3. Centroid And Centre of Gravity

Centre of Gravity:

The centre of gravity of a body is that the point through which the whole weight of the body acts. A body is having only one centre of gravity for all positions of the body. It is represented by "G" (or) "CG"

Centroid:

The point at which the total area of plane figure (like rectangle, square, triangle, circle etc) is assumed to be concentrated is known as Centroid of that area. It is also represented by "G" (or) "CG". The centriod and centre of gravity is at the same point.

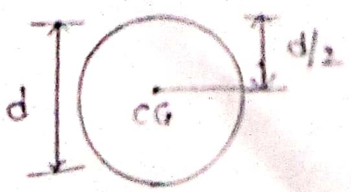
Methods for centre of Gravity:

→ The centre of gravity may be find out by following methods.

1. Geometrical Considerations
2. By methods of moments
3. By Graphical method.

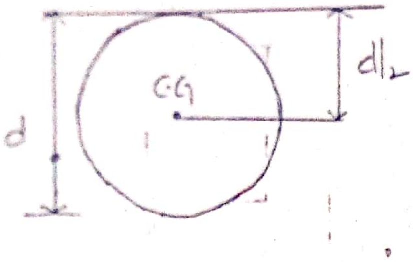
1. By Geometrical Considerations:

1 Uni-form Rod

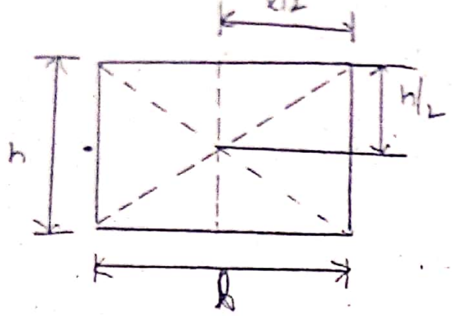


Centre of gravity	centroid
1) C.G applies to the bodies with masses and weight	1) centroid applies to the plane area.
2) It is the point where the whole weight of the body will act	2) It is the point where the whole area of the plane will act.
3) C.G of a body is a point through which the resultant gravitational force acts.	3) The moment of area about any axis through the point in a plane area is zero.
4) It is an irrespective of the orientation of the body	4) It is acceptable only the plane figures like the centre of a triangle
5) It is represented by C.G (or) G.	5) It is represented 'G'.

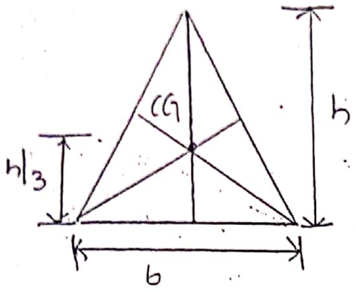
R.



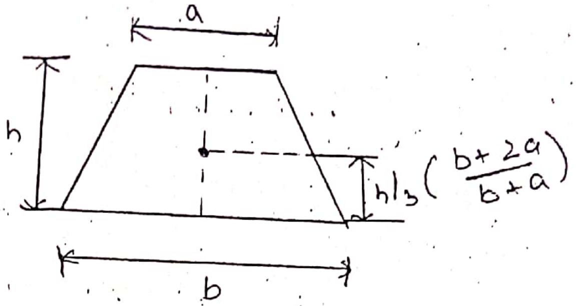
2. Rectangle



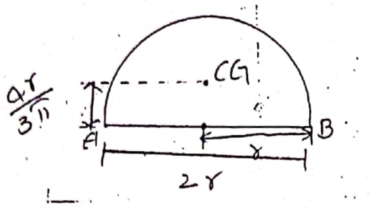
3. Triangle



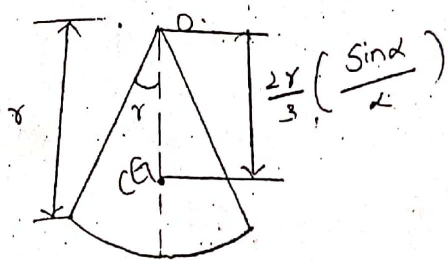
4. Trapezium



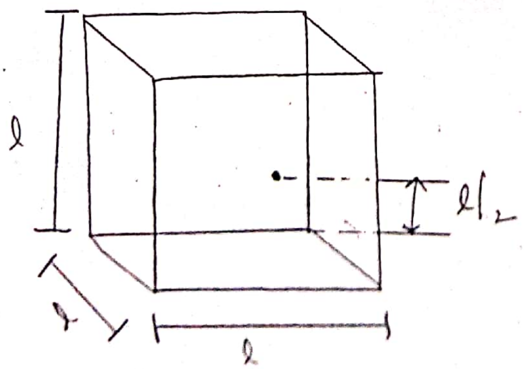
5. semicircular



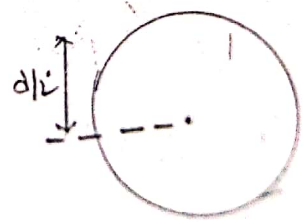
6. circular sector



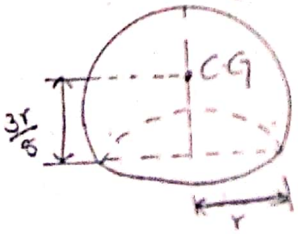
7. Cube



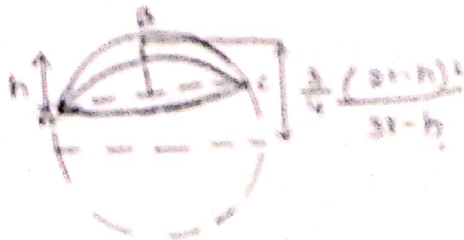
8. sphere



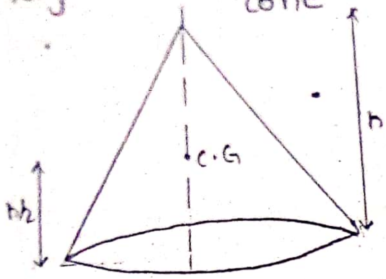
9. Hemisphere



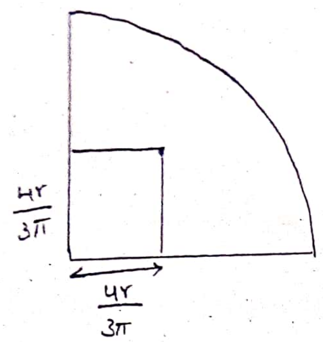
10. segment of sphere



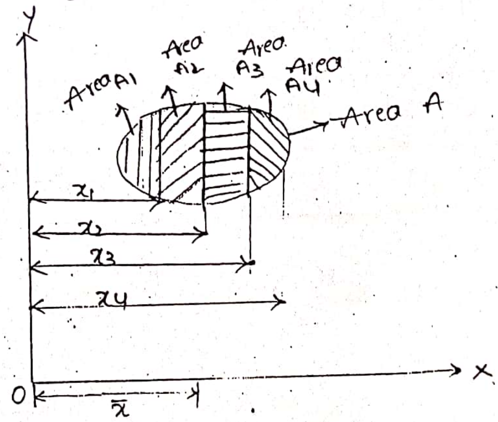
11. Right circular solid cone



12. Quarter circle



By the methods of moments



→ Shows a plane figure of total area 'A' whose centre of gravity is to be determined.

