UNIT-II

FLUID KINEMATICS

Fluid kinematics is the branch of fluid mechanics, which studies the fluid motion without considering the force causing the motion.

The methods used to study the fluid motion are:

1) Lagrangian method and
2) Eulerian method.

1) Lagrangian method:-

In this method a single fluid particle is followed during its motion and its velocity, acceleration and density etc, are described.

2) Eulerian method:-

It deals with the study of flow pattern of all particles simultaneously at one section. In this method the path traced by all the particles at one section and one-time are studied in detailed.

* Classification of fluid flow:-

The fluid flow is classified as:

i) Steady - unsteady flows
ii) uniform and non-uniform flows
iii) Laminar and Turbulent flows
iv) compressible and incompressible flows
v) Rotational and irrotational flow
vi) 1-D, 2-D and 3-D flow
vii) Vortex flow

Free Vortex flow | Forced Vortex flow.

i) <u>Steady flow</u>:- If discharge is constant at every point along the path of the flow. the flow is treated as steady flow.

(or)

the type of flow in which fluid characteristics like velocity, pressure, density etc, at a point does not changes with time.

Mathematically it can be expressed as

$$\frac{\partial v}{\partial t} = 0 \quad \frac{\partial p}{\partial t} = 0 \quad \& \quad \frac{\partial \rho}{\partial t} = 0$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \text{velocity} & \text{pressure} & \text{density.} \end{array}$$

<u>Ex</u>:- flow through constant diameter pipes, flow through tapering pipes.

* <u>unsteady flow</u>:- If discharge is not constant at every point along the path of the flow. the flow is treated as unsteady flow.

(or)

the type of flow in which fluid characteristics like velocity, pressure, density etc., at a point changes with respect to time.

<u>Ex</u>:- flow through channels during floods, flow through channels when the gates are just lifted.

ii) <u>uniform flow</u>:- If <u>velocity is constant</u> at every point along the path of the flow. the flow is treated as uniform flow.

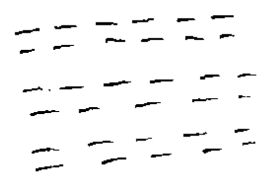
<u>Ex</u>:- flow through open channels with constant depth, flow through pipes with constant diameter.

* <u>Non-uniform flow</u>:- If <u>velocity is not constant</u> at every point along the path of the flow. the flow is treated as non-uniform flow.

<u>Ex</u>:- flow through tapering pipes, flow through channels during floods.

iii) Laminar flow:-

- The flow in which liquid particles move in layer such that one layer slides over the other layer



- the path lines of fluid particles are straight lines and they are parallel to each other.

No fluid particles will cross each other in laminar flow.

Laminar flow is observed in the case of highly viscous fluids.

Ex:- flow of thick oil in a small tube, flow of blood through the veins of human body.

* Turbulent flow:-

It is the type of flow in which the path lines of fluid particles are irregular curves crossing each other.

the flow is Erratic (unpredictable = the direction of the fluid particles cannot be identified)

Ex:- flow through rivers during floods, flow through open channels when the gates are just lifted.

iv) compressible flow:-

If the density of the fluid particles changes from point to point along the path of the flow. the flow is (treated) called compressible flow.

Ex:- flow of gas through the nozzle, flow of gas through the gas turbines

* In compressible flow:-

the type of flow in which the density of fluid particles does not change. (density constant) from point to point is called incompressible flow.

Ex:- pipe flow and channel flow.

V) Rotational flow :- If fluid particles rotates about its own mass centre while flowing. the flow is treated as Rotational flow.

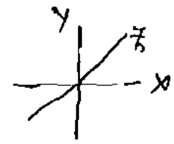
Ex: flow inside the casing of a centrifugal pump, flow of tidal waves.

* Irrotational flow:- If the fluid particles does not (ie) rotate about its own mass centre while flowing. the flow is treated as irrotational flow

Ex: All flows through the pipes and open channels are irrotational flows.

VI) 1-D, 2-D and 3-D flows:-

1-D flow:- the type of flow in which flow lines are Represented by a Straight line is known as one-dimensional flow.



(or)

the type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only.

2-D flow:- The type of flow in which stream lines or flow lines Represented by a curved line is known as 2-D flow

(or)

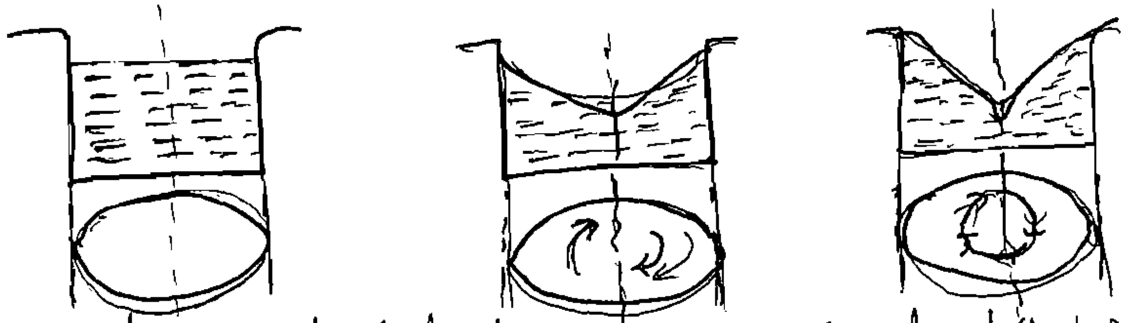
the type of flow in which the fluid parameter such as velocity is a function of time and two space coordinate. and other coordinates are neglected.

3-D flow:- the type of flow in which stream lines may be Represented in space along 3 mutual-lar direction is known as 3-D flow

(or)

the type of flow in which the fluid parameter such as velocity is a function of time and the three space coordinate.

viii) Vortex flow:-



Let us consider a cylindrical vessel containing some liquid and start rotating it about its vertical axis. We see that the liquid will also start revolving along with the vessel. After sometime we shall see that the liquid surface no longer remains level but it has been depressed down at the axis of the rotation and has risen up near the walls of vessel on all sides.

The type of flow in which the liquid flows continuously round a curved path about a fixed axis of rotation is called vortex flow.

1) Forced vortex flow:-

The type of vortex flow in which the external torque is required to rotate the fluid mass is known as forced vortex flow.

Ex:- Flow of liquid inside the impeller of a centrifugal flow, flow of water through the runner of a turbine, A vertical cylinder containing liquid which is rotated about its central axis with a constant angular velocity.

2) Free vortex flow:-

The type of vortex flow in which no external torque is required to rotate the fluid is known as free vortex flow.

Ex:- A whirlpool in a river (or) sea, flow of a liquid around a circular bend in a pipe, flow of liquid through a hole provided at the bottom of a container.

Classify the following type of flows:-

1) Flow through constant diameter pipe (steady uniform flow)
2) Flow through tapering pipe (steady non-uniform flow)
** 3) Flow through capillary tubes (laminar flow)
** 4) Flow of blood in veins of human beings (laminar flow)
5) Flow through open channels during floods (unsteady non-uniform flow/Turbulent flow)
6) Flow of Kerosene through the wicks of the stove (uniform flow)
7) Flow of water through the fountains (unsteady non uniform flow)
8) Flow of Tidal water (rotational flows)
9) Flow in a pipe considered as 1-D flow.
10) Flow between parallel plates of infinite Extent is known as 2-D flow.
11) The flow in a main stream of diverging pipe or channel is 3-D flow.
12) the flow in converging/diverging pipe is called as 3-D flow.
13) the flow in a prismatic open channel in which the width of the water depth are in the same order of Magnitude is considered as 3-D flow.
14) Motion of a liquid in a rotating tank is called rotational flow.
15) Flow of water in a wash basin is known as irrotational flow.

Classification of Flow Lines:

→ The fluid motion can be described in terms of path line, stream line, streak line

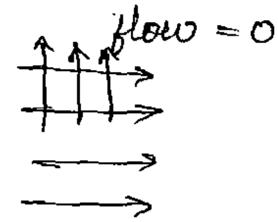
Path line :-

A path line is a curve traced by a single fluid particle during its motion. This is the outcome of lagrangian approac

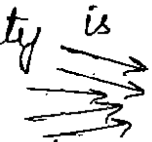
Stream line :-

It is the path traced by number of fluid particles is known as stream line. It is the outcome of Eulerian approa

Characteristics Of Stream line :-



1) The flow across a stream line is zero
2) Two stream lines won't cross each other
3) Stream lines are parallel
4) It is steady uniform flow
5) If stream lines are converging it indicates the velocity is increasing (accelerating flow)



6) If stream lines are diverging it indicates retarding flow i.e velocity is decreasing in the direction of the flow

Streak line :-

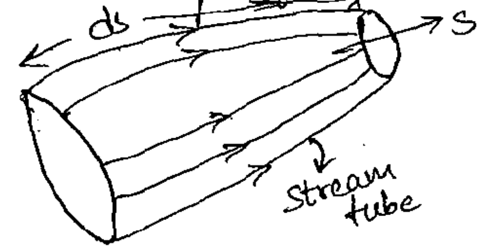
It is defined as a line i.e traced by a fluid particle passing through a fixed point in a flow field

Ex:- Smoke in the case of gases, smoke coming from a cigarette.

