UNIT – III

FLEXURAL STRESSES, SHEAR STRESSES

Theory of simple bending – Assumptions – Derivation of bending equation: M/I = f/y = E/R - Neutral axis – Determination of bending stresses – Section modulus of rectangular and circular sections (Solid and Hollow), I,T, Angle and Channel sections – Design of simple beam sections.

Derivation of formula – Shear stress distribution across various beam sections like rectangular, circular, triangular, I, T angle sections.

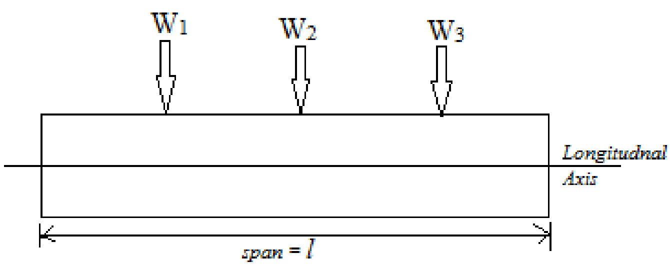
PART A: FLEXURAL STRESSES

Introduction

Beam: Beam is a structural member on which a system of external loads acts at right angles to its longitudinal axis.

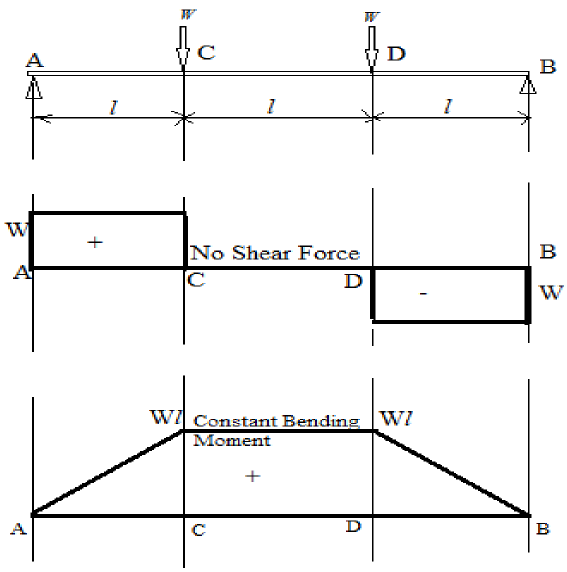
Due to these external loads, bending moments and shear forces are set-up at any point along the length of beam. Hence the beam has to resist the action of bending moment and shear force.

- The longitudinal stress produced at any section to resist the bending is known as the **bending stress or flexure.**



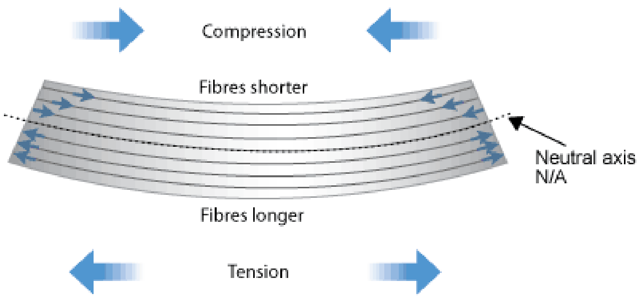
Pure bending or simple bending: If a length of beam is subjected to constant bending moment and no shear force, then stresses will be set-up in that length of the beam due to Bending Moment only and that length of beam is said to be pure bending or simple bending. Stresses are set-up in that length of the beam is known as Bending Stresses.

When a beam is bent due to application of constant bending moment, without being subjected to shear, it is said to be in a state of **simple bending or pure bending**.

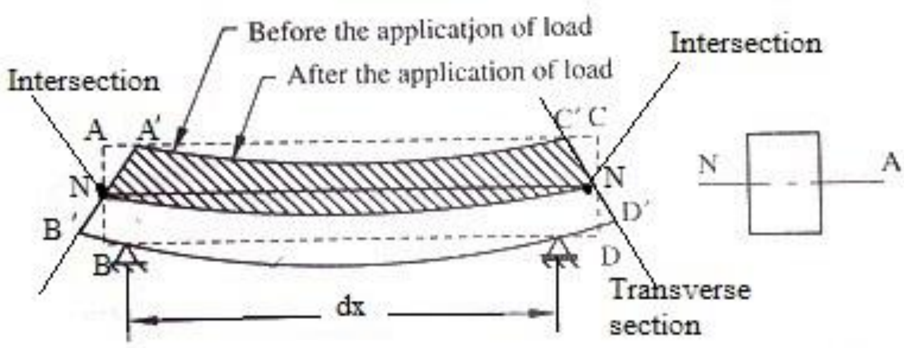

W
W
A
C
D
B
l
l
l
W
+
A
No Shear Force
C
D
B
-
W
Wl
Constant Bending
Moment
Wl
+
A
C
D
B

Theory of Simple Bending

- Consider a small length of simply supported beam subjected to bending moment.
- Consider two sections AB and CD, which are normal to axis of N-N.
- Due to action of bending moment, small length dx will deform.
- Layer of beam, which were originally of same length do not remain of same length.
- Top layer (layer above N-N) deformed from AC to A'C'. This top layer of beam has suffered compression and reduced to A'C'.



- Bottom layer (layer below N-N) deformed from BD to B'D'. This Bottom layer of beam has suffered Tension and elongated to B'D'.
- Between top and bottom of the beam, there will be longer which is neither shortened nor elongated. This layer is known as neutral layer or neutral surface.
- The line of intersection of neutral layer on a cross section (or with transverse section) of the beam is known as neutral axis (N.A.)



(a)

(b) Cross-section of the beam

- Top layer has been shortened maximum, that means compressive will be maximum at the top layer.

- The amount by which layer is compressed or stretched, depending upon the position of layer with respect to N-N.
- This theory is called theory of simple bending.

Assumptions in the theory of simple of bending

1. Material of beam is perfectly homogeneous (same material throughout) and isotropic (equal elastic property in all direction).
2. The beam material is stressed within its elastic limit and thus obeys Hooke's law.
3. The value of young's modulus (E) is same in tension and compression.
4. The transverse sections, which were plane before bending, remain plane after bending also.