→ Let 'A' is the area of composed of a number of small areas $A_1, A_2, A_3, \ldots$

$$\text{Total area } A = A_1 + A_2 + A_3 + \ldots$$

→ Let x_1 = distance of centre of gravity of area A_1 from OY axis

x_2 = distance of centre of gravity of area A_2 from OY axis

x_3 = distance of centre of gravity of area A_3 from OY axis

x_4 = distance of centre of gravity of area A_4 from OY axis

→ The moments of all small areas about OX axis is equal to

$$A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + \cdots \quad \text{——} (1)$$

→ Let $\bar{y}$ = distance of centre of gravity of total area 'A'

Moment of total area = $A\bar{y}$ —— (2)

Equating (1) & (2) equations

$$A\bar{y} = A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + \cdots$$

$$\boxed{\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + \cdots}{A}}$$

→ If we take moments of all areas about OX axis is equal to

$$A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + \cdots \quad \text{——} (3)$$

→ Let $\bar{x}$ = distance of centre of gravity of total area 'A'

Moment of total area = $A\bar{x}$ —— (4)

Equating (3) & (4) equations

$$A\bar{x} = A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + \cdots$$

$$\boxed{\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + \cdots}{A}}$$

By Graphical Method:

· Right angled Triangle

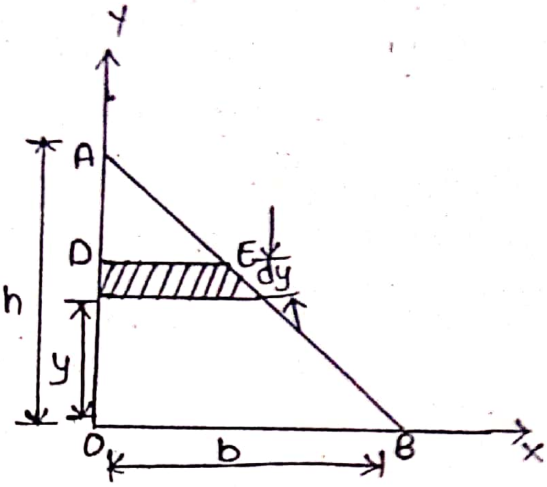


Figure shows a right angled triangle AOB of base width 'b' and height 'h'

Consider a small strip of thickness 'dy' at a distance 'y' from the OX axis.

Area of the strip dA = length of DE × dy

$$dA = DE \times dy \quad \text{—— (1)}$$

from similar triangles ADE and AOB

$$\frac{DE}{OB} = \frac{AD}{OA}$$

$$\frac{DE}{b} = \frac{h-y}{h}$$

$$DE = \frac{b}{h}(h-y)$$

from (1) $\quad dA = \frac{b}{h}(h-y) \times dy$

Moment of small strip dA about x-axis

$$= dA \times y$$

$$= \frac{b}{h}(h-y) \times dy \times y$$

$$= \frac{b}{h}(h-y)\, y \times dy$$

$$dA = \frac{b}{h}(hy - y^2)\,dy$$

The moment of total area A of the triangular section is obtained by integrating the above equation between the limits of 'o' to 'h'

$$\int_0^h dA = \int_0^h \frac{b}{h}(hy - y^2)\,dy$$

$$= \frac{b}{h}\int_0^h (hy - y^2)\,dy$$

$$= \frac{b}{h}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{b}{h}\left[\frac{h^3}{2} - \frac{h^3}{3}\right]$$

$$= \frac{b}{h}\left[\frac{h^3}{6}\right]$$

$$= \frac{bh^2}{6} \quad \text{——— } (2)$$

Let $\bar{y}$ = distance of centriod of total area 'A' from axis ar

Total Area

$$A = \int_0^h dA$$

$$= \int_0^h \frac{b}{h}(h-y)\,dy$$

$$= \frac{b}{h}\int_0^h (h-y)\,dy$$

$$= \frac{b}{h}\left[hy - \frac{y^2}{2}\right]_0^h$$

$$= \frac{b}{h}\left[h^2 - \frac{h^2}{2}\right]$$

$$= \frac{b}{h}\left[\frac{h^2}{2}\right]$$

$$\boxed{A = \frac{bh}{2}}$$

Total moment of triangular section about 'ox' axis is

$$= A \times \bar{y}$$

$$= \frac{bh}{2} \times \bar{y} \quad \text{---} (3)$$

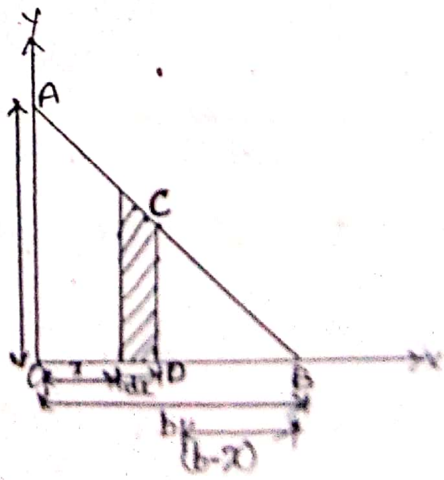
Equating (2) & (3) equations

$$\frac{bh^2}{6} = \frac{bh}{2} \times \bar{y}$$

$$\boxed{\bar{y} = \frac{h}{3}}$$

⇒ Similarly by considering vertical strip of thickness 'dx' at a distance 'x' from OY axis

Consider a small strip of thickness 'dx' at a distance of 'x' from x-axis



Area of the strip $dA = CD \times dx$ — (1)

from similar triangles CDB & AOB

$$\frac{CD}{AO} = \frac{BD}{OB}$$

$$\frac{CD}{h} = \frac{b-x}{b}$$

$$CD = \frac{h}{b}(b-x)$$

$$dA = \frac{h}{b}(b-x)\,dx$$

Moment of small strip dA about y axis

$$= dA \times x$$

$$= \frac{h}{b}(b-x)\,dx \times x$$

$$dA = \frac{h}{b}(bx-x^2)\,dx$$

The moment of total area A of the triangular section is obtained by ~~triangular~~ integrating the above equation betwee the limits of '0' to 'b'

$$= \int_0^b \frac{h}{b}(bx-x^2)\,dx$$

$$= \frac{h}{b}\int_0^b (bx-x^2)\,dx$$

$$= \frac{h}{b}\left[\frac{bx^2}{2} - \frac{x^3}{3}\right]_0^b$$

$$= \frac{h}{b}\left[\frac{b^3}{2} - \frac{b^3}{3}\right]$$

$$= \frac{hb^2}{6} \quad \text{——— (2)}$$

Let $\bar{x}$ = distance of centroid of total area 'A'

Total area

$$A = \int_0^b dA$$

$$= \int_0^b \frac{h}{b}(b-x)\,dx$$

$$= \frac{h}{b}\left[bx - \frac{x^2}{2}\right]_0^b$$

$$= \frac{h}{b}\left[b^2 - \frac{b^2}{2}\right]$$

$$= \frac{h}{b}\left[\frac{b^2}{2}\right]$$

$$\boxed{A = \frac{bh}{2}}$$

Total moment of triangular section about ox axis is

$$= A \times \bar{x}$$

$$= \cancel{\frac{bh}{2}} \times \cancel{\frac{b^2h}{6}}$$

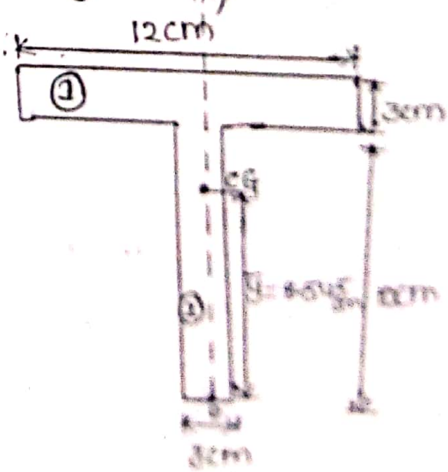
$$= \frac{bh}{2} \times \bar{x} \text{ --- } (3)$$

Equating (2) and (3) equations

$$\frac{hb^2}{6} = \frac{bh}{2} \times \bar{x}$$

$$\boxed{\bar{x} = \frac{b}{3}}$$

1. Find the centre of gravity of the following 'T' section



A. The given section is symmetrica about oy axis and it is divided into 2 parts

Area of part (1) $A_1 = 12 \times 3$

$= 36cm^2$

Area of part (2) $A_2 = 10 \times 3$

$= 30cm^2$

Total area $A = A_1 + A_2$

$= 36 + 30 = 66cm^2$

$$\bar{y}_1 = 10 + \frac{3}{2}$$

$$= 11.5\,cm$$

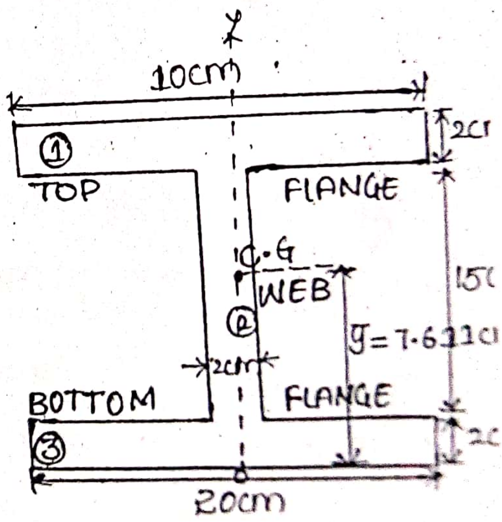
$$y_2 = \frac{10}{2}$$

$$= 5\,cm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A} = \frac{36 \times 11.5 + 5 \times 30}{66}$$

