6. Analysis of pin jointed plane Frames

plane frame:—

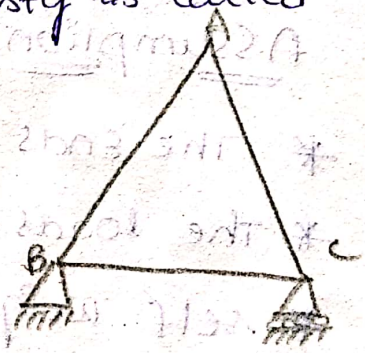
A frame in which all members lie in a single plane is called "plane frame". Ex:— Roof truss, Bridge trusses.

space frame:—

In all the members of frame do not lie in a single plane is called "space frame". Ex:— Tripod stand, transmission tower.

perfect frame:—

$m = 2j - 3$ condition satisfy is called "perfect frame"



Example-①:—

members (m) = 3.

joints (j) = 3

$m = 2j - 3$

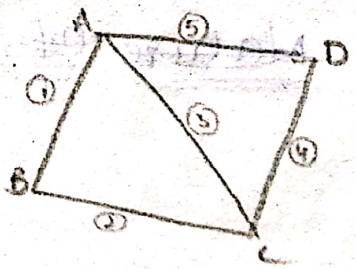
$3 = (2 \times 3) - 3 \Rightarrow 3 = 3$ (perfect frame)

Example-②:—

$m = 5, \quad j = 4.$

$m = 2j - 3$

$5 = (2 \times 4) - 3 \Rightarrow 5 = 5$

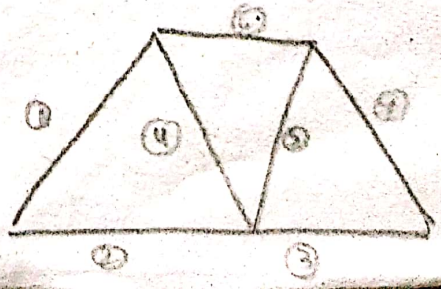


Example-3:—

$m = 7, \quad j = 5$

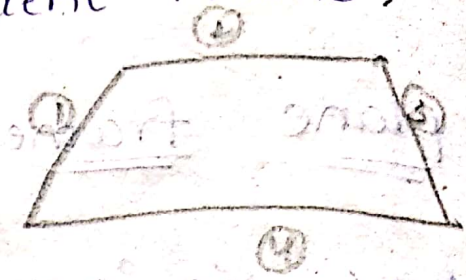
$m = 2j - 3$

$7 = (2 \times 5) - 3 \Rightarrow 7 = 7$



Deficient frame:-

$m < 2j-3$ is called "deficient frame".



Example-①

$m = 4, \quad j = 4$

$m = 2j-3$

$4 = (2 \times 4) - 3 \Rightarrow 4 < 5$ (Deficient frame)

Redundant frame:-

$m > 2j-3$ is called "redundant frame".

$m = 11, \quad j = 6$

$m = 2j-3$

$11 = (2 \times 6) - 3 \Rightarrow 11 > 9$

(Redundant frame)

Assumptions :-

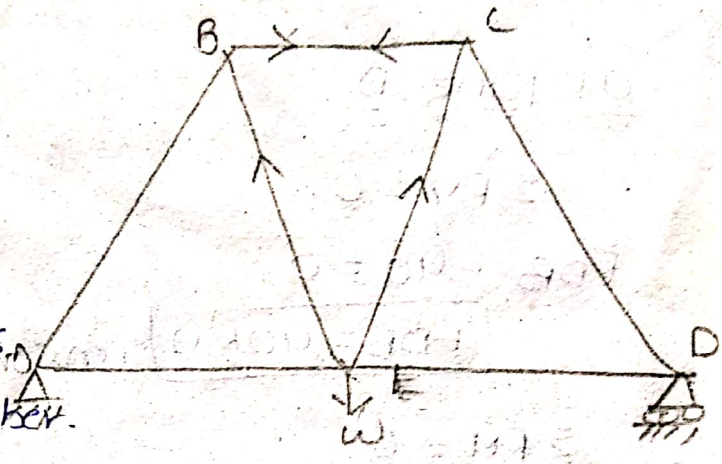
* The ends of the members are pin jointed (Hinged).
* The loads acts only at the joints.
* self weight of the members are negligible
* Members having uniform cross section throughout the span.

~~Nature of forces in member~~ :-

Nature of forces in members:—

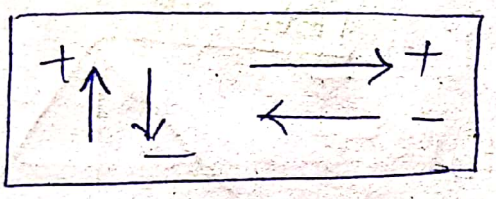
The members of pin jointed frames are subjected to either tensile (or) compressive forces a typical truss ABCDE loaded at joint 'E'

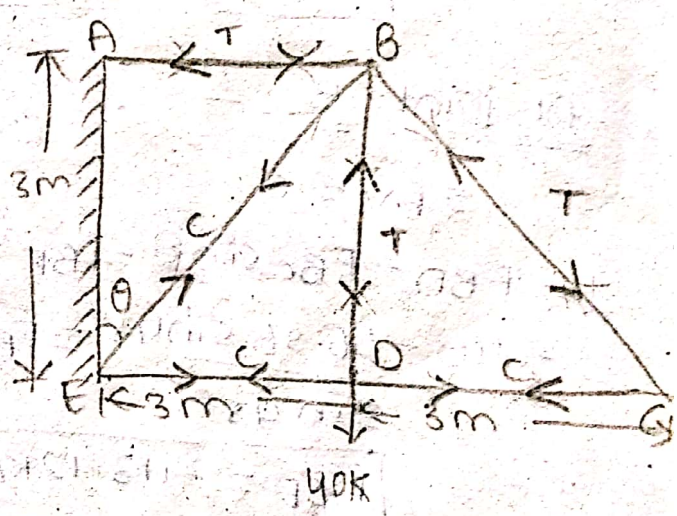
At joint 'E' load is applied downwards The opposite BE, CE are tension members, 'BC' is compression member.



Method of joints:—

① Find the forces in all members of truss as shown in fig





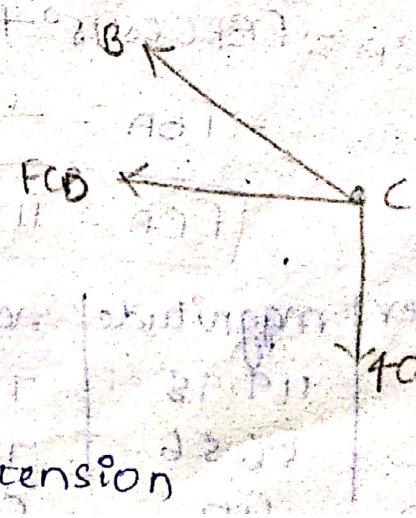
Sol:- $\tan\theta = \frac{AB}{DE}$

$\tan\theta = \frac{3}{3} = 1$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$

At join 'C'



