UNIT-1

System of forces

Classification of Mechanics :-

- Classification of Mechanics
  - Mechanics of deformable bodies
    - Strength of Materials
    - Theory of plasticity
    - Theory of Elasticity
  - Mechanics of Rigid bodies
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics
  - Mechanics of fluid
    - Ideal fluid
    - Viscous fluid
    - Compressible fluid

Statics :- It is the study of the effect of force system acting on a particle or rigid body which is at rest.

Dynamics :- It is the study of the effect of force system acting on a particle or a rigid body which is in motion.

Kinematics :- It is the study of geometry of motion without reference to the cause of motion i.e mass and force causing motion are not consider

Kinetics :- It is the study of geometry of motion with reference to the cause of motion (i.e mass and force causing motion are consid. i

particle :- It is a matter having considerable mass but negligible dimension.

Time :- It is the measure of duration between successive events.

Matter :- It is that which occupies space and can be perceived by our sences.

Mass :- It is the quantity of matter contained in a body. these quantities do not change on account of the position occupied by the body.

Scalar quantity :- A physical quantity which required only magnitude for it's compute description is known as scalar.

Ex :- distance, Area, volume, mass, work, power, energy, time, density, speed etc.

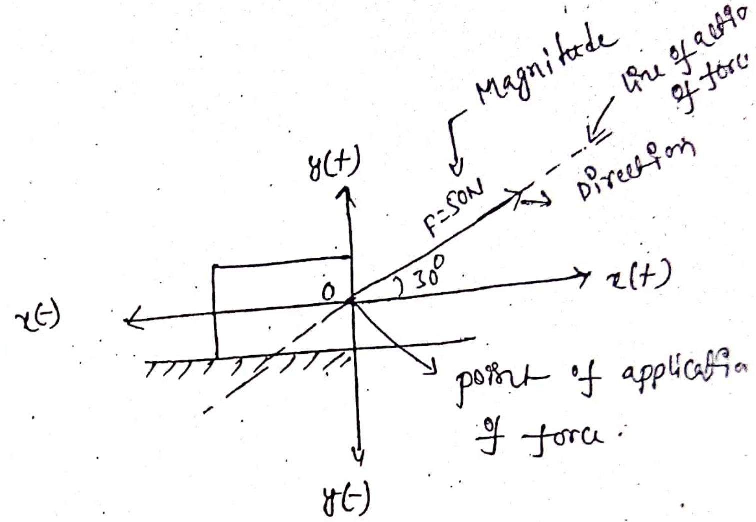
Vector quantity :- A quantity which required both magnitude and direction for it's complete description is known as vector quantity.

Rigid body :- It is defined as a body in which the particle does not change in relative position whatever large force may be applied

Force :- An external agency which changes or tends to change the state of rest or of uniform motion of a body upon which it acts is known as a force.

~~characteristics~~
Characteristics of force :-

a. Magnitude
b. Direction
c. point of application
d. line of action of force.



Laws of Mechanics

1. Newton's first Law :- Every body continues in it's state of rest or of uniform motion in a straight line unless an external unbalanced force acts on it.

2. Newton's Second Law :- The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of applied force.

$$\therefore\ f = \frac{mv - mu}{t} = \frac{m(v-u)}{t}$$

3. Newton's third law :- To every action, there is an equal and opposite reaction. The forces of action and reaction between interacting bodies are equal in magnitude.

Newton's law of gravitation :- The force of attraction between any two bodies in the universe is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

It is given by $F \propto \frac{m_1 m_2}{r^2}$ $\quad \therefore F = \frac{G m_1 m_2}{r^2}$

where G = universal gravitational constant

Principle of transmissibility of force :-

It states that the condition of equilibrium or uniform motion of rigid body will remain unchanged if the point of application of a force acting on a rigid body is transmitted to act at any other point along it's line of action.

Classification of force system :-

Force system
- coplanar
  - Concurrent
  - parallel
    - like
    - unlike
  - Non concurrent Non parallel
- Non coplanar
  - Concurrent
  - parallel
    - like
    - unlike
  - Non concurrent Non parallel

Force System :- when the number of forces act simultaneously on a body they are said to form a force system.

Depending upon whether the line of action of all the forces acting on the body lies in the same plane or in different plane, the force system may be classified as follows:

(i) **Coplanar force system** :-

If the line of action of all the forces in the system lies on the same plane then it is called a co-planar forces system.

(ii) **Coplanar concurrent force system** :-

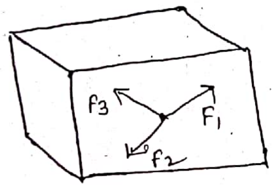


fig coplanar concurrent force.

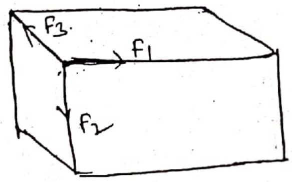
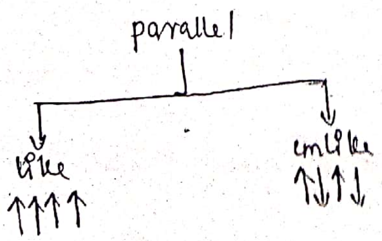
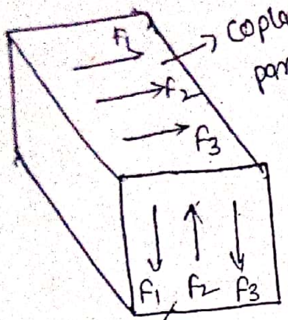


fig Non coplanar concurrent forces.

If the line of action of all the forces in the system passes through single point then it is called a concurrent force system.

(iii) **parallel force system** :- If the line of action of all the forces in the system are parallel to each other then it is called force system parallel force can be further classified into like and unlike.





→ coplanar like parallel force system.

coplanar unlike parallel force system

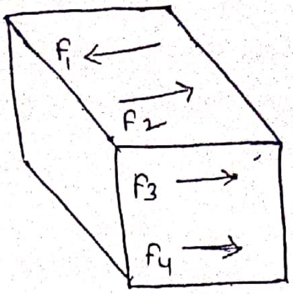
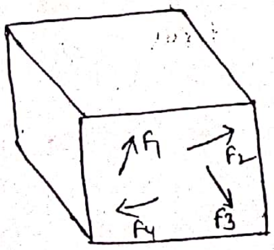


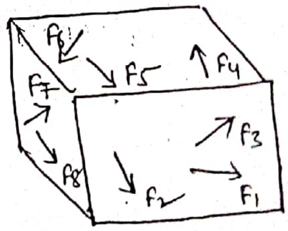
fig Non-coplanar unlike parallel force system

(iv) General force system ! -

If the line of action of all the forces in the system are neither concurrent not parallel then it is known as a non-concurrent non parallel force system.



coplanar non concurrent non parallel force system



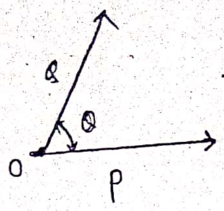
Non coplanar non concurrent and non parallel forces system

Composition of forces :-

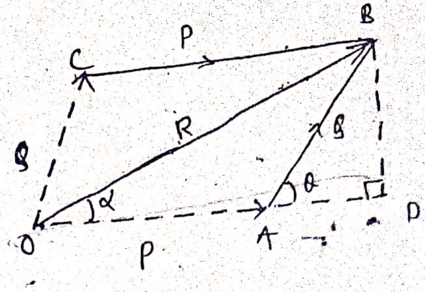
Forces may be combined (adding) to obtain a single force which produces the same effect as the original system of forces. this single force is known as resultant force. the process of finding the resultant of forces is called composition of forces.

the force is a vector quantity.

Law of parallelogram of forces :-



$\Rightarrow$



"If two forces acting simultaneously on a body at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of parallelogram which passes through the point of intersection of the two sides representing the forces.

Let P and Q be the two concurrent forces having included angle θ at point 'O' as shown in fig.

$OA = P$, $OC = Q$

Draw a ⊥ line from point B on OA extended, meeting at point D.

As $OC = AB = Q$ $\quad OB = R$

$OA = CB = P$

In ΔODB $\quad OB^2 = OD^2 + BD^2$

$OB^2 = (OA + AD)^2 + BD^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$R^2 = P^2 + 2PQ\cos\theta + Q^2\cos^2\theta + Q^2\sin^2\theta$

$R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$\therefore \Delta DAB \ \cos\theta = \frac{AD}{AB}$

$AD = AB\cos\theta$

$= Q\cos\theta$

and

$\therefore \Delta DAB \ \sin\theta = \frac{BD}{AB}$

$BD = AB\sin\theta$

$= Q\sin\theta$

Magnitude of resultant $\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$

In △OBD

let α be the angle made by R with P

$$\tan\alpha = \frac{BD}{OD} = \frac{BD}{OA + AB}$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Direction of resultant R.

$$\boxed{\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)}$$

Case-1 if two forces (P and Q) act at right angles, then

$$\theta = 90^0$$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^0}$$

$$\boxed{R = \sqrt{P^2 + Q^2}}$$

$$\tan\alpha = \frac{Q\sin 90^0}{P + Q\cos 90^0} \Rightarrow \boxed{\alpha = \tan^{-1}\left(\frac{Q}{P}\right)}$$

Case-2 if two forces are equal and acting at an angle θ

then

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{P^2 + P^2 + 2P\times P\cos\theta} \qquad (P = Q)$$

$$= \sqrt{2P^2 + 2P^2\cos\theta} \Rightarrow \sqrt{2P^2(1+\cos\theta)}$$

$$= \sqrt{2P^2 \times 2\cos^2\frac{\alpha}{2}}$$

$$= \sqrt{4P^2\cos^2\frac{\alpha}{2}} \Rightarrow 2P\cos\frac{\alpha}{2}$$

and $\alpha = \tan^{-1}\left(\frac{Q\sin\theta}{P+Q\cos\theta}\right)$

$= \tan^{-1}\left(\frac{P\sin\theta}{P+P\cos\theta}\right)$ $(P=Q)$

$= \tan^{-1}\frac{P\sin\theta}{P(1+\cos\theta)}$

$= \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$

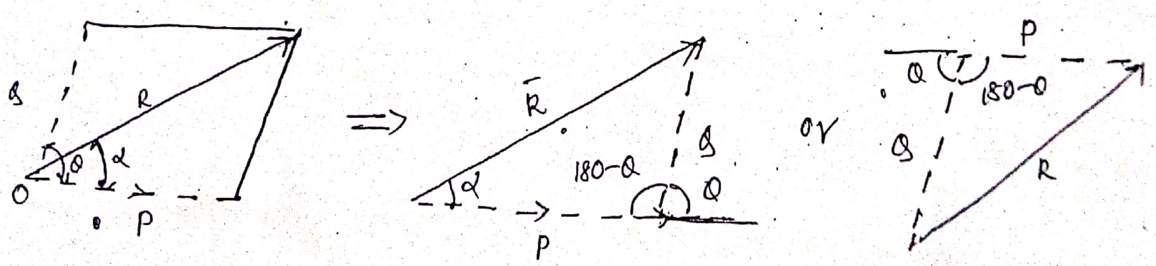
$= \tan^{-1}\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}}$ $(\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2})$

$= \tan^{-1}\left(\tan\frac{\alpha}{2}\right)$

$= \frac{\alpha}{2}$

Triangle of force :-

If two forces are represented by in magnitude and direction by the sides of a triangle taken in one direction, then their resultant is represented in magnitude and direction taken in reverse order by the ~~~~ third side of a triangle.

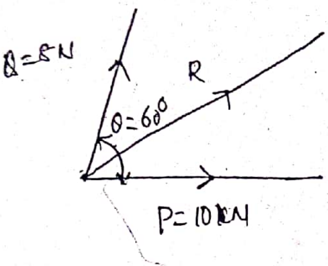


problems

① Two forces of magnitude 10 N and 8N are acting at a point. If the angle b/n the two forces is 60°. determine the magnitude of the resultant forces.

sol Given force $P = 10N$.

$Q = 8N$



Angle between the two forces $\theta = 60°$.

The magnitude of Resultant force (R)

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{10^2 + 8^2 + 2\times10\times8\times\cos60°}$$

$$\boxed{R = 15.62 N}$$

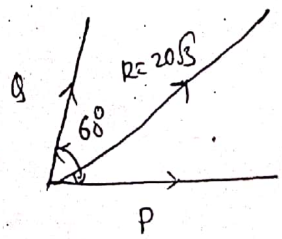
② Two equal forces are acting at a point with an angle of 60° between them. If the resultant force is equal to $20\times\sqrt{3}$, find magnitude of each force.

sol Given

Angle b/n the forces $\theta = 60°$

Resultant $R = 20\sqrt{3}$

The forces are equal i.e $P = Q$



$P = Q$

Magnitude of Resultant force $R = \sqrt{}$

$$R = 2P\cos\frac{\theta}{2}$$

$$20\sqrt{3} = 2\times P\times\cos\frac{60°}{2}$$

$$20\sqrt{3} = 2P\cos30°$$

$$P = \frac{20\times\sqrt{3}}{2\times\cos30°} \Rightarrow \boxed{P = 20N}$$

i.e $P = Q = 20N$

(3) The resultant of the two forces, when they act at an angle of 60° is 14 N. If the same forces are acting at right angles, their resultant is $\sqrt{136}$ N. Determine the magnitude of the two forces.