$$\boxed{\bar{y} = 8.545\,cm}$$

2. Find the centre of gravity of given 'I' section as shown in figure.



A. Given section is symmetrical about OY axis and it is divided into 3 parts

Area of part ① $A_1 = 10 \times 2$
$= 20\,cm^2$

Area of part ② $A_2 = 15 \times 2$
$= 30\,cm^2$

Area of part ③ $A_3 = 20 \times 2$
$= 40\,cm^2$

Total area $A = A_1 + A_2 + A_3$
$= 20 + 30 + 40$
$= 90\,cm^2$

$$y_1 = 2 + 15 + \frac{2}{2}$$

$$= 2 + 15 + 1$$

$$= 18\,cm$$

$$y_2 = 2 + \frac{15}{2}$$

$$= 2 + 7.5$$

$$= 9.5\,cm$$

$$y_3 = \frac{2}{2}$$

$$= 1\,cm$$

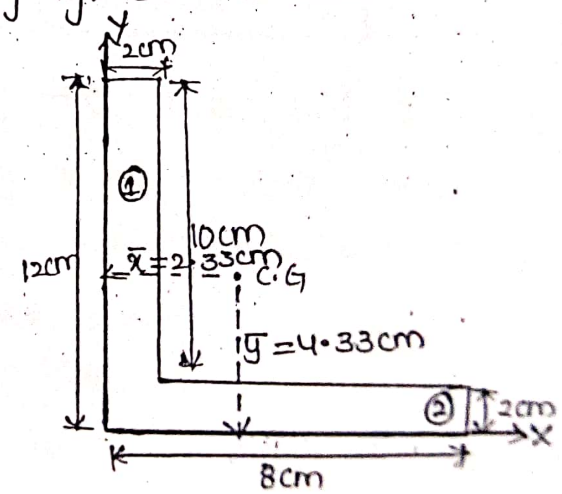
$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$\bar{y} = \frac{20 \times 18 + 30 \times 9.5 + 40 \times 1}{90}$$

$$\boxed{\bar{y} = 7.611\,cm}$$

3. Find the centre of gravity of given 'L' section

A: Given section is unsymmetrical and it is divided into 2 parts



Area of part ① $A_1 = 12 \times 2$

$= 24\,cm^2$

Area of part ② $A_2 = 8 \times 2$

$= 16\,cm^2$

Total area $A = A_1 + A_2$

$= 24 + 16$

$= 36\,cm^2$

$$y_1 = 2 + \frac{10}{2}$$

$$= 7\,cm$$

$$y_2 = \frac{2}{2}$$

$$= 1\,cm$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A}$$

$$= \frac{20 \times 7 + 16 \times 1}{36}$$

$$\boxed{\bar{y} = 4.33\,cm}$$

$$x_1 = \frac{2}{2} = 1\,cm$$

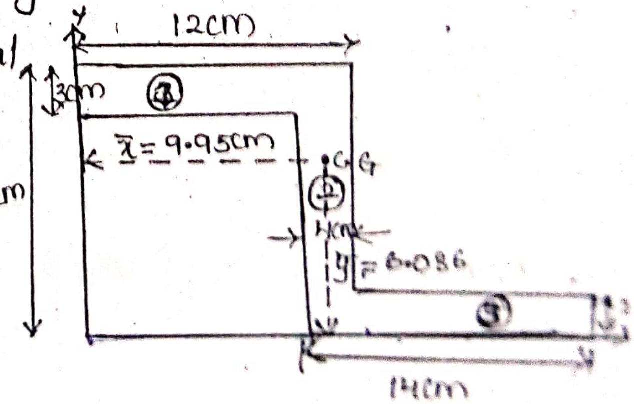
$$x_2 = \frac{8}{2} = 4\,cm$$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A}$$

$$\bar{x} = \frac{20 \times 1 + 16 \times 4}{36}$$

$$\boxed{\bar{x} = 2.33\,cm}$$

4. Find the centre of gravity given 'z' section



A: Given section is unsymmetrical and it is divided into 3 parts

Area of part ① $A_1 = 12 \times 3$
$= 36\,cm^2$

Area of part ② $A_2 = 7 \times 4$
$= 28\,cm^2$

Area of part ③ $A_3 = 14 \times 2$
$= 28\,cm^2$

Total area $A = A_1 + A_2 + A_3$
$= 36 + 28 + 28$
$= 92\,cm^2$

$x_3 = 8 + \frac{14}{2}$
$= 15\,cm$

$x_2 = 12 - 2$
$= 10\,cm$

$x_1 = \frac{12}{2}$
$= 6\,cm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

$$= \frac{36 \times 15 + 28 \times 10 + 28 \times 6}{92}$$

$$\boxed{\bar{x} = 9.95 cm}$$

$y_1 = 2 + 7 + \frac{3}{2}$ $= 10.5$ cm

$y_2 = 2 + \frac{7}{2}$ $= 5.5$ cm

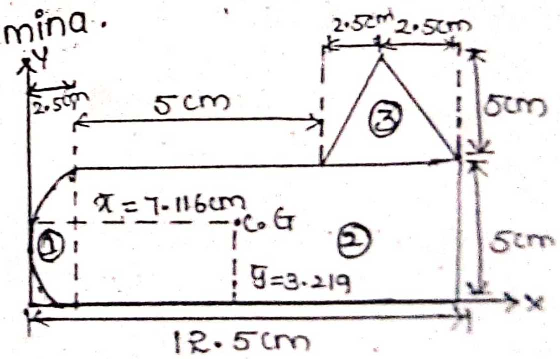
$y_3 = \frac{2}{2}$ $= 1$ cm

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A}$$

$$\bar{y} = \frac{36 \times 10.5 + 28 \times 5.5 + 28 \times 1}{92}$$

$$\boxed{\bar{y} = 6.086}$$

5. By using analytical method determine the centre of gravity of the plane uniform lamina.



A. Given section is unsymmetrical & divided into 3 parts

2. Rectangle

Area of part ② $A_2 = 10 \times 5$
$= 50 cm^2$

1. Semicircle

Area of part ① $A_1 = \frac{\pi}{2} \times r^2$
$= \frac{\pi}{2} \times (2.5)^2$
$= 9.812 cm^2$

3. Triangle

Area of part ③ $= \frac{1}{2} \times b \times h$
$= \frac{1}{2} \times 5 \times 5 = 12.5 cm^2$

Total area $A = A_1 + A_2 + A_3$

$= 9.812 + 50 + 12.5$

$= 72.312\ cm^2$

$x_1 = 2.5 - \frac{4r}{3\pi}$

$= 2.5 - \frac{4 \times 2.5}{3 \times 3.141}$

$= 1.489\ cm$

$x_2 = 2.5 + \frac{10}{2}$

$= 7.5\ cm$

$x_3 = 2.5 + 5 + 2.5$

$= 10\ cm$

$$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A}$$

$$\bar{x} = \frac{9.812 \times 1.489 + 50 \times 7.5 + 12.5 \times 10}{72.312}$$

$$\boxed{\bar{x} = 7.116\ cm}$$

$y_1 = \frac{5}{2} = 2.5\ cm$

$y_2 = \frac{5}{2} = 2.5\ cm$

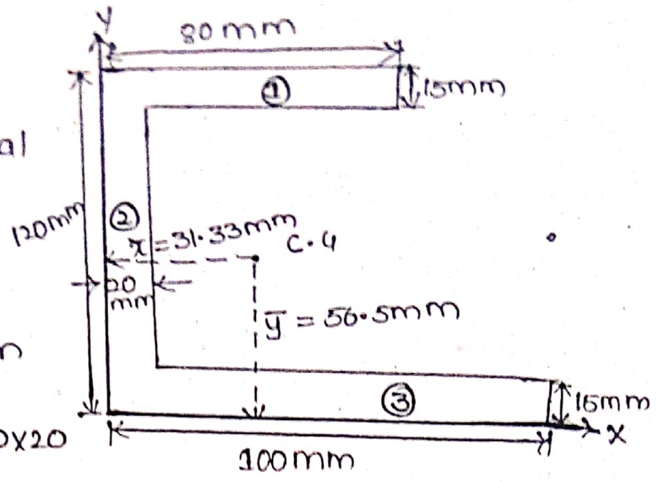
$y_3 = 5 + \frac{5}{3} = 6.666\ cm$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$$

$$\bar{y} = \frac{9.812 \times 2.5 + 50 \times 2.5 + 12.5 \times 6.666}{72.312}$$

$$\boxed{\bar{y} = 3.219\ cm}$$

. Find the centriod of the plane lamina as shown in figure.



