UNIT - VI

Introduction to Matrix Methods:-

Flexibility Methods: Introduction, Application to Continuous beams

Stiffness " " " " " " "

- Analysis of Indeterminate Structures is the major field in Structural Engg.
- Kani's Method is the best one among several Methods of Structural Analysis. But This also will not be convenient for present day multistorey Buildings.
- Hence there is a need for Matrix Methods.

Flexibility matrix Method:- (Force(s) Compatibility method)

→ Systematic development of consistent deformation method in matrix form has lead to the flexibility matrix method

→ The unknowns are forces, in this method.

→ Identifying basic determinate Structure and thereby identifying redundant forces is the key feature

→ No. of Redundant forces is equal to the Degree of Static Indeterminacy

→ Displacements in basic determinate Str. due to given loads and Redundants are found and conditions of consistency are formed

Approaches to Matrix Methods:

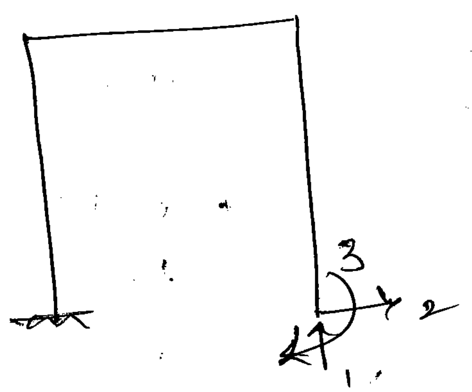
(i) Direct / Structure

(ii) Transformation matrix / Element.

Generalized coordinate systems:-

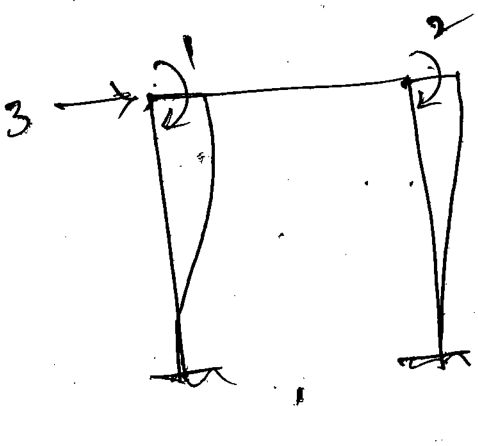
The directions of forces and disp. to determine the unknowns in the str. systems are known as the generalized coordinates.

eg:-



$$\{P\} = \begin{Bmatrix} V_D \\ H_D \\ M_D \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} \Delta V_D \\ \Delta H_D \\ \theta_D \end{Bmatrix}$$



$$\{P\} = \begin{Bmatrix} M_B \\ M_C \\ S \end{Bmatrix}$$

$$\{\Delta\} = \begin{Bmatrix} \theta_B \\ \theta_C \\ \delta \end{Bmatrix}$$

Flexibility matrix:- If a structure has 'n' no. of coordinates, Its displacement response to the forces is represented by, $\{\delta\} = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nn} \end{bmatrix}$

which is known as flexibility matrix

→ Element δ_{ij} represent, Displ. at coordinate i due to unit force at 'j'.

→ To develop flexibility matrix, a unit force is applied successively at coordinates 1, 2, 3 ... n. and displacements at all coordinates are computed

→ From Maxwell's Reciprocal Theorem $\delta_{ij} = \delta_{ji}$, Hence flexibility matrix has Diagonal Symmetry.

Flexibility matrix method :- procedure

(i) Determine Degree of static Indeterminacy

(ii) choose the Redundants and assign coordinates to them

(iii) Remove the Redundants and prepare basic determinate structure

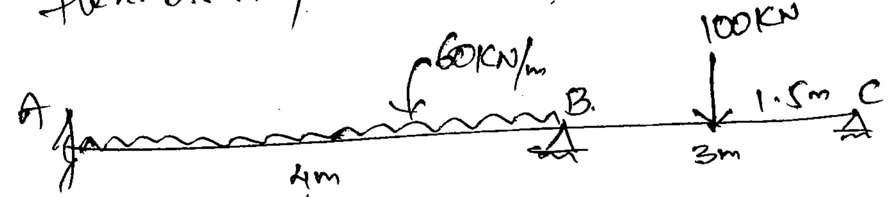
(iv) Determine deflections in the directions of coordinates due to given loads

(v) Determine the elements in flexibility matrix by applying unit force at each coordinate and ~~find~~, displ. at all the coordinates.

