Engineering Mechanics (Dynamics)

- Kinematics of particle,
- Rectilinear motion.
- Curvilinear motion.
- Rotational motion.
- Kinetics, force, mass and acceleration,
- Kinetic of particle Newton's 2^{nd} law.

General Description:

Kinetics of particle, rectilinear motion, curvilinear motion, rectangular components of curvilinear motion, Rotational motion, kinetics, force, mass and acceleration, kinetic of particle Newton's 2^{nd} law.

*Workshop Skills

The workshop training program is designed to satisfy the following:
Objectives Teaching safety rules and regulations on-site in an industrial environment proper use of working tools, instruments, and machines, introducing basic workshop practices, production, labor, and time-requirements of workshop operations. The students are introduced to training programs in six workshops: welding, forging, turning and milling, carpentry, and casting. The student is to spend 2 hours of training in every workshop

Notes:

- These lectures were prepared and used by me to conduct lectures for 1st year B. Tech. students as part of Engineering Mechanics course.
- Theories, Figures, Problems, Concepts used in the lectures to fulfill the course requirements are taken from the following references
- I take responsibility for any mistakes in solving the problems. Readers are requested to rectify when using the same.
- I thank the following authors for making their books & lectures available for reference

A. Ali

References: -

- Vector Mechanics for Engineers – Statics & Dynamics, Beer & Johnston; 10 edition.
- Engineering Mechanics Statics Vol. 1, Engineering Mechanics Dynamics Vol. 2, Meriam& Kraige; 6thedition.
- Engineering Mechanics – Statics, lectures by instructor, R. Ganesh Narayanan.
- Engineering Mechanics Dynamics, 14 ed., R. C. Hibbeler.
- Engineering Mechanics – Dynamics, lectures by instructor, Y. Wang.
- Lectures of other instructors in the department.
- Any other references in this field.

II-Dynamics

Previous sections were devoted to statics, i.e., to the analysis of bodies at rest.

We now begin the study of dynamics, the part of mechanics that deals with the analysis of **bodies in motion**.

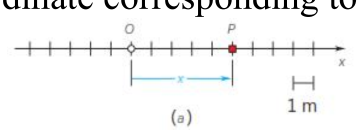
Dynamics Includes:

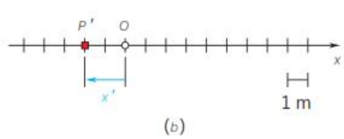
1. Kine**ma**tics, which is the study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time, ***without reference to the cause of the motion***.
2. Kinetics, which is the study of the relation existing between the forces acting on a body, the mass of the body, ***and the motion of the body***. Kinetics is used ***to predict the motion caused by given forces or to determine the forces required to produce a given motion.***

1- Rectilinear Motion of particles:

(Position, velocity and acceleration)

A particle moving along a straight line is said to be in rectilinear motion. At any instant (t), the particle will occupy a certain position on the straight line. The distance (position) x, with the Appropriate sign, completely defines the position of the particle; it is called the **Position Coordinate** of the particle. For example, the position coordinate corresponding to P in Fig. (a) is x=+5 m; the coordinate corresponding to $P^/$ in Fig. (b) is $x^/$ = -2 m.





If the motion of the particle may be given in the form of an equation in (x) and (t) such as:

$$x = 9t^2 - t^3$$

P: position occupied by the particle at time (t) and coordinate (x)

P`: position occupied by particle at time (t+Δt) and coordinate (x+Δx).

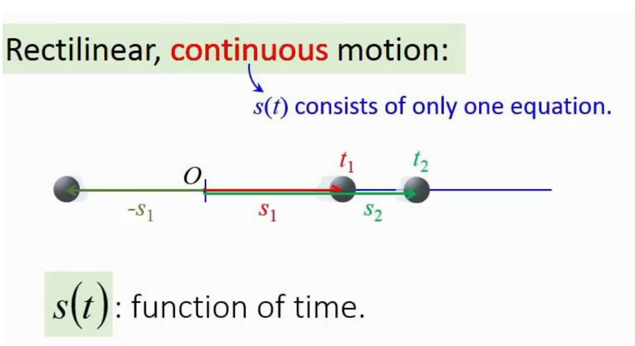
Average velocity = $\frac{\Delta x}{\Delta t}$ m/sec or ft/sec

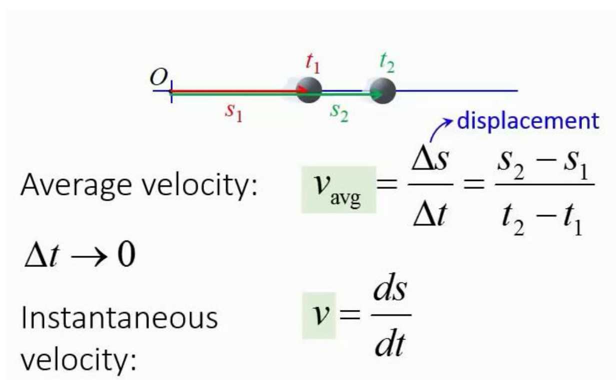
Instantaneous velocity: $v = \frac{dx}{dt}$

Average acceleration $= \frac{\Delta v}{\Delta t}$

Instantaneous acceleration $a = \frac{dv}{dt}$

Or $a = \frac{d^2x}{dt^2}$, $a = v\frac{dv}{dx}$





Example:-

If the position of a particle along x-axis varies in time as:-

$x = 2t^2 - 3t + 1$

1- What is the velocity at t=0?
2- When does velocity become zero?
3- What is the velocity at the origin?
4- Plot position – time plot.