   (i.e. Entire beam cross-section is assumed to rotate about neutral axis.
5. Each layer of beam is free to expand or contract independently of the layer, above or below it.
6. The radius of curvature of the beam is very large in comparison to cross-sectional dimension of the beam.
7. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

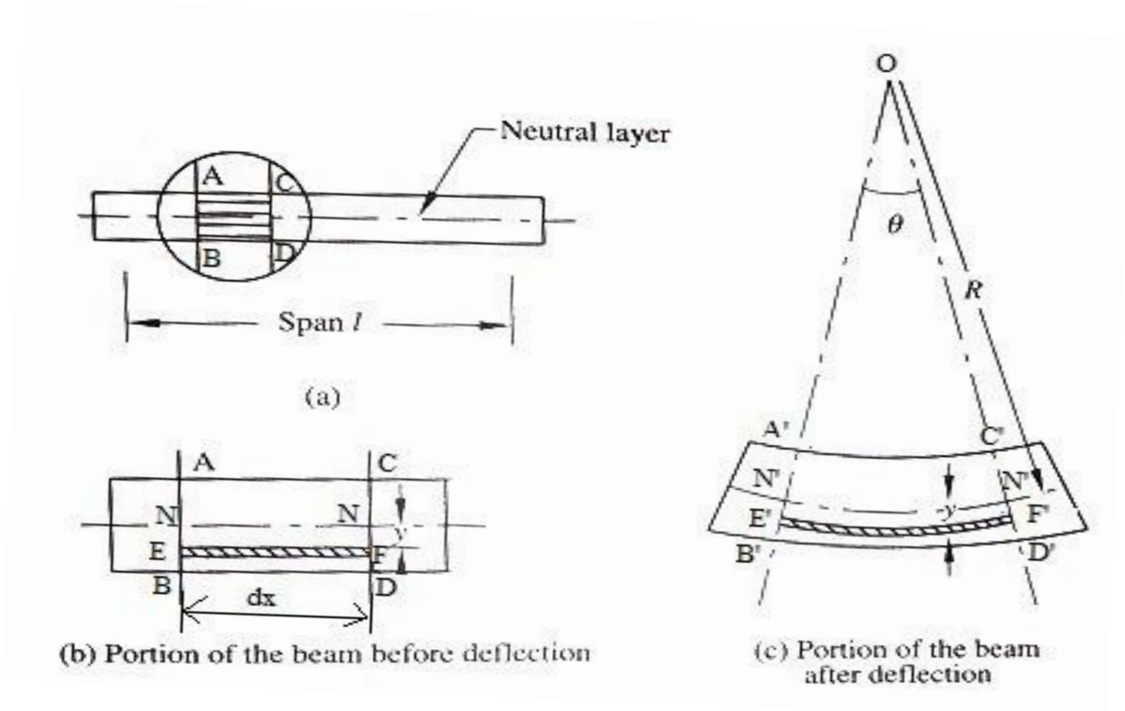
Derivation of bending equation and bending stress

Consider a portion of uniform beam subjected to pure bending. Due to action of bending, part of length dx will be deformed.

Let,

R = Radius of curvature of beam or Radius of neutral layer N'N'
θ = Angle subtended at o by A'B' and C'D'

Now consider a layer EF at a distance of y below the neutral layer NN, after the bending this layer will be elongated to E'F'



(b) Portion of the beam before deflection

(c) Portion of the beam after deflection

E'F' = NN = dx where EF = original of layer NN = length of neutral layer

After bending, the length of neutral layer N'N' will remain unchanged. But length of layer E'F' will increase.

NN = N'N' = dx

For ON'N' Angle = —— θ = ——

N'N' = R. θ

dx = R. θ

EF = R. θ

For OE'F' Angle = —— θ = ——

E'F' = (R + y). θ

Increase in length of the layer EF = E'F' – EF = (R + y). θ – R. θ = y. θ

Strain in layer EF = ———————— = —— ε = –

As R is constant, hence strain in a layer is proportional to its distance from neutral axis.

Young's modulus = —— Stress = Young's modulus × Strain

$\sigma = E \times \varepsilon$

σ = E × –

σ = y × (—)   σ α y

Since E & R both are constant, therefore stress in any layer is directly proportional to the distance of layer from the neutral layer.

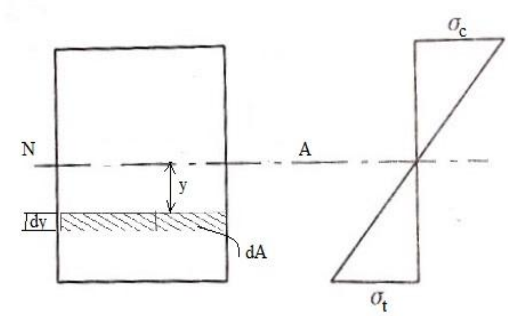
— —

- Maximum compressive stress will be experienced by top most fibre.
- Maximum tensile stress will be experienced by bottom most fibre.

Neutral Axis The neutral axis of any transverse section of beam is defined as the line of intersection of the neutral layer with transverse.

We have seen,

- If a section of beam is subjected to pure sagging moment, then the stress will be compressive at any point above the neutral axis
- And tensile below the neutral axis
- There will be no stress at neutral axis
- Stress at a distance y of neutral axis σ = y × (—)



Let N.A. is neutral axis

Consider a small layer at a distance of y from N.A.

Let dA = Area of that layer

Force on this layer = Stress on layer × Area of layer

= (y ×—)

Total force on beam = ∫(—

= — ∫

But for pure bending, there is no force on the section of beam

— dA = 0

E & R both are constant and it can't be 0

= 0

- It represents moment of area dA about N.A.
- We know that moment of any area about an axis passing through its centroid, is also equal to zero.
- Hence neutral axis coincides with centroidal axis.
- Centroidal axis of a section gives the position of N.A.

Moment of Resistance

- Due to pure bending, the layers above the N.A. are subjected to compressive stresses whereas layer below the N.A. are subjected to tensile stresses.
- These stresses form a couple (forces will act on these layers) whose moment must be equal to external moment (M).

The moment of this couple, which resist the external bending moment, is known as moment of resistance.

Force on layer at a distance y from N.A. = (y ×—) ×

Moment of this force about N.A. = Force on layer × y

= (y ×— × dA) × y

= —

Moment of forces on the section of beam (or moment of resistance) = — $\int$

Let M is external moment applied on this section.