(vi) ~~apply~~ the compatibility condition to compute $\{P\} = [f]^{-1} \left\{ \{\Delta\} - \{\Delta_L\} \right\}$

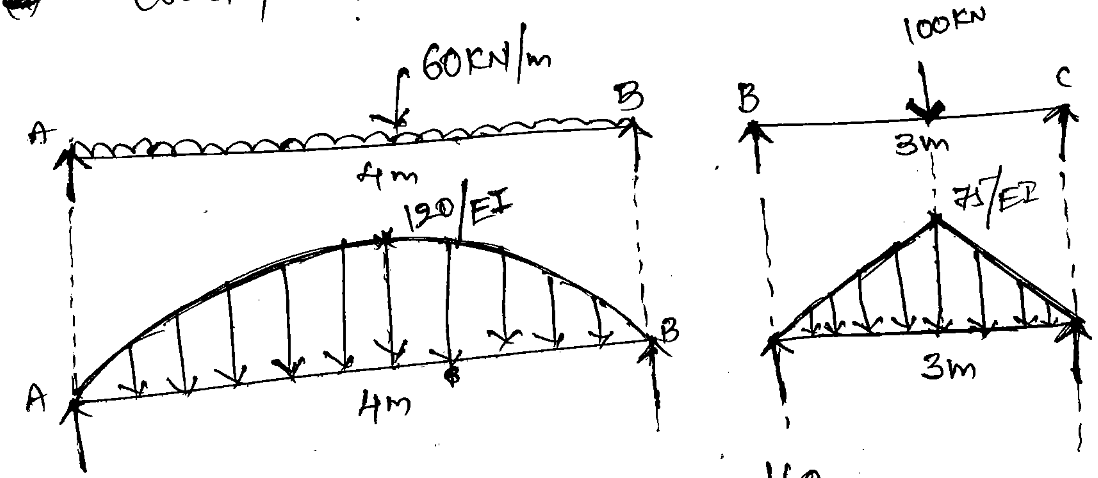
(vii) Knowing the Redund forces compute the member forces like SF & BM.

prob:- ① Analyse the continuous beam by flexibility method?



Sol: ① D.S.I → 2

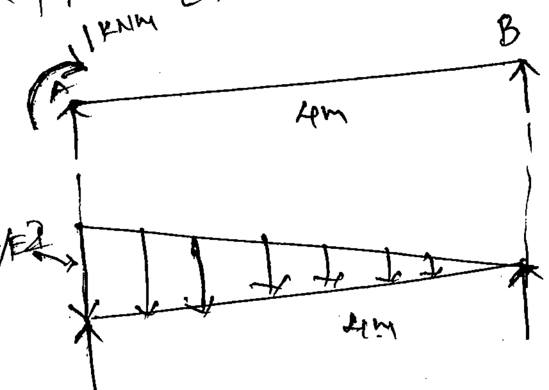
② choosing the Redundants as M_A & M_B



$$\Delta_{1L} = \theta_A = SF_A = \left(\frac{2}{3}\times 4\times \frac{120}{EI}\right)\times\frac{1}{2} = \frac{160}{EI}$$

$$\Delta_{2L} = \frac{160}{EI} + \left(\frac{1}{2}\times 3\times \frac{75}{EI}\right)\times\frac{1}{2} = \frac{216.25}{EI}$$

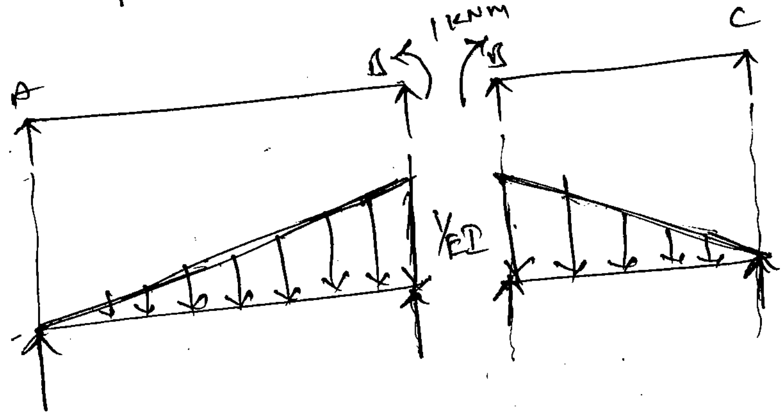
To find [f] matrix.