Solution:-

We first need to find out an expression for velocity by differentiating the given function of position with respect to time as:

$v = 4t - 3$

1- The velocity at t = 0

$$v = 4 * 0 - 3 = -3 m/s$$

2- When velocity becomes zero: $4t - 3 = 0; \quad \Rightarrow t = 3/4 = 0.75 sec.$

3-The velocity at the origin:

At origin, x = 0, $x = 2t^2 - 3t + 1 = 0 \quad \Rightarrow t = 0.5\,s, 1\,s$

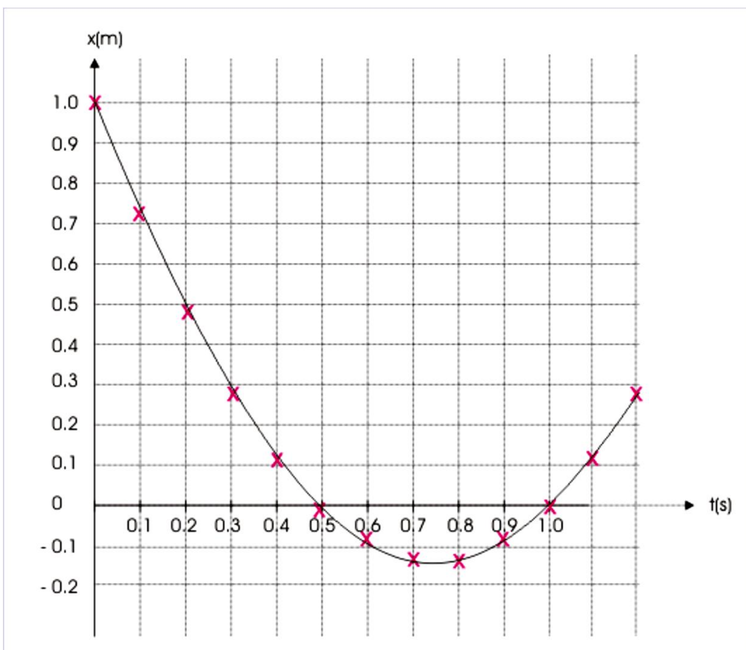
This means that particle is twice at the origin at t = 0.5 s and t = 1 s.

Now, $v_{(t=0.5s)} = 4t - 3 = 4 \times 0.5 - 3 = -1\,m/s$

Negative sign indicates that velocity is directed in the negative x _ direction.

$v_{(t=1s)} = 4t - 3 = 4 * 1 - 3 = 1\,m/s$

4-



Example:-

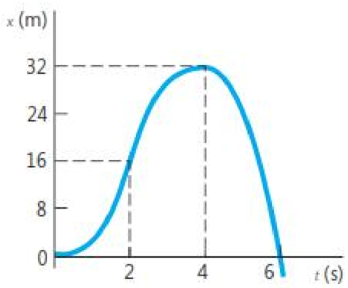
Consider a particle moving in a straight line, and assume that its position is defined by the equation: -

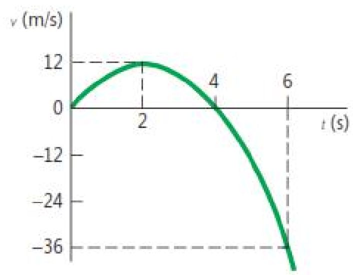
$x= 6\,t^2 - t^3$

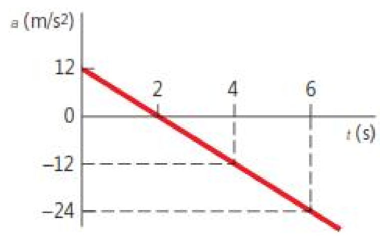
Find velocity & acceleration and draw it for t= 0 to t= 6 sec.