A. Given section is unsymmetrical and divided into 3 parts

Area of part ① $A_1 = 15 \times 80$
$= 1200mm$

Area of part ② $A_2 = 120 - 30 \times 20$
$= 1800mm$

Area of part ③ $A_3 = 15 \times 100$
$= 1500mm$

Total Area $A = A_1 + A_2 + A_3$
$= 1200 + 1800 + 1500$
$= 4500 mm$

$x_1 = \frac{80}{2} = 40mm$ $x_2 = \frac{20}{2} = 10mm$ $x_3 = \frac{100}{2} = 50mm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$$

$$\bar{x} = \frac{1200 \times 40 + 1800 \times 10 + 1500 \times 50}{4500}$$

$$\boxed{\bar{x} = 31.33 mm}$$

$y_1 = 15 + 90 + \frac{15}{2}$
$= 11.25mm$

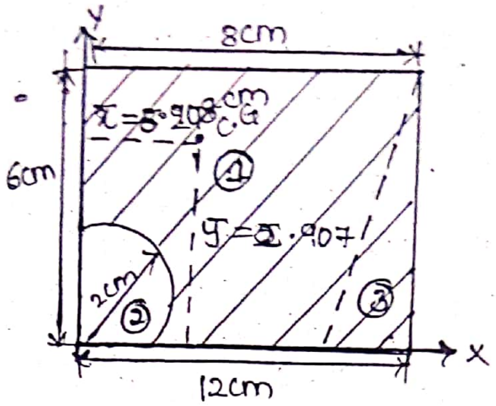
$y_2 = 15 + \frac{90}{2}$
$= 60mm$

$y_3 = \frac{15}{2}$
$y_3 = 7.5mm$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A} = \frac{1200 \times 11.25 + 1800 \times 60 + 1500 \times 7.5}{4500}$$

$$\boxed{\bar{y} = 56.5mm}$$

7. Determine the centre of gravity of the shaded area as shown in fig.



A. Given section is unsymmetrical and divided into ③ parts

1. Rectangle

Area of part ① $A_1 = 6 \times 8$
$= 48 cm^2$

2. Triangle

Area of part ② $A_2 = \frac{1}{2} \times 4 \times 6$
$= 12 cm^2$

3. Quater circle

Area of part ③ $A_3 = \frac{\pi}{4} \times r^2$
$= \frac{3.141}{4} \times (2)^2 = 3.14 cm^2$

Total area $A = A_1 + A_2 - A_3$
$= 48 + 12 - 3.14$
$= 56.86 cm^2$

$x_1 = \frac{8}{2} = 4cm$ $\quad x_2 = 8 + \frac{4}{3}$ $\quad x_3 = \frac{4r}{3\pi}$
$= 9.333cm$ $\quad = 0.848cm$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A}$$

$$\boxed{\bar{x} = 5.298 \text{ cm}}$$

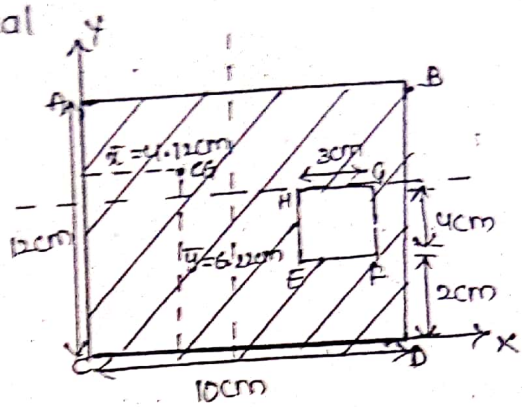
$y_1 = \frac{6}{2} = 3cm$ $\quad y_2 = \frac{6}{3} = 6cm$ $\quad y_3 = \frac{4r}{3\pi} = 0.848 cm$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A}$$

$$\boxed{\bar{y} = 2.907 \text{ cm}}$$

3. From a rectangle lamina ABCD 10cm x 12cm of a rectangular hole of 3cm x 4cm is cut as shown in fig.

4. The given section is unsymmetrical and divided into two parts



Area of part ① $A_1 = 12 \times 10$

$= 120 cm^2$

Area of part ② $A_2 = 4 \times 3$

$= 12 cm^2$

Total area $A = A_1 - A_2$

$= 120 - 12$

$= 108 cm^2$

$x_1 = \frac{10}{2} = 5cm$ $\quad$ $x_2 = 5 + 1 + 1.5$

$= 7.5cm$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A}$$

$$\boxed{\bar{x} = 4.72cm}$$

$y_1 = \frac{12}{2} = 6cm$ $\quad$ $y_2 = 2 + \frac{4}{2}$

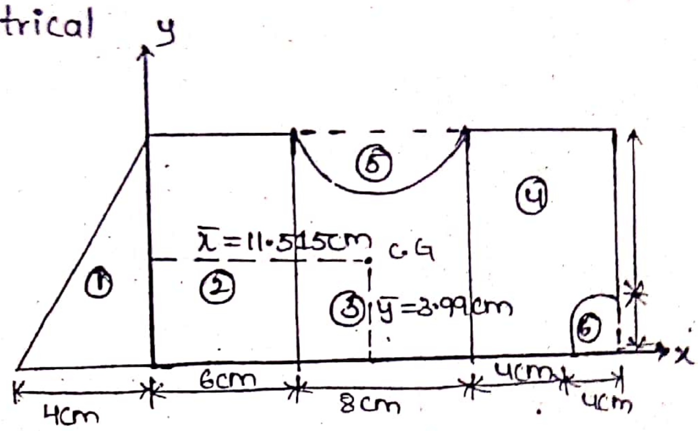
$= 4cm$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A}$$

$$\boxed{\bar{y} = 6.222 cm}$$

9. Find Centre of gravity of the composition figure given below.

A. Given section is unsymmetrical and divided into 6 parts



1. Right angle triangle

Area of part ① $A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 4$

$= 16cm^2$

2. Rectangle

Area of part ② $A_2 = 6 \times 8$

$= 48cm^2$

3. Square

Area of part ③ $A_3 = 8 \times 8$

$= 64cm^2$

4. Square

Area of part ④ $A_4 = 8 \times 8$

$= 64cm^2$

5. Semicircle

Area of part ⑤ $A_5 = \frac{\pi}{2} r^2$

$= \frac{\pi}{2} \times (4)^2 = 25.12cm^2$

6. Quater circle.

Area of part ⑥ $A_6 = \frac{\pi}{4} \times r^2$

$= 12.56cm^2$

Total area $A = A_2 + A_3 + A_4 - A_1 - A_5 - A_6$

$= 122.32cm^2$

$x_1 = \frac{b}{3} = \frac{4}{3}$ $\quad$ $x_2 = \frac{6}{2}$ $\quad$ $x_3 = \frac{8}{2} + 6$

$= 1.33\,cm$ $\quad$ $= 3\,cm$ $\quad$ $= 10\,cm$

$x_4 = 6+8+\frac{8}{2}$ $\quad$ $x_5 = 6+8+4+\left(4-\frac{4r}{3\pi}\right)$ $\quad$ $x_6 = 6+\frac{8}{2}$

$= 18\,cm$ $\quad$ $= 20.302\,cm$ $\quad$ $= 10\,cm$

$$\bar{x} = \frac{A_2x_2 + A_3x_3 + A_4x_4 - A_1x_1 - A_5x_5 - A_6x_6}{A}$$

$$\boxed{\bar{x} = 11.515\,cm}$$

$y_1 = \frac{8}{3} = 2.66\,cm$ $\quad$ $y_2 = \frac{8}{2}$ $\quad$ $y_3 = \frac{8}{2}$