$$f_{11} = \theta_A = \left(\frac{1}{2}\times 4\times\frac{1}{EI}\right)\frac{\left(\frac{2\times 4}{3}\right)}{4}$$

$$= \frac{2\times 2}{3EI} = \frac{4}{3EI}$$

$$f_{21} = \frac{2}{3EI}$$



$$f_{22} = \left(\frac{1}{2}\times 4\times\frac{1}{EI}\right)\times\frac{2}{3} + \left(\frac{1}{2}\times 3\times\frac{1}{EI}\right)\times\frac{2}{3}$$

$$= \frac{7}{3EI}$$

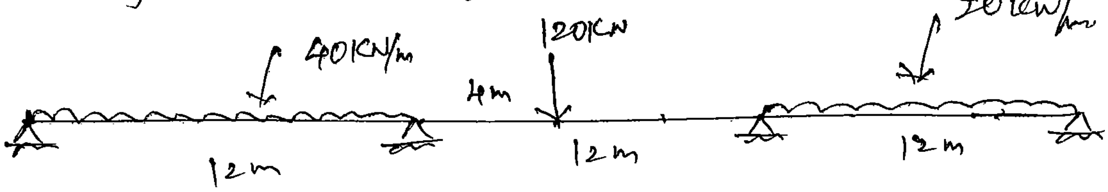
B.C's $\{\Delta\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$; $\{\Delta_L\} = \begin{Bmatrix} \frac{160}{EI} \\ \frac{296.25}{EI} \end{Bmatrix}$

$$[f] = \begin{bmatrix} \frac{4}{3EI} & \frac{2}{3EI} \\ 2/3EI & \frac{7}{3EI} \end{bmatrix} = \frac{1}{3EI}\begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

$$\therefore \{P\} = [f]^{-1}\{\{\Delta\}-\{\Delta_L\}\}$$

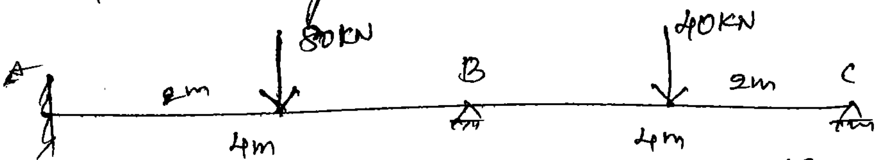
$$\therefore \{P\} = \begin{Bmatrix} M_A \\ M_B \end{Bmatrix} = \begin{Bmatrix} -85.875 \\ -68.25 \end{Bmatrix} \text{KNm}$$

prob:- (2) Analyse the continuous beam shown below by flexibility matrix method?



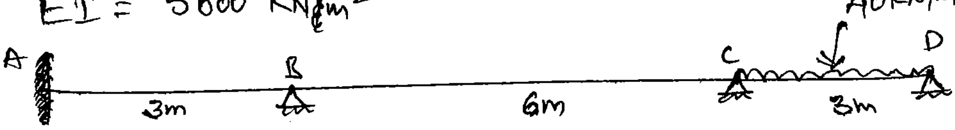
Sol $M_B = -449.78$ KNm ; $M_C = -174.22$ KNm.

prob:- (3) Analyse the continuous beam shown, if the downward settlement of supports B and C are 10mm and 5mm resp. Take $EI = 184 \times 10^{11}$ Nmm^2 use flexibility method?



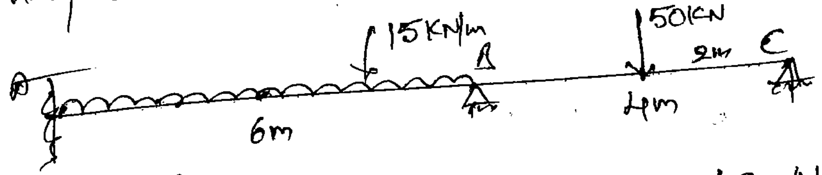
Sol $\{\Delta\} = \begin{Bmatrix} -0.01 \\ -0.005 \end{Bmatrix}$; $\{P\} = \begin{Bmatrix} R_B \\ R_C \end{Bmatrix} = \begin{Bmatrix} 18.485 \\ 27.105 \end{Bmatrix}$.