*** In the case of steady flow since there is no change in the fluid pattern stream line is same as that of streak line. In other words, for steady flow stream line, streak line and path line are identical

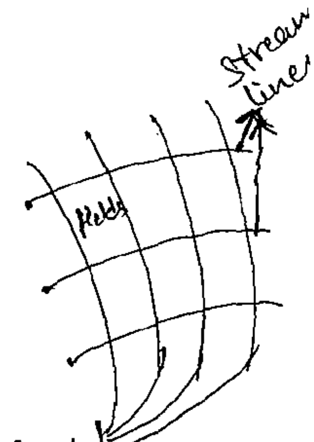
Stream Tube :

A stream tube is an imaginary tube consisting of bundle of stream lines passing through an small closed curve. A stream tube is completely bounded by stream lines at all sides except at ends. The flow across the stream tube is zero. The fluid can enter or leave the stream tube only at ends. The concept of stream tubes enternuely used in solving fluid problems.



** * Flow Net :-

A mesh or net formed by number of stream lines equipotential lines intersecting orthogonally (⊥er)



Stream lines (flow lines)

The line along which flow takes place is known as stream line (or) flow line

Water flows from points of high heads to points of low heads and makes smooth curves representing the paths followed by moving particles of water. These lines are called flow lines / stream lines

Equipotential lines :-

A line joining all the points of having equal total heads (or) potential head is called equipotential line

$$\text{Total head} = z + \frac{v^2}{2g} + \frac{p}{w}$$

** Characteristics of Flow net (or) properties of flow net :-

1) Flow lines & equipotential lines are orthogonal to each other
2) Two flow lines / two equipotential lines can never meet (or) cross each other
3) Fields are kept approximately squares

Applications :

1) For the given boundaries of flow if the velocity and pressure distribution at a section are known the velocity and pressure distribution at any other section can be calculated

$$A_1 V_1 = A_2 V_2$$

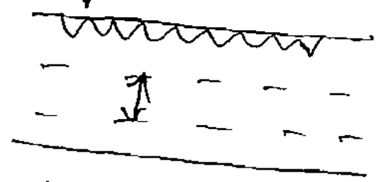
2) Loss of flow due to seepage in earthen dams and unlined canals can be determined
3) Uplift pressure under a dam or any hydraulic structure can be determined

Assumptions involved in flow net :-

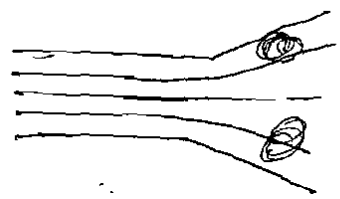
1) Flow is steady
2) Flow should be irrotational
3) Flow is governed by gravitational forces
4) The fluid weight does not control the flow phenomenon

Limitations of flow net :-

1) Flow net Analysis is not applicable close to the boundaries, because the effect of viscosity are permanant



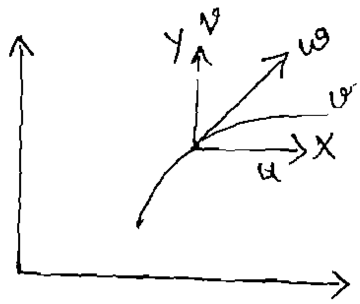
2) Flow net analysis is not applicable for sharp diverging flows because the flow pattern is not represented by flow net



Velocity and Acceleration :

Let V be the resultant velocity at any point in a fluid flow. Let u, v, w are its components in x, y and z directions.

The velocity components are the functions of space co-ordinate and Time.



Mathematically it can be expressed as

$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

$$w = f_3(x, y, z, t)$$

Resultant velocity is given by

$$V = \sqrt{u^2 + v^2 + w^2}$$

Let a_x, a_y and a_z are total acceleration in x, y and z directions respectively. Then by the chain rule of differentiation, we have

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t}.$$

$$\therefore \quad a_x = \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v + \frac{\partial u}{\partial z} \cdot w + \frac{\partial u}{\partial t}$$

$$a_y = \frac{dv}{dt} = u \cdot \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{dw}{dt} = u \cdot \frac{\partial w}{\partial x} + v \cdot \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

For Steady flow

$$\frac{\partial u}{\partial t} = 0 \;;\; \frac{\partial v}{\partial t} = 0 \;,\; \frac{\partial w}{\partial t} = 0 .$$

Hence accelerations in x, y and z directions becomes

$$a_x = \frac{\partial u}{\partial x} \cdot u + \frac{\partial u}{\partial y} \cdot v + \frac{\partial u}{\partial z} \cdot w$$

$$a_y = u \cdot \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \cdot \frac{\partial v}{\partial z} \quad \text{and}$$

$$a_z = u \cdot \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \cdot \frac{\partial w}{\partial z}$$

Total acceleration is $a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

local acceleration :

It is defined as the rate of ↑ce of velocity with respect to time at a given point in a flow field is Known as local acceleration. $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ are local accelerations.

Convective acceleration :

It is defined as the rate of change of velocity due to change of position of fluid particles in a fluid flow.

In the above equations the expression

$\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$, $\frac{\partial w}{\partial t}$ is called local accelerations.

In the above equations the expression other than $\frac{\partial u}{\partial t}$, $\frac{\partial v}{\partial t}$ and $\frac{\partial w}{\partial t}$ is called Convective acceleration.

Acceleration in any direction = Convective acceleration + local acceleration.

For Steady flow: Total acceleration = Convective acceleration.

For uniform flow: Total acceleration = local acceleration

For Steady - Uniform flow:

Total acceleration = 0

problems

1) The velocity vector in a fluid flow is given by $V = 4x^3 i - 10x^2 y j + 2t K$. Find the velocity and acceleration of a fluid particle at (2, 1, 3).

Sol:- Given

$$V = \underset{u}{4x^3} i - \underset{v}{10x^2 y} j + \underset{w}{2t} K$$

$u = 4x^3$ | $v = -10x^2 y$ | $w = 2t$

$u = 4 \times 2^3$ | $= -10 \cdot 2^2 \cdot 1$ | $= 2 \times 1$

$u = 32$ | $v = -40$ | $w = 2$

$$V = \sqrt{32^2 + (-40)^2 + 2^2} = 51.26$$

$$U = 4x^3$$

$$\frac{\partial u}{\partial x} = 12x^2 \qquad \frac{\partial u}{\partial y} = 0 \qquad \frac{\partial u}{\partial t} = 0 \qquad \frac{\partial u}{\partial z} = 0$$

$$a_x = u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$= 32 \times 12x^2$$

$$= 32 \times 12 \times 4$$

$$a_x = 1536$$

$$v = -10x^2y$$

$$\frac{\partial v}{\partial x} = -20xy \qquad \frac{\partial v}{\partial y} = -10x^2 \qquad \frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} = 0.$$

$$a_y = 32 \times (-20xy) + (-40) \times -10x^2$$

$$= -32(20)(2) + 400 \cdot 4$$

$$= 320 \text{ units.}$$

$$w = 2t$$

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial w}{\partial t} = 2$$

$$a_z = 2$$

$$\therefore a = \sqrt{(1536)^2 + (320)^2 + 2^2} = 1568 \cdot 9$$

2. A fluid flow field is given by $v = x^2 y i + y^2 z j - (2xyz + yz^2) k$. prove that it is a case of possible steady incompressable flow. Calculate the velocity and acceleration at a point (2,1,3)

Sol:- Given

$$v = \underset{u}{x^2 y} i + \underset{v}{y^2 z} j - \underset{w}{(2xyz + yz^2)} k$$

$u = x^2 y$ $\quad$ $v = y^2 z$ $\quad$ $w = -(2xyz + yz^2)$

$u = 4$ $\quad$ $v = 3$ $\quad$ $w = -(12+9) = -21$

$$\therefore \; V = \sqrt{4^2 + 3^2 + (21)^2}$$

$$= 21.587$$

$$u = x^2 y$$

$$\frac{\partial u}{\partial x} = 2xy \quad \frac{\partial u}{\partial y} = x^2 \quad \frac{\partial u}{\partial z} = 0 \quad \frac{\partial u}{\partial t} = 0.$$

$$\therefore \; a_x = 2xy \cdot (x^2 y) + 3(x^2)$$

$$= 4 \cdot 4 + 3 \cdot 4$$

$$= 16 + 12$$

$$= 28$$

$$v = y^2 z$$

$$\frac{\partial v}{\partial y} = 2yz \quad \frac{\partial v}{\partial z} = y^2 \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial v}{\partial t} = 0.$$

$$a_y = (3)2yz - 21(y^2)$$

$$= 18 - 21$$

$$= -3$$

$$w = -(2xyz + yz^2) \qquad a_z = -9$$

$$\frac{\partial w}{\partial x} = -2yz \qquad \frac{\partial w}{\partial y} = -(2xz + z^2) \qquad \frac{\partial w}{\partial z} = 2xy + 2yz$$

$$\frac{\partial w}{\partial t} = 0.$$

$$a_z = 4\cdot(-2yz) + (-3)(2xz + z^2) + 21(2xy + 2yz)$$

$$= 4(-6) + (-3)(12 + 9) + 21(4 + 6)$$

$$= -24 - 63 + 21(10)$$

$$a_z = 123$$

$$\therefore \quad a = \sqrt{(28)^2 + 9 + (123)^2}$$

$$a = 126.18.$$

Continuity Equation:-

Like Solid mechanics in fluid mechanics also the following three principles are used for the analysis of fluid motion. They are

1) conservation of mass
2) Conservation of energy
3) Conservation of momentum.

