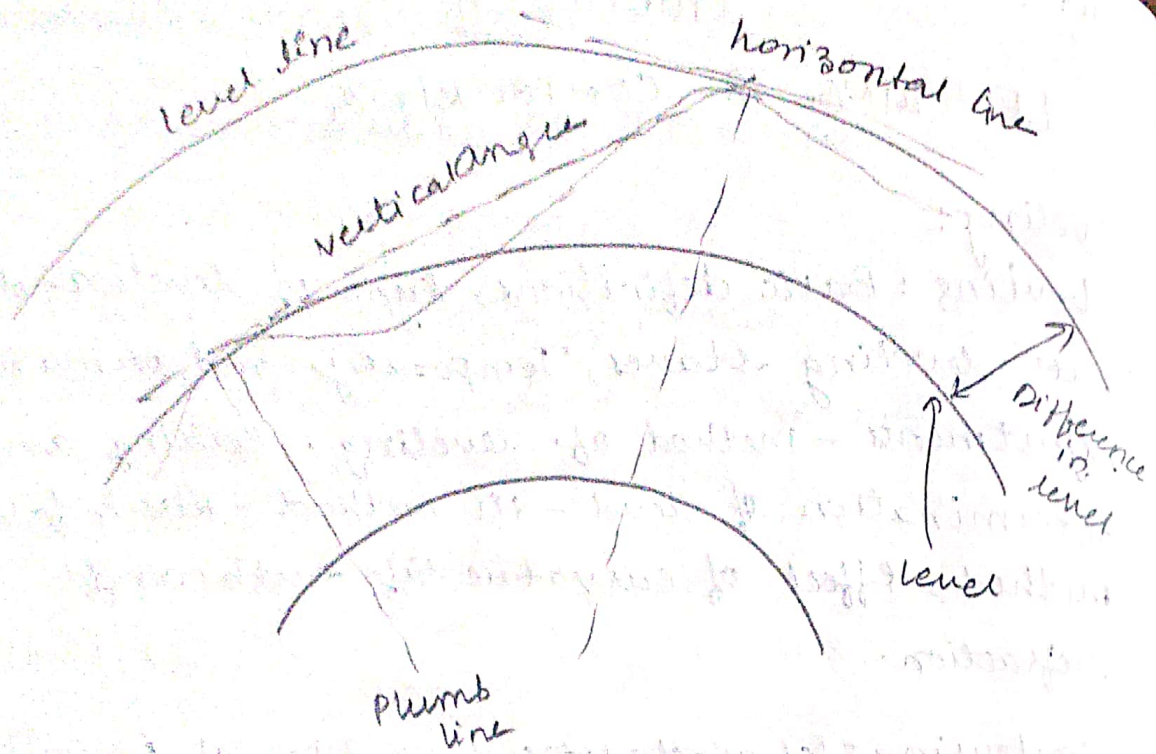
Contouring: characteristics and uses of contours, Direct and indirect methods of contouring, contour surveying, Interpolation and sketching of contours.

Leveling:

It is a branch of surveying. The object of which is: a) to find the elevations of given points wrt a given or assumed datum.

b) To establish points at a given elevation or at different elevations wrt to a given or assumed data.

- First operation is required to enable the works to be designed and second operation is required in setting out of all kinds of engineering works.
- Leveling deals with measurements in a vertical plane.



Level surface :

It is a curved surface which at each point is \perp to the direction of gravity at that point.

Eg: The surface of still water

Any surface \parallel to the mean spheroidal surface of the earth is a level surface.

Level line:

It is a line lying on level surface. \therefore

Horizontal line:

It is a straight line tangential to the level line at a point. It is also \perp to the plumb line.

vertical line:

It is normal to the level line at a point

Datum:

It is any surface to which elevations are referred. MSL is a convenient datum world over and elevations are commonly given as so much above or below MSL (mean sea level)

Bench Mark:

It is a relatively permanent point of reference whose elevation wrt some assumed datum is known.

Mean Sea Level:

It is the average height of sea for all stages of the tides. At any particular place it is derived by averaging the hourly tides heights over a long period of 19 years.

13/08/19

Elevation:

The elevation of a point on or near the surface of the earth is its vertical distance above or below an arbitrarily assumed level surface or datum.

Methods of leveling:

- 1) Barometric leveling
- 2) Trigonometric or indirect leveling
- 3) spirit leveling or direct leveling.

Types of levels & leveling states:

The instruments commonly used in direct leveling are a) level

b) leveling staff

c) Tripod.

a) Level:

The purpose of a level is to provide a horizontal line of sight. It consists of 4 parts.

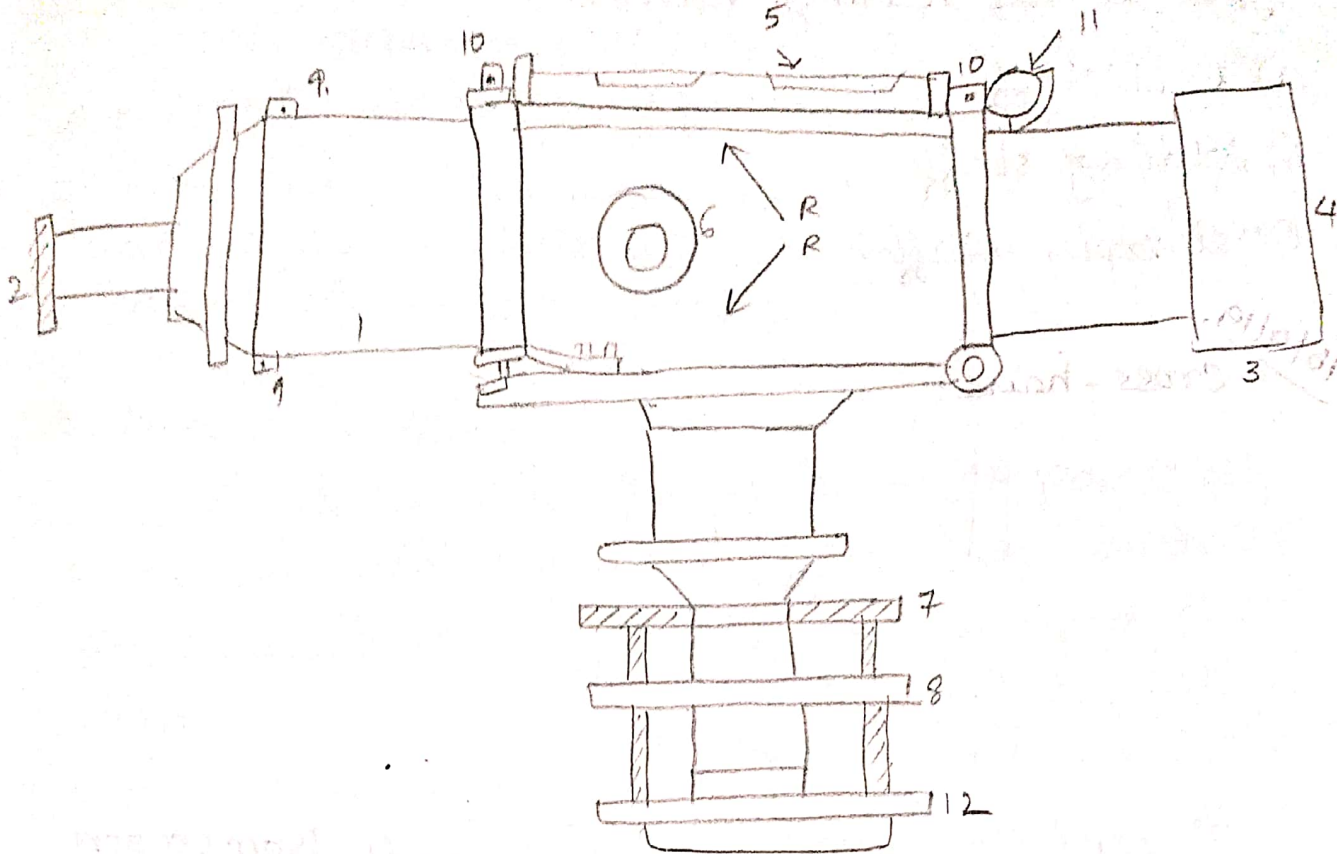
- (i) Telescope - To provide line of sight.
- (ii) Level tube - To make line of sight horizontal
- (iii) A leveling head - Tri brach & Trivet stage.
- (iv) Tripod - to support the instrument.

Types of levels:

- 1) Dumpy level
- 2) Wye (Y) level
- 3) Reversible level
- 4) Tilting level

Diagram of dumpy level & its component parts.

- 1) Telescope
- 2) Eye-piece
- 3) Ray shade
- 4) Objective end
- 5) Longitudinal bubble
- 6) Focusing screws
- 7) Foot screws
- 8) Upper parallel plate
- 9) Diaphragm Adjusting screws
- 10) Bubble tube adjusting screws
- 11) Transverse bubble tube
- 12) Foot plate.



Types of leveling staves :

1) A leveling staff is a straight rectangular rod having graduations, the foot of the staff represents '0' reading.

→ Purpose of leveling staff is to determine the amount by which the station i.e. the foot of the staff above or below the line of sight.

- 1) Self reading staff
- 2) Target staff.

Self Reading staff :

It is the one which can be read directly by the instrument man through telescope.

Target staff :

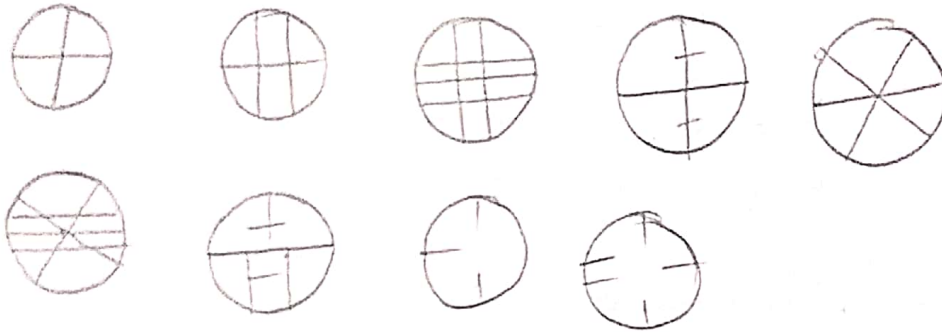
It contains a moving target against which the reading is taken by staff man.

Types of self reading staves:

- a) solid staff
- b) folding staff
- c) Telescopic staff.

16/9/19.

Cross-hairs:



These are made up of threads from cocoon of the brown spider, but may be of very fine platinum wire or filaments of silk.

*Temporary (or) station Adjustments of a level;

It consists of .

- 1) Setting of the level
- 2) leveling up
- 3) Elimination of parallax .
 - (i) focussing eye piece
 - (ii) focussing objective

Special methods of direct leveling: (spirit leveling)

- (1) Differential leveling / fly leveling
- (2) profile levelling
- (3) cross-sectioning
- (4) reciprocal leveling
- (5) precise levelling

(1) Differential levelling:

To determine difference in elevation of two points is called differential leveling.

→ when the points are apart, it is necessary to set up the instrument no. of times. This type is called fly ~~the~~ type levelling.

(2) Profile levelling:

To determine the elevation of points at measured intervals along a given line in order to obtain a profile of the surface along that line.

(3) Cross-sectioning:

It is also called as cross-levelling. It is the process of taking levels on each side of a main line at right angles to that line, in order to determine a vertical cross-section of the surface of the ground or of underlying strata or of both.

(4) Reciprocal levelling:

The difference in elevation between two points is accurately determined by two sets of reciprocal observations when it is not possible to setup the level between the points.

(5) Precised levelling:

It is the levelling in which the degree of precision required is too great to be attained by ordinary methods. Therefore special equipment or special precautions are required to eliminate sources of errors.

Terms & Abbreviations:

1) Station: It is the point where the level rod is held but not where the level is set up.

2) Height of Instrument:

It is the elevation of line of sight with ~~the~~^{the} assumed datum. It does not mean that height of telescope above the ground.

3) Back ^{sight} site:

It is the sight taken on a rod at a point of known elevation, to ascertain the amount by which the line of sight is above that point so that we can obtain height of instrument.

Back lighting \oplus is equal to measuring up from the point of known elevation to line of sight. It is also known as '+' sight.