prob(4):- Solve the beam shown below, if $\Delta_B = 30$mm(↓)
EI = 5600 KNm^2



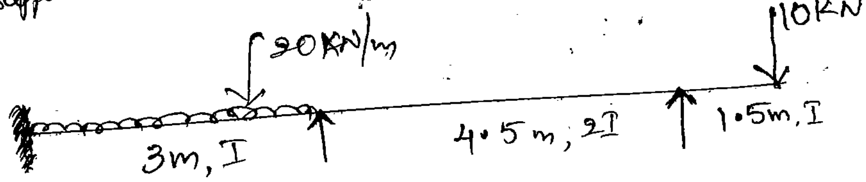
Sol $M_A = -84.21$, $R_B = -62.24$; $M_C = -43.14$ KNm.
KNm

Prob:- (5) Analyse the continuous beam by force method.



Sol) $M_A = -65.29$ KNm ; $M_B = -49.41$ KNm

Prob(6):- Analyse the continuous beam using compatibility method if the downward settlements of supports B & C are 100/EI and 50/EI resp.



SA - II

VI - 4

Stiffness Matrix Method :- (Disp. Method, Eqlm Method)

→ In this method, Basic unknowns are displacements.

→ Hence, the degree of Kinematic Indeterminacy is identified.

→ To start with, Joint displacement in all directions are restrained

→ Systematic development of slope deflection method in the matrix form has given rise to Stiffness matrix method

→ Equations of equilibrium are formed and solved for slopes and deflections at the Joints.

Stiffness matrix :- If a structure is having 'n' coordinates its force response to the displacement is represented by,
$$\begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1n} \\ K_{21} & K_{22} & \cdots & K_{2n} \\ \vdots & & & \\ K_{n1} & K_{n2} & \cdots & K_{nn} \end{bmatrix}$$
which is called as Stiffness matrix.

→ K_{ij} is the force at 'i' due to a unit disp. at 'j'

→ To develop stiffness matrix, unit displacement should be given successively at coordinates 1, 2, ...n and the force developed at all the coordinates are computed

→ from maxwell's reciprocal theorem $K_{ij} = K_{ji}$. Hence, stiffness matrix has diagonal symmetry.

Relation b/w flexibility and stiffness:-

$$\{\Delta\} = [f]\{P\} \Rightarrow \text{from } f = \frac{\Delta}{P}$$

$$\{P\} = [K]\{\Delta\} \Rightarrow \text{" } K = \frac{P}{\Delta}$$

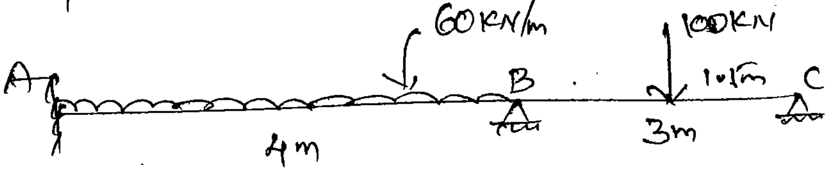
$$\{P\} = [K][f]\{P\}$$

$$\therefore [K][f] = [I] \leftarrow \text{Identity matrix.}$$

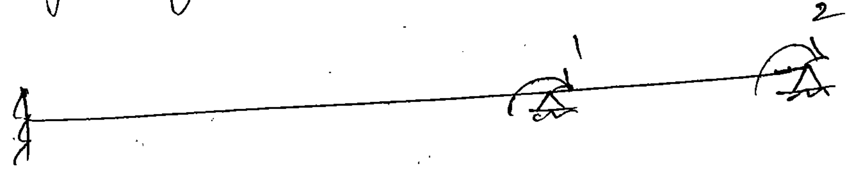
Hence flexibility and stiffness matrices are inverse of each other.

procedure:- (i) Det. DOF & Kinematic Indeterminacy
(ii) Assign the co-ordinates to unknown disp.
(iii) Impose restraints in all coordinate directions to get fully restrained str.
(iv) Det. forces developed in each of the coordinate directions of the fully restrained structure called $\{P_L\}$.
(v) Develop stiffness matrix $[K]$ by giving unit displ. to the restrained structure in each of the coordinate directions
(vi) write the B.C's $\{P\}$, i.e., total force in the directions of coordinates.
(vii) Solve for $\{\Delta\}$, from $\{\Delta\} = [K]^{-1}\{\{P\} - \{P_L\}\}$.
(viii) Calculate the member forces using the joint displacements calculated