sol:-

$$v = \frac{dx}{dt} = 12t - 3t^2 \quad , \quad a = \frac{dv}{dt} = 12 - 6t$$







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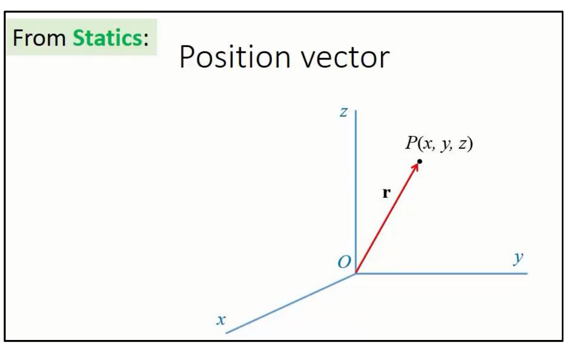
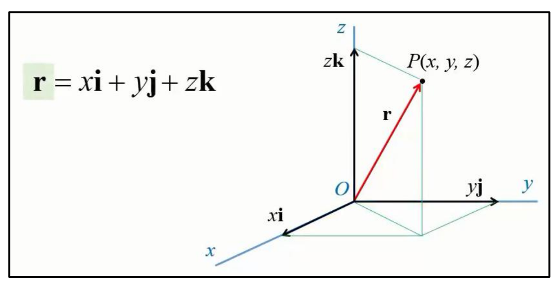
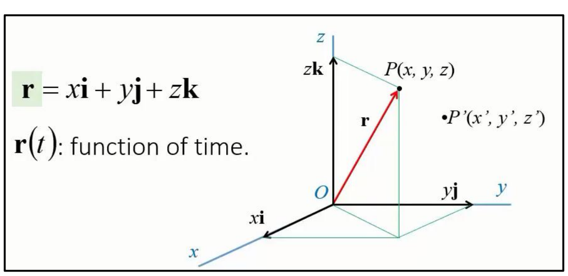
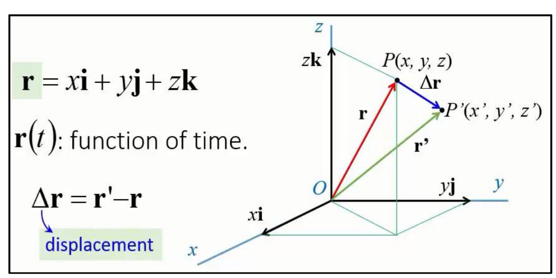
Particle Kinematics
Rectilinear, continuous motion









Velocity:

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

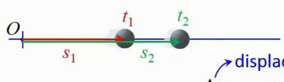
Acceleration:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

Rectilinear, continuous motion:

$s(t)$ consists of only one equation.

$s(t)$: function of time.



displacement

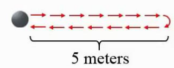
Average velocity: $v_{avg} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1}$

$\Delta t \to 0$

Instantaneous velocity: $v = \frac{ds}{dt}$

Speed: the magnitude of velocity, non-negative

Distance travelled: s_T



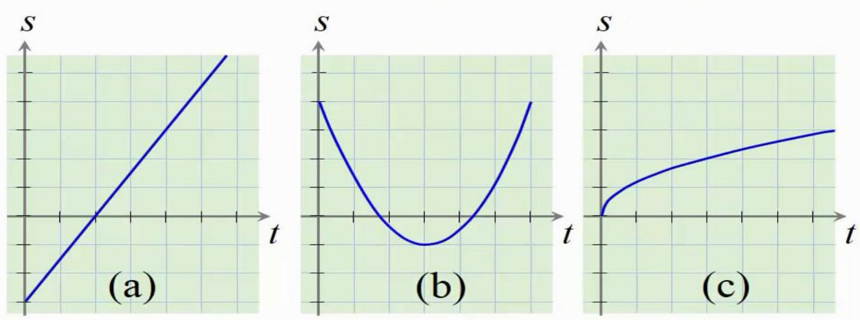
$\Delta s = 0$

$v_{avg} = \frac{\Delta s}{\Delta t} = 0$

$s_T = 10 \text{ m}$

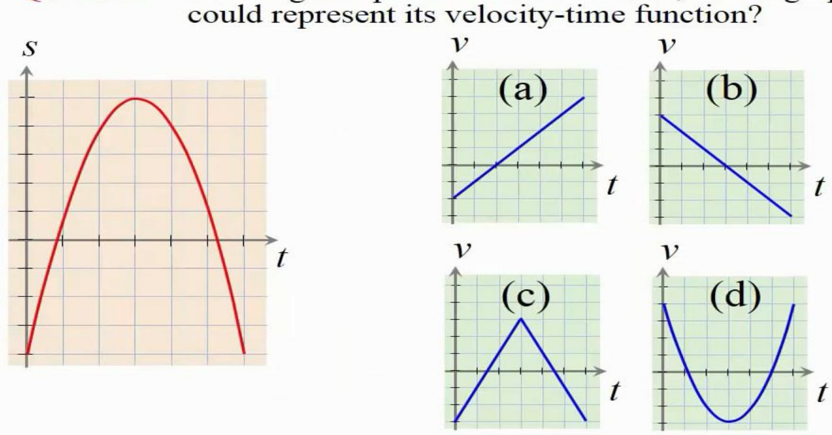
$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{10 \text{ m}}{5 \text{ s}} = 2 \text{ m/s}$

Question 2: Which graph could represent a position-time function for **rectilinear**, **continuous** motion?



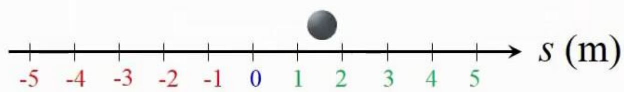
(d) All of the above.

Question 3: For the given position-time function, which graph could represent its velocity-time function?



Question 4: If the motion of the following ball takes place during 4 seconds, what is the average velocity of the ball?

