2. Equilibrium Of Forces

Equilibrant:

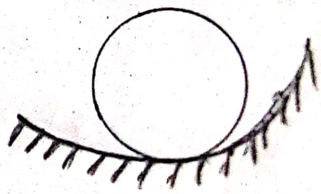
A force, which is equal, opposite and collinear to the resultant of a concurrent force system is known as the equilibrant is the force which, when appiled to a body acted by the concurrent force system, keeps the body in equilibrium.

Types of Equilibriums:

1. Stable equilibrium:

A body is said to be in equilibrium if

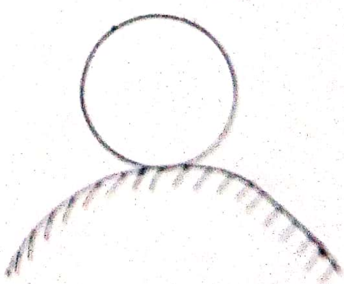
* It is initiary in a state of static equilibrium
* On giving a slight displacement an additional force is set up which tends to restore the original position of the body.



2. Unstable equilibrium:

A body is said to be in unstable equilibrium if

* It is intially in a state of static equilibrium
* An additional force is set up on sight displacement which tends to push it away from the original position of the body

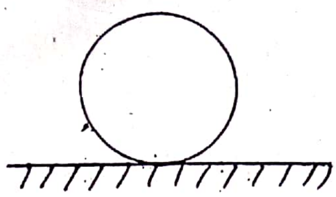


* It does not return back to it's original position after being slightly displaced by a force.

3. Neutral equilibrium:

A body is said to be in neutral equilibrium if

* It is intially in a static equilibrium
* No additional force is setup on slight displacement from intial position



* It occupies a new position and remains in static equilibrium in this new position.

Conditions of equilibrium:

When the resultant of force system acting on a body is zero, the body is in equilibrium.

Thus, the resultant force 'R' and resultant moment M_R both are zero.

$\therefore$ i, $R = \Sigma F = 0$

ii, $M_R = \Sigma M = 0$

Categories of Equilibrium:

1. Equilibrium of collinear force system:-

If forces are collinear then only one axis contains all the forces. Therefore only one force equation in the direction of the force is required

i.e $\Sigma d_x = 0$

Eg:

$\Sigma F_x = 0$

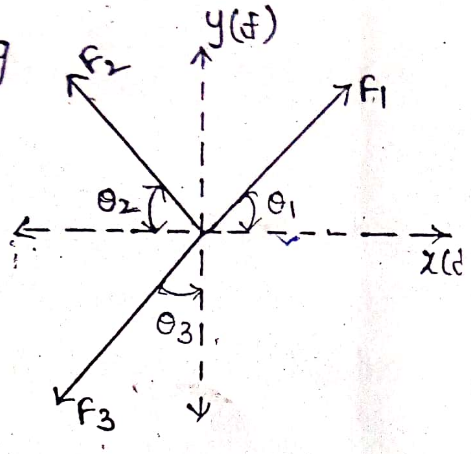
$F_1 - F_2 + F_3 = 0$

2. Equilibrium of concurrent force system:

If all the forces in coplanar force system are concurrent then the following equations can be used.

i, $\Sigma F_x = 0$

ii, $\Sigma F_y = 0$



Eg:

$$\Sigma F_x = 0$$

$$F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \sin\theta_3 = 0$$

$$\Sigma F_y = 0$$

$$F_1 \sin\theta_1 + F_2 - F_3 \cos\theta_3 = 0$$

3. Parallel force system:

If all the forces in coplanar force system are parallel, then

i, $\Sigma F = 0$

ii, $\Sigma M = 0$

4. General force system:

If all the force and couples acting in a plane form general force system.

i, $\Sigma F_x = 0$

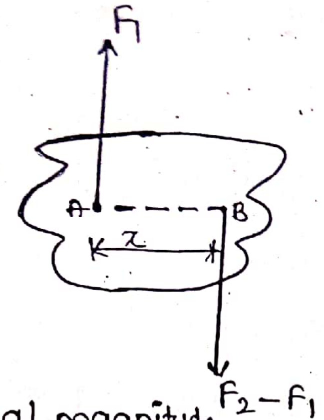
ii, $\Sigma F_y = 0$

iii, $\Sigma M = 0$

Two force system:

When a body is subjected of two forces, then the body will be in equilibrium if the two forces are collinear, equal and opposite

If the two forces acting on a body are equal and opposite but are parallel, as shown in fig,
then the body will not be in equilibrium



i, $\Sigma F_x = 0$

in this there is no horizontal forces then equation (i) is satisfied

ii, $\Sigma F_y = 0$

in this two vertical forces with equal magnitude but in opposite direction. i.e. $F_1 = F_2$

then eq^n (ii) is satisfied

iii, $\Sigma M = 0$

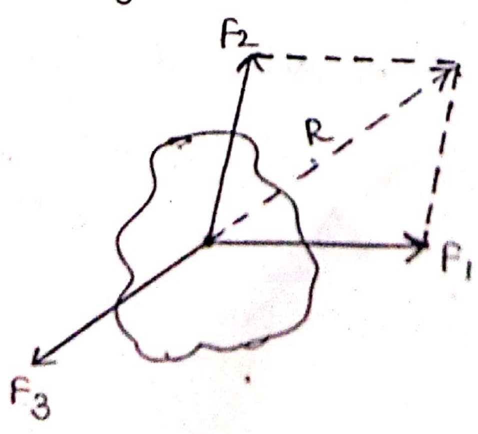
$$M_A = -F_2 \times x.$$

But M_A is not equal to zero. Hence (iii) condition is not satisfied. Hence the body will not be in equilibrium

Three force system:

The three forces acting on a body which is in equilibrium may be either concurrent or parallel.

Let us first consider that the body is in equilibrium when three forces, acting on the body, are concurrent.



a, when three forces are concurrent:

The three forces are F_1, F_2 and F_3 are acting on a body at point O and the body is in equilibrium.

The resultant of F_1 and F_2 is R, then the body is in equilibrium when R is equal to F_2. R is known as equilibrant Hence for three concurrent forces acting on a body when the body is in equilibrium, the resultant of the two forces should be equal and opposite to the third forces.

b, When three forces are parallel:

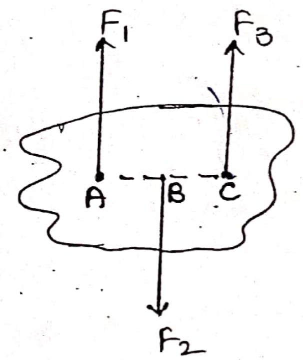


fig shows a body on which three parallel forces F_1, F_2 and F_3 are acting and the body is in equilibrium.

If three forces are acting in same direction, then the is not in equilibrium, there will be resultant $R = F_1 + F_2 + F_3$.

let us F_2 is acting in opposite direction as shown in fig.

i, $\Sigma F_x = 0$ as there is no horizontal forces

ii, $\Sigma F_y = 0$ i.e. $F_1 + F_3 = F_2$

iii, $\Sigma M = 0$ about any point

Taking moments about A

$$\Sigma M_A = -F_2 \times AB + F_3 \times AC$$

for equilibrium ΣM_A should be zero

$$\therefore -F_2 \times AB + F_3 \times AC = 0$$

Four Force system:

The body will be in equilibrium if the resultant force in horizontal direction is zero (i.e. $\Sigma F_x = 0$) resultant force in vertical direction ($\Sigma F_y = 0$) and moment of all forces about any point in the plane of forces is zero (i.e $\Sigma M = 0$)

Step by step procedure for drawing a F.B.D:-

1. Draw a neat sketch of the body assuming that all supports are removed.

2. Free body diagram may consists of an entire assembled structure or any combination or part of it.

3. Show all the relevant dimensions and angles on the sketch.

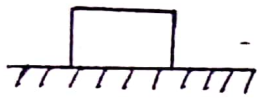
4. Show all the active forces on corresponding point of application and insert their magnitude and direction, if known.

5. Show all the reactive forces due to each support

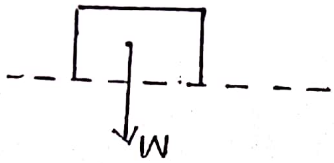
6. If the body is attached by a string, a rope or a cable then a force of tension must be shown at that point acting along the string or rope and away from the body.

1. Draw the F.B.D (Free Body Diagram) for the following cases.

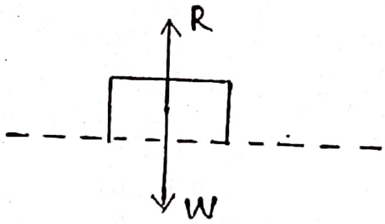
i) A block resting on a smooth horizontal plane



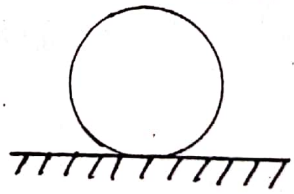
Step-1:



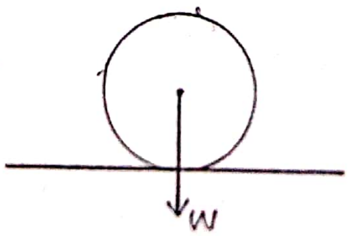
Step-2:



ii) A ball resting on a smooth horizontal plane



Step-1:



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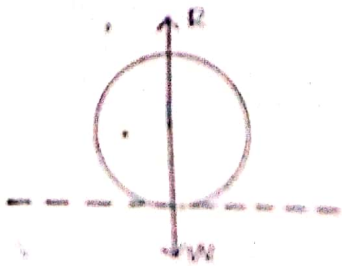
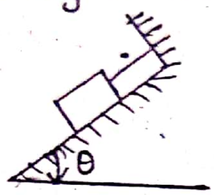


ing

2. A block on a smooth incline plane is restrained from moving downwards by a string attached to it.



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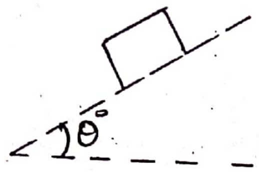

