# Moving Loads and Influence Lines

Rolling / moving Loads :- max S.F and BM at given Section, and Abs max. SF&BM due to Single point Load, UDL Longer than span and shorter than span. Two point Loads with fixed distance, Several point Loads - equivalent UDL - Focal length.

Intro:- Eg: Vehicle movement across the bridge girder.
(EOT) Electrically operated travelling overhead crane across Gantry girders in an industry.
Train on a bridge across a river. etc.

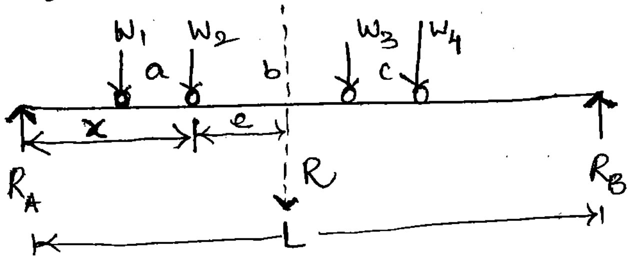
How to position Live Load to maximize the value of S.F, BM and Support reaction in ~~the beam~~.

## Max Shear force and Bending moment at a Given Section

In case of moving loads, BM and SF at a Section of a beam change as the loads roll from one position to the other.

(i) To determine the load positions for max. BM or S.F for a given section of a girder and to compute its value.

(ii) To determine the load positions so as to cause absolute maximum BM or S.F any where on the girder.



⇒ Let the position of load $W_2$ when bending moment under this load is maximum.

$$\therefore \; R_A = \frac{R.(L-x-e)}{L}$$

BM under Load $W_2$ is,

$$M_2 = R_A x - W_1 a$$
$$= \frac{R(L-x-e)x}{L} - W_1 a$$

for max, $M_2$, $\frac{dM_2}{dx} = 0$ ⇒ $\frac{R}{L}(L-e-2x) = 0$

$L-e-2x = 0$

⇒ $L-e = 2x$ ⇒ $\boxed{x = \frac{L}{2} - \frac{e}{2}}$

Bending moment under a particular load is maximum when the centre of the beam is midway between that load and the resultant of all loads there on the span".

* max. SF occurs at supports and is equal to the max. Reaction. maximum Reaction is the reaction to which the resultant load is nearest.

Absolute max. SF and BM :- Conditions

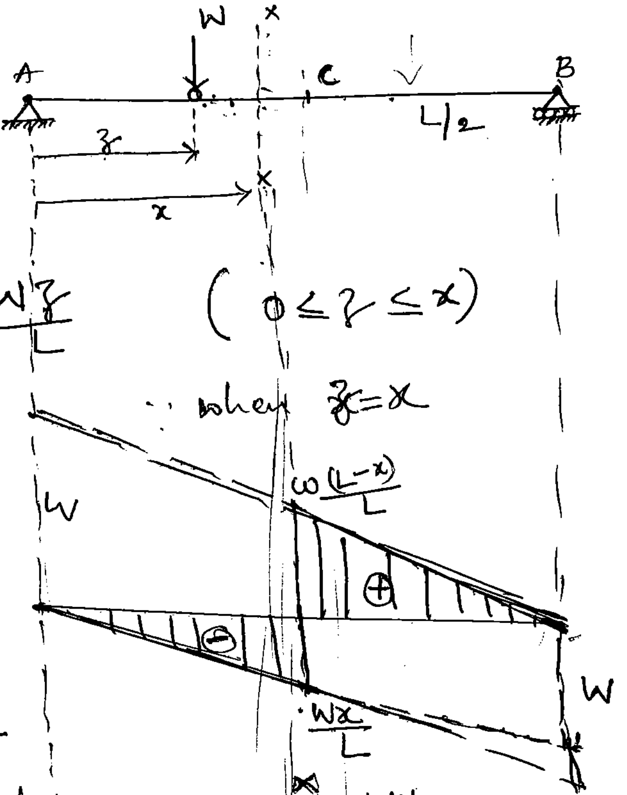
(i) Absolute maximum shear force $V_{abs}$ occurs at one of the two end supports and is numerically equal to the maximum support reactions.

(ii) By placing each load alternatively over a support and calculating the reactions, the absolute max. S.F $V_{abs}$ can be calculated. The critical position usually occurs when the largest load is on the support with as many remaining loads as possible on the span. It is to be borne in mind that the load can move in either direction.

(iii) Due to a system of concentrated loads moving across the span of a simple beam, the absolute maximum moment $M_{abs}$ always occurs under a large load located near the resultant of all the loads on the beam.

(iv) critical load that causes a max. moment must be obtained by the trial and error procedure. the Abs. max. moment can be found when the distance b/w the critical load and the resultant of the loads on the beam is bisected by the centre line of the beam.

## Single point load:-




$\Rightarrow$ if load is placed at a distance 'z' from A,

~~max~~. $-ve$ $S.F = R_B = -\frac{Wz}{L}$ $(0 \le z \le x)$

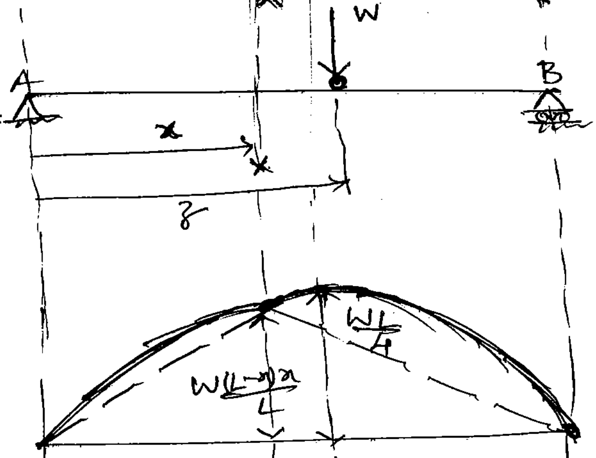
$\therefore$ when $z = x$

max. $-ve$ $S.F = -\frac{Wx}{L}$
$V_{x,max}$

$x = 0, \quad V_{x,max} = 0$

$x = L \quad V_{x,max} = -W = V_{abs}$

$\therefore$ Absolute maximum $-ve$ S.F occurs at right-hand support, its value is $\underline{-W}$




$\Rightarrow \quad V_x = R_A = \frac{(L-z)W}{L}$

when $z = x$, $V_{x,max} = \frac{W(L-x)}{L}$

when, $x = 0$, $V_{abs} = W$

$\therefore$ Absolute max $+ve$ S.F occurs at left support with a value "W".