Ball move (-2→ 4→ -4)



(a) 0.5 m/s (b) – 0.5 m/s
(c) 3.5 m/s (d) – 3.5 m/s

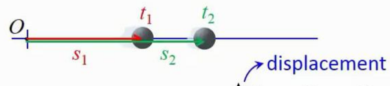
Question 5: If the motion of the following ball takes place during 4 seconds, what is the average speed of the ball?

Ball move (-2→ 4→ -4)

(a) 0.5 m/s (b) – 0.5 m/s
(c) 3.5 m/s (d) – 3.5 m/s

2d3b4b5c

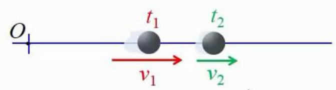
Rectilinear, continuous motion:



Average velocity: $v_{avg} = \dfrac{\Delta s}{\Delta t} = \dfrac{s_2 - s_1}{t_2 - t_1}$

$\Delta t \to 0$

Instantaneous velocity: $v = \dfrac{ds}{dt}$

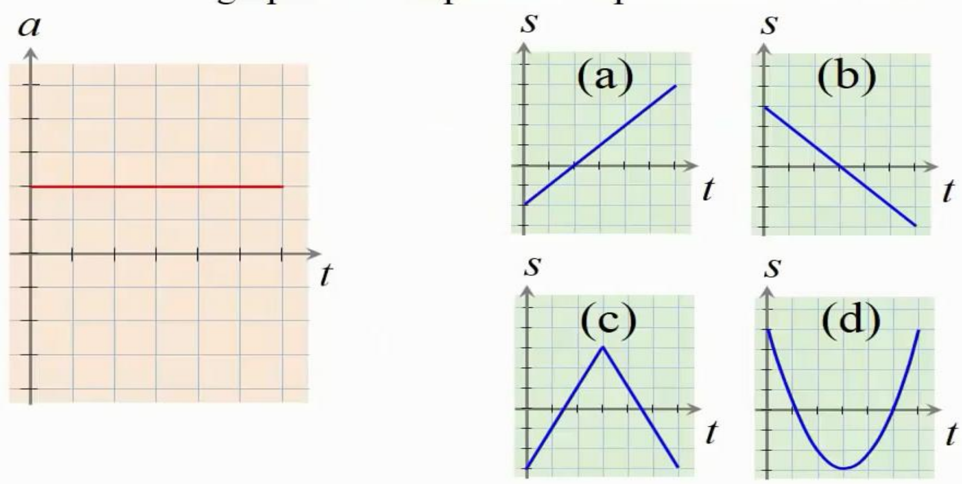


Average acceleration: $a_{avg} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$

$\Delta t \to 0$

Instantaneous acceleration: $a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$

Question 1: For the given acceleration-time function, which graph could represent its position-time function?



Three basic kinematic equations

① $v = \dfrac{ds}{dt}$

② $a = \dfrac{dv}{dt}$

$dt = \dfrac{ds}{v} = \dfrac{dv}{a}$

③ $a\,ds = v\,dv$

Example 1: The velocity of a particle moving along a straight line is $v = (t^2+2t)$ m/s, where t is in seconds. If its position $s = 0$ when $t = 0$, determine its **acceleration** and position when $t = 4$ s.

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad ads = vdv$$

$$a = \frac{dv}{dt} = \frac{d(t^2+2t)}{dt} = 2t + 2 = a(t)$$

$$@t = 4 \text{ s}, \quad a = 10 \text{ m/s}^2 \quad \text{Ans.}$$

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad ads = vdv$$

$$v = \frac{ds}{dt} \Rightarrow (t^2+2t)dt = ds \Rightarrow \int_{t=0}^{t} (t^2+2t)dt = \int_{s=0}^{s} ds$$

$$\Rightarrow s = \frac{1}{3}t^3 + t^2 = s(t)$$

Example 2: The acceleration of a particle moving along a straight line is $a = \sqrt{s}$ m/s^2, where s is in meters. If its position $s = 0$ and velocity $v = 0$ when $t = 0$, determine its **velocity** when $s = 16$ m. What time is that?

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad ads = vdv$$

$$ads = vdv \Rightarrow \sqrt{s}ds = vdv$$

$$\Rightarrow \int_{s=0}^{s} \sqrt{s}ds = \int_{v=0}^{v} vdv \quad \Rightarrow \frac{2}{3}s^{3/2} = \frac{1}{2}v^2$$

$$\Rightarrow v = \frac{2\sqrt{3}}{3}s^{3/4} = v(s)$$

$$@s = 16 \text{ m}, \quad v = 9.24 \text{ m/s} \quad \text{Ans.}$$

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt} \qquad ads = vdv$$

$$a = \frac{d^2 s}{dt^2}$$

$$v(s) = \frac{2\sqrt{3}}{3} s^{3/4}$$

$$v = \frac{ds}{dt} \Rightarrow \frac{2\sqrt{3}}{3} s^{3/4} = \frac{ds}{dt}$$

$$\Rightarrow dt = \frac{\sqrt{3}}{2} s^{-3/4} ds \Rightarrow \int_{t=0}^{t} dt = \int_{s=0}^{s} \frac{\sqrt{3}}{2} s^{-3/4} ds$$

$$\Rightarrow t = 2\sqrt{3} s^{1/4}$$

$$@ s = 16 \text{ m}, \quad t = 6.93 \text{ s}$$ Ans.