$= 4\,cm$ $\quad$ $= 4\,cm$

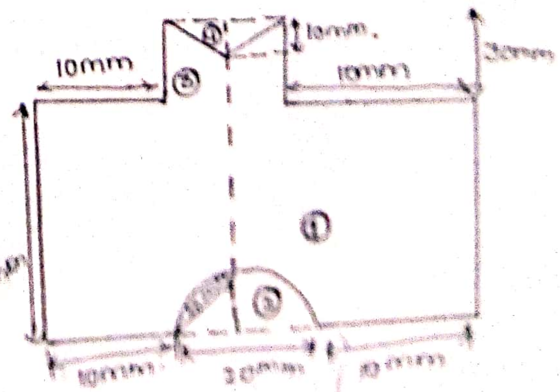
$y_4 = \frac{8}{2}$ $\quad$ $y_5 = 4+\left(4-\frac{4r}{3\pi}\right)$ $\quad$ $y_6 = \frac{4r}{3\pi}$

$= 4\,cm$ $\quad$ $= 6.804\,cm$ $\quad$ $= 1.698\,cm$

$$\bar{y} = \frac{A_2y_2 + A_3y_3 + A_4y_4 - A_1y_1 - A_5y_5 - A_6y_6}{A}$$

$$\boxed{\bar{y} = 3.99\,cm}$$

11. Locate the centriod for the shaded area as shown in fig. below



A. Given section is unsymmetrical and divided into 4 parts

1. Rectangle

Area of ① $A_1 = 40 \times 30$

$= 1200\,mm^2$

2. Semi circle

Area of ② $A_2 = \frac{\pi}{2} r^2$

$= \frac{\pi}{2} \times 100$

$= 157 mm^2$

3. Rectangle

Area of ③ $A_3 = 30 \times 20$

$= 600 mm^2$

4. Triangle

Area of ④ $A_4 = \frac{1}{2} \times 10 \times 20$

$= 100 mm^2$

Total area $A = A_1 - A_2 + A_3 - A_4$

$= 1200 - 157 + 600 - 100$

$= 1543 mm^2$

$y_1 = \frac{30}{2} = 15mm$ $\quad$ $y_2 = \frac{4r}{3\pi} = \frac{4 \times 10}{3.3141}$

$= 4.244mm$

$y_3 = 30 + \frac{30}{2}$

$= 45 mm$

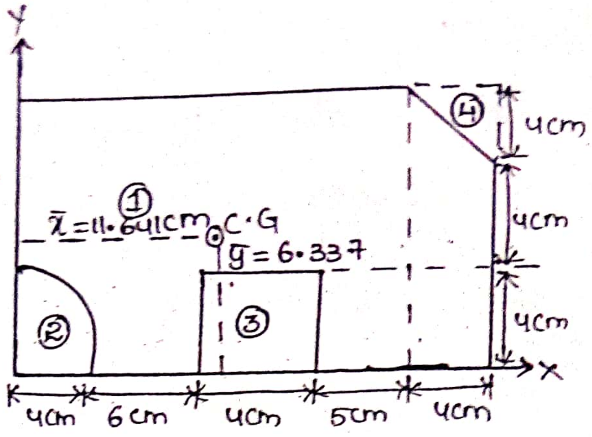
$y_4 = 50 + \frac{2}{3} \times 10$

$= 56.66mm$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2 + A_3 y_3 - A_4 y_4}{A}$$

$$\boxed{\bar{y} = 25.060mm}$$

2. Locate the centriod for the shaded area as shown in figure.



1. Given section is unsymmetrical and divided into 4 parts

1. Rectangle

Area of part ① $A_1 = 23 \times 12$

$= 276 cm^2$

2. Quater circle

Area of part ② $A_2 = \frac{\pi}{4} \times r^2$

$= \frac{\pi}{4} \times 4^2 = 12.56 cm^2$

3. Square

Area of part ③ $A_3 = 4 \times 4$

$= 16 cm^2$

4. Right angle triangle

Area of part ④ $A_4 = \frac{1}{2} \times 4 \times 4$

$= 8 cm^2$

Total area $A = A_1 - A_2 - A_3 - A_4$

$= 276 - 12.56 - 16 - 8$

$A = 239.434 cm^2$

Let

$x_1 = \frac{23}{2} = 11.5 cm$ $\qquad x_2 = \frac{4r}{3\pi} = 1.697 cm$

$x_3 = 4 + 6 + \frac{4}{2}$

$= 12 cm$

$x_4 = 4 + 6 + 4 + 5 + \frac{2 \times 4}{3}$

$= 21.666 cm$

$$\bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4}{A}$$

$$\boxed{\bar{x} = 11.641\,cm}$$

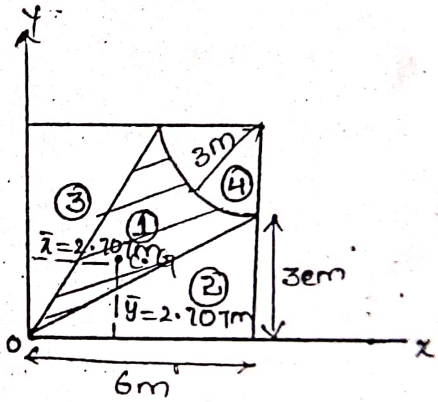
$y_1 = \frac{12}{2} = 6cm \qquad y_2 = \frac{4r}{3\pi} = 1.697cm \quad , y_3 = \frac{4}{2} = 2cm$

$$y_4 = 12 - \frac{4}{3}$$
$$= 10.67cm$$

$$\bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4}{A}$$

$$\boxed{\bar{y} = 6.337\,cm}$$

13. Locate the centriod of the shaded area as shown in figure



A. Given section is unsymmetrical and divided into 4 parts

1. Square

Area of part ① $A_1 = 6\times6$
$= 36cm^2$

2. Right angle triangle

Area of part ② $A_2 = \frac{1}{2}\times 6\times 3$
$= 9cm^2$

3. Right angle triangle

Area of part ③ $A_3 = \frac{1}{2}\times6\times3$
$= 9cm^2$

4. Quatercircle

Area of part ④ A_4
$= \frac{\pi}{4}\times9$
$= 7.065m^2$

Total area $A = A_1 - A_2 - A_3 - A_4$

$$A = 10.9356\,m^2$$

$x_1 = \frac{6}{2} = 3m$ $x_3 = \frac{3}{3} = 1m$

$x_2 = \frac{2b}{3} = \frac{2\times 6}{3} = 4m$ $x_4 = 6 - \frac{4r}{3\pi} = 4.727m$

$$\bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4}{A}$$

$$\boxed{\bar{x} = 2.707m}$$

$y_1 = \frac{6}{2} = 3m$ $y_3 = \frac{2h}{3} = 4m$

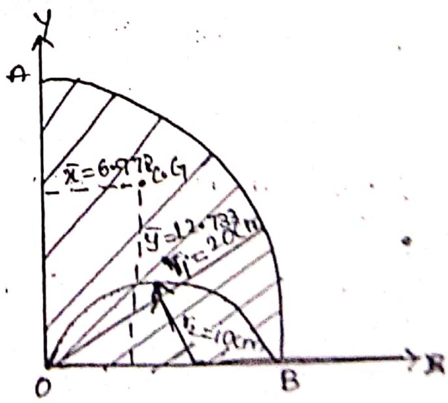
$y_2 = \frac{3}{3} = 1m$ $y_4 = 6 - \frac{4r}{3\pi} = 4.727m$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3 - A_4y_4}{A}$$

$$\boxed{\bar{y} = 2.707m}$$

4. Find the co-ordinates of the centriod of the area

A. Given section is symmetrical and it divided into 2 parts



1. Quadratic circle

Area of part ① $A_1 = \frac{\pi}{4} r^2$

$= \frac{\pi}{4}(20)^2$

$= 314.1\,cm^2$

2. Semi circle

Area of part ② $A_2 = \frac{\pi}{2} r^2$

$= \frac{\pi}{2} \times 10^2 = 157.05\,cm^2$

Total area $A = A_1 - A_2$

$= 157.05 cm$

$x_1 = \frac{4 \times 20}{3\pi}$ $\qquad$ $x_2 = \frac{20}{2}$

$\therefore x_2 = 10 cm$

$x_1 = 8.489 cm$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A}$$

$$\bar{x} = \frac{8.489 \times 314.1 - 157.05 \times 10}{157.05}$$

$$\boxed{\bar{x} = 6.978 cm}$$

$y_1 = \frac{4r}{3\pi}$ $\qquad$ $y_2 = \frac{4r}{3\pi}$

$= \frac{4 \times 20}{3 \times 3.141}$ $\qquad$ $= \frac{4 \times 10}{3 \times 3.141}$

