Instructional Objectives

After reading this chapter the student will be able to

1. Differentiate between various structural forms such as beams, plane truss, space truss, plane frame, space frame, arches, cables, plates and shells.
2. State and use conditions of static equilibrium.
3. Calculate the degree of static and kinematic indeterminacy of a given structure such as beams, truss and frames.
4. Differentiate between stable and unstable structure.
5. Define flexibility and stiffness coefficients.
6. Write force-displacement relations for simple structure.

1.1 Introduction

Structural analysis and design is a very old art and is known to human beings since early civilizations. The Pyramids constructed by Egyptians around 2000 B.C. stands today as the testimony to the skills of master builders of that civilization. Many early civilizations produced great builders, skilled craftsmen who constructed magnificent buildings such as the Parthenon at Athens (2500 years old), the great Stupa at Sanchi (2000 years old), Taj Mahal (350 years old), Eiffel Tower (120 years old) and many more buildings around the world. These monuments tell us about the great feats accomplished by these craftsmen in analysis, design and construction of large structures. Today we see around us countless houses, bridges, fly-overs, high-rise buildings and spacious shopping malls. Planning, analysis and construction of these buildings is a science by itself. The main purpose of any structure is to support the loads coming on it by properly transferring them to the foundation. Even animals and trees could be treated as structures. Indeed biomechanics is a branch of mechanics, which concerns with the working of skeleton and muscular structures. In the early periods houses were constructed along the riverbanks using the locally available material. They were designed to withstand rain and moderate wind. Today structures are designed to withstand earthquakes, tsunamis, cyclones and blast loadings. Aircraft structures are designed for more complex aerodynamic loadings. These have been made possible with the advances in structural engineering and a revolution in electronic computation in the past 50 years. The construction material industry has also undergone a revolution in the last four decades resulting in new materials having more strength and stiffness than the traditional construction material.

In this book we are mainly concerned with the analysis of framed structures (*beam, plane truss, space truss, plane frame, space frame and grid*), arches, cables and suspension bridges subjected to static loads only. The methods that we would be presenting in this course for analysis of structure were developed based on certain energy principles, which would be discussed in the first module.

1.2 Classification of Structures

All structural forms used for load transfer from one point to another are 3-dimensional in nature. In principle one could model them as 3-dimensional elastic structure and obtain solutions (response of structures to loads) by solving the associated partial differential equations. In due course of time, you will appreciate the difficulty associated with the 3-dimensional analysis. Also, in many of the structures, one or two dimensions are smaller than other dimensions. This geometrical feature can be exploited from the analysis point of view. The dimensional reduction will greatly reduce the complexity of associated governing equations from 3 to 2 or even to one dimension. This is indeed at a cost. This reduction is achieved by making certain assumptions (like Bernoulli-Euler' kinematic assumption in the case of beam theory) based on its observed behaviour under loads. Structures may be classified as 3-, 2- and 1-dimensional (see Fig. 1.1(a) and (b)). This simplification will yield results of reasonable and acceptable accuracy. Most commonly used structural forms for load transfer are: beams, plane truss, space truss, plane frame, space frame, arches, cables, plates and shells. Each one of these structural arrangement supports load in a specific way.

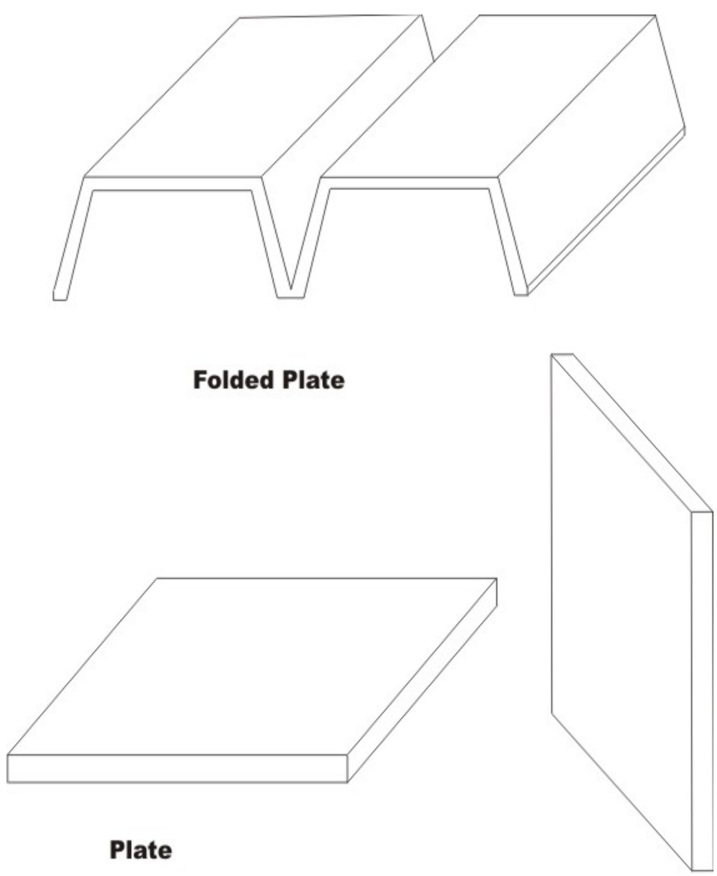


Fig 1.1(a) 2D and 3D Structural Forms

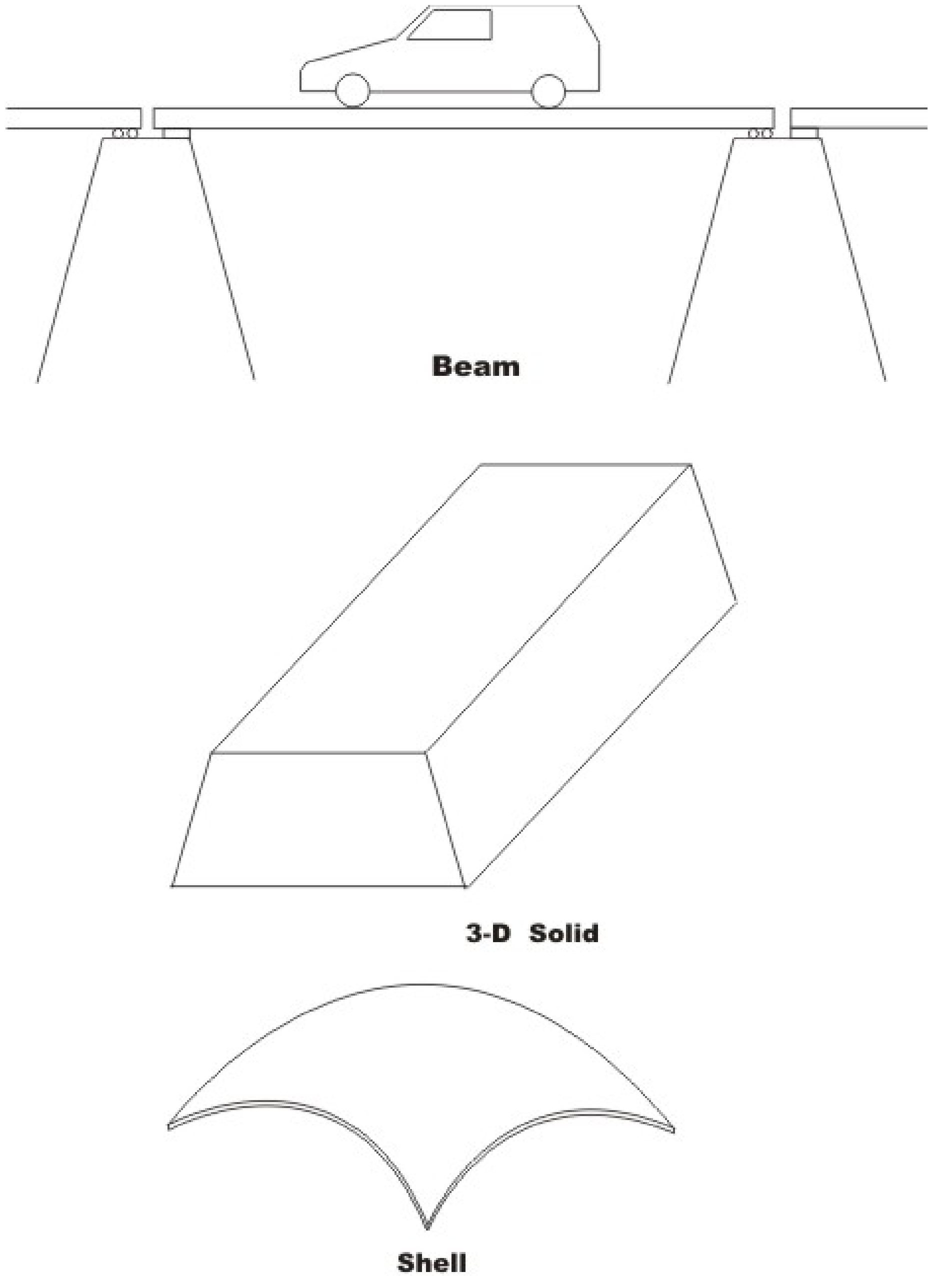


Fig 1.1(b) Commonly Used Structural Forms

Beams are the simplest structural elements that are used extensively to support loads. They may be straight or curved ones. For example, the one shown in Fig. 1.2 (a) is hinged at the left support and is supported on roller at the right end. Usually, the loads are assumed to act on the beam in a plane containing the axis of symmetry of the cross section and the beam axis. The beams may be supported on two or more supports as shown in Fig. 1.2(b). The beams may be curved in plan as shown in Fig. 1.2(c). Beams carry loads by deflecting in the same plane and it does not twist. It is possible for the beam to have no axis of symmetry. In such cases, one needs to consider unsymmetrical bending of beams. In general, the internal stresses at any cross section of the beam are: bending moment, shear force and axial force.

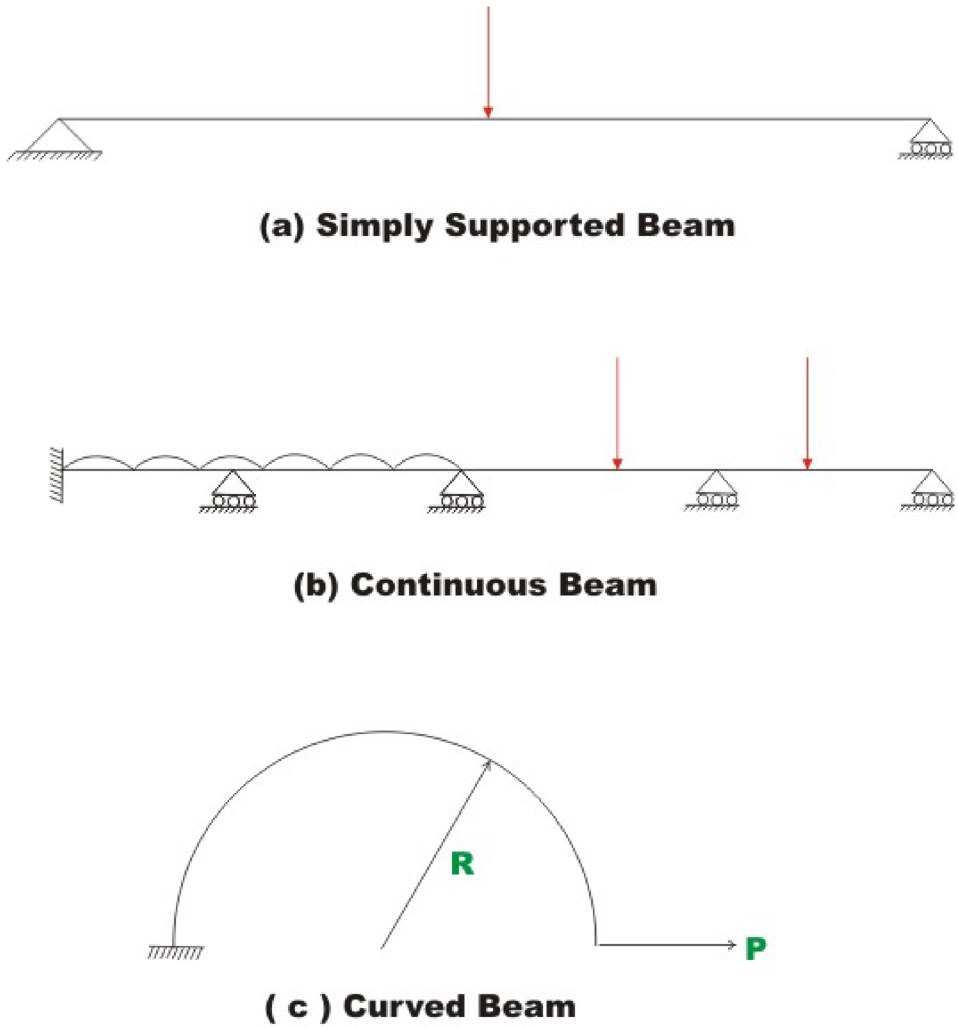
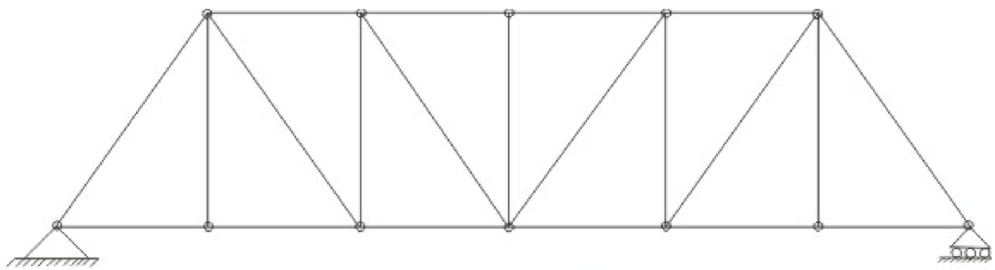
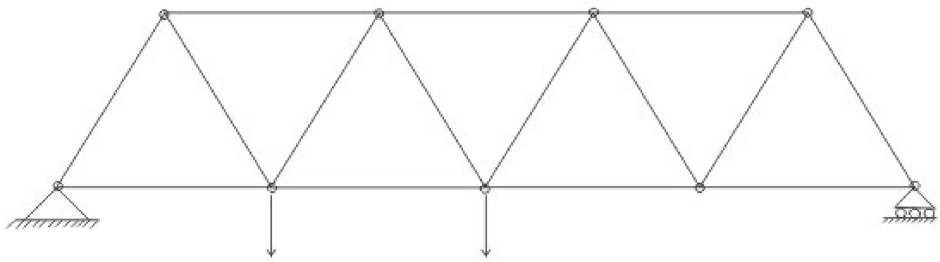


Fig 1.2 Beams

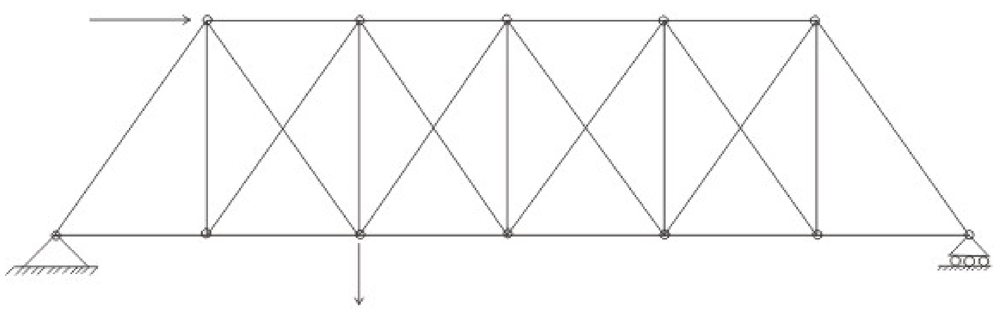
In India, one could see ***plane trusses*** (vide Fig. 1.3 (a),(b),(c)) commonly in Railway bridges, at railway stations, and factories. Plane trusses are made of short thin members interconnected at hinges into triangulated patterns. For the purpose of analysis statically equivalent loads are applied at joints. From the above definition of truss, it is clear that the members are subjected to only axial forces and they are constant along their length. Also, the truss can have only hinged and roller supports. In field, usually joints are constructed as rigid by welding. However, analyses were carried out as though they were pinned. This is justified as the bending moments introduced due to joint rigidity in trusses are negligible. Truss joint could move either horizontally or vertically or combination of them. In ***space truss*** (Fig. 1.3 (d)), members may be oriented in any direction. However, members are subjected to only tensile or compressive stresses. Crane is an example of space truss.



(a) Pratt Truss

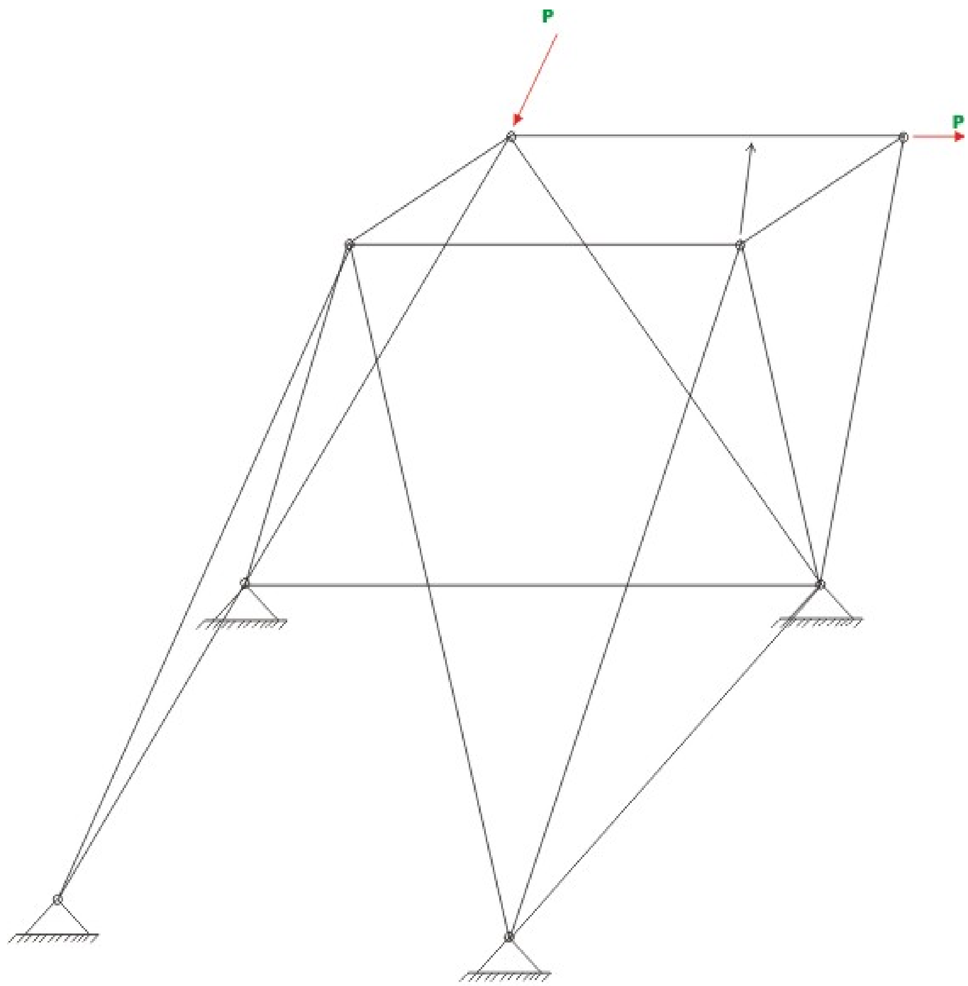


(b) Warren Truss



(c) Double Warren Truss

Fig 1.3 Trusses



(d) Space Truss

Plane frames are also made up of beams and columns, the only difference being they are rigidly connected at the joints as shown in the Fig. 1.4 (a). Major portion of this course is devoted to evaluation of forces in frames for variety of loading conditions. Internal forces at any cross section of the plane frame member are: bending moment, shear force and axial force. As against plane frame, ***space frames*** (vide Fig. 1.4 (b)) members may be oriented in any direction. In this case, there is no restriction of how loads are applied on the space frame.