$$t = 2\sqrt{3} s^{1/4} \Rightarrow s(t) = \frac{1}{144} t^4$$

$$\Rightarrow v(t) = \frac{1}{36} t^3$$

$$\Rightarrow a(t) = \frac{1}{12} t^2$$

Question 2: For problem: "The acceleration of a particle moving along a straight line is $a = -0.2v^2$ m/s^2, where v is in m/s. If its initial velocity $v = 80$ m/s, determine its velocity when $t = 2$ s. ", which equation should you start with?

(a) $v = \dfrac{ds}{dt}$ (b) $a = \dfrac{dv}{dt}$

(c) $ads = vdv$ (d) It doesn't matter.

Question 3: When correctly solving problem: "The acceleration of a particle moving along a straight line is $a = -0.2v^2$ m/s², where v is in m/s. If its initial velocity $v = 80$ m/s, determine its velocity when $t = 2$ s. ", which of the following you might encounter?

(a) $\int_{t=0}^{t} dt = \int_{v=0}^{v} -0.2v^2 dv$ (b) $\int_{t=0}^{t} dt = \int_{v=0}^{v} -5v^{-2} dv$

(c) $\int_{t=0}^{t} dt = \int_{v=80}^{v} -0.2v^2 dv$ (d) $\int_{t=0}^{t} dt = \int_{v=80}^{v} -5v^{-2} dv$

Question 4: Based on the hint you get from the previous questions, solve this problem: "The acceleration of a particle moving along a straight line is $a = -0.2v^2$ m/s², where v is in m/s. If its initial velocity $v = 80$ m/s, determine its velocity when $t = 2$ s. "

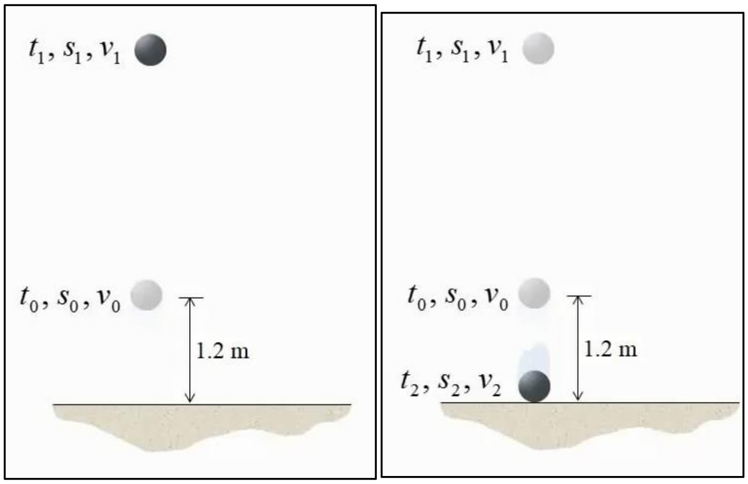
ans.: 1d2b3d

Particle Kinematics
Rectilinear motion with *constant* acceleration

Constant acceleration, a_c

$v = \dfrac{ds}{dt}$	➡	$s = s_0 + v_0 t + \dfrac{1}{2} a_c t^2$
$a = \dfrac{dv}{dt}$	➡	$v = v_0 + a_c t$
$a ds = v dv$	➡	$v^2 = v_0^2 + 2a_c (s - s_0)$

Example: A ball was thrown straight up in the air from 1.2 meter above the ground. After 3 seconds the ball returns to the ground. Determine the maximum height it reached.



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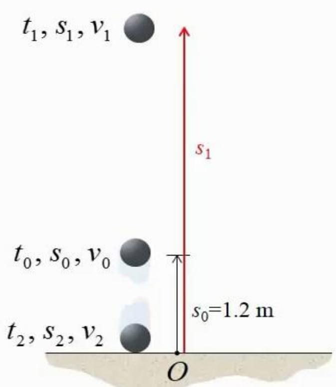
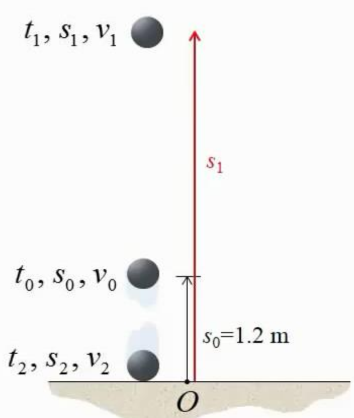