$= 8.489 cm$ $\qquad$ $= 4.2449 cm$

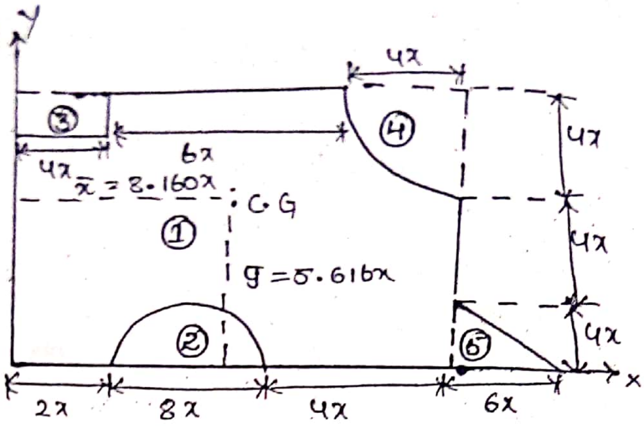
$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A}$$

$$\bar{y} = \frac{8.489 \times 314.1 - 4.2449 \times 157.05}{157.05}$$

$$\boxed{\bar{y} = 12.733 cm}$$

15. Determine the co-ordinates of the C.G of the plane area with reference to the axes shown in figure. Take $x = 40mm$

A- Given section is unsymmetrical and it is divided into 5 parts



1. Rectangle

Area of part ① $A_1 = 14 \times 12$

$= 168 x^2 cm^2$

2. semicircle

Area of part ② $A_2 = \frac{\pi}{2} \times r^2$

$= 25.12 x^2 cm^2$

3. Square

Area of part ③ $A_3 = 4x \times 4x$

$= 16x^2 cm^2$

4. quater circle

Area of part ④ $A_4 = \frac{\pi}{4} \times 4x^2$

$= 12.56 x^2 cm^2$

5. Right angle triangle

Area of part ⑤ $A_5 = \frac{1}{2} \times 6x \times 4x$

$= 12x^2$

Total area $A = A_1 - A_2 - A_3 - A_4 + A_5$

$= 126.301 x^2 mm^2$

$x_1 = \frac{14x}{2} = 7x$

$x_3 = \frac{4x}{2} = 2x$

$x_2 = 2x + \frac{8x}{2} = 6x$

$x_4 = 4x + 6x + \left(4x - \frac{4r}{3\pi}\right)$

$= 12.302x$

$$x_5 = 2x + 8x + 4x + \left(\frac{6x}{2}\right)$$

$$= 16x$$

$$\bar{x} = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 + A_5x_5}{A}$$

$$\boxed{\bar{x} = 8.16x\,mm}$$

$$y_1 = \frac{12}{2} = 6x \qquad y_2 = \frac{4(4x)}{3\pi} = 1.6979x \qquad y_3 = 8x + \frac{4x}{2} = 10x$$

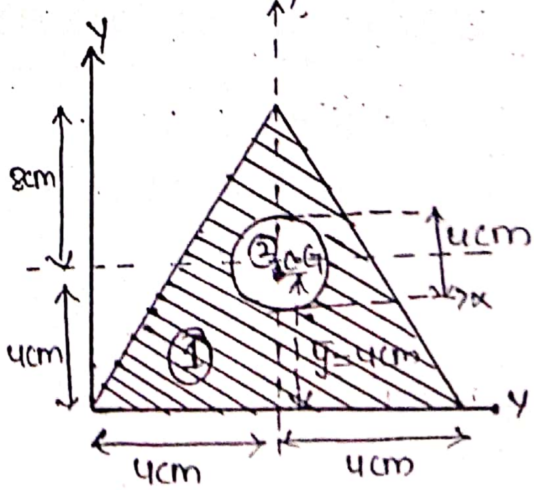
$$y_4 = 12x - \frac{4(4x)}{3\pi} \qquad y_5 = \frac{h}{3} = \frac{4x}{3}$$

$$= 10.302x \qquad y_5 = 1.333x$$

$$\bar{y} = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 + A_5y_5}{A}$$

$$\boxed{\bar{y} = 5.616x}$$

16. Locate the centroid for the shaded area as shown in figure.



A. The given section is symmetrical about y-axis

1. Triangle

Area of part ① $A_1 = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 8 \times 12$$

$$= 48\,cm^2$$

2. Circle

Area of part ② $A_2 = \pi r^2$

$= \pi (2)^2$

$= 12.564 cm^2$

Total area $A = A_1 - A_2$

$= 48 - 12.564$

$= 35.436 cm^2$

$y_1 = \frac{12}{3} = 4cm$

$y_2 = 2 + \frac{4}{2} = 4cm$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A}$$

$$\bar{y} = \frac{4 \times 48 - 12.564 \times 4}{35.436}$$

$$\boxed{\bar{y} = 4cm}$$

17. Locate centriods for the shaded area shown in figure

A. Given section is unsymmetrical and can split up into 5 parts

1. Triangle

Area of part ① $A_1 = \frac{1}{2} \times 60 \times 50$

$= 1500 mm^2$

2. Square

Area of part ② $A_2 = 60 \times 60$

$= 3600 mm^2$.

3. Semicircle

Area of part ③ $A_3 = \frac{\pi r^2}{2}$

$= \frac{\pi}{2} \times (20)^2$

$= 628 mm^2$

4. Semi circle

Area of part ④ $A_4 = \frac{\pi r^2}{2}$

$= 628 mm^2$

5. Semicircle

Area of part ⑤ $A_5 = \frac{\pi r^2}{2}$.

$= 1413 mm^2$

Total area $A = A_2 - A_1 - A_3 - A_4 + A_5$

$= 2257 mm^2$

$x_1 = \frac{50}{3}$

$= 16.66 mm$

$x_2 = \frac{60}{2}$

$= 30 mm$

$x_3 = 40 + \left(20 - \frac{4(20)}{3\pi}\right)$

$= 51.507 mm$

$x_4 = 40 + 20 + \frac{4(20)}{3\pi}$

$= 68.492 mm$

$x_5 = 40 + 20 + \frac{4(30)}{3\pi}$

$= 72.738 mm$

$$\bar{x} = \frac{A_2 x_2 - A_1 x_1 - A_3 x_3 - A_4 x_4 + A_5 x_5}{A}$$

$$\bar{x} = \frac{\cancel{\bar{x}} - 3600(16.66) + 3600(30) - 628(51.507) - 628(68.492) + 1413(72.78)}{2257}$$

$$\boxed{\bar{x} = 48.9275 mm}$$

y_1

$\bar{y}$

18.

in

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A

$y_1 = \frac{60}{3}$ $\qquad$ $y_2 = \frac{60}{2}$ $\qquad$ $y_3 = 10 + \frac{40}{2}$