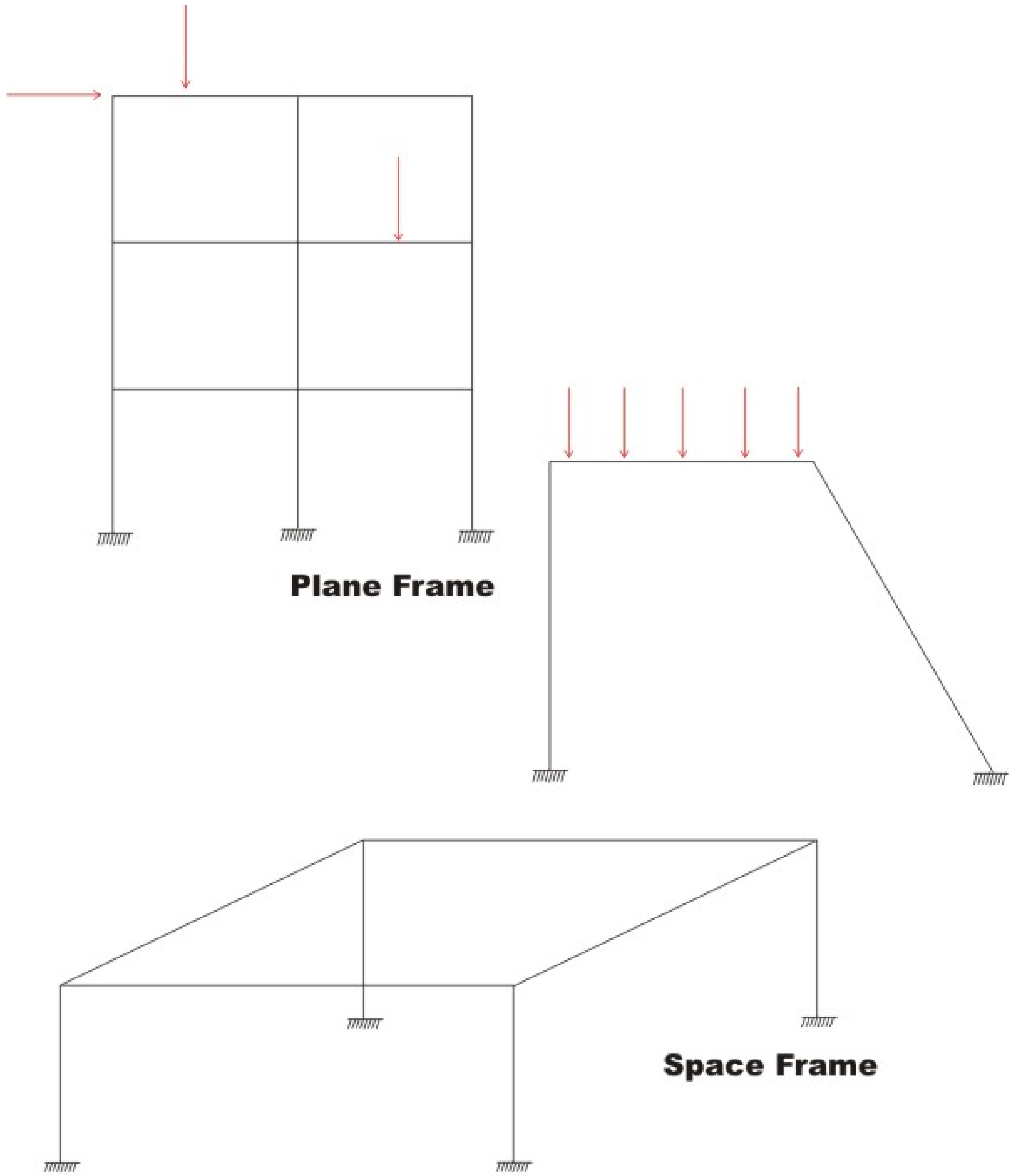


Fig 1.4 Frames

1.3 Equations of Static Equilibrium

Consider a case where a book is lying on a frictionless table surface. Now, if we apply a force F_1 horizontally as shown in the Fig.1.5 (a), then it starts moving in the direction of the force. However, if we apply the force perpendicular to the book as in Fig. 1.5 (b), then book stays in the same position, as in this case the vector sum of all the forces acting on the book is zero. When does an object move and when does it not? This question was answered by Newton when he formulated his famous second law of motion. In a simple vector equation it may be stated as follows:

$$\sum_{i=1}^{n} F_i = ma \qquad (1.1)$$

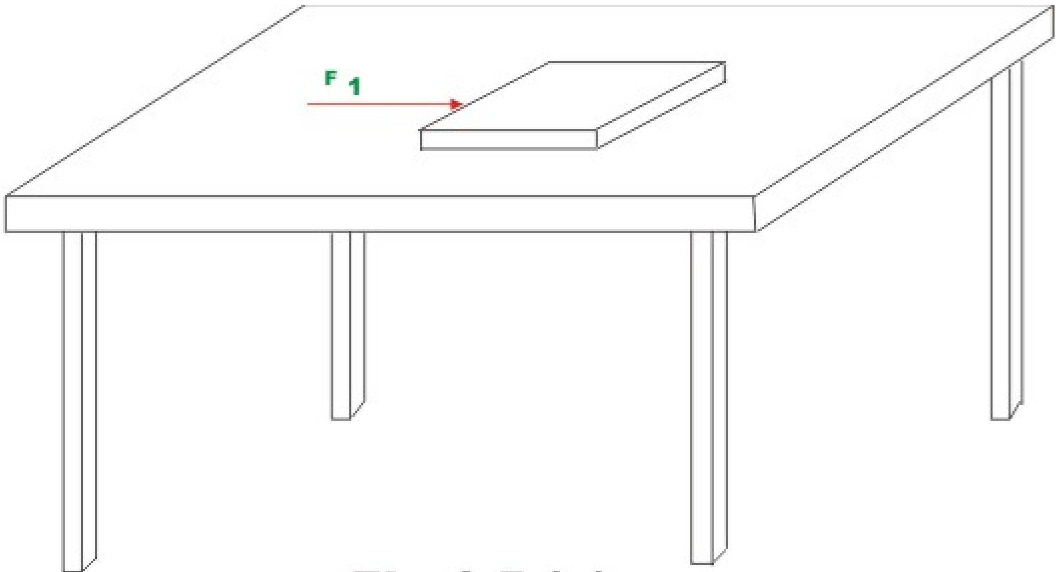


Fig 1.5 (a)

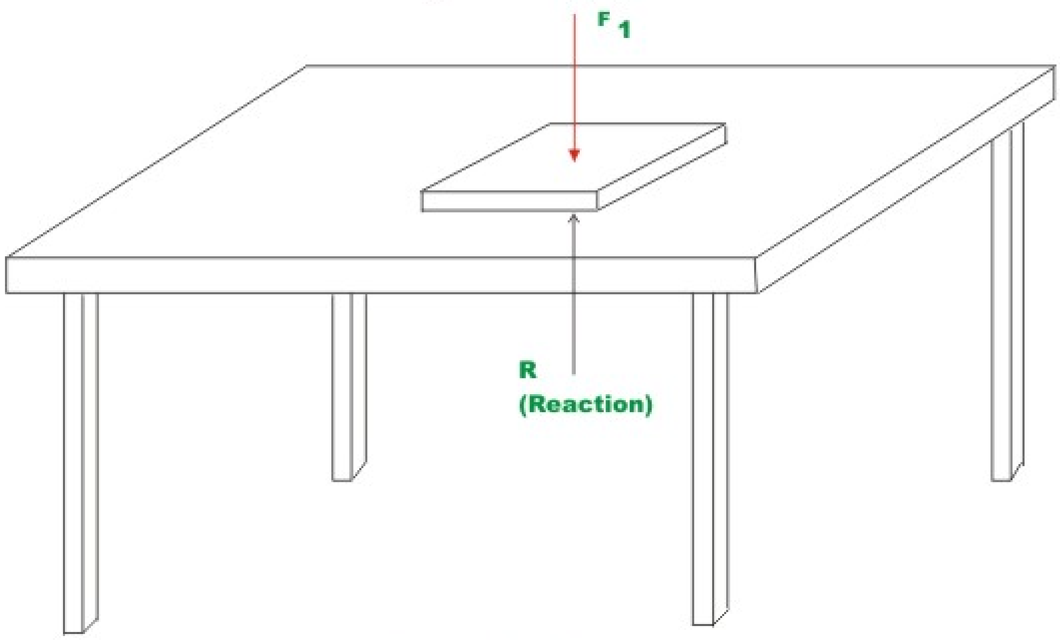


Fig 1.5(b)

where $\sum_{i=1}^{n} F_i$ is the vector sum of all the external forces acting on the body, m is the total mass of the body and a is the acceleration vector. However, if the body is in the state of static equilibrium then the right hand of equation (1.1) must be zero. Also for a body to be in equilibrium, the vector sum of all external moments ($\sum M = 0$) about an axis through any point within the body must also vanish. Hence, the book lying on the table subjected to external force as shown in Fig. 1.5 (b) is in static equilibrium. The equations of equilibrium are the direct consequences of Newton's second law of motion. A vector in 3-dimensions can be resolved into three orthogonal directions viz., *x, y* and *z* (Cartesian) co-ordinate axes. Also, if the resultant force vector is zero then its components in three mutually perpendicular directions also vanish. Hence, the above two equations may also be written in three co-ordinate axes directions as follows:

$$\sum F_x = 0\,;\ \sum F_y = 0\,;\ \sum F_z = 0 \tag{1.2a}$$

$$\sum M_x = 0\,;\sum M_y = 0\,;\sum M_z = 0 \tag{1.2b}$$

Now, consider planar structures lying in $xy -$plane. For such structures we could have forces acting only in x and y directions. Also the only external moment that could act on the structure would be the one about the z-axis. For planar structures, the resultant of all forces may be a force, a couple or both. The static equilibrium condition along x-direction requires that there is no net unbalanced force acting along that direction. For such structures we could express equilibrium equations as follows:

$$\sum F_x = 0\,;\sum F_y = 0\,;\sum M_z = 0 \tag{1.3}$$

Using the above three equations we could find out the reactions at the supports in the beam shown in Fig. 1.6. After evaluating reactions, one could evaluate internal stress resultants in the beam. Admissible or correct solution for reaction and internal stresses must satisfy the equations of static equilibrium for the entire structure. They must also satisfy equilibrium equations for any part of the structure taken as a free body. If the number of unknown reactions is more than the number of equilibrium equations (as in the case of the beam shown in Fig. 1.7), then we can not evaluate reactions with only equilibrium equations. Such structures are known as the statically indeterminate structures. In such cases we need to obtain extra equations (*compatibility equations*) in addition to equilibrium equations.

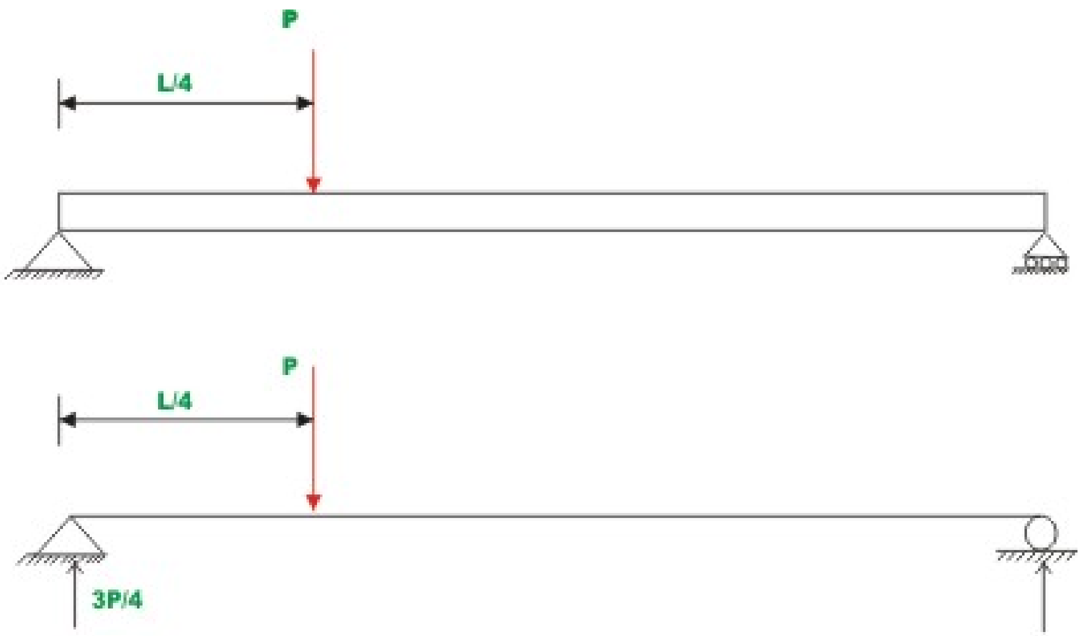


Fig 1.6 Statically Determinate Beam

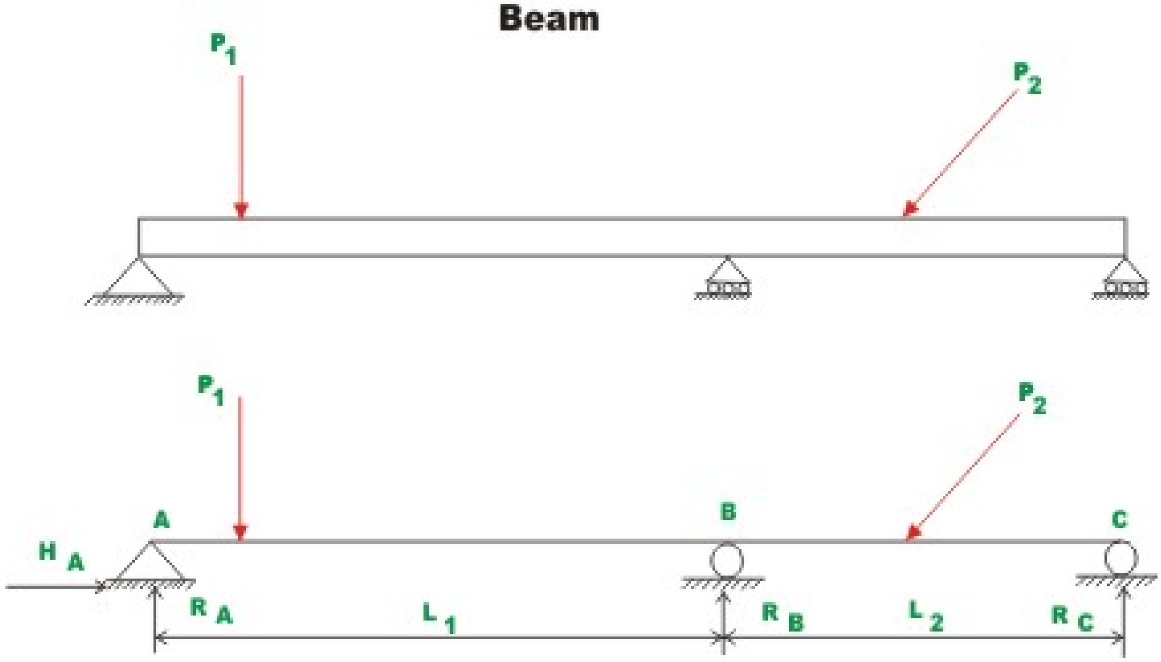


Fig 1.7 Statically Indeterminate Beam

1.4 Static Indeterminacy

The aim of structural analysis is to evaluate the external reactions, the deformed shape and internal stresses in the structure. If this can be accomplished by equations of equilibrium, then such structures are known as determinate structures. However, in many structures it is not possible to determine either reactions or internal stresses or both using equilibrium equations alone. Such structures are known as the statically indeterminate structures. The indeterminacy in a structure may be external, internal or both. A structure is said to be externally indeterminate if the number of reactions exceeds the number of equilibrium equations. Beams shown in Fig.1.8(a) and (b) have four reaction components, whereas we have only 3 equations of equilibrium. Hence the beams in Figs. 1.8(a) and (b) are externally indeterminate to the first degree. Similarly, the beam and frame shown in Figs. 1.8(c) and (d) are externally indeterminate to the 3^{rd} degree.