For equilibrium the moment of resistance offered by section should be equal to external bending moment.

M = — $\int$

= Second moment of area or moment of inertia of section about N.A.

M = —

— —

We know that — —

— — —

This equation is known as bending equation or Bernoulli-Euler equation.

Section modulus

It is the ratio of moment of inertia of a section about the neutral axis to the distance of outermost layer from the neutral axis. It is denoted by Z.

Z = ——

I = Moment of inertia about N.A.

Distance of outermost layer from the neutral axis

Maximum bending moment (or moment of resistance offered by the section)

— ——

M = Z

- Moment of resistance offered by the section is maximum when section modulus Z is maximum.
- Section modulus represents strength of the section.

Section modulus for various shapes

a) Rectangular Section:

Moment of inertia of a rectangular section about an axis through its C.G.(or through N.A.)

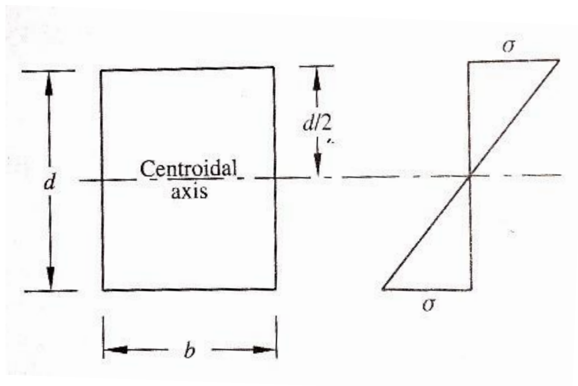
I = ——

Distance of outermost layer from N.A.

—

Z = —— —— —

Z = ——



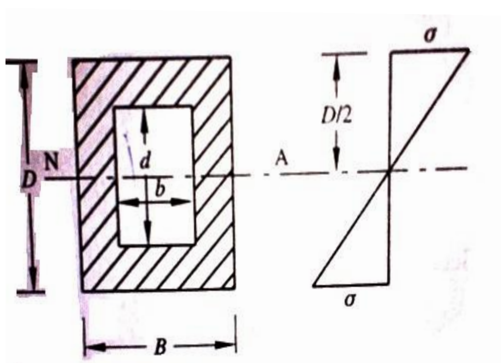
b) Hollow rectangular section:

I =—— — —— = ————

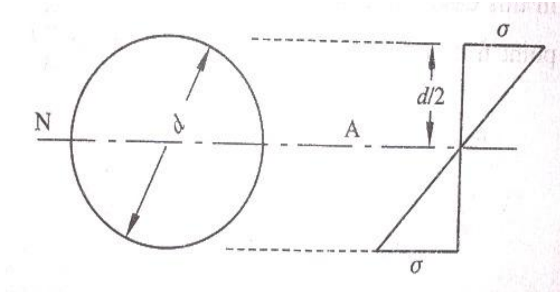
—

Z = —— ———— —

Z = ————



c) Solid Circular Section:



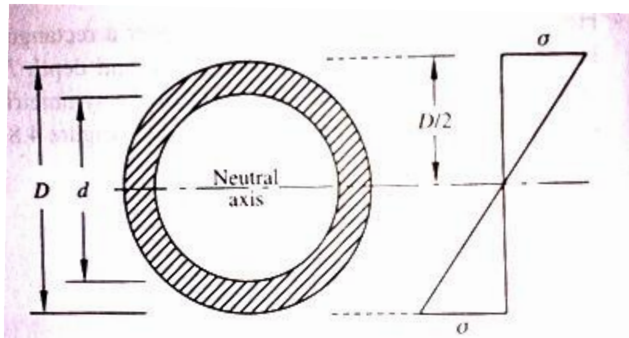
I = ——

—

Z = —— = ——   –

Z = ——

d) Hollow Circular Section:



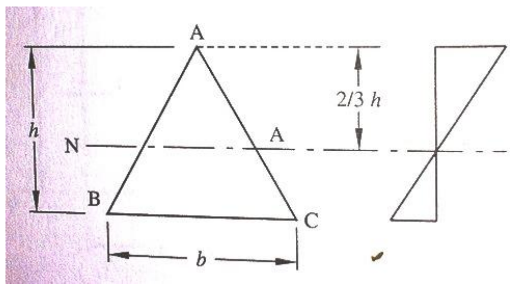
I = ——   —— = ————

—

————   –

————

e) Triangular Section:



b = base width of triangle

h = height of triangle

I = ——

—

Z = —— = ——   —

Z = ——

Strength of section: It means the moment of resistance offered by the section.

$M = \sigma . Z$

- Moment of resistance depends upon section modulus.
- Greater the value of section modulus, stronger will be the section.

— — σ = —

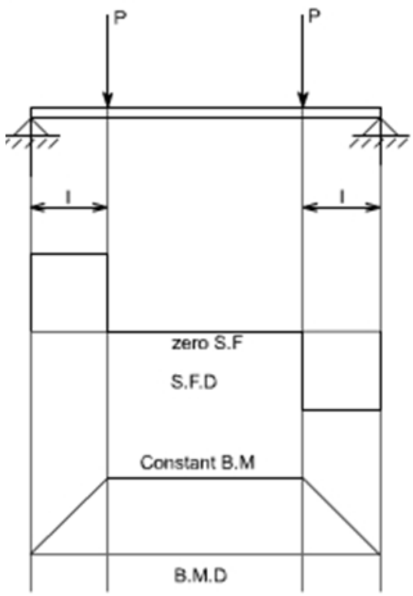
σ = — σ = —

- If section modulus is small, then stress will be more.

PART A: FLEXURAL STRESSES (CONCEPTS REVIEW)

The normal stresses developed in structural members due to bending are called flexure stresses. When a beam member is subjected to transverse loads, it bends. Depending on the loading, twisting and buckling effects may also occur. In this section, we are interested to study the bending effects alone, and not the combined effects of bending twisting and buckling.

Theory of simple bending or pure bending



Due to the eternal loading, the internal reactions developed on any cross-section of a beam may consist of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the only bending effects are prominent, we assume that the loading is such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the beam member.