$= 20mm$ $\qquad$ $= 30mm$ $\qquad$ $= 30mm$

$y_4 = 10 + \frac{40}{2}$ $\qquad$ $y_5 = \frac{60}{2}$

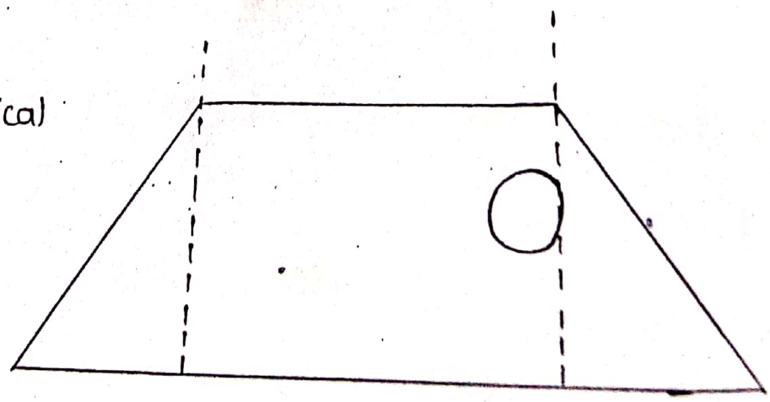
$= 30mm$ $\qquad$ $= 30mm$

$$\bar{y} = \frac{-A_1 y_1 + A_2 y_2 - A_3 y_3 - A_4 y_4 + A_5 y_5}{A}$$

$$\bar{y} = \frac{-(1500)(20) + 3600(30) - 628(30) - 628(30) + 1413(30)}{2257}$$

$$\boxed{\bar{y} = 36.6459mm}$$

18. Locate the centriod for the shaded area as shown in figure



→ Given section is unsymmetrical and can split up into 4 parts

1. Triangle

Area of part ① $A_1 = \frac{1}{2} \times 4 \times 8$

$= 56cm^2$

2. Rectangle

Area of part ② $A_2 = l \times b$

$= 10 \times 28$

$= 280cm^2$

3. Triangle

Area of part ③ $A_3 = \frac{1}{2} \times 4 \times 28$

$= 36\,cm^2$

4. Circle

Area of part ④ $A_4 = \pi r^2$

$= \pi (4)^2$

$= 50.256$

Total Area $= A_2 + A_3 - A_4 - A_1$

$= 280 + 56 - 50.256 - 56$

$= 229.76\,cm^2$

$x_1 = \frac{b}{3} = \frac{4}{3}$

$= 1.33\,cm$

$x_2 = \frac{10}{2}$

$= 5\,cm$

$x_3 = 10 + \frac{4}{3}$

$= 11.33\,cm$

$x_4 = 2 + \left(4 - \frac{4 \times 4}{3\pi}\right)$

$= 4.3015$

$$\bar{x} = \frac{A_2 x_2 + A_3 x_3 - A_1 x_1 - A_4 x_4}{A}$$

$$\boxed{\bar{x} = 1.59\,cm}$$

$y_1 = \frac{h}{3} = \frac{28}{3}$

$= 9.33\,cm$

$y_2 = \frac{28}{2}$

$= 14\,cm$

$y_3 = \frac{28}{3}$

$= 9.33\,cm$

$y_4 = 16 + 4$

$= 20\,cm$

$$\bar{y} = \frac{A_2 y_2 + A_3 y_3 - A_1 y_1 - A_4 y_4}{A}$$

$$\boxed{\bar{y} = 12.688\,cm}$$

Centre of gravity of plane figures by integration method

We know $\bar{x} = \dfrac{A_1x_1 + A_2x_2 + A_3x_3 + \cdots\cdots}{A}$ ——— (i)

and

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + \cdots\cdots}{A} \text{ ——— (ii)}$$

where

$$A = A_1 + A_2 + A_3 + \cdots\cdots$$

The above equations can be written as

$$\bar{x} = \frac{\Sigma A_i x_i}{\Sigma A_i}$$

$1-\frac{4\times 1}{3\pi}$ and $\bar{y} = \dfrac{\Sigma A_i y_i}{\Sigma A_i}$

15

where $i = 1, 2, 3, 4, \cdots\cdots$

By integrating $\bar{x} = \dfrac{\int x^* dA}{\int dA}$

$$\bar{y} = \frac{\int y^* dA}{\int dA}$$

where $\int x^* dA = \Sigma A_i x_i$

$$\int y^* dA = \Sigma A_i y_i$$

Centre of gravity of a line:

The centre of gravity of a line which may be striaght (or) curve maybe obtained by dividing the line into a large no. of small lengths as shown in figure.

The centre of gravity of a line is by replacing dA by dL

$$\bar{x} = \frac{\int x^* dL}{\int dL}$$

$$\bar{y} = \frac{\int y^* dL}{\int dL}$$

where x^* = distance of centre of gravity of length dL from OY axis

y^* = distance of centre of gravity of length dL from OX axis

→ If the lines are strength then the above equation can be written as

$$\boxed{\bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3 + \cdots}{L_1 + L_2 + L_3 + \cdots}}$$

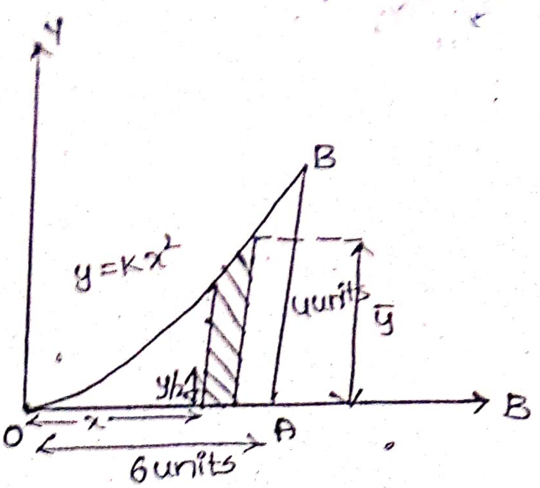
$$\boxed{\bar{y} = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3 + \cdots}{L_1 + L_2 + L_3}}$$

Problem:

Determine the co-ordinates of centre of gravity of the area AOB. If the curve OB represents the equation of parabola given by $y = Kx^2$. In which OA = 6 units, AB = 4 units.

The equation of parabola

$$y = kx^2 \quad \text{——— (1)}$$



length of $OA = 6$ units $= x$

length of $AB = 4$ units $= y$

substituting x, y values in (1)

$$4 = k(6)^2$$

$$k = \frac{1}{9}$$

Substituting k value in equation (1)

$$y = \frac{1}{9}x^2$$

$$x^2 = 9y$$

$$x = 3\sqrt{y}$$

Consider a vertical strip of height y and with dx thickness as shown in fig. The area of the strip

$$dA = dx \times y$$

distance of centre of gravity of area dA, from y axis

$$dA = x$$

distance of centre of gravity of area dA from y axis

$$dA = y/2$$

Let

$\bar{x}$ = distance of centre of gravity of total area AOB from y axis

$\bar{y}$ = distance of centre of gravity of total area AOB from x axis

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

and

$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

$$\bar{x} = \frac{\int x\,dA}{\int dA}$$

$$= \frac{\int_0^6 x \times dx \times y}{\int_0^6 dx \times y} = \frac{\int_0^6 x \times \frac{x^2}{9} \times dx}{\int_0^6 \frac{x^2}{9}\,dx}$$

$$= \frac{\frac{1}{9}\int_0^6 x^3\,dx}{\frac{1}{9}\int_0^6 x^2\,dx} = \frac{\left[\frac{x^4}{4}\right]_0^6}{\left[\frac{x^3}{3}\right]_0^6}$$

$$= \frac{\left[\frac{6^4}{4}\right]}{\left[\frac{6^3}{3}\right]} = \frac{324}{72}$$

$$\boxed{\bar{x} = 4.5}$$

$$\bar{y} = \frac{\int y\,dA}{\int dA}$$

$$= \frac{\int_0^6 y/2\,dA}{\int_0^6 dA} = \frac{\int_0^6 y/2 \times y\,dx}{\int_0^6 y\,dx}$$

$$= \frac{\int_0^6 \frac{x^2}{18} \cdot \frac{x^2}{9}\,dx}{\int_0^6 \frac{x^2}{9}\,dx}$$

$$= \frac{\frac{1}{162}\int_0^6 x^4 dx}{\frac{1}{9}\int_0^6 x^2 dx} = \frac{\frac{1}{162}\left[\frac{x^5}{5}\right]_0^6}{\frac{1}{9}\left[\frac{x^3}{3}\right]_0^6}$$

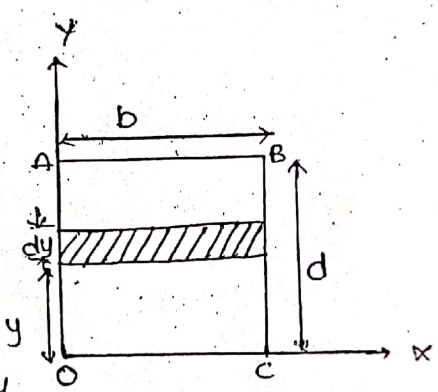
$$= \frac{9}{162}\left[\frac{1555.2}{72}\right]$$

$$\boxed{\bar{y} = 1.2}$$

Centre of gravity (or) centroid of sections from integration method :

1. Rectangle:

The figure shows a rectangular section ABCD having width 'b' and depth 'd'



Consider a rectangular elementary strip of thickness "dy" at a distance "y" from the axis "ox".