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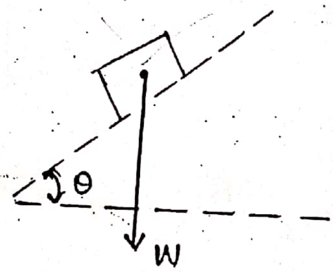

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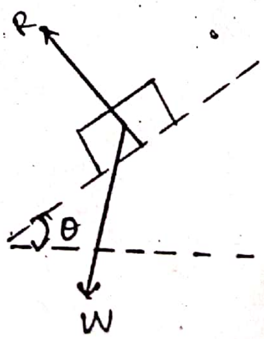
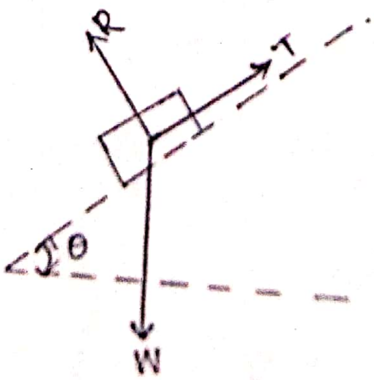
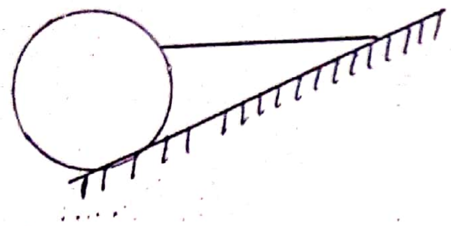


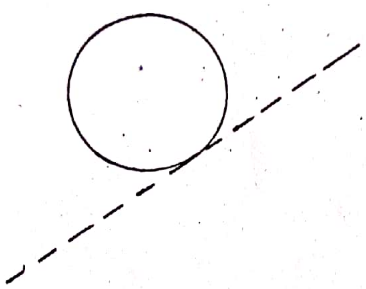
Step-4:



3. A sphere on a smooth inclined plane is restrained from moving downwards by a string attached to the sphere whose other end is attached to the inclined plane.



Step 1:



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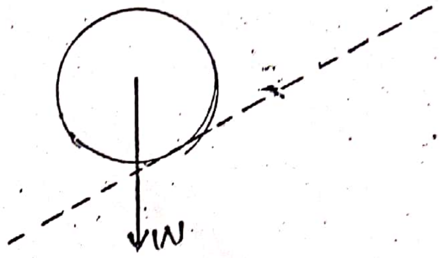

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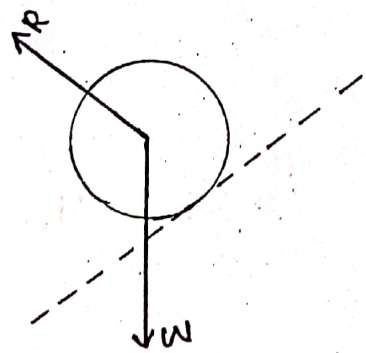

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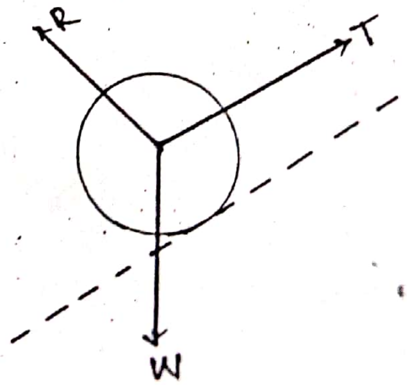
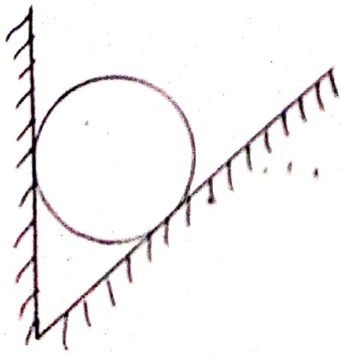
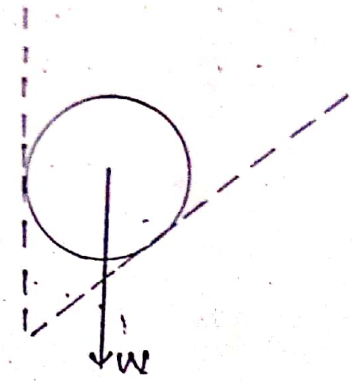


4. A sphere on a smooth inclined plane is restrained from moving downwards by a vertical plane.



Step 1:

Step2:



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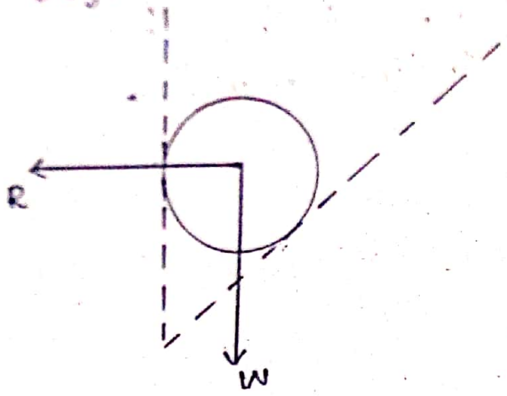

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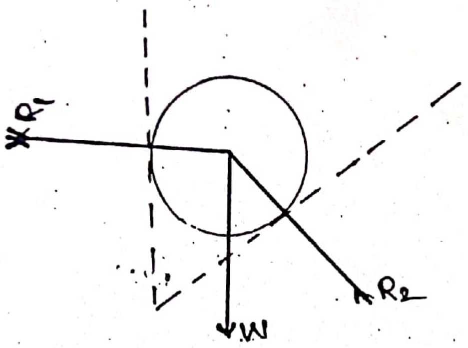
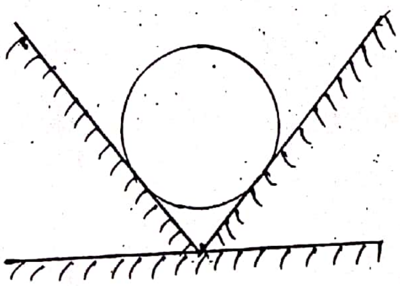


5. A sphere resting in a through



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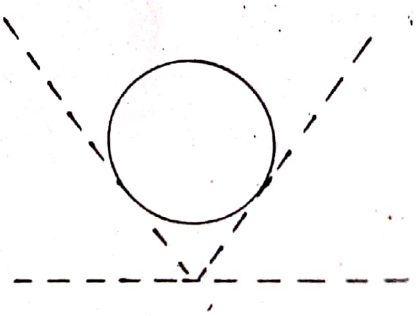
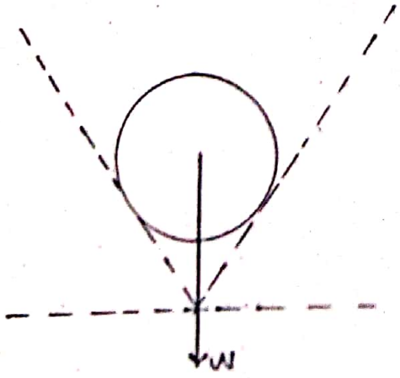
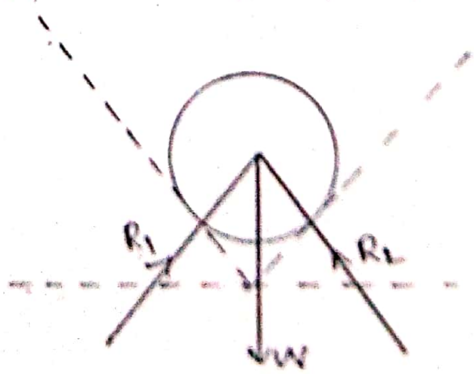


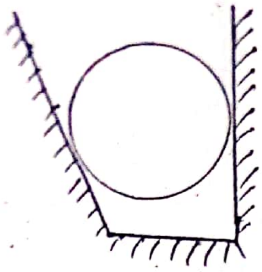
Step2:



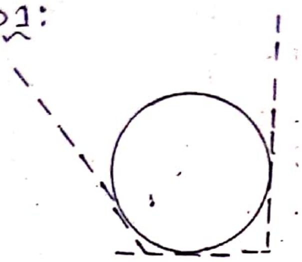
Step3:



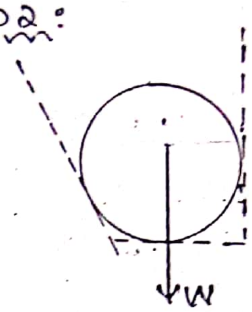
6. Draw the F.B.D of a cylinder resting in a channel as show in fig. assume all contact surfaces to be smooth



Step 1:



Step 2:



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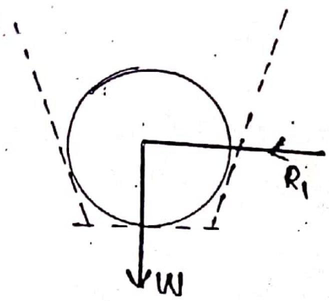
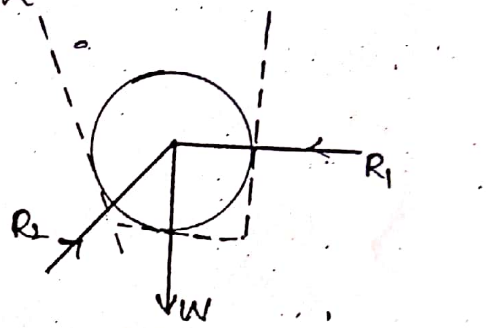
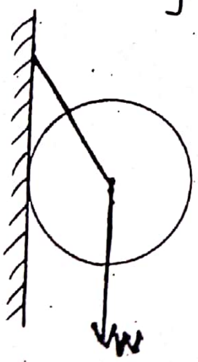


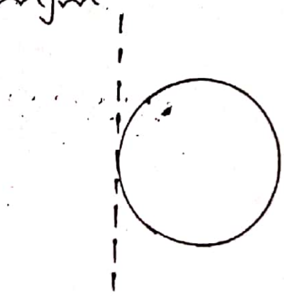
Step 4:



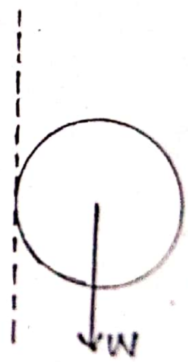
7. Draw the F.B.D of ball of weight w supported by a string as resting aganist a smooth vertical wall at c as shown in fig



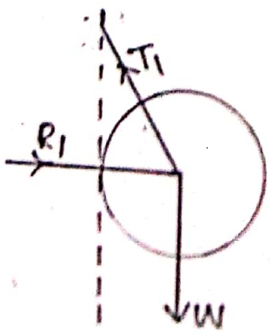
Step 1:



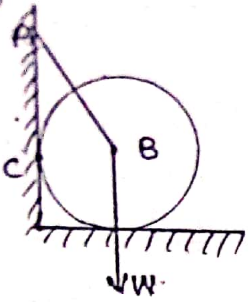
Step 2:



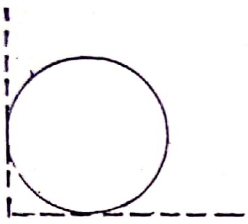
Step 3:



1. Draw the F.B.D. of a ball of weight W supported by a string AB and resting aganist a smooth vertical wall at C and also aganist a smooth horizontal floor at D as shown in fig.