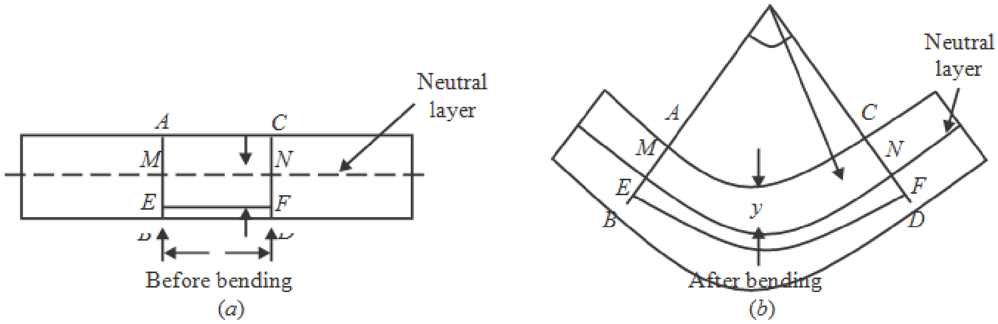
When the loading is such that a beam section experiences only constant moment and has zero shear force, then it is said to be subjected to **pure bending** or **simple bending**. For example, for the loading pattern shown, the span between the two point loads is in the state of pure bending.

Assumptions for the theory of pure bending:

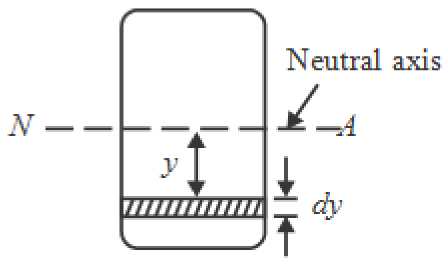
1. The beam is initially straight and has a constant cross-section.
2. Beam material is homogeneous and isotropic
3. The beam has a longitudinal plane of symmetry.
4. Resultant of the applied loads lies in the plane of symmetry.
5. The geometry of the overall member is such that bending not buckling is the primary cause of failure.
6. Elastic limit is nowhere exceeded and 'E' is same in tension and compression.
7. Plane cross - sections remains plane before and after bending.
8. Shear force at the beam cross-section is zero.

Derivation of the bending equation

Consider the beam section as shown in the figure below. Consider any two normal sections *AB* and *CD* of a beam at small distance δL apart (that is, $AC = BD = \delta L$). Let *AB* and *CD* intersect neutral layer at the points *M* and *N* respectively. When the beam is subjected to bending as shown, the top layers of the beam are subjected to compression and the bottom layers are subjected to tension. The neutral axis or the neutral layer is the layer which is neither subjected to tension or compression.



σ/y = ***E*/*R***



Figure

Let;

M = bending moment acting on beam

θ = Angle subtended at centre by the arc.

R = Radius of curvature of neutral layer *M′ N′* .

At any distance '*y*' from neutral layer *MN*, consider layer *EF*.

As shown in the figure the beam because of sagging bending moment. After bending, *A′ B′*, *C′ D′* , *M′ N′* and *E′F′* represent final positions of *AB*, *CD*, *MN* and *EF* in that order. When produced, *A′ B′* and *C′ D′* intersect each other at the *O* subtending an angle θ radian at point *O*, which is centre of curvature. As *L* is quite small, arcs *A′ C′* , *M′ N′* , *E′ F′* and *B′ D′* can be taken as circular.

Now, strain in layer *EF* because of bending can be given by *e* = (*E' F'* - *EF*)/*EF* = (*E' F'* - *MN*)/*MN*

As *MN* is the neutral layer, *MN* = *M′ N′*

$$e = \frac{E'F' - M'N'}{M'N'} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y\theta}{R\theta} = \frac{y}{R}$$

…………………………………………(i)

Let; σ = stress set up in layer EF because of bending

E = Young's modulus of material of beam.

$$E = \frac{\sigma}{e} \qquad \text{or,} \quad e = \frac{\sigma}{e}$$

……………………………………………(ii)

Equate the equation (i) and (ii);

$$\frac{y}{R} = \frac{\sigma}{E}$$

……………………………………………(iii)

At distance '*y*', let us consider an elementary strip of quite small thickness *dy*. We have already assumed that 'σ ' is bending stress in this strip.

Let *dA* = area of the elementary strip. Then, force developed in this strip = σ.*dA.*

Then the, elementary moment of resistance because of this elementary force can be given by *dM* = *f.dA.y*

Total moment of resistance because of all such elementary forces can be given by

$$\int dM = \int \sigma \times dA \times y$$

$$M = \int \sigma \times dA \times y$$

From the Equation (iii),

$$\sigma = y \times \frac{E}{R}.$$

By putting this value of f in Equation (iv), we get

$$M = \int y \times \frac{E}{R} \times dA \times y = \frac{E}{R} \int dA \times y^2$$

But

$$\int dA \cdot y^2 = 1$$

where I = Moment of inertia of whole area about neutral axis N-A.

$$M = (E/R)\ .\ I$$

$$M/I = E/R$$

$$M/I = \sigma/y = E/R$$

Where,

M = Bending moment

I = Moment of Inertia about axis of bending that is; I_{xx}

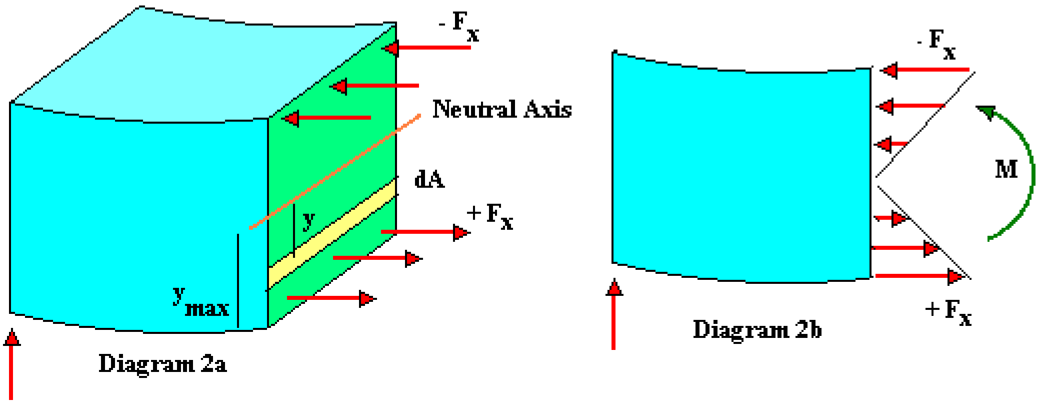
y = Distance of the layer at which the bending stress is consider

(We take always the maximum value of y, that is, distance of extreme fiber from N.A.)