$\Sigma F_V = 0$

$F_{CB} \sin\theta = 40$

$F_{CB} = \frac{40}{\sin 45^\circ}$

$\therefore \boxed{F_{CB} = 56.56\text{kN}}$ tension

$\Sigma F_H = 0$

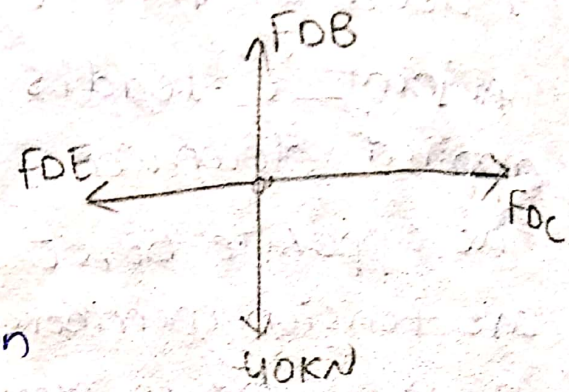
$-F_{CB}\cos\theta - F_{CD} = 0$

$F_{CD} = -F_{CB} \times \cos 45°$

$F_{CD} = -56.56 \times \cos 45°$

$\boxed{F_{CD} = -40\ kN.}$ compression

<u>at joint D</u>



$\Sigma F_V = 0$

$F_{DB} - 40 = 0$

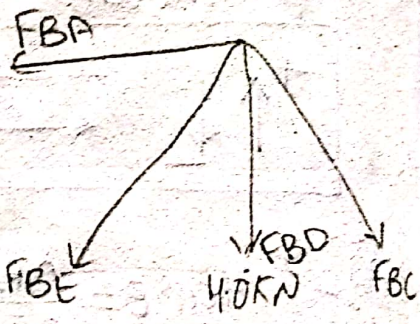
$\boxed{F_{DB} = 40kN}$ Tension

$\Sigma F_H = 0$

$-F_{DE} + F_{DC} = 0$ $\quad F_{DC} = F_{CD} = -40kN$

$\boxed{F_{DE} = -40\ kN}$ compression.

<u>At joint B :-</u>



$\Sigma F_V = 0$

$-F_{BD} - F_{BC}\sin\theta - F_{BE}\sin\theta = 0$

$-40 - 56.56\sin 45° = F_{BE}$

$\sin 45°$

$\boxed{F_{BE} = -113.12kN}$ compression

$\Sigma F_H = 0$

$-F_{BA} - F_{BE}\cos 45° + F_{BC}\cos 45° = 0$

$+F_{BA} = -(-113.12\cos 45°) + 56.56\cos 45°$

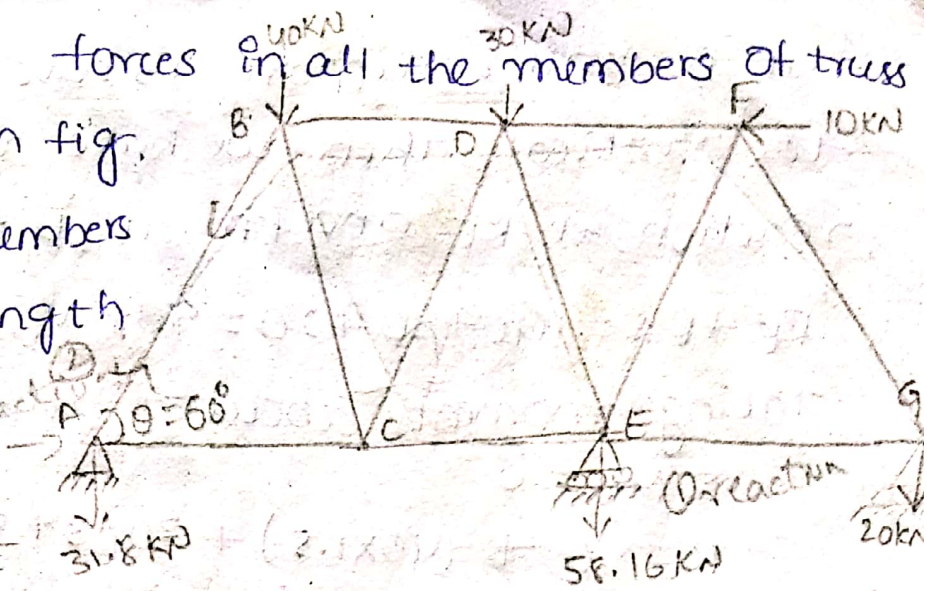
$\boxed{F_{BA} = 119.98\ kN}$ Tension

Member	magnitude	nature			
AB	119.98	T	BD	40	T
BC	56.56	T	BE	113.12	C
CD	40	C	DE	40	C

② Find the forces in all the members of truss as shown in fig.

All the members are 3m length

$\theta = 60^\circ$

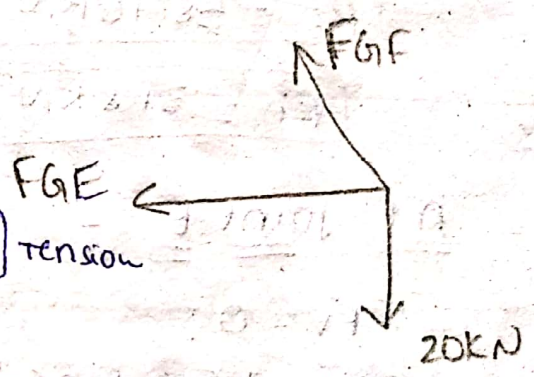


Sol:- At joint G :-

$\Sigma F_V = 0$

$F_{GF} \sin 60^\circ = 20$

$F_{GF} = 23.09\,KN$



FGE tension

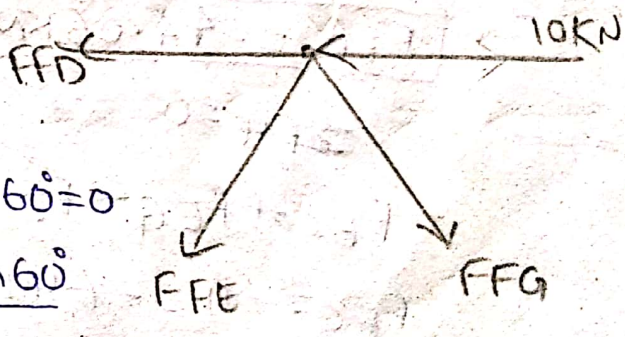
$\Sigma F_H = 0$

$-F_{GE} - F_{GF} \cos 60^\circ = 0$

$-F_{GE} = 23.09 \times \cos 60^\circ$

$F_{GE} = -11.5\,KN$ comp

at joint F :-



$\Sigma F_V = 0$

$-F_{FG} \sin 60^\circ - F_{FE} \sin 60^\circ = 0$

$F_{FE} = \frac{-23.09 \times \sin 60^\circ}{\sin 60^\circ}$

$F_{FE} = -23.09\,KN$ comp

$\Sigma F_H = 0$

$-F_{FD} - 10 - F_{FE} \cos 60^\circ + F_{FG} \cos 60^\circ = 0$

$F_{FD} = -10 - (-23.09 \times \cos 60^\circ) + (23.09 \times \cos 60^\circ)$

$F_{FD} = 13.09\,KN$ tension

At 'E' roller support is have that indicates vertical then we have to calculate reactions at 'RE' and 'RA' at 'E' and 'A'.