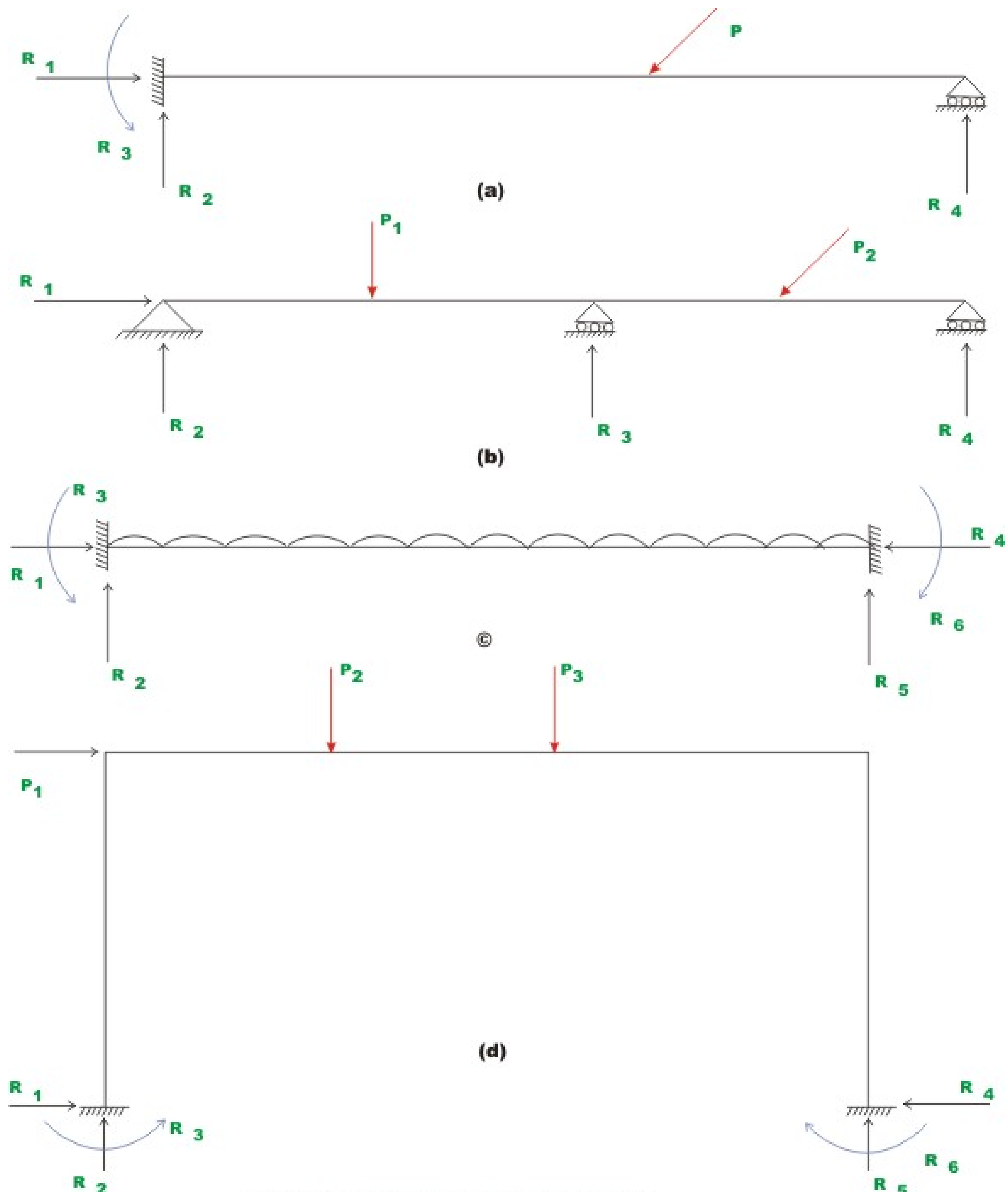
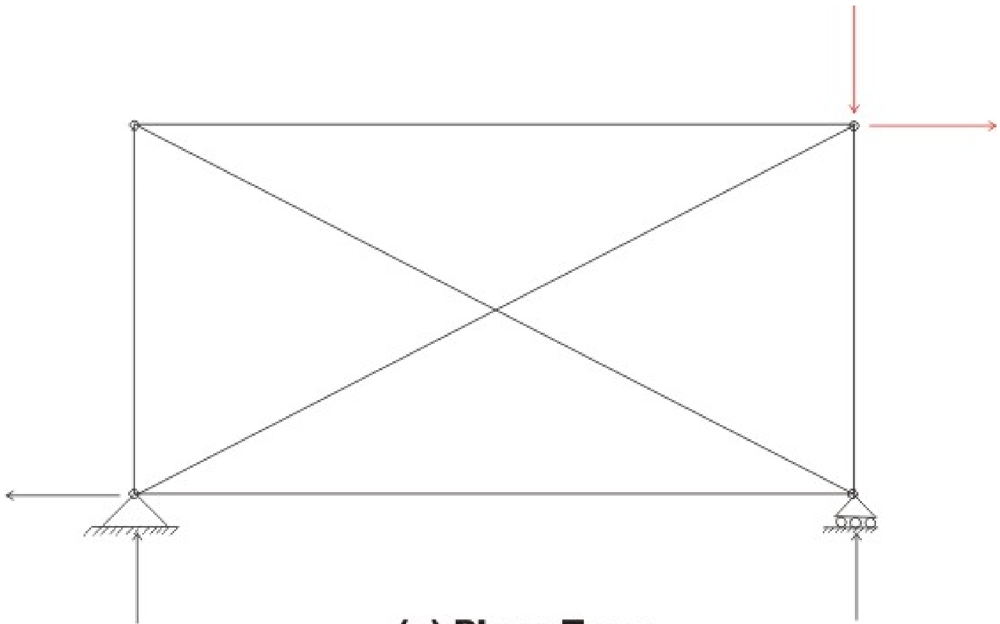
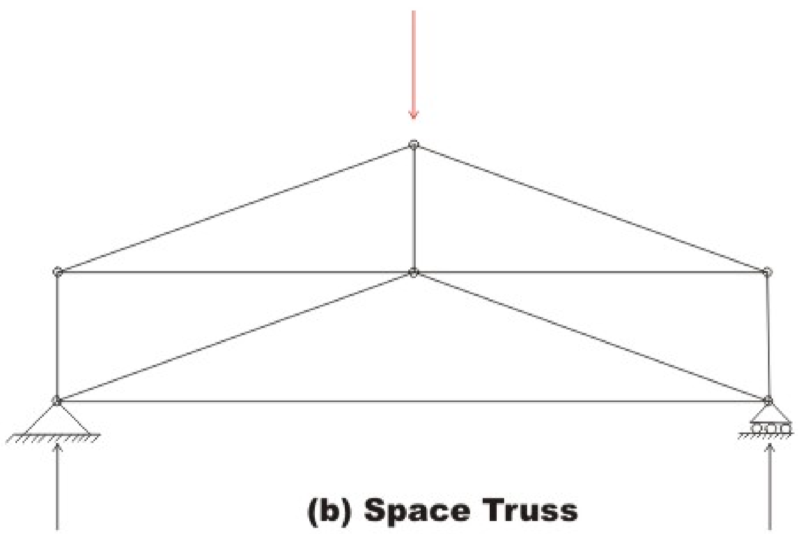


Fig 1.8 Externally Statically Indeterminate Structures

Now, consider trusses shown in Figs. 1.9(a) and (b). In these structures, reactions could be evaluated based on the equations of equilibrium. However, member forces can not be determined based on statics alone. In Fig. 1.9(a), if one of the diagonal members is removed (cut) from the structure then the forces in the members can be calculated based on equations of equilibrium. Thus, structures shown in Figs. 1.9(a) and (b) are internally indeterminate to first degree.The truss and frame shown in Fig. 1.10(a) and (b) are both externally and internally indeterminate.

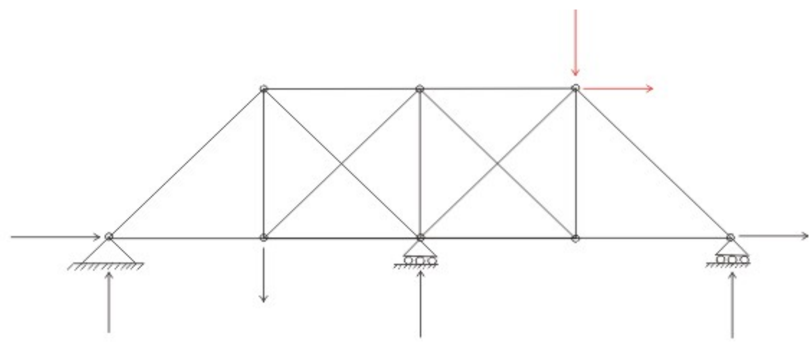


(a) Plane Truss

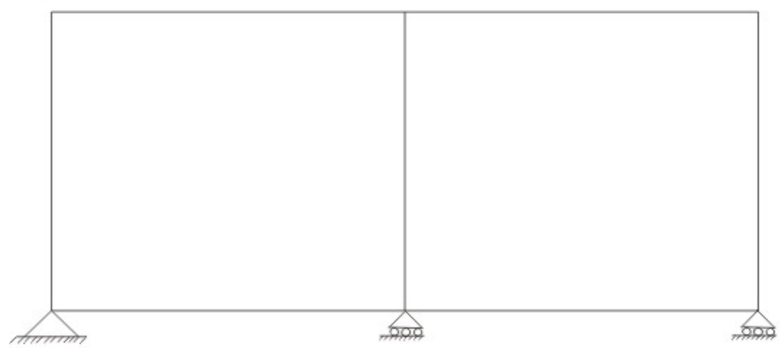


(b) Space Truss

Fig 1.9 Internally Statically Indeterminate Structures



(a) Plane Truss



(b) Plane Frame

Fig 1.10 Externally and Internally Indeterminate Structures

So far, we have determined the degree of indeterminacy by inspection. Such an approach runs into difficulty when the number of members in a structure increases. Hence, let us derive an algebraic expression for calculating degree of static indeterminacy.

Consider a planar stable truss structure having m members and j joints. Let the number of unknown reaction components in the structure be r. Now, the total number of unknowns in the structure is $m+r$. At each joint we could write two equilibrium equations for planar truss structure, viz., $\sum F_x = 0$ and $\sum F_y = 0$. Hence total number of equations that could be written is $2j$.

If $2j = m + r$ then the structure is statically determinate as the number of unknowns are equal to the number of equations available to calculate them.

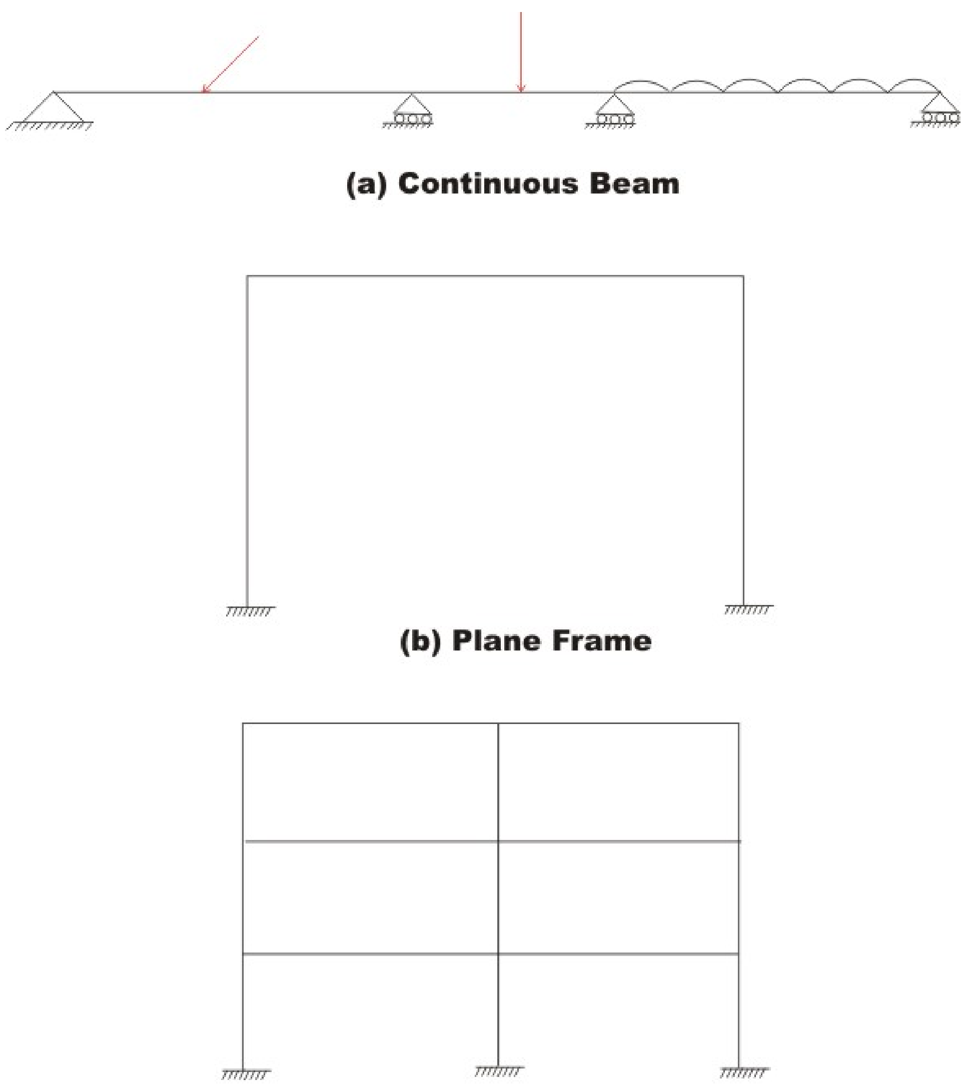
The degree of indeterminacy may be calculated as

$$i = (m+r) - 2j \tag{1.4}$$

We could write similar expressions for space truss, plane frame, space frame and grillage. For example, the plane frame shown in Fig.1.11 (c) has 15 members, 12 joints and 9 reaction components. Hence, the degree of indeterminacy of the structure is

$$i = (15 \times 3 + 9) - 12 \times 3 = 18$$

Please note that here, at each joint we could write 3 equations of equilibrium for plane frame.



(a) Continuous Beam

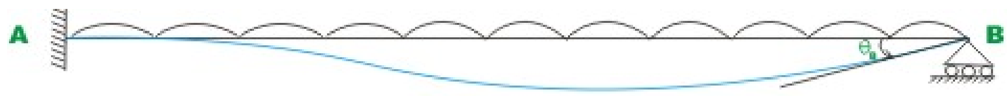
(b) Plane Frame

(c) Plane Frame

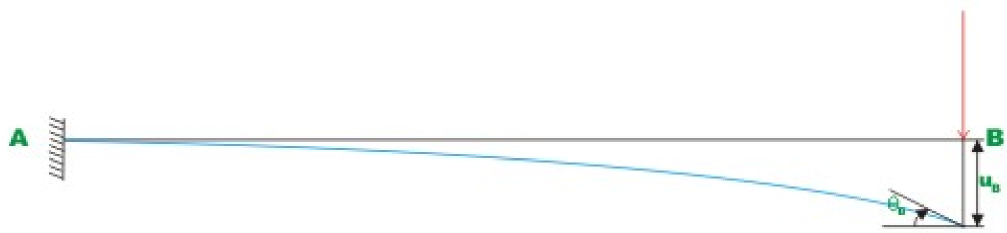
Fig 1.11 Indeterminate Structures

1.5 Kinematic Indeterminacy

When the structure is loaded, the joints undergo displacements in the form of translations and rotations. In the displacement based analysis, these joint displacements are treated as unknown quantities. Consider a propped cantilever beam shown in Fig. 1.12 (a). Usually, the axial rigidity of the beam is so high that the change in its length along axial direction may be neglected. The displacements at a fixed support are zero. Hence, for a propped cantilever beam we have to evaluate only rotation at B and this is known as the kinematic indeterminacy of the structure. A fixed fixed beam is kinematically determinate but statically indeterminate to 3^{rd} degree. A simply supported beam and a cantilever beam are kinematically indeterminate to 2^{nd} degree.



(a) Propped Cantilever Beam



(b) Cantilever Beam

(c) Simply Supported Beam

Fig 1.12 Kinematically Indeterminate Structures

The joint displacements in a structure is treated as independent if each displacement (translation and rotation) can be varied arbitrarily and independently of all other displacements. The number of independent joint displacement in a structure is known as the degree of kinematic indeterminacy or the number of degrees of freedom. In the plane frame shown in Fig. 1.13, the joints B and C have 3 degrees of freedom as shown in the figure. However if axial deformations of the members are neglected then $u_1 = u_4$ and u_2 and u_4 can be neglected. Hence, we have 3 independent joint displacement as shown in Fig. 1.13 i.e. rotations at B and C and one translation.

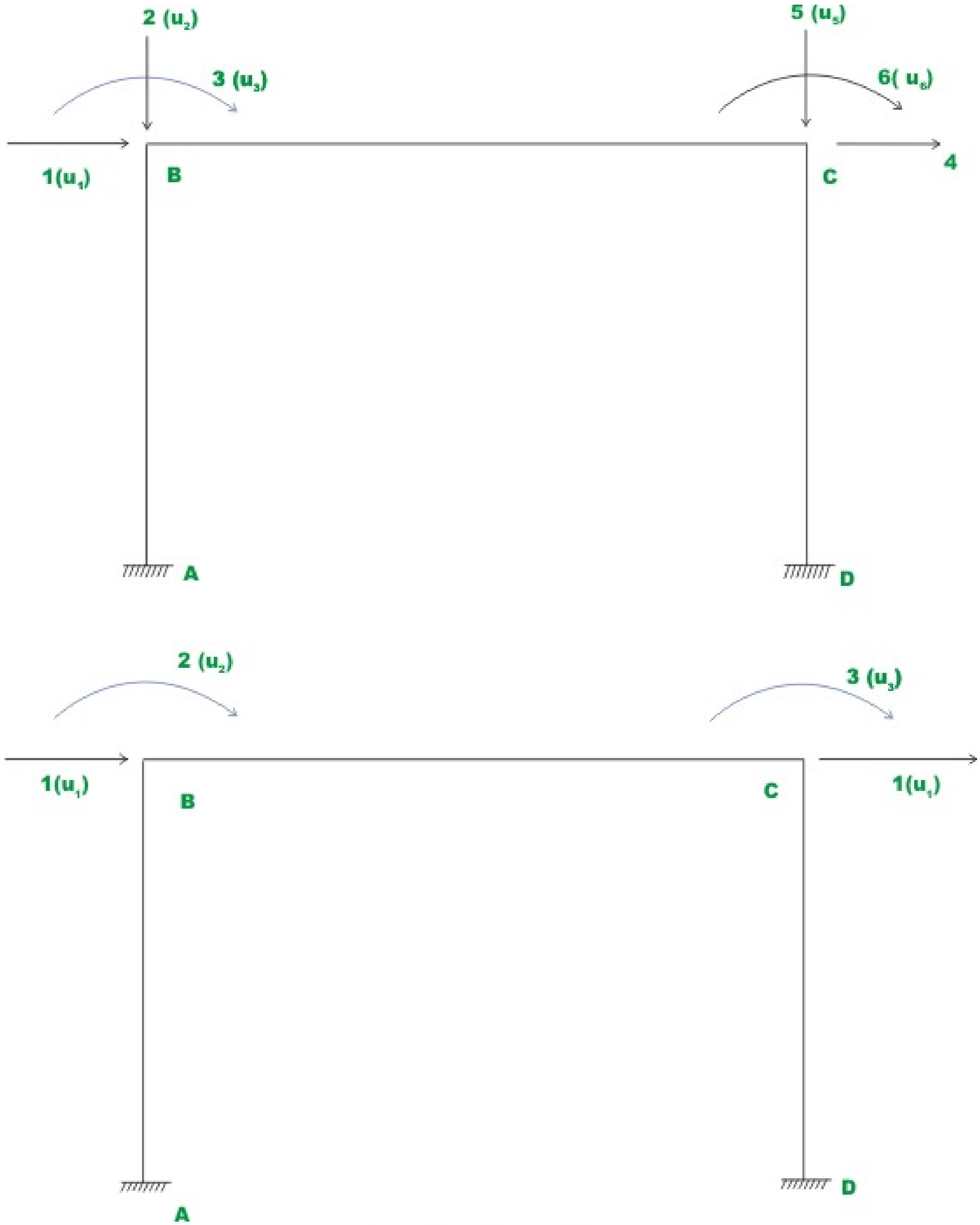


Fig 1.13 Rigid Frame

1.6 Kinematically Unstable Structure

A beam which is supported on roller on both ends (vide. Fig. 1.14) on a horizontal surface can be in the state of static equilibrium only if the resultant of the system of applied loads is a vertical force or a couple. Although this beam is stable under special loading conditions, is unstable under a general type of loading conditions. When a system of forces whose resultant has a component in the horizontal direction is applied on this beam, the structure moves as a rigid body. Such structures are known as kinematically unstable structure. One should avoid such support conditions.