$\Rightarrow$ max. BM:- when W is b/w A & X, $BM_x$ is equal to $R_B \cdot (L-x) = \frac{Wz(L-x)}{L}$

when $z = x$, $\quad M_{x,max} = \frac{Wx(L-x)}{L}$ ; when $z = 0$, $M_x = 0$

when Load is between X & B.

$M_x = R_A x = \frac{W(L-z)x}{L}$ $\quad$ when $z = L$, $M_x = 0$

$M_{x,max} = \frac{W(L-x)x}{L}$ | $M_{x,max} = \frac{WxL}{L} - \frac{Wx^2}{L} = Wx - \frac{Wx^2}{L}$

$\frac{dm_x}{dx} = 0 \Rightarrow \quad W - \frac{2Wx}{L} = 0$

$\boxed{x = L/2}$

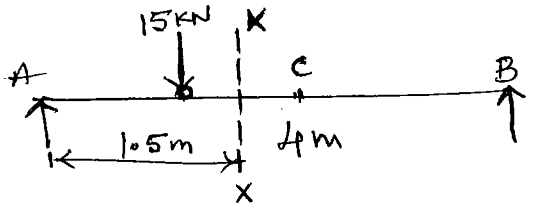
$\therefore$ Abs Max. BM $= \frac{WL}{4}$

Prob:- Det max the ... -ve ...

1.5 m in a Simple beam of Span 4m when a Concentrated Load of 15 KN rolls across the beam. Also Calculate the Absolute -ve and +ve shears and BMs?

Sol:-

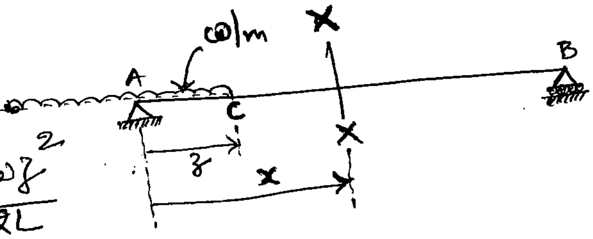
$$V_{x,max^-} = \frac{-15\times1.5}{4} = -5.625 \text{ KN}$$




$$V_{x,max^+} = \frac{15(4-1.5)}{4} = 9.375 \text{ KN}.$$

$$M_{max} = \frac{15\times1.5\times2.5}{4} = 14.0625 \text{ KNm}$$

$$V_{abs} = \pm 15 \text{ KN}; \quad M_{abs} = \frac{15\times4}{4} = 15 \text{ KNm}$$




**UDL Longer than Span:-**

$$V_x = -R_B = -\frac{wz\times(z/2)}{L} = -\frac{wz^2}{2L}$$

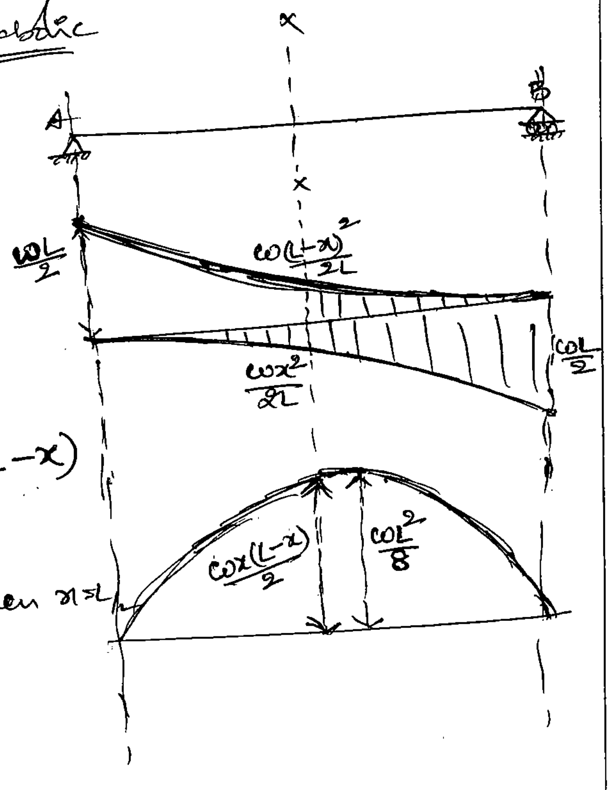
when, head of load reach the Section, $z = x$

$$V_{max^-} = -\frac{wx^2}{2L}; \quad \text{when } x=0, \ V=0; \quad x=L, \ V=-\frac{wL}{2} \ \Big\} \text{ parabolic}$$

$$V_x = R_A = \frac{w(L-x)^2}{2L} \Rightarrow V_{x,max^+} = \frac{w(L-x)^2}{2L}$$

when $x=0$, $V = \frac{wL}{2}$; parabolic

$x=L$, $V=0$




Max. BM:- $M_x = R_B\times(L-x)$

$$= \frac{wz^2}{2L}(L-x)$$

$$M_x = \frac{wx^2(L-x)}{2L}$$

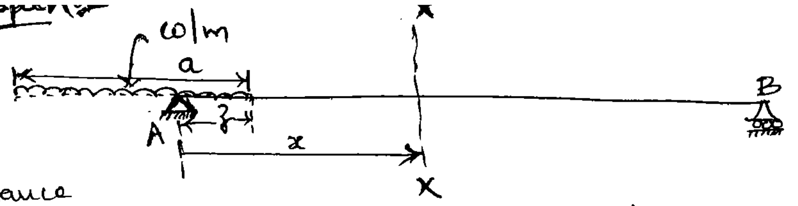
$$M_{x,max} = \frac{wLx}{2} - \frac{wx^2}{2} = \frac{wx}{2}(L-x)$$