Sol Given

Case-I : $R_1 = 14\,N$

$\theta = 60^{\circ}$

Case-II : $R_2' = \sqrt{136}\,N$

$\theta = 90^{\circ}$

Let the two forces are P and Q

Case-I

$$R' = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$14 = \sqrt{P^2 + Q^2 + 2PQ\cos 60^{\circ}}$$

$$14 = \sqrt{P^2 + Q^2 + 2PQ \times \frac{1}{2}} \Rightarrow 14^2 = P^2 + Q^2 + PQ$$

$$196 = P^2 + Q^2 + PQ \longrightarrow (i)$$

Case-II

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90^{\circ}}$$

$$R = \sqrt{P^2 + Q^2} \Rightarrow (\sqrt{136})^2 = P^2 + Q^2$$

$$136 = P^2 + Q^2 \longrightarrow (ii)$$

Subtracting eqn (ii) from eqn (i)

$$196 - 136 = \cancel{P^2} + \cancel{Q^2} + PQ - \cancel{P^2} - \cancel{Q^2}$$

$$PQ = 60 \longrightarrow (iii)$$

Multiplying the above eqn by 2, we get

$$120 = 2PQ \longrightarrow (iv)$$

adding eqn (iv) to eqn (ii), we get.

$$136 + 120 = P^2 + Q^2 + 2PQ \Rightarrow 256 = P^2 + Q^2 + 2PQ = (P+Q)^2$$

$16 = P + Q$

$P = (16 - Q) \longrightarrow (v)$

substituting the value of P in eqn (iii), we get

$60 = (16 - Q) \times Q$

$60 = 16Q - Q^2$ or $Q^2 - 16Q + 60 = 0$

$Q = 10$ and 6

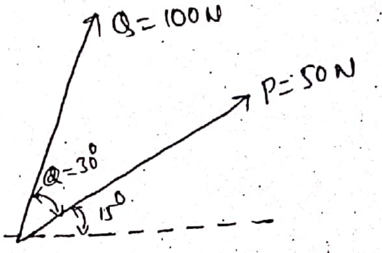
substituting the value of Q in eqn (v) we get

$P = (16 - 10)$ $\quad$ $P = 16 - 6$

$P = 6$ $\quad$ $P = 10$

∴ Hence the two forces are 10N and 6N.

(4) Two forces are acting at a point O as shown in fig. Determine resultant in magnitude and direction.

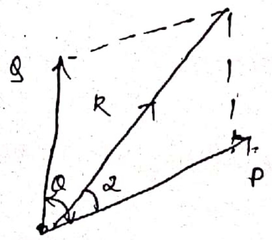


sol Given

$P = 50N$

$Q = 100N$

Angle between the two forces are $\theta = 30^0$



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$= \sqrt{50^2 + 100^2 + 2 \times 50 \times 100 \times \cos 30^0}$

$\boxed{R = 145.46N}$

The angle made by the resultant with the direction of P

$\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

$\alpha = \tan^{-1}\left(\dfrac{100\sin 30^0}{50 + 100 \times \cos 30^0}\right)$

$\alpha = \tan^{-1}(0.366)$

$\boxed{\alpha = 20.10^0}$

∴ Angle made by resultant with x-axis $= \alpha + 15^0$

$= \cancel{30^0}\ \alpha + 15^0 = 20.10 + 15$

$= \cancel{20.10+15} = 35.10^0$

(3) The resultant of two concurrent forces is 1500N and the angle between the forces is 90^0. The resultant makes an angle of 36^0 with one of the force. Find the magnitude each force.

sol Resultant $R = 1500N$

Angle b/n the forces, $\theta = 90^0$

Angle made by the resultant with one of the force, $\alpha = 36^0$

Let P and Q are two forces.

$$\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} = \tan 36^0 = \frac{Q\sin 90^0}{P+Q\cos 90^0}$$

$$= \frac{Q\times 1}{P+Q\times 0} \quad \text{or} \quad 0.726 = \frac{Q}{P}$$

$$\boxed{Q = 0.726P} \quad \text{—— (1)}$$

Now $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$$1500^2 = P^2+Q^2+2PQ\cos 90^0$$

$$= P^2+(0.726P)^2+2P(0.726P)\times\cos 90^0$$

$$1500^2 = P^2+0.527P^2+0$$

$$1500^2 = 1.527P^2$$

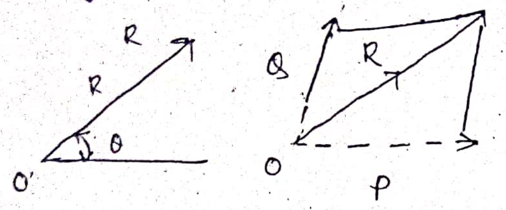
$$P = \sqrt{\frac{1500^2}{1.527}} \Rightarrow \boxed{P = 1213.86N}$$

$$\boxed{Q = 881.26N}$$

Resolution of force :-

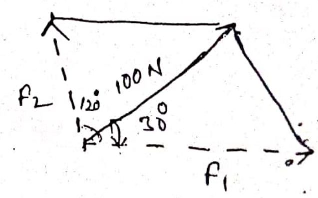
The process of breaking the force into a number of component which are equivalent to the given forces is called resolution of force.

the law of parallelogram shows how to combine two forces into a resultant force.



5) Resolve the 100N force acting a 30° to horizontal into two component on along horizontal and other along 120° to horizontal.

Sol Given



$R = 100\,N$

$\theta = 120^\circ$

$\alpha = 30^\circ$

Method-1 : By parallelogram Law

$$100 = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120^\circ}$$

$$10000 = F_1^2 + F_2^2 - F_1F_2 \quad \text{—— (i)}$$

$$\tan 30^\circ = \frac{F_2 \sin 120^\circ}{F_1 + F_2\cos 120^\circ}$$

$$0.5774 F_1 - 0.2887 F_2 = 0.866 F_2$$

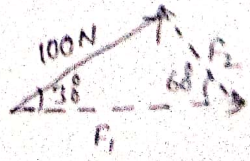
$$F_1 = 2F_2 \quad \text{—— (ii)}$$

Solving eqn (i) & (ii)

$F_1 = 115.47\,N$, $F_2 = 57.76\,N$.

Method-II By Triangular Law

By Sine rule

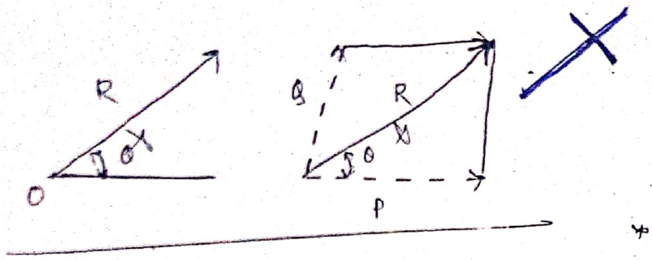


$$\frac{100}{\sin 60^\circ} = \frac{F_1}{\sin 90^\circ} = \frac{F_2}{\sin 30^\circ}$$

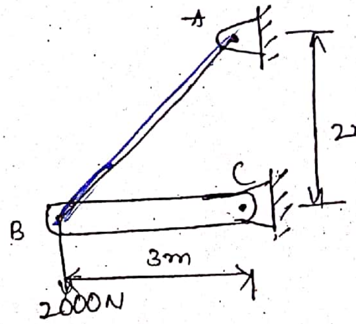
$F_1 = 115.47\,N$, $F_2 = 57.76\,N$

Resolution of force :-

the process of breaking the force into a number of components which are equivalent to the given forces is called resolution of force.



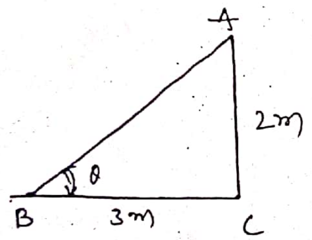
(i) Resolve the 200N force into two oblique components one acting along AB and the other acting along BC shown in fig -



Sol Given

BC = 3m

AC = 2m

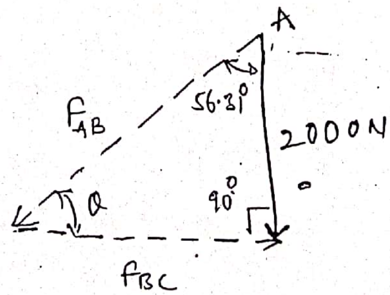


$\tan\theta = \frac{2}{3}$

$\theta = \tan^{-1}(2/3)$

$\theta = 33.69^0$

using sine Rule



$$\frac{F_{AB}}{\sin 98} = \frac{2000}{\sin 33.69} = \frac{F_{BC}}{\sin 56.31^0}$$

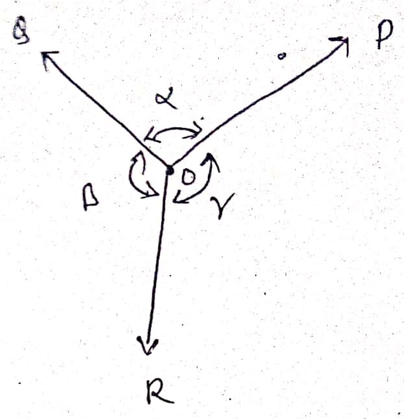
$F_{AB} = 3605.56N$

$F_{BC} = 3000N$

Lami's theorem :-

It states that "If three forces acting at a point are in equilibrium, each force will be proportional to the sine of the angle between the other two forces."

Suppose the three forces P and Q, R are acting at point O and they are in equilibrium as shown in fig.



Let α = Angle b/n the forces P and Q

β = Angle b/n Q and R

γ = Angle b/n R and P

$P \propto$ sine of angle b/n Q and R

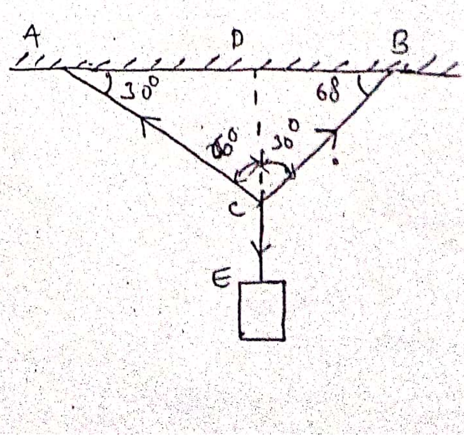
i.e. $P \propto \sin\beta \Rightarrow \dfrac{P}{\sin\beta} = \text{constant}$

Similarly $Q \propto \sin\gamma \Rightarrow \dfrac{Q}{\sin\gamma} = \text{const}$

$R \propto \sin\alpha \Rightarrow \dfrac{R}{\sin\alpha} = \text{const}$

$$\boxed{\dfrac{P}{\sin\beta} = \dfrac{Q}{\sin\gamma} = \dfrac{R}{\sin\alpha}}$$

7) A weight of 1000N is supported by two chains as shown in fig. Determine the tension in each chain.



Sol Given

Weight at C = 1000 N

$\angle CAB = 30^\circ$

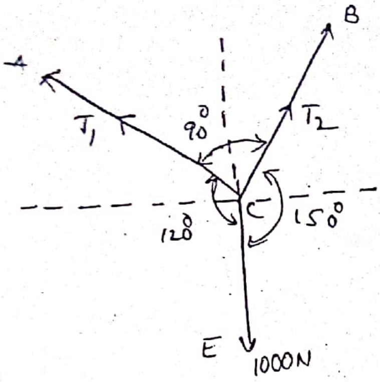
$\angle CBA = 60^\circ$

$\angle ACB = 90^\circ$

let T_1 = Tension in chain 1

T_2 = Tension in chain 2

Applying Lami's theorem at point C



$$\frac{T_1}{\sin 150^0} = \frac{T_2}{\sin 120^0} = \frac{1000}{\sin 90^0}$$

$$\frac{T_1}{\sin 150^0} = \frac{1000}{\sin 90^0}$$

$T_1 = 1000 \times \sin 150^0$

$\boxed{T_1 = 500N}$

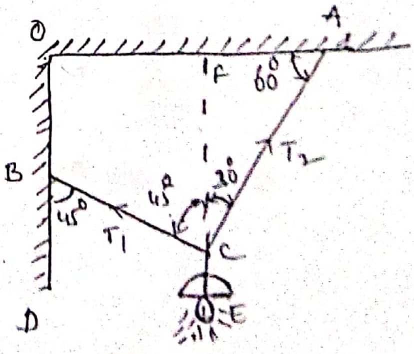
$$\frac{T_2}{\sin 120^0} = \frac{1000}{\sin 90^0}$$

$T_2 = 1000 \times \sin 120^0$

$\boxed{T_2 = 866N}$

(1) An electric light fixture weighing 15N hangs from a point C by two strings AC and BC, AC is inclined at 60^0 to the horizontal and BC at 45^0 to the vertical as shown in fig. using Lami's theorem or otherwise determine the forces in the string AC and BC

Sol Given



weight at C = 15N

$\angle OAC = 60^0$

$\angle CBD = 45^0$

let T_1 = force in string BC

T_2 = force in string AC

using Lami's eqn at C

$$\frac{15}{\sin 75^0} = \frac{T_1}{\sin 150} = \frac{T_2}{\sin 135^0}$$

$$\frac{T_1}{\sin 150} = \frac{15}{\sin 75^0}$$

$$T_1 = \frac{15 \times \sin 150}{\sin 75^0} = 7.76N$$

$$\boxed{T_1 = 7.76N}$$

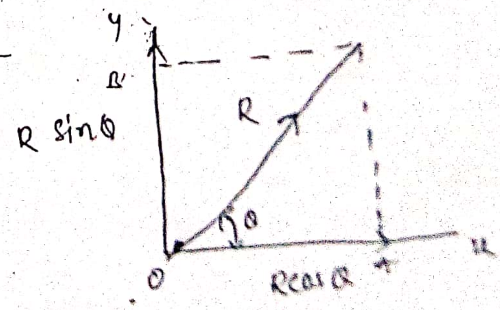
$$\frac{T_2}{\sin 135^0} = \frac{15}{\sin 75^0}$$

$$T_2 = \frac{15 \times \sin 135^0}{\sin 75^0} = 10.98N$$

$$\boxed{T_2 = 10.98N}$$

Resolution of force into Rectangular components :-

Resolution of a force means "finding the components of a given force in two given sections".



Let R = A force.

θ = Angle made by the R with x-axis.