1) Conservation of mass:-

It states that mass can neither be created nor be destroyed on the basis of this principle the continuity equation is derived.

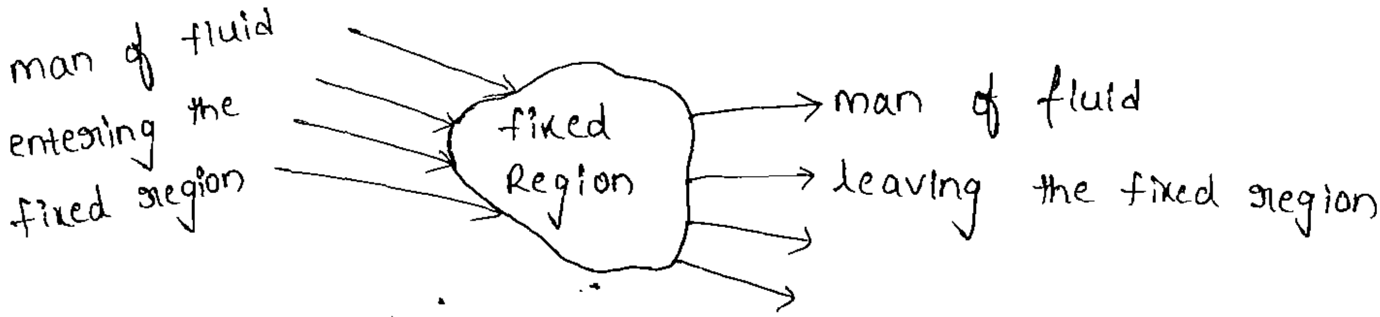
2) conservation of energy:-

It states that energy can neither be created nor destroyed. Based on this principle energy equation can derived.

3) Conservation of momentum:-

The resultant impulse (force x time) acting on the fluid is equal to change in momentum of the fluid. Based on this principle momentum equation is derived.

In fm the conservation of mass mathematically expressed as

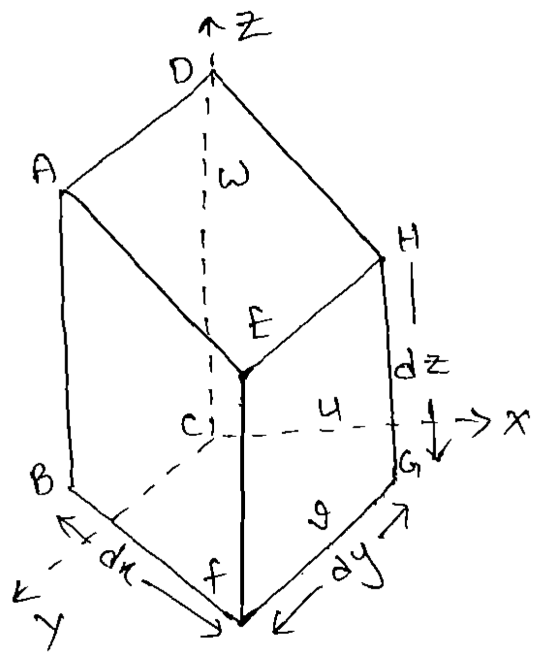


Rate of increase of fluid mass with in the fixed region = mass of fluid leaving the fixed (region) Region - mass of fluid entering the fixed region

for steady flow : The rate of ↑ce of fluid mass with in the fixed region = 0

Hence, the rate at which the fluid mass enters the fixed region = The rate at which the fluid mass leaves the fixed region.

Continuity Equation for 3-D flow :-



Consider a fluid element of length dx, dy and dz in the direction of x, y and z. Let u, v, w are inlet velocity components in x, y and z directions.

mass of the fluid entering the face ABCD is =

$$= \text{Density} \times \text{volume of fluid } (A \times h)$$

$$= \rho \times Q$$

(Volume = discharge)

$$= \rho \times A \times v$$

$$= \rho \times (dz\,dy) \times u$$

Change of fluid mass per unit distance =

$$= \frac{\partial}{\partial x}(\rho u \cdot dz \cdot dy) \cdot dx$$

mass of fluid leaving the face EFGH per second is

$$= (\rho \cdot dz \cdot dy \cdot u) + \frac{\partial}{\partial x}(\rho \cdot u \cdot dz\,dy)\,dx$$

Net gain in fluid mass in x-direction = - EFGH + ABCD

$$= -\frac{\partial}{\partial x}(\rho \cdot u \cdot \partial z \cdot \partial y) \cdot dx$$

Net gain in mass in y-direction $= -\frac{\partial}{\partial y}(\rho \cdot v \cdot dx \cdot dy)\,dz$

Net gain in mass in z-direction $= -\frac{\partial}{\partial z}(\rho \cdot w \cdot dx \cdot dy \cdot dz)$

Total net gain in mass of fluid $= -\left(\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho \cdot v) + \frac{\partial}{\partial z}(\rho \cdot w)\right) \cdot dx \cdot dy \cdot dz$

Rate of increase of mass of fluid elements with respect to time $= \frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz)$

$\rho \cdot v$

According to principle of conservation of mass the rate of ↑ce of mass of fluid element with respect to time = difference of fluid man entering the fixed region and leaving the fixed region.

$$\frac{\partial}{\partial t}(\rho \cdot dx \cdot dy \cdot dz) = -\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx \cdot dy \cdot dz$$

$$\frac{\partial \rho}{\partial t} = -\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]$$

$$\frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right) = 0$$

This is a general continuity equation.

<u>case (i):-</u> If flow is steady continuity equation becomes

$$\left(\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right) = 0$$

<u>case (ii):-</u> for steady in compressible flow
↓
(density constant)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

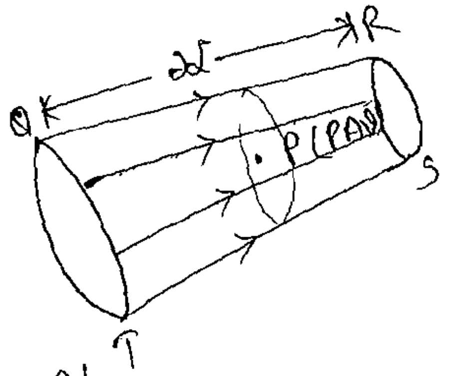
<u>case (iii):-</u> for <u>2-D</u> <u>flow</u> i.e., <u>only</u> <u>in</u> <u>x</u> <u>and</u> <u>y</u> <u>directions</u>:-

for steady - incompressible flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

Continuity equation in 1-D flow:-

Let us consider an elementary stream tube of length ∂S as shown in figure.



let us consider a point P at the centre of the tube let the man density, velocity and area at the section be ρ, v and A respectively.

The man rate of flow exactly at $P = \rho \times Q$

$$= \rho \times A \times V$$

The man rate of flow entering the tube through the face TQ $(m_1) = \rho AV - \frac{\partial}{\partial S}(\rho AV)\frac{\partial S}{2}$

The man rate of flow leaving the tube through the face RS $(m_2) = \rho AV + \frac{\partial}{\partial S}(\rho AV)\frac{\partial S}{2}$

Net gain in mass rate of flow $= m_1 - m_2$

$$= 1 - \frac{\partial}{\partial S}(\rho AV)\partial S \text{ --- (1)}$$

Rate of Ice of man with respect to time $= \frac{\partial}{\partial t}(\rho \cdot A \cdot \partial S)$ — (2)

According to law of conservation of man (1) and (2) are equal. on equating them, we get

$$\frac{\partial}{\partial t}(\rho \cdot A \cdot \partial S) = \frac{-\partial}{\partial S}(\rho VA)\partial S$$

$$\therefore \frac{\partial \rho A}{\partial t} + \frac{\partial}{\partial d}(\rho A v) = 0$$

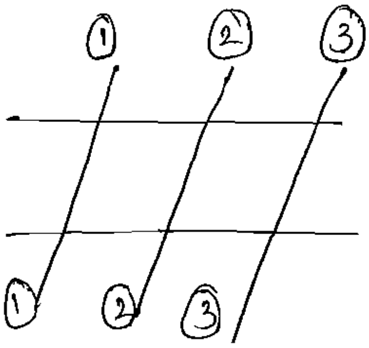
∴ It is the general continuity equation for 1-D flow.

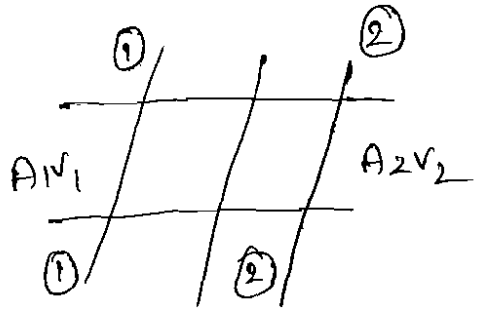
Case (i):- for steady flow the continuity equation becomes $\frac{\partial}{\partial d}(\rho A v) = 0$

Case (ii):- for steady-incompressible flow the continuity equation becomes $\frac{\partial}{\partial d}(A v) = 0$

from the above equation the equation can be written as

$A_1 v_1 = A_2 v_2 = A_3 v_3$, the subscripts 1,2,3 represents cross sectional areas and velocity of fluid at three different sections.