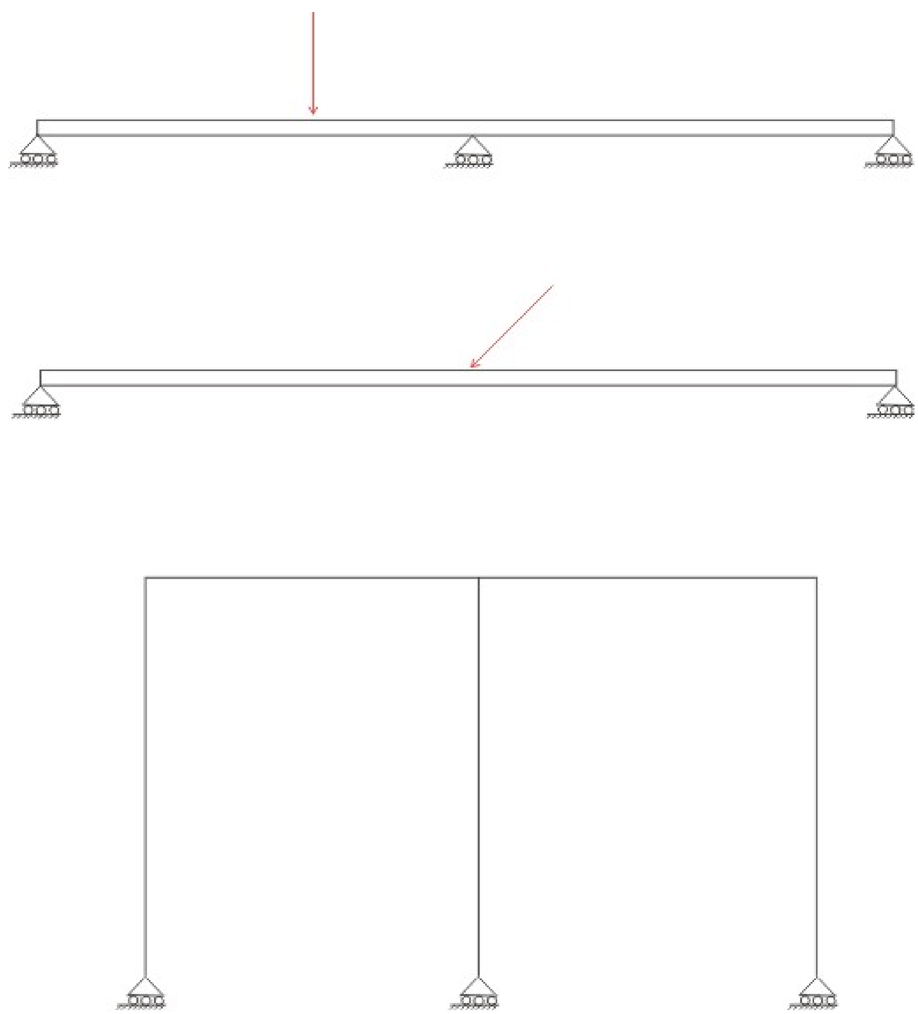


Fig 1.14 Kinematically Unstable Structures

1.7 Compatibility Equations

A structure apart from satisfying equilibrium conditions should also satisfy all the compatibility conditions. These conditions require that the displacements and rotations be continuous throughout the structure and compatible with the nature supports conditions. For example, at a fixed support this requires that displacement and slope should be zero.

1.8 Force-Displacement Relationship

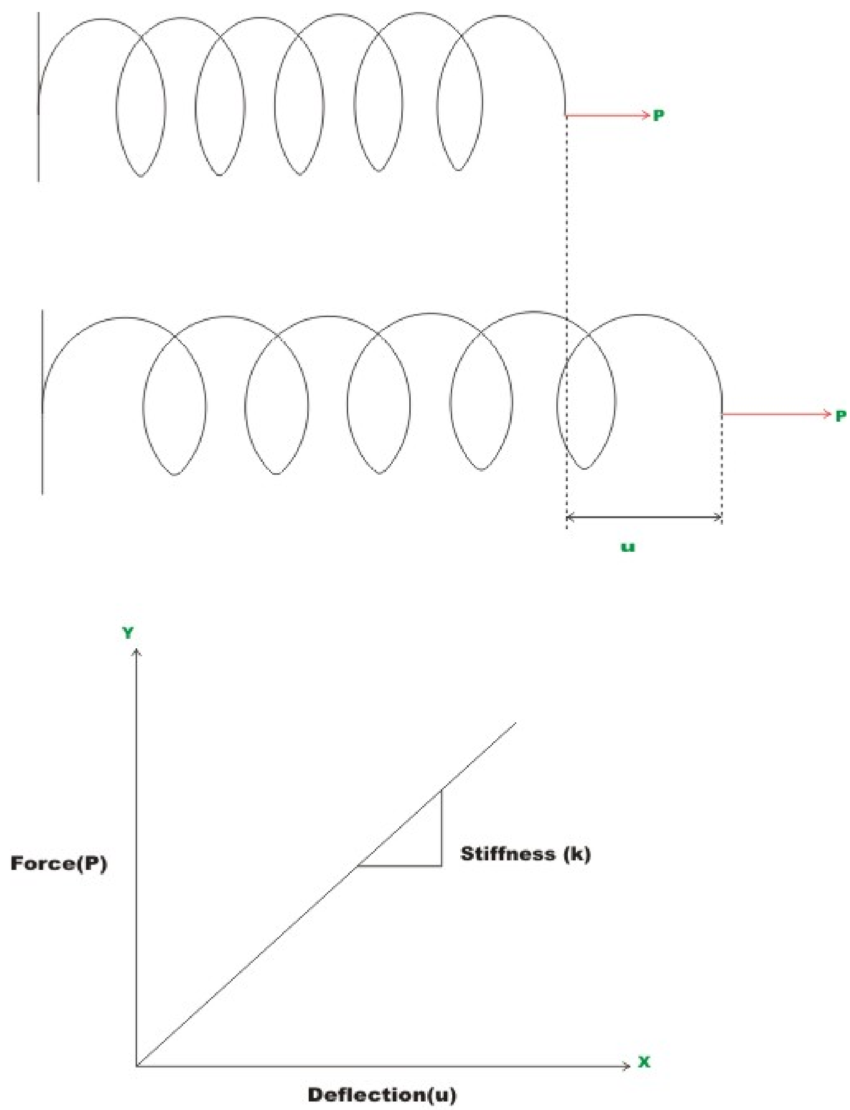


Fig 1.15 Force displacement Relationship

Consider linear elastic spring as shown in Fig.1.15. Let us do a simple experiment. Apply a force P_1 at the end of spring and measure the deformation u_1. Now increase the load to P_2 and measure the deformation u_2. Likewise repeat the experiment for different values of load $P_1, P_2, ..., P_n$. Result may be represented in the form of a graph as shown in the above figure where load is shown on y-axis and deformation on abscissa. The slope of this graph is known as the stiffness of the spring and is represented by k and is given by

$$k = \frac{P_2 - P_1}{u_2 - u_1} = \frac{P}{u} \tag{1.5}$$

$$P = ku \tag{1.6}$$

The spring stiffness may be defined as the force required for the unit deformation of the spring. The stiffness has a unit of force per unit elongation. The inverse of the stiffness is known as flexibility. It is usually denoted by a and it has a unit of displacement per unit force.

$$a = \frac{1}{k} \tag{1.7}$$

the equation (1.6) may be written as

$$P = ku \Rightarrow \qquad u = \frac{1}{k} P = aP \tag{1.8}$$

The above relations discussed for linearly elastic spring will hold good for linearly elastic structures. As an example consider a simply supported beam subjected to a unit concentrated load at the centre. Now the deflection at the centre is given by

$$u = \frac{PL^3}{48EI} \quad \text{or} \quad P = \left(\frac{48EI}{L^3}\right) u \tag{1.9}$$

The stiffness of a structure is defined as the force required for the unit deformation of the structure. Hence, the value of stiffness for the beam is equal to

$$k = \frac{48EI}{L^3}$$

As a second example, consider a cantilever beam subjected to a concentrated load (P) at its tip. Under the action of load, the beam deflects and from first principles the deflection below the load (u) may be calculated as,

$$u = \frac{PL^3}{3EI_{zz}} \tag{1.10}$$

For a given beam of constant cross section, length L, Young's modulus E, and moment of inertia I_{ZZ} the deflection is directly proportional to the applied load. The equation (1.10) may be written as

$$u = a \, P \tag{1.11}$$

Where a is the flexibility coefficient and is $a = \frac{L^3}{3EI_{zz}}$. Usually it is denoted by a_{ij} the flexibility coefficient at i due to unit force applied at j. Hence, the stiffness of the beam is

$$k_{11} = \frac{1}{a_{11}} = \frac{3EI}{L^3} \tag{1.12}$$

S.A-I

Unit-I: propped cantilevers

* $D_s = 1$, if reaction at prop end is treated as a Redundant force, the resulting basic determinate structure is a cantilever, if fixed end moment is the redundant, then the basic determinate structure is a simply supported beam.

procedure:- Step 1:- Remove the prop and determine the deflection of the prop end for given loads.

Step 2:- Apply the redundant and find the deflection of the prop end without considering given loads

Step 3:- use the consistency condition for prop end to find prop reaction

Step 4:- Solve for reactions at the fixed end from equilibrium equations

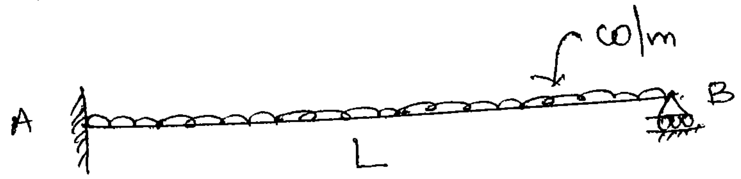
Step 5:- Draw SFD and BMD.

cantilever	propped cantilever
1. Determinate beam	1. Indeterminate beam.
2. Equilibrium equations are sufficient for solving the beam	2. One ~~Additional~~ condition called consistency condn is required for solving
3. One end fix and other end free in a beam	3. One end fix and other end simply supported
4. POCFs are zero	4. POCFs are one

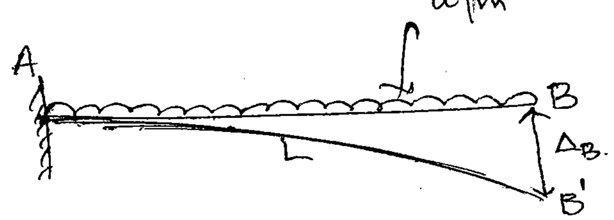
what is a propped cantilever? what is the degree of indeterminacy?

Name a method for deriving the compatibility Equation for the propped cantilever?

prob:- propped cantilever subjected to UDL throughout:-

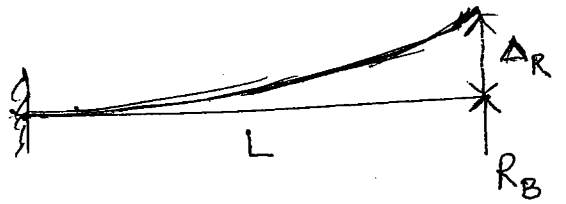


sol:- Removing prop,



$$\Delta_{B,L} = \frac{\omega L^4}{8EI}$$

Applying Redundant R_B,



$$\Delta_{BR} = \frac{R_B L^3}{3EI} \quad (-ve)$$

from consistency condition, $\Delta_B = 0$

$$\therefore \quad \Delta_{B,L} + \Delta_{B,R} = 0$$

$$\therefore \quad \frac{\omega L^4}{8EI} - \frac{R_B L^3}{3EI} = 0 \Rightarrow \boxed{R_B = \frac{3\omega L}{8}}$$

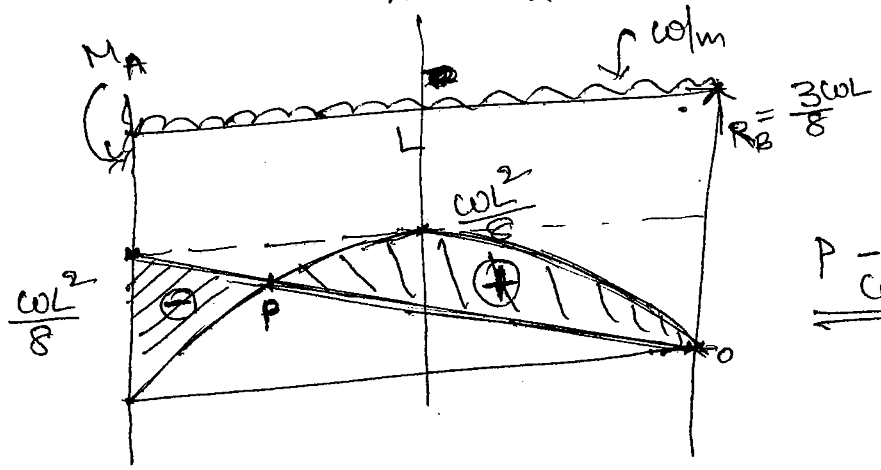
from eq^lm, $\Sigma F_y = 0$ $\quad R_A + R_B = \omega L$

$$R_A = \omega L - \frac{3}{8}\omega L = \boxed{\frac{5\omega L}{8} = R_A}$$

$$\Sigma M_A = 0 \Rightarrow M_A + R_B \times L - \omega L \times \frac{L}{2} = 0$$

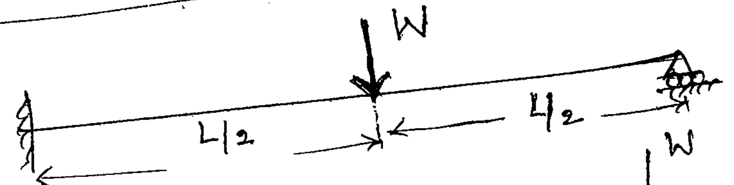
$$M_A = \frac{\omega L^2}{2} - \frac{3\omega L^2}{8} = \boxed{\frac{\omega L^2}{8} = M_A}$$

BMD:-

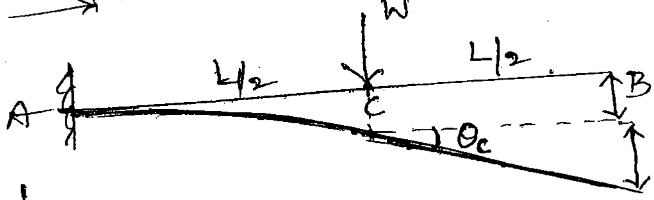


P - point of contra flexure

prob:- Cantilever with prop subjected to point load at centre -II



sol:- Removing prop,

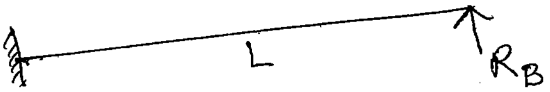


$$\Delta_{B,L} = \Delta_c + \theta_c \times \frac{L}{2}$$

$$= \frac{W(L/2)^3}{3EI} + \frac{W(L/2)^2}{2EI} \times \left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{24EI} + \frac{WL^3}{16EI} = \frac{5WL^3}{48EI}$$

applying Redundant:-



$$\therefore \Delta_{BR} = \frac{R_B L^3}{3EI} \text{ (-ve)}$$

from Consistency:- $\Delta_B = 0 \Rightarrow \Delta_{B,L} + \Delta_{B,R} = 0$

$$\frac{5WL^3}{48EI} - \frac{R_B L^3}{3EI} = 0 \Rightarrow \frac{5WL^3}{\cancel{48}_{16}EI} = \frac{R_B L^3}{\cancel{3}EI}$$

$$\therefore R_B = \frac{5}{16}W = \boxed{\frac{5W}{16} = R_B}$$

from Equilibrium Equations:-

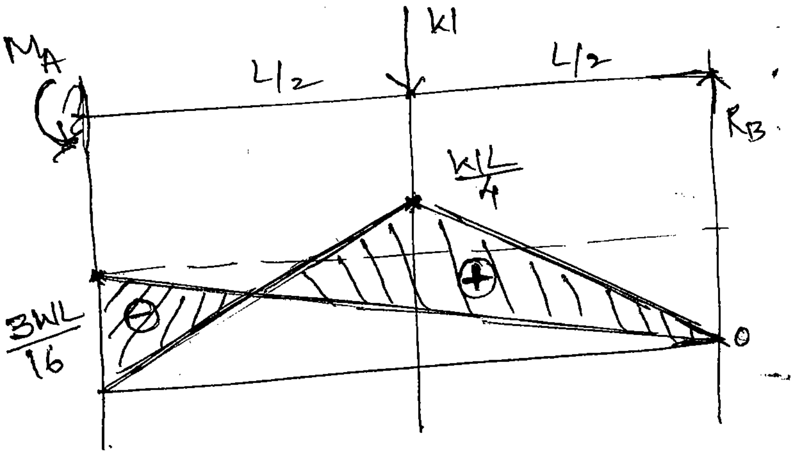
$$R_A + R_B = W \Rightarrow R_A = W - \frac{5W}{16}$$

$$\boxed{R_A = \frac{11W}{16}}$$

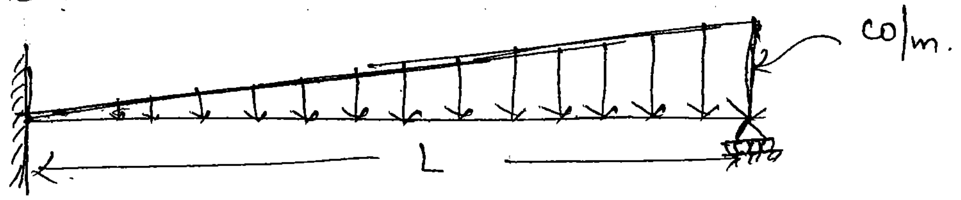
$$\circlearrowleft + \Sigma M_A = 0 \quad M_A + R_B \times L - W \times \frac{L}{2} = 0$$

$$M_A = \frac{WL}{2} - \frac{5WL}{16} = \boxed{\frac{3WL}{16} = M_A}$$

BMD:-

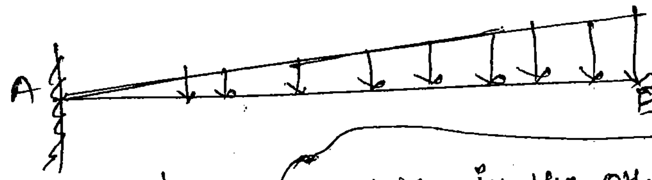


Prob:- Find the Support moment for the propped cantilever carrying UVL ω/unit length from A to B. Draw BMD?



Sol:-

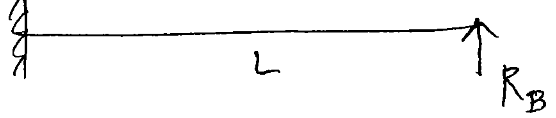
Removing the prop,



$$\Delta_{B,L} = \frac{11\omega L^4}{120EI}.$$

∵ UVL in the otherway $\Delta_B = \frac{\omega L^4}{30EI}$

Applying the Redundant,



$$\Delta_{B,R} = \frac{R_B L^3}{3EI} \text{ (-ve)}$$

$\Delta_{B,L} + \Delta_{BR} = 0$

from consistency condition, $\Delta_B = 0$

$$\therefore \frac{11\omega L^4}{120EI} - \frac{R_B L^3}{3EI} = 0$$

$$\therefore \boxed{\frac{11\omega L}{40} = R_B}$$

$R_A + R_B = \frac{\omega L}{2}$

from Equilibrium Equations,

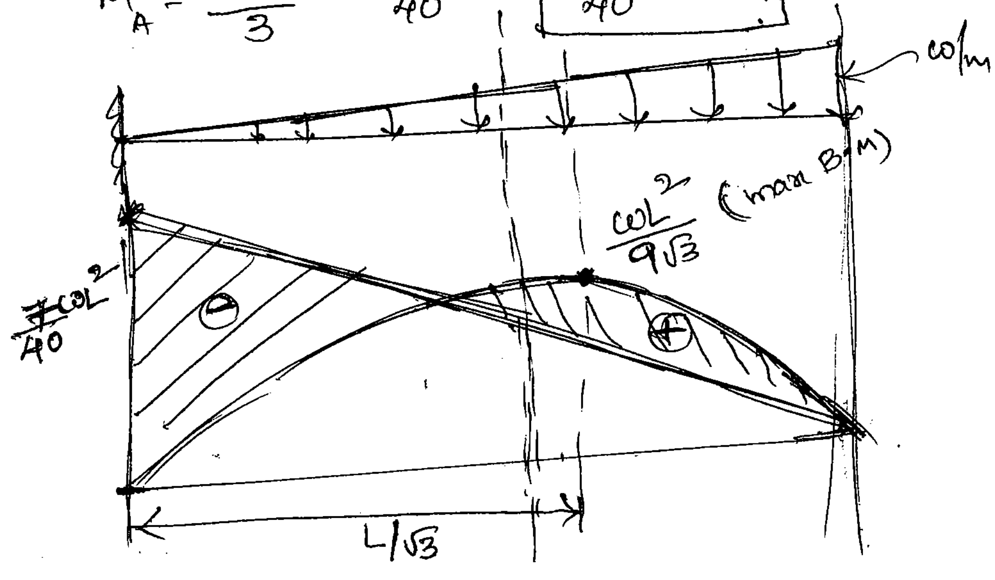
$R_A = \frac{\omega L}{2} - \frac{11\omega L}{40}$

$\boxed{R_A = \frac{9\omega L}{40}}$

$\circlearrowright \Sigma M_A = 0 \quad M_A + R_B \times L - \frac{1}{2} \times L \times \omega \times \frac{2L}{3} = 0$

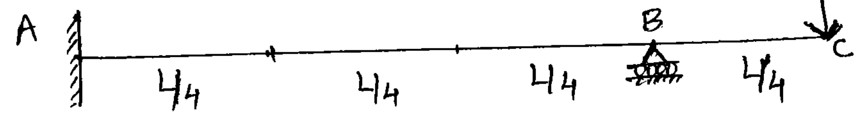
$$M_A = \frac{\omega L^2}{3} - \frac{11\omega L^2}{40} = \boxed{\frac{7\omega L^2}{40} = M_A}$$

BMD:-

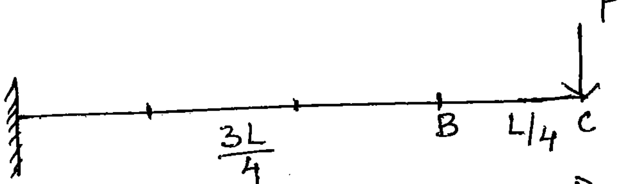


Probl: Draw BMD for a propped cantilever with an overhang at L/4 from free end carrying a point load at free end ?