Component of R along x-axis = $R\cos\theta$

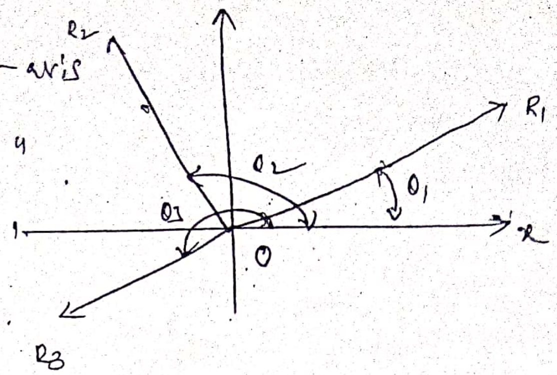
component of R along y-axis = $R\sin\theta$

Resolution of No. of Coplanar forces :-

θ_1 = Angle made by R_1 with x-axis

θ_2 = " " R_2 " "

θ_3 = " " R_3 " "



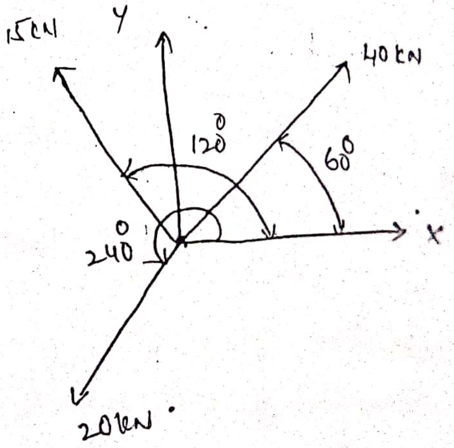
$$\Sigma H = R_1\cos\theta_1 + R_2\cos\theta_2 + R_3\cos\theta_3 + \cdots$$

$$\Sigma V = R_1\sin\theta_1 + R_2\sin\theta_2 + R_3\sin\theta_3 \cdots$$

Resultant of all forces, $R = \sqrt{H^2 + V^2}$

Angle made by R with x-axis $\tan\theta = \frac{\Sigma V}{\Sigma H}$

① Three forces of magnitude 40 kN, 15 kN and 20 kN are acting at a point O as shown in fig. The angles made by 40 kN, 15 kN and 20 kN forces with x-axis are 60°, 120° and 240° respectively. Determine the magnitude and direction of the resultant force.



Sol: Given $\theta_1 = 60°$, $R_1 = 40$ kN

$\theta_2 = 120°$, $R_2 = 15$ kN

$\theta_3 = 240°$, $R_3 = 20$ kN

Sum of components of all forces along x-axis is given by

$$\Sigma H = R_1\cos\theta_1 + R_2\cos\theta_2 + R_3\cos\theta_3$$

$$= 40\times\cos 60° + 15\cos 120° + 20\cos 240°$$

$$\Sigma H = 2.5 \text{ kN}$$

Sum of components of all forces along y-axis

$$\Sigma V = R_1\sin\theta_1 + R_2\sin\theta_2 + R_3\sin\theta_3$$

$$= 40\sin 60° + 15\sin 120° + 20\sin 240°$$

$$\Sigma V = 30.31 \text{ kN}$$

the magnitude of Resultant force

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \sqrt{2.5^2 + 30.31^2}$

$\boxed{R = 30.41\ kN}$

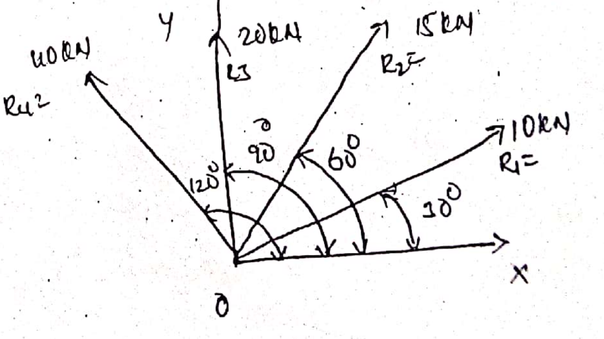
the direction of resultant force

$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{30.31}{2.5} = 12.124$

$\theta = \tan^{-1}(12.124)$

$\boxed{\theta = 85.28^0}$

(10) four forces of magnitude 10 kN, 15 kN, 20 kN and 40 kN are acting at a point O as shown in fig. the angles made by 10 kN, 15 kN, 20 kN and 40 kN with x-axis are 30^0, 60^0, 90^0 and 120^0 respectively. find the magnitude and direction of the resultant force.



sol

$\Sigma H = -3.84\ kN$

$\Sigma V = 72.63\ kN$

$R = \sqrt{\Sigma H^2 + \Sigma V^2} = 72.73\ kN$

$\theta = \tan^{-1}\left(\frac{V}{H}\right)$

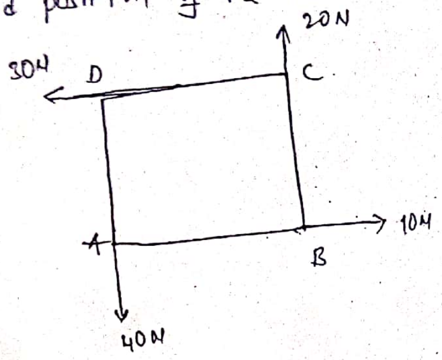
$= \tan^{-1}(-18.91)$

$\theta = -86.97$

$\theta = 180 - 86.97$

$= 93.03^0$

(11) Four forces of magnitudes 10 N, 20 N, 30 N and 40 N are acting respectively along the four sides of a square ABCD as shown in fig. Determine the magnitude, direction and position of the resultant force.



Sol Given force along AB = 10 N

BC = 20 N

CD = 30 N

DA = 40 N

(i) Sum of horizontal forces

$$\Sigma H = 10 - 30 = -20\ N$$

(ii) Sum of vertical forces

$$\Sigma V = 20 - 40 = -20\ N$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{(-20)^2 + (-20)^2}$$

$$= \sqrt{400 + 400} = \sqrt{2 \times 400}$$

$$R = 20\sqrt{2}\ N$$

Direction of resultant force.

$$\tan\theta = \frac{V}{H} = \frac{-20}{-20} = 1$$

$$\theta = 45^0$$

Since H and V are -ve, hence θ lies between 180° and 270°.

It is clear that

$$\theta = 180 + 45^0 = 225^0$$

(iii) position of the resultant force :-

The position of the resultant force is obtained by equating the clockwise moments and anticlockwise moments about A

let L = ⊥ distance b/n A and line of action of the resultant force and

a = side of the square ABCD

Taking moments of all forces about A

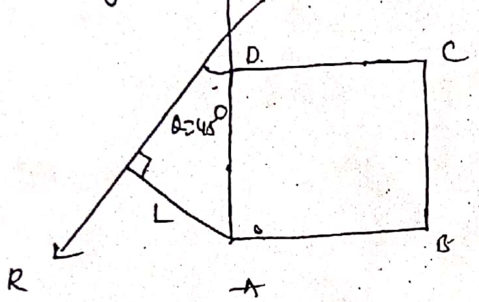
$40\times 0 + 10\times 0 + 20\times a + 30\times a = R \times \perp$ distance of R from A

$= 20\sqrt{2} \times L$

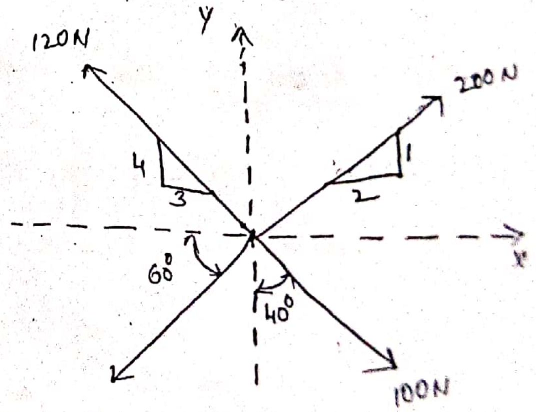
$20a + 30a = 20\sqrt{2} \times L$

$L = \frac{50a}{20\sqrt{2}} = \frac{5a}{2\sqrt{2}}$

The position of the resultant force is shown in fig.



(11) A system of four forces acting at a point on a body is as shown in fig(a). Determine the resultant.



sol

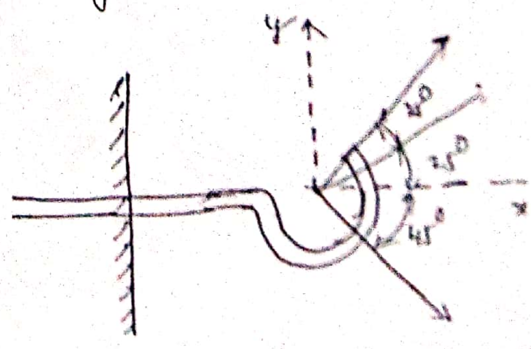
$\Sigma H = 146.2N, \ \Sigma V = 65.5N$

$\boxed{R = 160.2N}$

$\alpha = \tan^{-1}\left(\frac{65.5}{146.2}\right)$

$\boxed{\alpha = 24.1^\circ}$

(12) Determine the resultant of the three forces acting on a hook as shown in fig.



sol

$\Sigma H = 152.8N$

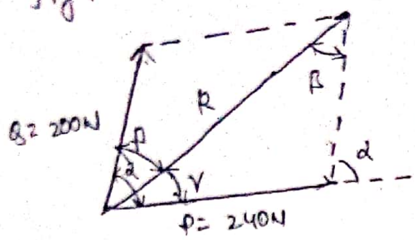
$\Sigma V = 52.1$

$R = 161.5N$

$\alpha = 18.83^\circ$

(13) Two forces of magnitude 240N and 200N are acting at a point as shown in fig. If the angle b/n the forces is 60°, determine the magnitude of the resultant force. Also determine the angle β and γ as shown in fig.

Sol

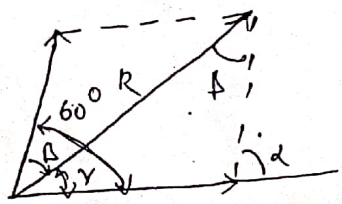


Ans $\alpha = 60^\circ$

$\gamma = 26.995^\circ$

$\beta = 33^\circ$

(14) Two forces P and Q are acting at a point O as shown in fig. The resultant force is 400N and angles β and γ are 35° and 25° respectively. Find the two forces P and Q ?

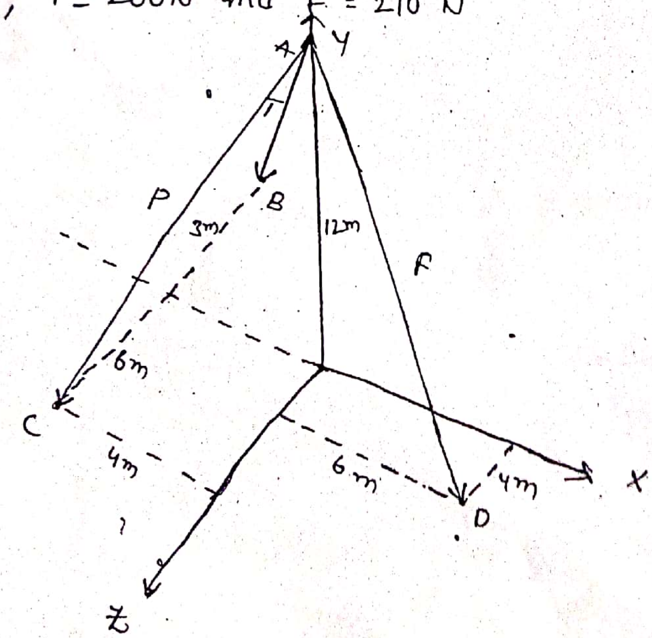


P = 264.93 N

Q = 195.19N

Force system in space :-

15. Find the resultant to the force system shown in fig. Take P = 280N, T = 260N and F = 210 N



Sol

$A\,(0,12,0) \Rightarrow \vec{OA} = 12\vec{j}$

$B\,(-4,0,-3) \Rightarrow \vec{OB} = -4\vec{i} - 3\vec{k}$

$C\,(-4,0,6) \Rightarrow \vec{OC} = -4\vec{i} + 6\vec{k}$

$D\,(6,0,4) \Rightarrow \vec{OD} = 6\vec{i} + 4\vec{k}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AB} = -4\vec{i} - 12\vec{j} - 3\vec{k}$

$\vec{AC} = \vec{OC} - \vec{OA}$

$= -4\vec{i} - 12\vec{j} + 6\vec{k}$

$\vec{AD} = \vec{OD} - \vec{OA}$

$= 6\vec{i} - 12\vec{j} + 4\vec{k}$

calculation of unit vectors along AB, AC and AD

$$\hat{n}_{AB} = \frac{\vec{AB}}{|\vec{AB}|} \Rightarrow \frac{-4\vec{i} - 12\vec{j} - 3\vec{k}}{\sqrt{(-4)^2 + (-12)^2 + (-3)^2}}$$

$$\vec{T} = T \times \hat{n}_{AB}$$

$$= 260 \times \frac{-4\vec{i} - 12\vec{j} - 3\vec{k}}{\sqrt{169}} \Rightarrow 20 \times (-4\vec{i} - 12\vec{j} - 3\vec{k})$$

$$\vec{T} = 80\vec{i} - 240\vec{j} - 60\vec{k}$$

$$\hat{n}_{AC} = \frac{\vec{AC}}{|\vec{AC}|} \Rightarrow \frac{-4\vec{i} - 12\vec{j} + 6\vec{k}}{\sqrt{(-4)^2 + (-12)^2 + (6)^2}}$$

$$\hat{n}_{AC} = \frac{-4\vec{i} - 12\vec{j} + 6\vec{k}}{14}$$

$$\vec{P} = P \times \hat{n}_{AB} \Rightarrow \frac{280}{14}(-4\vec{i} - 12\vec{j} + 6\vec{k})$$

$$\vec{P} = 20(-4\vec{i} - 12\vec{j} + 6\vec{k})$$

$$= -80\vec{i} - 240\vec{j} + 120\vec{k}$$

$$\hat{n}_{AD} = \frac{\vec{AD}}{|\vec{AD}|} \Rightarrow \frac{6\vec{i}-12\vec{j}+4\vec{k}}{\sqrt{(6)^2+(-12)^2+(4)^2}}$$

$$\Rightarrow \frac{6\vec{i}-12\vec{j}+4\vec{k}}{14}$$

$$\vec{F} = F \times \hat{n}_{AD}$$

$$= \frac{210}{14}(6\vec{i}-12\vec{j}+4\vec{k})$$

$$\vec{F} = 15 \times (6\vec{i}-12\vec{j}+4\vec{k})$$

$$\vec{F} = 90\vec{i}-180\vec{j}+60\vec{k}$$

$$\vec{R} = \vec{T}+\vec{P}+\vec{F}$$

$$= \cancel{90}\vec{i} - 660\vec{j}+120\vec{k}$$

$T_x = 90$

$P_y = -660$

$F_z = 120$

$$|\vec{R}| = \sqrt{(90)^2+(-660)^2+(120)^2}$$

$$\boxed{R = 676.830N}$$

$$\cos\theta_x = \frac{\vec{T_x}}{R} \Rightarrow \theta_x = \cos^{-1}\left(\frac{90}{676.830}\right)$$

$$\theta_x = 82.358^0$$

$$\theta_y = \cos^{-1}\left(\frac{-660}{676.830}\right)$$

$$\theta_y = 167.196^0$$

$$\theta_z = \cos^{-1}\left(\frac{120}{676.830}\right)$$

$$\theta_z = 79.787^0$$

$R_x = -70$

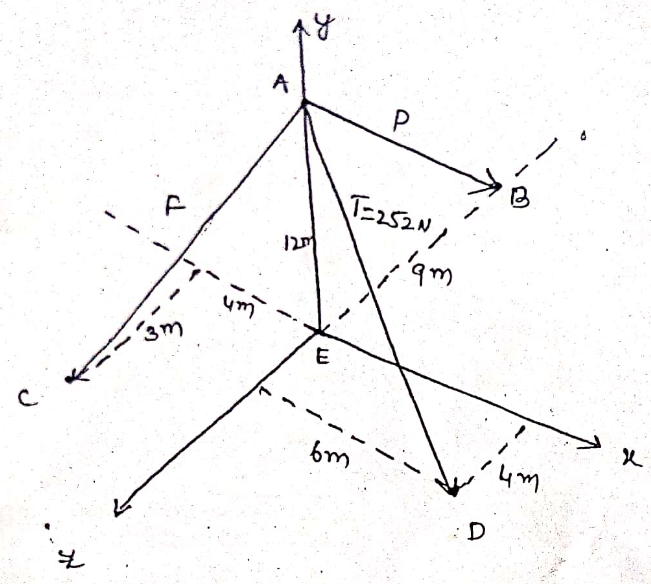
$R_y = -660$

$R_z = 120$

$$\cos\theta_x = \frac{R_x}{R}$$

$$\theta_x = -70$$

(15) In fig, a vertical boom AE is supported by guy wires from A to B, C and D. If the tensile load in AD = 252 N, find the forces in AC and AB so that the resultant force on A will be vertical