4) Fore Sight:

It is a sight taken on a rod held at a point of unknown elevation to ascertain the amount by which the point is below the line of sight, and thus to obtain the elevation of the station.

It is also known as minus sight ('-' sight) except in special case of tunnel survey.

5) Turning point or change point:

T.P or C.P is a point on which both '+' sight and '-' sight are taken. The '-' sight is taken on the point in one set of instrument to ascertain the elevation of the point where as '+' sight is taken on the same point in other set of

the instrument to establish new height of instrument.

b) Intermediate sight :

It is a point / these are the points intermediate between back sight and fore sight on which the minus sight is taken to determine the elevations of intermediate stations.

(i) $H.I = R.L + B.S$

(Elevation of bench mark + back sight)

(ii) Elevation of station point = $H.I - I.S / F.S$

Hand signals during observations :

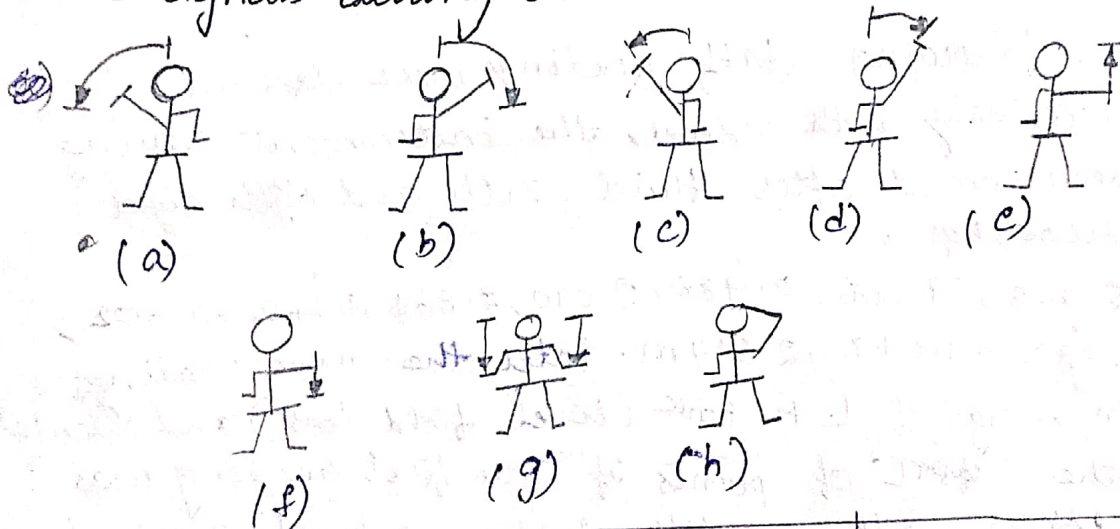


Fig	signal	Message
(a)	Movement of left arm over 90°	Move to my left
(b)	Movement of right arm over 90°	Move to my right
(c)	Movement of left arm over 30°	Move top off staff to my left
(d)	Movement of right arm over 30°	Move top off staff to my right
(e)	Extension of arm horizontally and moving hand up wards	Right Rise height peg or staff
(f)	Extension of arm horizontally and moving hand down wards	Reduce height of Peg or staff.

(g)	Extension of both arms and slightly thrusting downwards	Establish the position.
(h)	Extension of arms and placement of hand on top of head.	Returns to me

Booking and Reducing levels:

There are 2 methods

- (i) collimation or height of instrument method.
- (ii) Rise and fall method.

⊗

Collimation or height of instrument method:

Problem:

1) The following staff readings were observed successively with a level, the instrument having been moved after third, sixth and eighth readings.

2.228, 1.606, 0.988, 2.090, 2.864, 1.262, 0.602, 1.982, 1.044, 2.684 m. Enter the above readings in a page of L.F. Book (Level field book) and calculate the R.L. of points if the first reading was taken with a staff held on a bench mark of 432.384 m

Sol:- method I - H.I method
or Collimation method.

Station	B.S	I.S	F.S	H.I	R.L	Remarks
A	2.228	1.606		434.612	432.384 433.006	RL of BM
B	2.090	2.864	0.988 1.262	435.714	433.624 432.85	CP1
C	0.602		1.262 1.482	432.79 435.054	431.588 434.452	CP2
D	1.044	2.684	1.982 2.684	431.252 434.116	430.208 ^{3.072} 428.568 431.432	CP3

$$\sum BS - \sum FS = L.R.L - F.R.L$$

$$5.964 - 6.916 = 431.432 - 432.384$$

$$-0.952 = -0.952$$

∴ There is a fall of 0.952 m.

Station	B.S	I.S	F.S	Rise Fall	Fall	R.L	Remarks
1.	2.228			0.622		432.384	RL of BM
2.		1.606		0.618		433.006	
3.	2.090		0.988		-0.774	433.624	C-P1
4.		2.864		1.602		432.85	
5.	0.602		1.262		1.38	434.452	CP2
6.	1.044		1.982		1.64	433.072	
7.			2.684			431.432	CP3

$$\sum Rise - \sum Fall = L.R.L - F.R.L$$

$$2.842 - 3.794 = 431.432 - 432.384$$

$$-0.952 = -0.952$$

Comparison of H.I method & Rise & fall method:

H.I method is more rapid less tedious & simple. However since the check on calculations for intermediate sights is not available. The mistakes in their levels pass unnoticed.

→ The Rise & Fall method even though more tedious, it provides a full check in calculations for all sides.

→ However H.I method is more suitable where it is required to take number of readings ~~to~~ from the same instrument setting such as for constructional work, profile leveling etc.

Problem:

The following figures were extracted from a levelled field book, some of the entries being illegible owing to exposure to rain. Insert the missing figures and check your results. Rebook all the figures by rise & fall method.

Station	BS	IS	FS	Rise	Fall	R.L	Remarks
1.	2.285					232.460	BM-1
2.	1.650		2.065 2.020	0.0200		232.480	
3.		2.105			0.455	232.025	
4.	2.025 1.625		1.960	0.145		232.17	
5.	2.050		1.925		0.300	231.87	
6.		1.665		0.385		232.255	BM-2
7.	1.690		1.325	0.340		232.595	
8.	2.865		2.100		0.41	232.185	
9.			1.625	1.24		233.425	BM-3

Check :

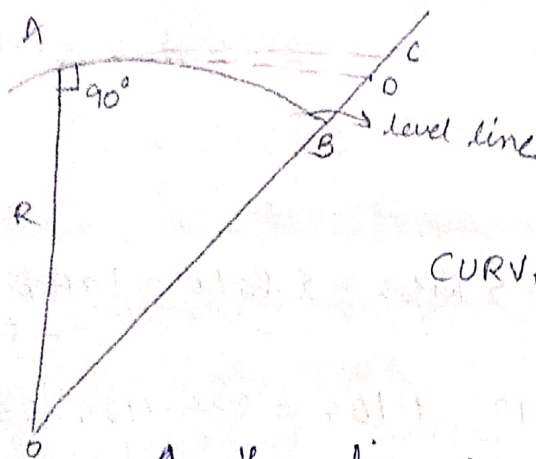
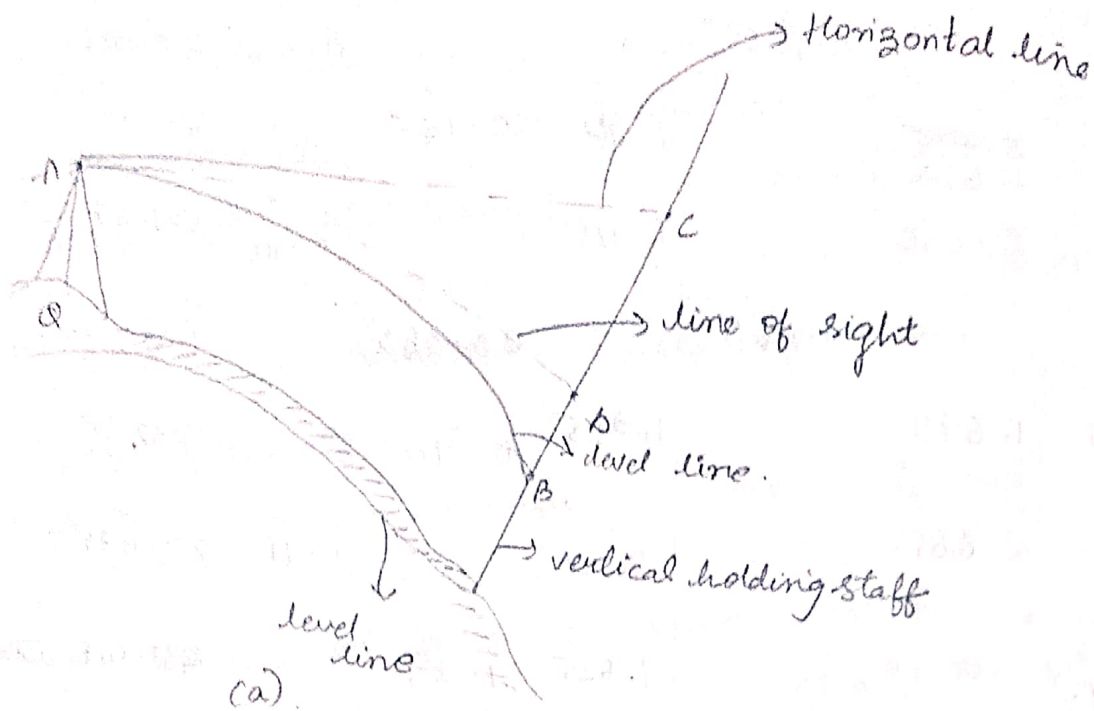
$$\sum B.S - \sum F.S = \sum Rise - \sum Fall = Last R.L - First R.L$$

$$\Rightarrow 12.165 - 11.2 = 2.13 - 1.165 = 233.425 - 232.460$$

$$\Rightarrow 0.965 = 0.965 = 0.965 \text{ m (Rise)}$$

Hence checked.

Curvature & Refraction corrections:



CURVATURE & REFRACTION

In the fig AC is horizontal line which deflects upwards from level line AB by an amount of BC. AD is the actual line of sight.

Curvature:
 Correction: $OC^2 = OA^2 + AC^2$

$$\angle CAO = 90^\circ$$

$$BC = C_c \text{ (correction for curvature)}$$

$$AB = d = \text{horizontal distance between A and B}$$

$$OB = AO = R = \text{Radius of earth in the same units as that of 'd'}$$

$$(R+C_c)^2 = R^2 + d^2$$

$$R^2 + C_c^2 + 2RC_c = R^2 + d^2$$

$$C_c^2 = d^2 - 2RC_c$$

$$C_c(C_c + 2R) = d^2$$

$$C_c = \frac{d^2}{C_c + 2R}$$

$$C_c = \frac{d^2}{2R + C_c}$$

$C_c \rightarrow$ negligible when compared to $2R$

$$\therefore C_c \approx \frac{d^2}{2R}$$

Note: Both R and d should be in same units.

2. Take radius of earth $R = 6370 \text{ km}$

$$C_c = \frac{d^2}{2 \times 6370} = 0.07849 d^2 \text{ m}$$

$$C_c = 0.07849 d^2 \text{ m}$$

In the above formula d value is in km while C_c will be in meters.