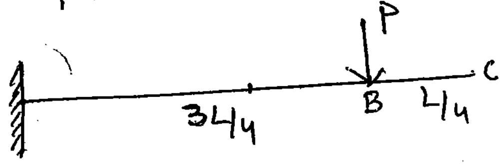
Sol:



Sol:- Removing the prop.



from maxwell Reciprocal theorem,



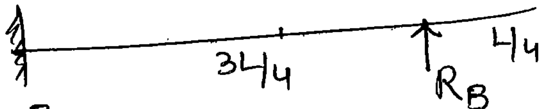
$$\Delta_B = \Delta_C$$

$$\therefore \text{ finding } \Delta_c = \Delta_B + \theta_B \times \frac{L}{4} = \frac{P\left(\frac{3L}{4}\right)^3}{3EI} + \frac{P\left(\frac{3L}{4}\right)^2}{2EI} \times \frac{L}{4}$$

$$= \frac{P\,\cancel{27}^{\,9} L^3}{64 \times \cancel{3}EI} + \frac{P\,9L^2}{16 \times 2 \times EI} \times \frac{L}{4}$$

$$= \frac{9PL^3}{64EI}\left(1+\frac{1}{2}\right) = \frac{9PL^3}{64EI} \times \frac{3}{2} = \frac{27PL^3}{128\,EI} = \Delta_{B,L}$$

Applying Redundant,



$$\Delta_{B,R} = \frac{R_B \times \left(\frac{3L}{4}\right)^3}{3EI} = R_B \times \frac{9L^3}{64EI} \quad (-ve)$$

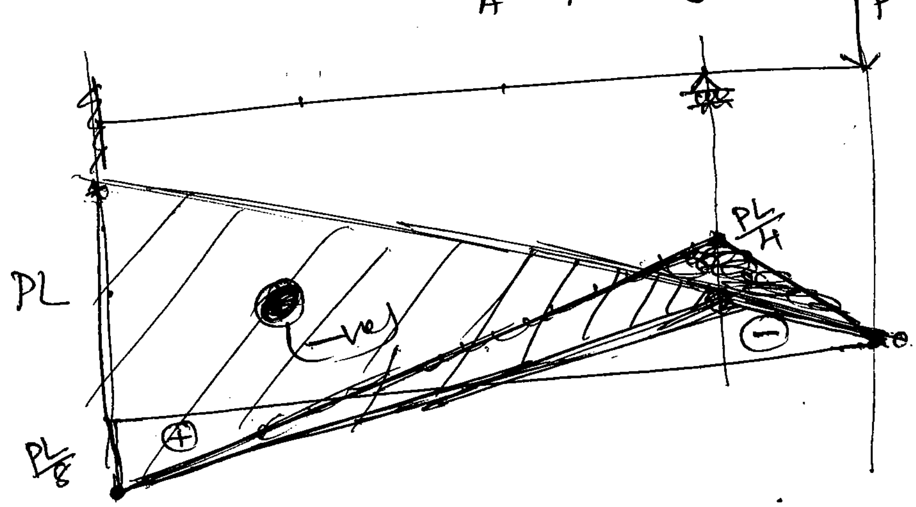
from consistency, $\Delta_B = 0 \Rightarrow \frac{\cancel{27}^{\,3} PL^{\cancel{3}}}{\cancel{128}_{\,2} \cancel{EI}} = R_B \times \frac{\cancel{9}L^{\cancel{3}}}{\cancel{64}\cancel{EI}}$

$$\boxed{\therefore R_B = \frac{3P}{2}}$$

$$M_A + \frac{3P}{2} \times \frac{3L}{4} - PL = 0$$

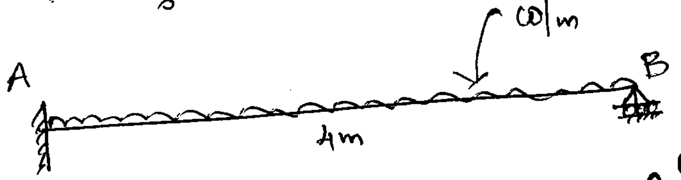
$$M_A = PL - \frac{9}{8}PL = \frac{-PL}{8} \quad (c.w)$$

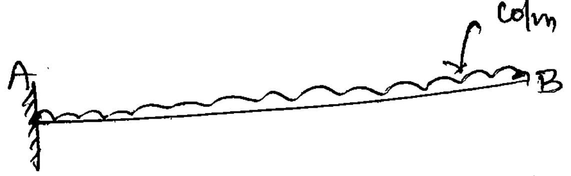
BMD:-



Prob:- A timber beam 12x20Cm and 4m long is loaded with a UDL. It is fixed at left end and simply supported at right end. If maximum allowable fibre stress is 10N/mm² and right support settles by an amount equal to $\frac{\omega l^4}{24EI}$, Determine permissible value of load(ω)

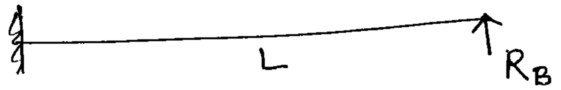
Sol:-





Removing prop,

$$\Delta_{B,L} = \frac{\omega L^4}{8EI}$$



Applying Redundant,

$$\Delta_{B,R} = \frac{R_B L^3}{3EI} \quad (-ve)$$

Consistency:- $\Delta_B = \Delta_{B,L} - \Delta_{B,R} = \frac{\omega L^4}{24EI}$

$$\Rightarrow \frac{\omega L^4}{8EI} - \frac{R_B L^3}{3EI} = \frac{\omega L^4}{24EI}$$

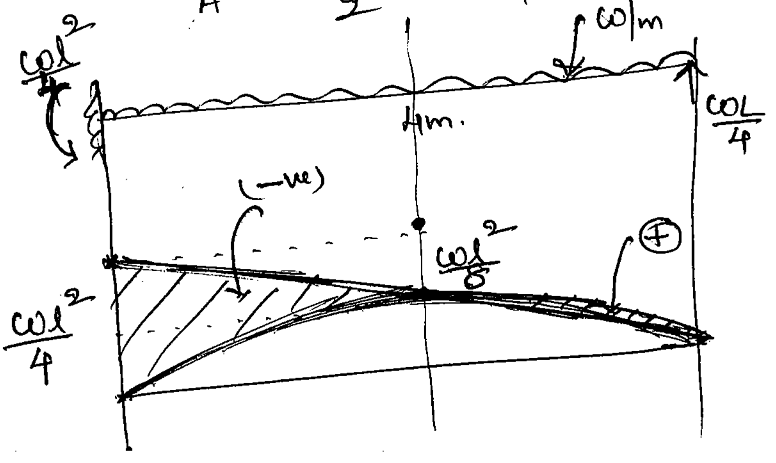
$$\frac{\omega L^4}{8EI}\left(1-\frac{1}{3}\right) = \frac{R_B L^3}{3EI}$$

$$\frac{\omega L^{\cancel{4}}}{4\cancel{8EI}} \times \frac{\cancel{2}}{\cancel{3}} = \frac{R_B \times \cancel{L^3}}{\cancel{3EI}} \Rightarrow \boxed{R_B = \frac{\omega L}{4}}$$

$\circlearrowright \Sigma M_A = 0$ $\quad M_A + R_B \times L - \omega \times l \times \frac{l}{2} = 0$

$$M_A = \frac{\omega l^2}{2} - \frac{\omega l \times l}{4} = \frac{\omega l^2}{2}\left(1-\frac{1}{2}\right) = \frac{\omega l^2}{4}$$

BMD:-



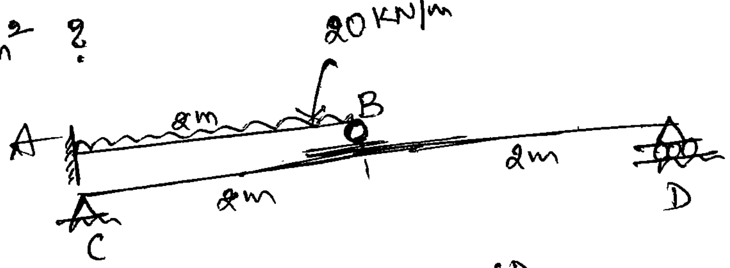
$M_{max} = \frac{wl}{4}$, Bending stress, $\frac{M}{I} = \frac{\sigma}{y}$

$$M = \sigma \times \frac{I}{y} \Rightarrow \frac{wl^2}{4} = 10 \times \frac{\left(\frac{bd^3}{12}\right)}{(d/2)}$$

$$w = 10 \times \frac{120 \times 200^2}{6} \times \frac{4}{(4000)^2} \qquad \left(\because \sigma_{allowable} = 10 N/mm^2\right)$$

$$\boxed{w = 2 \ KN/m}$$

prob:- A cantilever AB of length 2m is fixed at end A and B rests on SSB of span 4m at centre. Find Support moment at A and deflection at B when cantilever is loaded with UDL of 20,000 N/m. EI for cantilever is 1×10^7 Nm². EI for SSB is 2×10^7 Nm² ?



sol:- If AB is not Resting on CD,

$$\Delta = \frac{wL^4}{8EI} = \frac{20 \times 2^4}{8 \times 10,000} = 4 \times 10^{-3} \ m.$$

Beam AB will be pushed up by a force 'P'.

$$\therefore \quad \Delta_1 = \frac{PL^3}{3EI} = \frac{P \times 2^3}{3EI}$$

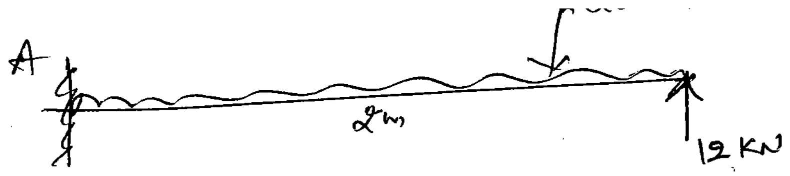
force 'p' on CD will bring down B, by Δ_2,

$$\Delta_2 = \frac{PL^3}{48EI} = \frac{P \times 4^3}{48 \times 2EI}$$

$$\therefore \ 4 \times 10^{-3} = \frac{20 \times 2^4}{8EI} = \frac{P \times 2^3}{3EI} + \frac{P(4^3)}{48 \times 2EI}$$

$$\frac{20 \times 2^4}{8} = P\left(\frac{2}{3}\right)(5)$$

$$P = \frac{20 \times 2 \times 3}{10} = 12 \ KN$$



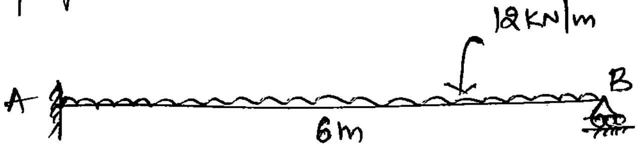
$$M_A = 12\times2 - 20\times2\times1 = 24-40 = \underline{\underline{-16\ \text{KNm}}}$$

$$\Delta_B = 4\times10^{-3}\ \text{m} = \underline{\underline{4\ \text{mm}}}$$

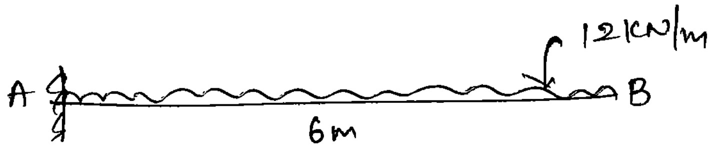
supply Dec/Jan 2016

prob:- A cantilever of 6m length carries an UDL of 12 KN/m over the full span. If the free end is supported by a prop, find the Reaction at the prop and also draw the S.F and B.M diagrams?

sol:-

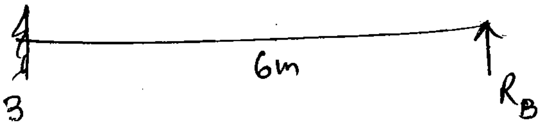


Removing prop:-



$$\Delta_{B,L} = \frac{\omega L^4}{8EI} = \frac{12\times6^4}{8\times EI}$$

Applying Redundant:-



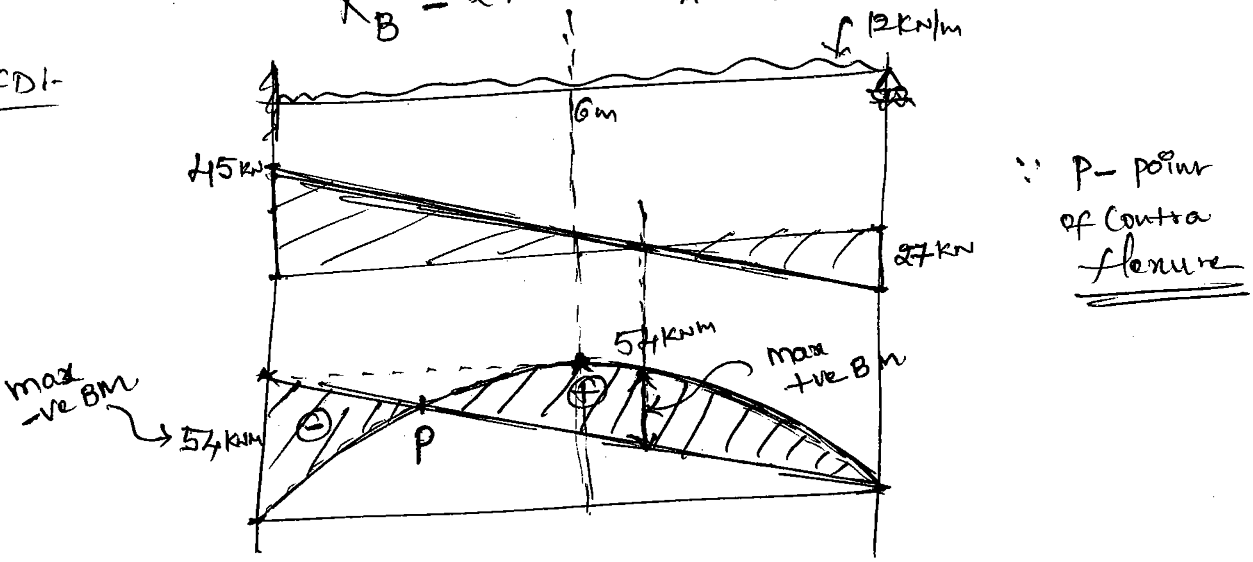
$$\Delta_{B,R} = \frac{R_B\times L^3}{3EI} \quad (-ve)$$

from consistency:- $\Delta_B = 0$ $\quad \Delta_{B,L} + \Delta_{BR} = 0$

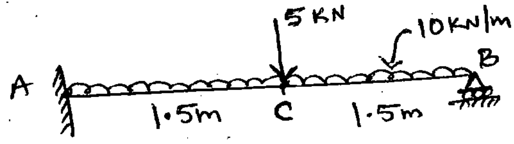
$$\frac{\omega L^4}{8EI} - \frac{R_B L^3}{3EI} = 0 \Rightarrow \frac{\cancel{12}^{3}\times\cancel{6^4}^{3}}{\cancel{8}_{2}\cancel{EI}} = \frac{R_B\times \cancel{6^3}}{3\cancel{EI}}$$

$R_B = 27$ KN $\quad R_A = 45$ KN ; $M_A = -54$ KNm.