Sol force T on AD (T) = 252 N

the resultant force on A will be vertical

$\therefore \vec{R} = 0\vec{i} + R_V\vec{j} + 0\vec{k}$

coordinates of A (0, 12, 0) => $+12\vec{j}$

B (0, 0, −9) => $-9\vec{K}$

C (−4, 0, 3) => $-4\vec{i} + 3\vec{k}$

D (6, 0, 4) => $6\vec{i} + 4\vec{k}$

$\vec{AB} = \vec{EB} - \vec{EA}$

$= -12\vec{j} - 9\vec{K}$

$\vec{AD} = \vec{ED} - \vec{EA}$

$= 6\vec{i} - 12\vec{J} + 4\vec{k}$

$\vec{Ac} = \vec{EC} - \vec{EA}$

$= -4\vec{i} - 12\vec{j} + 3\vec{k}$

calculation of unit vector along AB, AC, AD

$$\bar{T} = T \times \frac{\vec{AD}}{|\vec{AD}|} \Rightarrow \bar{T} = 252 \times \frac{6\vec{i} - 12\vec{j} + 4\vec{k}}{\sqrt{(6)^2 + (-12)^2 + (4)^2}}$$

$$\bar{T} = 18\,(6\vec{i} - 12\vec{j} + 4\vec{k})$$

$$\bar{T} = 108\vec{i} - 216\vec{j} + 72\vec{k}$$

$$\vec{P} = P \times \frac{\vec{AB}}{|\vec{AB}|} \Rightarrow P \times \frac{(0\vec{i} - 12\vec{j} - 9k)}{\sqrt{(0)^2 + (-12)^2 + (-9)^2}}$$

$$= \quad \vec{P} = 0i - 0.8Pj - 0.6Pk$$

$$\vec{F} = F \times \frac{\bar{AC}}{|AC|} = F \times \frac{(-4\vec{i} - 12j + 3k)}{\sqrt{(-4)^2 + (-12)^2 + (3)^2}}$$

$$\bar{F} = 0.307Fi - 0.923Fj + 0.23Fk$$

$$\bar{R} = \bar{P} + \bar{T} + \bar{F}$$

$$0i + R_y j + 0k = (108 + 0 - 0.3F)i + (-216 - 0.8P - 0.923F)j + (72 - 0.6P + 0.23F)k$$

i: $0 = 108 - 0.3F$ $\quad F = 360N$

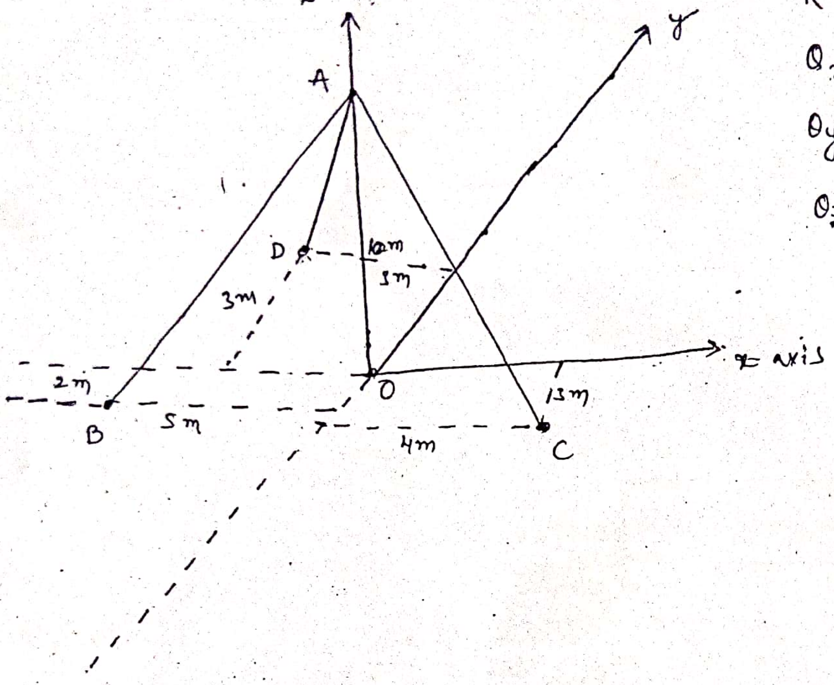
K: $0 = 72 - 0.6P + 0.25F$ $\quad P = 270N$

j: $R_y = -216 - 0.8P - 0.923F = -763N$

1. Determine the resultant of a system of three concurrent forces passing through the origin and points (10, −5, 8), (−5, 5, 7) and (6, −4, −3) respectively the respective magnitudes of the forces are 1500N, 2500N, and 2000N.

—Ans R = 2270.89N

(16) Determine the resultant of the Tension forces acting at point A of the transmission tower. The magnitudes of tensions along cables AB, AC and AD are respectively 1000N, 2000N and 1800N.



$R = 4337.38N$

$\theta_x = 92.93^0$

$\theta_y = 92.85^0$

$\theta_z = 175.91^0$

(17) The resultant of four forces which are acting at a point O as shown in fig. is along Y-axis. The magnitude of forces F_1, F_3, and F_4 are 10kN, 20kN and 40kN respectively. The angles made by 10kN, 20kN and 40kN with x-axis are 30^0, 90^0 and 120^0 respectively. Find the magnitude and direction of force F_2 if resultant is 72kN.

sol Given $F_1 = 10kN$, $\theta_1 = 30^0$

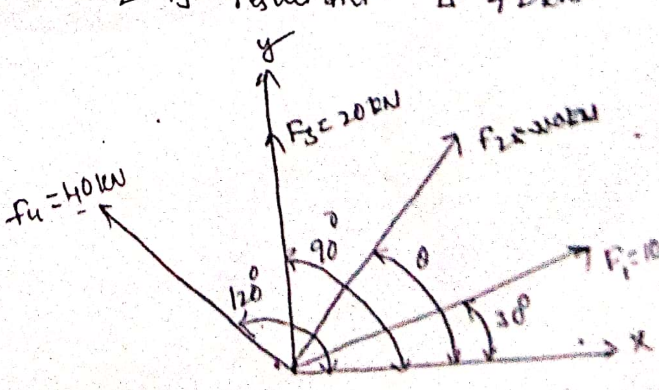
$f_2 = ?$, $\theta_2 = \theta$

$f_3 = 20kN$, $\theta_3 = 90^0$

$f_4 = 40kN$, $\theta_4 = 120^0$

Resultant $R = 72kN$

Resultant is along y-axis



Hence the algebraic sum of horizontal component should be zero and algebraic sum of vertical components should be equal to the resultant.

$\Sigma H = 0$ and $\Sigma V = R = 72\,kN$.

But $\Sigma H = F_1 \cos 30^0 + F_2 \cos \theta + F_3 \cos 90^0 + F_4 \cos 120^0$

$= 10 \times 0.866 + F_2 \cos \theta + 20 \times \cos 0 + 40 \times (-1/2)$

$= 8.66 + F_2 \cos \theta + 0 - 20$

$\Sigma H = F_2 \cos \theta - 11.34$

but $\Sigma H = 0$ (or) $F_2 \cos \theta - 11.34 = 0 \longrightarrow$ (i)

but $\Sigma V = R$

$\Sigma V = R = F_1 \sin \theta_1 + F_2 \sin \theta + F_3 \sin \theta_3 + F_4 \sin \theta_4$

$= 10 \times \sin 30^0 + F_2 \sin \theta + 20 \times \sin 90^0 + 40 \times \sin 120^0$

$72 = F_2 \sin \theta + 59.64$ ——

$F_2 \sin \theta = 72 - 59.64 = 12.36$

$F_2 \sin \theta = 12.36 \longrightarrow$ (ii)

Dividing eqn (ii) and eqn (i)

$$\frac{F_2 \sin \theta}{F_2 \cos \theta} = \frac{12.36}{11.34} \Rightarrow \theta = \tan^{-1}(1.0899)$$

$\theta = 47.46^0$

substituting the value of θ in eqn (ii), we get

$F_2 \times \sin(47.46^0) = 12.36$

$$\boxed{F_2 = 16.77\,kN}$$

Moment of a force :-

The production of a force and the perpendicular distance of the line of action of the force from a point is known as moment of the force about that point.

Let F = A force acting on a body

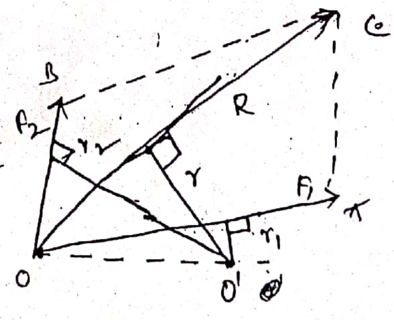
r = perpendicular distance from the point O

Moment of a force = $F \times r$

→ The tendency of this moment is to rotate the body in the clockwise direction about O. Hence this moment is called clockwise moment.

Principle of Moments (or) Varignon's principle :-

Principle of moments states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.



~~Proof~~

From fig shows two forces F_1 and F_2 acting at point O; these forces are represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OC which is that diagonal of parallelog. m OACB. Let O' is the point in the plane about which moments of F_1, F_2 and R are to be determined.

From point O', draw perpendiculars on OA, OC and OB

Let r_1 = ⊥ distance b/n F_1 and O'.

r = ⊥ distance b/n R and O'.

r_2 = ⊥ distance b/n F_2 and O'.

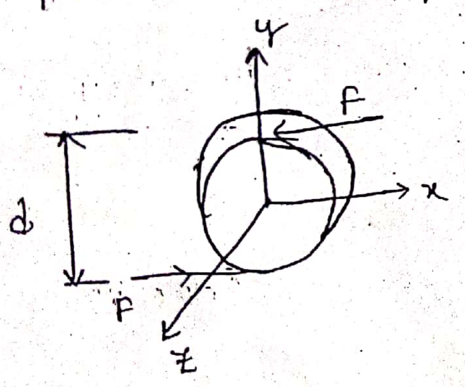
then according to varignon's principle; :-

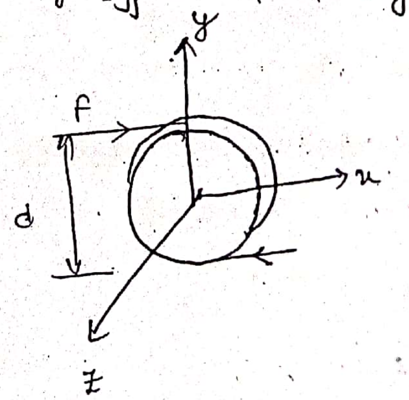
Moment of R about O' must be equal to algebraic sum of moments of F_1 and F_2 about O'.

$$R \times r = F_1 \times r + F_2 \times r_2$$

couple :-

Two non - collinear parallel forces of equal magnitude and in opposite direction form a couple. It is a special case of parallel forces which produces the rotary effect on a rigid body.





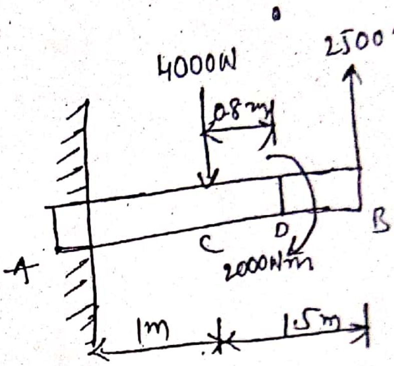
Moment of couple :-

The magnitude of rotation known as the moment of couple is the product of common magnitude of the two forces F and of the perpendicular distance d (arm of the couple) between the lines of action.

(15) Fig shows two vertical forces and a couple of moment 2000 Nm acting on a horizontal rod which is fixed at end A.

(i) Determine the resultant of the system.

(ii) Determine an equivalent system through A.



Given

Force at C = F_C = 4000N

B = F_B = 2500 N

Moment at D = 2000 Nm

AC = 1 m, BC = 1.5 m

CD = 0.8 m, BD = 0.7 m.

(i) Resultant of the system :-

Resultant force R = 4000 − 2500

= 1500 N acting downward

The point at which the resultant is acting is obtained by taking moments about point A.

Moment of force 4000N about point A = 4000 × 1

= 4000 Nm (clockwise)

Moment of force 2500N about point A

= 2500 (1+1.5)

= 6250 Nm (anti-clockwise)

Moment at D = 2000 Nm (clockwise)

Sum of all moment about A

= 4000 − 6250 + 2000

= −250 (anti-clockwise)

Let x = Distance of resultant force from A

$\therefore$ Moment of resultant force $= R \times x$

$= 1500x$

$\therefore 1500x = 250$

$$x = \frac{250}{1500}$$

$$\boxed{x = 0.166 m}$$

(ii) equivalent system through A

Single resultant force, $R = 1500 N$

Single moment through A $= 250 Nm$.