$R_E + R_A = 40 + 30 + 20 = 90\ KN$

taking moments about 'A'

$\Sigma M_A = 0$

$-R_E \times 6 + (40 \times 1.5) + 30\left(3 + \frac{3}{2}\right) - 10 \times 3 \sin 60^\circ + (20 \times 9) = 0$

$\boxed{R_E = 58.16\ KN}$

$\boxed{R_A = 31.8\ KN}$

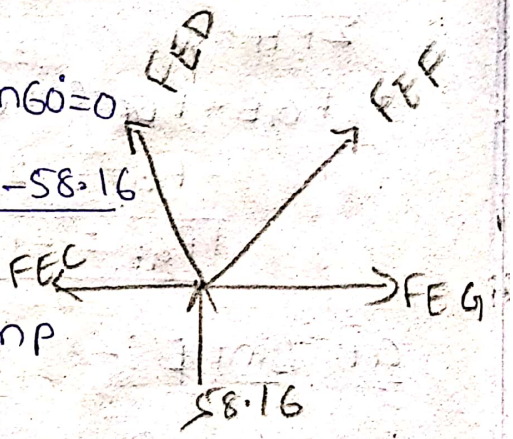
At joint E :-

$\Sigma F_V = 0$

$58.16 + F_{EF} \sin 60^\circ + F_{ED} \sin 60^\circ = 0$

$F_{ED} = \frac{-(-23.09 \times \sin 60^\circ) - 58.16}{\sin 60^\circ}$

$\boxed{F_{ED} = -44.06\ KN}$ comp



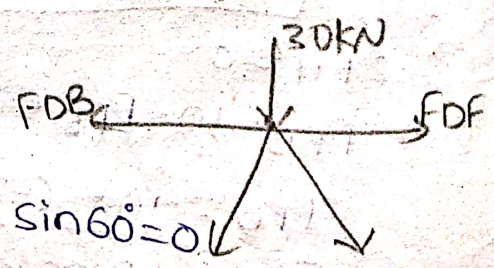
$\Sigma F_H = 0$

$-F_{EC} + F_{EG} - F_{ED}\cos 60^\circ + F_{EF}\cos 60^\circ = 0$

$F_{EC} = -11.5 - (-44.6 \times \cos 60^\circ) + (-23.09 \times \cos 60^\circ)$

$\boxed{F_{EC} = -1.015\ KN}$

At joint D :-



$\Sigma F_V = 0$

$-30 - F_{DE} \sin 60^\circ - F_{DC} \sin 60^\circ = 0$

$F_{DC} = \frac{-(-44.06 \sin 60^\circ) - 30}{\sin 60^\circ}$

$F_{DC} = 9.41\,kN$ Tension

$\Sigma f_H = 0$

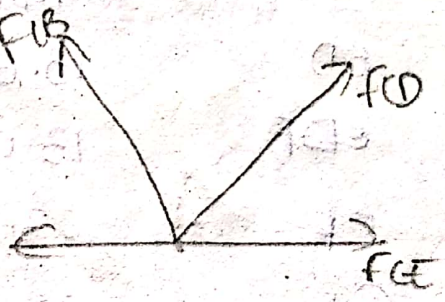
$-F_{OB} + F_{OF} - F_{DC}\cos60^\circ + F_{DE}\cos60^\circ = 0$

$F_{DB} = 13.09 - 9.41 \times \cos60^\circ + (-44.06 \times \cos60^\circ)$

$F_{DB} = -13.645\,kN$ comp

At joint 'C'

$\Sigma f_V = 0$



$F_{CD}\sin60^\circ + F_{CB}\sin60^\circ = 0$

$\frac{9.41 \times \sin60^\circ}{\sin60^\circ} = -F_{CB}$

$F_{CB} = -9.41\,kN$ comp

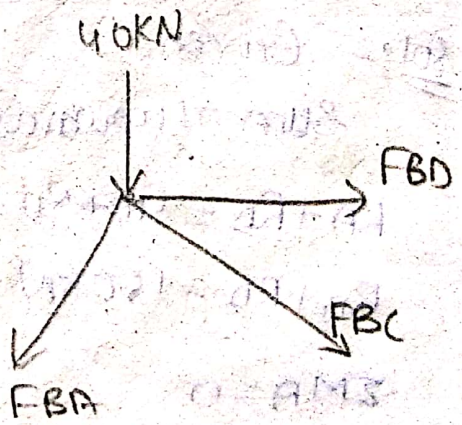
$\Sigma f_H = 0$

$-F_{CB}\cos60^\circ - F_{CA} + F_{CD}\cos60^\circ + F_{CE} = 0$

$-(-9.41 \times \cos60^\circ) + (9.41 \times \cos60^\circ) + (-1.015) = F_{CA}$

$F_{CA} = 8.395\,kN$ tension

At joint B :-



$\Sigma f_V = 0$

$-40 - F_{BA}\sin60^\circ - F_{BC}\sin60^\circ = 0$

$\frac{-40 - (-9.41 \times \sin60^\circ)}{\sin60^\circ} = F_{BA}$

$F_{BA} = -36.7\,kN$ comp

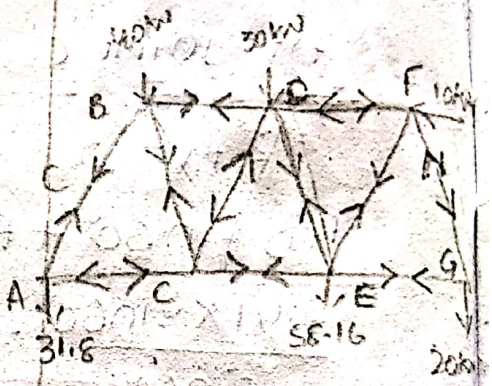
$\Sigma f_H = 0$

$-F_{BA}\cos60^\circ + F_{BD} + F_{BC}\cos60^\circ = 0$

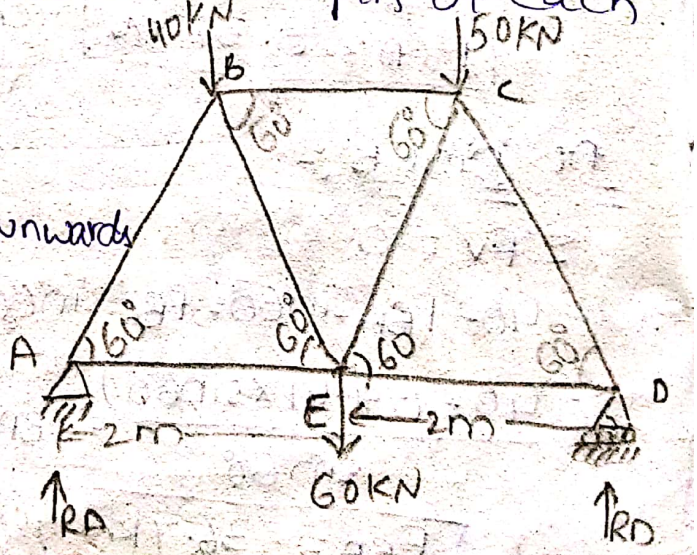
$-(-36.7\cos60^\circ) + (-13.645) + (-9.41 \times \cos60^\circ) = 0$

$0 = 0$

Member	magnitude	nature
AB	36.7	C
BC	9.41	C
AC	8.39	T
CE	1.015	C
CD	9.41	T
DE	44.06	C
BD	13.64	C
DF	13.09	T
EF	23.09	C
GF	23.09	T
EG	11.54	C



① Determine the forces in all the members of truss as shown in fig indicate the magnitude and nature of forces all inclined members are 60° to the horizontal and length of each member is 2m.