Step1:



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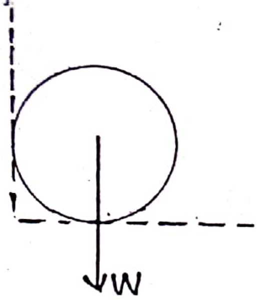

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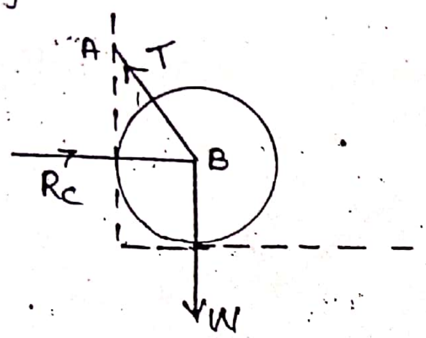
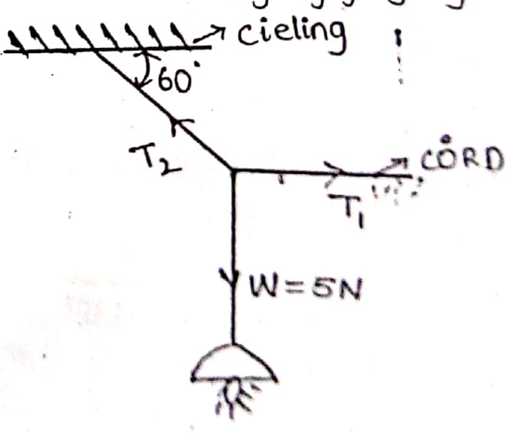


⇒ A lamp weighing 5N is suspendend from the ceiling by a chain. It is pulled a side by a horizontal cord until the chain makes an angle of 60° with the cieling as shown in fig. Find the tensions in the chain and the cord by applying lami's theorem.



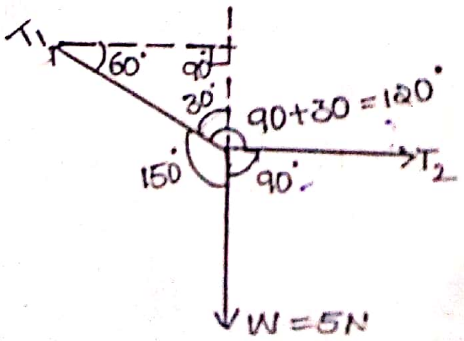
sol: Given,

weight $\omega = 5N$

Let

T_1 = Tension in the chain

T_2 = Tension in the cord



Applying lami's theorem

$$\frac{T_1}{\sin 90^\circ} = \frac{T_2}{\sin 150} = \frac{W}{\sin 120^\circ}$$

$$\frac{T_1}{\sin 90} = \frac{W}{\sin 120^\circ}$$

$$T_1 = 5 \times \frac{1}{0.866}$$

$$\boxed{T_1 = 5.773N}$$

$$\frac{T_2}{\sin 150} = \frac{W}{\sin 120^\circ}$$

$$T_2 = 5 \times \frac{0.5}{0.866}$$

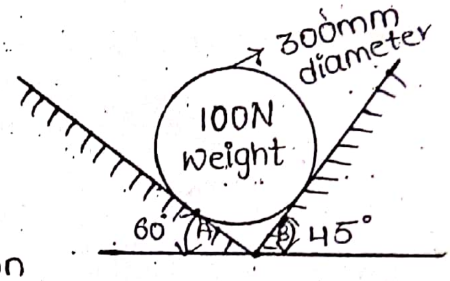
$$\boxed{T_2 = 2.866N}$$

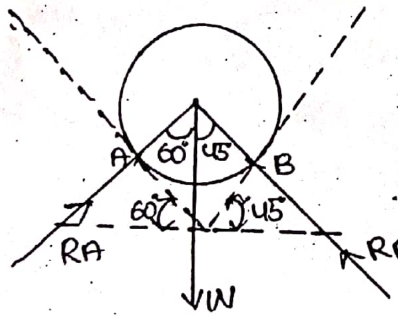
9. Determine the reactions at A and B as shown in fig.

Sol: Given,

Weight = 100N

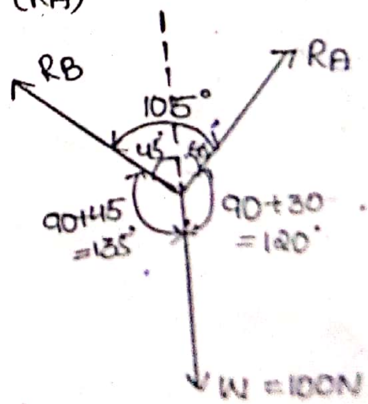
Diameter = 300mm





Force acting on the ball

1. Weight of ball (W)
2. Reaction at B (R_B)
3. Reaction at A (R_A)



Applying lami's theorem

$$\frac{W}{\sin 105^\circ} = \frac{R_A}{\sin 135^\circ} = \frac{R_B}{\sin 120^\circ}$$

$$R_A = W \times \frac{\sin 135^\circ}{\sin 105^\circ}$$

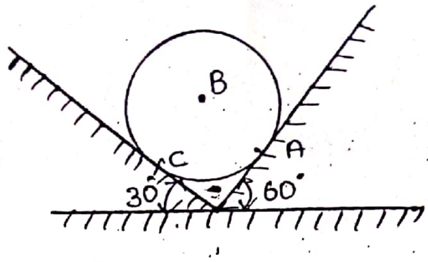
$$= 100 \times \frac{0.707}{0.965} \rightarrow \boxed{R_A = 73.194N}$$

$$R_B = W \times \frac{\sin 120^\circ}{\sin 105^\circ}$$

$$= 100 \times \frac{0.866}{0.965} \rightarrow \boxed{R_B = 89.654N}$$

10. A ball of weight 120N rests in a right angled grooves as shown in fig. The slides of the groove are inclined to an angle of 30° and 60° to the horizontal. If the surfaces are smooth, then determine the reactions R_A and R_C at the point of contact.

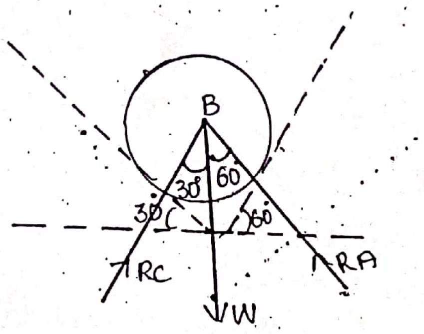
Sol: Weight of ball $W = 120N$



Let

Reaction at $A = R_A$

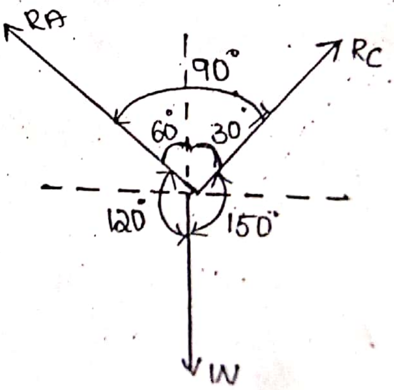
Reaction at $C = R_C$



Applying lamis theorem

$$\frac{W}{\sin 90^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{R_C}{\sin 120^\circ}$$

$$\frac{W}{\sin 90^\circ} = \frac{R_A}{\sin 150^\circ}$$



$$R_A = 120 \times \frac{0.5}{1}$$

$$\boxed{R_A = 60N}$$

$$\frac{R_C}{\sin 120^\circ} = \frac{W}{\sin 90^\circ}$$

$$R_C = 120 \times \frac{0.866}{1}$$

$$\boxed{R_C = 103.92N}$$

11. Two identical rollers, each of weight $w = 1000N$, are supported by an inclined plane and a vertical wall as shown in fig. Find the reactions at the points of supports A, B and C. Assume all the surfaces to be smooth.

Sol: weight of each roller $W = 1000N$

Radius of each roller is same. Hence line EF will be parallel to AB.