$t_0 = 0$ $\quad s_0 = 1.2$ m $\quad v_0 = ?$

$t_1 = ?$ $\quad s_1 = ?$ $\quad v_1 = 0$ $\quad a_c = -9.81$ m/s^2

$t_2 = 3$ s $\quad s_2 = 0$ $\quad v_2 = ?$

↑ Positive



$t_0 = 0$ $\quad s_0 = 1.2$ m $\quad v_0 = ?$

$t_1 = ?$ $\quad s_1 = ?$ $\quad v_1 = 0$ $\quad a_c = -9.81$ m/s^2

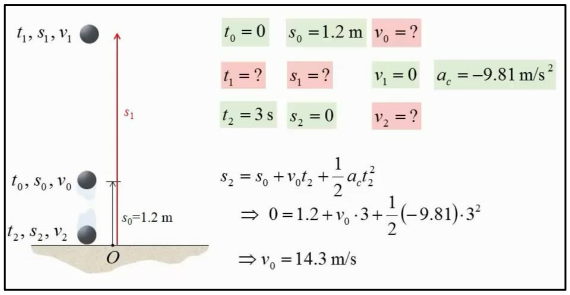
$t_2 = 3$ s $\quad s_2 = 0$ $\quad v_2 = ?$

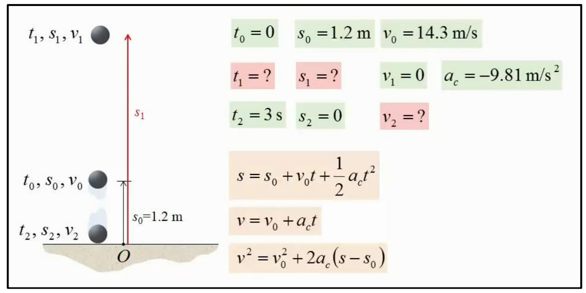
↑ Positive

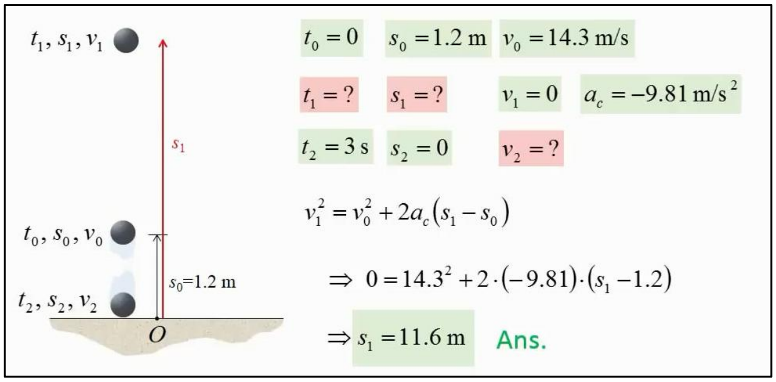
$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v = v_0 + a_c t$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$







Question 3: A ball is thrown straight upward from ground level. Its initial velocity is 20 ft/s. Determine the maximum height it would reach.

(a) 6.21 m (b) 20.4 m

(c) 6.21 ft (d) 20.4 ft

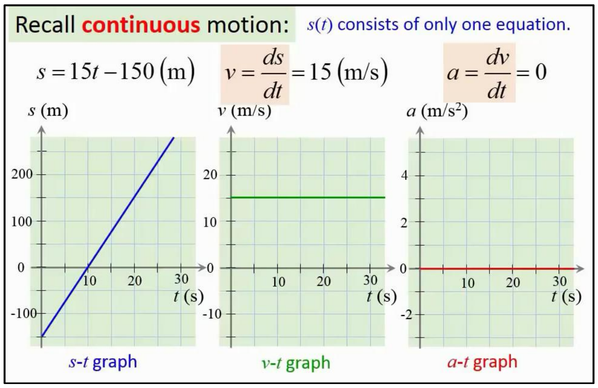
Question 4: A car starts from rest and travels at a constant acceleration of 3000 mi/h^2 along a straight path. How long does it take for it to reach a distance of 1 mile? What is its velocity at that point?

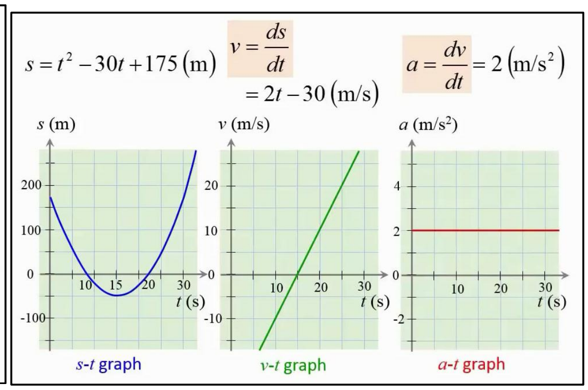
(a) 1.55 min, 60 mi/h. (b) 3.33 min, 77.5 mi/h.

(c) 3.33 min, 60 mi/h. (d) 1.55 min, 77.5 mi/h.

Ans.: 3c4d

Example:





Determination of the motion of a particle

The motion of a particle is said **to be known if** the position of the particle is known for every value of the time *t*. In general, the acceleration of the particle can be expressed as a function of one or more of the variables ***x, v,*** *and* ***t***. In order to determine the position coordinate **x** in terms of **t**, it will thus be necessary to perform two successive integrations.

