(b) *With the aid of a compass* (fig. 5-35).

(i) Draw a line *AB* equal to the given length.

(ii) At *A*, draw a line *AE* perpendicular to *AB*. (Refer problem 5-3, Method III, fig. 5-5).

(iii) With centre *A* and radius *AB*, draw an arc cutting *AE* at *D*.

(iv) With centres *B* and *D* and the same radius, draw arcs intersecting at *C*.

(v) Draw lines joining *C* with *B* and *D*.

Then *ABCD* is the required square.

5-12. TO CONSTRUCT REGULAR POLYGONS

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 9 for the following problem.

Problem 5-26. *To construct a regular polygon, given the length of its side.*

Let the number of sides of the polygon be seven (i.e. heptagon).

Method I: (fig. 5-36 and fig. 5-37):

(i) Draw a line *AB* equal to the given length.

(ii) With centre *A* and radius *AB*, draw a semi-circle *BP*.

(iii) With a divider, divide the semi-circle into seven equal parts (same as the number of sides). Number the division-points as 1, 2, etc. starting from *P*.

(iv) Draw a line joining *A* with the second division-point 2.

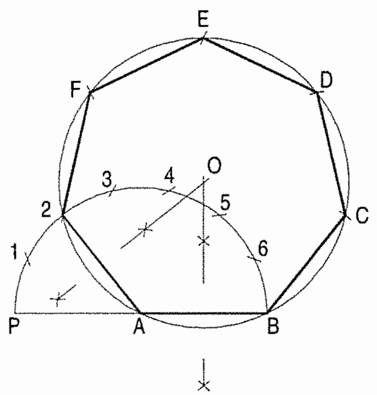

FIG. 5-36

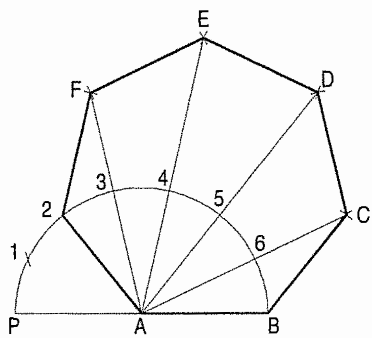

FIG. 5-37

(a) *Inscribe circle method* (fig. 5-36).

(i) Draw perpendicular bisectors of *A*2 and *AB* intersecting each other at *O*.

(ii) With centre *O* and radius *OA*, describe a circle.

(iii) With radius *AB* and starting from *B*, cut the circle at points, *C*, *D*.....2.

(iv) Draw lines *BC*, *CD* etc. thus completing the required heptagon.

(b) *Arc method* (fig. 5-37).

(i) With centre *B* and radius *AB*, draw an arc cutting the line *A*6-produced at *C*.

(ii) With centre *C* and the same radius, draw an arc cutting the line *A*5-produced at *D*.

(iii) Find points *E* and *F* in the same manner.

(iv) Draw lines *BC*, *CD* etc. and complete the heptagon.

Method II: General method for drawing any polygon (fig. 5-38):

(i) Draw a line *AB* equal to the given length.

(ii) At *B*, draw a line *BP* perpendicular and equal to *AB*.

(iii) Draw a line joining *A* with *P*.

(iv) With centre *B* and radius *AB*, draw the quadrant *AP*.

(v) Draw the perpendicular bisector of *AB* to intersect the straight line *AP* in 4 and the arc *AP* in 6.

(a) A *square* of a side equal to *AB* can be inscribed in the circle drawn with centre 4 and radius *A*4.

(b) A regular *hexagon* of a side equal to *AB* can be inscribed in the circle drawn with centre 6 and radius *A*6.

(c) The mid-point 5 of the line 4-6 is the centre of the circle of the radius *A*5 in which a regular pentagon of a side equal to *AB* can be inscribed.

(d) To locate centre 7 for the regular *heptagon* of side *AB*, step-off a division 6-7 equal to the division 5-6.

(i) With centre 7 and radius equal to *A*7, draw a circle.

(ii) Starting from *B*, cut it in seven equal divisions with radius equal to *AB*.

(iii) Draw lines *BC*, *CD* etc. and complete the heptagon.

Regular polygons of any number of sides can be drawn by this method.

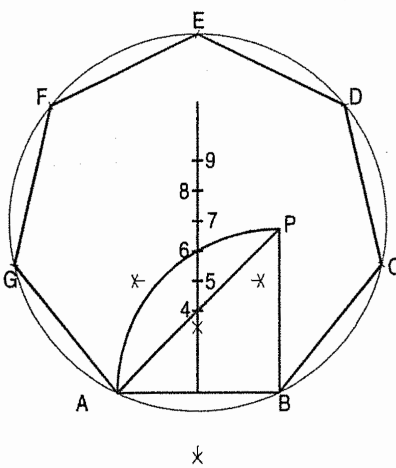

FIG. 5-38

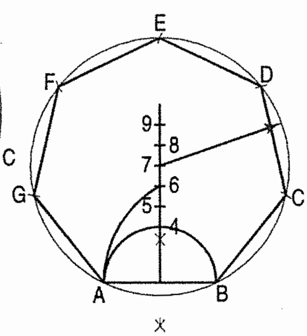

FIG. 5-39

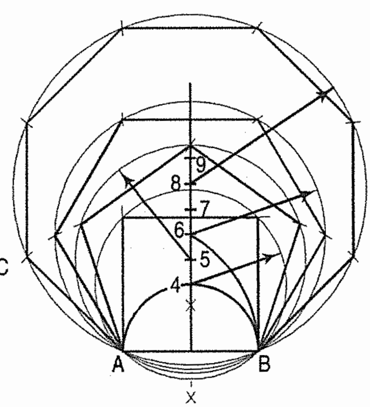

FIG. 5-40

Alternative method (fig. 5-39 and fig. 5-40):

(i) On *AB* as diameter, describe a semi-circle.