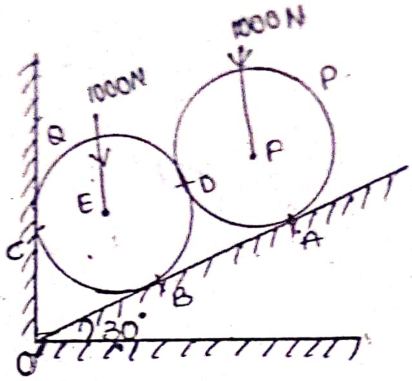
let

R_A = reaction at point A

R_B = reaction at point B

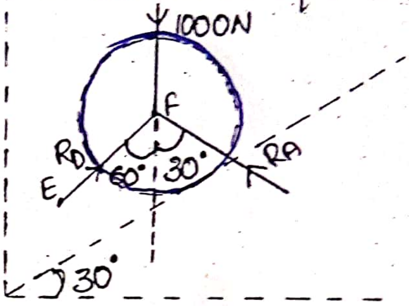
R_C = reaction at point C

R_D = reaction at point D



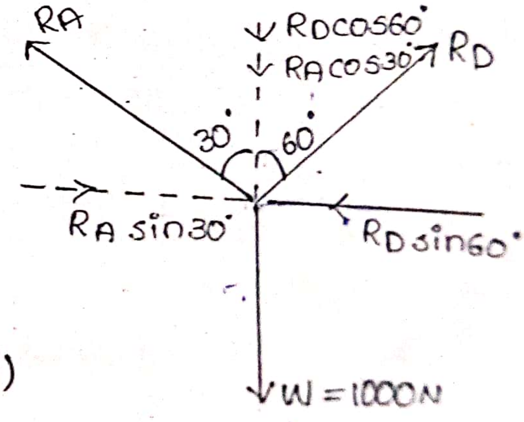
Case(i):

Consider the equilibrium of roller P, which is in equilibriu under the following forces



1. Weight of the roller P
2. Reaction at point A R_A
3. Reaction at point D R_D

For equilibrium



$$\Sigma F_x = 0$$

$$-R_D \sin 60° + R_A \sin 30° = 0$$

$$R_A \sin 30° - R_D \sin 60° = 0$$

$$R_A \frac{\sin 30°}{\sin 60°} = R_D$$

$$R_A \times 0.577 = R_D \quad \text{——(i)}$$

Vertically $\Sigma f_y = 0$

$$R_A \cos 30° + R_D \cos 60° - W = 0$$

$$R_A \cos 30° + R_A \times 0.577 \times \cos 60° - 1000 = 0$$

$$R_A \times 0.866 + R_A \times 0.577 \times 0.5 = 1000$$

$$R_A (0.866 + 0.577 \times 0.5) = 1000$$

$$R_A = \frac{1000}{1.1545}$$

$$\boxed{R_A = 866.175\text{ N}}$$

from (i) $R_D = R_A \times 0.577$

$= 866.175 \times 0.577$

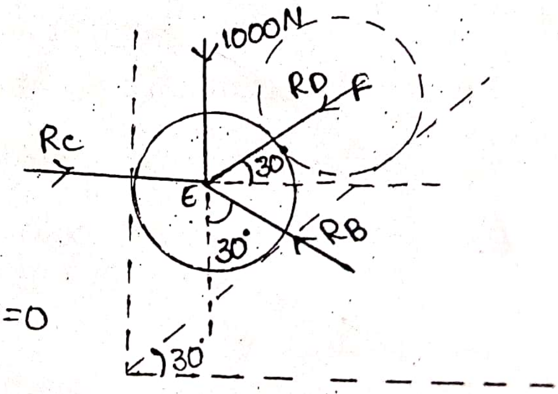
$$\boxed{R_D = 499.782\,N}$$

Case(ii):

Consider the equilibrium of roller Q, which is in equilibrium under the following forces

1. Weight of roller Q (W_Q)
2. Normal reaction at point B (R_B)
3. Normal reaction at point C (R_C)
4. Reaction between two rollers (R_D)

for equilibrium



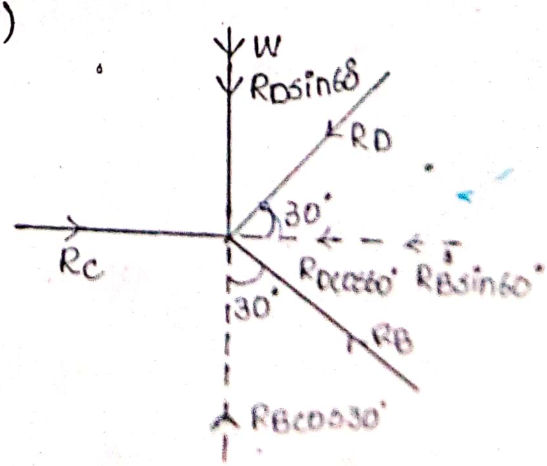
$\Sigma F_x = 0$

$R_C - R_D \cos 60° - R_B \sin 60° = 0$

$R_C - 499.782 \times 0.866 - R_B \times 0.5 = 0$

$R_C - R_B \times 0.5 = 432.823$

$R_C = 432.823 + R_B \times 0.5$ — (ii)



$\Sigma F_y = 0$

$R_B \cos 30° - R_D \sin 30° - 1000 = 0$

$R_B \cos 30° = 249.891 + 1000$

$R_B = \dfrac{1249.89}{1000}$

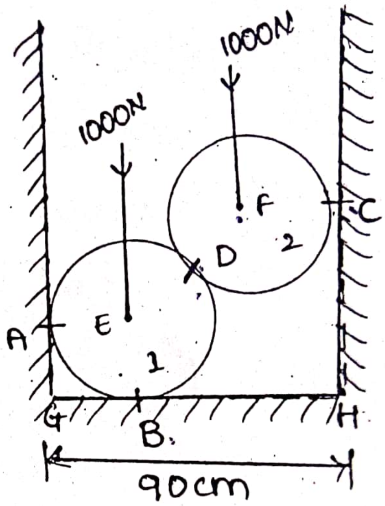
$$\boxed{R_B = 1443.292\,N}$$

from (ii)

$R_C = 1448.292 \times 0.5 + 432.823$

$\boxed{R_C = 1154.469N}$

12. Two spheres of weight of 1000N and of radius 25cm in a horizontal channel of width 90cm as shown in fig. find the reactions on the points of contact A, B and C.



A. Given,

weight of each sphere = 1000N

radius of sphere r = 25cm = 250mm

width of channel (GH) = 90cm
= 900mm

Let

R_A = Reaction at point A

R_B = Reaction at point B

R_C = Reaction at point C

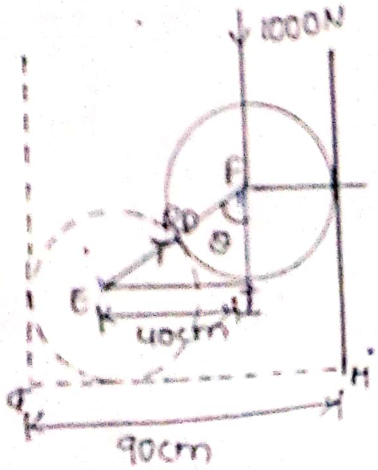
Case(i):

Considering the equilibrium of sphere 2 which is in equilibrium of under following forces

1. Weight of sphere (W)

2. Reaction at point C (R_C)

3. Reaction between two sphere (R_D)



Radius of sphere = 250mm

length of EF = ED+DF

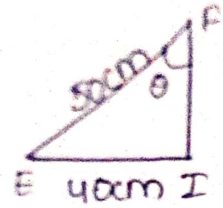
= 25 + 25 = 50cm

= 500mm

length of EI = GH − GE − IH = 90−25−25

= 40cm = 400mm

from right angle Δ^{le} EFI

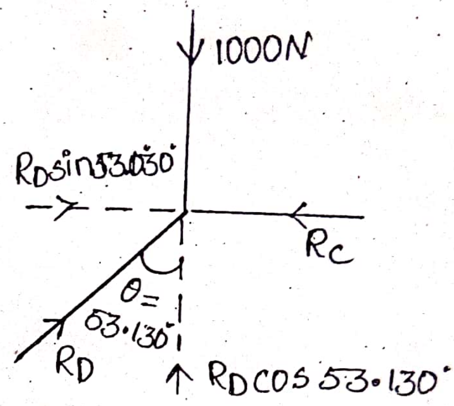


$$\sin\theta = \frac{EI}{EF}$$

$$\theta = \sin^{-1}\left(\frac{4}{5}\right)$$

$$\boxed{\theta = 53.130^\circ}$$

for equilibrium



$$\Sigma F_x = 0$$

$$R_C = R_D \times \sin 53.130^\circ$$

$$R_C = R_D \times 0.799 \quad \text{——— (i)}$$

$$\Sigma F_y = 0$$

$$1000 = R_D \times \cos 53.130^\circ$$

$$R_D = \frac{1000}{\cos 53.130^\circ}$$

$$\boxed{R_D = 1666.662N}$$

from (i),

$$R_C = 1666.662 \times 0.799$$

$$\boxed{R_C = 1331.662\,N}$$

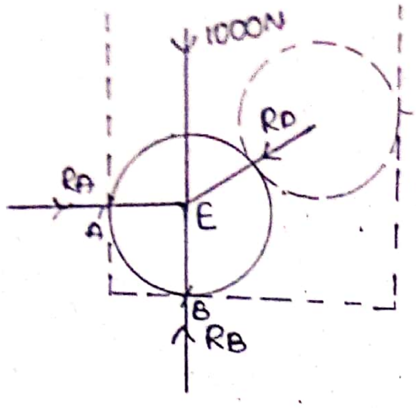
Case(ii):

Considering the equilibrium of sphere 1 which is in equilibrium under following forces

1. Weight of sphere 1 (W_1)
2. Normal reaction at point A (R_A)
3. Normal reaction at point B (R_B)
4. Reaction between two sphere (R_D)

for equilibrium

$\Sigma fx = 0$

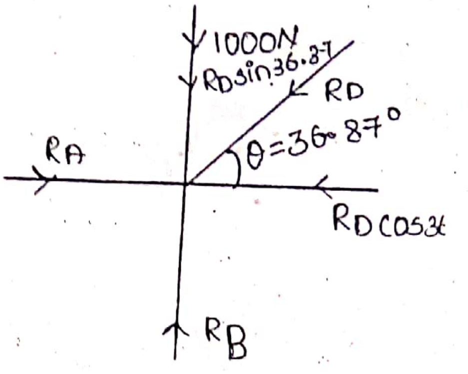


$R_A = R_D \cos 36.87$

$R_A = 1666.662 \times 0.799$

$$\boxed{R_A = 1333.327 N}$$

$\Sigma fy = 0$

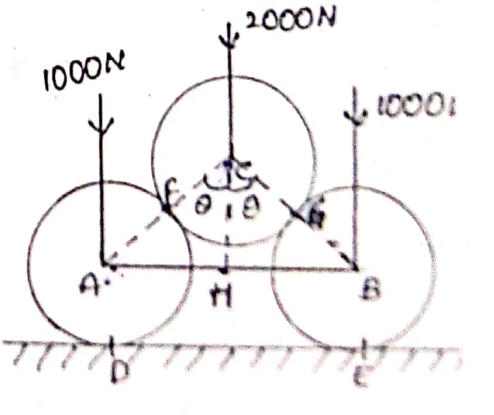


$R_B = 1000 + R_D \sin 36.87$

$R_B = 1000 + 1666.662 \times 0.600$

$$\boxed{R_B = 1999.999 N}$$

13. Two smooth circular cylinder each of weight is 1000N and radius 15cm are connected at their centres by a string AB of length = 40cm and rest upon a horizontal planes, supporting above them, a third cylinder of weight = 2000N and radius 15 as shown in fig. find the force s in the string AB and the pressure produced on the floor at the points of contact D and