E = Modulus of elasticity of beam material.

R = Radius of curvature

Determination of bending stresses



The above figure shows the distribution of bending stresses in the beam subjected to moment. The bending stress distribution along the cross-section is given in the second figure.

The bending stress, σ, is calculated by using the bending equation as

—

We can see that the bending stress is directly proportional to 'y', the distance of the fibre in consideration from the neutral axis. Thus, the bending stress is maximum at the extreme fibres of the beams.

—

Section modulus

The section modulus is defined as:

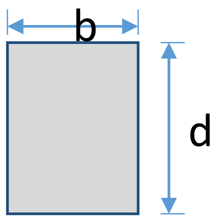
——

where I is the moment of inertia taken about the neutral axis. Thus, in the bending equation, the maximum bending stress or,

Section modulus is a pure geometric property for a given cross-section. It is most useful in the design of beams or flexural members. For general design, the elastic section modulus is used, applying up to the yield point for most metals and other common materials. It is also often used to determine the yield moment (M_y) such that $M_y = Z \; \sigma_y$, where σ_y is the yield strength of the material. The moment of resistance offered by the section is maximum if the section modulus is maximum. Thus, ***the section modulus thus represents the strength of the section***.

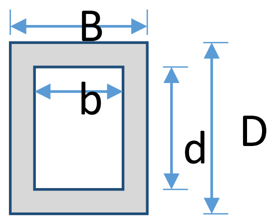
Section modulus for some of the typical sections is as shown below:

A) Rectangular section



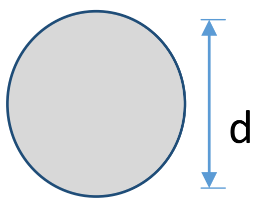
$$I = \frac{bd^3}{12}; \; Z = \frac{bd^2}{6}$$

B) Hollow Rectangular Section



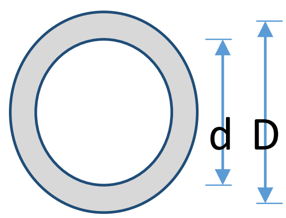
$$I = \frac{1}{12}\left(BD^3 - bd^3\right); \; Z = \frac{1}{6D}\left(BD^3 - bd^3\right)$$

C) Circular Section



$$I = \frac{\pi d^4}{64}; \; Z = \frac{\pi d^3}{32}$$

D) Hollow Circular Section



$$I = \frac{\pi}{64}\left(D^4 - d^4\right); \; Z = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

PART B: SHEAR STRESSES

Transverse and Longitudinal Shear Stresses in Beam Sections

In the previous section, we computed the bending stress (normal stress) on a cross section of a beam when the beam members are subjected to simple or pure bending. This means that we or,

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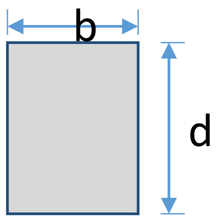
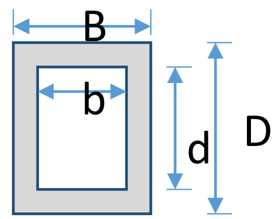


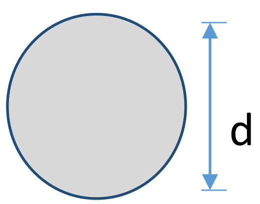
$$I = \frac{bd^3}{12};\ Z = \frac{bd^2}{6}$$

B) Hollow Rectangular Section



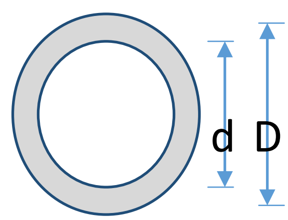
$$I = \frac{1}{12}\left(BD^3 - bd^3\right);\ Z = \frac{1}{6D}\left(BD^3 - bd^3\right)$$

C) Circular Section



$$I = \frac{\pi d^4}{64};\ Z = \frac{\pi d^3}{32}$$

D) Hollow Circular Section

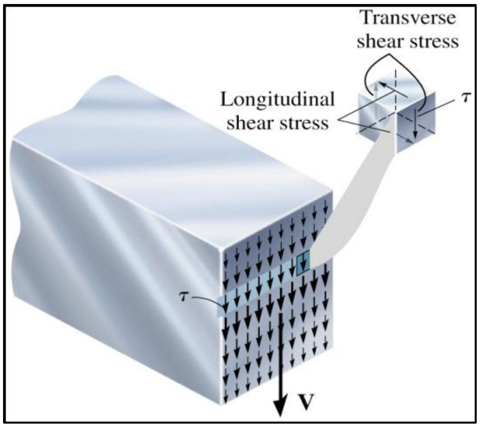


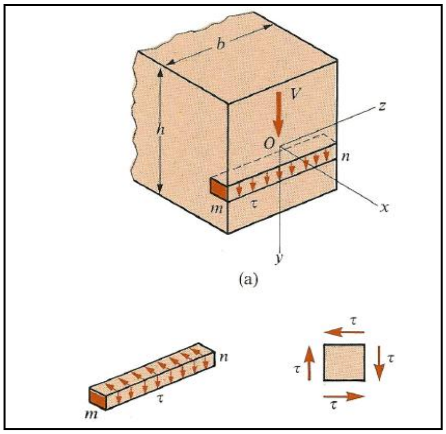
$$I = \frac{\pi}{64}\left(D^4 - d^4\right);\ Z = \frac{\pi}{32D}\left(D^4 - d^4\right)$$

PART B: SHEAR STRESSES

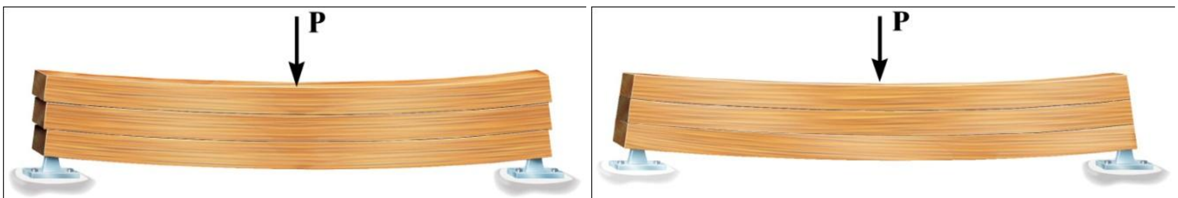
Transverse and Longitudinal Shear Stresses in Beam Sections

In the previous section, we computed the bending stress (normal stress) on a cross section of a beam when the beam members are subjected to simple or pure bending. This means that we assumed bending moment to be constant and the shear force on that section to be zero. However, when beam members are subjected to loads, in general we find that bending moment and shear forces exist together at a section. In those sections, where the bending is associated with shear force, we need to compute the shearing stress as well. In this section, we shall learn how to compute the value of the shearing stress and their distributions over a cross-section.