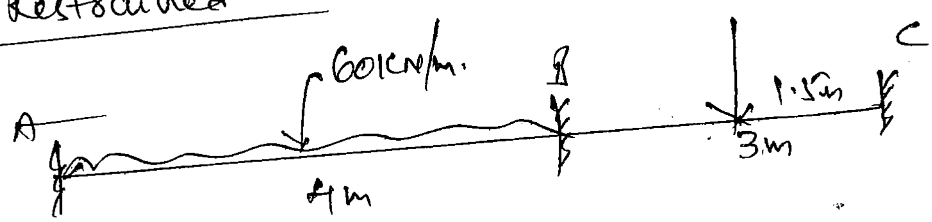
Prob(1):- Analyse the continuous beam shown below using disp. method. Take EI is const.



sol: neglecting axial deformations $D_K = 2$



fully Restrained str:-



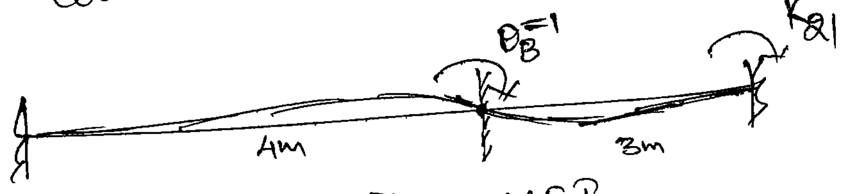
$M_{FBA} = 80\,KNm.$

$M_{FAB} = -80\,KNm$; $M_{FCB} = 37.5\,KNm.$

$M_{FBC} = -37.5\,KNm$;

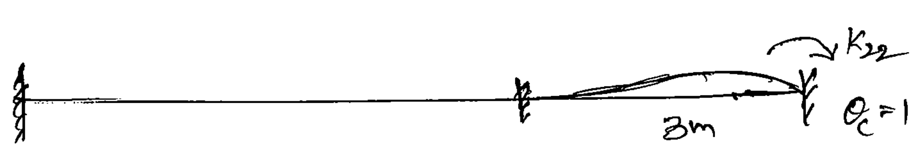
$$\{P_L\} = \begin{Bmatrix} P_{L1} \\ P_{L2} \end{Bmatrix} = \begin{Bmatrix} M_B \\ M_C \end{Bmatrix} = \begin{Bmatrix} 80-37.5 \\ 37.5 \end{Bmatrix} = \begin{Bmatrix} 42.5 \\ 37.5 \end{Bmatrix}$$

To develop stiffness matrix:- give unit rotation in coordinate direction 1.



$$K_{11} = \frac{4EI}{4} + \frac{4EI}{3} = \frac{7EI}{3}$$

K_{21} = force at '2' due to disp at '1'. $= \frac{2EI}{3}$



$$K_{22} = \frac{4EI}{3}$$

Final forces acting in coordinate directions at 1 & 2 are zero. $\{P\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

$$\therefore [K] = \begin{bmatrix} \frac{7EI}{3} & \frac{2EI}{3} \\ \frac{2EI}{3} & \frac{4EI}{3} \end{bmatrix}$$

$$\{\Delta\} = [K]^{-1} \left\{ \{P\} - \{P_L\} \right\}$$

$$\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \frac{3}{EI(7\times 4 - 2^2)} \begin{bmatrix} 4 & -2 \\ -2 & 7 \end{bmatrix} \left\{ \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 42.5 \\ 37.5 \end{Bmatrix} \right\}$$

$$= \begin{Bmatrix} -\frac{11.875}{EI} \\ -\frac{22.188}{EI} \end{Bmatrix}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{L}(2\theta_A + \theta_B)$$

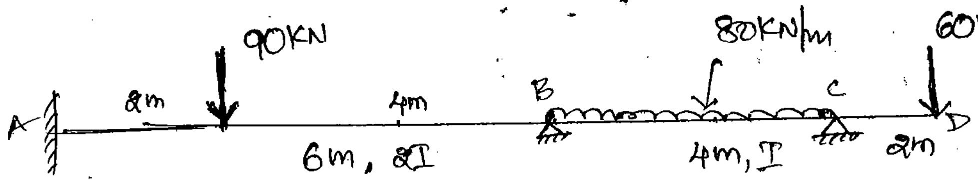
$$= -80 + \frac{2EI}{4}\left(2\times 0 - \frac{11.875}{EI}\right) = -85.938 \text{ KNm}$$

$M_{BA} = 68.13$ KNm.