A. Given,

weight of each sphere ~~AB~~ = 1000N

weight of sphere C = 2000N

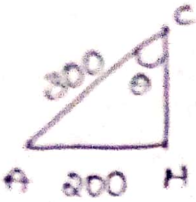
Radius of each cylinder = 15cm
= 150mm

length of string AB = 40cm = 400mm

length of AH = $\frac{AB}{2}$ = 20cm = 200mm

length of AC = AF + CF
= 15 + 15 = 30cm = 300mm

from ΔACH



$$\sin\theta = \frac{AH}{AC}$$

$$\theta = \sin^{-1}\left(\frac{2}{3}\right)$$

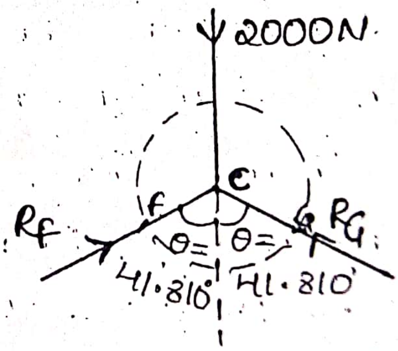
$$\boxed{\theta = 41.810}$$

Case(i):

Considering the equilibrium of sphere c which is in equilibrium of under following forces

1. Weight of sphere c (W_c)
2. Reaction between A and C (R_F)
3. Reaction between B and C (R_G)

for equilibrium



$\Sigma f_x = 0$

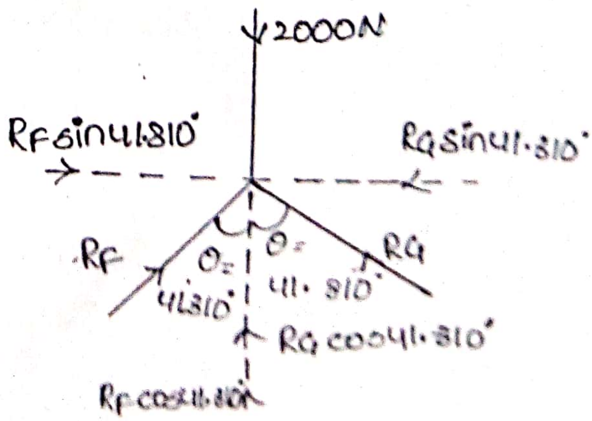
$$R_F \times \sin 41.810 = R_G \times \sin 41.810^\circ$$

$$R_F = R_G \times \frac{\sin 41.810}{\sin 41.810}$$

$$R_F = R_G \quad \text{—— (i)}$$

$\Sigma f_y = 0$

$$R_F \cos 41.810^\circ + R_G \cos 41.810^\circ = 2000N$$



$$R_G \cos 41.810^\circ + R_G \cos 41.810^\circ = 2000N$$

$$R_G = \frac{2000}{\cos 41.810^\circ + \cos 41.810^\circ}$$

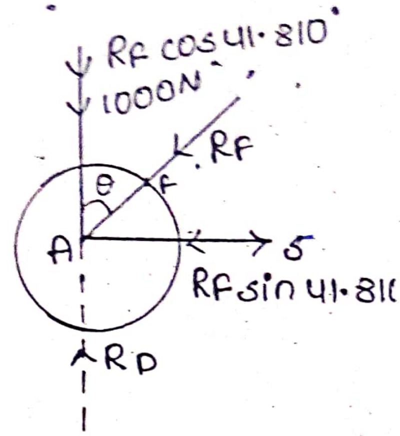
$$\boxed{R_F = R_G = 1341.957N}$$

Case(ii):

Considering the equilibrium of sphere A

let S = forces in the direction of string AB

for equilibrium



$\Sigma fx = 0$

$$S = R_F \sin 41.810^\circ$$

$$S = 1341.957 \times \sin 41.810^\circ$$

$$\boxed{S = 894.632\,N}$$

$\Sigma fy = 0$

$$R_D = 1000 + R_F \times \cos 41.810^\circ$$

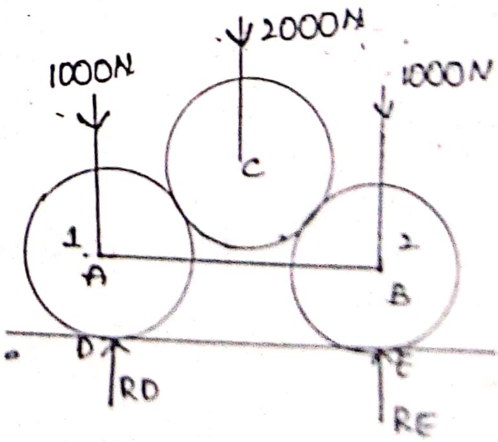
$$R_D = 1000 + 1341.957 \times 0.745$$

$$\boxed{R_D = 2000.240\,N}$$

Case(iii):

Consider the equilibrium spheres A, B and C

In this case only vertical forces are exists.



$$R_D + R_E = 1000 + 1000 + 2000$$

$$2000.240 + R_E = 4000$$

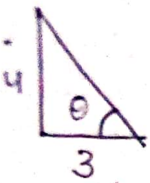
$$R_E = 4000 - 2000.240$$

$$R_E = 1999.76$$

$$\boxed{R_E \cong 2000\,N}$$

14. Find the tension in each ~~sphere~~ rope in the given fig.

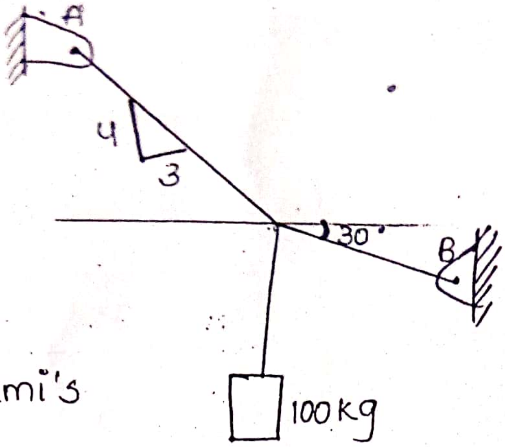
A.

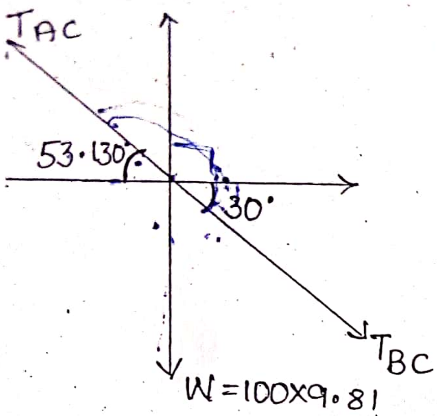


$\tan\theta = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\boxed{\theta = 53.130^\circ}$





By applying lami's theorem

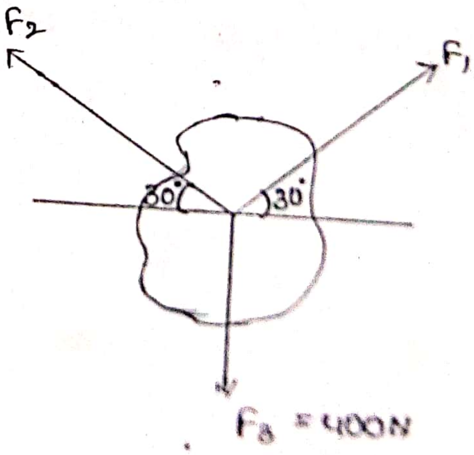
$$\frac{100\times9.81}{\sin(156.87)} = \frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin(143.130^\circ)}$$

$\boxed{T_{AC} = 2162.755\,N}$

$\boxed{T_{BC} = 1498.404\,N}$

5. Three forces F_1, F_2 and F_3 are acting on a body a show in fig. and the body is in equilibrium if the magnitude of force F_3 is 400N. find the magnitudes of forces F_1 and F_2.

A. Method-I:



As the body is in equilibrium

i) $\Sigma F_x = 0$

$F_1\cos 30^\circ - F_2\cos 30^\circ = 0$

$F_1 - F_2 = 0$

$\Rightarrow F_1 = F_2$ —— (i)

ii) $\Sigma F_y = 0$

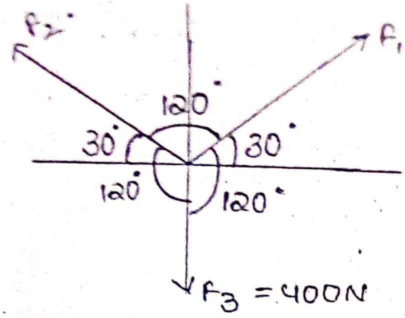
$F_1\sin 30^\circ + F_2\sin 30^\circ - 400 = 0$

$F_1\times0.5 + F_2\times0.5 = 400$

$\therefore \boxed{F_2 = F_1 = 400N}$

Method-2:

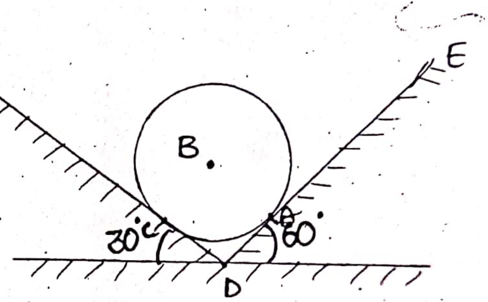
$$\frac{F_1}{\sin 120^\circ} = \frac{F_2}{\sin 120^\circ} = \frac{400}{\sin 120^\circ}$$



$$\therefore \boxed{F_1 = F_2 = 400N}$$

16. A ball of weight 120N rests in a right-angled groove, as shown in fig. The sides of the groove are inclined to an angle of 30° and 60° to the horizontal if all the surfaces are smooth, they determine the reactions R_A and R_C at the points of contact