Let us consider three common classes of motion:

1- $a = f(t)$. **The acceleration is a given function of t.**

$$dv = adt$$
$$dv = f(t)dt$$

Then

$$\int dv = \int f(t)dt$$

$$\int_{v_0}^{v} dv = \int_{0}^{t} f(t)dt$$

$$v - v_0 = \int_{0}^{t} f(t)dt$$

(note: we can write ***v = dx / dt***)

2- $a = f(x)$. **The acceleration is a given function of x.**

$$vdv = adx$$
$$vdv = f(x)dx$$

$$a := \frac{dv}{dt} \qquad v := \frac{dx}{dt} \qquad dt := \frac{dv}{a}$$

Then

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} f(x)\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)\,dx$$

3- $a = f(v)$. **The acceleration is a given function of v.**

$$f(v) = \frac{dv}{dt} \qquad f(v) = v\frac{dv}{dx}$$

$$dt = \frac{dv}{f(v)} \qquad dx = \frac{v\,dv}{f(v)}$$

Example:-

The position of a particle which moves along a straight line is defined by the relation $\boldsymbol{x = t^3 - 6t^2 - 15t + 40}$, where x is expressed in feet and t in seconds. Determine (a) the time at which the velocity will be zero, (b) the position and distance travelled by the particle at that time, (c) the acceleration of the particle at that time, (d) the distance travelled by the particle from $\boldsymbol{t = 4\text{ s to }t = 6\text{ s}}$., **Sol.**

The equations of motion are

$$x = t^3 - 6t^2 - 15t + 40 \qquad (1)$$

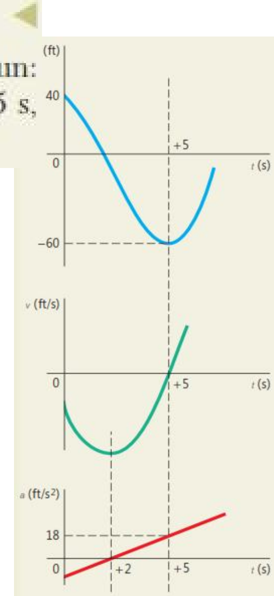
$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \qquad (2)$$

$$a = \frac{dv}{dt} = 6t - 12 \qquad (3)$$

a. Time at Which v = 0. We set $v = 0$ in (2):

$$3t^2 - 12t - 15 = 0 \qquad t = -1\text{ s} \quad \text{and} \quad t = +5\text{ s}$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$, the particle moves in the negative direction; for $t > 5$ s, $v > 0$, the particle moves in the positive direction.



b. Position and Distance Traveled When v = 0. Carrying $t = +5$ s into (1), we have

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60\text{ ft}$$

The initial position at $t = 0$ was $x_0 = +40$ ft. Since $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, we have

$$\text{Distance traveled} = x_5 - x_0 = -60\text{ ft} - 40\text{ ft} = -100\text{ ft}$$

$$\text{Distance traveled} = 100\text{ ft in the negative direction}$$

c. Acceleration When v = 0. We substitute $t = +5$ s into (3):

$$a_5 = 6(5) - 12 \qquad a_5 = +18\text{ ft/s}^2$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals will be computed separately.

From $t = 4$ s to $t = 5$ s: $x_5 = -60$ ft

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

$$\text{Distance traveled} = x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft}$$
$$= 8 \text{ ft in the negative direction}$$

From $t = 5$ s to $t = 6$ s: $x_5 = -60$ ft

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$

$$\text{Distance traveled} = x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft}$$
$$= 10 \text{ ft in the positive direction}$$

Total distance traveled **from $t = 4$ s to $t = 6$ s is 8 ft + 10 ft** = 18 ft ◀

HW : Read Exs. 11.2-11.3 & Solve problem 11.CQ1 – 11.32 page 613 in ref.1

Uniform Rectilinear Motion

In this motion, the acceleration (a) of the particle is zero for every value of t.

$$\frac{dx}{dt} = v = cons.$$

The position coordinate x is obtained by integrating this equation. Denoting by x_0 the initial value of x, we write

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

This equation can be used only if the velocity of the particle is known to be constant.

Uniformly Accelerated Rectilinear Motion

The acceleration is therefore constant and:-

$$\frac{dv}{dt} = a = const.$$

the velocity; $\boldsymbol{v}$; of the particle is obtained by integrating;

$$\int_{v_0}^{v} dv = a\int_{0}^{t} dt$$

$$v - v_0 = at$$

$$v = v_0 + at$$

Where (v_0) is the initial velocity. Substituting for (v), then we write

$$\frac{dx}{dt} = v_0 + at$$

$$\int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)dt$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

$$\Rightarrow x = x_0 + v_0 t + \frac{1}{2}at^2$$

We can also write

$$v\frac{dv}{dx} = a = const.$$

$$vdv = adx$$

$$\int_{v_0}^{v} vdv = a\int_{x_0}^{x} dx$$

$$\frac{1}{2}(v^2 - v_0^2) = a(x - x_0)$$

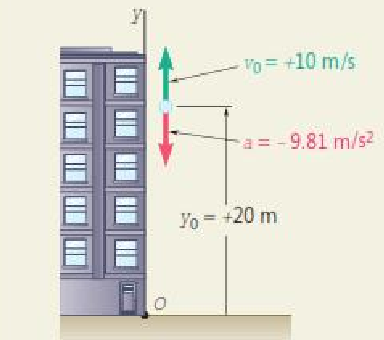
$$\Rightarrow v^2 = v_0^2 + 2a(x - x_0)$$