Sol:- Given.

Sum of upwards = downwards

$R_A + R_D = 40 + 50 + 60$

$R_A + R_D = 150\,KN$

$\Sigma M_A = 0$

$-R_D \times 4 + 50 \times \left(2 + \frac{2}{2}\right) + 40\left(\frac{2}{2}\right) + (60 \times 2) = 0$

$R_D = \frac{310}{4}$

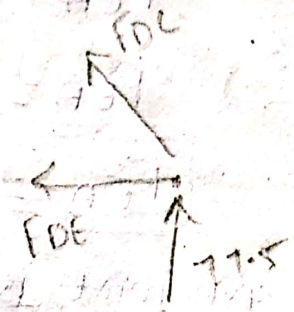
$\therefore \boxed{R_D = 77.5\,KN}$

↑+ →+
← −
↓ −

$R_D = 150 - 77.5$

$\boxed{R_A = 72.5\ KN}$

at joint D :-



$\Sigma F_V = 0$

$F_{DC}\sin 60^\circ + 77.5 = 0$

$\boxed{F_{DC} = -89.48\ KN}$

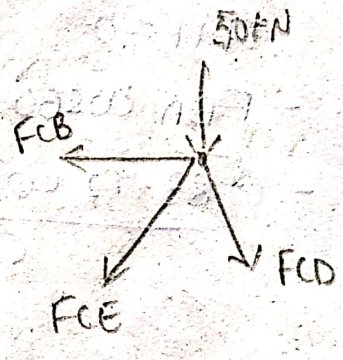
$\Sigma F_H = 0$

$-F_{DE} - F_{DC}\cos 60^\circ = 0$

$F_{DE} = -(-89.48 \times \cos 60^\circ)$

$\therefore \boxed{F_{DE} = 44.74\ KN}$

at joint C :-



$\Sigma F_V = 0$

$-50 - F_{CE}\sin 60^\circ - F_{CD}\sin 60^\circ = 0$

$\dfrac{-50 - (-89.48\sin 60^\circ)}{\sin 60^\circ} = F_{CE}$

$\boxed{F_{CE} = 31.74\ KN}$

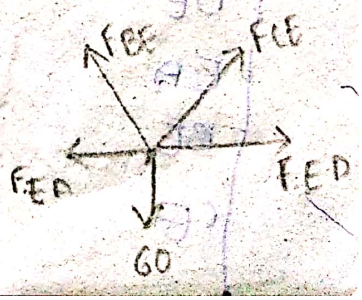
$\Sigma F_H = 0$

$-F_{CB} - F_{CE}\cos 60^\circ + F_{CD}\cos 60^\circ = 0$

$F_{CB} = -31.74\cos 60^\circ + (-89.48 \times \cos 60^\circ)$

$\boxed{F_{CB} = -60.61\ KN}$

at joint E :-



$\Sigma F_V = 0$

$-60 + F_{BE}\sin 60^\circ + F_{CE}\sin 60^\circ = 0$

$-60 + 31.74\sin 60^\circ / \sin 60^\circ = F_{BE}$

$F_{BE} = +37.54\,KN$

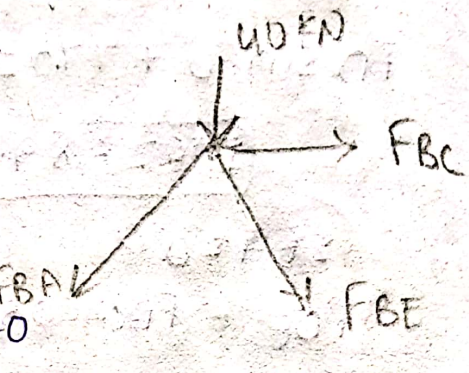
at $\Sigma F_H = 0$

$-F_{EA} + F_{ED} - F_{BE}\cos60^\circ + F_{CE}\cos60^\circ = 0$

$F_{EA} = 44.74 - (+37.5\cos60^\circ) + (31.74\cos60^\circ)$

$F_{EA} = 41.86\,KN$

At joint B :-



$\Sigma F_V = 0$

$-40 - F_{BA}\sin60^\circ - F_{BE}\sin60^\circ = 0$

$\frac{-40 - 37.54\sin60^\circ}{\sin60^\circ} = F_{BA}$

$F_{BA} = -83.72\,KN$