Load occupies the whole span.

$$M_{max} = \frac{wx}{2}(L-x) \quad \text{when } x = L/2$$

$$M_{max} = \frac{wL^2}{8}$$

UDL Shorter than span




Negative S.F :-

Case(1):- When the distance of the Section is Shorter than the length of load ($x<a$)

SF at X is $= V_x = -R_B = -\dfrac{\omega z(z/2)}{L} = -\dfrac{\omega z^2}{2L}$

at $z=x$, $V_{max} = -R_B = -\dfrac{\omega x^2}{2L}$

$x=0$, $V_x = 0$

$x=a$, $V_{max} = -\dfrac{\omega a^2}{2L}$

Case(2):- $x>a$ ; when head of load reach 'X'.

$$V_{max} = -R_B = -\frac{\omega a}{L}\left(x-\frac{a}{2}\right)$$

$x = a/2$ $V_{max} = 0$

$x = a$ $V_{max} = \dfrac{-\omega a^2}{2L}$

$x = L$ $V_{max} = \dfrac{-\omega a(L-a/2)}{L}$

$x = \left[L + (a/2)\right] \Rightarrow V_{max} = -\dfrac{\omega a}{L}\left(L + \dfrac{a}{2} - \dfrac{a}{2}\right)$

$= -\omega a$

$x = L + (a/2)$

* max. negative S.F at a Section occurs when head of the load is on the Section. (Load to the left of the Section)

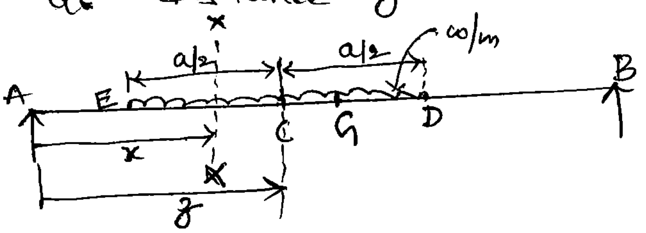
* max. positive S.F at a Section occurs when the tail of the load is on the Section

* Absolute max. negative S.F occurs at Support B when head of the load is at B.

* Absolute max. positive S.F occurs at Support A when tail of the load is at A.

BM:- when load is in AX portion, BM at Section 'X' is expressed as $M_x = R_B(L-x)$

to get maximum BM at the Section X the load has to be arranged in Such a manner that CG of the load is at distance $z$ from 'A'.




$R_B = \omega a(z/L)$

$XD = (z - x + a/2)$

$M_x = R_B(L-x) - \dfrac{\omega(XD)^2}{2}$

$M_x = \frac{\omega a}{L}(L-x)\,z - \frac{\omega}{2}\left(z - x + \frac{a}{2}\right)^2$

for $M_x$ to be maximum, $\frac{dM_x}{dz} = 0$

$$\Rightarrow \frac{\omega a}{L}(L-x) - \frac{\omega}{\cancel{2}}\left(z - x + \frac{a}{2}\right) \times \cancel{2} = 0$$

$$\frac{a}{L}(L-x) = \left(z - x + \frac{a}{2}\right)$$

$$\frac{ED}{AB} \times XB = XD \quad \text{or} \quad \frac{XB}{XD} = \frac{AB}{ED} = \frac{AB - XB}{ED - XD} = \frac{AX}{Ex}$$

$$\therefore \boxed{\frac{Ex}{XD} = \frac{AX}{XB}}$$

* BM at a section will be maximum when the position of load is such that, the section devides the span and the load in the same ratio.

$$z = \frac{a(L-x)}{L} + x - \frac{a}{2} = \frac{a}{2} + x - \frac{ax}{L}$$

$$M_{max} = \frac{\omega a}{L}(L-x)\left(\frac{a}{2} + x - \frac{ax}{L}\right) - \frac{\omega}{2}\left(\frac{a}{L}(L-x)\right)^2$$

$$= \frac{\omega a x}{L}(L-x)\left(1 - \frac{a}{2L}\right)$$

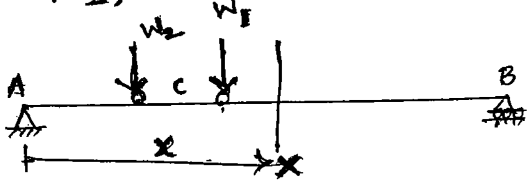
Absolute max BM. occurs at centre, $x = L/2$

$$M_{abs} = \frac{\omega a}{4}\left(L - \frac{a}{2}\right)$$

(4)-VI

Assuming $W_1$ is lighter than $W_2$,

**SF:-** case (i) Both Loads to the left of Section 'x'



case (ii) $W_1$ to the right of 'x' and $W_2$ to the left of it.

case (iii) Both Loads to the right of Section

Case (i):- Both Loads to the left of 'x': $V_x = R_B$

(a) when $x < c$, only $W_1$ will be on the beam, $W_2$ out side the span with $W_1$ on 'x'.

$$\therefore \quad V_{01x} = -R_B = -\frac{W_1 x}{L}$$

(b) when $x > c$, both Loads will be on the beam with $W_1$ at 'x'.