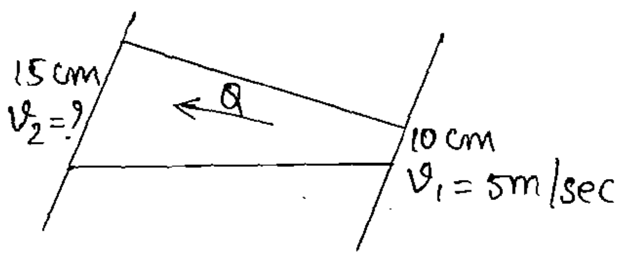




$A v = 0$

problems:

1) The diameter of a pipe at the sections ① and ② are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section ① is 5m/sec. Determine also the velocity at section ②.



$$Q = A \times V$$

$$Q = \frac{\pi \times \left(\frac{10}{100}\right)^2}{4} \times 5 = 0.039 \ m^3/sec$$

$$Q = A_2 V_2$$

$$0.039 = \frac{\pi \times \left(\frac{15}{100}\right)^2}{4} \times V_2$$

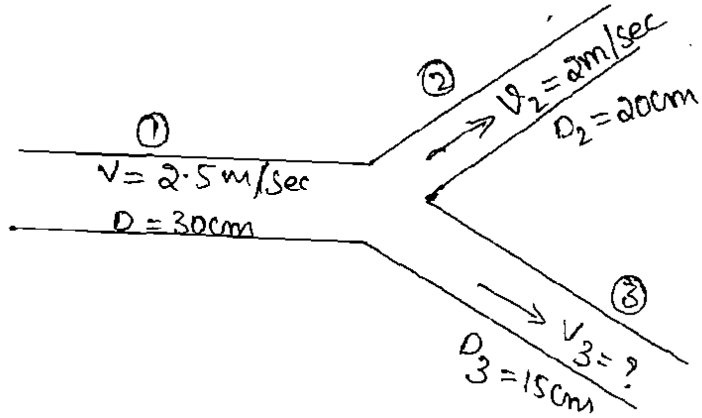
$$\therefore V_2 = 2.22 \ m/sec.$$

2) A 30 cm diameter pipe conveying water branches into two pipes of diameters 20cm and 15cm respectively. If the average velocity in the 30cm diameter pipe is 2.5m/sec. Find the discharge in this pipe also determine the velocity in 15cm pipe if the average velocity in 20 cm diameter pipe is 2m/sec.

Sol:

$$Q = \frac{\pi \times \left(\frac{30}{100}\right)^2 \times 2.5}{4}$$

$(A \times v)$



$$= 0.1767\ m^3/Sec$$

$$Q = Q_2 + Q_3$$

$$0.1767 = \frac{\pi \times \left(\frac{20}{100}\right)^2}{4} \cdot 2 + \frac{\pi \times \left(\frac{15}{100}\right)^2}{4} \times v_3$$

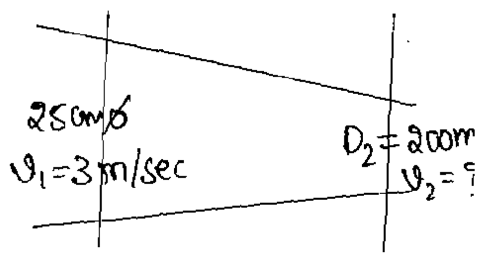
$(A_2 v_2)$ $(A_3 v_3)$

$$\therefore v_3 = 6.44\ m/sec.$$

3) A 25cm diameter pipe carries oil of specific gravity 0.9 at a velocity of 3m/sec. at another section the diameter is 20cm. Find the velocity at this section and also the mass rate of flow of oil.

Sol:

$$A_1 V_1 = A_2 V_2$$



$$\frac{\pi}{4} \times \left(\frac{25}{100}\right)^2 \times 3 = \frac{\pi}{4} \times \left(\frac{20}{100}\right)^2 \times v_2$$

$$\therefore v_2 = 4.68\ m/sec.$$

$$Q = A_1 V_1 = A_2 V_2$$

Mass rate of flow $= \rho \times Q$

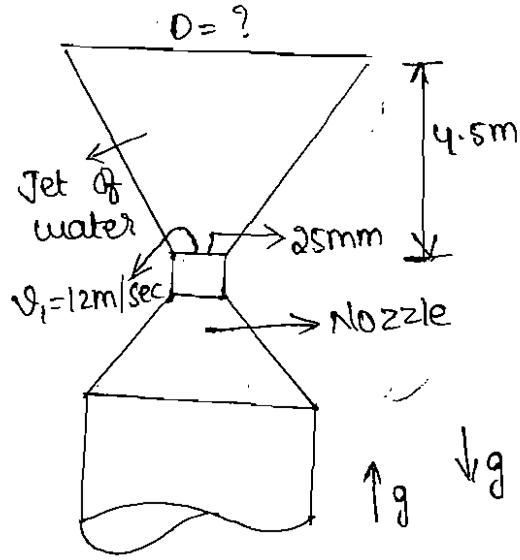
$$= \rho \times A_1 v_1$$

$$= (0.9 \times 1000) \times \frac{\pi}{4} \times \left(\frac{25}{100}\right)$$

$$= 132.23\ kg/sec.$$

4) A Jet of water from a 25mm diameter nozzle is directed vertically upwards. Assuming that the jet remain circular and neglecting any loss of energy. what will be the diameter at a point 4.5m above the nozzil. If the velocity with which the jet leaves the nozzle is 12m/se

Sol:



$$v^2 - u^2 = \pm 2gh$$

$$v^2 - u^2 = -2gh$$

$$v_2^2 - v_1^2 = -2gh$$

$$v_2^2 = -2 \times 9.81 \times 4.5 + (12)^2$$

$$\therefore v_2 = 7.46\ m/sec$$

Continuity equation is $A_2 V_2 = A_1 V_1$

$$\frac{\pi}{4} \cdot D_2^2 \times 7.46 = \frac{\pi}{4} \times \left(\frac{25}{1000}\right)^2 \times 12$$

$$D_2 = 31.7\ mm$$

Velocity potential Function:

It is defined as scalar function of space and time such that its partial -ve derivative w.r.t any direction gives the fluid velocity in that direction.

Mathematically it is defined as.

u, v, w are velocity components in x, y and z respectively.

$$\therefore u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$w = -\frac{\partial \phi}{\partial z}$$

The -ve Sign indicates that the velocity potential ↓ces in the case of flow.

In other words it indicates that the flow is always in the direction of ↓cing pipe ϕ ↓

For steady-incompressible 3D flow the continuity equ is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

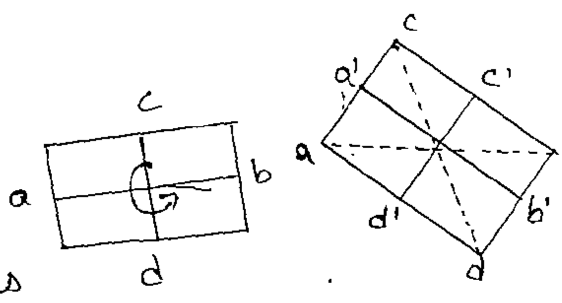
Substituting u, v, w values in above equ. we get

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

The above equ is called as laplace equation For any function ϕ that Satisfies laplace equ will Correspond to some case of fluid flow (it may be rotational / Irrotational flow)

Rotational Function properties:

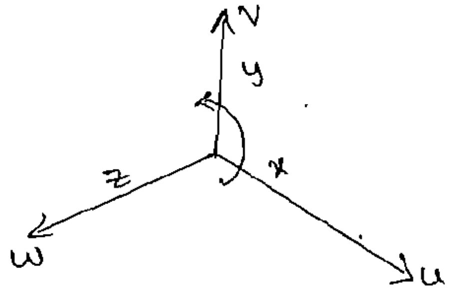
A fluid element is said



to be a Rotation w.r.t z-axis If its both axis horizontal (x-axis) as well as vertical

www.Jntufastupdates.com

(Y-axis) Rotates in the same direction x, y, z.



$z \to x-y$

Conditions for Rotation:

The conditions for Rotation / Rotational components Rotation about z, x, y.

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$u = \frac{-\partial \phi}{\partial x} \;;\quad v = \frac{\partial \phi}{\partial y} \;;\quad w = \frac{-\partial \phi}{\partial z}.$$

$$\omega_z = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x\,\partial y} + \frac{\partial^2 \phi}{\partial x\,\partial y}\right]$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial^2 \phi}{\partial z\,\partial x} - \frac{\partial^2 \phi}{\partial x\,\partial z}\right]$$

$$\left.\begin{array}{l}\omega_x = 0\\ \omega_y = 0\\ \omega_z = 0\end{array}\right\} \text{Irrotational}$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial^2 \phi}{\partial x\,\partial y} - \frac{\partial^2 \phi}{\partial x\,\partial y}\right]$$

$\omega_x \neq \omega_y \neq \omega_z = 0 \longrightarrow$ Rotational when the Rotational components are zero the flow is Called irrotational. Hence the properties of the potential functions are.