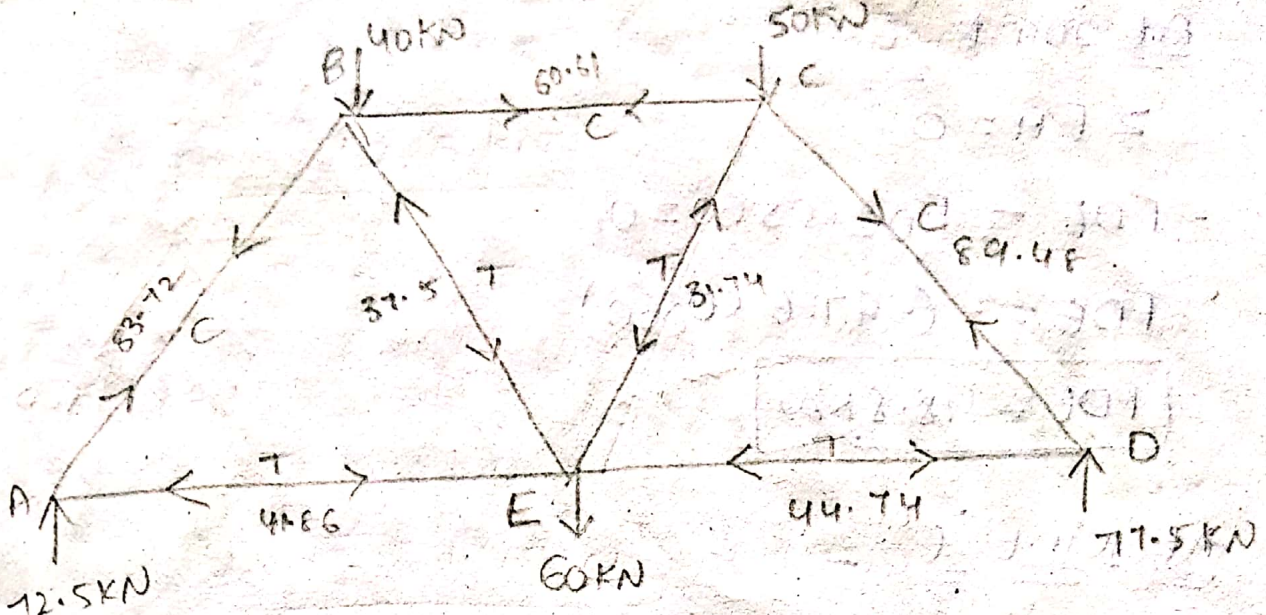
$\Sigma F_H = 0$

$-F_{BA}\cos60^\circ + F_{BE}\cos60^\circ + F_{BC} = 0$

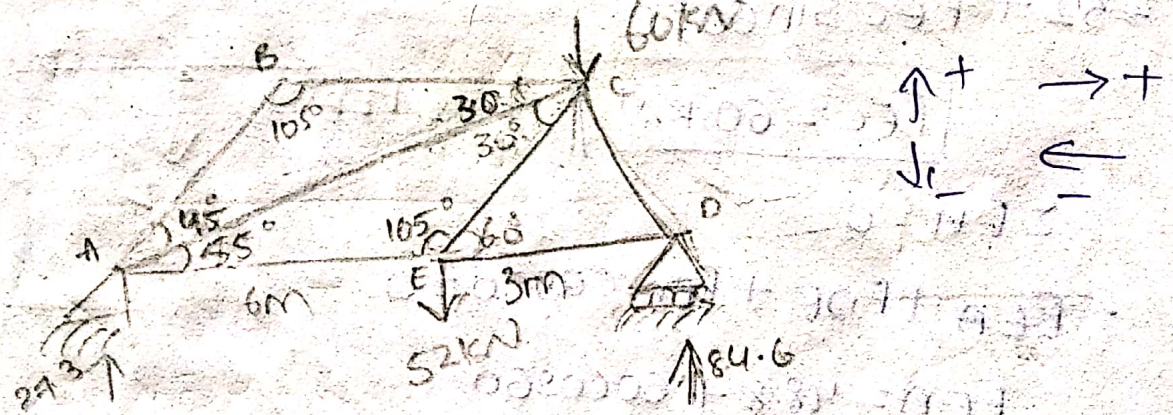
$\frac{-(-83.72\cos60^\circ) + (-60.61)}{\cos60^\circ} = -F_{BE}$

$\therefore F_{BE} = +37.5\,KN$

member	magnitude	nature
AB	83.72	C
BC	60.61	C
CD	89.48	C
DE	44.74	T
EA	41.86	T
BE	37.5	T
CE	31.74	T



①



$R_A + R_D = 60 + 52 = 112\,kN$

$\Sigma M_A = 0$

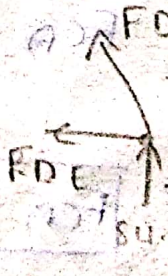
$-R_D \times 9 + 60\left(6 + \frac{3}{2}\right) + 52(6) = 0$

$R_D = 84.6\,kN$

$\therefore R_A = 112 - 84.6$

$\therefore \boxed{R_A = 27.3\,kN}$

At joint 'D' :—



$\Sigma F_V = 0$

$84.6 + F_{DC} \sin 60° = 0$

$\boxed{F_{DC} = -97.6\,kN}$

At joint 'D'

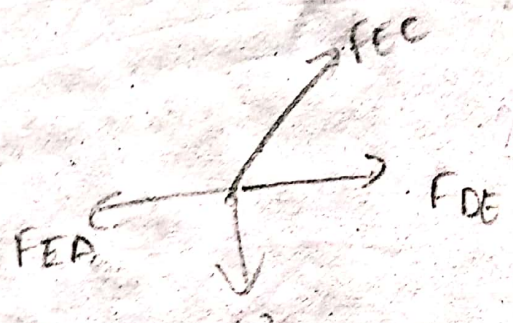
$\Sigma F_H = 0$

$-F_{DE} - F_{DE}\cos 60^\circ = 0$

$F_{DE} = -(-97.6\cos 60^\circ)$

$\boxed{F_{DE} = 48.8\ KN}$

At joint 'E' :-



$\Sigma F_V = 0$

$-52 + F_{EC}\sin 60^\circ = 0$

$\boxed{F_{EC} = 60\ KN}$

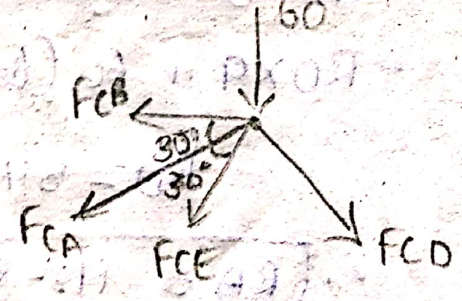
$\Sigma F_H = 0$

$-F_{EA} + F_{DE} + F_{EC}\cos 60^\circ = 0$

$F_{EA} = 48.8 + 60\cos 60^\circ$

$\therefore \boxed{F_{EA} = 78.8\ KN}$

At joint 'C'



$\Sigma F_V = 0$

$-60 - F_{CE}\sin 60^\circ - F_{CD}\sin 60^\circ - F_{CA}\sin 90^\circ = 0$

$F_{CA} = \dfrac{-60 - 60\sin 60^\circ - (-97.6\sin 60^\circ)}{\sin 90^\circ}$

$\therefore \boxed{F_{CA} = -27.43\ KN}$

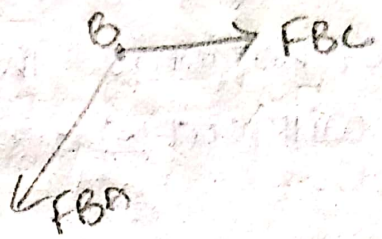
$\Sigma F_H = 0$

$-F_{CB} - F_{CA}\cos 30^\circ - F_{CE}\cos 30^\circ + F_{CD}\cos 60^\circ = 0$

$F_{CB} = -(-27.43 \times \cos 30^\circ) - 60 \times \cos 30^\circ + (-97.6 \cos 90^\circ)$