$$V_{21x} = -R_B = -\frac{W_1 x + W_2 (x-c)}{L}$$

case (ii) $W_1$ to the right and $W_2$ to the left of 'x'

$$V_x = -R_B + W_1$$

(c) $(L-x) > c$, both $W_1$ and $W_2$ will be on the beam $W_2$ on the 'x'.

$$V_{22x} = -R_B + W_1 = -\frac{W_2 x + W_1 (x+c)}{L} + W_1$$

(d) $(L-x) < c$, $W_1$ will be out side the girder, $W_2$ at x

$$V_{02x} = -R_B = -\frac{W_2 x}{L}$$

One of the above four expressions will give max. -ve S.F,

(i) $x=0$ to $x=c$ ; (ii) $x=c$ to $x=L-c$ ; (iii) $x=(L-c)$ to $L$

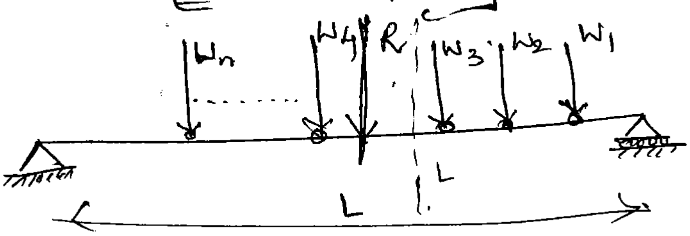
* max. -ve S.F occurs only when both the loads are to the left of the section with $W_1$ first approaching it.

* max +ve S.F occurs only when both the loads are to the right of the section.

Max BM: Avg load on the left of the section is equal to the avg load on right of section

Abs. max BM: at $x = \frac{1}{2}\left[L - \frac{W_1 c}{W_1+W_2}\right]$

Several point Loads:-



max. S.F:- Trial and Error.

max BM under a given loads when centre of beam is midway b/w the load and Resultant of all loads!

Max BM at a given section: avg load on left = avg load on Right.

Abs. max BM:- Abs. max BM occurs under one of the loads. It occurs under heavier load nearer the mid span.

Equivalent UDL:- Single point load:-

$$M_{max} = \frac{Wx}{L}(L-x);$$

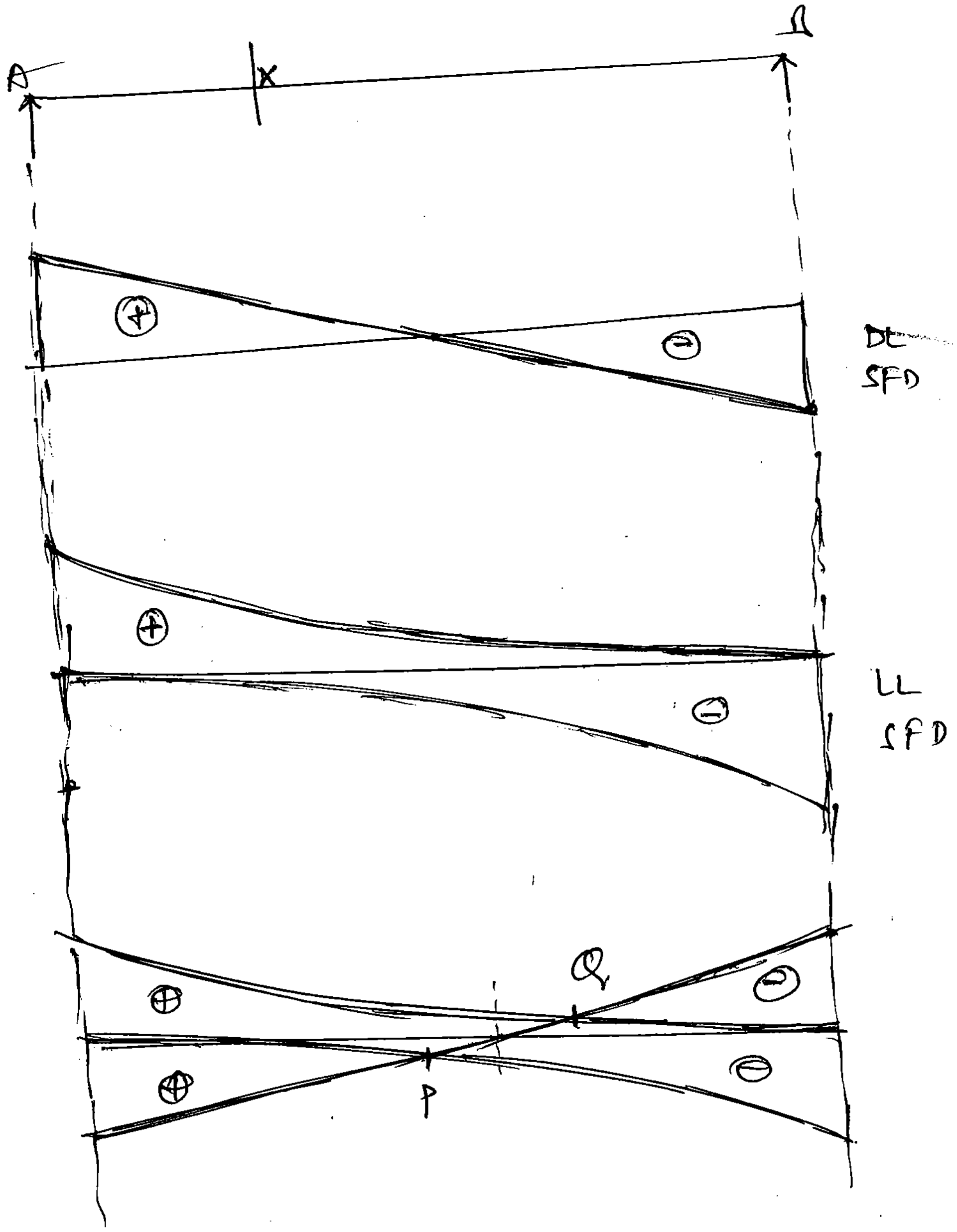
$$M = \frac{w_{eq}L}{2}x - \frac{w_{eq}x^2}{2} = \frac{w_{eq}x}{2}(L-x)$$

$$\therefore \quad w_{eq} = \frac{2W}{L},$$

UDL shorter than span:- $M_{abs} = \frac{wa}{4}(L - a/2)$

$$M = \frac{w_{eq}L^2}{8} \Rightarrow \quad w_{eq} = \frac{2wa}{L^2}(L - a/2)$$

focal lengths :- The portion over which shear force changes sign due to ~~dead~~ load and live load (moving) is called the focal length.