$M_{BC} = -68.13$ KNm

$M_{CB} = 0$

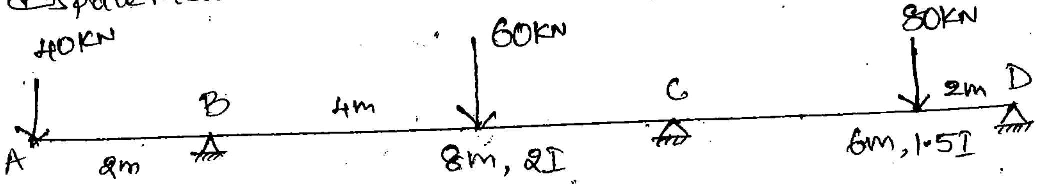
Analyse the beam shown below using stiffness matrix method?



Sol: $\{P\} = \begin{Bmatrix} 0 \\ 120 \end{Bmatrix}$; $\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} \frac{28.802}{EI} \\ \frac{-1.017}{EI} \end{Bmatrix}$

$M_{AB} = -60.8$; $M_{BA} = 78.403$ kNm ; $M_{BC} = -78.403$; $M_{CB} = 120$ kNm

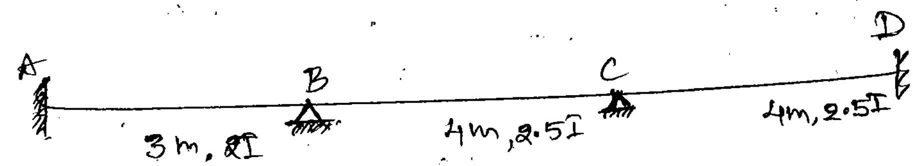
Prob ③:- Analyse the continuous beam shown below by displacement method?



Sol:- $\{P\} = \begin{Bmatrix} -80 \\ 0 \\ 0 \end{Bmatrix}$; $\begin{Bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{Bmatrix} = \begin{Bmatrix} \frac{-27.035}{EI} \\ \frac{14.07}{EI} \\ \frac{-78.145}{EI} \end{Bmatrix}$

$M_{BC} = -80$; $M_{CB} = 60.55$; $M_{CD} = -60.55$; $M_{DC} = 0$

Prob ④:- Analyse the continuous beam shown below, if the support B sinks by 10mm. Use disp. method. Take $EI = 6000$ kNm^2 ?



Sol: $\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{Bmatrix} \frac{7.079}{EI} \\ \frac{-13.198}{EI} \end{Bmatrix}$

$M_{AB} = -69.613$; $M_{BA} = -59.227$

$M_{BC} = 59.227$; $M_{CB} = 33$ kNm.

prob:- Difference between flexibility & stiffness method?

Flexibility method	Stiffness Method
(i) Also called as force method, compatibility method	(i) Also called as Displ. method, Equilibrium method.
(ii) unknowns are forces	(ii) unknowns are displacements.
(iii) Degree of static indeterminacy is to be calculated	(iii) Degree of kinematic indeterminacy is to be calculated
(iv) It is developed based on consistent deformation method	(iv) It is developed based on slope-deflection method
(v) consistency conditions are formed and solved to get unknown forces	(v) Equilibrium equations are formed and solved to get unknown displ.
(vi) Basic structure in this method is Determinate str	(vi) Basic str. is fully restrained structure which is indeterminate
(vii) D.S.I is the size of flexibility matrix	(vii) D.K.I is the size of stiffness matrix.
(viii) flexibility matrix is developed by applying unit force at each coordinate.	(viii) Stiffness matrix is developed by applying unit displacement at each coordinate
(ix) Flexibility matrix is inverse of stiffness matrix	(ix) Stiffness matrix is the inverse of flexibility matrix
(x) flexibility matrix has diagonal symmetry.	(x) Stiffness matrix has diagonal symmetry.
(xi) uses generalized coordinate system to solve indeterminate structure	(xi) uses generalized coordinate system to analyse indeterminate str
(xii) f_{ij} represent displ. at 'i' due to unit force at 'j'.	(xii) K_{ij} represent force at 'i' due to unit disp. at 'j'.
(xiii) compatibility conditions $\{P\} = [f]^{-1}\{\{\Delta\}-\{\Delta_L\}\}$.	(xiii) Equilibrium conditions. $\{\Delta\} = [K]^{-1}\{\{P\}-\{P_L\}\}$.