Corrections for refraction:

$$C_r = \frac{1}{7} \frac{d^2}{2R} = 0.01121 d^2 \text{ m}$$

$$C_r = 0.01121 d^2 \text{ m}$$

Combined correction for curvature and Refraction:

$$C = C_c - C_r$$

$$C = \frac{d^2}{2R} - \frac{1}{7} \frac{d^2}{2R}$$

$$C = \frac{6d^2}{7 \times 2R}$$

$$C = ~~0.0679~~ 0.06728 d^2 \text{ m}$$

$$C = 0.06728 d^2 \text{ m}$$

Problem:

Find the correction for curvature, refraction and combined correction for curvature and refraction for a distance of a) 1200m, b) 2.48 km

sol: a) $d = 1200 \text{ m}$

$$C_c = 0.07849 d^2 = 113025.6 \text{ m}$$

$$C_r = 0.01121 d^2 = 16142.4 \text{ m}$$

$$C = 0.06728 d^2 = 96883.2 \text{ m}$$

b) $2.48 \text{ km} \Rightarrow 2480 \text{ m}$

$$C_c = 0.07849 d^2 = 482744.89 \text{ m}$$

$$C_r = 0.01121 d^2 = 68945.98 \text{ m}$$

$$C = 0.06728 d^2 = 413798.912 \text{ m}$$

- 2) Find C , C_c , C_r for a distance of a) 3400m
 b) 1.29 km

Sol:- a) 3400m .

$$C_c = 0.07849d^2 = 907344.4 \text{ m}$$

$$C_r = 0.01121d^2 = 129587.6 \text{ m}$$

$$C = 0.06728d^2 = 777756.8 \text{ m}$$

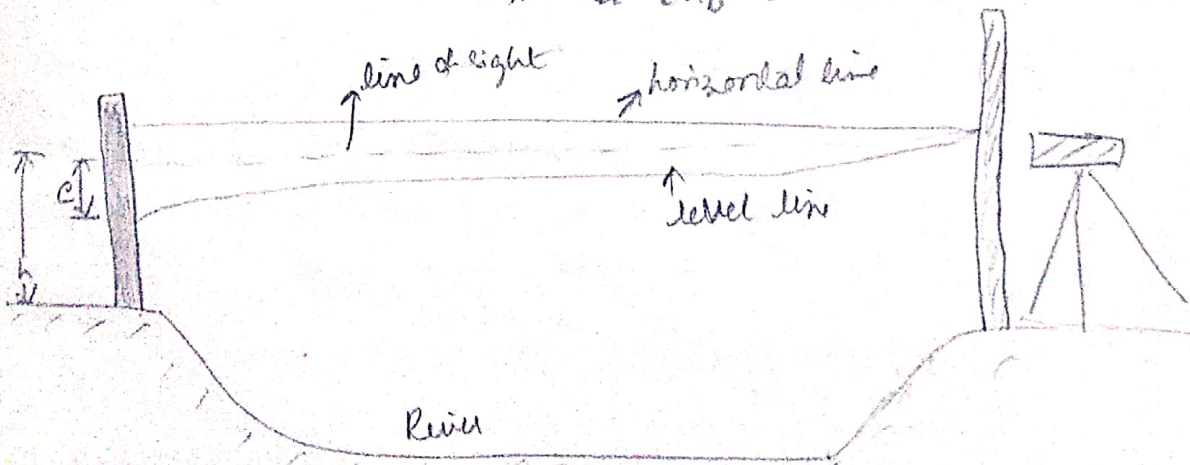
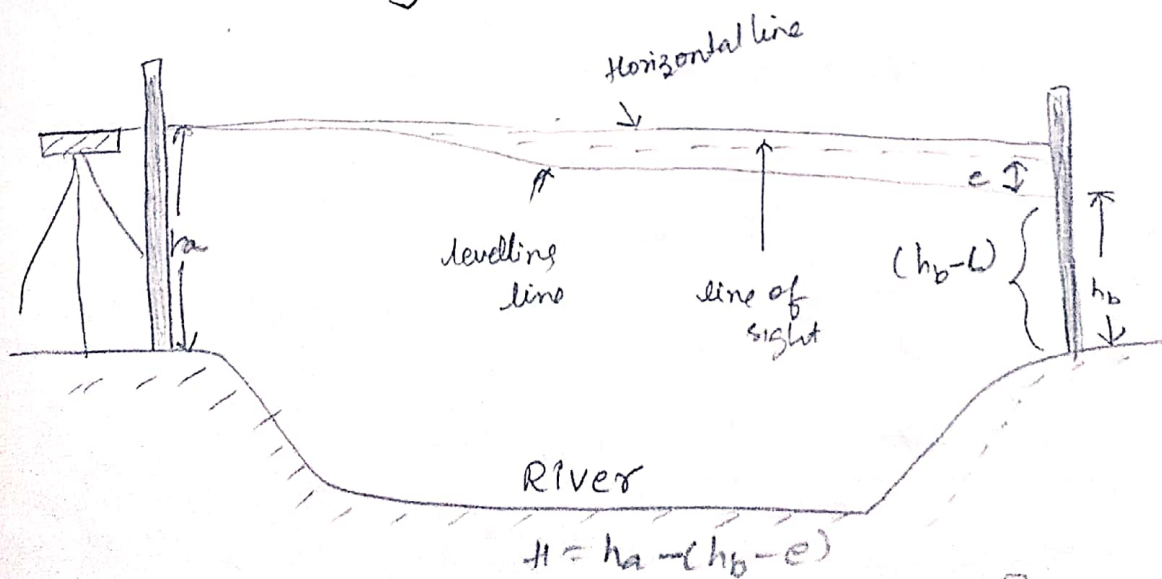
b) 1.29 km = 1290 m

$$C_c = 0.07849d^2 = 130615.20 \text{ m}$$

$$C_r = 0.01121d^2 = 18654.56 \text{ m}$$

$$C = 0.06728d^2 = 11960.648 \text{ m}$$

Reciprocal levelling:



Cross-sectioning :

Cross-sections are run at right angle to the longitudinal profile and on either side of it for the purpose of lateral outline to the ground surface. They provide the data for estimating quantities of earth work and for other purposes.

Sta	Distance			BS	IS	FS	HI	RL	Remarks
	L	C	R						
BM				1.325			101.325	100	
0		0			1.865			99.460	at
L ₁	3				1.905			99.420	at 5m
L ₂	6				2.120			99.205	channel
L ₃	9				2.825			99.500	
R ₁		3	3		1.705			99.620	
R ₂		7.5	7.5		1.520			99.805	
R ₃		10	10		1.955			99.370	at
I		20			1.265			100.060	at
L ₁	3				1.365			99.96	20m channel
L ₂	6				0.725			100.600	
L ₃	9				2.125			99.20	
R ₁			3		1.925			99.400	
R ₂			7		2.250			99.025	
R ₃			10		0.890			100.435	
						2.120		99.205	

$$\sum BS - \sum FS = \text{Last RL} - \text{First RL}$$

$$= 99.205 - 100$$

$$0.795 \quad \dots \quad 0.795 \text{ (fall)}$$

(fall)

Checked.

27/8/17

Errors in levelling:

Instrumental Error	Natural Error	Personal error
1) The error due to imperfect adjustment	1) Earth's curvature	1) Mistakes in manipulation
2) Sluggish bubble	2) Atmospheric refraction	2) Mistakes in rod handling
3) Error due to movement of objective slide	3) Variations in temperature	3) Errors in sighting
4) Rod not of standard length	4) Settlement of tripod or turning points	4) Mistakes in reading the rod
5) Error due to defective joint	5) error due to wind vibrations defect	5) Mistakes in recording and computing

Contouring:

Contour: It is an imaginary line on the ground joining the points of equal elevation.

Contour interval:

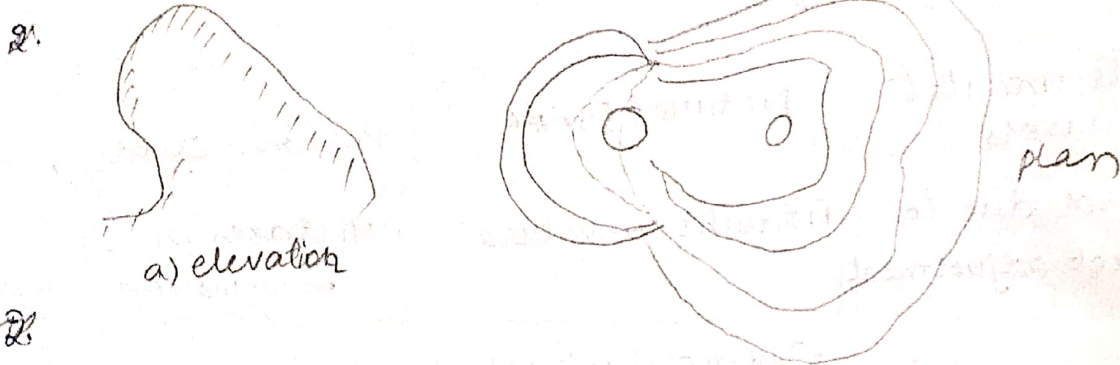
The vertical distance between any two consecutive contours is called contour interval.

Horizontal Equivalent :

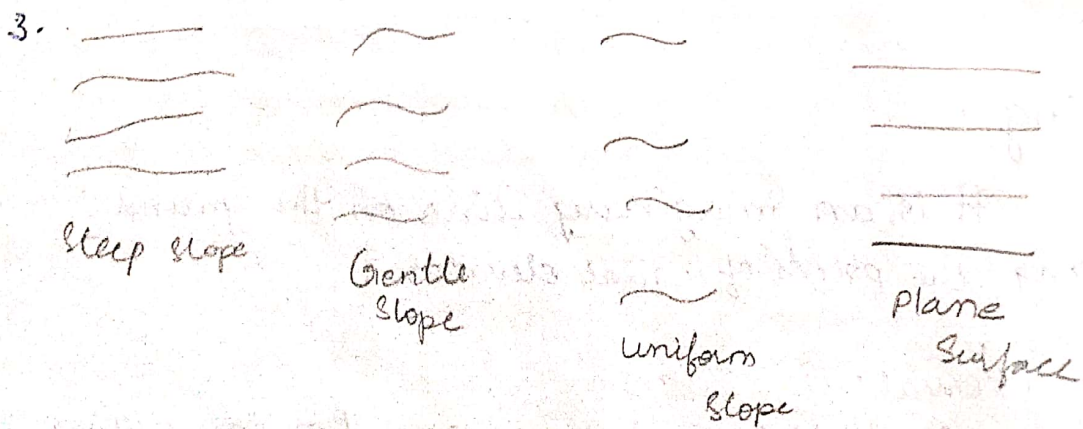
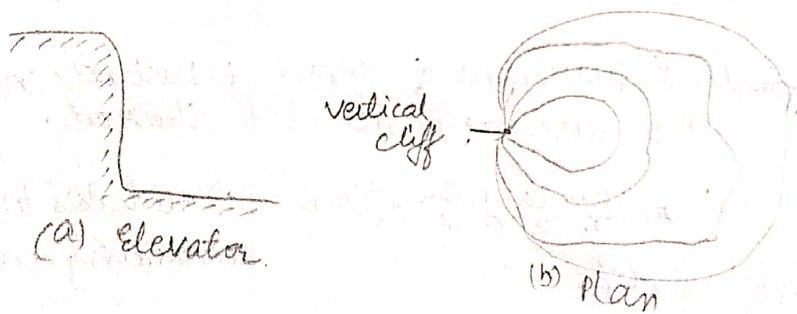
The horizontal distance between 2 points on 2 consecutive contours is called horizontal equivalent.

Characteristics of Contour :

1. Two contour lines of different elevations cannot cross each other, except in case of overhanging cliff & cave.



2. Contour lines of different elevations can unite to form one line only in case of vertical cliff.



Contour lines close together indicates steep slope if they are far apart it indicates gentle slope. If they are equally spaced, it indicates uniform slopes.

A series of straight, parallel, equally spaced contours represents a plane surface.

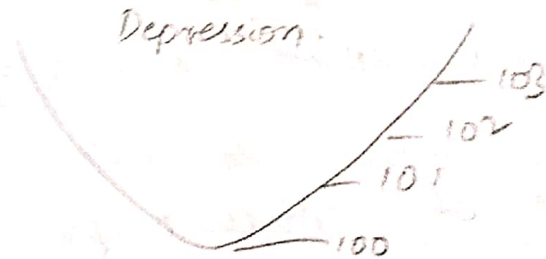
4. A contour passing through any point is \perp to the line of steepest slope at that point. A closed contour line with one or more higher ones inside it represents a hill.



Hill

— 0 M.S.