A. Given,



weight of sphere = 120N

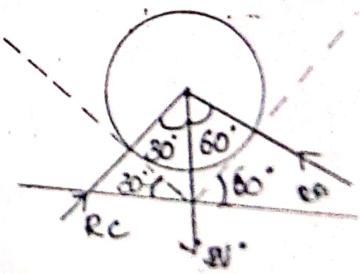
Angle of groove = 90°

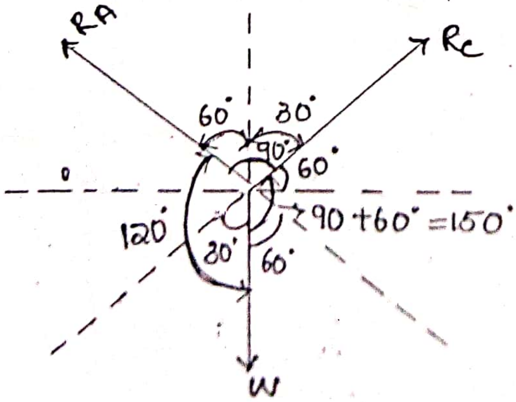
Angle made by FD with horizontal = 30°

Angle made by ED with horizontal = 60°

Let R_A = Reaction R_A at A normal to DE

R_C = Reaction R_C at C normal to FD





Applying lami's theorem

$$\frac{W}{\sin 90^\circ} = \frac{R_A}{\sin 150^\circ} = \frac{R_C}{\sin 120^\circ}$$

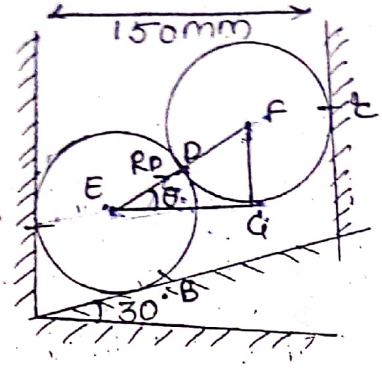
$$\frac{R_A}{0.5} = \frac{120}{1}$$

$$\boxed{R_A = 60N}$$

$$\frac{R_C}{0.866} = \frac{120}{1}$$

$$\boxed{R_C = 103.923N}$$

Two cylinders each of diameter 100mm and each weighing 200N are placed as shown in fig. Assuming that all the contact surfaces are smooth, find the reactions at A, B and C.



Given,

diameter of each cylinder = 100mm

Radius of each cylinder $= \frac{100}{2}$

$= 50\,mm$

weight of each cylinder = 200N

let: R_A = Reaction at contact point A

R_B = Reaction at contact point B

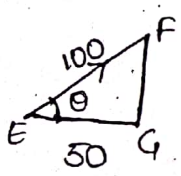
R_C = Reaction at contact point C

width of the channel = 150mm

$\Rightarrow EG = 150 - 50 - 50 = 50\,mm$

$\Rightarrow EF = 50 + 50 = 100\,mm$

from ΔFEG



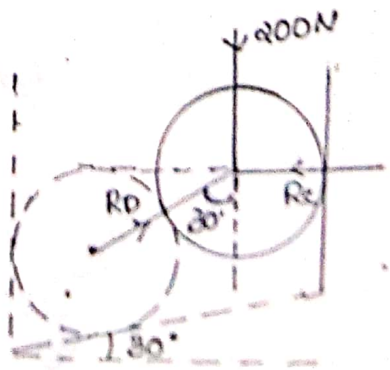
$$\cos\theta = \frac{EG}{EF}$$

$$\theta = \cos^{-1}\left(\frac{5}{10}\right)$$

$$\boxed{\theta = 60^\circ}$$

Case(i): Considering the equilibrium of cylinder 2 which is equilibrium under following forces.

1. Weight of the cylinder 2 (W_2)
2. Reaction at point C (R_C)
3. Reaction at point D (R_D)

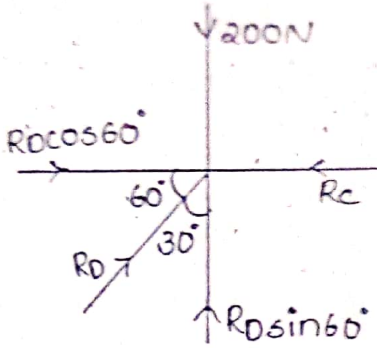


for equilibrium

$$\Sigma F_x = 0$$

$$R_C = R_D \cos 60^\circ$$

$R_C = R_D \times 0.5$ ——— (i)



$\Sigma f_y = 0$

$R_D \sin 60° = 200$

$$R_D = \frac{200}{0.866}$$

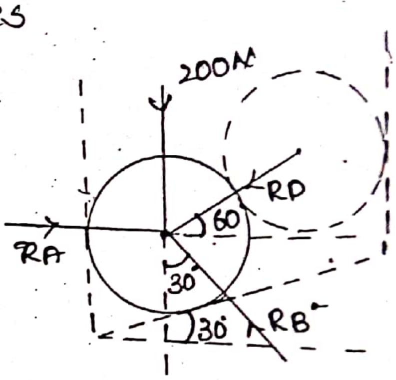
$$\boxed{R_D = 230.946N}$$

from (i) $R_C = 230.946 \times 0.5$

$$\boxed{R_C = 115.473N}$$

Case(ii): Considering the equilibrium of cylinder 1 which is in equilibrium under following forces

1. Weight of cylinder 1 (W_1)
2. Reaction at point A (R_A)
3. Reaction at point B (R_B)
4. Reaction at point D (R_D)



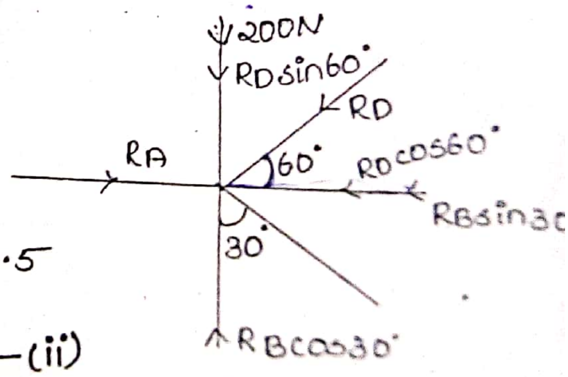
for equilibrium

$\Sigma f_x = 0$

$R_A - R_B \sin 30° - R_D \cos 60° = 0$

$R_A = R_B \times 0.5 + 230.946 \times 0.5$

$R_A = R_B \times 0.5 + 115.473$ ——(ii)



$\Sigma f_y = 0$

$R_B \cos 30° - 200 - R_D \sin 60° = 0$

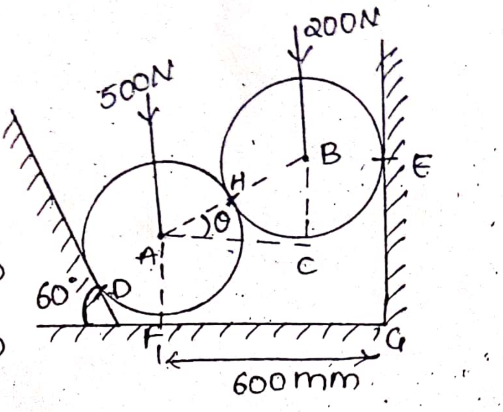
$R_B \cos 30° = 200 + 230.946 \times 0.866$

$$\boxed{R_B = 461.899N}$$

from (ii) $R_A = 461.899 \times 0.5 + 115.473$

$$\boxed{R_A = 346.422N}$$

8. Two spheres A and B are resting in a smooth through as shown in fig. Draw the free body diagrams of A and B showing all the forces acting on them, both in magnitude and direction. Radius of spheres A and B are 250 mm and 200 mm respectively.



7. Given,

Weight of sphere A $W_A = 500N$

Weight of sphere B $W_B = 200N$

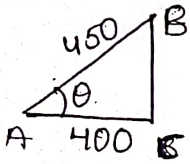
Radius of sphere A $R_A = 250mm$

Radius of sphere B $R_B = 200mm$

length of AC = FG − BE

$= 600 - 200 = 400mm$

from Δ ABC



$$\cos\theta = \frac{AC}{AB}$$

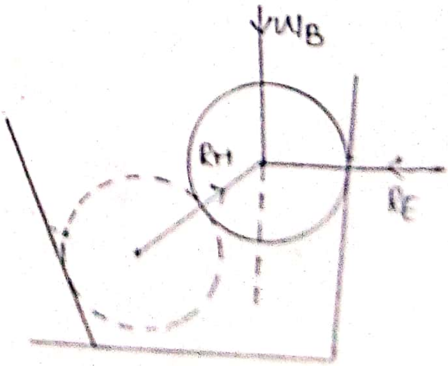
$$\theta = \cos^{-1}\left(\frac{400}{450}\right)$$

$$\boxed{\theta = 27.266^\circ}$$

Case(i):

Considering the equilibrium of sphere B under the following forces

1. Weight of sphere B(W_B)
2. Reaction at H (R_H)
3. Reaction at E (R_E)



for equilibrium

$\Sigma F_x = 0$

$R_E = R_H \cos 27.266^\circ$

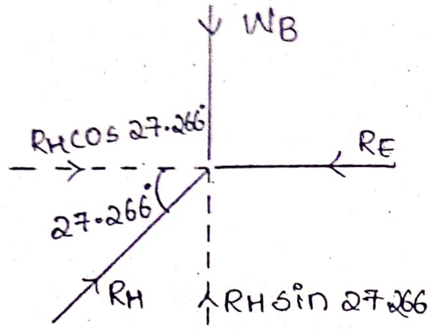
$R_E = R_H \times 0.888$ ——(i)