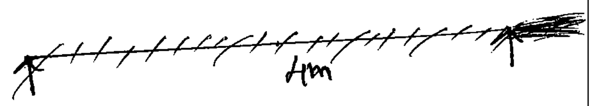

PQ is focal lengths of girder AB.

To find focal length, $V_{DL} + V_{LL} = 0$ gives 'x' which is half of the ~~focal~~ length outside focal length

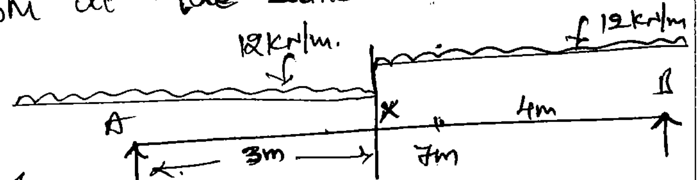
due to symmetry $= 2x$.

$\therefore$ focal length $= \underline{L - 2x}$

prob: Get max. positive and negative shear force and bending moment at a Section 1·5m in a Simple beam of span 4m when a Concentrated load of 15 KN rolls across the beam. Also, calculated the absolute negative and positive shears and bending moments?




prob: A UDL of intensity 12 KN/m and length more than 7m moves across a girder of span of 7m. Find the maximum positive and negative shear force at a section 3m from left support as well as its absolute value. Similarly, determine the maximum BM at the same section and the absolute value?




Sol: max -ve S.F occurs when head of the UDL is on X,

$$V_{max} = \frac{12\times3\times1.5}{7} = -7.714\text{ KN}$$

max. +ve S.F occurs when the tail of the UDL is on 'X',

$$V_{max}^{+} = \frac{12\times4\times2}{7} = 13.714\text{ KN}$$

Absolute maximum S.F occurs when the Head of UDL is on B (or) the tail is on A.

$$\therefore V_{abs} = \pm\frac{12\times7}{2} = 42\text{ KN}.$$

max BM at X:- when load occupies the whole span,

$$= \frac{1}{\cancel{2}}\times\overset{1.5}{\cancel{3}}\times\left(\frac{12\times3\times4}{7}\right)\times1\cancel{2} + \frac{1}{\cancel{2}}\times\overset{2}{\cancel{4}}\times\left(\frac{12\times3\times4}{7}\right)\times1\cancel{2}$$

Abs. BM $= \frac{wl^2}{8} = 73.5$ KNm

$$= \frac{12\times3\times4}{7}\times1\cancel{2}\left[3.5\right] = 72\text{ KNm}$$

Influence Lines:- IL for SF & BM - Load position for max S.F at a Section - Load position for max. BM at a Section, Single Point Load, UDL Longer than Span, UDL Shorter than Span - IL for forces in members of pratt and Warren trusses.

⑥ - VI

Introduction:- Loads acting on a beam may be broadly classified as dead loads and Live loads.

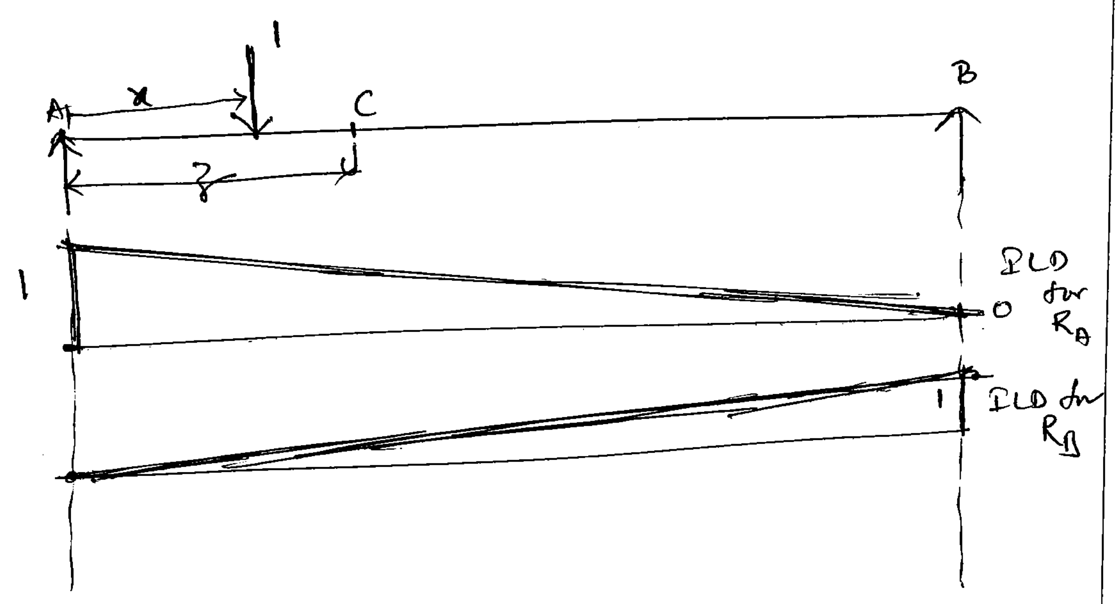
* Loads which do not change their position during the life of the beam are D.L.