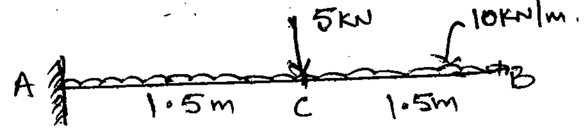
SFD:-



May/June 2015 (a) prob:- Analyse the propped cantilever beam shown below. Draw S.F.D and BMD. Assume EI constant throughout? (8M)



Sol:- Removing prop :-

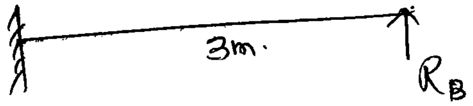


$$\Delta_{B,L} = \frac{\omega L^4}{8EI} + \frac{\omega L^3}{3EI} + \theta_c \times (L/2)$$

$$= \frac{10\times 3^4}{8EI} + \frac{5\times(1.5)^3}{3EI} + \frac{5\times(1.5)^2}{2EI}\times(1.5)$$

$$= \frac{101.25}{EI} + \frac{5.625}{EI} + \frac{8.4375}{EI} = \frac{115.3125}{EI}$$

Applying Redundant,



$$\Delta_{BR} = \frac{R_B \times 3^3}{3EI} = \frac{9R_B}{EI} \text{ (-ve)}$$

from consistency, $\Delta_B = 0 \Rightarrow \Delta_{B,L} + \Delta_{B,R} = 0$

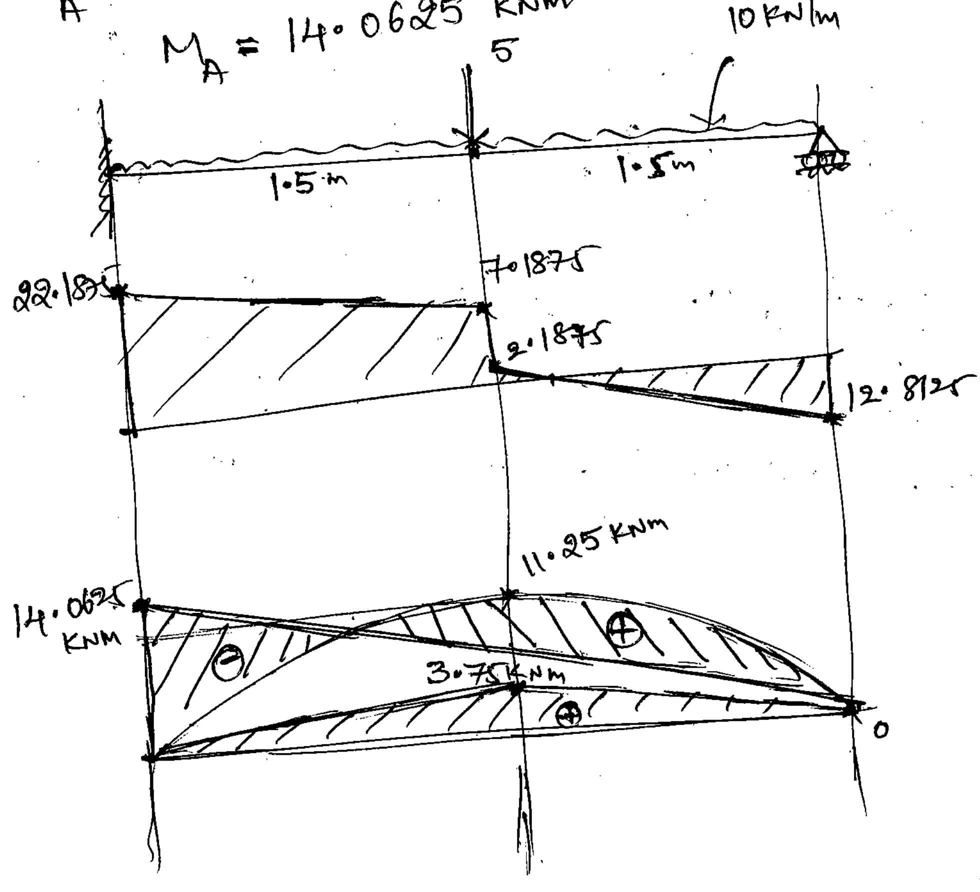
$$\therefore \frac{115.3125}{EI} = \frac{R_B \times 9}{EI} \Rightarrow \boxed{\begin{array}{l} R_B = 12.8125\text{ KN} \\ R_A = 22.1875\text{ KN} \end{array}}$$

$\curvearrowright \Sigma M_A = 0 \quad M_A + 12.8125\times 3 - 5\times 1.5 - 10\times 3\times 1.5 = 0$

$M_A = 14.0625$ KNm

SFD:-

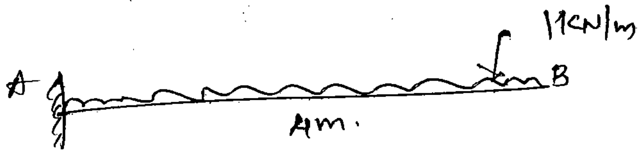
BMD



(b) A cantilever of length 4m carries a UDL of 1 kN/m length over the whole length. Free end of the cantilever is supported on a prop. If $E = 2\times10^5$ N/mm² and $I = 10^8$ mm⁴, then (i) find the prop reaction (ii) deflection at the centre of cantilever? (8M)

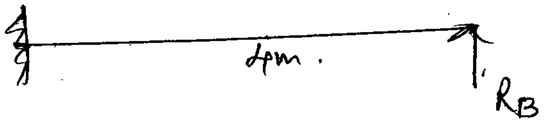
Sol:-

Removing prop:-



$$\Delta_{B,L} = \frac{wL^4}{8EI} = \frac{1\times4^4}{8EI}$$

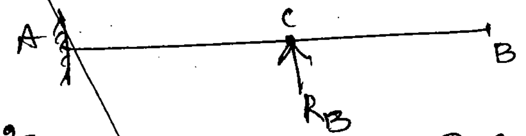
Applying Redundant,



$$\Delta_{B,R} = \frac{R_B 4^3}{3EI} \quad (-ve)$$

from consistency:- $\Delta_B = 0 = \Delta_{B,L} + \Delta_{B,R}$

$$\therefore \frac{4^4}{8EI} = \frac{R_B\times4^3}{3EI} \Rightarrow R_B = \frac{3}{2} = 1.5\text{ kN}$$



$$\Delta_C = \Delta_{C,UDL} + \Delta_{C,Pt}$$

$$= \frac{w(4)^4}{8EI} + \frac{R_B\times(L/2)^3}{3EI} + \frac{R_B\times(L/2)^2}{2EI}\times(L/2)$$ [Maxwell Reciprocal theorem]

Use Moment Area

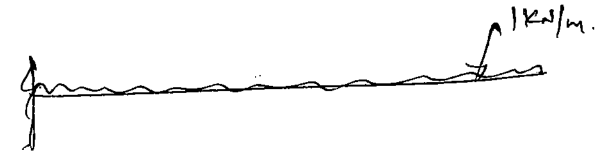
$$= \frac{1\times(2)^4}{8EI} + \frac{(1.5)\times2^3}{3EI} + \frac{1.5\times(2)^2}{2EI}\times(2)$$

$$= \frac{1}{EI}\left[2+4+6\right] = \frac{12}{EI} = \frac{12\times10^3\times(10^3)^3}{2\times10^5\times10^8} = 0.6\text{ mm } (\downarrow)$$

$\delta_c = 0$ //

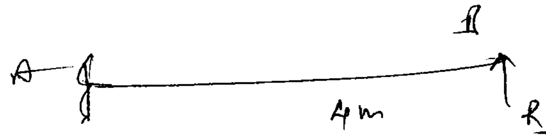
(b) A cantilever of length 4m carries a UDL of 1KN/m length over the whole length. free end of the cantilever is supported on a prop. If $E = 2\times10^5\ N/mm^2$ and $I = 10^8\ mm^4$ then find (i) the prop reaction (ii) deflection at centre? 5(i)-I

Removing prop :-



$$\Delta_{B,L} = \frac{\omega L^4}{8EI} = \frac{1\times4^4}{8EI}\ (+ve)$$

Applying Redundant

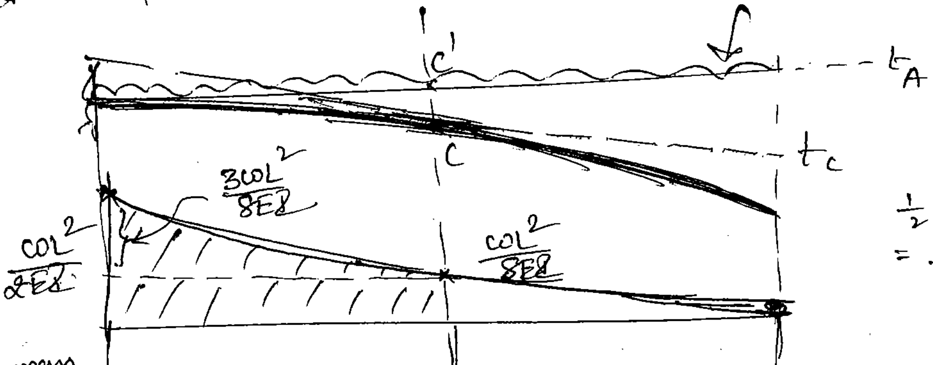


$$\Delta_{B,R} = \frac{R_B L^3}{3EI} = \frac{R_B(4)^3}{3EI}\ (-ve)$$

from consistency cond^n Δ_B so

$$\frac{1\times4^4}{8EI} = \frac{R_B\times4^3}{3EI} \Rightarrow R_B = 1.5\ KN$$

To find deflection at Centre



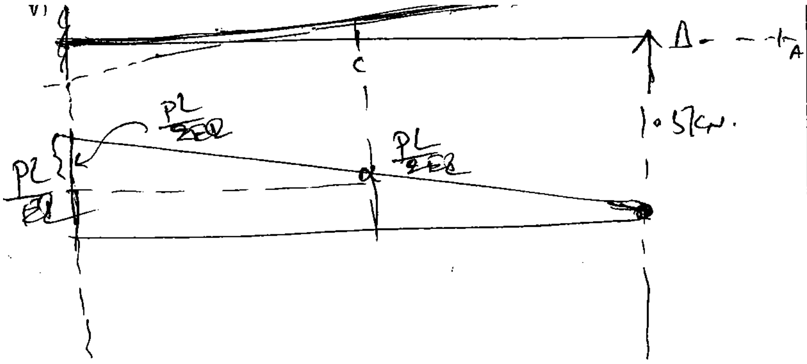
$\frac{1}{2}-\frac{1}{8} = \frac{4-1}{2\times8} = 3/8$

Mohr's theorem - II

$$\therefore\ \delta_{C,UDL} = CC' = \left(\frac{\omega L^2}{8EI}\right)\times\frac{L}{2}\times\frac{L}{4} + \left(\frac{1}{2}\times\frac{L}{2}\times\frac{3\omega L^2}{8EI}\right)\times\left(\frac{2}{3}\times\frac{L}{2}\right)$$

$$= \left(\frac{\omega L^4}{64EI} + \frac{3\omega L^4}{16\times8EI}\right) = \frac{\omega L^4}{64EI}\left(1+\frac{3}{2}\right) = \frac{\omega L^4}{EI}\times\left(\frac{5}{64\times2}\right) = \frac{5\omega L^4}{128EI}$$

$\delta_{C,\,pt\ load}$ =



$$\delta_{C,pt} = \frac{PL}{2EI}\times\frac{L}{2}\times\frac{L}{4} + \left(\frac{1}{2}\times\frac{L}{2}\times\frac{PL}{2EI}\right)\times\left(\frac{2}{3}\times\frac{L}{2}\right)$$

$$= \frac{PL^3}{16EI} + \frac{PL^3}{24} = \frac{PL^3}{4EI}\left(\frac{1}{4}+\frac{1}{6}\right)$$

$$= \frac{PL^3}{4EI}\,\frac{\cancel{10}\,5}{\cancel{4}\,2\times 6} = \underline{\underline{\frac{5PL^3}{48EI}}}$$

$$\therefore\ \delta_C = \delta_{C,UDL} - \delta_{C,pt.}$$

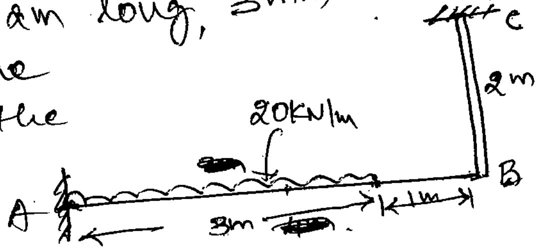
$$= 0.5 - 0.5 = \underline{\underline{0\ \text{mm}}}$$

SSB:-

(1) Beam AB of span 3m rest over another beam BC of span 2m as shown below. Find the reactions at the supports A and C, given that the flexural rigidity of beam AB is twice that of BC?

Ans:- $V_A = 40.46$ kN
$M_A = 61.38$ kNm
$V_C = 19.54$ kN
$M_C = 39.08$ kNm.

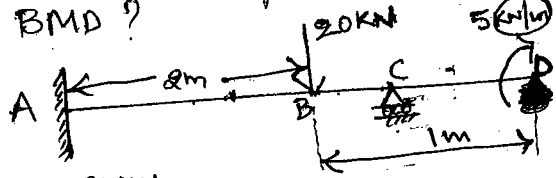
(2) The cantilever beam shown below, of span 4m is supported by a 2m long, 3mm diameter wire CB. Determine the force developed in the wire due to loading shown in the figure.



If $EI = 5000\ \text{kNm}^2$ and $E_{wire} = 200\ \text{kN/mm}^2$.

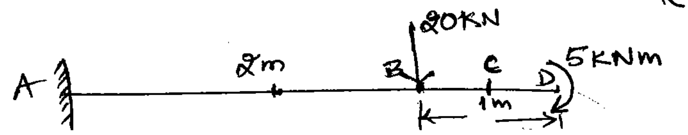
Ans:- $P = \underline{10.3\ \text{kN}}$

Set-2 (a) Determine the reactions of the propped cantilever beam and draw SFD and BMD? (5) -I

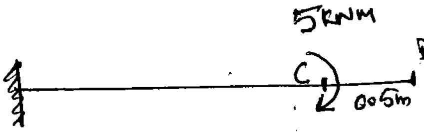


Sol:-

Removing prop:-



$$\Delta_{c,L} = \frac{20 \times (2)^3}{3EI} + \frac{20 \times 2^2}{2EI} \times (0.5) + (\ )$$



$$= \left[\Delta_B + \theta_B \times BC\right]_{Pt\ load} + \frac{5 \times 2.5^2}{2EI} + \left(\frac{5 \times 2.5}{EI}\right) \times 0.5$$

$$= \quad " \quad + \left[\Delta_c + \theta_c \times CD\right]_{moment}.$$

$$= \frac{160}{3EI} + \frac{20}{EI} + \frac{15.625}{EI} + \frac{6.25}{EI}$$

$$= \frac{95.2083}{EI}$$

$$\Delta_{c,R} = \frac{R_c \times (2.5)^3}{3EI} = \frac{R_c \times 5.2083}{EI} \ (-ve)$$

from Consistency, $\Delta_c = 0 \Rightarrow \Delta_{c,L} + \Delta_{c,R} = 0$

$$\frac{95.2083}{EI} = \frac{R_c \times 5.2083}{EI}$$

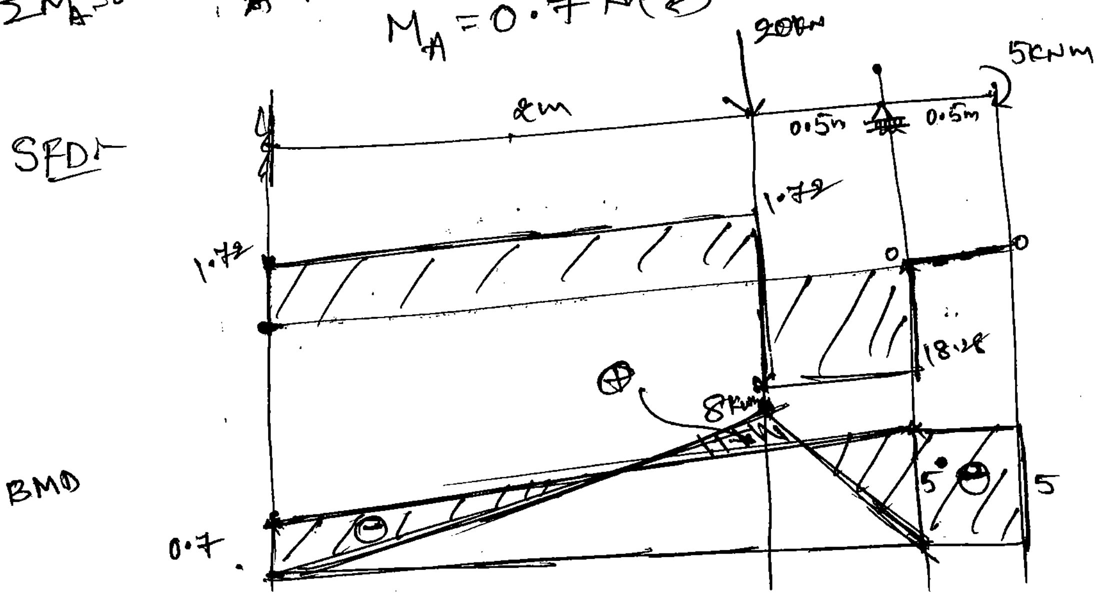
$R_c = 18.28KN$

$R_A = 1.72KN$

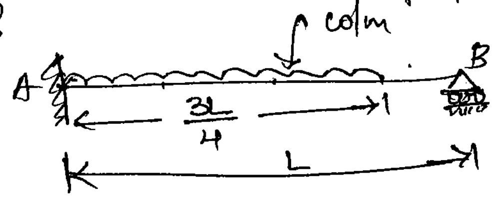
$R_A = 20 - 18.28 = 1.72KN$

$\Sigma M_A = 0 \quad M_A + 18.28 \times 2.5 - 20 \times 2 - 5 = 0$

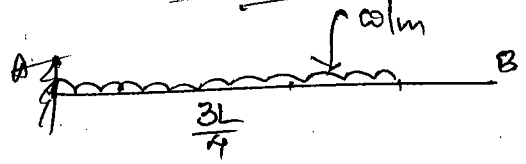
$M_A = 0.7 \text{ KN}(\circlearrowright)$



Set- prob: A propped cantilever beam of length L is subjected to UDL of ω/m length over three-fourth of its span from the fixed support. Determine the prop reaction and sketch the BMD?

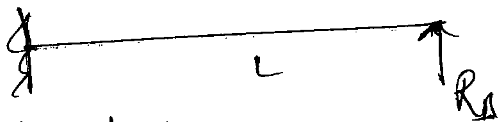


Sol: Remove the prop:



$$\Delta_{B,L} = \frac{\omega (3L/4)^4}{8EI} + \frac{\omega (3L/4)^3}{6EI} \times (L/4) \times \frac{3}{3}$$

$$= \frac{\omega \left(\frac{3L}{4}\right)^4}{EI}\left(\frac{1}{8} + \frac{1}{18}\right)$$



Applying Redundant: $\Delta_{B,R} = \frac{R_B L^3}{3EI}$ (-ve)

from Consistency, $\Delta_B = 0$ $\Delta_{B,L} + \Delta_{B,R} = 0$

$$\therefore \frac{\omega \left(\frac{3L}{4}\right)^4}{EI}\left(\frac{1}{8} + \frac{1}{18}\right) = \frac{R_B \times L^3}{3EI}$$

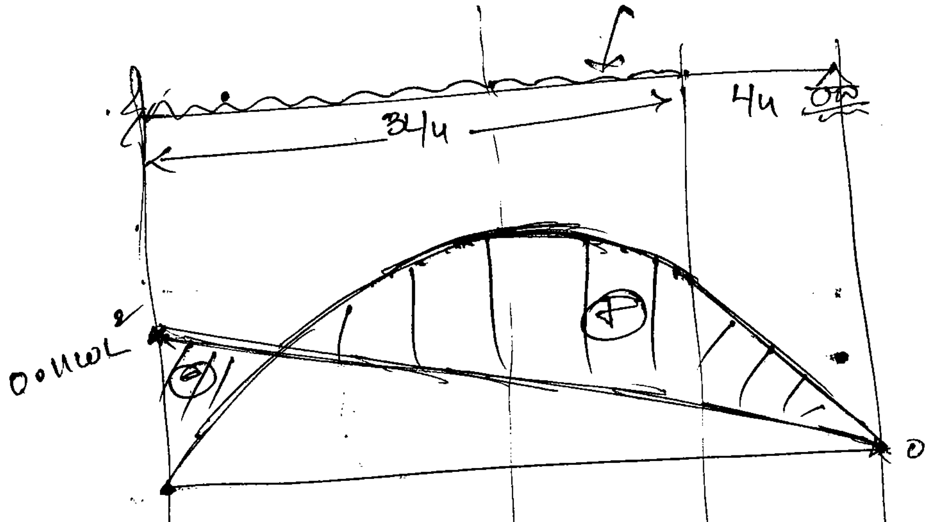
$$\frac{3\omega \times 3^4 \times L}{4^4} \times \left(\frac{26}{8 \times 18}\right) = R_B = \frac{351\,\omega L}{2048}$$

BMD: $\Sigma M_A = 0$ $M_A + \frac{351\,\omega L}{2048} \times L - \omega \times \left(\frac{3L}{4}\right)\left(\frac{3L}{8}\right) = 0$

$$M_A = \omega L^2 (0.28125) - \omega L^2 (0.1714)$$

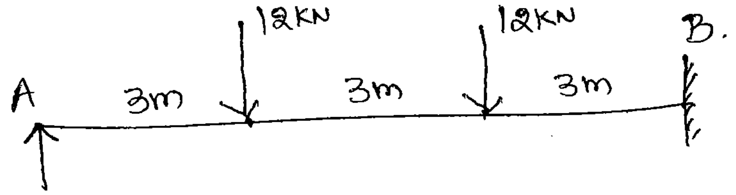
$$= \omega L^2 (0.11)$$

BMD:

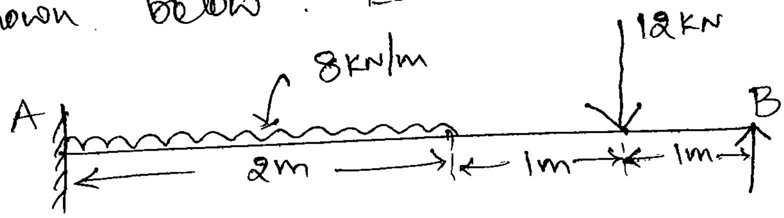


Exercise Questions from T.S.T.M

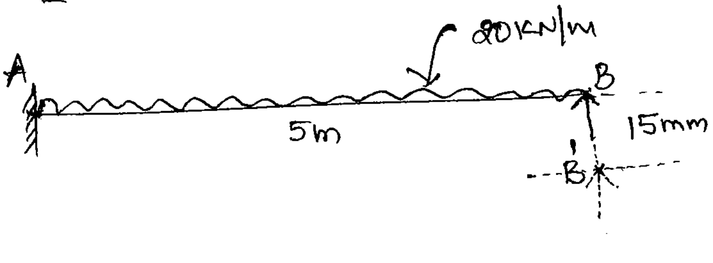
1. What is a propped cantilever?
2. Is it statically determinate or Indeterminate?
3. If it is Indeterminate, what is the degree of indeterminacy?
4. How is the prop reaction determined? Explain!
5. Explain the consistent deformation method of analysing a propped Cantilever?
6. Determine the prop reaction in the beam shown below?. EI is Constant.