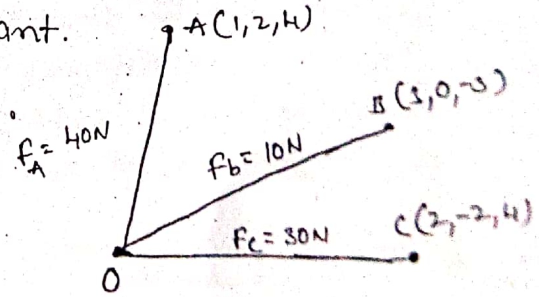
* The lines of actions of three forces concurrent at origin O passes respectively through point A, B, C having coordinates

$x_a = 1 \quad y_a = +2 \quad z_a = +4$

$x_b = +3 \quad y_b = 0 \quad z_b = -3$

$x_c = +2 \quad y_c = -2 \quad z_c = +4$

The magnitude of the forces are $F_A = 40N$, $F_b = 10 N$, and $F_c = 30N$. Find the magnitude and direction of their resultant.



Sol Given

$F_a = 40N, F_b = 10 N, F_c = 30 N$

$\vec{OA} = i + 2j + 4k$

$\vec{OB} = 3i + 0j - 3k$

$\vec{OC} = 2i + 2j + 4k$

$$\vec{F_a} = F_a \times \hat{n}_{\vec{OA}} \Rightarrow 40 \times \frac{i + 2j + 4k}{\sqrt{(1)^2 + (2)^2 + (4)^2}}$$

$$\vec{F_a} = 8.73\vec{i} + 17.46j + 34.92k.$$

$$\vec{F_b} = F_b \times \hat{n}_{\vec{OB}} = 10 \times \frac{3i + 0j - 3k}{\sqrt{(3)^2 + (0)^2 + (-3)^2}}$$

$$\vec{F_b} = 7.08i + 0j - 7.08k$$

$$\vec{F_c} = F_c \times \hat{n}_{\vec{OC}} = 30 \times \frac{2i - 2j + 4k}{\sqrt{(2)^2 + (-2)^2 + (4)^2}}$$

$$\vec{F_c} = 12.24i - 12.24j + 24.48k$$

Resultant vector $\vec{R} = \vec{F_a} + \vec{F_b} + \vec{F_c}$

$$\vec{R} = (8.73 + 7.08 + 12.24)i + (17.46 + 0 - 12.24)j + (34.92 - 7.08 + 24.48)k$$

$$\vec{R} = 28.05i + 5.22j + 53.32k$$

Magnitude of resultant

$$|\vec{R}| = R = \sqrt{(28.05)^2 + (5.22)^2 + (53.32)^2}$$

$$R = 59.59\,N$$

Direction of R_x, R_y, R_z

we know $\vec{R} = R_x i + R_y j + R_z k$

$$\theta_x = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\left(\frac{28.05}{59.59}\right)$$

$$\theta_x = 61.92^0$$

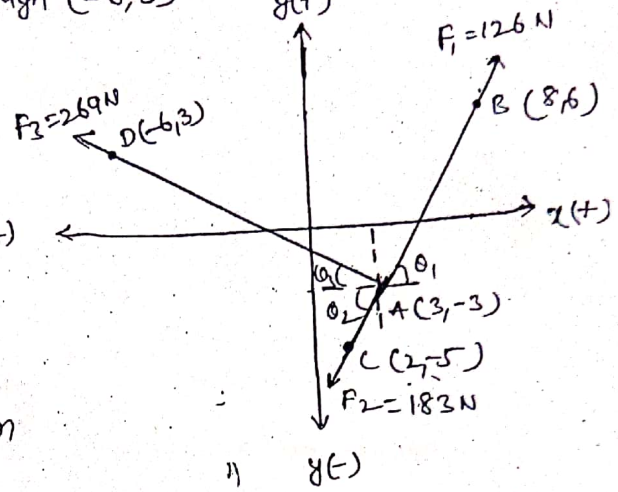
$$\theta_y = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\left(\frac{5.22}{59.59}\right)$$

$$\theta_y = 84.97^0$$

$$\theta_z = \cos^{-1}\left(\frac{R_z}{R}\right) = \cos^{-1}\left(\frac{53.31}{59.5}\right)$$

$$\theta_z = 26.52^0$$

* Determine the resultant of the three forces originating at point (3, -3) and passing through the point indicated: 126 N through (8,6), 183 N through (2,-5) and 269 N through (-6,3).



sol **To find**

(i) θ_1, θ_2, θ_3

(ii) Resultant of the three forces

(i) θ_1, θ_2, θ_3

$$\tan\theta = \left|\frac{y_2-y_1}{x_2-x_1}\right| \quad \text{— slope (m)}$$

$$\theta_1 = \tan^{-1}\left|\frac{6-(-3)}{8-3}\right|$$

$$\theta_1 = 60.95^0$$

$$\theta_2 = \tan^{-1}\left|\frac{y_3-y_1}{x_3-x_1}\right| \Rightarrow \theta_2 = \tan^{-1}\left|\frac{-5-(-3)}{2-3}\right|$$

$$\theta_2 = 63.44^0$$

$$\theta_3 = \tan^{-1}\left|\frac{y_4-y_1}{x_4-x_1}\right| \Rightarrow \tan^{-1}\left|\frac{3-(-3)}{-6-3}\right|$$

$$\theta_3 = 33.69^0$$

$$\Sigma F_x = 126\cos\theta_1 - 183\cos\theta_2 - 269\cos\theta_3$$

$$= 126\cos 60.93^0 - 183\cos 63.44^0 - 269\cos 33.69^0$$

$$\Sigma F_x = -244.47\,N$$

$$\Sigma F_y = 126\sin\theta_1 - 183\sin\theta_2 + 269\sin\theta_3$$

$$= 126\sin 60.93^0 - 183\sin 63.44^0 + 269\sin 33.69^0$$

$$\Sigma F_y = 95.68\,N$$

Magnitude of Resultant R

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(244.47)^2 + (95.68)^2}$$

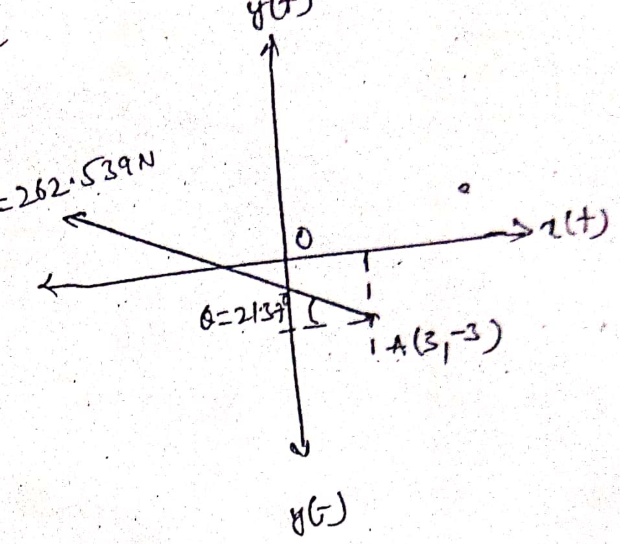
$$R = 262.53\ N$$

Direction of Resultant force

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{95.68}{244.47}\right)$$

$$\theta = 21.37^0$$



Types of parallel Forces :-

1. Like parallel forces :-

The parallel forces which are acting in the same direction, are known as like parallel forces.

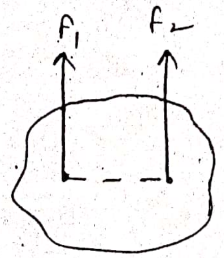
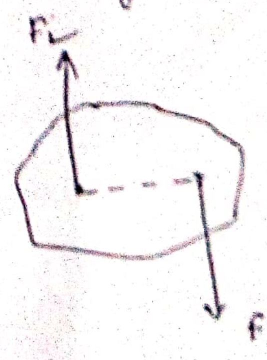


Fig shows F_1 and F_2 are two parallel forces. They are acting in the same direction. Hence they are called like parallel forces.

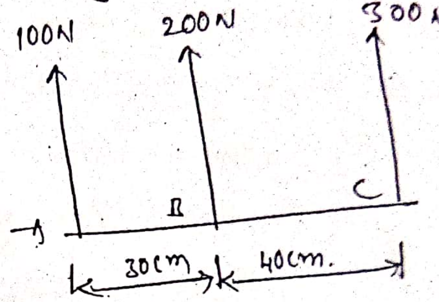
These forces may be ~~param~~ equal or un equal in magnitude

2. Un like parallel forces :-

The parallel forces which are acting in the opposite direction are known as un like parallel forces.



* three like parallel forces 100N, 200N, and 300N are acting at points A, B and C respectively on a straight line ABC as shown in fig. The distances are AB = 30 cm and BC = 40cm. Find the resultant and also the distance of the resultant from point A on line ABC



Sol Given

$f_A = 100N$ $AB = 30cm$

$f_B = 200N$ $BC = 40cm$

$f_C = 300N$

Resultant $R = F_A + f_B + F_C$

$= 100 + 200 + 300$

$R = 600\ N$

Let x = distance of Resultant acting from point A

Take moments about point A

$\Sigma M = 100 \times 0 + 200 \times 30 + 300 \times 70$

$= 27000$ N·cm (anticlockwise)

Moment of Resultant R about A

$= R \times x$

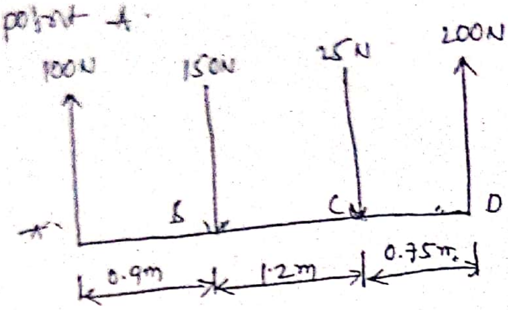
But Resultant moment $= \Sigma M$

$R \times x = 27000$

$x = \dfrac{27000}{600}$

$\boxed{x = 45cm}$

* Four parallel forces of magnitudes 100N, 150N, 25N and 200N are shown in fig. Determine the magnitude of the resultant and also the distance of the resultant from point A.

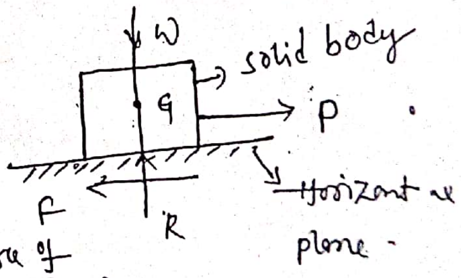


Sol Ans: 125N
x = 3.06m

Friction :-

When a solid body slides over a stationary solid body, a force is exerted at the surface of contact by the stationary body on the moving body. This force is called force of friction, and is always acting in the direction opposite to the direction of motion.

Limiting force of friction :-



W = weight of body acting through C.G

R = Normal Reaction.

P = Force acting on the body through C.G.

(F = force of friction)

If P is small, the body will not move as the force of friction acting on the body is more than P. But if magnitude of P goes on increasing, a stage comes when the solid body is on point of motion. At this stage, the force of friction acting on the body is called limiting force of friction. It is denoted by F.

Resolving the forces on the body vertically and horizontally

$R = W$

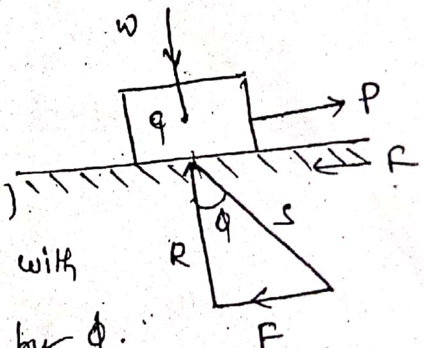
$P = F$

Co-efficient of friction :-

It is defined as the ratio of the limiting force of friction (F) to the normal reaction (R) between two bodies. It is denoted by μ.

$$\mu = \frac{\text{limiting force of friction}}{\text{Normal Reaction}} = \frac{F}{R}$$

Angle of Friction (ϕ)



It is defined as the angle made by the resultant of the normal reaction (R) and the limiting force of friction (F) with the normal Reaction (R). It is denoted by ϕ.

Let S = Resultant of the Normal reaction (R) and limiting force of friction

Angle of friction ϕ = Angle b/n R and S

$$\tan\phi = \frac{F}{R} = \frac{\mu R}{R}$$

$$\boxed{\phi = \tan^{-1}(\mu)}$$

Cone friction :-

It is defined as the right circular cone with vertex at the point of contact of the two bodies (or surfaces), axis in the direction of normal reaction (R) and semi vertical angle equal to angle of friction (ϕ).

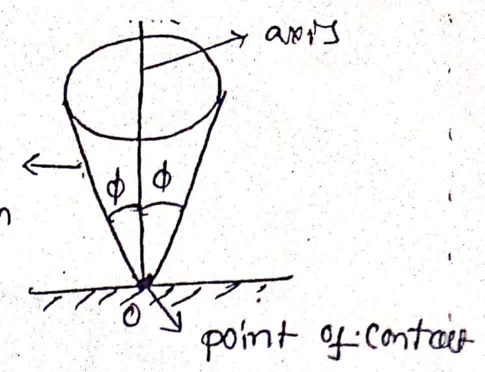


fig: cone friction.

O = point of contact b/n two bodies.

R = Normal Reaction and also axis of the cone friction.

ϕ = Angle of friction.

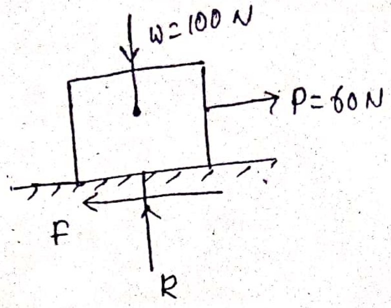
Angle of Repose :-

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Dry friction (or) coulomb friction :-

Dry friction develops when the unlubricated surface of two solids are in contact under a condition of sliding or a tendency to slide. The direction of the friction force always opposes the relative motion or impending motion. This type of friction is also known as coulomb friction.