* Loads which can change their position during the life of the beam are L.L

ILD:- ILD for a Stress resultant is the one in which ordinate represent the value of the stress resultant for the corresponding position of unit load at the abscissa

ILDs for SSB:-

(i) ILD for reaction $R_A$ :-



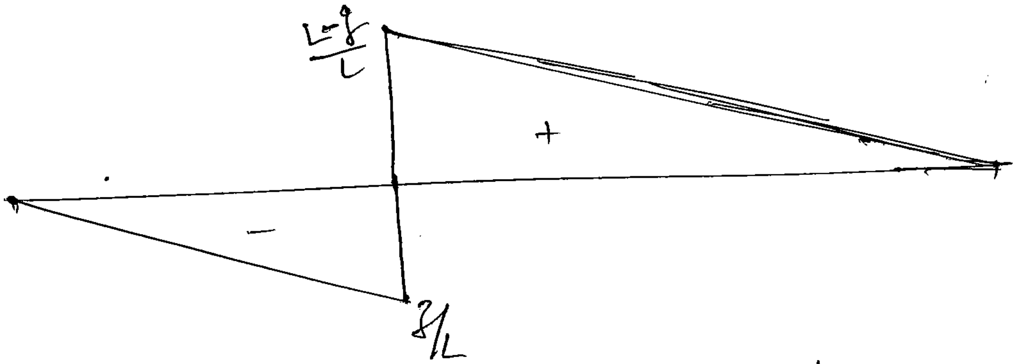
$$R_A = \frac{1(L-x)}{L} = 1-\frac{x}{L}$$ $x=0 \quad R_A=1$; $x=L \quad R_A=0$

for $R_B$:- $R_B = \frac{x}{L}$ ; $x=0 \quad R_B=0$; $x=L \quad R_B=1$

ILD for $SF_c$:-

a) when $x<z$, $SF_c = \frac{-x}{L}$ ; $x=0 \quad SF=0$; $x=z \quad SF = -\frac{z}{L}$

b) " $x>z$, $SF_c = R_A = \frac{L-x}{L}$ ; $x=z$, $SF_c = \frac{L-z}{L}$; $x=L$, $SF_c = 0$




BM$_c$:- when $x<z$ : $M_c = R_B \times (L-z)$

$$= \frac{x}{L}(L-z)$$

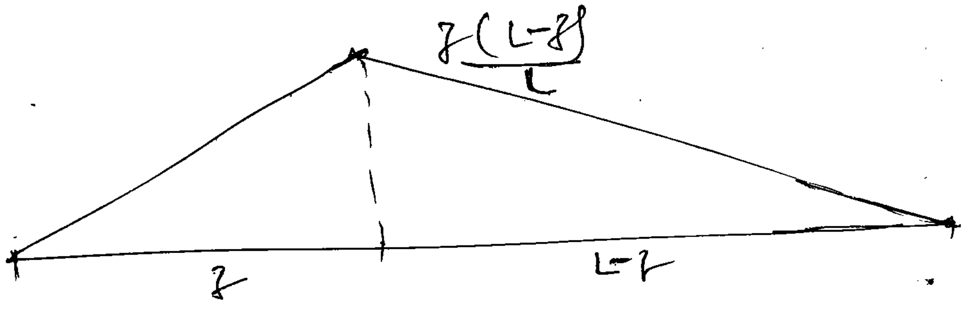
$x=0 \quad M_c=0$

$x=z \quad M_c = \frac{z(L-z)}{L}$

$x>z$ :- $M_c = R_A z = \frac{(L-x)}{L} \times z$

$x=z \quad M_c = \left(\frac{L-z}{L}\right) \times z$

$x=L \quad M_c = 0$



1) Draw ILD for the bending moment at a Section x for a simply supported beam? (3M) (7)-VI

2) Draw ILD for the reaction at A for a SSB ~~AB~~ AB? (4M)

3) Two point loads of 6000N and 3000N spaced at 4m apart cross a girder of 10m span from left to right, with smaller load leading. Draw SF and BM ~~diagrams~~. Find the position and amount of Abs. max. BM? (16M)

4) A uniform load of 2000N/m, 5m long crosses a girder of 20m span from left to right. Cal. the max. S.F and BM at a section 8m from left support? (16M)

5) Draw ILD for the ~~reaction at~~ B S.F at a Section x for a simply supported beam AB?. (3M)

6) Draw ILD for reaction at B for SSB.? (3m)

7) Draw ILD for forces in the members of a Warren Truss.? (16m)

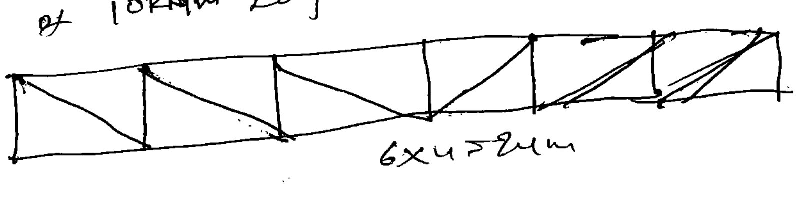
Supply 8) What are the positions of a single load for maximum BM at section and Absolute BM in the span? (4M)

Supply 9) A SSB of span 8m is loaded with three point loads of 5KN, 10KN and 15KN at a distance 2m, 4m and 6m resp from right end. It also carries UDL of 10KN/m throughout the span. Find position and magnitude of max. deflection and calculate max S.F? of max. deflection and calculate max S.F?

2015 10) Define ILD and draw ILD? 4M.

11) A UDL of 40KN/m and of length 3m transverse across the span of SSB of length 16m. Compute max BM at 4m from left support and absolute BM? (16M)

12) Find max force in the members shown in fig. when UDL of 10KN/m longer than span crosses the bridge.