The three equations we have derived provide useful relations among position coordinate, velocity, and time in the case of a uniformly accelerated motion, an important application of uniformly accelerated motion is the ***motion of a freely falling body***. The acceleration of a freely falling body (usually denoted by g) is equal to 9.81 m/s^2 or 32.2 ft./s^2.

It is important to keep in mind that the three equations can be used ***only when the acceleration of the particle is known to be constant****.*

Example: - A ball is tossed with a velocity of 10 m/s directed vertically upward from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine (a) the velocity v and elevation y of the ball above the ground at any time t, (b) the highest elevation reached by the ball and the corresponding value of t, (c) the time when the ball will hit the ground and the corresponding velocity. Draw the v−t and y−t curves.

SOLUTION



a. Velocity and Elevation. The y axis measuring the position coordinate (or elevation) is chosen with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated. Substituting for a in $a = dv/dt$ and noting that at $t = 0$, $v_0 = +10$ m/s, we have

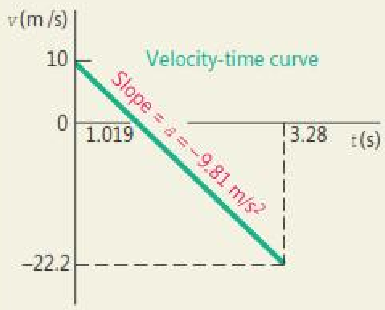
$$\frac{dv}{dt} = a = -9.81\,\text{m/s}^2$$

$$\int_{v_0=10}^{v} dv = -\int_0^t 9.81\,dt$$

$$[v]_{10}^{v} = -[9.81t]_0^t$$

$$v - 10 = -9.81t$$

$$v = 10 - 9.81t \quad (1)$$



Substituting for v in $v = dy/dt$ and noting that at $t = 0$, $y_0 = 20$ m, we have

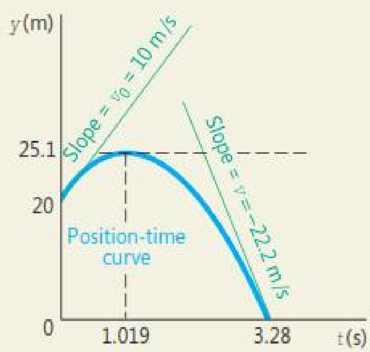
$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0=20}^{y} dy = \int_0^t (10 - 9.81t)\,dt$$

$$[y]_{20}^{y} = [10t - 4.905t^2]_0^t$$

$$y - 20 = 10t - 4.905t^2$$

$$y = 20 + 10t - 4.905t^2 \quad (2)$$



b. Highest Elevation. When the ball reaches its highest elevation, we have $v = 0$. Substituting into (1), we obtain

$$10 - 9.81t = 0 \qquad t = 1.019 \text{ s}$$

Carrying $t = 1.019$ s into (2), we have

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \qquad y = 25.1 \text{ m}$$

c. Ball Hits the Ground. When the ball hits the ground, we have $y = 0$. Substituting into (2), we obtain

$$20 + 10t - 4.905t^2 = 0 \qquad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s}$$

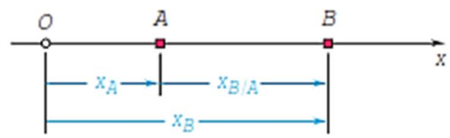
Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into (1), we have

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \qquad v = 22.2 \text{ m/s w}$$

Motion of Several Particles: Dependent Motion

Relative Motion of Two Particles. Consider two particles A and B moving along the same straight line. If the position coordinates x_A and x_B are measured from the same origin, the difference $x_B - x_A$ defines the relative position coordinate of B with respect to A and is denoted by $x_{B/A}$. We write

$$x_{B/A} = x_B - x_A \quad \text{or} \quad x_B = x_A + x_{B/A}$$



The rate of change of $x_{B/A}$ is known as the relative velocity of B with respect to A and is denoted by $v_{B/A}$. Differentiating above, we write

$$v_{B/A} = v_B - v_A \quad \text{or} \quad v_B = v_A + v_{B/A}$$

A positive sign for $v_{B/A}$ means that B is observed from A to move in the positive direction;

a negative sign means that it is observed to move in the negative direction.

The rate of change of $v_{B/A}$ is known as the relative acceleration of B with respect to A and is denoted by $a_{B/A}$. Differentiating above equ., we obtain

$$a_{B/A} = a_B - a_A \quad \text{or} \quad a_B = a_A + a_{B/A}$$

Dependent Motions:

Position of a particle may *depend* on position of one or more other particles.

$$x_A + 2x_B = \text{constant}$$