As we have studies from the earlier chapter of stresses, the transverse shear stresses at a section will always be accompanied by a complimentary horizontal shear stresses acting on the longitudinal layers of the beam. This is depicted in the above figures. This can also be demonstrated by applying transverse load to a set-up of layers of wooden boards, not bonded together and bonded together, as shown below.



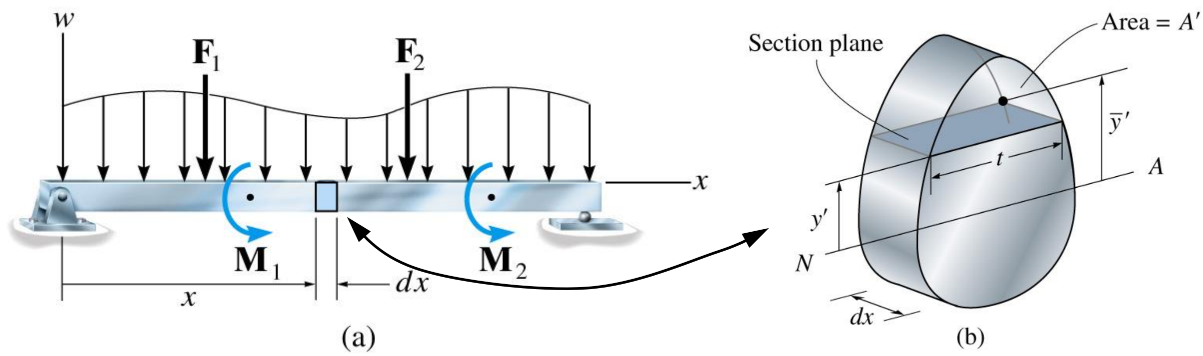
Boards not bonded together
(a)

Boards bonded together
(b)

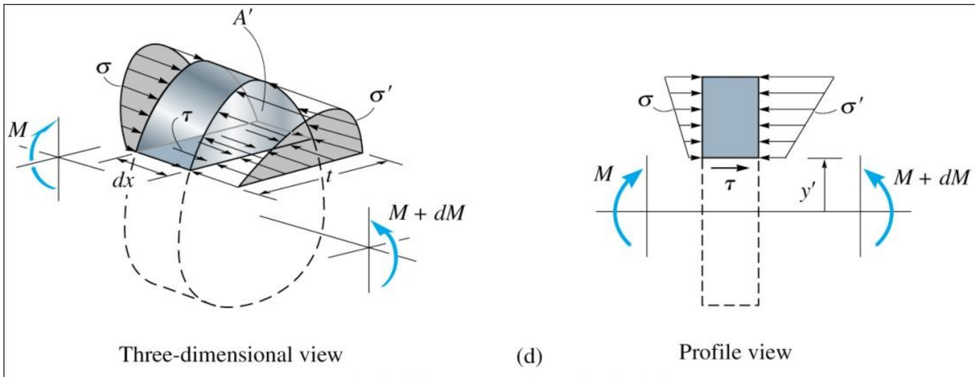
When boards are not glued together, they slip relative to each other at the layers of their separation. Thus, bending action tends to produce longitudinal displacement in the material. When boards are glued together, the slippage action is prevented and shear stress is developed at the inter layer surfaces.

Shear Stress at a Section in a Beam

Consider a section of a beam subject to a general case of loading as shown in the figure below.



The normal stresses due to bending (flexure) on the sections are as shown below.



By applying the force equilibrium in the longitudinal direction, we get

$$\int_{A'} \sigma' dA - \int_{A'} \sigma dA - \tau(tdx) = 0$$

$$\Rightarrow \int_{A'} \left(\frac{M+dM}{I}\right) ydA - \int_{A'} \left(\frac{M}{I}\right) ydA - \tau(tdx) = 0$$

Simplifying and substituting, dM/dx = V, the shear force, we get

$$\tau = \frac{1}{It}\left(\frac{dM}{dx}\right)\int_{A'} ydA$$

$$\tau = \frac{VA'\bar{y}}{It}$$

The above shear-formula has to be understood as follows. The above formula gives the shear stress at the layer of the cross-section of the beam, located at a distance of y from the neutral axis.

V is the shear force acting at that particular section of the beam.

I is the moment of inertia of the entire cross-section of the beam.

't' is the width of the cross-section, at the layer of interest, i.e. at a distance of y from the neutral axis.

A' is the portion of the area of the cross-section of the beam, above the layer of interest.

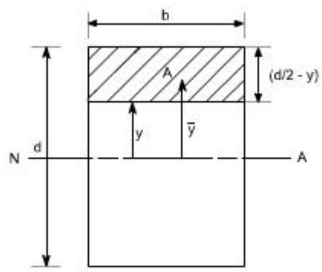
$\overline{y}$ is the distance of the centroid of the area A' from the neutral axis.