5. A closed contour line with one or more lower ones inside it indicates a depression without an outlet.

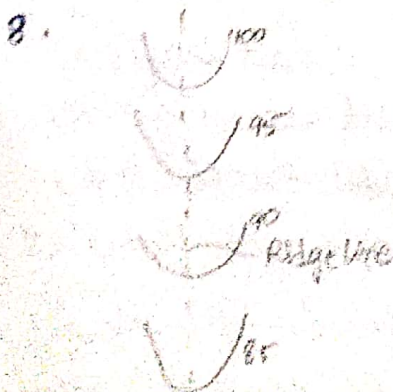


Depression

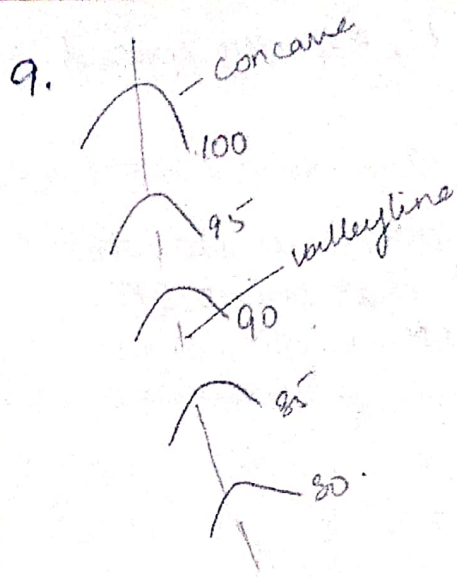
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6. Two contour lines having same elevation cannot unite and continue as one line. Similarly a single contour cannot split into 2 lines.

7. A contour line must close up on itself, though not necessarily within the limits of the map.



Contour lines cross a water shed or ridge line at right angles. They form U-shaped curves of concave side of curve towards higher ground.



Contour lines cross a valley line at right angles. They form sharp curves of V-shape across it with convex side of the curve towards higher ground.

10. The same contour appears on either side of a ridge or a valley for the highest horizontal plane that intersects the ridge must cut in both sides. The same is true of lower horizontal plane that cuts a valley.

Uses of Contour maps:

1. These are used to find out the nature of the ground.
2. To find out the profile of the ground along that line. It helps in finding out depth of cutting and depth of filling, if formation level of road or railway track is decided.
3. Inter-visibility of any two points can be found by drawing profile of the ground along that line.
4. To decide the route of railway, roadway, canal, sewer lines, can be decided to minimize the earth work and balancing the earthwork.
5. catchment area and quantity of water flow at any point of river can be formed. This study is very important in locating bunds, dams and also

to do find out flood levels.

6. From the contours we can calculate the capacity of reservoir.

Methods of locating Contours:

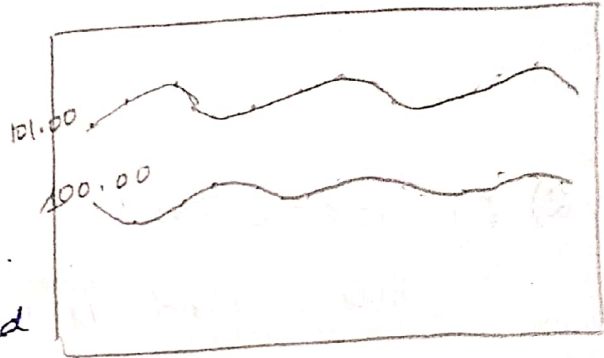
1. Direct Method
2. Indirect Method.

1. Direct method:

In the direct method, the contour to be plotted is to be traced on the ground.

Only those points are surveyed

which happens to be plotted. After having surveyed those points, they are plotted and contours are drawn through them. This method is slow and tedious so it is used for small areas and where great accuracy is required.

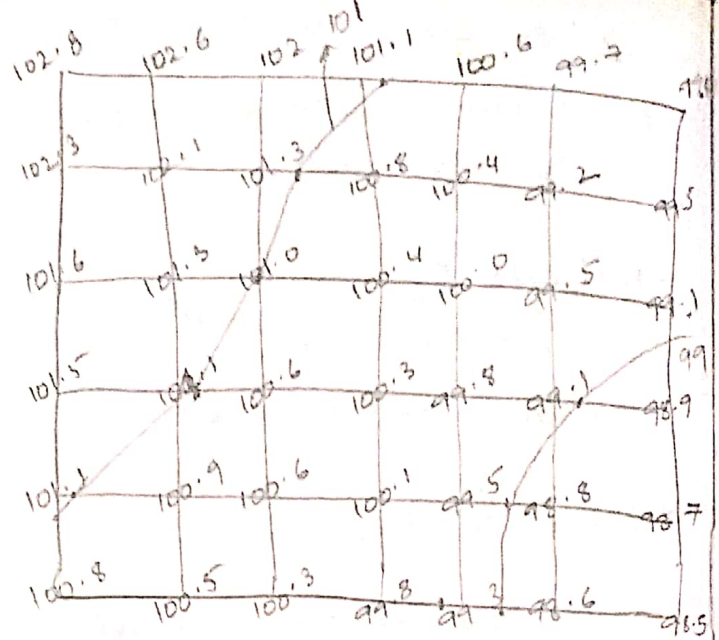


2. Indirect method:

In indirect method, some suitable guide points are selected and surveyed. The guide points need not necessarily be on the contours. These guide points having being plotted, serve as basis for the interpolation of contours. This is the method most commonly used in engineering surveys.

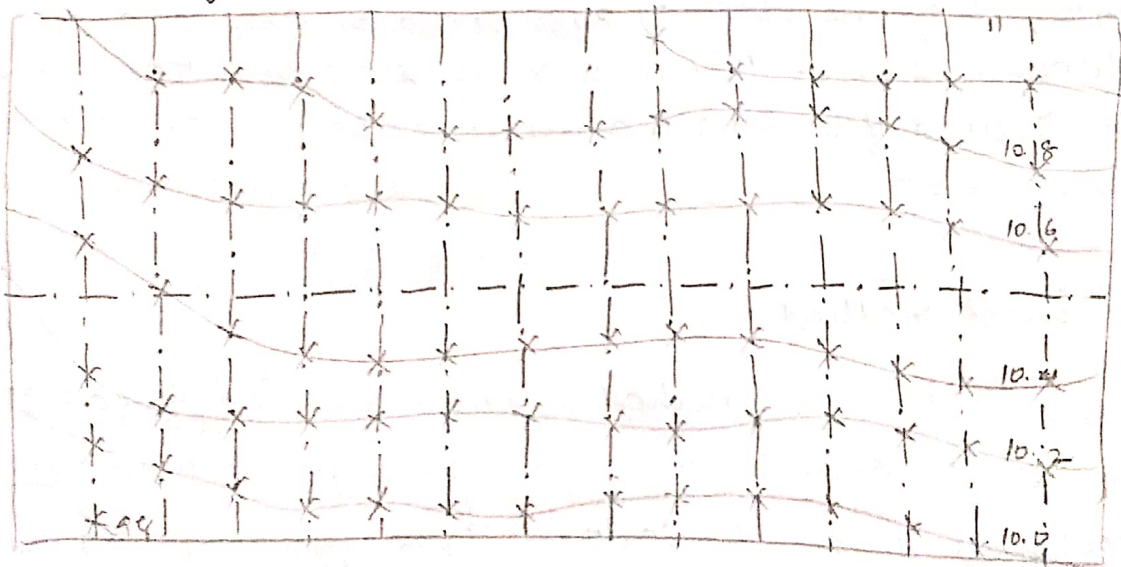
1) By squares:

(5m x 5m)

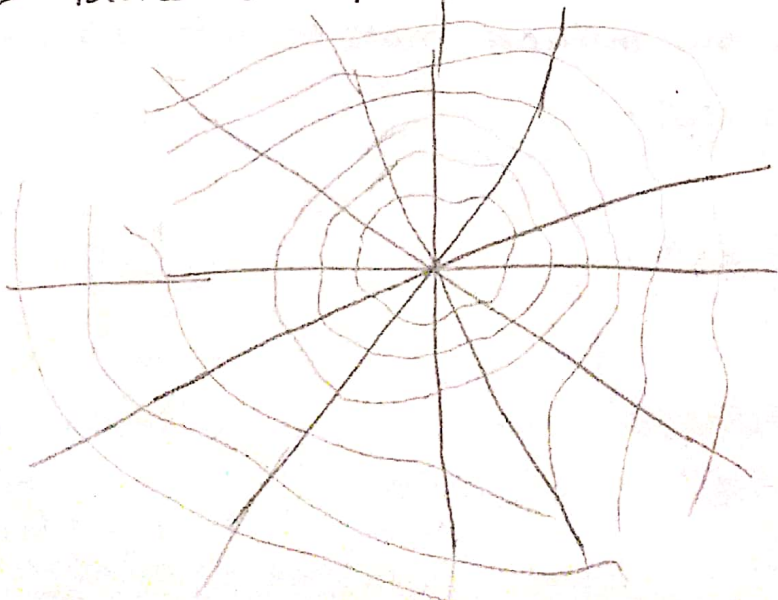


2) By cross-sections:

This method is suitable for road projects and railway projects



3) Tachometric / Radial line method:

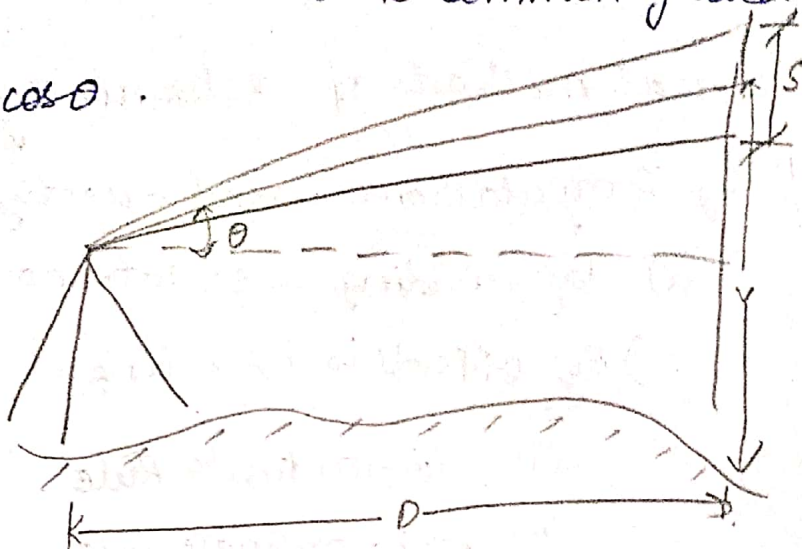


This method is suitable for hilly areas. In this method theodolite with tachometer is commonly used..

$$D = K_1 S \cos^2 \theta + K_2 \cos \theta$$

$$V = D \tan \theta$$

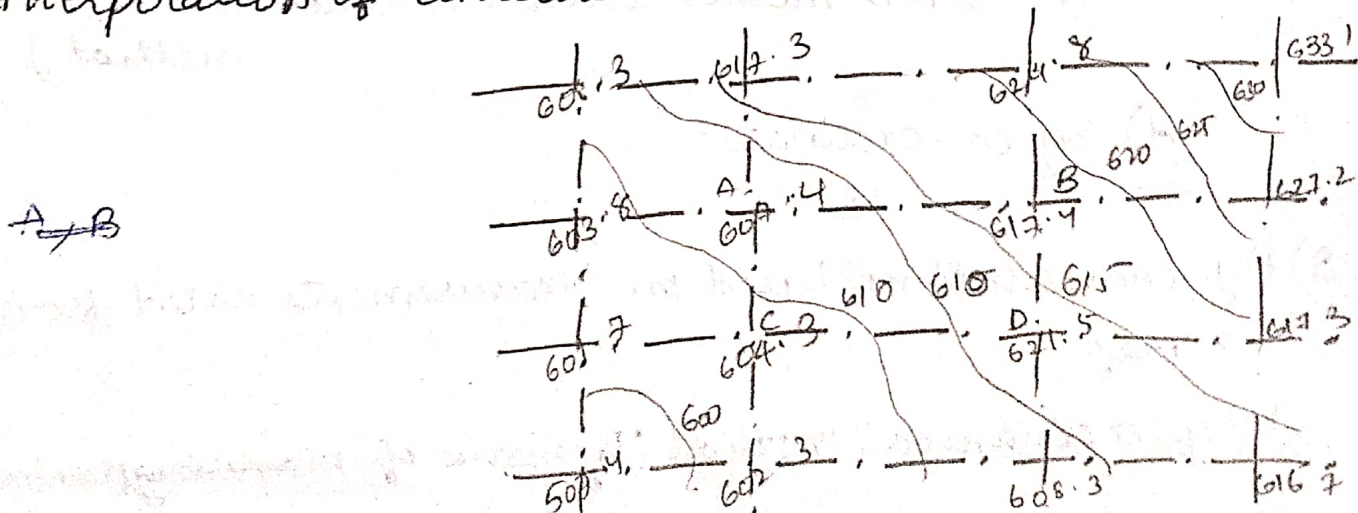
where K_1, K_2 are instrument constants



Contour gradient:

It is a line lying throughout on the surface of the ground and preserving a constant inclination to the horizontal.