[1] If velocity potential components are zero the flow is called irrotational. Hence, the properties of the potential functions are]

1) If velocity potential exists the flow should be irrotational.

2) If velocity potential satisfies the laplace equation it represents the possible steady - incompressible irrotational flow.

3) If ϕ = Constant at every point along the flow line is called equipotential line.

(or)

Along equipotential line the ϕ is constant.

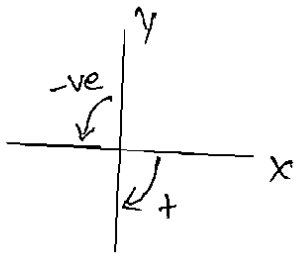
Stream Function:

It is defined as the scalar function of space and time such that its partial derivative w.r.t any direction gives the velocity compound at right angles to the direction. It is denoted by ψ (psi)

It is defined only for 2-Dimensional flow so, Mathematically for steady-flow it is defined as $\psi(x,y)$ such that

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$



For 2-Dimensional flow the Continuity equ is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{\partial \psi}{\partial y}$$

$$v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0 \implies 0 = 0$$

Hence existence of stream function ψ is a possible case of fluid flow. The flow may be Rotational or irrotational flow.

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$= \frac{1}{2}\left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right]$$

$\omega_z \neq 0 \longrightarrow$ rotational

$\omega_z = 0 \longrightarrow$ irrotational

The properties of stream functions are:

1) If stream function ψ exists it is a possible case of fluid flow which may be rotational or irrotational.

2) If stream function satisfies the Laplace equation It is a possible case of irrotational flow.

Relationship b/w velocity potential function and stream function:

velocity potential	stream Function	Relation
$u = \frac{-\partial \phi}{\partial x}$	$u = -\frac{\partial \psi}{\partial y}$	$\boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$
$v = \frac{-\partial \phi}{\partial y}$	$v = +\frac{\partial \psi}{\partial x}$	$\boxed{-\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}}$

Equi-potential line:

A line along which the velocity potential function is constant is known as Equipotential line.

$\phi \longrightarrow$ constant

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$d\phi = -u\,dx - v\,dy$$

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$\frac{\partial \phi}{\partial x} = -u \; ; \; \frac{\partial \phi}{\partial y} = -v$$

For equipotential line.

$$0 = -u \cdot dx - v \cdot dy$$

$$u\,dx = -v\,dy$$

$$\therefore \frac{dy}{dx} = \frac{-u}{v}$$

The above equation represents the slope of equipotential line.

Steam line / Flow line:

A line along which steam line (ψ) is constant is known as stream line / Flow line.

$$\psi \longrightarrow \text{constant.}$$

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

$$d\psi = 0$$

$$d\psi = \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot \partial y$$

$$0 = v \cdot dx - u \cdot dy$$

$$\frac{dy}{dx} = \frac{v}{u}$$

$\therefore$ The above equation represents the slope of stream line or flow line.

Note: The product of slope of equipotential line and straight line at the point of intersection is $\underline{-1}$

Problems

1) Is the flow represented by $u = 2x$; $v = -2y$ physically possible if so, obtain the expression for stream function. Is the flow is irrotational if so obtain velocity potential function.

Sol:

$$u = 2x \quad ; \quad v = -2y$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$2 - 2 = 0$$

$$0 = 0 \text{ (True)}$$

∴ The flow is possible

$$\frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial x} = 2y$$

$$\int \partial \psi = \int 2y \cdot dx$$

$$\psi = 2xy + f(y)$$

As integration is done w.r.t "x" the constant term must be in terms of y or a numerical constant.

$$\frac{\partial \psi}{\partial y} = 2x + f'(y)$$

$$u = 2x + f'(y)$$

$$2x = 2x + f'(y)$$

$$f'(y) = 0 \Rightarrow f(y) = 0$$

$$\psi = 2xy + \underbrace{f(y)}_{0}$$

$$\therefore \psi = 2xy$$

∴ The above equation is expression for stream function

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

$u = 2x;\ v = -2y$

$\frac{\partial v}{\partial x} = 0;\ \frac{\partial u}{\partial y} = 0$

$\frac{1}{2}(0-0) = 0$

$0 = 0$ (True)

$\therefore$ The flow is irrotational.

$$\frac{\partial \phi}{\partial x} = -u$$

$$\frac{\partial \phi}{\partial x} = -2x$$

$$\int \partial\phi = \int -2x \cdot dx$$

$$\phi = -\frac{2x^2}{2} + f(y)$$

$$\phi = -x^2 + f(y)$$

$$\frac{\partial \phi}{\partial y} = f'(y)$$

$$\frac{\partial \phi}{\partial y} = -v$$

$$-v = f'(y)$$

$$2y = f'(y)$$

$$y^2 = f(y)$$

$$\phi = -x^2 + y^2$$

$$\therefore \phi = y^2 - x^2$$

2) The velocity components in a 2-D flow for an incompressible fluid are as follows $u = \frac{y^3}{3} + 2x - x^2y$ and $v = xy^2 - 2y - \frac{x^3}{3}$ obtain an expression for stream function ~~ψ~~.

Sol(:) Given

$$u = \frac{y^3}{3} + 2x - x^2y$$

$$v = xy^2 - 2y - \frac{x^3}{3}$$

$$\frac{\partial \psi}{\partial x} = -v;\ \frac{\partial \psi}{\partial y} = u$$

$$\frac{\partial \psi}{\partial x} = -\left(xy^2 - 2y - \frac{x^3}{3}\right)$$

$$\frac{\partial \psi}{\partial x} = -xy^2 + 2y + \frac{x^3}{3} \rightarrow (1)$$

$$\frac{\partial \psi}{\partial y} = \frac{y^3}{3} + 2x - x^2y \rightarrow (2)$$

from (1) $\Rightarrow$

$$d\psi = \left(-xy^2 + 2y + \frac{x^3}{3}\right) \cdot dx$$

$$\therefore \psi = \frac{-x^2y^2}{2} + 2xy + \frac{x^4}{12} + f(y)$$

$$\frac{\partial \psi}{\partial y} = -\frac{x^2}{2} \cdot 2y + 2x + f'(y)$$

$$= -x^2y + 2x + f'(y)$$

$$\left(-x^2y + 2x + \frac{y^3}{3}\right)$$

$$\therefore f'(y) = \frac{y^3}{3}$$

$$f(y) = \frac{y^4}{12}$$

$$\therefore \psi = -\frac{x^2y^2}{2} + 2xy + \frac{x^4}{12} + \frac{y^4}{12}.$$

3) The velocity components in a 2-D flow are $u = \frac{y^3}{3} + 2x - x^2y$; $v = xy^2 - 2y - \frac{x^3}{3}$ show that these functions represents a possible case of irrotational flow

Sol:)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\cancel{2} - 2xy + 2xy - \cancel{2} = 0$$

$$0 = 0 \text{ (true)}$$

∴ The flow is possible

~~$\omega_z = \frac{1}{2}[\frac{\partial v}{}$~~

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

$$= \frac{1}{2}\left[\cancel{y} - \cancel{x} - \cancel{y} + \cancel{x}\right] = 0$$

$$0 = 0 \text{ (True)}$$

∴ The flow is irrotational.

UNIT - III

FLUID DYNAMICS

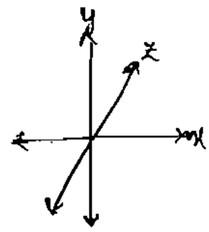
Fluid Dynamics:-

The branch of fluid mechanics which deals with the behaviour of the fluid in motion by considering the forces causing the motion fluid dynamics analyse the fluid motion by Newton's second law $(F = ma)$

$$F_x = ma_x$$

$$F_y = ma_y$$

$$F_z = ma_z$$



Types of forces acting on the fluid (body):-

- Gravity force
- pressure force
- Force due to viscosity
- Force due to Turbulence
- Force due to compressibility

If $F = F_g + F_p + F_v + F_T + F_c$

$$\therefore F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_T)_x + (F_c)_x$$

$$\therefore F_y = (F_g)_y + (F_p)_y + (F_v)_y + (F_T)_y + (F_c)_y$$

$$\therefore F_z = (F_g)_z + (F_p)_z + (F_v)_z + (F_T)_z + (F_c)_z$$

Condition-1 :-

If force due to compressibility is neglegible, the resultant equation is known as Reynold's equation.

$$(F_c)_x = 0$$

$$(F_c)_y = 0$$

$$(F_c)_z = 0$$

condition-2 :-

If force due to turbulance is neglegible the resultant equation is known as Navier stokes equation

$$(F_t)_x = 0$$

$$(F_t)_y = 0$$

$$(F_t)_z = 0$$

condition-3 :-

If the flow is assumed to be ideal. viscous forces are neglegible, the resulting equation is known as Euler's equation.

$$(F_v)_x = 0$$

$$(F_v)_y = 0$$

$$(F_v)_z = 0$$

Derivation of Euler's equation:-

The equation of motion in which process due to gravity and force due to pressure are taken into consideration is known as Euler's equation.