Shearing stress distribution in typical cross-sections

Let us consider few examples to determine the sheer stress distribution in a given X- sections

Shear stress distribution in beams of a rectangular section

Consider a rectangular x-section of dimension b and d



A is the area of the x-section cut off by a line parallel to the neutral axis. $\bar{y}$ is the distance of the centroid of A from the neutral axis

$$\tau = \frac{F.A.\bar{y}}{I.z}$$

$$\text{for this case, } A = b\left(\frac{d}{2} - y\right)$$

$$\text{While} \qquad \bar{y} = \left[\frac{1}{2}\left(\frac{d}{2} - y\right) + y\right]$$

$$\text{i.e} \qquad \bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right) \text{ and } z = b\,;\, I = \frac{b.d^3}{12}$$

substituting all these values, in the formula

$$\tau = \frac{F.A.\bar{y}}{I.z}$$

$$= \frac{F.b.\left(\frac{d}{2} - y\right).\frac{1}{2}.\left(\frac{d}{2} + y\right)}{b.\frac{b.d^3}{12}}$$

$$= \frac{\frac{F}{2}.\left\{\left(\frac{d}{2}\right)^2 - y^2\right\}}{\frac{b.d^3}{12}}$$

$$= \frac{6.F.\left\{\left(\frac{d}{2}\right)^2 - y^2\right\}}{b.d^3}$$

This shows that there is a parabolic distribution of shear stress with y. The maximum value of shear stress would obviously beat the location y = 0.

$$\text{Such that} \quad \tau_{max} = \frac{6.F}{b.d^3} \cdot \frac{d^2}{4}$$

$$= \frac{3.F}{2.b.d}$$

$$\text{So} \quad \boxed{\tau_{max} = \frac{3.F}{2.b.d}} \text{ The value of } \tau_{max} \text{ occurs at the neutral axis}$$

The mean shear stress in the beam is defined as

$$\tau_{mean} \text{ or } \tau_{avg} = F/A = F/b.d$$

$$\text{So} \quad \tau_{max} = 1.5\ \tau_{mean} = 1.5\ \tau_{avg}$$

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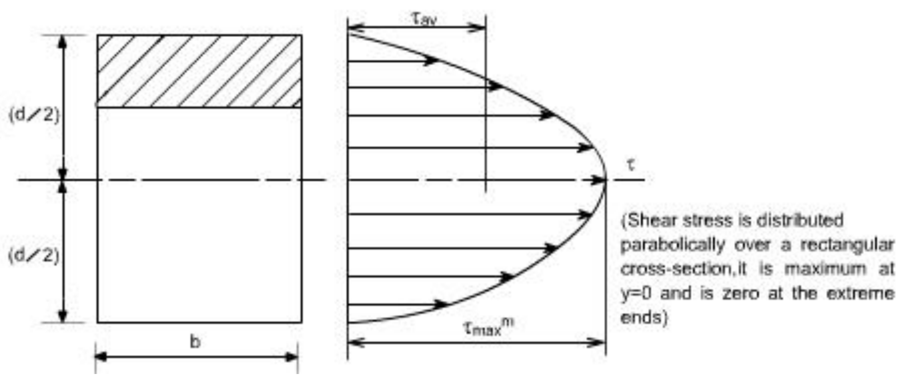
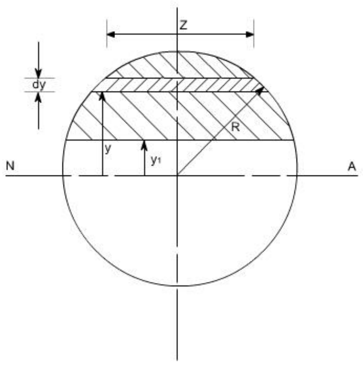
$$\text{So} \quad \tau_{max} = 1.5\ \tau_{mean} = 1.5\ \tau_{avg}$$

Therefore the shear stress distribution is shown as below.



It may be noted that the shear stress is distributed parabolically over a rectangular cross-section, it is maximum at y = 0 and is zero at the extreme ends.

Shear stress distribution in beams of circular cross-section



Let us find the shear stress distribution in beams of circular cross-section. In a beam of circular cross-section, the value of Z width depends on y.

Using the expression for the determination of shear stresses for any arbitrary shape or a arbitrary section.

$$\tau = \frac{FA\,\bar{y}}{Z\,I} = \frac{FA\int y\,dA}{Z\,I}$$

Where $\int y\,dA$ is the area moment of the shaded portion or the first moment of area.

Here in this case 'dA' is to be found out using the Pythagoras theorem

$$\left(\frac{Z}{2}\right)^2 + y^2 = R^2$$

$$\left(\frac{Z}{2}\right)^2 = R^2 - y^2 \text{ or } \frac{Z}{2} = \sqrt{R^2 - y^2}$$

$$Z = 2\sqrt{R^2 - y^2}$$

$$dA = Z\,dy = 2.\sqrt{R^2 - y^2}\,.dy$$

$I_{N.A}$ for a circular cross-section $= \dfrac{\pi R^4}{4}$

Hence,

$$\tau = \frac{FA\bar{y}}{ZI} = \frac{F}{\dfrac{\pi R^4}{4} 2\sqrt{R^2 - y^2}} \int_{y_1}^{R} 2\,y\sqrt{R^2 - y^2}\,dy$$

Where R = radius of the circle.

[The limits have been taken from y_1 to R because we have to find moment of area the shaded portion]

$$= \frac{4F}{\pi R^4 \sqrt{R^2 - y^2}} \int_{y_1}^{R} y\sqrt{R^2 - y^2}\,dy$$

The integration yields the final result to be

$$\tau = \frac{4F\left(R^2 - y_1^2\right)}{3\pi R^4}$$

Again this is a parabolic distribution of shear stress, having a maximum value when $y_1 = 0$

$$\tau_{max^m}|_{y_1 = 0} = \frac{4F}{3\pi R^2}$$

Obviously at the ends of the diameter the value of $y_1 = \pm R$ thus $\tau = 0$ so this again a parabolic distribution; maximum at the neutral axis

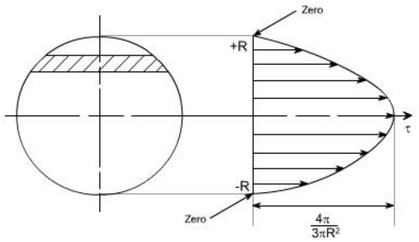
Also

$$\tau_{avg} \text{ or } \tau_{mean} = \frac{F}{A} = \frac{F}{\pi R^2}$$

Hence,

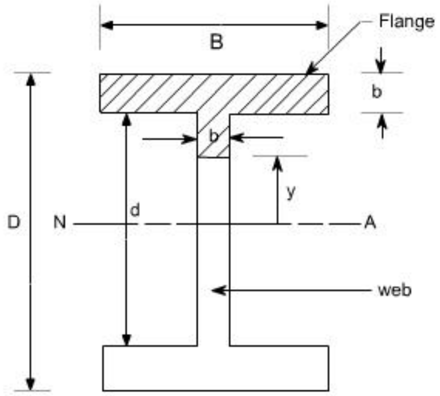
$$\boxed{\tau_{max^m} = \frac{4}{3}\tau_{avg}}$$