$\Sigma f_y = 0$

$W_B = R_H \sin 27.266^\circ$

$200 = R_H \times 0.548$

$\boxed{R_H = 436.564N}$



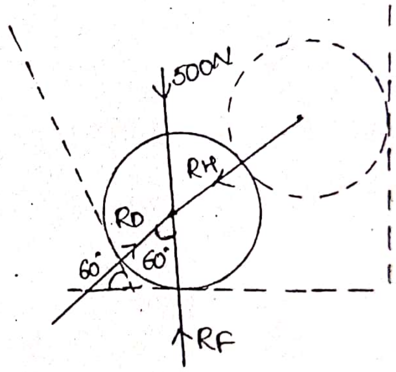
from(i) $R_E = R_H \times 0.888$

$= 436.564 \times 0.888$

$\boxed{R_E = 387.669N}$

Case(ii): Considering the equilibrium sphere A which is in equilibrium under following forces

1. Weight of sphere A
2. Reaction at point D R_D
3. Reaction at point F R_F



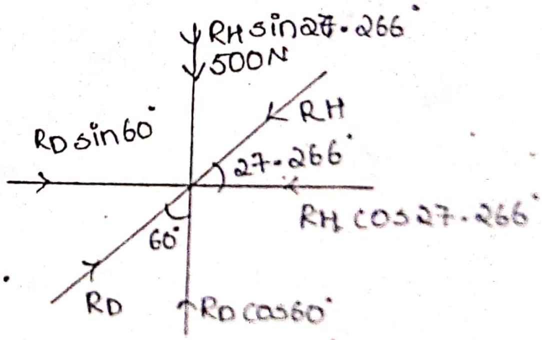
for equilibrium

$\Sigma f_x = 0$

$R_H \cos 27.266^\circ = R_D \sin 60^\circ$

$R_H = R_D \times \dfrac{\sin 60^\circ}{\cos 27.266^\circ}$

$R_H = R_D \times 0.974$



$R_D = \dfrac{R_H}{0.974} = \dfrac{436.564}{0.974}$

$\boxed{R_D = 448.102N}$

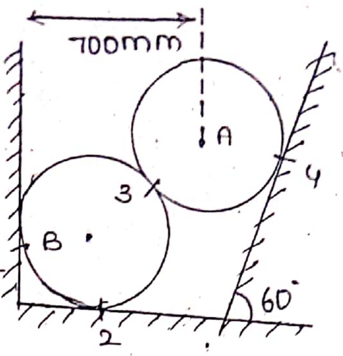
$\Sigma F_y = 0$

$R_F + R_D \cos 60^\circ = 500 + R_H \sin 27.266^\circ$

$R_F + 448.102 \times 0.5 = 500 + 436.564 \times 0.458$

$\boxed{R_F = 475.895N}$

19. Two spheres A and B of weight 1000N and 750N, respectively are kept as shown in fig. Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A = 400mm and B = 300mm.



A. Given,

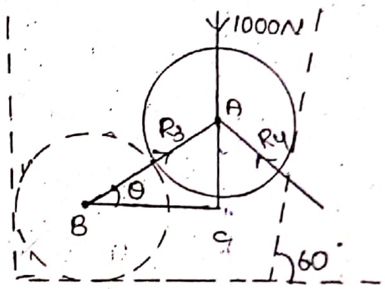
weight of sphere A $W_A = 1000N$

weight of sphere B $W_B = 750N$

Radius of sphere A $r_A = 400mm$

Radius of sphere B $r_B = 300mm$

Case(i): Considering the equilibrium of sphere A which is in equilibrium under following forces

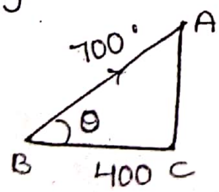


1. Weight of sphere (W_A)
2. Reaction at point 3 (R_3)
3. Reaction at point 4 (R_4)

⇒ length of AB = 300 + 400
= 700mm

⇒ length of BC = 700 − 300
= 400mm

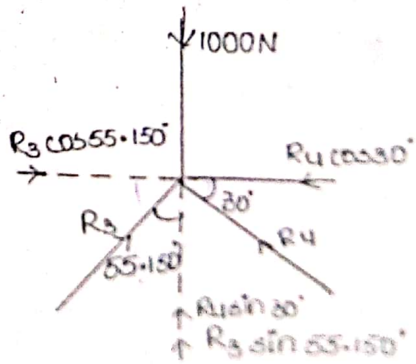
from Δ ABC



$\cos\theta = \frac{BC}{AB}$

$\theta = \cos^{-1}\left(\frac{4}{7}\right)$

$\boxed{\theta = 55.150°}$



for equilibrium

$\Sigma f_x = 0$

$R_3 \cos 55.150° - R_4 \cos 30° = 0$

$R_3 = R_4 \times 1.515$ —— (1)

$\Sigma f_y = 0$

$R_4 \sin 30^\circ + R_3 \sin 55.150^\circ - 1000 = 0$

$R_4 \times 0.5 + R_4 \times 1.515 \times 0.820 = 1000$

$R_4 = \dfrac{1000}{(0.5 + 1.515 \times 0.820)}$

$\boxed{R_4 = 573.953N}$

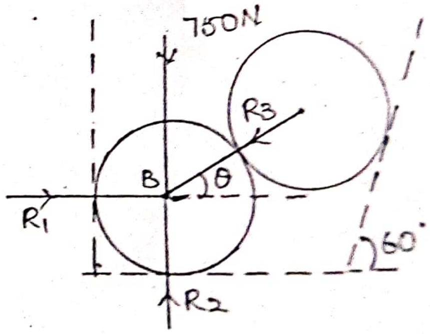
$R_3 = R_4 \times 1.515$

$R_3 = 573.953 \times 1.515$

$\therefore \boxed{R_3 = 869.540N}$

Case(ii): Considering the equilibrium sphere B which is in equilibrium under following forces.

1. Weight of sphere B (W_B)
2. Reaction at point 1 (R_1)
3. Reaction at point 2 (R_2)
4. Reaction at point 3 (R_3)



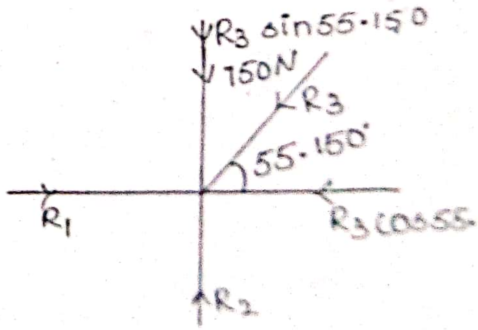
for equilibrium

$\Sigma f_x = 0$

$R_1 = R_3 \cos 55.150^\circ$

$R_1 = 869.540 \times 0.571$

$\boxed{R_1 = 496.881N}$



$\Sigma F_y = 0$

$R_2 - 750 - R_3 \sin 55.150^\circ = 0$

$R_2 = 750 + 869.540 \times 0.820$

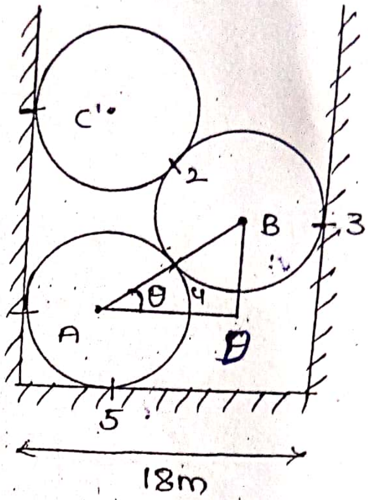
$\boxed{R_2 = 1463.388N}$

10. Three cylinders are piled up in a rectangular channel as shown in fig. Determine the reactions at point 6 between the cylinder A and the vertical wall of the channel.

a, Cylinder A radius = 4cm, m = 15kg

b, cylinder B radius = 6cm, m = 40kg

c, cylinder C radius = 5cm, m = 20kg



1. Given,

Radius of cylinder $r_A = 4cm$

$= 40mm$

~~Radi~~ weight of cylinder $w_A = m \times 9.81$

$= 15 \times 9.81$

$= 147.15N$

Radius of cylinder $r_B = 6cm = 60mm$

weight of cylinder $w_B = 40 \times 9.81$

$= 392.4N$

Radius of cylinder $r_C = 5cm = 50mm$

weight of cylinder $w_C = 20 \times 9.81$

$= 196.2N$

Case(i): Considering F.B.D of the entire system

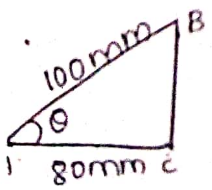
$\Rightarrow$ length of $AB = 4+6 = 10cm$

$= 10mm$

length of $AD = 18-4-6$

$= 8cm = 80mm$

from Δ BAD



$\cos\theta = \frac{AD}{AB}$

$\theta = \cos^{-1}(8/10)$

$\boxed{\theta = 36.869^\circ}$

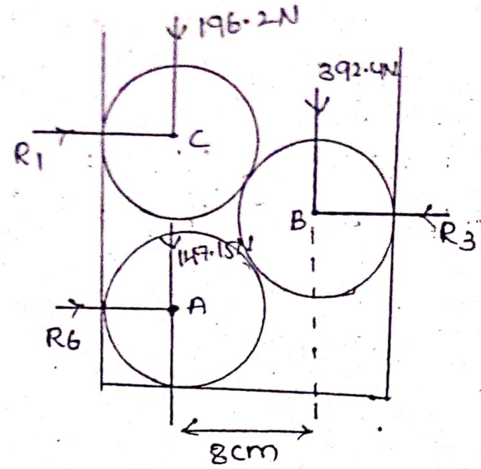
for equilibrium

$\Sigma F_y = 0$

$R_5 = W_A + W_B + W_C$

$R_5 = 147.15 + 392.4 + 196.2$

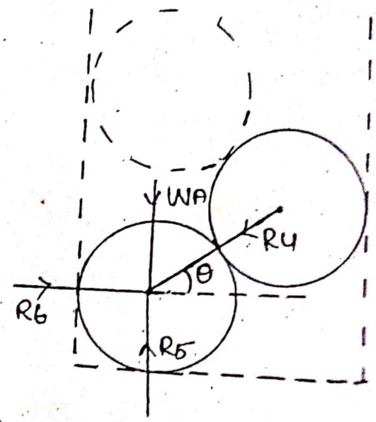
$\boxed{R_5 = 735.75N}$



Case(ii):