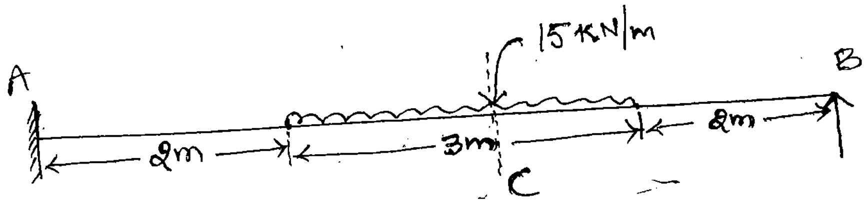
7. Using consistent deformation method, evaluate the prop reaction in the beam. Shown below? EI is constant.



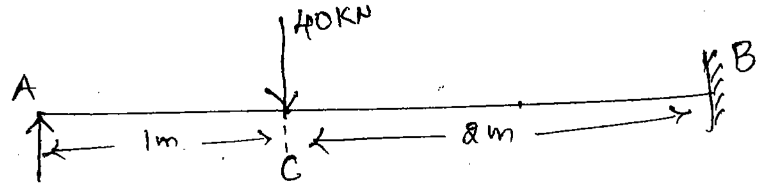
8. In the beam shown below, the prop has Sunk by 15mm. Calculate the prop reaction. Take $E = 200 \times 10^6 \ KN/m^2$ and $I = 5 \times 10^{-6} \ m^4$.



⑨ Find the deflection at C in the beam shown below. Take $EI = 9000\ kNm^2$.

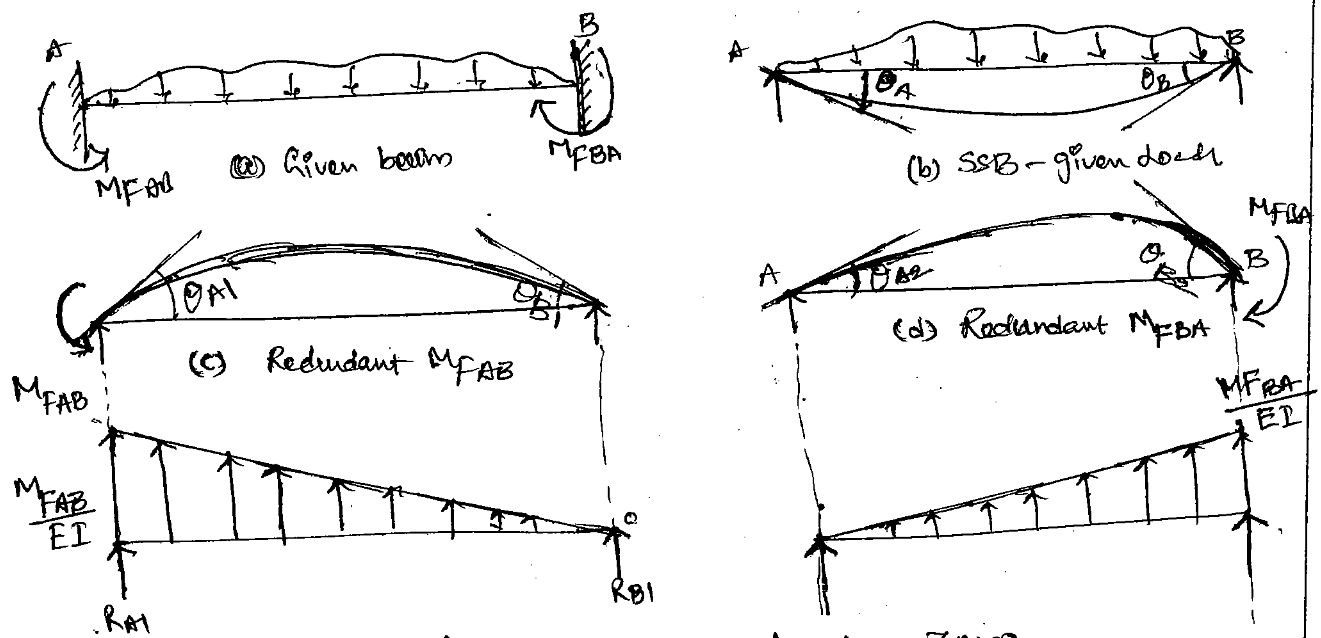


⑩ Determine the deflection at C in a propped cantilever shown below?



Fixed Beams

Consider the fixed beam shown below. Let the end moments developed be M_{FAB} and M_{FBA}. To find these end moments, the consistent deformation method may be used. Taking a simply supported beam as a basic determinate structure, the redundant forces are M_{FAB} and M_{FBA}.



From consistency :- (i) slope at A should be zero.
(ii) " B "

$$\therefore \theta_A - \theta_{A1} - \theta_{A2} = 0 \qquad \theta_B - \theta_{B1} - \theta_{B2} = 0$$

$$\therefore \boxed{\theta_A = \theta_{A1} + \theta_{A2}} \qquad \boxed{\theta_B = \theta_{B1} + \theta_{B2}}$$

$$\theta_{A1} = \left(\frac{1}{2}\times L\times\frac{M_{FAB}}{EI}\right)\left(\frac{2L}{3}\right)\times\frac{1}{L} = \frac{M_{FAB}L}{3EI}$$

$$\theta_{A2} = \left(\frac{1}{2}\times L\times\frac{M_{FBA}}{EI}\right)\times\frac{L}{3}\times\frac{1}{L} = \frac{M_{FBA}L}{6EI}$$

$$\theta_{B2} = \frac{M_{FBA}L}{3EI}$$

$$\theta_{B1} = \frac{M_{FAB}L}{6EI}$$

$$\therefore \theta_A = \frac{L}{6EI}\left(2M_{FAB} + M_{FBA}\right) \; ; \quad \theta_B = \frac{L}{6EI}\left(2M_{FBA} + M_{FAB}\right)$$

procedure :-

Step 1 :- Remove the Redundants (2 Nos) and prepare a basic determinate beam (simply supported beam) and calculate slope at both the ends θ_A, θ_B due to given loads.

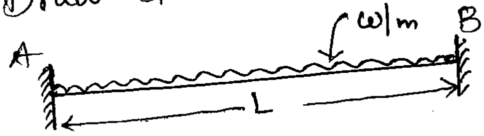
Step 2 :- Apply the Redundants and find the slopes at the ends of the SSB.

Step 3 :- Apply consistency conditions to solve ~~for~~ the Redundants (M_{FAB}, M_{FBA}).

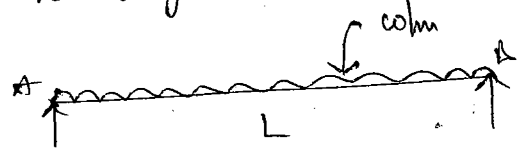
Step 4 :- Apply equilibrium equations to solve for the support Reactions.

Step 5 :- Draw SFD and BMD and also the required deflections.

prob :- Find the fixed end moments developed in the fixed beam shown below? Draw SFD and BMD?



sol :- Removing Redundants,



$$\theta_A = \theta_B = \frac{wL^3}{24EI}$$

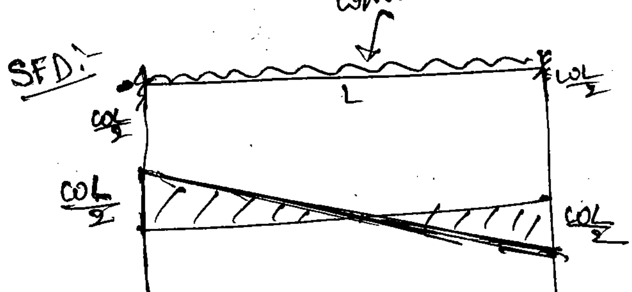
from consistency,

$$\theta_A = \frac{1 \times L}{6EI}(2M_{FAB} + M_{FBA})$$

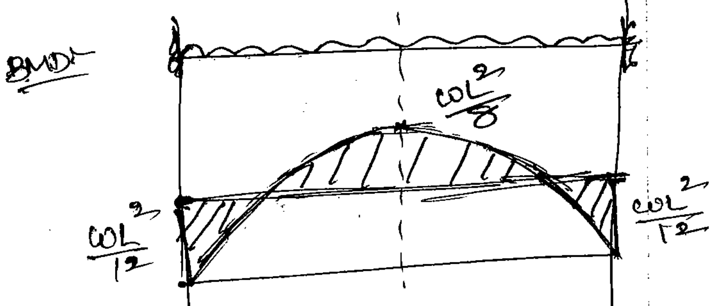
$$\frac{wL^{\cancel{3}}}{\cancel{24}_{12}E\cancel{I}} = \frac{\cancel{L}}{\cancel{6}_{2}E\cancel{I}}(\cancel{3}M_{FAB})$$ (∵ due to Symmetry $M_{FAB} = M_{FBA}$)

$$\therefore M_{FAB} = M_{FBA} = \frac{wL^2}{12}$$

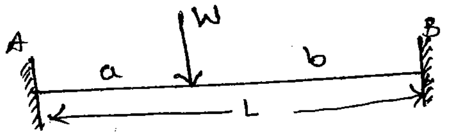
SFD :-



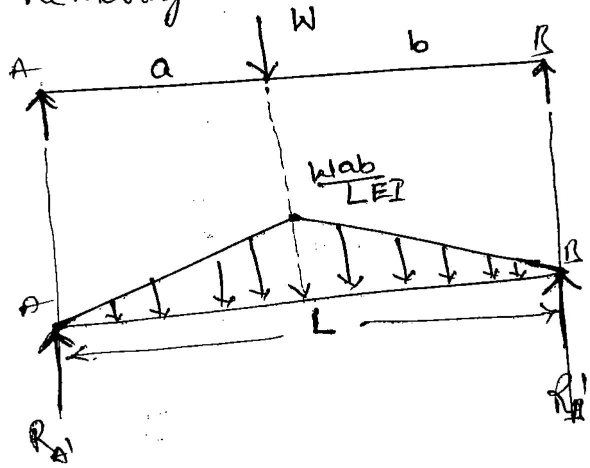
BMD :-



Prob.- A Fixed beam of span L is subjected to a concentrated (Q)-II
Load of W at a distance 'a' from end A. Determine end moments?



Sol:- Removing the Redundants.



$$\theta_A = R_A' = \left(\frac{1}{2} \times \cancel{L} \times \frac{Wab}{\cancel{L}EI}\right) \times \left(\frac{L+b}{3}\right) \cdot \frac{1}{L}$$

$$= \frac{Wab}{6EI}\left(\frac{L+b}{L}\right)$$

lly $\theta_B = \frac{Wab}{6EI}\left(\frac{L+a}{L}\right)$

Applying Redundants,

$$\theta_A = \frac{L}{6EI}\left(2M_{FAB} + M_{FBA}\right)$$

$$\theta_B = \frac{L}{6EI}\left(2M_{FBA} + M_{FAB}\right)$$

from consistency :-

$$\frac{Wab}{6\cancel{EI}}\left(\frac{L+b}{L}\right) = \frac{L}{6\cancel{EI}}\left(2M_{FAB} + M_{FBA}\right)$$

$$2 \times \left(\frac{Wab}{L^2}(L+b)\right) = 2 \times 2M_{FAB} + M_{FBA} \times 2$$

$$(-)\frac{Wab}{\cancel{6EI}}\frac{(L+a)}{L^2} = (-)M_{FAB} \overset{(-)}{+} 2M_{FBA}$$

$$\therefore \frac{Wab}{L^2}\left(2\cancel{L} + 2b - \cancel{L} - a\right) = 3M_{FAB}$$

$$\frac{Wab}{L^2}\left(\cancel{a} + b + 2b - \cancel{a}\right) = 3M_{FAB}$$

$$\frac{Wab}{L^2}(\cancel{3}b) = \cancel{3}M_{FAB}$$

$$\therefore \boxed{M_{FAB} = \frac{Wab^2}{L^2}}$$

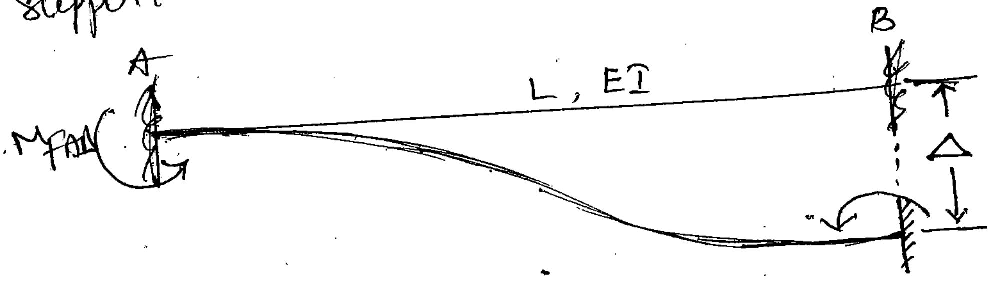
$$\boxed{M_{FBA} = \frac{Wba^2}{L^2}}$$

$\because$ if $a = b = L/2$

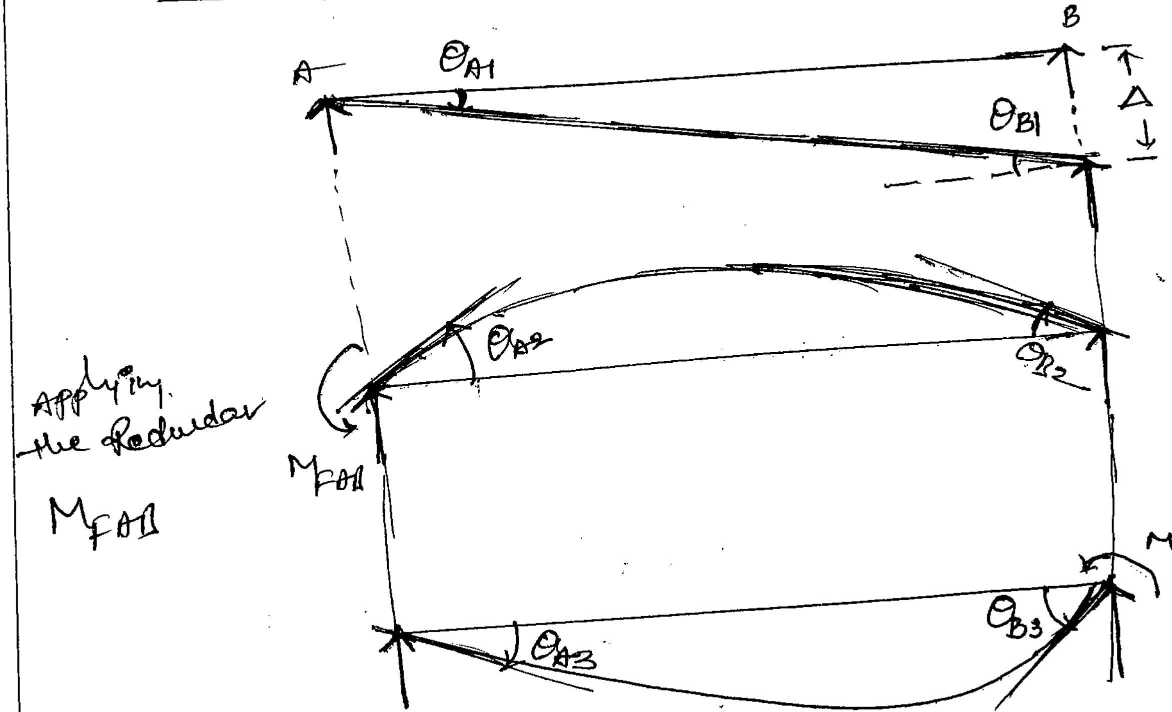
$M_{FAB} = M_{FBA}$

$= \frac{WL}{8}$

Prob:- Determine ... in a fixed beam of span L and flexural Rigidity EI when the right hand side support settles down by 'Δ'?



Sol:- Removing the Redundants:-



Applying the Redundant M_{FAB}

$$\theta_{A1} = \theta_{B1} = \frac{\Delta}{L} \quad \text{(clock wise)}$$

$$\theta_{A2} = \frac{M_{FAB}L}{3EI} \text{ (-ve)} \quad ; \quad \theta_{B2} = \frac{M_{FAB}L}{6EI}$$

$$\theta_{A3} = \frac{M_{FBA}L}{6EI} \text{ (+ve)} \quad ; \quad \theta_{B3} = \frac{M_{FBA}L}{3EI} \text{ (-ve)}$$

from consistency, $\theta_A = 0$; $\theta_B = 0$

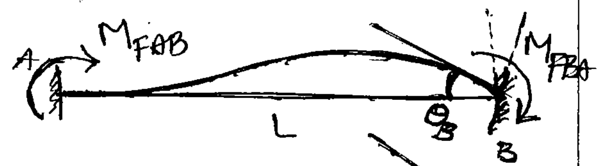
$$\frac{\Delta}{L} - \frac{M_{FAB}L}{3EI} + \frac{M_{FBA}L}{6EI} = 0 \quad \text{—— (1)}$$

$$\frac{\Delta}{L} + \frac{M_{FAB}L}{6EI} - \frac{M_{FBA}L}{3EI} = 0 \quad \text{—— (2)}$$

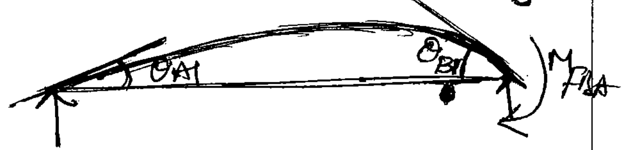
from (1) & (2) $M_{FAB} = M_{FBA} \Rightarrow$ Substituting in (1) or (2)

$$M_{FAB} = M_{FBA} = \frac{6EI\Delta}{L^2}$$

prob.- Determine the fixed end moments developed in a fixed beam of length 'L' and flexural Rigidity EI when the right hand side support is rotated by an angle 'θ'? (3)-II



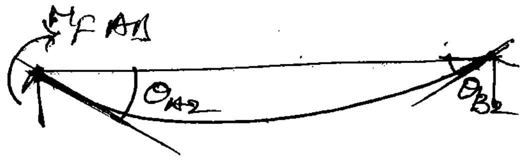
sol:- Removing the Redundants



applying M_{FBA},

$$\theta_{A1} = \frac{M_{FBA}L}{6EI}$$

$$\theta_{B1} = \frac{M_{FBA}L}{3EI}$$



$$\theta_{A2} = \frac{M_{FAB}L}{3EI} \quad ; \quad \theta_{B2} = \frac{M_{FAB}L}{6EI}$$

∴ from consistency $\theta_A = 0$ $\theta_{A1} + \theta_{A2} = 0$

$$-\frac{M_{FBA}L}{6EI} + \frac{M_{FAB}L}{3EI} = 0 \Rightarrow \frac{M_{FBA}\cancel{L}}{\cancel{6}_2 \cancel{EI}} = \frac{M_{FAB}\cancel{L}}{\cancel{3EI}}$$

$$\boxed{M_{FAB} = \frac{M_{FBA}}{2}}$$

$$\theta_B = \theta_{B1} - \theta_{B2} = \frac{M_{FBA}L}{3EI} - \frac{M_{FAB}L}{6EI}$$

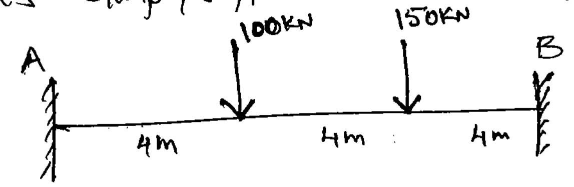
$$\theta_B = \frac{M_{FBA}L}{3EI} - \frac{M_{FBA}L}{2 \times 6EI}$$

$$= \frac{M_{FBA}L}{EI}\left(\frac{1}{3} - \frac{1}{12}\right) = \frac{M_{FBA}L}{EI} \times \frac{\cancel{9}^{3}}{\cancel{3} \times \cancel{12}_{4}}$$

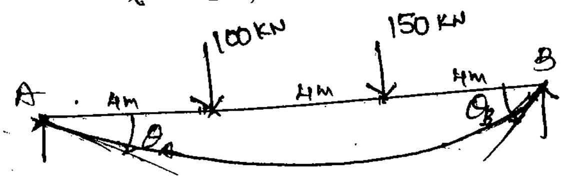
$$\boxed{\therefore M_{FBA} = \frac{4EI\theta_B}{L}}$$

$$\boxed{\therefore M_{FAB} = \frac{2EI\theta_B}{L}}$$

Q. 1 — A fixed beam of length ... carries two point loads at a distance of 4m and 8m from left end respectively. Find the fixed end moments under the loads when the beam is Simply Supported. Draw BMD.