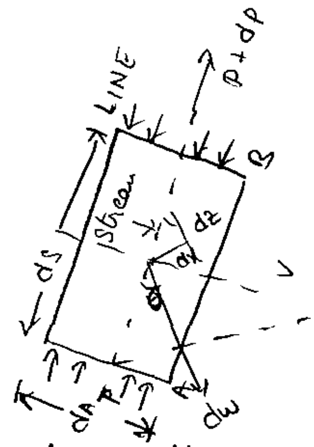
It is based on Newton's second law of motion ($F = ma$)

By integrating Euler's equation Bernoulli's equation is obtained.

Assumptions made in Euler's equation:-

- Fluid is assumed as homogeneous (i.e., Density is constant) and incompressible.
- The fluid is ideal (i.e, non-viscous) force due to viscosity is neglegible.
- The velocity of flow is uniform over the section.
- No energy force ~~(except)~~ except gravity and pressure forces involved in the flow.

Derivation:



consider a steady flow of an ideal fluid along a stream line. Now, consider a small element AB of the flowing fluid as shown in figure.

Let, dA = Cross-sectional area of the fluid element.

ds = Length of the fluid element.

dw = wt. of the fluid element.

P = pressure on the element ~~of A.~~ at A.

P+dP = pressure on the element at B.

Let v = velocity of the fluid element.

θ = Angle b/n the direction of flow and line of action of the wt. of the element.

we know the Euler's equation is based on Newton's ~~and~~ second law of motion

$$\therefore F = ma$$

$$m = \rho \times v$$
$$= \rho \times dA \times ds$$

$$a = \frac{dv}{dt}$$
$$= \frac{dv}{ds} \cdot \frac{ds}{dt}$$
$$= v \cdot \frac{dv}{ds}$$

$$F = ma$$

$$= \rho . dA . \cancel{ds} . v \frac{dv}{\cancel{ds}}$$

$$= \rho . v . dA . dv$$

$\therefore$ The net force acting on the fluid element.

$$= p . dA - (p + dp) dA - dw . \cos\theta$$

$$= p . dA - (p + dp) dA - \rho g . dA . \cancel{ds} . \frac{dz}{\cancel{ds}}$$

$$= -dp dA - \rho g dA . dz$$

$$F = \rho . v . dA . dv$$

$$dw = w \times v$$
$$dw = w . dA . ds$$
$$dw = \rho . g . dA . ds$$
$$\cos\theta = \frac{dz}{ds}$$

$$\therefore \; -dp\, dA - \rho g . dA . dz = \rho . v . dA . dv$$

Dividing both sides by " $-\rho . dA$ "

$$+ \frac{dp}{\rho} + g . dz = - v . dv$$

$$\therefore \; \frac{dp}{\rho} + g . dz + v . dv = 0$$

The above equation is called Euler's equation.

Integration of Euler's equation is the Bernoulli's equation.

$$\frac{1}{\rho} \int dp + g . \int dz + \int v . d\theta = \int 0$$

$$\frac{p}{\rho} + g . z + \frac{v^2}{2} = \text{constant}$$

Multiplying with ρ on both sides, we get

$$p + \rho . g . z + \frac{\rho v^2}{2} = \text{constant}$$

$$p + w . z + \frac{w}{g} . \frac{v^2}{2} = \text{constant}$$

Dividing the entire equation by w.

$$\frac{p}{w} + z + \frac{v^2}{2g} = \text{constant}$$

$$\boxed{\frac{p}{w} + z + \frac{v^2}{2g} = \text{Constant}}$$

$z \rightarrow$ Datum head

The above equation is Bernoulli's equation for section 1 and 2, we get

$$\frac{p_1}{w} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{v_2^2}{2g} = \text{constant.}$$

Bernoulli's equation states that in a steady ideal and incompressible fluid the total energy remains constant at every point along the path of the flow.

Assumptions made in Bernoulli's equation:

- The fluid is ideal (viscous forces / Frictional forces are ~~neglee~~ neglegible).
- The liquid is incompressible and density is constant.
- The flow is steady (discharge constant).
- Flow is uniform across the cross section. (discharge constant)
- No other force ~~concep~~ except gravitational force should act on a liquid.

$$= -1072 - 16633.836$$

$$= -171914 \text{ N}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(18607.70)^2 + (17914)^2}$$

$$= 25.82 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left[\frac{17914}{18607.7}\right]$$

$$= 43^\circ 54'$$

Ideal - Non - Viscous

Real - Viscous

* Bernoulli's Equation to real fluids:-

Bernoulli's Equation derived based on the assumption that.

1. The fluid is ideal (Non - Viscous) shear forces and frictional forces are neglected but in practical all are real fluids. In the case of real real fluids when the flow takes place from one section to

another section loss of energy takes place. The loss of energy takes place. The loss of energy should be taken in to consideration in Bernoulli's Equation

The Bernoulli's equation for real fluids can be expressed as

1. $$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L$$

(Flow takes place from section 1-1 to 2-2)

2. $$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 + h_L = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2$$

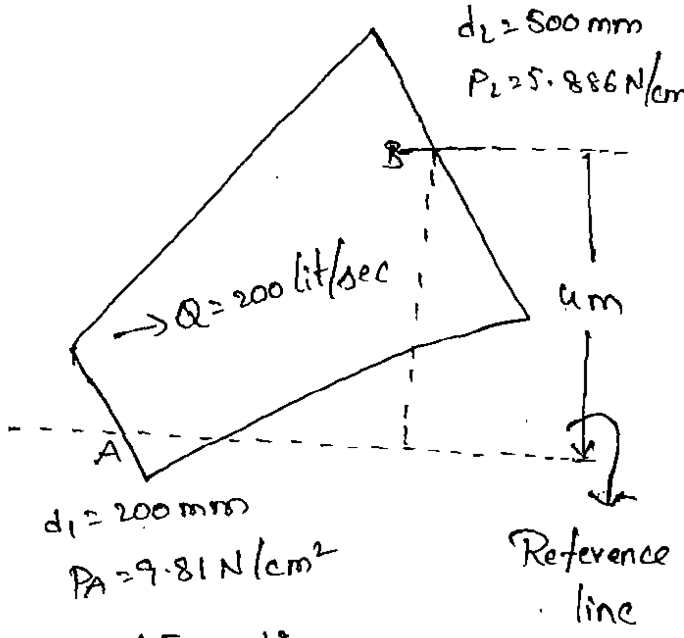
(Flow takes place from section 2-2 to 1-1)

* Problems:-

1. A pipe line carrying oil of specific gravity 0.87 changes in diameter from 200mm at position 'A' to 500mm at position B which is 4m at higher level. If the pressure at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 lit/sec. Determine the loss of head and direction of flow.

sol:-

$$G = \frac{P_{oil}}{P_w}$$



$$P_{oil} = 0.87 \times 100$$

$$w = \rho \cdot g$$

$$\therefore w = 0.87 \times 1000 \times 9.81$$

$$Q = A_1V_1 = A_2V_2$$

$$\frac{200}{1000} = \frac{\pi}{4} \times \left(\frac{200}{1000}\right)^2 v_1 = \frac{\pi}{4}\left(\frac{500}{1000}\right)^2 v_2$$

$$\therefore v_2 = 1.018 \text{ m/sec}$$

$$\therefore v_1 = \frac{20}{3.14}$$

$$\therefore v_1 = 6.369 \text{ m/s}$$

$$E_A = \frac{P_A}{w} + \frac{v_A^2}{2g} + z_A$$

$$= \frac{\cancel{9.81} \times 10^4}{0.87 \times \cancel{1000} \times \cancel{9.81}} + \frac{v_1^2}{2 \times 9.81} + 0$$

$$= \frac{10}{0.87} + \frac{(6.369)^2}{2 \times 9.81} = 13.56 \text{ m}$$

$$E_B = \frac{P_B}{w} + \frac{v_B^2}{2g} + z_B$$

$$= \frac{5.886 \times 10^4}{0.87 \times 1000 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4$$

$$= 10.94 \text{ m}$$

$\therefore E_A = 13.56\,m$ and $E_B = 10.94\,m$

water always flows from higher energy to lower energy. Hence water flows from section A to section B.

$$E_A = E_B + h_2$$

$$\therefore h_2 = E_A - E_B = 13.56 - 10.94$$

$$= 2.62\,m$$

(1) To determine the resultant force acting on the boundary of a flow passage by the stream of fluid. As the stream changes its direction or magnitude of velocity or both.

The problems of these types are:

1. pipe bends
2. Stationary and moving vanes
3. Jet propulsion
4. Reducers etc...,

Limitations on Bernoulli's equation:

Bernoulli's theorem is derived based on some assumptions which are rarely possible. Hence it has the following limitations.

- Bernoulli's theorem is assumed that velocity is uniform across the section. But in real the velocity is maximum at centre and minimum at edges. The velocity is reduced to minimum at edges due to frictional force offered by the pipe material.
- Bernoulli's equation has been derived under the assumption that no external force ~~accep~~ except the gravitational force is acting on the liquid. But in practical some external forces such as pipe friction, centrifugal force when pipe takes curved path are acting on the fluid. Bernoulli's equation does not considered these external forces.
- Bernoulli's equation has been derived under the assumption that there is no loss of energy while fluid flowing. But in actual practise, some K.E ~~KE~~ is converted into heat energy in turbulent flow. Hence there is loss of energy. In the case of viscous force there is some loss of energy.

due to the shear forces but the Bernoulli's equation neglects.