Considering the F.B.D of cylinder A which is in equilibriu under following forces

1. Weight of cylinder A (W_A)
2. Reaction at point 6 (R_6)
3. Reaction at point 5 (R_5)
4. Reaction at point 4 (R_4).

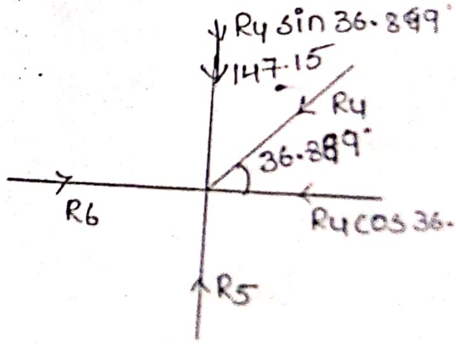


for equilibrium

$\Sigma F_x = 0$

$R_6 = R_4 \times \cos 36.869$

$R_6 = R_4 \times 0.800$ ——(i)



$\Sigma F_y = 0$

$R_5 - R_4 \sin 36.869 - 147.15 = 0$

$735.75 = R_4 \times 0.599 + 147.15$

$\boxed{R_4 = 982.637N}$

from (i)

$R_6 = 982.637 \times 0.800$

$\boxed{R_6 = 786.109N}$

Q. Two persons lift a mass 100kg by cables passing over two pulleys as shown in fig. Determine the force P and Q that must be appiled by two persons. If the body is in equilibrium at the position shown.

Given,

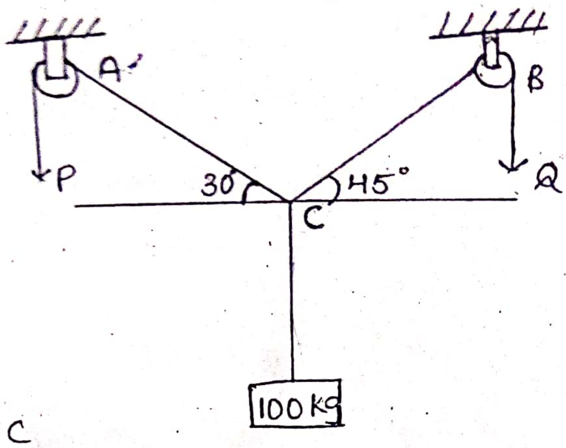
$\boxed{m = \frac{W}{g}}$

mass $m = 100 \text{kg}$

weight $W = m \times 9.81$

$= 100 \times 9.81$

$= 981 \, N$

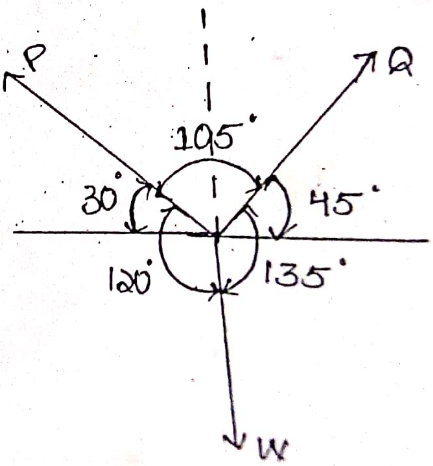


Let P = Tension in the string AC

Q = Tension in the string BC

Applying lami's theorem

$$\frac{P}{\sin 135^\circ} = \frac{Q}{\sin 120^\circ} = \frac{W}{\sin 105^\circ}$$



$$\Rightarrow \frac{P}{0.707} = \frac{981}{0.965}$$

$$P = 718.722 N$$

$$P = \frac{718.722}{9.81} \Rightarrow \boxed{P = 73.264 \text{Kg}}$$

$$\Rightarrow \frac{Q}{0.866} = \frac{981}{0.965}$$

$$Q = \frac{879.513}{9.81}$$

$$\boxed{Q = 89.654 \text{kg}}$$

Q. A light string ABCDE who's extremity A is fixed, as weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300N at the free end E as shown in fig. If in the equilibrium position BC is horizontal and AB and CD makes 150° & 120° with BC, find

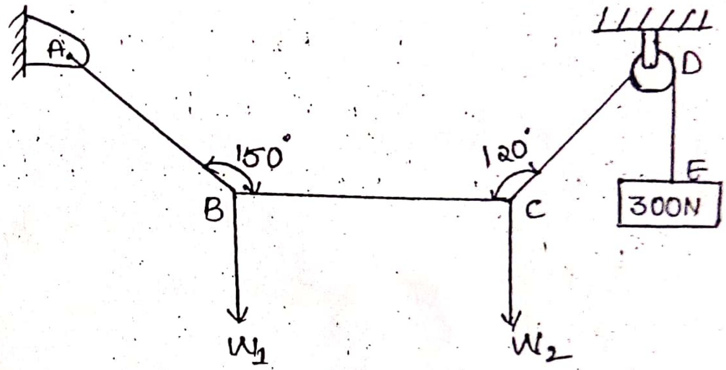
1. Tension in the position AB, BC and CD of the string
2. Magnitudes of W_1 and W_2

A: Given,

weight = 300N

W_1 = Weight at point B

W_2 = Weight at point C



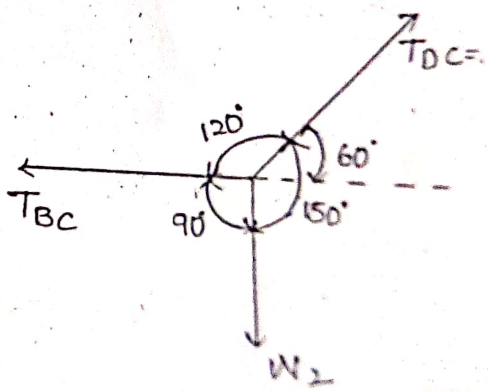
Case(i):

Consider the string BCD

$$\frac{T_{DC}}{\sin 90^\circ} = \frac{T_{BC}}{\sin 150} = \frac{W_2}{\sin 120^\circ}$$

$$\frac{T_{BC}}{0.5} = \frac{300}{1}$$

$$\boxed{T_{BC} = 150N}$$



$$\frac{W_2}{0.866} = \frac{300}{1}$$

$$W_2 = 300 \times 0.866$$

$$\boxed{W_2 = 259.8N}$$

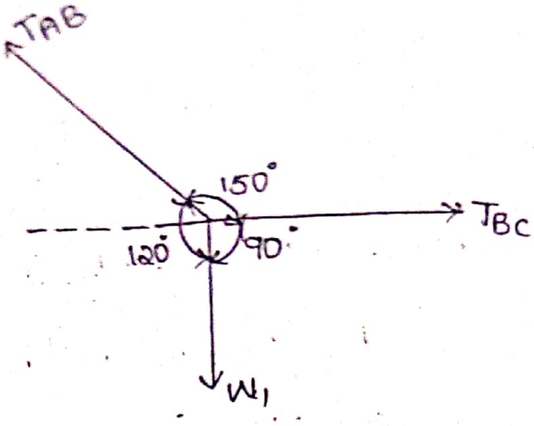
Case(ii):

Consider the string ABC

$$\frac{T_{BC}}{\sin 120^\circ} = \frac{W_1}{\sin 150} = \frac{T_{AB}}{\sin 90^\circ}$$

$$\frac{150}{0.866} = \frac{T_{AB}}{1}$$

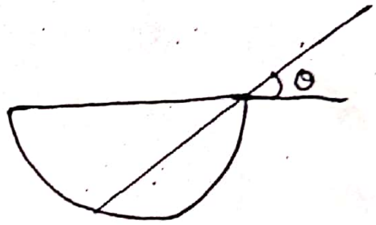
$$\boxed{T_{AB} = 173.210}$$



$$\frac{W_1}{0.5} = 173.210$$

$$\boxed{W_1 = 86.605}$$

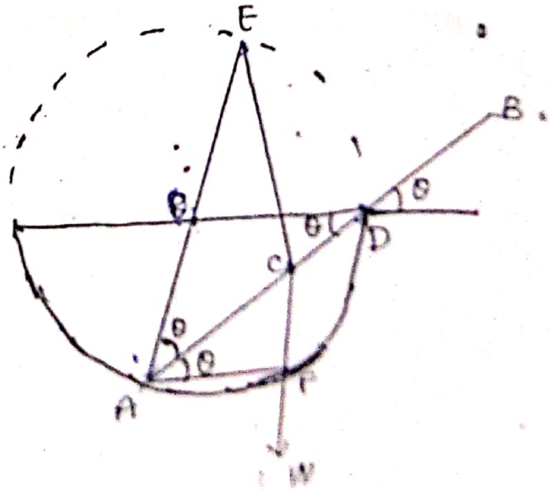
13. A uniform rod AB of length 3R and weight W rest inside a semi spherical bowl of radius 'R' as shown in fig. Neglecting friction, determine angle 'θ' corresponding to equilibrium.



A. length of AB = 3R

R = radius of bowl

W = weight of the rod which is acting downwards through its centriod (or) Midpoint



$\angle ODA = \angle CAE = \angle OAD = \theta$

length of AE = R+R = 2R

length of AC = $\frac{AB}{2}$ = 1.5R

from ΔCAF

$$\cos\theta = \frac{AF}{AC}$$

$$AF = AC\cos\theta$$

$$= 1.5\cos\theta \text{ ——— (1)}$$

from ΔEAF

$$\cos\theta = \frac{AF}{AE}$$

$$AF = AE\cos 2\theta$$

$$AF = 2R\cos 2\theta$$

from ①

$$2\cancel{R}\cos 2\theta = 1.\cancel{5}\cancel{R}\cos\theta$$

$$2\cos 2\theta = 1.5\cos\theta$$

$$2(2\cos^2\theta - 1) = 1.5\cos\theta$$

$$4\cos^2\theta - 2 = 1.5\cos\theta$$

$$4\cos^2\theta - 1.5\cos\theta - 2 = 0$$

$$\cos\theta = 0.919$$

$$\boxed{\theta = 23.219^\circ}$$