Interpolation of contours:



MODULE - III

COMPUTATION OF AREAS & VOLUMES

General methods of determining areas:

1) By computations based directly on field measurements.

a) By dividing area into no. of triangles -

b) By offsets to base line.

(i) Mid-ordinate Rule

(ii) Avg-ordinate rule

(iii) Trapezoid " "

(iv) Simpson's one third rule

} at irregular intervals
at regular intervals.

c) By latitudes & departures

(i) D.M.D method (Double Meridian distance method)

(ii) D.P.D method (Double parallel distance method)

d) By co-ordinates

2) By computations based on measurements scaled from a map.

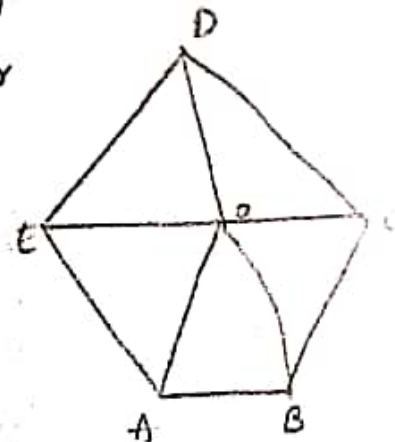
3) By mechanical method (By means of planimeter)

1) By dividing area into no. of triangles:

This method is suitable only for the work of small & layout

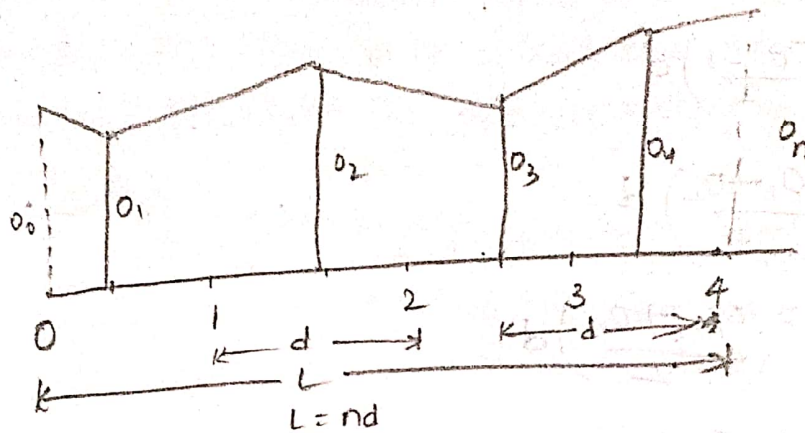
$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2}$$



1. b) By offsets to a base line: (at n at regular intervals:

(i) Mid - ordinate rule :



Area = Δ = Avg ordinate \times length of base

$$= \frac{o_0 + o_1 + o_2 + o_3 + \dots + o_n}{n} (L)$$

$$= \frac{o_1 + o_2 + o_3 + \dots + o_n}{n} (d)$$

$$\Delta = \sum o (d)$$

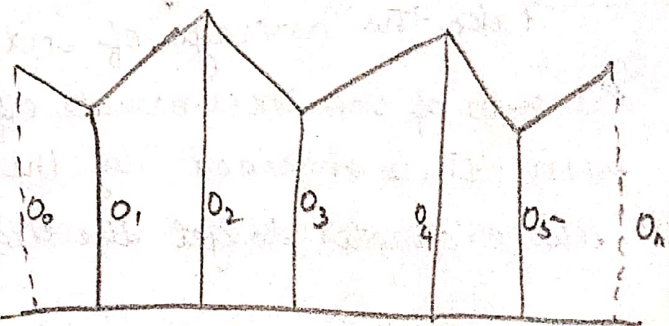
(ii) Avg - ordinate rule :

Δ = Avg ordinate \times length of base

$$\Delta = \left(\frac{o_0 + o_1 + o_2 + \dots + o_n}{n+1} \right) L$$

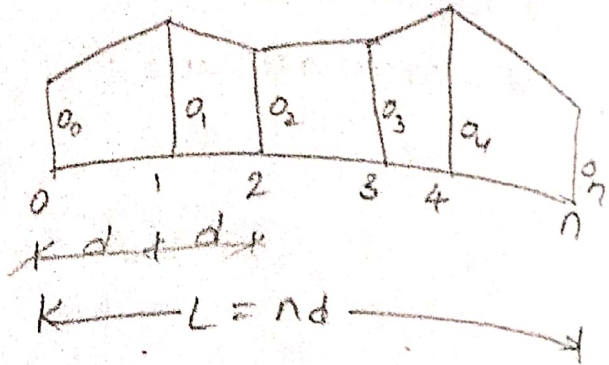
$$= \left(\frac{\sum o}{n+1} \right) L$$

$$\Delta = \left(\frac{L}{n+1} \right) \sum o$$



(iii) Trapezoidal Rule:

This rule is more accurate than above 2 rules.



$$\Delta_1 = \left(\frac{o_0 + o_1}{2} \right) d$$

$$\Delta_2 = \left(\frac{o_1 + o_2}{2} \right) d$$

$$\Delta_n = \left(\frac{o_{n-1} + o_n}{2} \right) d$$

$$\Delta = \Delta_1 + \Delta_2 + \dots + \Delta_n$$

$$\Delta = \left(\frac{o_0 + o_1}{2} \right) d + \left(\frac{o_1 + o_2}{2} \right) d + \dots + \left(\frac{o_{n-1} + o_n}{2} \right) d$$

$$= \left[\frac{2(o_1 + o_2 + o_3 + \dots + o_{n-1}) + o_0 + o_n}{2} \right] d$$

$$\Delta \Rightarrow \left[\frac{o_0 + o_n}{2} + o_1 + o_2 + o_3 + \dots + o_{n-1} \right] d$$

Take the average of end offsets and add them to the sum of the intermediate offsets. Multiply the total sum thus obtained by the common distance between the ordinates to get the required area.

(iv) Simpson's one-third Rule:

This rule is applicable only the number of divisions of area is even, i.e. total no. of ordinates are odd.

Statement: The area is equal to sum of two end ordinates + 4 times the sum of even intermediate ordinates + twice the sum of odd intermediate

ordinates, the whole is multiplied by $\frac{1}{3}$ the common interval between them.

→ If there is an odd number of divisions, (resulting in even number of ordinates). The area of the last division must be calculated separately and it has to be added the previous area Δ which is obtained by applying Simpson's $\frac{1}{3}$ rule.

$$\Delta = \frac{d}{3} \left[(O_0 + O_n) + 4(O_1 + O_3 + O_5 + \dots + O_{n-1}) + 2(O_2 + O_4 + O_6 + \dots + O_{n-2}) \right]$$

Problem:

The following 11 offsets were taken at 10 m interval from a survey line to an irregular boundary line. Offsets are: 3.25, 5.60, 4.20, 6.65, 8.75, 6.20, 3.25, 4.20, 5.65. Calculate the area enclosed between the survey line, irregular boundary line and the first and last offsets by application of a) mid ordinate b) avg. ordinate c) trapezoidal d) Simpson's $\frac{1}{3}$ rule.

Sol:-

3.25	5.6	4.20	6.65	8.75	6.20	3.25	4.20	5.65
O_0	O_1	O_2	O_3	O_4	O_5	O_6	O_7	O_8

a) $\Delta = \sum O d = (47.75) 10 = 477.5 \text{ m}^2$

b) $\Delta = \left(\frac{L}{n+1} \right) \sum O \Rightarrow \frac{80}{(8+1)} \times 47.75$ $L = nd = 8 \times 10 = 80$
 $= 424.44 \text{ m}^2$

c) trapezoidal = $\Delta = \left[\frac{3.25 + 5.65}{2} + 5.6 + 4.20 + 6.65 + 8.75 + 6.20 + \frac{3.25 + 4.20}{2} \right] \times 10$
 $\Delta = [4.45 + 38.85] 10 = 433 \text{ m}^2$

$$d) \Delta = \frac{d}{3} [(O_0 + O_n) + 4(O_1 + O_3 + \dots) + 2(O_2 + O_4 + \dots)]$$

$$\frac{10}{3} [(3.25 + 5.65) + 4(5.6 + 6.65 + 6.2 + 4.2) + 2(4.2 + 8.75 + 3.25)]$$

$$\Rightarrow \frac{10}{3} [8.9 + 90.6 + 32.4]$$

$$= 439.67 \text{ m}^2$$

$$\Delta = \frac{10}{3} [(3.25 + 4.2) + 4(5.6 + 6.65 + 6.2) + 2(4.2 + 8.7 + 3.25)]$$



Measurement of volume:

a) List out the methods of measuring volume and explain the purposes.

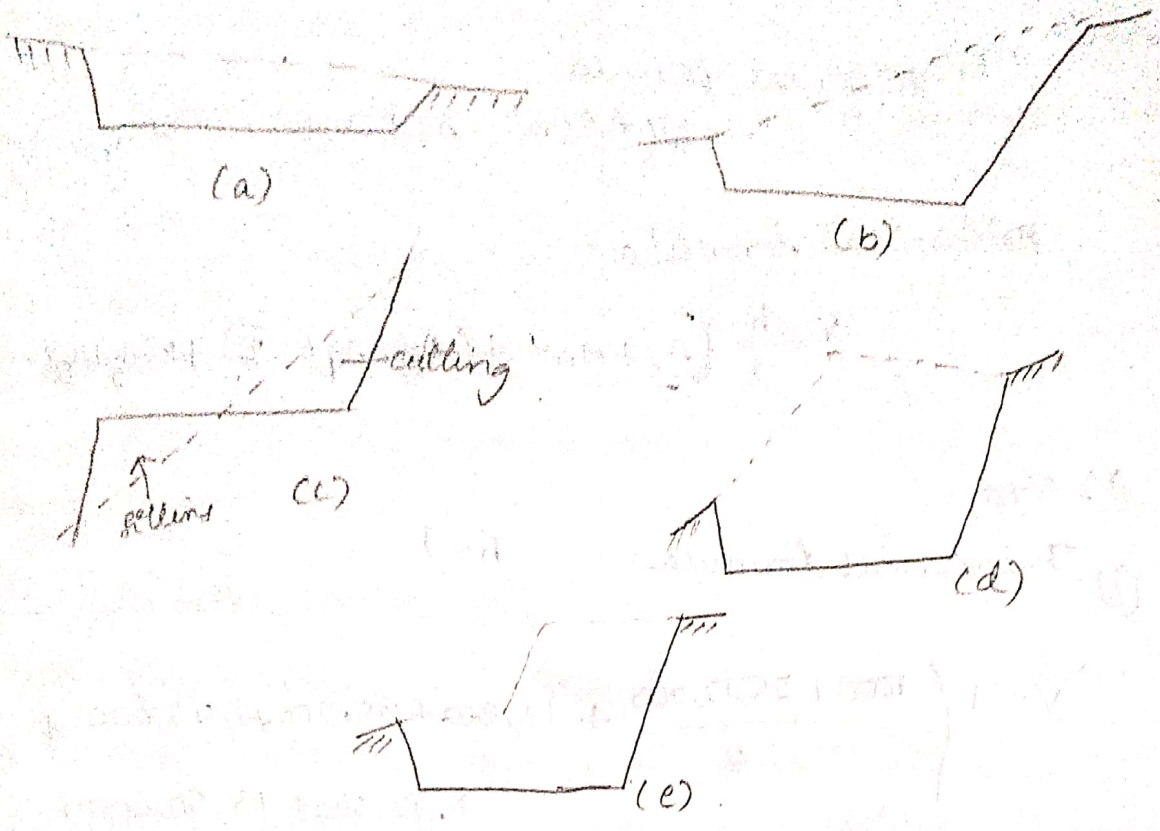
Ans: There are three methods to measure the volume.

- 1) From cross-sections
- 2) From spot levels
- 3) From contours

For the first two methods used for calculation of earthwork while the third method is adopted for calculation of reservoir capacities.