$\therefore F_{CB} = -77\ KN$

at joint B :-



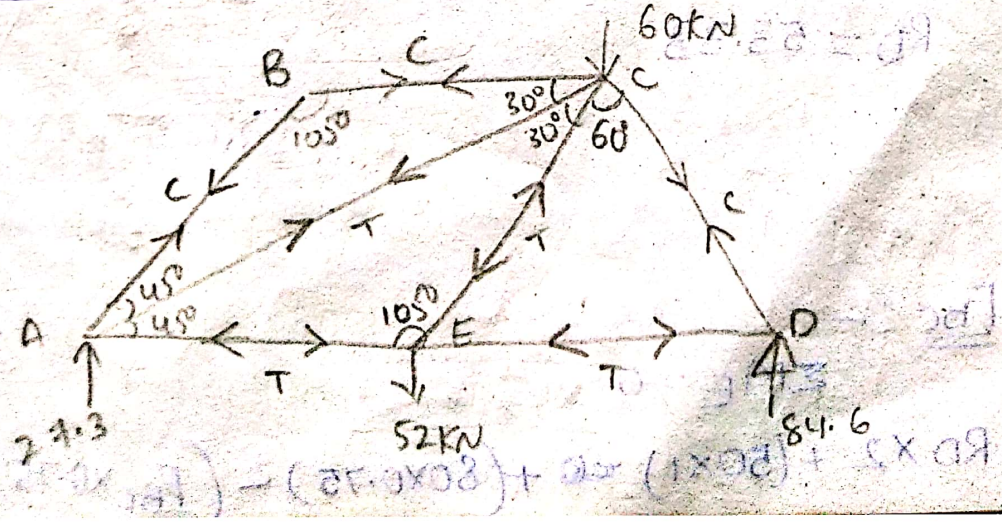
$\Sigma FM = 0$

$F_{BC} - F_{BA} \cos 30^\circ = 0$

$F_{BA} = \frac{-77}{\cos 30^\circ}$

$F_{BA} = -88.9\ KN$

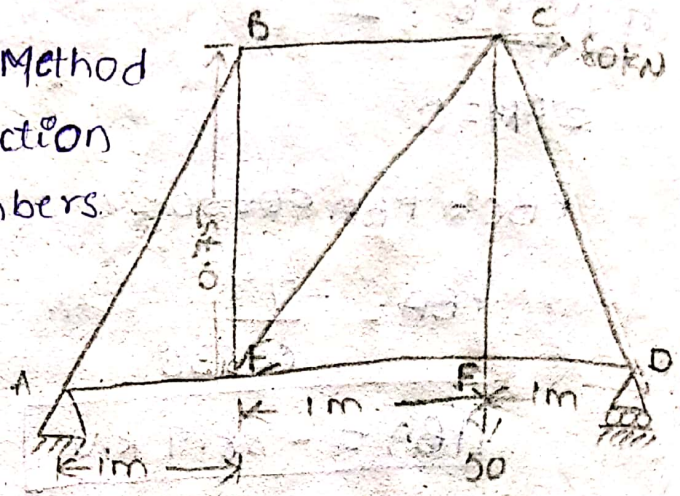
member	magnitude	nature
AB	88.9	C
BC	77	C
CD	97.6	C
DE	48.8	T
EA	78.8	T
AC	27.43	C
CE	60	T



Method of sections :-

① Determine the forces in members BC, CE and EF from the figure shown.

the condition for Method of section is. The section will cut ≤ 3 members.

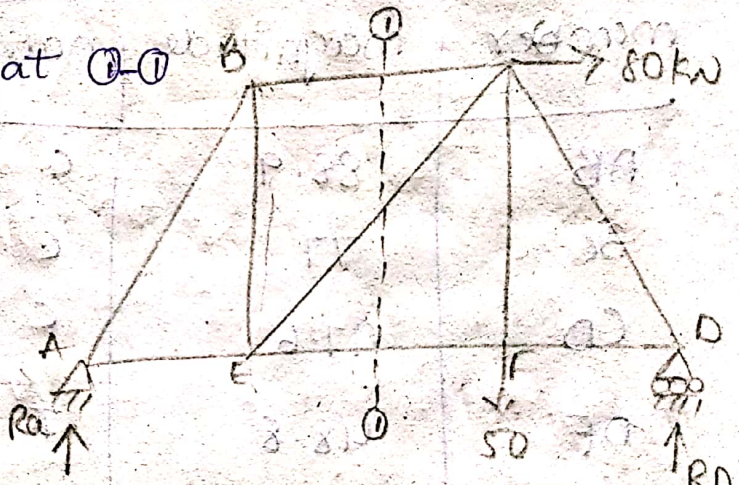


Sol :-

Take a section at ①-①

BC, EF, CE

$R_a + R_d = 50\ kN$

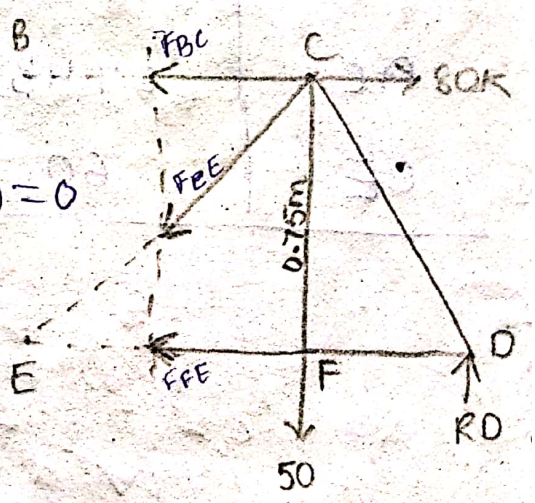


Consider right portion of the section.

$\Sigma M_A = 0$

$-(R_D \times 3) + (50 \times 2) + (80 \times 0.75) = 0$

$R_D = 53.33$



F_{BC} :-

$\Sigma M_E = 0$

$-R_D \times 2 + (50 \times 1) + (80 \times 0.75) - (F_{BC} \times 0.75) = 0$

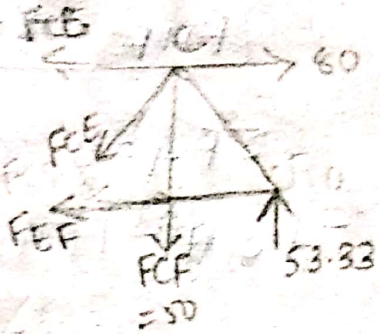
$F_{BC} = 4.45\,kN$

F_{EF} :—

$\Sigma M_C = 0$

$-R_D \times 1 + (8.0 \times 0.) + (F_{EF} \times 0.75) = 0$

$F_{EF} = +71.10\,kN$



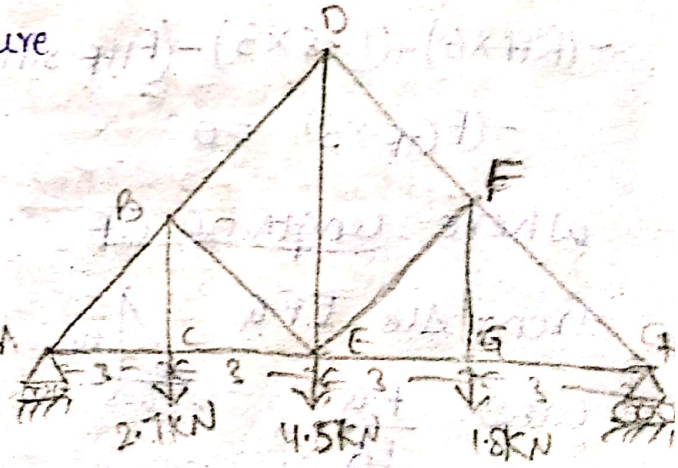
$\Sigma F_V = 0$

$+53.33 - 50 - F_{CE} \sin 45^\circ = 0$

$F_{CE} = 4.70\,kN$

② Find the forces in members. by using of Method of sections. (HF, HG), (GF, GE) (FD, FE)

member	magnitude	nature
HF	8.55	C
GH	7.4	T
GF	1.8	T
GE	7.41	T
EF	2.07	T
FD	11.70	C