The distribution of shear stresses is shown below, which indicates a parabolic distribution

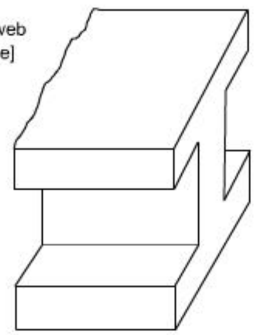


Shear stress distribution in beams of I - section

Consider an I - section of the dimension shown below

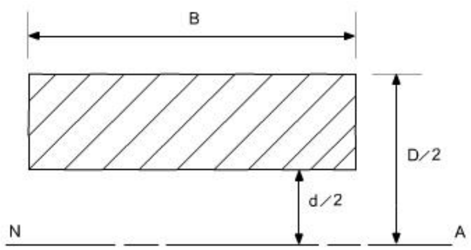


[Here flange and web thickness are same]



The shear stress distribution for any arbitrary shape is given as $\tau = \frac{F A \bar{y}}{Z I}$

Flange:



$$\text{Area of the flange} = B\left(\frac{D-d}{2}\right)$$

Distance of the centroid of the flange from the N.A

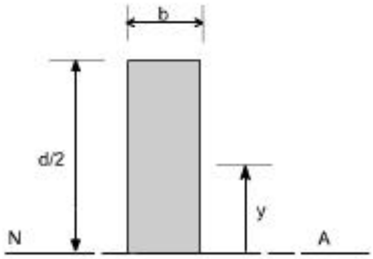
$$\bar{y} = \frac{1}{2}\left(\frac{D-d}{2}\right) + \frac{d}{2}$$

$$\bar{y} = \left(\frac{D+d}{4}\right)$$

Hence,

$$A\bar{y}\big|_{Flange} = B\left(\frac{D-d}{2}\right)\left(\frac{D-d}{4}\right)$$

Web area:



Area of the web

$$A = b\left(\frac{d}{2} - y\right)$$

Distance of the centroid from N.A

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} - y\right) + y$$

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

Therefore,

$$A\bar{y}\big|_{web} = b\left(\frac{d}{2} - y\right)\frac{1}{2}\left(\frac{d}{2} + y\right)$$

Hence,

$$A\bar{y}\big|_{Total} = B\left(\frac{D - d}{2}\right)\left(\frac{D + d}{4}\right) + b\left(\frac{d}{2} - y\right)\left(\frac{d}{2} + y\right)\frac{1}{2}$$

Thus,

$$A\bar{y}\big|_{Total} = B\left(\frac{D^2 - d^2}{8}\right) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Therefore shear stress,

$$\tau = \frac{F}{bI}\left[\frac{B\left(D^2 - d^2\right)}{8} + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

To get the maximum and minimum values of t substitute in the above relation.

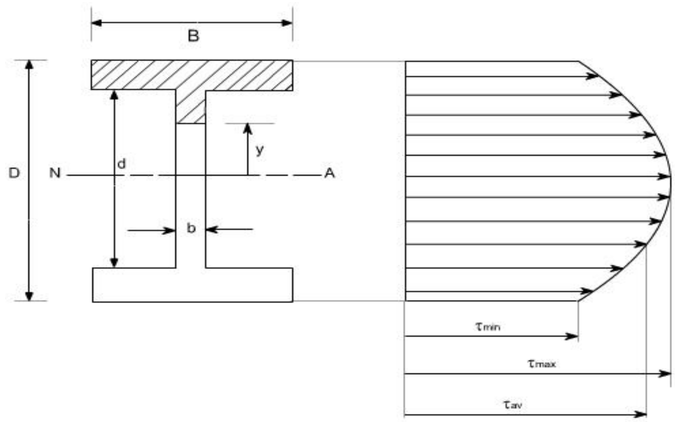
y = 0 at N. A. And y = d/2 at the tip. The maximum shear stress is at the neutral axis. i.e. for the condition y = 0 at N. A.

$$\tau_{max} \text{ at } y = 0 = \frac{F}{8bI}\left[B\left(D^2 - d^2\right) + bd^2\right]$$

The minimum stress occur at the top of the web, the term bd 2 goes off and shear stress is given by the following expression

$$\tau_{min} \text{ at } y = d/2 = \frac{F}{8bI}\left[B\left(D^2 - d^2\right)\right]$$

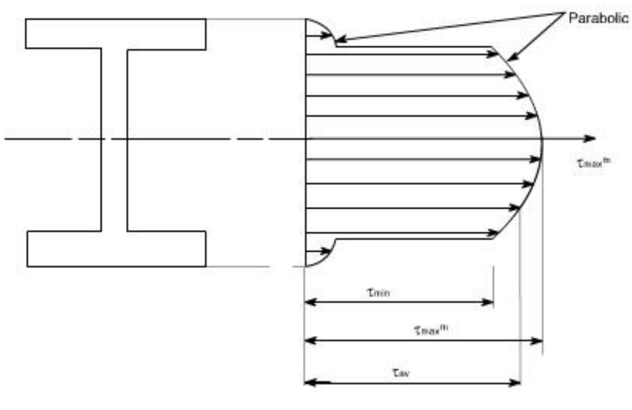
The distribution of shear stress may be drawn as below, which clearly indicates a parabolic distribution.



$$\tau_{max^m} = \frac{F}{8bI}\left[B\left(D^2 - d^2\right) + bd^2\right]$$

Note: from the above distribution we can see that the shear stress at the flanges is not zero, but it has some value, this can be analyzed from equation (1). At the flange tip or flange or web interface y = d/2.Obviously than this will have some constant value and than onwards this will have parabolic distribution.

In practice it is usually found that most of shearing stress usually about 95% is carried by the web, and hence the shear stress in the flange is neglible however if we have the concrete analysis i.e. if we analyze the shearing stress in the flange i.e. writing down the expression for shear stress for flange and web separately, we will have this type of variation.



This distribution is known as the “top – hat” distribution. Clearly the web bears the most of the shear stress and bending theory we can say that the flange will bear most of the bending stress.