* A body of weight 100N is placed on a rough horizontal plane. Determine the co-efficient of friction if a horizontal force of 60 N just causes the body to slide over the horizontal plane.



sol weight of body, $w = 100N$

Horizontal force $P = 60N$

$F = P = 60\ N$

$R = W = 100\ N$

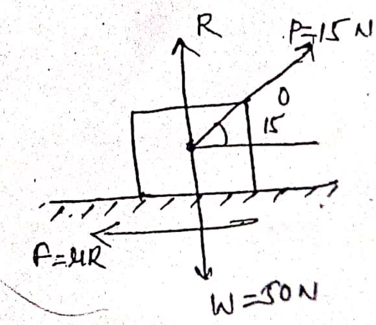
but $F = \mu R$

$$\mu = \frac{F}{R} = \frac{60}{100}$$

$$\boxed{\mu = 0.6}$$

* The force required to pull a body of wiegut 50N on a rough horizontal plane is 15N. Determine the co-efficient of friction if the force is applied at an angle of 15^0 with the horizontal.

sol Given



$W = 50N$

$P = 15N$

$\mu = ?$

$\theta = 15^0$

Resolving the ~~plane~~ forces along the plane

$= P\cos 15^0$

$= 15\times\cos 15^0$

Resolving the force Normal to the plane

$W = R + P\sin 15^0 \Rightarrow R = W - P\sin 15^0$

$R = 46.12N$

but $f = 15\cos 15^0$

$\mu \times R = 15\cos 15^0 \Rightarrow \mu\times 46.12 = 15\cos 15^0 \Rightarrow \mu = \frac{15\cos 15^0}{46.12} = 0.311$

* A pull of 20N, inclined at 25^0 to the horizontal plane, is required just to move a body placed on a rough horizontal plane. But the push required to move the body is 25N. If the push is inclined at 25^0 to the horizontal, find the weight of the body and co-efficient of friction.

Sol Given

pull required $P = 20N$,

Inclination of pull $\theta = 25^0$

push required to $P^1 = 25N$

Inclination of pull $\theta^1 = 25^0$

let W = weight of the body.

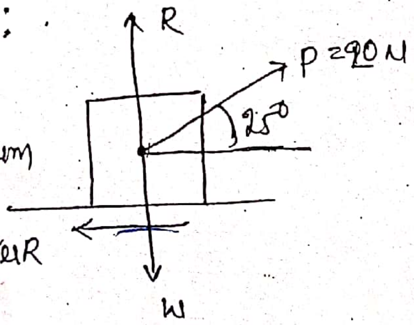
μ = co-efficient of friction.

R = Normal reaction when body is pulled

R^1 = Normal Reaction when body is pushed.

1st case :-

when body is pulled :



the body is in equilibrium under the following forces :

Resolving the forces along the plane

$F = P\cos 25^0$

but $F = \mu R \Rightarrow \mu R = P\cos 25^0$

$= 20 \times \cos 25^0$

$\mu R = 18.126 \longrightarrow (i)$

Resolving the forces Normal to the plane

$R + P\sin 25^0 = W$

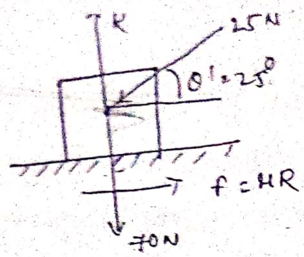
$\Rightarrow R = W - P\sin 25^0$

~~Subst~~ (ii) in (i)

$\mu (W - P\sin 25^0) = 18.126 \longrightarrow (ii)$

$\mu (W - 8.452) = 18.126 \longrightarrow (ii)$

2nd case :- when body is pushed.



Resolving forces along the plane,

$F = 25\cos 25^\circ$

but $F = \mu R \Rightarrow \mu R = 25\cos 25^\circ$

$\mu R = 22.657$ ⟶ (iii)

Resolving forces Normal to the plane

$R = W + 25\sin 25$

$= W + 10.565$

Subst R value in eqn (iii)

$\mu(W + 10.565) = 22.657$ ⟶ (iv)

Dividing eqn (ii) by eqn (iv)

$$\frac{\mu(W - 8.452)}{\mu(W + 10.565)} = \frac{18.126}{22.657}$$

$W = 84.547$.

substituting the value of W in eqn (ii.)

$\mu(84.547 - 8.452) = 18.126$

$\boxed{\mu = 0.238}$

* A block of weight W is placed on a rough horizontal plane surface as shown in fig and a force P is applied at an angle θ with the horizontal such that the block just tends to move. prove that the force P will be the least if the angle θ is equal to the angle of friction.

Sol Given

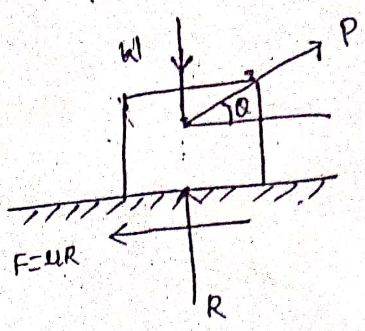
Weight of Block = W Let R = Normal Reaction.

force applied = P

Inclination of force = θ

μ = co-efficient of friction.

F = force of friction = μR



Resolving forces vertically,

$$R + P\sin\theta = W$$

$$R = W - P\sin\theta \longrightarrow (i)$$

Resolving forces horizontally.

$$P\cos\theta = F = \mu R \longrightarrow (ii)$$

Substituting the eqn (i) in eqn (ii)

$$P\cos\theta = \mu(W - P\sin\theta) \longrightarrow (iii)$$

but $\mu = \tan\phi$

where ϕ = Angle of friction.

substituting the value of μ in eqn (iii)

$$P\cos\theta = \tan\phi\,(W - P\sin\theta)$$

$$P\cos\theta = \frac{\sin\phi}{\cos\phi}(W - P\sin\theta)$$

$$P\cos\theta\cos\phi = W\sin\phi - P\sin\theta\sin\phi$$

$$P\cos\theta\cos\phi + P\sin\theta\sin\phi = W\sin\phi$$

$$P\cos(\theta - \phi) = W\sin\phi$$

$$P = \frac{W\sin\phi}{\cos(\theta-\phi)}$$

from above eqn force P will be max when $\cos(\theta-\phi)$ is minimum.

But $\cos(\theta-\phi)$ will be maximum if

$$\cos(\theta-\phi) = 1 \Rightarrow \theta - \phi = 0$$

$$\Rightarrow \theta = \phi$$

$\therefore P_{least} = W\sin\phi$ or $W\sin\theta$

* A body of wieght 500N is pulled up on inclined plane, by a force of 350N. The inclination of the plane is 30° to the horizontal and the force is applied parallel to the plane. Determine the co-efficient of friction.

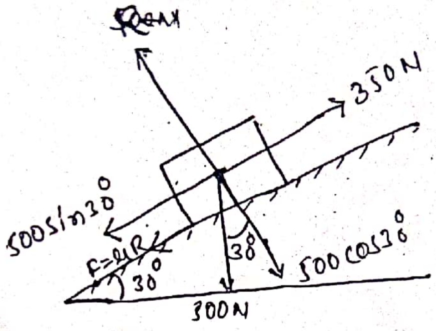


fig moving up

sol Given

$W = 500\ N$

$P = 350\ N$

Inclination $\alpha = 30^\circ$

μ = co-efficient of friction

R = Normal Reaction

F = force of friction = μR

Resolving the forces along the plane

$500 \sin 30^\circ + F = 350$

$500 \sin 30^\circ + \mu R = 350$ ——— (i)

Resolving the forces Normal to the plane

$R = 500 \cos 30^\circ = 500 \times 0.866$

$R = 433\ N$

Substituting R value in eqn (i)

$500 \sin 30^\circ + \mu \times 433 = 350$

$500 \times 0.5 + 433\mu = 350$

$433\mu = 350 - 250$

$\mu = \frac{100}{433}$

$\boxed{\mu = 0.23}$

* A body of weight 450N is pulled up along an inclined plane having inclination 30° to the horizontal at a steady speed. Find the force required if the co-efficient of friction between the body and the plane is 0.25 and force is applied parallel to the inclined plane. If the distance travelled by the body is 10m along the plane, find the workdone on the body

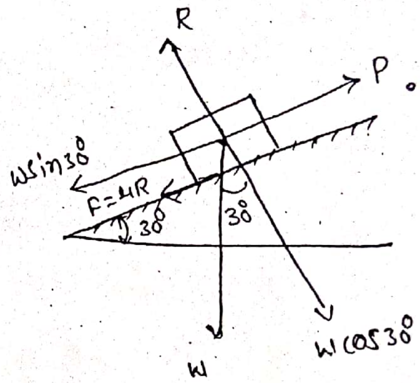
Sol given $W = 450\,N$

Inclination of plane $\alpha = 30^0$

$\mu = 0.25$

Distance travelled by body = 10m

let the force required = P.



Resolving the forces along the plane

$P = W\sin 30^0 + F$

$= 450 \times \sin 30^0 + 0.25 \times R$

$P = 225 + 0.25R$ —————— (i)

Resolving forces Normal to the plane

$R = W\cos 30^0 = 450 \times 0.866$

$R = 389.7\,N$

Substituting the value of R in eqn (i)

$P = 225 + 0.25 \times 389.7$

$\boxed{P = 322.425\,N}$

workdone on the body = Force x Distance travelled in the direction of force

$= 322.425 \times 10$

$= 3224.25\,Nm$

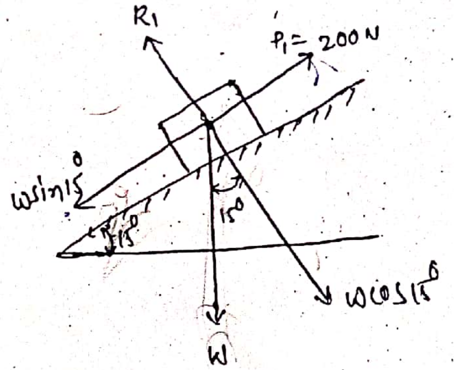
$= 3224.25\,J$

* An effort of 200N is required just to move a certain body up an inclined plane of angle 15°, the force acting parallel to the plane. If the angle of inclination of the plane is made 20°, the effort required, again applied parallel to the plane, is found to be 230 N. Find the weight of the body and co-efficient of friction.

Sol Given

Effort required when $P_1 = 200N$, $\theta_1 = 15^0$

Effort required when $P_2 = 230N$, $\theta_2 = 20^0$



1st case

$P_1 = 200N$, $\theta_1 = 15^0$

w = weight of body

μ = co-efficient of friction

R_1 = Normal Reaction

F_1 = force of friction

$= \mu R_1$

Resolving the forces normal to the plane,

$$R_1 = w\cos 15^0$$

Resolving the forces along the plane

$$w\sin 15^0 + F_1 = P_1$$

$$w\sin 15^0 + \mu R_1 = 200 \longrightarrow (i)$$

substituting the value of R_1 in eqn (i)

$$w\sin 15^0 + \mu(w\cos 15^0) = 200$$

$$w(\sin 15^0 + \mu\cos 15^0) = 200 \longrightarrow (ii)$$

2^{nd} case :-

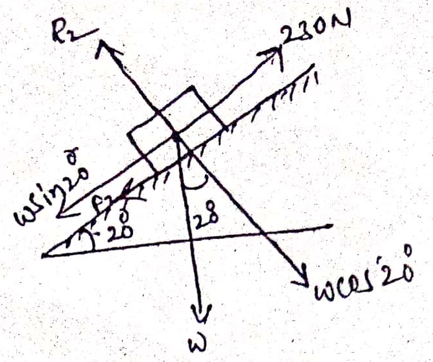
$P_2 = 230N$

$\theta_2 = 20^0$

R_2 = Normal Reaction.

F_2 = force of friction.

$= \mu R_2$



Resolving the forces along the plane,

$$w \sin 20^0 + F_2 = 230$$

$$w \sin 20^0 + \mu R_2 = 230 \longrightarrow (iii)$$

Resolving the forces normal to the plane

$$R_2 = w \cos 20^0$$

Substituting the value of R_2 in eqn (iii) we get

$$w \sin 20^0 + \mu \times w\cos 20^0 = 230 \longrightarrow (iv)$$

Dividing eqn (iv) by eqn (ii)

$$\frac{w(\sin 20^0 + \mu \cos 20^0)}{w(\sin 15^0 + \cos 15^0)} = \frac{230}{200} = 1.15$$

$$\boxed{\mu = 0.26}$$

weight of the body (w)

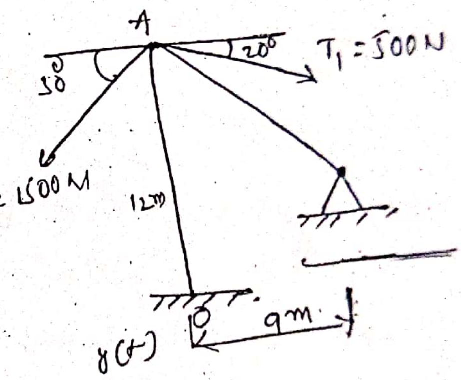
the weight of the body is obtained by substituting the value in eqn (iv).

$$w(\sin 20^0 + 0.26 \times \cos 20^0) = 230^0$$

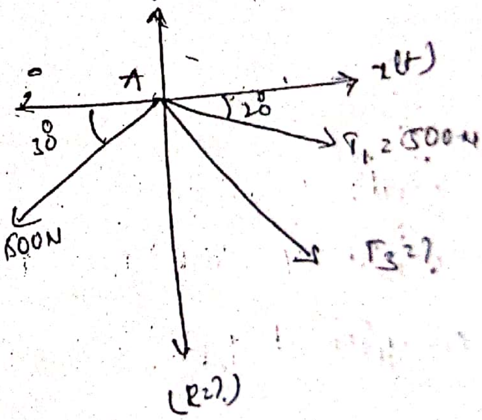
$$w(0.342 + 0.26 \times 0.9397) = 230$$

$$\boxed{w = 392.3N}$$

* The top end of vertical boot is connected by two cables having tension $T_1 = 500N$ and $T_2 = 1500N$ as shown in fig. The third cable AB is used as a guy wire. Determine the tension in cable AB if the resultant of the three concurrent forces acting at A is vertical. Also find the Resultant.