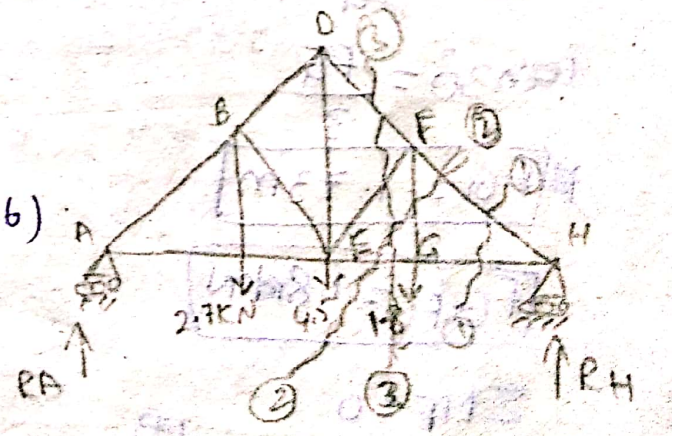
Sol :—

Reactions

$\Sigma M_A = 0$

$-(R_H \times 12) + (1.8 \times 9) + (4.5 \times 6) + (2.7 \times 3) = 0$

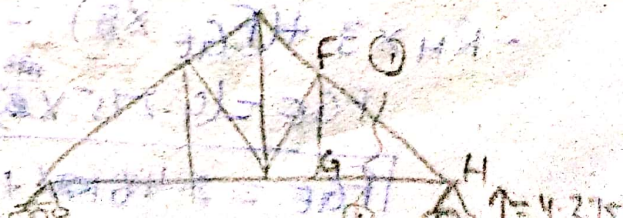
$\therefore R_H = 4.275\,kN$



At section ①-①

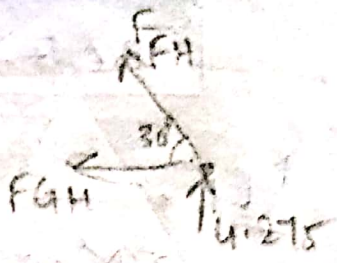
$\Sigma F_V = 0$

$+R_H + F_{HG} \sin 30^\circ = 0$



$$\frac{4.275}{\sin 30^\circ} = -F_{HF}$$

$$\boxed{F_{HF} = -8.55\,kN}$$



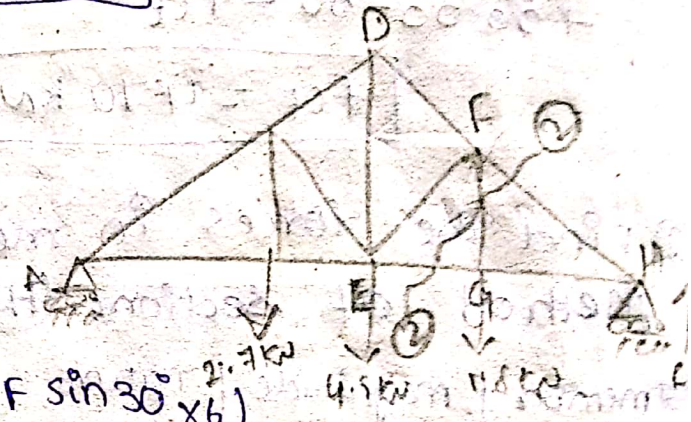
$$\Sigma F_H = 0$$

$$-F_{GH} - F_{FH}\cos 30^\circ = 0$$

$$F_{GH} = -(-8.55 \times \cos 30^\circ)$$

$$\boxed{F_{GH} = 7.4\,kN}$$

At section ②-②



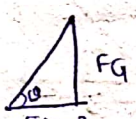
~~$\Sigma F_V = 0$~~

$$\Sigma M_E = 0$$

$$-(R_H \times 6) + (1.8 \times 3) - (F_{HF}\sin 30^\circ \times 6) - (F_{GF} \times 3) = 0$$

where, length of GF.

from Δle EFG

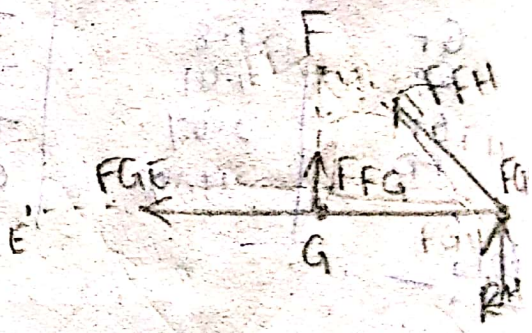


$$\tan\theta = \frac{FG}{EG}$$

$$\tan 30^\circ = \frac{FG}{3}$$

$$\boxed{FG = 1.73\,m}$$

$$\boxed{F_{GF} = 1.84\,kN}$$



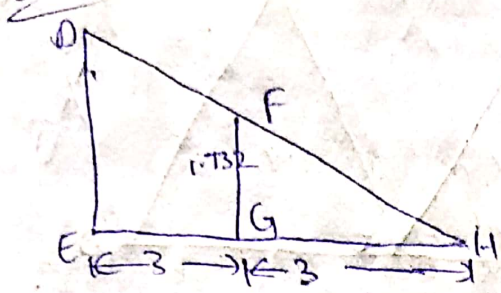
$$\Sigma M_F = 0$$

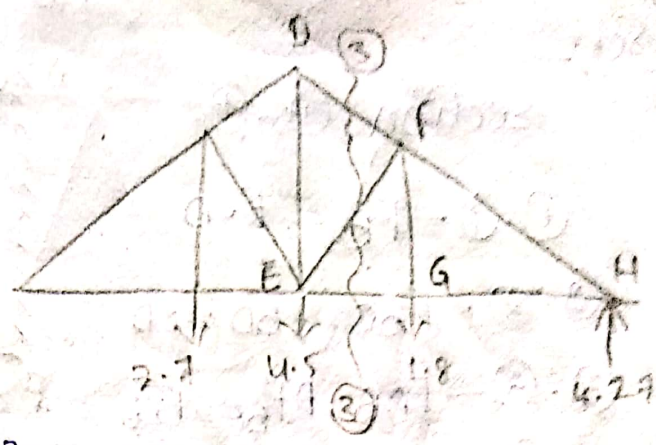
$$-R_H \times 3 + (F_{GE} \times 1.73) = 0$$

$$F_{GE} = (4.275 \times 3)/1.73$$

$$\boxed{F_{GE} = 7.41\,kN}$$

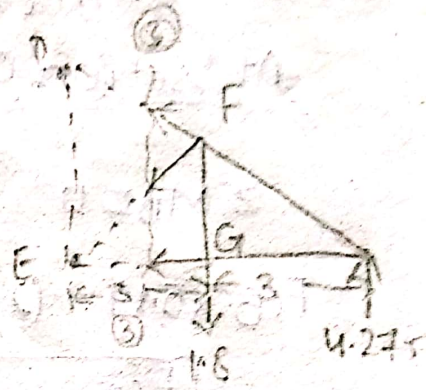
At section ③-③





$$\frac{DE}{EH} = \frac{FG}{GH} \Rightarrow \frac{DE}{6} = \frac{1.73}{3}$$

$$DE = 3.46m$$



$\Sigma M_D = 0$

$$-(RH \times 6) + (1.8 \times 3) - F_{EF} \sin 60^\circ \times 3 + F_{GE} \times 3.46$$