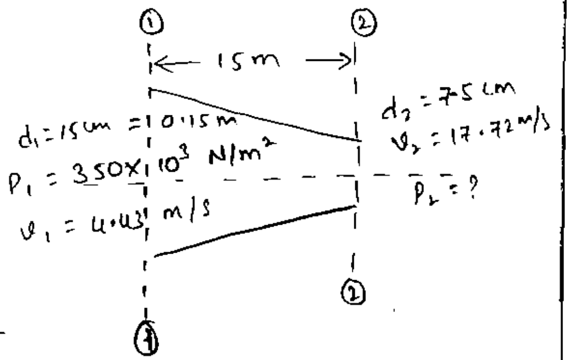
Use of Bernoulli's equation :-

Bernoulli's equation can be applied between the two points in order to determine the pressure & velocity at a section between two points. If pressure and velocity are known at the other section.

* Problems

1. Water flows in a pipe line at certain point where the diameter is 15 cm. pressure and velocity 350 kN/m² and 4.43 m/s respectively. At a point 15 m away diameter of the pipe reduces to 7.5 cm If the flow occurs 15 cm to 7.5 cm. Find the pressure at 7.5 cm section. If the flow is a, Horizontal; b, vertical.

Sol Horizontal.



$$A_1 V_1 = A_2 V_2$$

$$\frac{\pi (0.15)^2}{4} (4.43) = \frac{\pi (0.075)^2}{4} V_2$$

$$\therefore V_2 = 17.72 \text{ m/s}$$

$$\frac{p_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{w} + \frac{V_2^2}{2g} + Z_2$$

Taking the centre of the pipe line as reference line $z_1 = z_2$

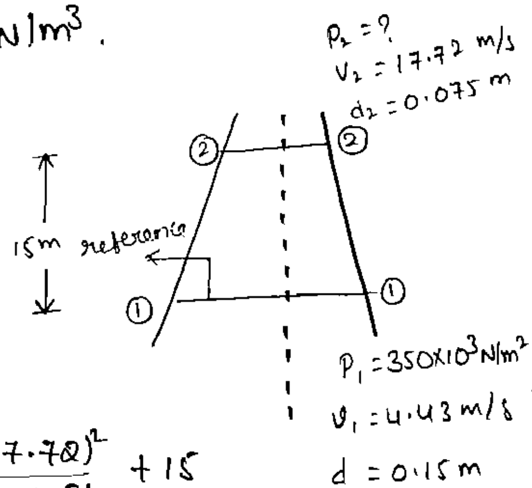
$$\therefore \quad \frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g}$$

$$\frac{350 \times 10^3}{9810} + \frac{(4.43)^2}{2 \times 9.81} = \frac{P_2}{9810} + \frac{(17.72)^2}{2 \times 9.81}$$

$$35.677 + 1.000 = \frac{P_2}{9810} + 16.003$$

$$P_2 = 202.77 \text{ kN/m}^3.$$

<u>vertical</u>



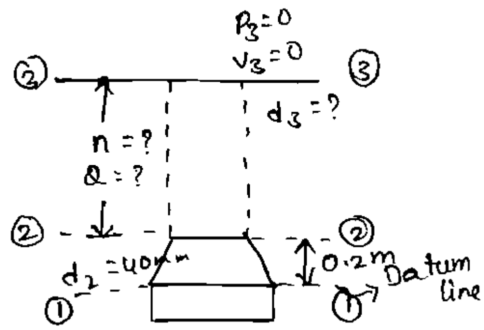
$$\frac{P_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{v_2^2}{2g} + z_2$$

$$\frac{350 \times 10^3}{9810} + \frac{(4.43)^2}{2 \times 9.81} + 0 = \frac{P_2}{9810} + \frac{(17.72)^2}{2 \times 9.81} + 15$$

$$P_2 = 55.661 \text{ kN/m}^3$$

3. A Jet of water issues vertically upwards from a 0.2 m high nozzle whose inlet and outlet diameters are 100mm and 40mm respectively. If the pressure at the inject is 20 kN/m² above the atmospheric pressure. Determine the discharge and the height to which the jet will rise.

sol:- Applying Bernoullis Equation between section ①-① and ②-②



$d_1 = 100mm$

$P_1 = 20 kN/m^2$

$V_1 = ?$

$$\frac{P_1}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\omega} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{20\times10^3}{9810} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0.2$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = 1.838$$

Continuity Equation

$$A_1V_1 = A_2V_2$$

$$\frac{\pi}{4}\left(\frac{\cancel{100}}{\cancel{1000}}\right)^2 \times V_1 = \frac{\pi}{4}\left(\frac{4\cancel{0}}{100\cancel{0}}\right)^2 \times V_2$$

$$\frac{100}{16} = \frac{V_2}{V_1}$$

$$\Rightarrow V_2 = 6.25\, V_1$$

$$\Rightarrow V_1 = 0.16\, V_2$$

$$\frac{1}{2g}\left[V_2^2 - (0.16)^2 V_2^2\right] = 1.838$$

$$V_2^2\,[0.9744] = 1.838 \times 2 \times 9.81$$

$$V_2 = 6.08 \text{ m/s}$$

$$V_1 = 0.9 \text{ m/s}$$

$$Q = A_1V_1 = A_2V_2$$

$$= \frac{\pi}{4} \times (100 \times 10^{-3})^2 \times 0.97$$

$$= 7.64 \times 10^{-3} \text{ m}^3/\text{sec}$$

Applying Bernoulli's Equation between section 2 - 2 and 3 - 3

$$\frac{P_2}{w} + \frac{v_2^2}{2g} + z_2 = \frac{P_3}{w} + \frac{V_3^2}{2g} + z_3$$

$$0 + \frac{(6.08)^2}{2 \times 9.81} + 0 = 0 + 0 + h$$

$(\because z = \text{height})$

$$\therefore \boxed{h = 1.88\text{m}}$$

→ Modification of Bernoulli's Equation to account for non-uniformity in velocity distribution & kinetic Energy correction factor :-

Kinetic energy correction factor :-

Bernoulli's Equation derived based on the assumption that velocity distribution is uniform across the section of pipe but in actual practice velocity distribution is maximum at centre and minimum at edges.

The velocity head in Bernoulli's equation is effected by this assumption. Hence a correction factor is applied to the velocity head which is known as kinetic Energy correction factor.

It is defined as ratio of k.E based on Actual velocity to the k.E based on Average velocity.

$$k.E = \alpha = \frac{k.E/sec \text{ based on Actual velocity}}{k.E/sec \text{ based on Average velocity}}$$

$$\alpha = \frac{1}{A} \cdot \int \left(\frac{v_{act}}{\bar{v}_{avg}}\right)^3 dA.$$

After Applying the Bernoulli's Equation it can be expressed as

$$\frac{P}{w} + \alpha\left(\frac{v^2}{2g}\right) + z = k$$

In general $\alpha > 1$

$\alpha = 1$ Velocity distribution is uniform

$\alpha = 2$ laminar flow

$\alpha = 1.01$ to 1.2 for Turbulent flow

Momentum Equation :-

Momentum Equation is based on laws of the conservation of momentum it states that the net force acting on a fluid mass is equal to change in momentum of the flow per unit time in that direction

$$F = ma$$

$$a = \frac{dv}{dt}$$

$$F = m \cdot \frac{dv}{dt}$$

$$F \cdot dt = m \cdot dv$$

$$F = \frac{d(mdv)}{dt}$$

(d)

Impluse momentum Equation states that the impulse of a force (F) acting on a fluid mass (m) in a short interval of time (dt) is equal to change in momentum of the fluid (d(mv))

$$\boxed{F \times dt = d(mv)}$$

→ Application of Impulse- momentum Equation Impulse-momentum Equation is used in the following two types of problem.

1) Pipe bends
2) Stationary & Moving Vanes
3) Jet propulsion
4) Reducers etc.

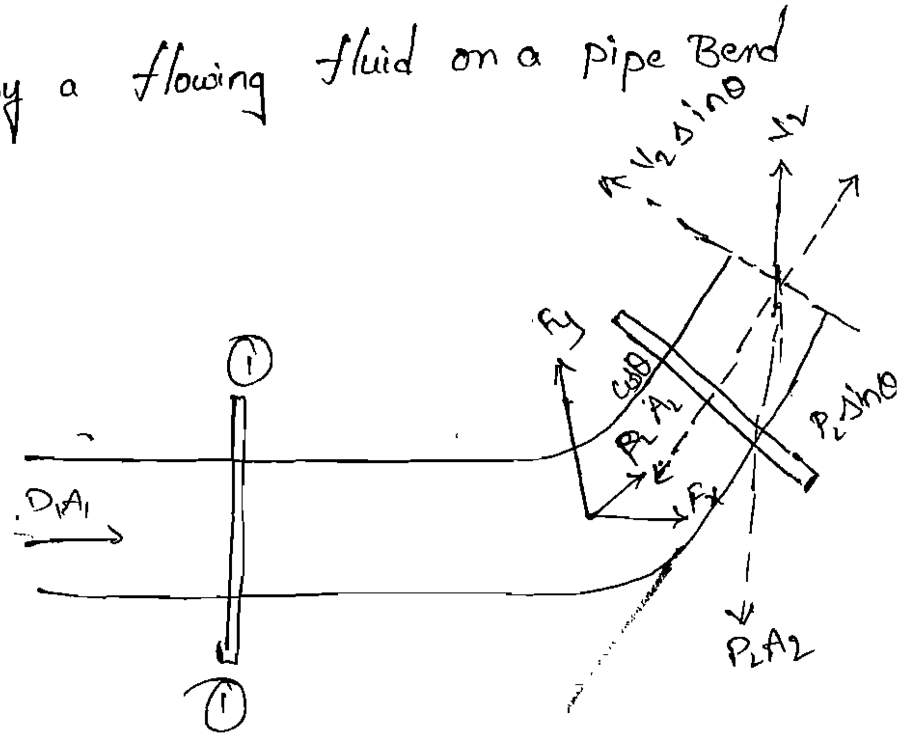
→ To determine the characteristics of flow when there is a abrupt change of flow section (non-uniform flow)

→ problems of this type includes

a) sudden enlargement in pipes

b) Hydraullic jump in the case of dam.