Sol) It is given that the resultant of the force system is vertical. It means that algebraic sum of all the force component in horizontal x-axis direction must be zero.



(i) $\Sigma F_x = 0$

$$500\cos 20^\circ - 1500\cos 30^\circ + T_3 \sin 36.87^\circ = 0$$

$$T_3 = 1381.98N$$

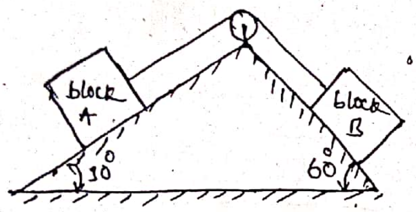
(ii) ∵ Resultant is vertical

$$\therefore R = \Sigma F_y$$

$$R = -500\sin 20^\circ - 1500\sin 30^\circ - 1381.98\cos 36.87^\circ$$

$$\boxed{R = -2026.59\,N}$$

* Two blocks A and B are placed on inclined planes as shown in fig. The block A weighs 1000 N. Determine minimum weight of the block B for maintaining the equilibrium of the system. Assume that the blocks are connected by an inextensible string passing over a frictionless pulley. Co-efficient of friction μ_A between the block A and the plane is 0.25. Assume the same value for μ_B.



Sol. Given

Weight of block A = 1000 N

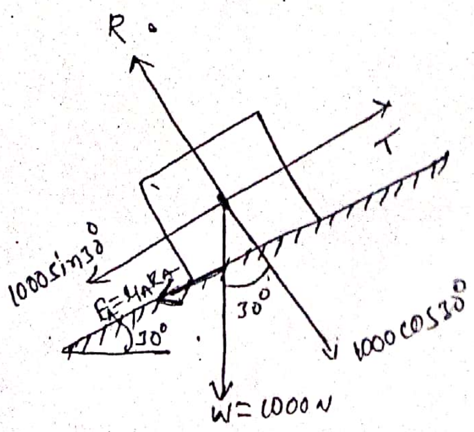
weight of block B = ?

Co-efficient between block A and the plane $\mu_A = 0.25$

$\mu_B = 0.25$

for minimum weight of block B in limiting equilibrium condition, tendency of block B will be to impend downwards i.e impending motion of block A will be upward.

(i) Consider the free body diagram of block A ; —



$\Sigma F_y = 0$

$R_A - W\cos 30^\circ = 0$

$R_A = W\cos 30^\circ \Rightarrow 1000 \times \cos 30^\circ$

$$\boxed{R_A = 866.03\,N}$$

$\Sigma F_x = 0$

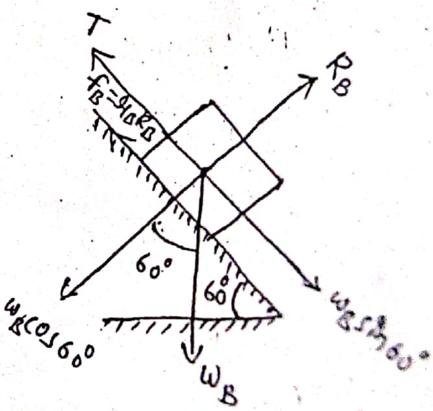
$T - W\sin 30^\circ - F_A = 0$ $\quad (\because f = \mu_A R_A)$

$T - 1000 \times \sin 30^\circ - 0.25 \times 866.03 = 0$

$T = 1000 \sin 30^\circ + 0.25 \times 866.03$

$$\boxed{T = 716.51\,N}$$

(ii) consider the free body diagram of block B



$\Sigma F_y = 0$

$R_B - W_B \cos 60^\circ = 0$

$R_B = W_B \times \cos 60^\circ$

$R_B = 0.5 \times W_B$

$\Sigma F_x = 0$

$T + F_B - W_B \sin 60^\circ = 0$

$T + \mu_B R_B - W_B \sin 60^\circ$

$716.51 + 0.25 \times 0.5 W_B - W_B \sin 60^\circ = 0$

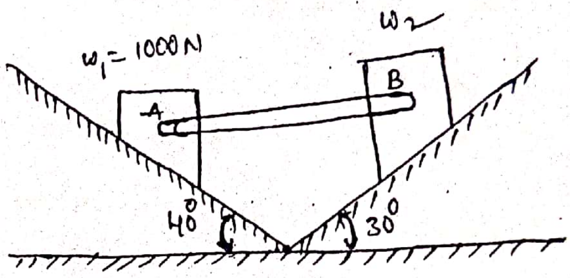
$$W_B = \frac{716.51}{\sin 60^\circ - 0.25 \times 0.5}$$

$W_{B(min)} = 966.92\,N$

* Two blocks w_1 and w_2 resting on two inclined planes are connected by a horizontal bar AB as shown in fig. If w_1 equal 1000 N, determine the maximum value of w_2 for which the equilibrium can exist. the angle of limiting friction is 20° at all rubbing surf is

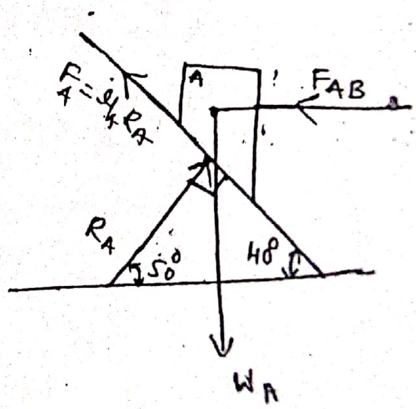
sol Given

$W_1 = 1000$ N



For maximum weight of the block B in limiting equilibrium condition, tendency of block B will be to impend upwards.

(i) consider the free body diagram of block A :-



$\Sigma F_y = 0$

$R_A \sin 50^\circ + \mu R_A \sin 40^\circ - 1000 = 0$

$R_A = \dfrac{1000}{\sin 50^\circ + \tan 20^\circ \times \sin 40^\circ}$

$\boxed{R_A = 1000 \text{ N}}$

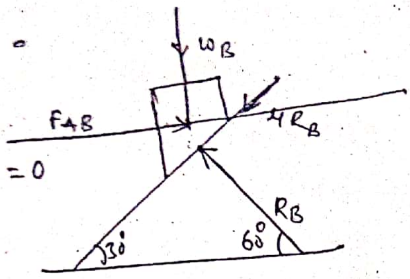
$\Sigma F_x = 0$

$R_A \cos 50^\circ - \mu R_A \cos 40^\circ - F_{AB} = 0$

$F_{AB} = 1000 \times \cos 50^\circ - \tan 20^\circ \times 1000 \cos 40^\circ$

$\boxed{F_{AB} = 363.97 \text{ N}}$

Consider the free body diagram of block B. :-



$\Sigma F_x = 0$

$F_{AB} - \mu R_B \cos 30^\circ - R_B \cos 60^\circ = 0$

$363.97 - \tan 20^\circ R_B \cos 60^\circ - R_B \cos 60^\circ = 0$

$$R_B = \frac{363.97}{(\cos 60^\circ + \tan 20^\circ \cos 30^\circ)}$$

$R_B = 448.48 N$

$\Sigma F_y = 0$

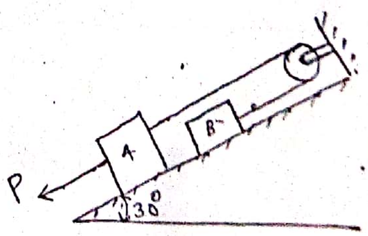
$R_B \sin 60^\circ - \mu R_B \sin 30^\circ - W_B = 0$

$W_B = 446.48 \sin 60^\circ - \tan 20^\circ \times 446.48 \sin 30^\circ$

$$\boxed{W_{B\,(max)} = 305.41 N}$$

* Determine the force P to cause motion to impend. Take masses of block, A and B as 9 kg and 4 kg respectively and the co-efficient of sliding friction as 0.25. The force P and rope are parallel to the inclined plane as shown in fig. Assume pulley to be frictionless.

Given $m_A = 9 kg$, $m_B = 4 kg$, $\mu = 0.25$



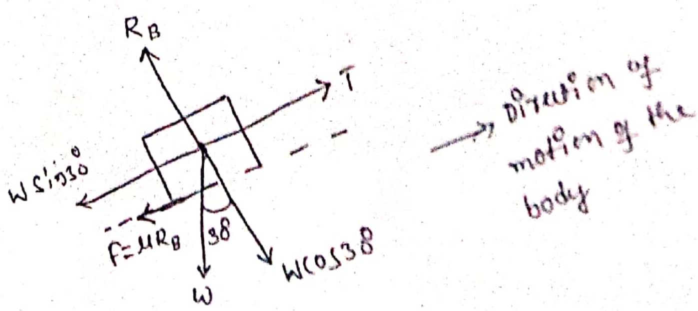
Sol (i) Consider the F.B.D of block B

$\Sigma F_y = 0$

$R_B = W_B \cos 30^\circ$

$R_B = 4 \times 9.81 \times \cos 30^\circ$

$R_B = 33.98 N$



$\Sigma F_x = 0$

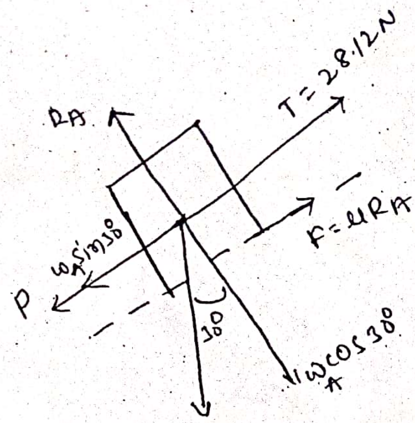
$T = F + W_B \sin 30^\circ$

$T = \mu \times R_B + W_B \sin 30^\circ$

$= 0.25 \times 33.98 + 4 \times 9.81 \times \sin 30^\circ$

$\boxed{T = 28.12\,N}$

(ii) Consider the F.B.D of block A



$\Sigma F_y = 0$

$R_A = W_A \cos 30^\circ$

$= 9 \times 9.81 \times \cos 30^\circ$

$\boxed{R_A = 76.46\,N}$

$\Sigma F_x = 0$

$T + F = P + W_A \sin 30^\circ$

$T + \mu R_A = P + W_A \sin 30^\circ$

$28.12 \times 0.25 \times 76.46 = P + 9 \times 9.81 \times \sin 30^\circ$

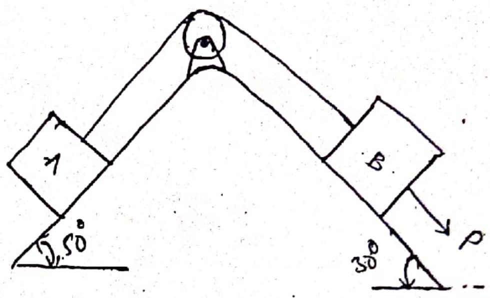
$\boxed{P = 3.09\,N}$

* Two blocks A and B of weight 500N and 750N respectively are connected by a cord that passes over a frictionless pulley as shown in fig. The co-efficient of friction between the block A and the inclined plane is 0.4 and that between the block B and the inclined plane is 0.3. Determine the force P to be applied to block B to produce the impending motion of block B down the plane.

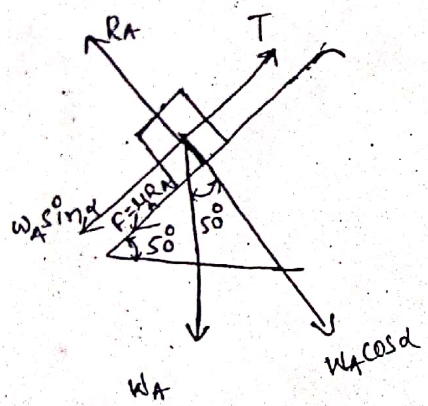
<u>sol</u> <u>Given</u>

$W_A = 500N$, $W_B = 750N$

$\mu_A = 0.4$, $\mu_B = 0.3$



(i) consider the F.B.D of block A



$\Sigma F_y = 0$

$R_A = W_A \cos\alpha \Rightarrow 500 \times \cos 50^\circ$

$\boxed{R_A = 321.393N}$

$\Sigma F_x = 0$

$T = F + W_A \sin\alpha$

$= \mu_A R_A + W_A \sin\alpha$

$T = 0.4 \times 321.393 + 500 \times \sin 50^\circ$

$\boxed{T = 511.579 N}$

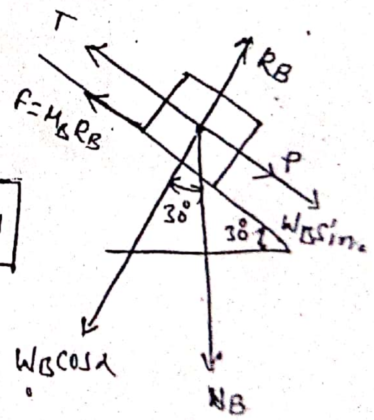
(ii) consider the F.B.D of block B

$\Sigma F_y = 0$

$R_B = W_B \cos\alpha$

$= 750 \times \cos 30^\circ$

$\boxed{R_B = 649.519 N}$



$\Sigma F_x = 0$

$P + W_B \sin\alpha = T + \mu_B R_B$

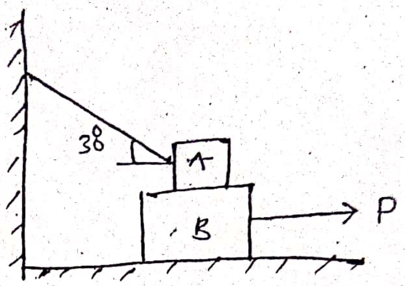
$P + 750 \times \sin 30^\circ = 511.579 + 0.3 \times 649.5$

$\boxed{P = 331.44 N}$

* Find force P required to pull block B shown in fig. Co-efficient of friction between A and B is 0.3 and between B and floor is 0.25. Weights of A = 20 kg and B = 30 kg.