1) From cross sections:

1. Level section
2. Two-level section
3. Side hill two level section
4. Three level section
5. Multi level section



problem:

1) The areas within the contour line at the sight of reservoir and the face of the proposed dam are as follows:

Contour	Area (m ²)
101	1000
102	12,800
103	95,200
104	1,47,600
105	8,72,500
106	13,50,000
107	19,85,000
108	22,86,000
109	25,12,000

Taking 101 as the bottom level of reservoir and 109 as top level calculate.

Capacity of reservoir.

Sol:-

Trapezoidal formula:

$$V = h \left(\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right)$$

Prismoidal formula:

$$V = \frac{h}{3} \left[A_1 + A_n + 4(A_2 + A_4 + \dots + A_{n-1}) + 2(A_3 + A_5 + \dots) \right]$$

(1) Tra

(i) Trapezoidal formula: $h = 1$

$$V = 1 \left(\frac{1000 + 25,12,000}{2} + 12,800 + 95,200 + 1,47,600 + 8,72,500 + 13,50,000 + 19,85,000 + 22,86,000 \right)$$

$$V = (256500 + u)$$

$$V = 8005600 \text{ m}^3$$

(ii) Prismoidal formula:

$$V = \frac{1}{3} \left[1000 + 25,12,000 + 4(12,800 + 1,47,600 + 13,50,000 + 22,86,000) + 2(95,200 + 8,72,500 + 19,85,000) \right]$$

$$= \frac{1}{3} \left[2513000 + 4(3796400) + 2(2952700) \right]$$

$$= \frac{1}{3} \left[2513000 + 15185600 + 5905400 \right]$$

$$= 78,68,000 \text{ m}^3$$

C) By Latitudes & Departures

(i) By double meridian distance method (D.M.D)

Area by D.M.D:

This method is most often used for connecting the area of a closed traverse. This method is known as DMD method.

To calculate area by this method the latitudes and the departures of each line of the traverse is balanced, a reference meridian is then assumed to pass through the most westerly station of the traverse and the double meridian distances of the lines are computed.

→ Meridian distances

The meridian distance of any point in a traverse is the distance of that point to the reference meridian measured at right angles to the meridian.

The meridian distance of a survey line is defined as the meridian distance of its mid point. The meridian distance (abbreviated as M.D) is also sometimes called as longitude. In the fig the reference meridian is chosen through the most westerly station A.

The meridian distance represented by
 m' of AB = half of its departure. In the same
 way the meridian distance of second line BC,
 $m_1 = D_1/2$
 will be given by.

$$m_2 = m_1 + D_1/2 + D_2/2$$

Similarly, the third line CD's meridian dist
 is calculated by

$$m_3 = m_2 + D_2/2 - D_3/2$$

The meridian distance of last line DA is
 given by $m_4 = m_3 + (-D_3/2) + (-D_4/2)$

$$m_4 = D_4/2$$

Statement:

Hence, the rule for meridian distance may be
 stated as follows, the M.D of any line is equal
 to the meridian dist of preceding line +
 half the departure of preceding line + half
 the departure of the line itself.

NOTE:

Acc to the above statement, the meridian
 dist of 1st line will be equal to half of its
 departure. In applying the rule proper attention
 to be paid to the signs of the departures i.e
 +ve sign for eastern departure and -ve sign
 for western departure.

Meridian distances / longitude:

x Area by latitude and meridian distances:

In the above fig east, west lines are drawn from each station to the reference meridian. Thus, getting triangles and trapeziums. One side of each triangle or trapezium. So formed will be one of the lines, the base of the triangle or trapezium will be latitude of that line, height of the triangular (or) trapezium will be the meridian distance of the line. Therefore area of each triangle (or) trapezium =

Latitude of the line x meridian distance of line

$$A_1 = L_1 \times m_1$$

$$A_2 = L_2 \times m_2$$

In the above fig the area of traverse ABCD = algebraic sum of areas of dDCc

traverse ABCD = Algebraic sum of areas of dDCc,

$$C = Bb, dDA, ABb$$

The latitude (L) will be taken +ve if it is Northing and -ve if it is Southing then

$$\text{Area (A)} = \text{Area of ADcC} + \text{Area CcBb} - \text{Area of dDA} - \text{Area of ABb}$$

$$A = L_3 m_3 + L_2 m_2 - L_4 m_4 - L_1 m_1$$

$$\boxed{A = \sum L_i m_i}$$

① The following table gives corrected latitudes and departures in m of the sides of a closed traverse ABCD

Side	Latitude		Departure	
	N northing	S southing	E easting	W westing
AB	+108 (L ₁)	-	+4 (D ₁)	-
BC	+15 (L ₂)	-	+249 (D ₂)	-
CD	-	-	+4 (D ₃)	-
DA	0 (L ₄)	-	-	-257 (D ₄)

Compute area by meridian distances and latitude

Sol:

$$m_1 = D_1/2 = 4/2 = 2$$

$$m_2 = m_1 + \frac{D_1}{2} + \frac{D_2}{2} = 2 + \frac{4}{2} + \frac{249}{2} = 128.50$$

$$m_3 = m_2 + \frac{D_2}{2} - \frac{D_3}{2} = 128.5 + \frac{249}{2} - \frac{4}{2} = 251$$

$$m_4 = D_4/2 = -257/2 = -128.50$$

$$\text{Area} = L_1 m_1 + L_2 m_2 + L_3 m_3 + L_4 m_4$$

$$= 108(2) + 15(128.50) + (-123)(251) + (-128.5)(0)$$

$$\text{Area} = 216 + 15(128.50) - 123(251)$$

$$\text{Area} = 28729.5 \text{ m}^2$$

Side	Latitude	Departure
PQ	+128 (L ₁)	+9 (D ₁)
QR	+15 (L ₂)	+258 (D ₂)
RS	-143 (L ₃)	+9 (D ₃)
SP	0 (L ₄)	-276 (D ₄)

Calculate area by latitudes and MD method.

Sol:- $m_1 = D_1/2 = 9/2 = 4.5 \text{ m}$

$$m_2 = m_1 + D_1/2 + D_2/2 = 4.5 + 4.5 + 258/2 = 138 \text{ m}$$

$$m_3 = m_2 + D_2/2 - D_3/2 = 138 + 258/2 - 9/2 = 262.5 \text{ m}$$

$$m_4 = D_4/2 = \frac{-276}{2} = -138 \text{ m}$$

$$\text{Area} = L_1 m_1 + L_2 m_2 + L_3 m_3 + L_4 m_4$$

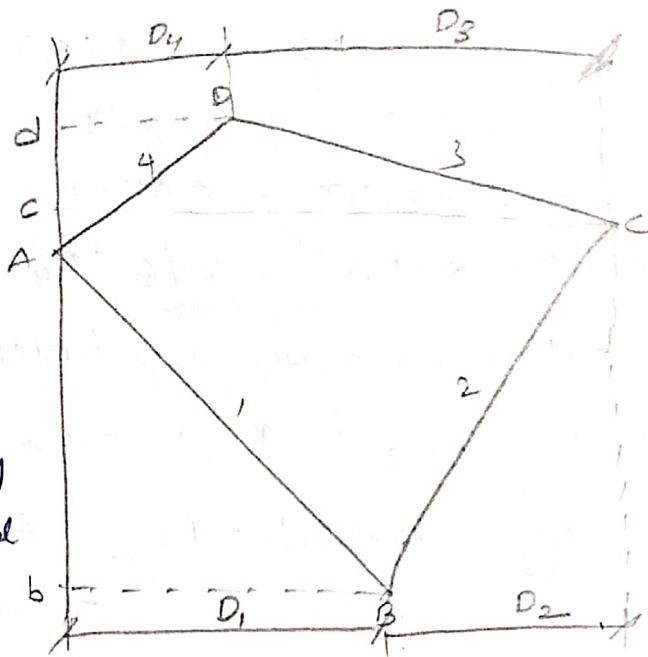
$$= 128(4.5) + 15(138) + (-143)(262.5) + 0$$

$$\text{Area} = 576 + 2070 - 37537.5$$

$$= +34891.5 \text{ m}^2$$

DMD (Double Meridian distance)

The DMD of a line is equal to sum of the meridian distances of two extremities.



Statement:

The rule for finding DMD of any line stated as follows. The DMD of any line is equal to DMD of preceding line + departure of the preceding line + departure of the line itself.

Attention should be paid to the sign of the departure. The DMD of the first line will be equal to its departure. The DMD of last line is also equal to its departure. But this fact should be used as a check.

→ DMD is represented by 'M'.

$$\begin{aligned} \text{DMD of AB} &= M_1 = \text{m of A} + \text{m of B} \\ &= 0 + D_1 \end{aligned}$$

$$M_1 = D_1$$

$$\text{DMD of BC} = M_2 = \text{m of B} + \text{m of C}$$

$$M_2 = D_1 + D_3 + D_4 = D_1 + D_1 + D_2$$

$$M_2 = M_1 + D_1 + D_2$$

$$\text{DMD of CD} = M_3 = \text{m of C} + \text{m of D}$$

$$= D_1 + D_2 + D_4 = M_1 + D_1 + D_2 + D_3 - D_3$$

$$M_3 = M_1 + D_1 + D_2 + D_2 - D_3$$

$$M_3 = M_2 + D_2 - D_3$$

$$\text{DMD of DA} = M_4 = M_1 + D_1 + D_2 + D_3 - D_3 - D_3 - D_4$$

$$M_4 = M_3 - D_3 - D_4$$

Area by Latitudes and DMD:

In the above fig, the area of traverse ABCD =
 Area of DdCc + area of CcBb + - Area of dDA
 - Area ABb .

$$\text{Area} = \frac{1}{2} (dD + cC) \times L = \frac{1}{2} [M_3 L_3 + M_2 L_2 - M_4 L_4 - M_1 L_1]$$

Methodology:

1. Multiply DMD of each line with its latitude
2. Find the algebraic sum these products
3. The required area will be $\frac{1}{2}$ of the sum.

Line	L	D	DMD (M)	Area (M x L) (m ²)
AB	+108	+4	+4	432 m ²
BC	+15	+249	4+4+249 = 257	3855
CD	+123	+4	257+249+4 = 510	-62730
DA	0	-257	257	0

$$\text{Area} = \frac{\sum A}{2} = \frac{1}{2} (432 + 3855 - 62730)$$

$$= +19867.5 - \frac{1}{2} (58443)$$

$$A = 29221.5 \text{ m}^2$$

Q) line	L(m)	D(m)	DMD(M)	Area (M x L) m ²
PQ	+128	+9	+9	1152
QR	+15	+258	9 + 9 + 258 = 276	4140
RS	-143	+9	276 + 258 + 9 = 543	-77649
SP	0	-276	543 + 9 - 276 = 276	0

$$\Sigma A = -72357$$

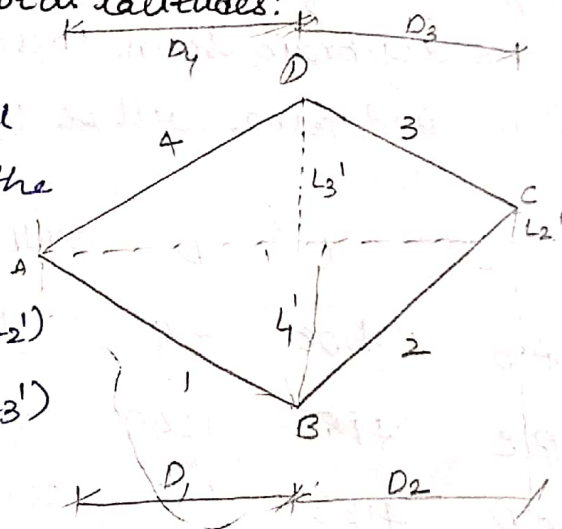
$$\text{Area} = \frac{1}{2} \Sigma A = 36178.5 \text{ m}^2$$

21/9/19

Area from departures & total latitudes:

If L_1', L_2', L_3' are total latitudes of the ends of the lines. Then area =

$$\text{area} = \frac{1}{2} [D_1(0 - L_1') + D_2(-L_1' + L_2') + (D_3)(L_2' + L_3') + (D_4)(L_3' + 0)]$$



$$\frac{1}{2} [0 - D_1 L_1' + D_2 L_1' + D_2 L_2' + D_3 L_2' + D_3 L_3']$$

$$= \frac{1}{2} [-D_1 L_1' + D_2 L_1' + D_2 L_2' - D_3 L_2' - D_3 L_3' - D_4 L_3']$$

$$= \frac{1}{2} [-L_1' (D_1 + D_2) + L_2' (D_2 - D_3) - L_3' (D_3 + D_4)]$$

$$= -\frac{1}{2} [L_1' (D_1 + D_2) - L_2' (D_2 - D_3) + L_3' (D_3 + D_4)]$$

$$= -\frac{1}{2} [L_1' (D_1 + D_2) + L_2' (D_3 - D_2) + L_3' (D_3 + D_4)]$$

Note 1

The -ve. sign to the area has no significance. So we can neglect the '-' sign.