* → Force exerted by a flowing fluid on a pipe Bend



Consider 2 sections ① and ② as shown in figure

v_1 = Velocity of flow at section ①

P_1 = Pressure intensity at section ①

A_1 = Cross sectional area of pipe at section-①

$P_2 A_2 V_2$ = Corresponding value of velocity, Pressure and area of section at ②..

→ Let F_x and F_y be the components of forces exerted by the flowing fluid on the bend in x and y directions respectively.

→ Then the force exerted by the bend on the fluid in the direction of x and y will be equal to F_x and F_y but in opposite directions.

→ Hence the component of the force exerted by the pipe bend on fluid in x-direction $= -F_x$

In the direction of $y = -F_y$

→ the other external of y are acting on the fluid are P_1A_1 and P_2A_2 on the sections 1 and 2 respectively

→ the Applying Impulse momentum equation in x-direction

Net force = Mass per sec (change in Velocity)

$= \rho Q$

$$\boxed{F\,dt = M(V_2 - V_1)}$$

$$P_1 A_1 - P_2 A_2 \cos\theta - F_x = \rho Q (v_2 \cos\theta - v_1)$$

$$\boxed{F_x = P_1 A_1 - P_2 A_2 \cos\theta + \rho Q (v_1 - v_2 \cos\theta)} \longrightarrow (1)$$

Applying Impulse - Moment Equation in y-direction

$$0 - P_2 A_2 \sin\theta - F_y = \rho Q (v_2 \sin\theta - 0)$$

$$\boxed{F_y = \rho Q (-v_2 \sin\theta) - P_2 A_2 \sin\theta} \longrightarrow (2)$$

Resultant Force

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$\tan\theta = \frac{F_y}{F_x}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)}$$

In General

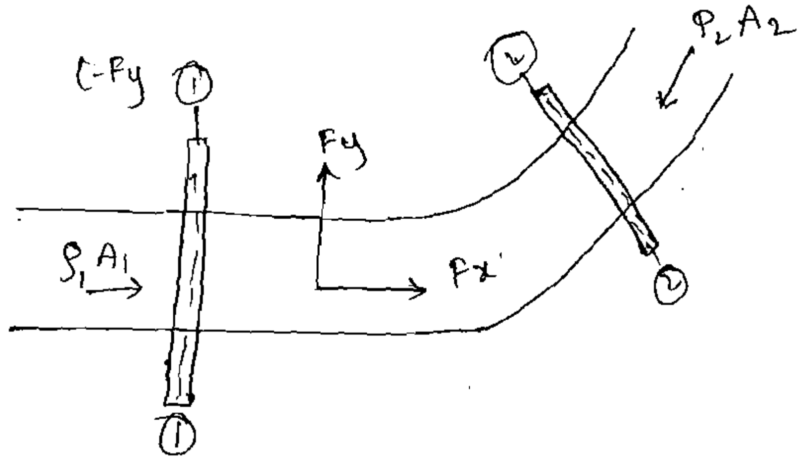
$$F_x = \rho Q (v_{1x} - v_{2x}) + P_1 A_1 - P_2 A_2 \cos\theta$$

$$F_y = \rho Q (v_{1y} - v_{2y}) - P_2 A_2 \sin\theta$$

* Problems On a Pipe Bend:-

$$F_x = \rho Q (v_{1x} - v_{2x}) + (P_1 A_1)_x - (P_2 A_2)_x$$

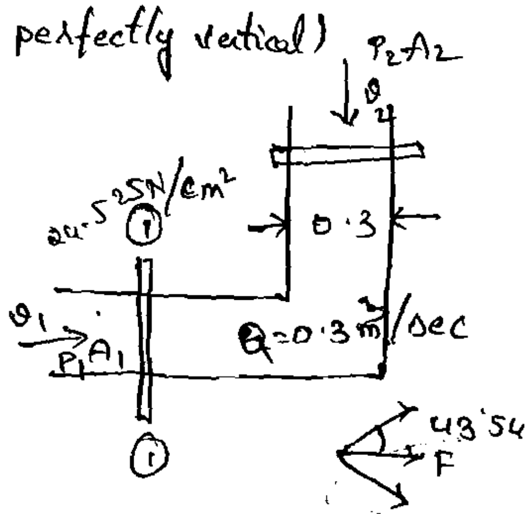
$$F_y = \rho Q (v_{1y} - v_{2y}) - (P_1 A_1)_y - (P_2 A_2)_y$$



* Problems:-

1. A pipe of 300mm diameter conveying 0.30 m^3/sec of water has a right angled bend in a horizontal plane. Find the resultant force entered by the bend if the pressure at inclined and outlined at the bend are 24.525 N/cm^2 and 23.544 N/cm^2

(∵ it is perfectly vertical)



Sol:-

$$F_x = 1000 \times 0.30\,(4.24-0)$$

$$\frac{24.525 \times 10^4 \times \pi \times (0.3)^2}{4} - 0$$

$$= 1272 + 17326.9125$$

$$= 18607.70N$$

$$Q = A_1 \times v_1$$

$$v_1 = \frac{Q}{A_1} = \frac{0.3}{\frac{\pi(0.3)^2}{4}}$$

$$= 4.24 \; m/sec = v_2 \quad (1)$$

$$F_y = (1000 \times 0.3)(0-4.24) + 0 - \frac{23.544 \times 10^{-4} \times \pi (0.3)^2}{4}$$

$= -1272 - 16633.836$

$= -171914\ N$

$F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(18607.70)^2 + (17914)^2}$

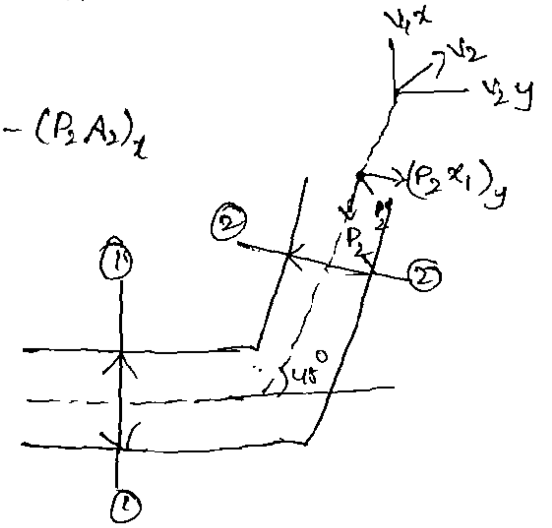
$= \sqrt{(18607.70)^2 + (17914)^2}$

$= 28.82 kN$

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left[\frac{17914}{18607.70}\right]$

$= 43°54'$

2. A 300m diameter pipe carries water under head of 20m with a velocity 3.5 m/s if the axis of the pipe turns through 45° magnitude and direction of resultant forces



$F_x = \rho Q(V_{1x} - V_{2x}) + (P_1A_1)_x - (P_2A_2)_x$

$Q = A_1V_1 = A_2V_2$

$V_1 = V_2 = 3.5\ m/sec$

$Q = \frac{\pi}{4}(0.3)^2 \times 3.5$

$= 0.247\ m^3/sec$

$p = wh$

$= 9810 \times 20$

$= 196200\ N/m^2$

$$F(x) = \rho Q\,(V_{1x} - V_{2x}) + (P_1A_1)_x - (P_2A_2)_x$$

$$= 1000 \times 0.247\,(3.5 - 3.5\cos 45^\circ) + \left(196200 \times \frac{\pi}{4}(0.3)^2\right) - \left(196200\cos 45^\circ \cdot \frac{\pi}{4}(0.3)^2\right)$$

$$= 4310.69\,N$$

$$F(y) = \rho Q\,(\cancelto{0}{V_{1y}} - V_{2y}) - \cancelto{0}{(P_1A_1)_y} - (P_2A_2)_y$$

$$= 1000 \times 0.247\,(0 - 3.5\sin 45^\circ) - 0 - \left(196200\sin 45^\circ \frac{\pi}{4}(0.3)^2\right)$$

$$= -10405.9\,N$$

$$F = \sqrt{(Fx)^2 + (Fy)^2}$$

$$= 11263.4\,N$$

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

$$= 67^\circ 29'$$