Sol: Removing Redundants M_A & M_B;



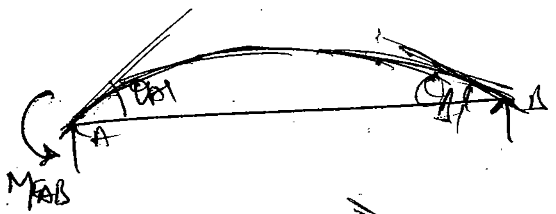
$$\theta_A = \frac{Wab(L+b)}{6LEI} \; ; \; \theta_B = \frac{Wab(L+a)}{6LEI}$$

$$\theta_A = \frac{100 \times 4 \times 8\,(12+8)}{6 \times 12\, EI} + \frac{150 \times 8 \times 4\,(12+4)}{6 \times 12\, EI}$$

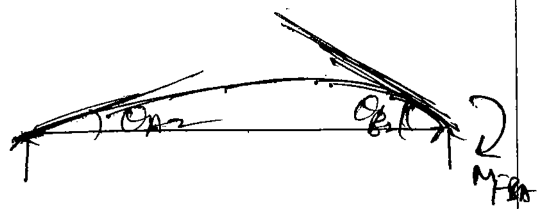
$$= \frac{888.89}{EI} + \frac{1066.67}{EI} = \frac{1955.56}{EI}$$

$$\theta_B = \frac{100 \times 4 \times 8\,(12+4)}{6 \times 12 \times EI} + \frac{150 \times 8 \times 4\,(12+8)}{6 \times 12 \times EI}$$

$$= \frac{147200}{6 \times 12 \times EI} = \frac{2044.44}{EI}$$



Applying Redundants



from consistency,

$$\theta_A = \frac{L}{6EI}(2M_{FAB} + M_{FBA})$$

$$\theta_B = \frac{L}{6EI}(M_{FAB} + 2M_{FBA})$$

$$\therefore \frac{1955.56}{\cancel{EI}} = \frac{\cancel{12}^{\,2}}{\cancel{6EI}}(2M_{FAB} + M_{FBA})$$

$$\frac{2044.44}{\cancel{EI}} = \frac{12}{6\cancel{EI}}(M_{FAB} + 2M_{FBA})$$

$(1955.56 = 4M_{FAB} + 2M_{FBA}) \times 2$

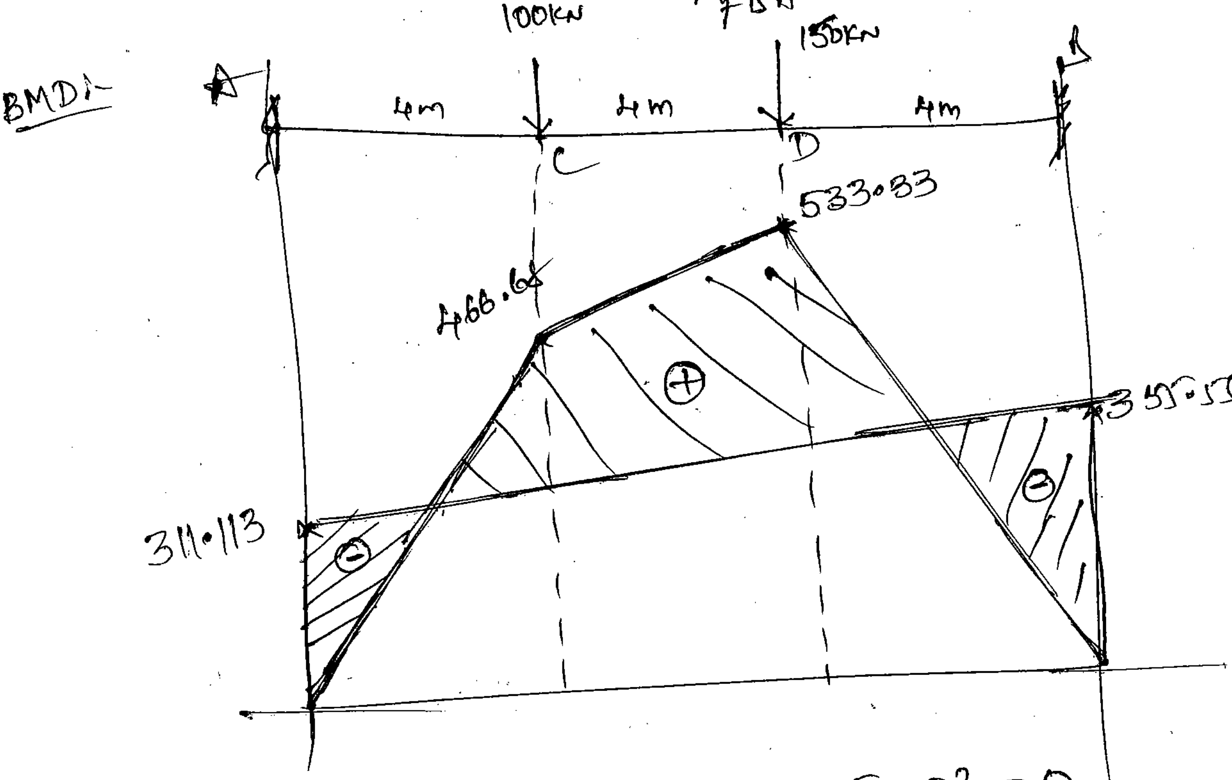
$2044.44 = 2M_{FAB} + 4M_{FBA}$

$3911.12 = 8M_{FAB} + 4M_{FBA}$

$1866.68 = 6M_{FAB} \Rightarrow M_{FAB} = 311.113$ KNm.

$M_{FBA} = 355.553$ KNm

BMD:-



$R_A \times 12 - 100 \times 8 - 150 \times 4 - 311.113 + 355.553 = 0$

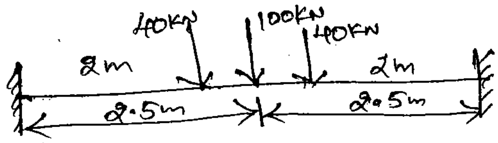
$R_A = 112.963$ KN
$R_B = 137.037$ KN } Including FEMs

$R_A = \dfrac{100 \times 8}{12} + \dfrac{150 \times 4}{12} = 116.67$ KN.

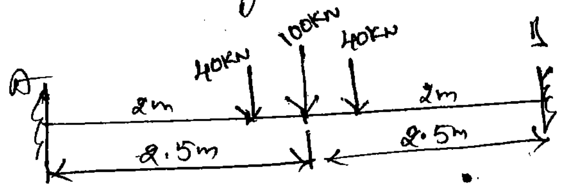
$R_B = (\ ) + (\ ) = 133.33$ KN

SS, $BM_C = 116.67 \times 4 = 466.68$ KNm

SS $BM_D = 133.33 \times 4 = 533.33$ KNm prob:- A fixed beam is shown below. Solve the beam and draw SFD and BMD?



Sol:- Removing Redundants,



$$\theta_A = \theta_B = \frac{100 \times (5)^2}{16EI} + \frac{40 \times 2 \times 3}{6 \times 5 \times EI}(5+3) + \frac{40 \times 2 \times 3 \times (5+2)}{6 \times 5 \times EI}$$

$$= \frac{1}{EI}\left[\cancel{181.25}\ 156.25 + 64 + 56\right] = \frac{\cancel{901.25}}{EI} = \frac{276.25}{EI}$$

Applying Redundants, $\theta_{A1+A2} = \frac{L}{6EI}(2M_{FAB} + M_{FBA})$

$$\theta_{B1+B2} = \frac{L}{6EI}(M_{FAB} + 2M_{FBA})$$

From consistency, $\frac{\cancel{901.25}\ 276.25}{EI} = \frac{5}{6EI}\left[2M_{FAB} + M_{FBA}\right]$

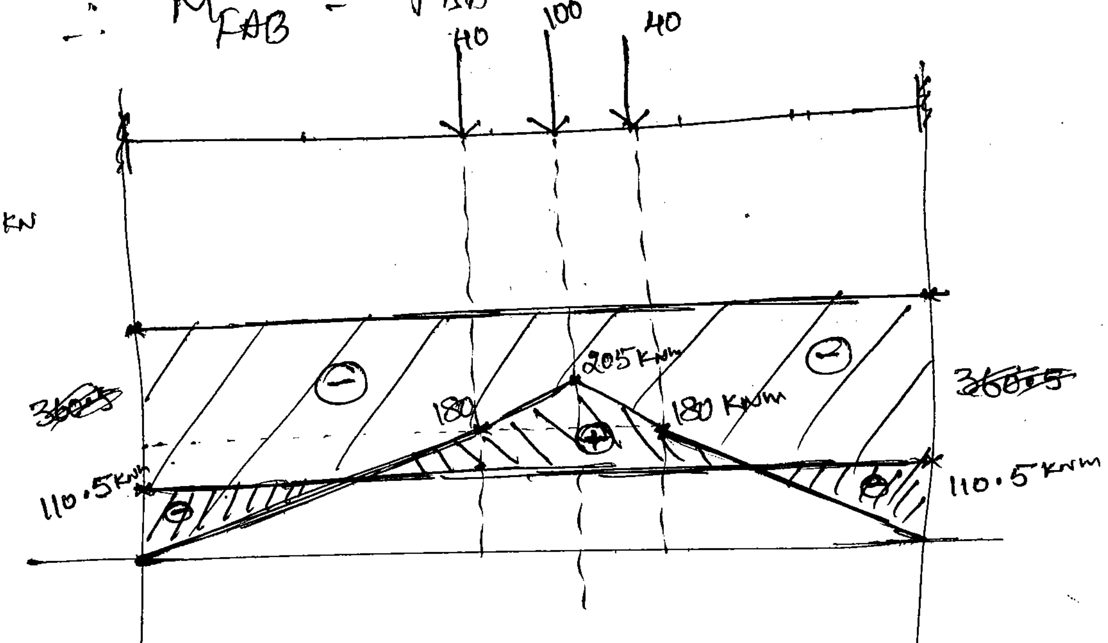
$\because M_{FAB} = M_{FBA} \Rightarrow \frac{901.25}{\cancel{EI}} \times \frac{\cancel{6EI}\ 2}{5} = \cancel{3}M_{FAB}$

$\therefore M_{FAB} = M_{FBA} = \cancel{360.5}$ KNm. $= 110.5$

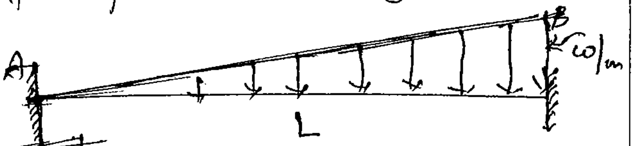
BMD:-

SSB

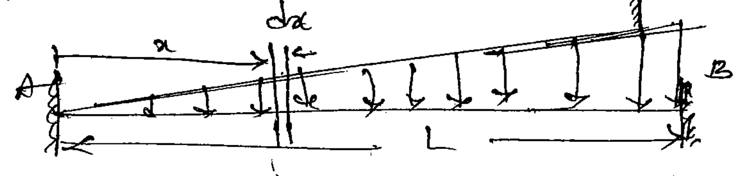
$R_A = R_B = 90$ KN



prob:- Determine the fixed end moments developed in the fixed beam of span L subjected to UVL ?



sol:-



$$w_x = \frac{wx}{L}$$

$$M_{FAB} = \int_0^L \frac{wx}{L} \cdot \frac{x(L-x)^2}{L^2}\, dx$$

$$= \int_0^L \frac{w}{L^3} x^2 (L^2 + x^2 - 2Lx)\, dx$$

$$= \frac{w}{L^3} \int_0^L (x^2L^2 + x^4 - 2Lx^3)\, dx$$

$$= \frac{w}{L^3} \left[L^2\left(\frac{x^3}{3}\right) + \frac{x^5}{5} - 2L\frac{x^4}{4} \right]_0^L$$

$$= \frac{w}{L^3} \left[L^2\left(\frac{L^3}{3}\right) + \frac{L^5}{5} - \frac{\cancel{2}L}{\cancel{4}_2} \times L^4 \right]$$

$$= \frac{w}{L^3} \left[\frac{L^5}{3} + \frac{L^5}{5} - \frac{L^5}{2} \right]$$

$$= \frac{w \times L^5}{L^3} \left[\frac{1}{3} + \frac{1}{5} - \frac{1}{2} \right]$$

$$= wL^2 \left[\frac{10+6-15}{30} \right]$$

$$\boxed{M_{FAB} = \frac{wL^2}{30}}$$

$$M_{FBA} = \int_0^L \left(\frac{\omega x}{L}\right) \times \frac{(L-x)x^2}{L^2}\,dx$$

$$= \int_0^L \frac{\omega}{L^3}\, x^3 (L-x)\, dx$$

$$= \int_0^L \frac{\omega}{L^3}(Lx^3 - x^4)\, dx$$

$$= \frac{\omega}{L^3}\left[L\left(\frac{x^4}{4}\right) - \frac{x^5}{5}\right]_0^L$$

$$= \frac{\omega}{L^3}\left[\frac{L^5}{4} - \frac{L^5}{5}\right]$$

$$= \frac{\omega L^5}{L^3}\left[\frac{1}{4} - \frac{1}{5}\right]$$

$$= \omega L^2\left[\frac{5-4}{20}\right] = \frac{\omega L^2}{20}$$

$$\therefore \boxed{M_{FBA} = \frac{\omega L^2}{20}}$$

2015 Set 4 A fixed beam AB of length 3m carries a point load of 45 KN at a distance of 2m from A. If flexural rigidity of the beam is 1×10^4 KNm² Determine (i) Fixed end moments
(ii) Deflection under the load
(iii) maximum deflection

Fixed Beams

Exercise Questions from TSTM.

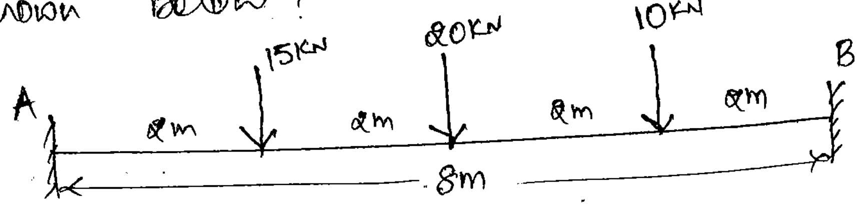
① what is an encastre beam?

② what is the degree of indeterminacy of a constrained beam?

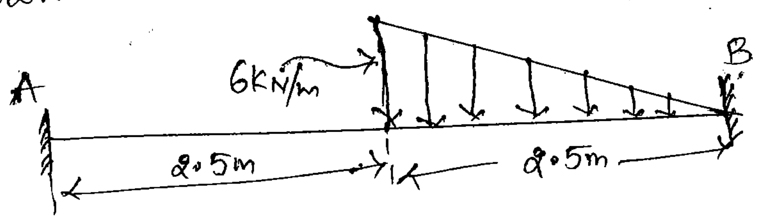
③ why the ends in a fixed beam are called "Direction - fixed ends"? ($\because \theta = 0 ; \delta = 0$ fixed in direction)

④ Derive expressions for fixed-end moments in a fixed beam of span 'L' carrying UDL of ω KN/m by consistent deformation method?

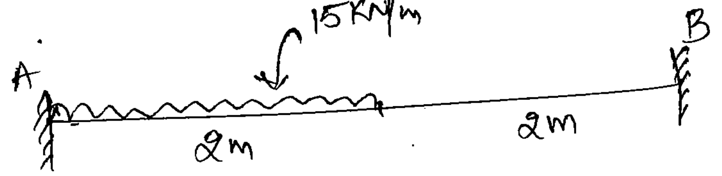
⑤ Determine the fixed-end moments in the beam shown below?



⑥ Evaluate the fixed end moments in the beam shown below?

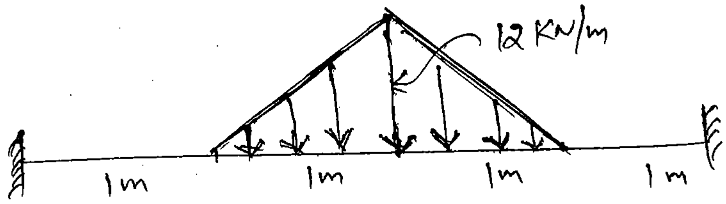


⑦ Draw the SFD and BMD of the beam shown below

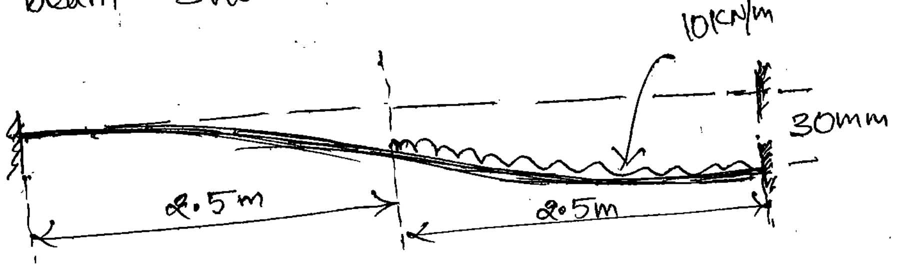


P.T.O

⑧ Calculate the deflection at the centre of the beam shown below?



⑨ ~~Determine~~ the fixed end moments in the beam shown below?



⑩. Evaluate the fixed-end moments in the beam shown below. Take $EI = 11{,}000\ KNm^2$