Step wise procedure to find out area by this method:

- 1) Find total latitude (L') of each station of traverse.
- 2) Find algebraic sum of departures of two lines meeting at that station.
- 3) Multiply the total latitude of each station by corresponding algebraic sum of departures which are found in step-2.
- 4) Half the algebraic sum of total Latitudes and departures will give the required area.

Problem:

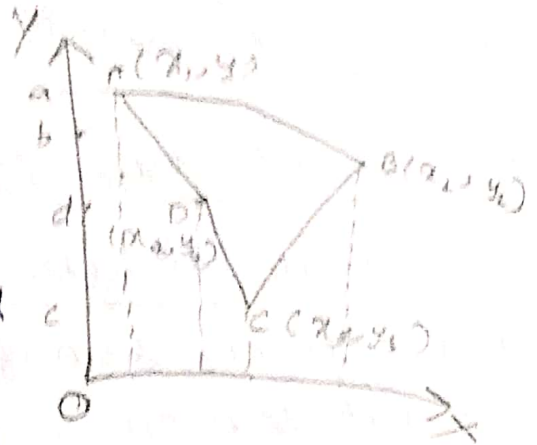
Line	2 L	3 D	4 Stn	5 Total latitude (L')	6 Algebraic sum of adjoining departures	Double area (5×6)
AB	+108	+4	B		253 108	253 27324
BC	+15	+249	C		253 123	253 31119
CD	-123	+4	D		253 0	-253 0
DA	0	-257	A		0	-253 0
						<hr/> ΣA = 58443

$$\therefore \text{Area} = \frac{1}{2} \Sigma A$$

$$= \frac{1}{2} \times 58443 = 29221.5$$

Area by Co-ordinates:

Let $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ be the co-ordinates of the stations A, B, C, D resp of a traverse ABCDA. If A is the total area of the traverse, then area



$$A = \frac{1}{2} \begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$$

$$= \frac{1}{2} [(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_4 - x_4 y_3) + (x_4 y_1 - x_1 y_4)]$$

Problem:

Line	(N) Latitude	(E) Departure	Station	Independent co-ordinates	
				North (y)	East (x)
AB	+108	+4	A	100 y_1 +208	100 x_1 104
BC	+15	+249	B	208 y_2	353 y_2 104
CD	-123	+4	C	100 223	357
DA	0	-257	D	100-100	100-357
			A	100	100

$$= \frac{1}{2} \begin{vmatrix} 100 & 104 & 353 & 357 & 100 \\ 100 & 208 & 223 & 100 & 100 \end{vmatrix}$$

$$= \frac{1}{2} \left[(100 \times 209 - 100 \times 104) + (104 \times 223 - 209 \times 353) \right. \\ \left. + (353 \times 100 - 223 \times 357) + (357 \times 100 - 100 \times 100) \right]$$

$$= \frac{1}{2} \left[10400 + (-50232) + (-44311) + 25700 \right]$$

$$= \frac{1}{2} \Rightarrow 29291.5 \text{ m}^2$$

problem:

Coordinates of A (100, 802), B (711, 802), C (635, 852)

D (994, 902) E (241, 952)

F (884, 1002)

G (266, 1052)

H (811, 1102)

I (100, 1102)

$$\text{Sol: } -\frac{1}{2} \begin{vmatrix} 802 & 802 & 852 & 902 & 952 & 1002 & 1052 & 1102 \\ 100 & 711 & 635 & 994 & 241 & 884 & 266 & 811 \\ & & & & & & & 100 \end{vmatrix}$$

$$\frac{1}{2} \left[(802 \times 711 - 802 \times 100) + (802 \times 635 - 711 \times 852) + (852 \times 994 - 902 \times 635) \right. \\ \left. + (902 \times 241 - 994 \times 952) + (952 \times 884 - 241 \times 1002) + \right. \\ \left. (1002 \times 884 - 1052 \times 266) + (1052 \times 811 - 266 \times 1102) \right. \\ \left. + (1102 \times 100 - 811 \times 1102) + (1102 \times 100 - 802 \times 100) \right]$$

$$= \frac{1}{2} \left[490022 + (-96502) + (274118) + (-728906) \right. \\ \left. + (600086) + (605936) + (560040) + (-783522) \right. \\ \left. + (300000) \right]$$

$$= \frac{1}{2} (951272) = 475636$$

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VOLUMES

SINGLE LEVEL SECTION:

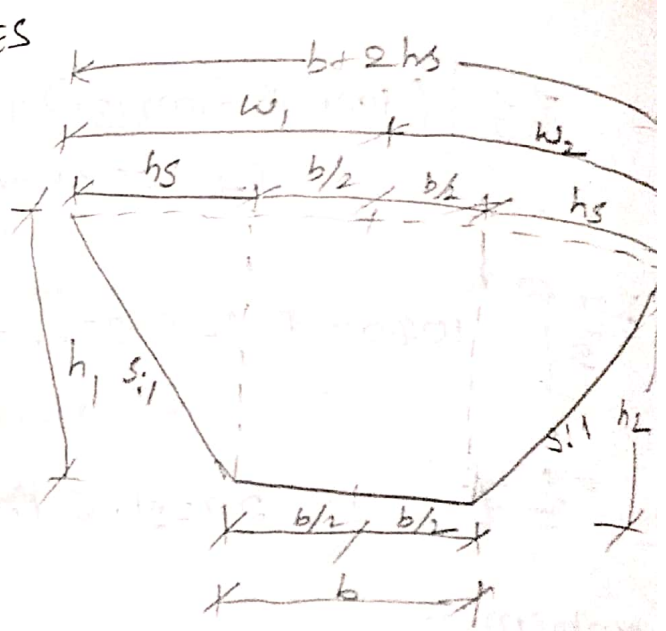
$$w_1 = w_2 = \frac{b}{2} + hs$$

$$w_1 + w_2 = b + 2hs$$

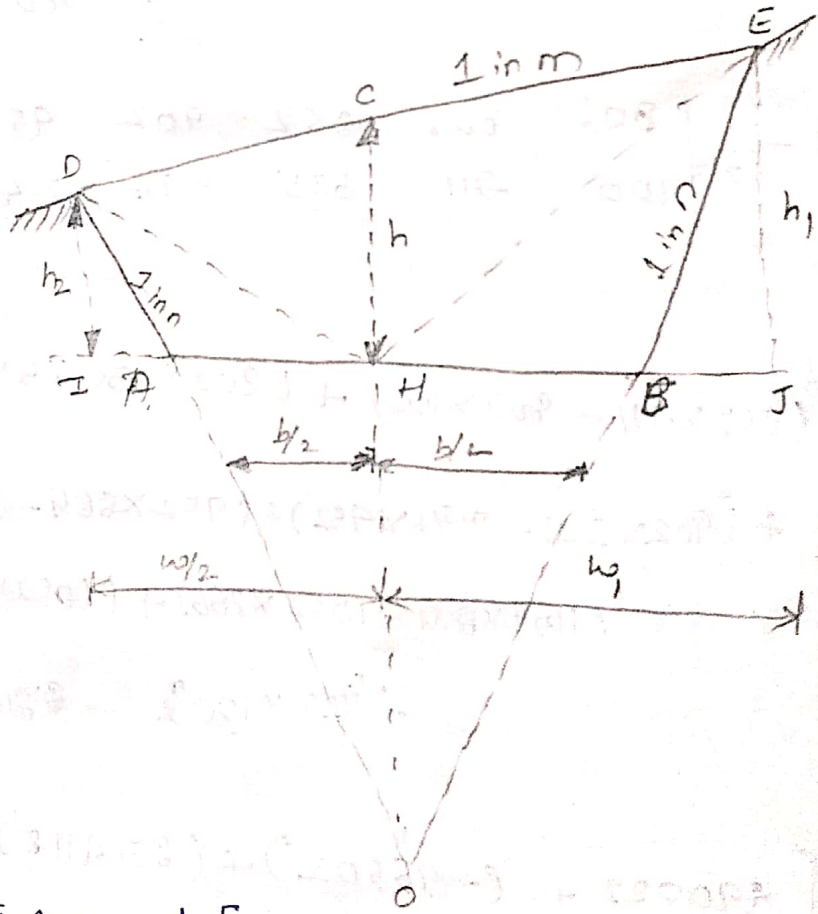
$$\text{Area} = \left[\frac{(b + 2hs) + b}{2} \right] h$$

$$= \frac{2(b + hs)}{2} \times h$$

$$A = (b + hs) h$$



Two-level section:



Area of DCEBA = Area of [$\Delta DAH + \Delta EBH + \Delta DCH + \Delta DECH$]

$$= \left(\frac{1}{2} \times \frac{b}{2} \times h_2 \right) + \left(\frac{1}{2} \times \frac{b}{2} \times h_1 \right) + \left(\frac{1}{2} \times w_2 \times h \right) + \left(\frac{1}{2} \times w_1 \times h \right)$$

$$\Rightarrow \frac{1}{2} \times \frac{b}{2} (h_1 + h_2) + \frac{1}{2} h (w_1 + w_2)$$

$$\frac{1}{2} \left[\frac{b}{2} (h_1 + h_2) + h (w_1 + w_2) \right]$$

$$BJ = nh_1 \longrightarrow (i)$$

↓

$$HJ - HB$$

$$w_1 - \frac{b}{2} = nh_1$$

$$\boxed{w_1 = nh_1 + \frac{b}{2}}$$

$$\text{Also } w_1 = (h_1 - h)m$$

$$(h_1 - h)m = nh_1 + \frac{b}{2}$$

$$nh_1 + \frac{b}{2} = (h_1 - h)m$$

$$nh_1 = h_1 m - hm - \frac{b}{2}$$

$$nh_1 - mh_1 + mh + \frac{b}{2} = 0$$

$$h_1 (n - m) + mh + \frac{b}{2} = 0$$

$$h_1 = \frac{-\frac{b}{2} - mh}{n - m}$$

$$\boxed{h_1 = \frac{\frac{b}{2} + mh}{m - n}}$$

$$\boxed{h_1 = \frac{m}{m - n} \left(\frac{b}{2m} + h \right)}$$

sub h_1 value in w_1

$$\text{we have } w_1 = n \times \frac{m}{m - n} \left(\frac{b}{2m} + h \right) + \frac{b}{2}$$

$$= \frac{bn}{2(m - n)} + \frac{hmn}{m - n} + \frac{b}{2}$$

$$w_1 = \frac{b}{2} + \frac{mn}{m-n} \left(h + \frac{b}{2m} \right)$$

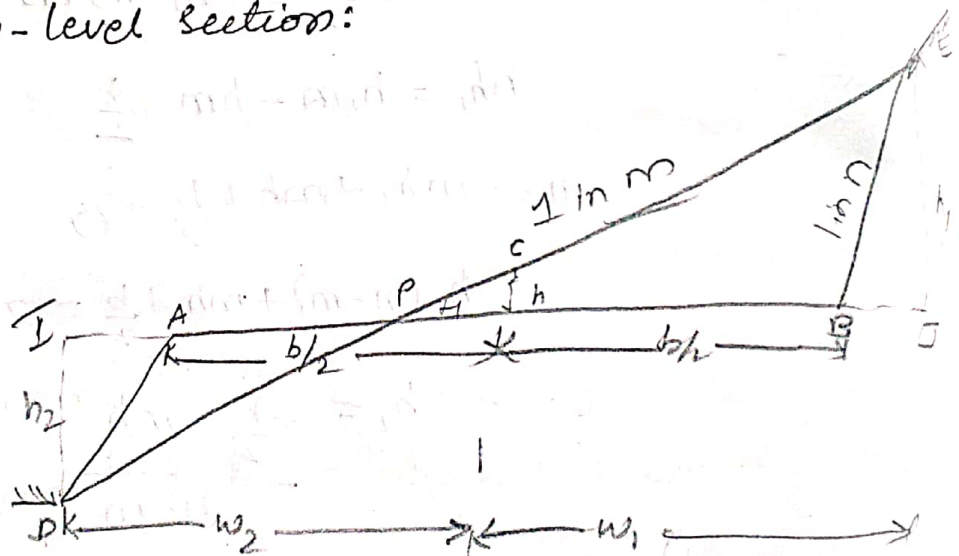
$$w_2 = \frac{b}{2} + \frac{mn}{m+n} \left(h - \frac{b}{2m} \right)$$