Sol

(i) Consider the F.B.D of Block A



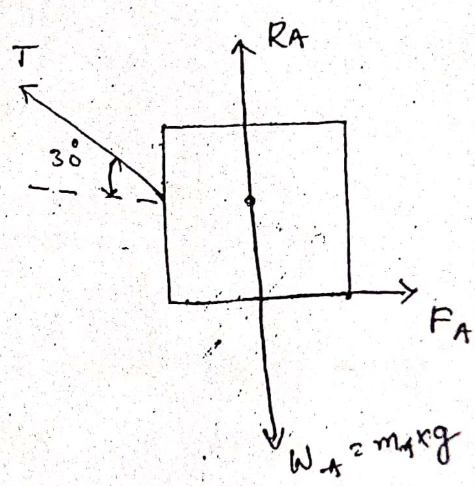
Resolving the forces vertically

$R_A + T\sin 30° = m_A \times g$

$R_A = 20 \times 9.81 - T \times \sin 30°$

Resolving the forces horizontally

F.B.D of Block A



$T\cos 30° = F_A$

but $F_A = \mu_A R_A$

$T\cos 30° = 0.3 \times 20 \times 9.81 - T \times \sin 30°$

$\boxed{T = 57.93\ N}$

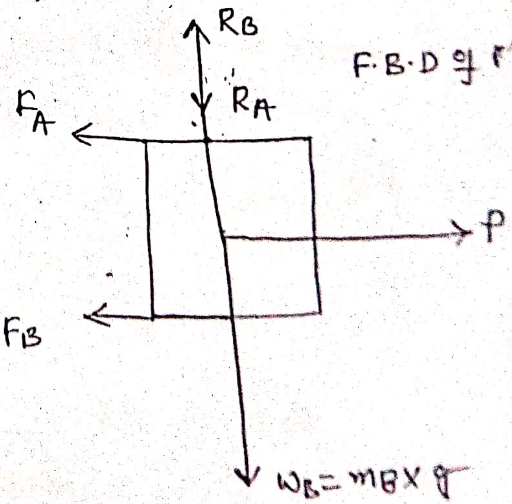
$\boxed{R_A = 167.235\ N}$

(ii) Consider the F.B.D of Block B

$R_B = 30 \times 9.81 + 167.235$

$\boxed{R_B = 461.535\ N}$

F.B.D of blocks

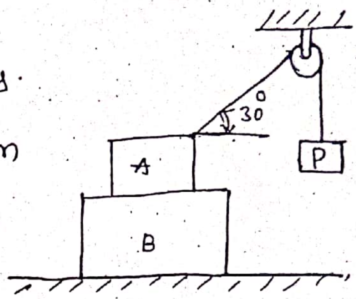


$P = \mu_A R_A + \mu_B R_B$

$= 0.3 \times 167.235 + 0.25 \times 461.535$

$\boxed{P = 165.55\ N}$

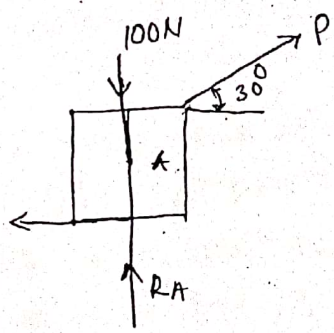
Two blocks $A = 100N$ and $B = 150N$ are resting on ground as shown in fig. co-efficient of friction between ground and block B is 0.10 and that between block B and A is 0.30. Find the minimum value of weight P in the pan so that motion starts. Find whether B is stationary w.r.t ground and A moves or B is stationary w.r.t A.



Sol: Case-I : B is stationary w.r.t ground and A moves.

consider given A is in limiting equilibrium which means block A moves over the surface of B.

Consider block A F.B.D



$R_A + P\sin 30° = 100$

$R_A = 100 - P\sin 30°$

$P\cos 30° = F_A$

$P\cos 30° = \mu_A R_A$

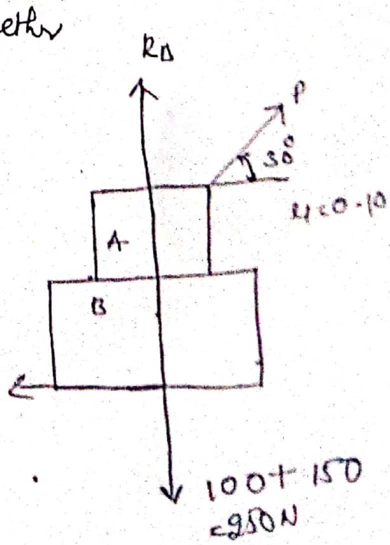
$P\cos 30° = 0.3 \times (100 - P\sin 30°)$

$\boxed{P = 29.53N}$

Case-II Block B is stationary w.r.t A

consider both blocks A and B moving together

consider the F.B.D of A and B together



$R_B + P\sin 30° = 250$

$R_B = 250 - P\sin 30°$

$P\cos 30° = \mu \times R_B$

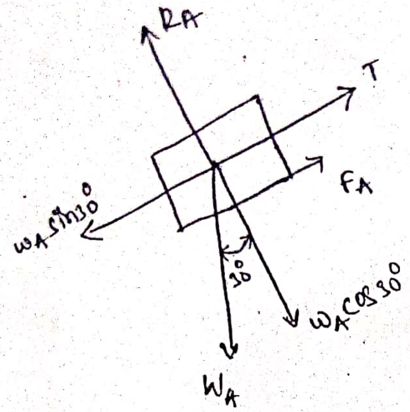
$P\cos 30° = \mu \times 250 - P\sin 30°$

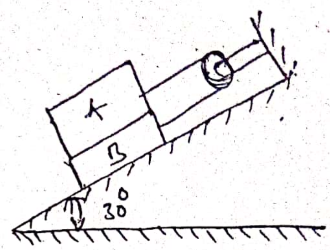
$P \times \cos 30° = 0.10 \times 250 - P\sin 30°$

$\boxed{P = 27.29N}$

* Block A has mass of 20 kg and block B has a mass of 10 kg. Knowing that the co-efficient of static friction is 0.15, between the two blocks and zero between block B and the slope, find the magnitude of frictional force between the two masses. What is the force in the string tying the blocks.

Sol: Consider the F.B.D of Block A





$R_A = w_A \cos 30^0$

$= m_A \times g \times \cos 30^0$

$R_A = 20 \times 9.81 \times \cos 30^0$

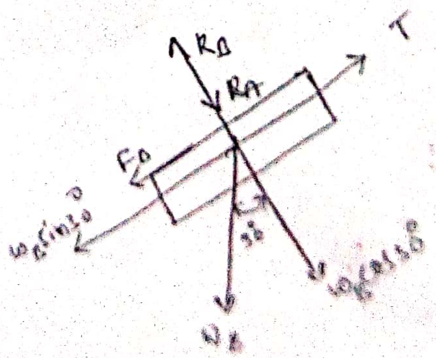
$\boxed{R_A = 169.91N}$

$T + F_A = w_A \sin 30^0$

$T + F_A = 20 \times 9.81 \times \sin 30^0$

$T + F_A = 98.1$ —— (i)

Consider the F.B.D of Block B



$R_B = R_A + w_B \cos 30^0$

$= 169.91 + 10 \times 9.81 \times \cos 30^0$

$R_B = 254.87N$

$T = F_B + W_B \sin 30^0$

$T - F_B = 10 \times 9.81 \times \sin 30^0$

$T - F_A = 49.05$ —— (ii)

Adding eq^n (i) & (ii)

$2T = 147.15$

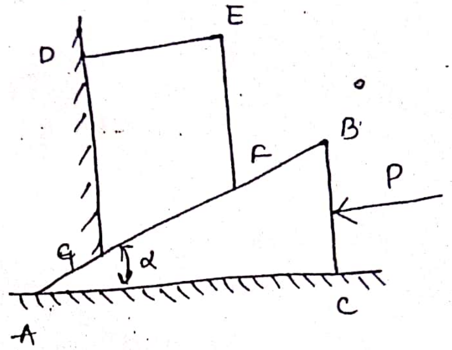
$\boxed{T = 73.56N}$ and

From eq^n i

$\boxed{F = 24.53N}$

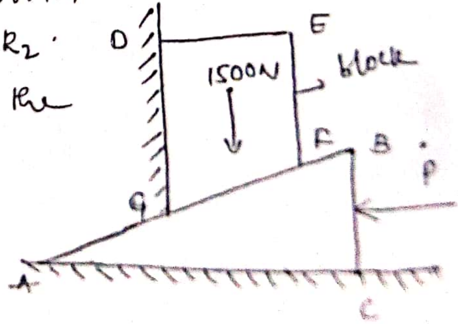
Wedge Friction :-

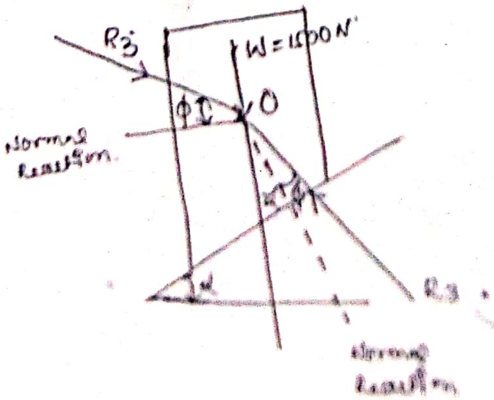
A wedge is a piece of metal or wood which is usually of a triangular or trapezoidal in cross section. It is used for either lifting loads or used for slight adjustments in the position of a body i.e for tightening fits, Keys for shafts.



* A block over lying a 10° wedge on a horizontal floor and leaning against a vertical wall and weighing 1500N is to be raised by applying a horizontal force to the wedge. Assuming co-efficient of friction between all the surfaces in contact to be 0.3, determine the minimum horizontal force to be ~~raise~~ applied to raise the block.

Sol: The block is shown in fig is in equilibrium under the action of three forces W, R_3 and R_2. These forces when produced are meeting at the point O.





$\mu = \tan\phi$

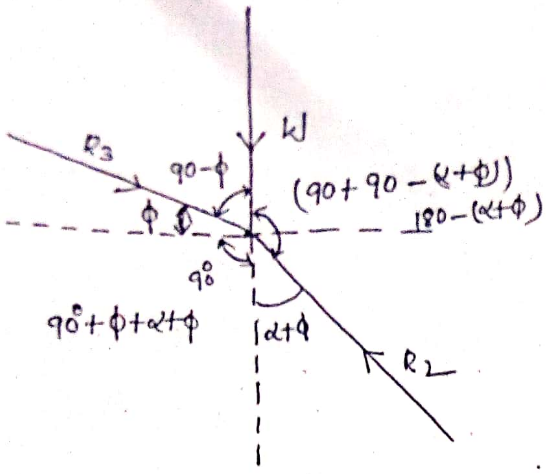
$\phi = \tan^{-1}(\mu)$

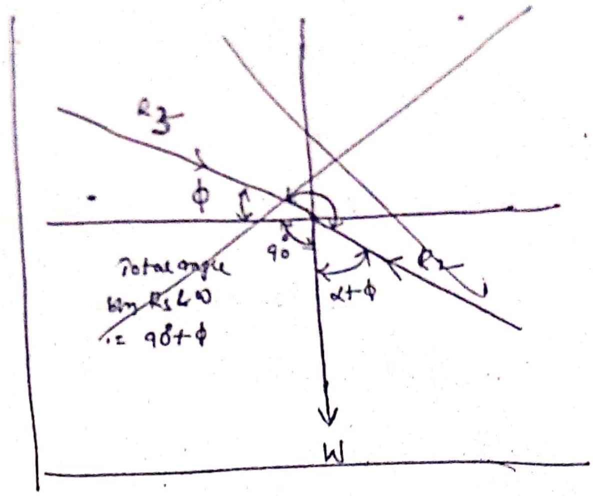
$= \tan^{-1}(0.3)$

$= 0.291^\circ$

$= 0.291 \times \left(\frac{180}{\pi}\right)$

$\phi = 16^\circ 41'$





$$\frac{W}{\sin(90^\circ+\phi+\alpha+\phi)} = \frac{R_2}{\sin(90-\phi)} = \frac{R_3}{\sin(180-(\alpha+\phi))}$$

$$\frac{1500}{\sin(16^\circ 42'+90+10^\circ+16^\circ 42')} = \frac{R_2}{\sin(90-16^\circ 42')} = \frac{R_3}{\sin(180-(10+16^\circ 42'))}$$

$$\frac{1500}{\sin(133^\circ 24')} = \frac{R_3}{\sin(153^\circ 18')} = \frac{R_2}{\sin(72^\circ 18')}$$

$$\frac{1500}{0.7265} = \frac{R_3}{0.4493} = \frac{R_2}{0.9578}$$

$$R_3 = 1500 \times \frac{0.4493}{0.7265}$$

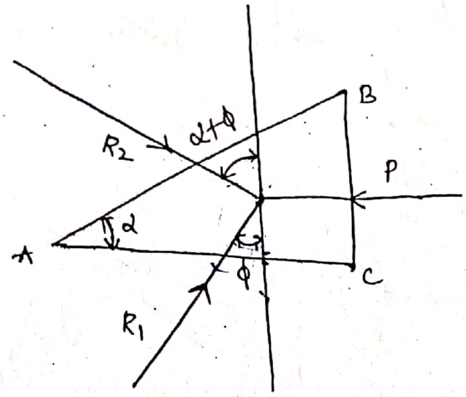
$$\boxed{R_3 = 927.66N}$$

$$R_2 = 1500 \times \frac{0.9578}{0.7265}$$

$$\boxed{R_2 = 1977.56N}$$

Now consider the equilibrium of the wedge as shown in fig. three forces R_1, R_2 and ~~R_3~~ P when produced are meeting at the point L.

Applying Lami's theorem, we get



Angle between P & R_2
$= 90+(\alpha+\phi)$

Angle between R_2 & R_1
$= [180-\phi-(\alpha+\phi)]$

Angle between R_1 & P
$= 90+\phi$

$$\frac{R_1}{\sin[90+(\alpha+\phi)]} = \frac{R_2}{\sin(90+\phi)} = \frac{P}{\sin[180-\phi-(\alpha+\phi)]}$$

$$\frac{R_1}{\sin[90+(10+16^\circ42')]} = \frac{R_2}{\sin(90+16^\circ42')} = \frac{P}{\sin[180-16^\circ42'-(10+16^\circ42')]}$$

from the last two parts, we get

$$\frac{R_2}{\sin(106^\circ42')} = \frac{P}{\sin(136^\circ36')}$$

$$P = R_2 \times \frac{\sin(136^\circ36')}{\sin(106^\circ42')}$$

$$\boxed{P = 1418.44\,N}$$

1. Laws of Mechanics
2. Resoultion of forces
3. Friction
   - wedge friction
   - ladder friction