(1 of DE)

$$F_{EF} = 2.07 KN$$

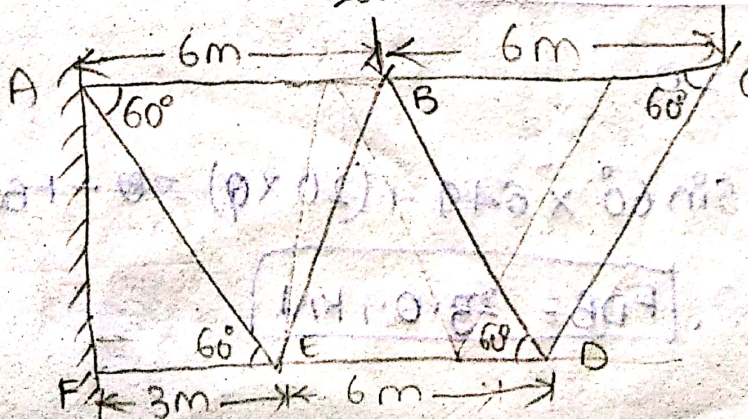
$\Sigma M_E = 0$

$$-RH \times 6 + (1.8 \times 3) - F_{FD} \cos 60^\circ \times 3.46 = 0$$

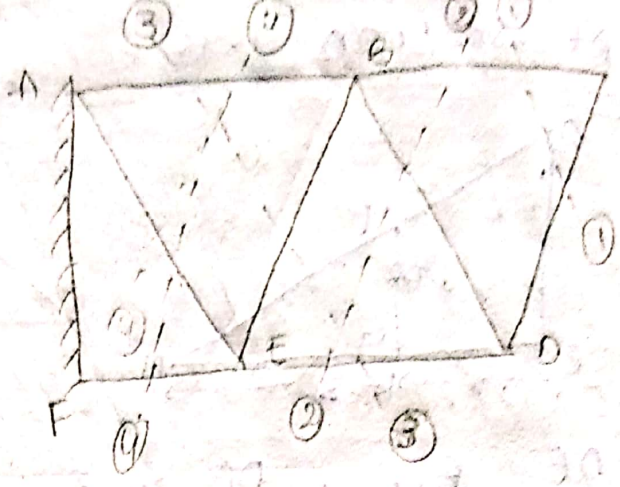
$$F_{FD} = -11.70 KN$$

① The truss is cantilever beam

Find the forces in members BC, CD, BD, BE, BA and EA, ED.



01:-



Section ①-①

①-① - F_{BC}, F_{CD}

②-② - F_{BC}, F_{BD}, F_{ED}

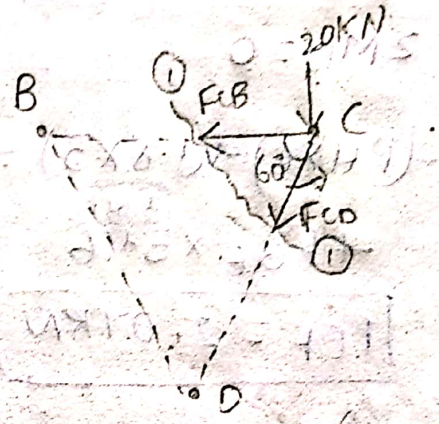
③-③ - F_{AB}, F_{BE}, F_{DE}

④-④ - F_{AB}, F_{AE}, F_{FE}

From Δle AEF

$\tan 60^\circ = \frac{AF}{EF} \Rightarrow AF = 5.19$ m

At Section ①-①



$\Sigma M_D = 0$

$-F_{CB} \times 5.19 + (20 \times 3) = 0$

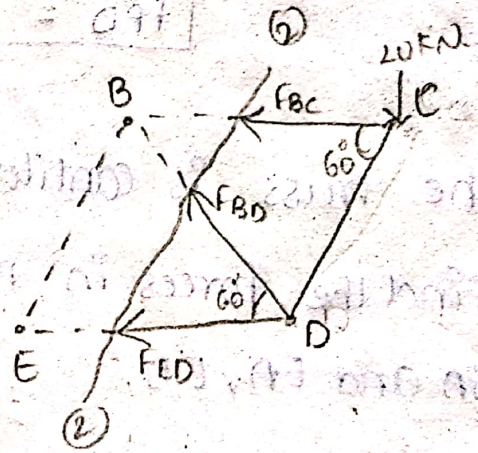
$\therefore \boxed{F_{CB} = 11.56\,KN}$

$\Sigma M_B = 0$

$F_{CD} \sin 60^\circ \times 6 + (20 \times 6) = 0$

$\therefore \boxed{F_{CD} = -23.09\,KN}$

At Section ②-②:-



$\Sigma M_B = 0$

$F_{ED} \times 5.19 + (20 \times 6) = 0$

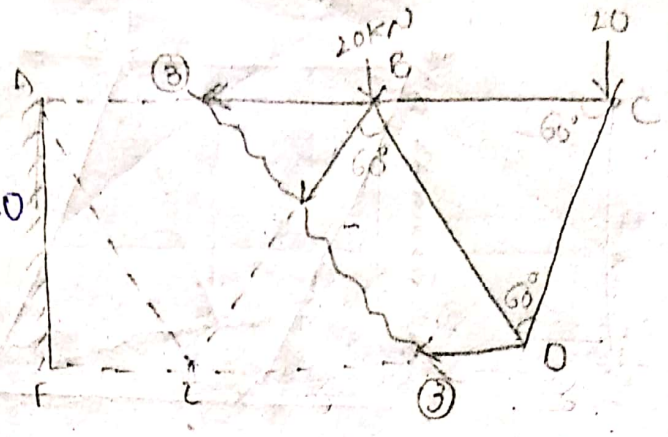
$\therefore \boxed{F_{DE} = -23.12\,KN}$

$\Sigma M_E = 0$

$-F_{BD} \times \sin 60^\circ \times 6.19 + (20 \times 6) - F_{BC} \times 5.19 = 0$

$\therefore \boxed{F_{DB} = 23.09\,KN}$

At section 3-3:-



$\Sigma M_E = 0$

$+(20\times3)$

$-F_{BA}\times5.19+(20\times6)=0$

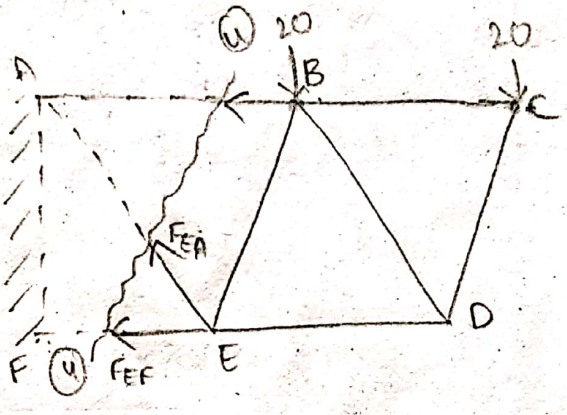
$\therefore F_{BA} = 46.24\,KN$

$\Sigma M_A = 0$

$(+20\times12)+(F_{BE}\times\sin60^\circ\times6)+(F_{DE}\times9)+(20\times6)=0$

$F_{BE} = -49.06\,KN$

At section (4)-(4)



$\Sigma M_F = 0$

$-F_{EA}\sin60^\circ\times3+(20\times12)$

$+(20\times6)-(F_{BA}\times5.19)=0$

$F_{AE} = 46.19\,KN$

$\Sigma M_A = 0$

$-F_{EF}\times5.19-(20\times12)-(20\times6)=0$

$\therefore F_{EF} = -69.36\,KN$

Member	Magnitude	nature
AB	46.24	T
BC	11.56	T
CD	23.09	C
DE	23.12	C
EF	69.36	C
AE	46.19	T
BE	49.06	C
BD	23.09	T