$$h_2 = \frac{m}{m+n} \left(h - \frac{b}{2m} \right)$$

$$A = \frac{1}{2} \left[\frac{b}{2} (h_1 + h_2) + h (w_1 + w_2) \right]$$

$$A = \frac{n \left(\frac{b}{2} \right)^2 + m^2 (bh + nh^2)}{(m^2 - n^2)}$$

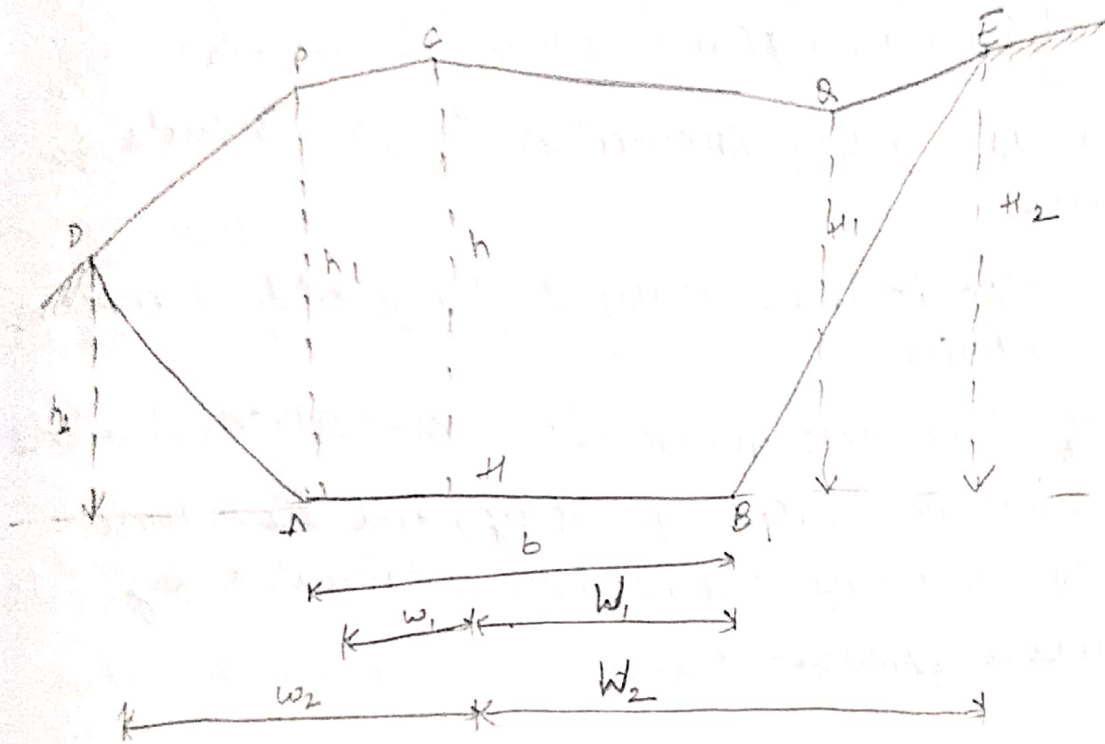
Side Hill Two-level Section:



$$A_1 = \frac{1}{2} \left(\frac{b}{2} + mh \right) \left\{ \frac{m}{m-n} \left(\frac{b}{2m} + h \right) \right\} = \frac{\left(\frac{b}{2} + mh \right)^2}{2(m-n)}$$

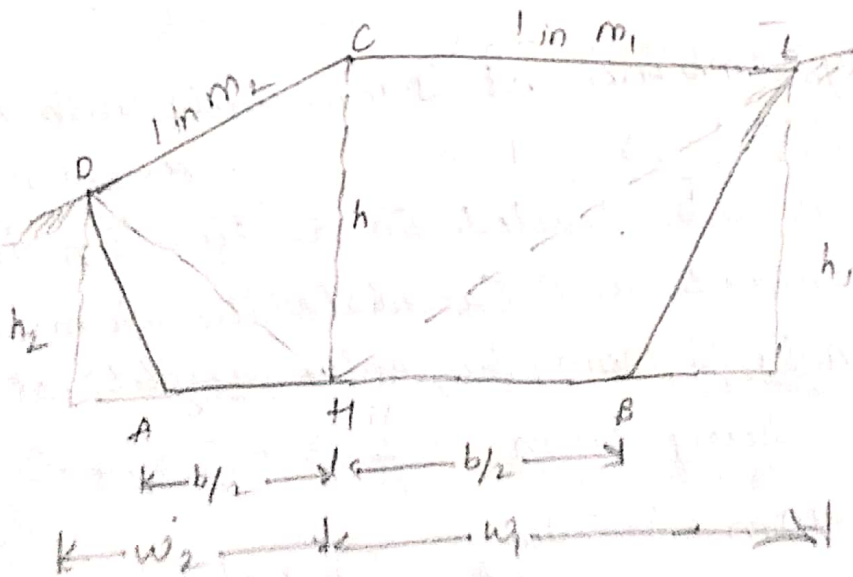
$$A_2 = \frac{\left(\frac{b}{2} - mh \right)^2}{2(m-n)}$$

Multi level section:



$$A = \frac{1}{2} \left[h_2 \left(+\frac{b}{2} - w_1 \right) + h_1 (w_2 + 0) + h (w_1 + W_1) + H_1 (w_2) + H_2 \left(-w_1 + \frac{b}{2} \right) \right]$$

Three level section:



$$A = \left[\frac{b}{4} (h_1 + h_2) + \frac{h}{2} (w_1 + w_2) \right]$$

Prismoidal formula:

$$V = \frac{d}{3} [(A_1 + A_n) + 4(A_2 + A_4 + \dots + A_{n-1}) + 2(A_3 + A_5 + \dots + A_{n-2})]$$

This is also known as Simpson's rule for volume.

Here also it is necessary to have odd number of cross-sections.

If there are even number of cross-sections, the end strip must be treated separately, and the volume between remaining sections may be calculated by Prismoidal formula.

Trapezoidal formula (Avg end area method):

$$V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + A_4 + \dots + A_{n-1} \right]$$

Problem:

A railway embankment is 10m wide with side slope $1\frac{1}{2} : 1 \Rightarrow \frac{3}{2} : 1 \Rightarrow 3 : 2$. Assuming the ground is to be leveled in a direction transverse to the centered line. Calculate the volume content in a length of 120m, the centre heights at 20m intervals being in m 2.2, 3.7, 3.8, 4, 3.8, 2.8, 2.5

Sol: $b = 10\text{m}$ $d = 20\text{m}$

$n = 1.5$

$A = (b + nh)h$

~~Let~~ $h_1, h_2, h_3, \dots, h_7 = 2.2, 3.7, 3.8, \dots, 2.5$

$A_1 = \frac{2 \cdot 2}{2} (1.5(2.2) + 10) = 29.26$

$$A_2 = 3.7 (10 + 1.5(3.7)) = 57.53$$

$$A_3 = 3.8 (10 + 1.5(3.8)) = 59.66$$

$$A_4 = 4 (10 + 1.5(4)) = 64$$

$$A_5 = 3.8 (10 + 3.8(1.5)) = 59.66$$

$$A_6 = 2.8 (10 + 1.5(2.8)) = 39.76$$

$$A_7 = 2.5 (10 + 1.5(2.5)) = 34.375$$

prismoidal:

$$V = \frac{20}{3} \left[(29.26 + 34.375) + 4(57.53 + 64 + 39.76) + 2(59.66 + 59.66) \right]$$

$$= \frac{20}{3} \left[63.635 + 4(161.03) + 2(119.32) \right]$$

$$= 6306.56 \text{ m}^3$$

Trapezoidal:

$$V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right]$$

$$= 20 \left[\frac{29.26 + 34.375}{2} + 57.53 + 59.66 + 64 + 59.66 + 39.76 \right]$$

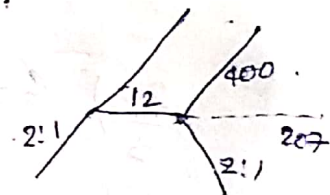
$$= 6248.55 \text{ m}^3$$

Q) A railway embankment 400m long is 12m wide at formation level and has side slope 2:1. The ground levels at every 100m along the centre line are as follows

Distance	RL	FL	Depth of filling
0	204.80	207	2.2m h_1
100	206.20	208	1.8m h_2
200	207.50	209	1.5m h_3
300	207.20	208 210	2.8 h_4
400	208.30	211	2.7 h_5

The formation level at '0' chainage is 207m and embankment has a rising gradient of 1 in 100. The ground is level across the centre line. Calculate Volume of earthwork.

Sol:- $b = 12, d = 100$
 $n = 2$



$$A = h(b + nh)$$

$$A_1 = 2.2(12 + 2(2.2)) = 36.08 \text{ m}^2$$

$$A_2 = 1.8(12 + 2(1.8)) = 28.08 \text{ m}^2$$

$$A_3 = 1.5(12 + 2(1.5)) = 22.5 \text{ m}^2$$

$$A_4 = 2.8(12 + 2(2.8)) = 49.28 \text{ m}^2$$

$$A_5 = 2.7(12 + 2(2.7)) = 46.98 \text{ m}^2$$

Prismoidal

$$V = \frac{100}{3} \left[36.08 + 46.98 + 4(28.08 + 49.28) + 2(22.5) \right]$$

$$= 14583.33$$

Trapezoidal :-

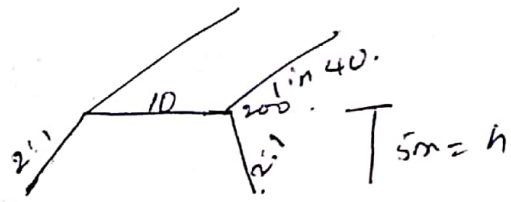
$$V = 100 \left[\frac{36.08 + 46.98}{2} + 28.08 + 22.5 + 49.22 \right]$$

$$= 14139 \text{ m}^3.$$

Q) A road embankment 10m wide at formation level with side slopes 2:1 and with an avg height of 5m is constructed with an average gradient 1 in 40 from contour 220m to 280m find the volume of earth work.

Sol: -

Diff in level between both the ends of the road = 280 - 220 = 60m



$$60 \times 40 = 2400 \text{ m} = L$$

$$A = (b + nh) h$$

$$= (10 + 2(5)) 5$$

$$= 100 \text{ m}^2$$

$$V = A \times L$$

$$= (100 \times 2400)$$

$$= 24 \times 10^4 \text{ m}^3.$$

Q) Find out the volume of earthwork in a road cutting ^{non-long} along the centre from the following data, formation between 10m, side slope 1 to 1 or 1:1. The avg ^{depth} earth of cutting along the centre of

Line is 5m, slope of ground in C/S 10:1.

Sol:-

$$A = \frac{n\left(\frac{b}{2}\right)^2 + m^2(bh + nh^2)}{m^2 - n^2}$$

$$L = 120\text{m}, \quad n = 1, \quad m = 10.$$

$$h = 5\text{m}, \quad b = 10\text{m}$$

$$A = \frac{1\left(\frac{10}{2}\right)^2 + (10)^2(10(5) + 1(5)^2)}{(10)^2 - (1)^2}$$

$$= \frac{7525}{99} = 76.01 \text{ m}^2$$

$$V = A \times L$$

$$= 76.01 \times 120 = 9121.2 \text{ m}^3.$$