Max of ... due to moving loads :-

Single point Loads

(i) Max -ve S.F occurs when load is just to the left of section 'C'. $= \frac{Wz}{L}$

(ii) max +ve S.F occurs when Load is just to the right of section 'C' $= W\left(\frac{L-z}{L}\right)$

(iii) max BM will occur when the load is on the section itself $= W\frac{z(L-z)}{L}$.

Abs. max S.F = W -ve at B. and +ve at A.

" " BM = $\frac{WL}{4}$ for $z = L/2$ "

UDL Longer than Span :-

Max. -ve S.F, $= \frac{wz^2}{2L}$ (load b/w A & C)

" +ve S.F $= \frac{w(L-z)^2}{2L}$ (load b/w C & B)

max BM, UDL on whole span, $= w\frac{z(L-z)}{2}$

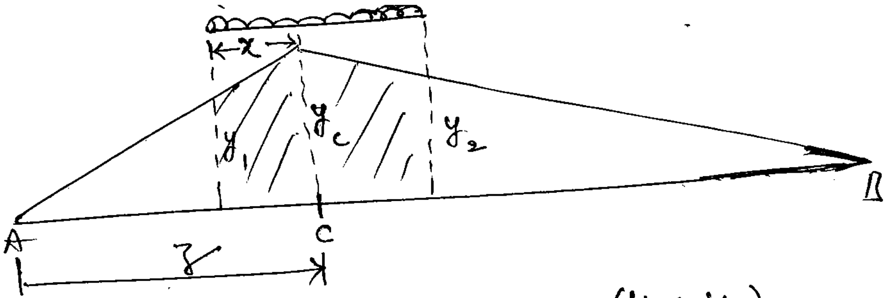
Abs. max S.F $= \frac{wL}{2}$.

" " BM $= \frac{wL^2}{8}$

UDL smaller than span

(i) for max +ve S.F the tail of UDL should reach the section.

(ii) for max ~~+ve~~ the head of the UDL reach the section




$$M_C = \omega \times x\left(\frac{y_1+y_c}{2}\right) + \omega(d-x)\frac{(y_c+y_2)}{2}$$

for $M_c$ to be maximum, $\frac{dM_c}{dx} = 0$,

$$\omega\left(\frac{y_1+y_c}{2}\right) - \omega\frac{(y_c+y_2)}{2} = 0$$

$$\boxed{y_1 = y_2}$$

$$\therefore \frac{(z-x)}{z} \times y_c = \frac{(L-z)-(d-x)}{(L-z)} \times y_c$$

$$Lx = dz$$

$$\boxed{\frac{x}{d} = \frac{z}{L}}$$

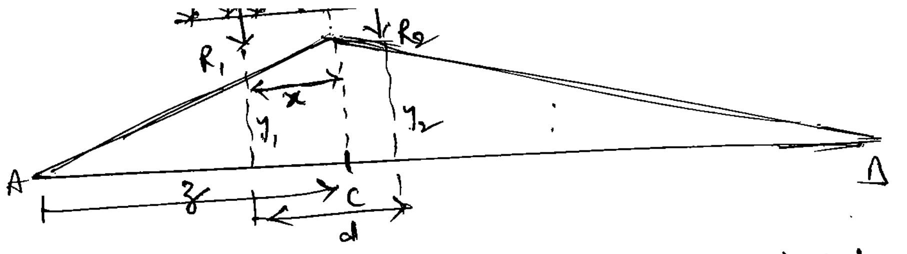
* B.M is maximum at a section when the load is so placed that the section divides the load in the same ratio as it divides the span.

* For absolute maximum moment C.G of the load will be at the mid-span

Train of point loads–

* for max −ve S.F at a section, most of the loads are to be to the left of section

* for maximum +ve S.F at a section, most of the loads are to be to the right of the section




$$M_c = R_1 y_1 + R_2 y_2 = R_1 \frac{(z-x)}{z} \times y_c + R_2 \times \frac{(L-z)-(d-x)}{(L-z)} \times y_c$$

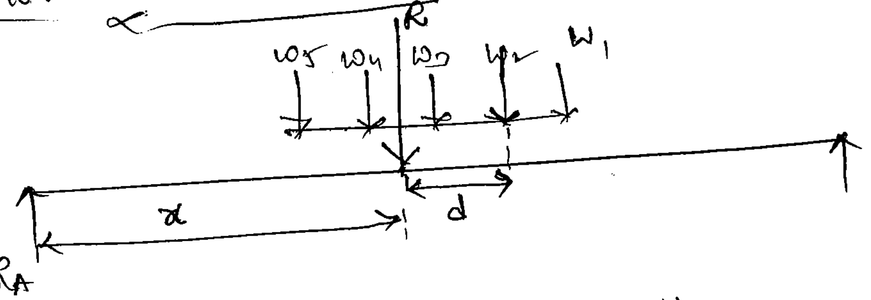
for $M_c$ to be maximum $\frac{dM_c}{dx} = 0$ so

$$\boxed{\frac{R_1}{z} = \frac{R_2}{l-z}}$$

"Avg load on left portion of the ~~beam~~ is same as the Avg load on right side portion.

Absolute Max S.F :- happens at the supports.

Maximum moment under a load




$$R_A = \frac{R(L-x)}{L} \quad ; \quad M_{w_2} = R_A(x+d) - R(d)$$

$$= \frac{R(L-x)(x+d)}{L} - Rd$$

$$\frac{dM}{dx} = 0 \Rightarrow \frac{R}{L}(L - x - d) = 0$$

$$x = \frac{L}{2} - \frac{d}{2}$$

distance of $W_2$ from 'A' $= x + d = \frac{L}{2} + \frac{d}{2}$

"For moment to be max under a load, the load and Resultant should be equidistant from the mid-span"!.

**Absolute maximum BM:-** Absolute maximum moment occur under one of the loads when the resultant of all the loads and the load under consideration are equidistant from the centre of the beam.