Let dA = Area of rectangular elementary strip

$\therefore dA = b \times dy$

Moment of the area dA about axis $\overline{ox} = dA \times y$

$= b \times dy \times y$

The moment of whole area about axis $\overline{ox}$ is obtained by integrating the above equation with the limits '0' to 'd'

$$\int_0^d b \times dy \times y = b\int_0^d y\,dy$$

$$= b\left[\frac{y^2}{2}\right]_0^d$$

$$= b \cdot \frac{d^2}{2} \text{ ——— } (1)$$

Let "A" = Area of rectangular section

$$= \int_0^d dA$$

$$= \int_0^d b\,dy$$

$$= b\left[y\right]_0^d$$

$$A = b \times d$$

Let $\bar{y}$ = distance of the centroid of the rectangular section from axis "OX"

$\therefore$ Moment of total area = $A \times \bar{y}$ ——— (2)

Equating eqn's (1) & (2) we get

$$A \times \bar{y} = \frac{bd^2}{2}$$

$$\bar{y} = \frac{bd^2}{2A} = \frac{\cancel{b}d^{\cancel{2}}}{2} \times \frac{1}{\cancel{b} \times \cancel{d}}$$

$$\boxed{\bar{y} = \frac{d}{2}}$$

Similarly considering a vertical rectangular elementary strip of thickness "dx" at a distance of "x" from the axis "OY"

Let dA = Area of vertical rectangular elementary strip

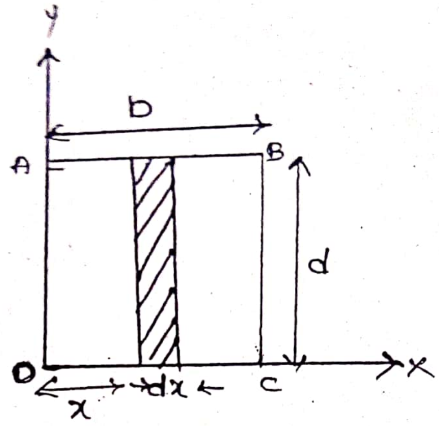
$\therefore$ dA = d×dx

Moment of the area about axis $\overline{OY} = dA \times x$

$= d \times dx \times x$

The moment of whole area about $\overline{OY}$ axis is obtained by integrating the above equation with the limits 'O' to 'b'

$$= \int_0^b dx\, dx$$



$$= d \int_0^b x\, dx$$

$$= d\left[\frac{x^2}{2}\right]_0^b = d \cdot \frac{b^2}{2} \quad \text{--- } (1)$$

Let A = Area of rectangular section

$$A = \int_0^b dA$$

$$= \int_0^b d \times dx = d\int_0^b dx$$

$$= d\left[x\right]_0^b$$

$$A = d \times b$$

Let $\bar{x}$ = distance of centroid of the rectangular section from axis "$\overline{OY}$".

$\therefore$ Moment of total area $= A \times \bar{x}$ —— (2)

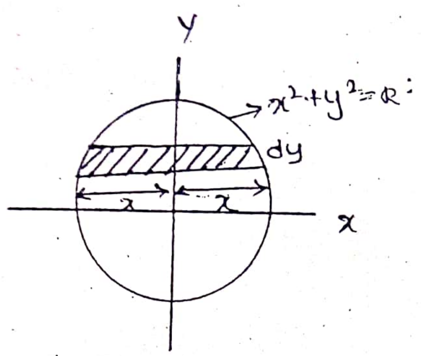
Equating eq^{n}s. (1) & (2) we get

$$A \times \bar{x} = \frac{db^2}{2}$$

$$\bar{x} = \frac{d b^2}{2A}$$

$$\bar{x} = \frac{\cancel{d} b^{\cancel{2}}}{2} \times \frac{1}{\cancel{d}\cancel{b}}$$

$$\boxed{\bar{x} = \frac{b}{2}}$$



2. Circular section:

Consider a circular section of radius "R" with "O" as centre, the equation of the circle is $x^2+y^2=R^2$

Consider a rectangular elementary strip of thickness "dy" at a distance of "y" from the axis "OX". The area of elementary strip $dA = 2x \times dy$ —— (1)

Moment of this area "dA" about "x"- axis $= dA \times y$

$= 2x \times y \times dy$

$= 2\sqrt{R^2-y^2}\, dy \times y$ —— (2)

[∵ but from eq^n of circle $x^2+y^2=R^2$, $x=\sqrt{R^2-y^2}$]

Moment of total area "A" about x axis will be obtained by integrating the above equation between the limits from $-R$ to $+R$

∴ Moment of Area about x-axis $= \int_{-R}^{R} 2\sqrt{R^2-y^2} \times y \times dy$

$$= 2\int_{-R}^{R} \sqrt{R^2-y^2} \times y \times dy$$

$$= 2\int_{-R}^{R} (R^2-y^2)^{1/2} \times y \times dy$$

$$= -\frac{2}{3}\left[\sqrt{R^2-y^2}\ \ (R^2-y^2)\right]_{-R}^{R}$$

$$= -\frac{2}{3}\left[(R^2-y^2)^{3/2}\right]_{-R}^{R}$$

$$= -\frac{2}{3}[0-0] = 0$$

Similarly by considering a vertical rectangular strip of thickness "dx" at a distance "x" from the "OY" axis. The area of elementary strip $dA = 2y \times dx$ —— ①

Moment of this area "dA" about "Y" axis $= dA \times x$

$$= 2y \times dx \times x$$

$$= 2\sqrt{R^2-x^2} \times dx \times x \quad \text{—— ②}$$

Moment of the total area "A" about "OY" axis will be obtained by integrating the above equation between the limits "−R" to "+R"

∴ Moment of Area A about y-axis

$$= \int_{-R}^{R} 2\sqrt{R^2-y^2} \times x \times dx$$

$$= 2\int_{-R}^{R} \sqrt{R^2-y^2} \times x \times dx \quad = 2\int_{-R}^{R} (R^2-y^2)^{1/2} \times x \times dx$$

$$= -\frac{2}{3}\left[\sqrt{R^2-y^2}\ \ (R^2-y^2)\right]_{-R}^{R}$$

$$= -\frac{2}{3}(0-0)$$

$$= 0$$

Let $\bar{x}$ = distance at centre of gravity of total area fro

yaxis

Moment of total area $= A \times \bar{x}$ ——— (2)

$$A \times \bar{x} = 0$$

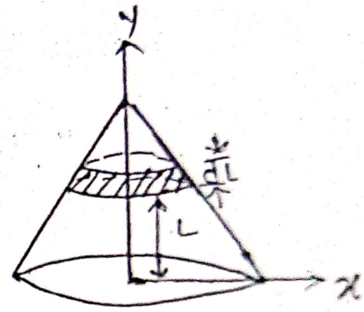
$$\boxed{\bar{x} = 0}$$

3. Right angle triangle :

Pappu's theorem (I) (or) Area of revolution:

"The area of a surface of revolution is equal to the length of the generating curve, times the distance travelled by the centroid of the generating curve while the surface is generated."

Proof:



Consider a right angled triangle having hypotenuse as a straight line 'L' generating curve.

Let vertical line along "y" axis be imaginary axis and horizontal line along "x" axis as base radius of cone.

If it revolves completely, i.e., 0° to 360° about y axis then the hypotenuse 'L' will generate the surface of revolution cone is obtained.

$$\therefore \; dA = 2\pi x \times dL$$

$$\int dA = \int 2\pi x \, dL$$

$$\therefore \; \boxed{A = 2\pi x_G L}$$

Pappu's theorem (II) (or) Volume of revolution :

" The volume of a body of revolution is equal to the generating area times the distance travelled by the centroid of the area while the body is generated."

Proof:

Consider an elemental area (dA) of the area "A" generates a volume (dv) when revolue about y-axis.

$$\therefore dv = dA \times 2\pi x$$

Integrating the above equation by given area

$$\int dv = \int dA \times 2\pi x$$

$$\therefore \boxed{V = 2\pi x_G A}$$

where

x_G = distance of centre of gravity of area A from $\bar{o}y$ axis.