2016
**Prob:-** Two point loads of 6KN and 3KN spaced at 4m apart cross a girder of 10m span from left to right, with smaller load leading. Draw SF and BM diagrams. Find the position and amount of Abs max. BM? (16M)

**Sol:-** $W_1 = 3KN \quad W_2 = 6KN, \quad c = 4m \quad L = 10m$

Condition, $c < \frac{W_1 L}{W_1 + W_2} \Rightarrow \frac{(3 \times 10)}{(3+6)} = \frac{30}{9} = 3.333$

Hence $c > \frac{W_1 L}{W_1 + W_2}$

$$\boxed{eq. 18.26}$$

$$\cancel{V_{22x}} \cancel{=} -\frac{W_1 x \cancel{+ W_2(x-c)}}{\cancel{L}} = -\cancel{3x4+}$$

$$V_{22x} = -\frac{W_2 x + W_1(x+c)}{L} + W_1$$

$$V_{02x} = -\frac{W_2 x}{L} \quad \text{— Eqn 2}$$

$(L-x) \cancel{<} C \Rightarrow L > \cancel{x+c} \Rightarrow \cancel{x+c}$

$x = 0 \quad V_{02x} = 0$

$x = 4m \quad V_{02x} = -\frac{6 \times 4}{10} = -2.4\ KN$

$(L-x) \cancel{>} C \Rightarrow x = 4m, \quad V_{22x} = -\frac{6 \times 4 + 3(4+4)}{10} + 3$

$= \cancel{1}.8\ KN$

$x = 5m, \quad V_{22x} = \frac{-6 \times 5 + 3(4+5)}{10} \cancel{+} 3$

$= -2.7$

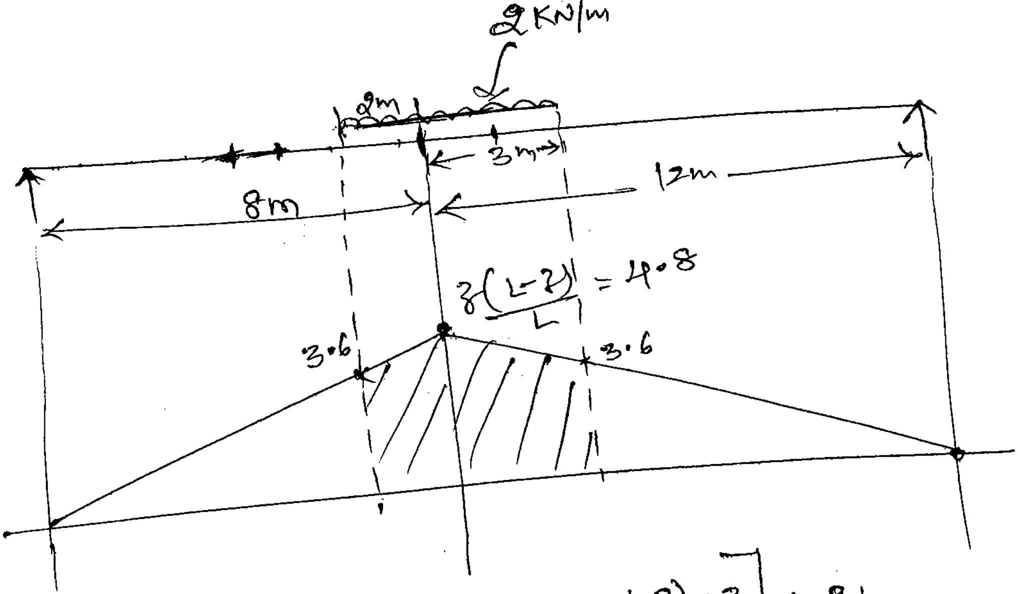
$x = 10, \quad V_{22x} = -\frac{6 \times 10 + 3(4+10)}{10} + 3 = -7.2$

Probl- A uniform load of 2000 N/m, 5m long crosses a girder of 20m span from left to right. Cal. max S.F and BM at a section 8m from left support?

Sol:-

$$\boxed{\frac{x}{d} = \frac{z}{L}}$$

$$\frac{x}{5} = \frac{8}{20} \Rightarrow x = \frac{8 \times 5}{20} = 2m$$

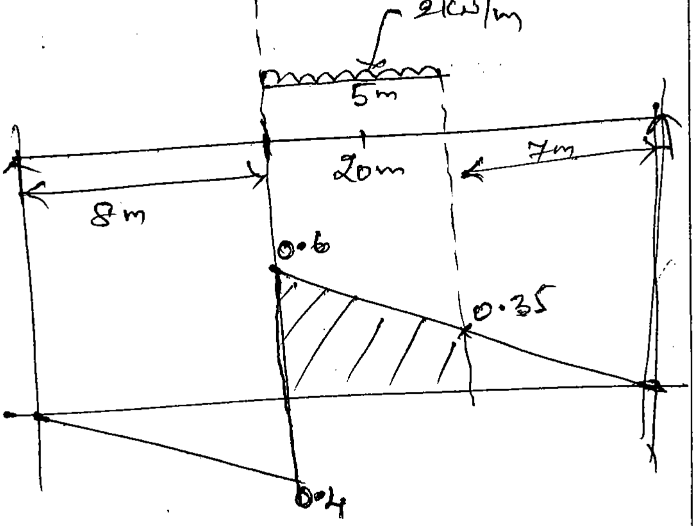



$$\text{max BM}_{at\,8m} = \left[\left(\frac{3.6+4.8}{2}\right)\times 2 + \left(\frac{3.6+4.8}{2}\right)\times 3\right] \times 2$$

$$= \left(\frac{3.6+4.8}{2}\right) \times 5 \times 2$$

$$= 42 \text{ KNm}$$

For max SF:-




$$= \left(\frac{0.6+0.35}{2}\right) \times 5 \times 2$$

$$= 4.75 \text{ KN}$$