(ii) With either *A* or *B* as centre and *AB* as radius, describe an arc on the same side as the semi-circle.

(iii) Draw a perpendicular bisector of *AB* cutting the semi-circle at point 4 and the arc at point 6.

(iv) Obtain points 5, 7, 8 etc. as explained in method II.

Fig. 5-40 shows *a square, a regular pentagon, a regular hexagon and a regular octagon*, all constructed on *AB* as a common side.

5-13. SPECIAL METHODS OF DRAWING REGULAR POLYGONS

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This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 10 for the following problem.

Problem 5-27. *To construct a pentagon, length of a side given.*

Method I: (fig. 5-41):

(i) Draw a line *AB* equal to the given length.

(ii) With centre *A* and radius *AB*, describe a circle-1.

(iii) With centre *B* and the same radius, describe a circle-2 cuttting circle-1 at *C* and *D*.

(iv) With centre *C* and the same radius, draw an arc to cut circle-1 and circle-2 at *E* and *F* respectively.

(v) Draw a perpendicular bisector of the line *AB* to cut the arc *EF* at *G*.

(vi) Draw a line *EG* and produce it to cut circle-2 at *P*.

(vii) Draw a line *FG* and produce it to cut circle-1 at *R*.

(viii) With *P* and *R* as centres and *AB* as radius, draw arcs intersecting each other at *Q*.

(ix) Draw lines *BP*, *PQ*, *QR* and *RA*, thus completing the pentagon.

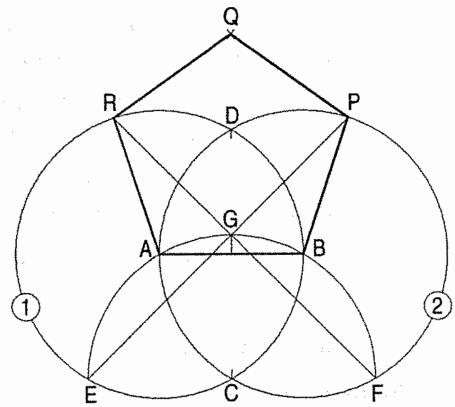

FIG. 5-41

Method II: (fig. 5-42):

(i) Draw a line *AB* equal to the given length.

(ii) Bisect *AB* in a point *C*.

(iii) Draw a line *BD* perpendicular and equal to *AB*.

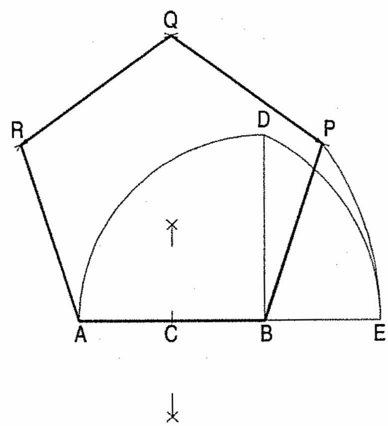

FIG. 5-42

(iv) With centre C and radius CD, draw an arc to intersect the line *AB*-produced at *E*.

(v) Then *AE* is the length of the diagonal of the pentagon.

(vi) Therefore, with centre *A* and radius *AB*, draw an arc intersecting the arc drawn with centre *B* and radius *AE* at *R*.

(vii) Again with centre *A* and radius *AE*, draw an arc intersecting the arc drawn with centre *B* and radius *AB* at *P*.

(viii) With centres *A* and *B* and radius *AE*, draw arcs intersecting each other at *Q*.

(ix) Draw lines *BP*, *PQ*, *QR* and *RA*, thus completing the pentagon.

Problem 5-28. *To construct a hexagon, length of a side given* (fig. 5-43 and fig. 5-44).

(a) *With T-square and 30°-60° set-square only* (fig. 5-43).

(i) Draw a line *AB* equal to the given length.

(ii) From *A*, draw lines *A*1 and *A*2 making 60° and 120° angles respectively with *AB*.

(iii) From *B*, draw lines *B*3 and *B*4 making 60° and 120° angles respectively with *AB*.

(iv) From *O* the point of intersection of *A*1 and *B*3, draw a line parallel to *AB* and intersecting *A*2 at *F* and *B*4 at *C*.

(v) From *F*, draw a line parallel to *BC* and intersecting *B*3 at *E*.

(vi) From *C*, draw a line parallel to *AF* and intersecting *A*1 at *D*.

(vii) Draw a line joining *E* and *D*.

Then *ABCDEF* is the required hexagon.

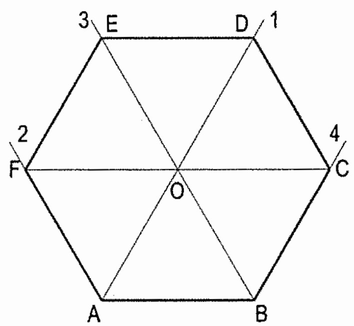

FIG. 5-43

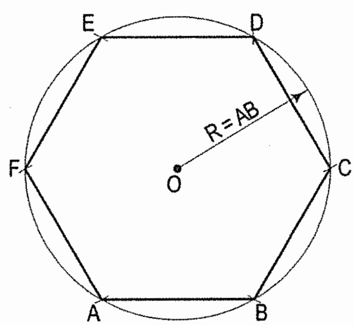

FIG. 5-44

(b) *With the aid of a compass* (fig. 5-44).

(i) With Point *O* as centre, draw a circle of radius equal to the given side length of the required polygon.

(ii) Draw a horizontal line passing through the centre of the circle and cutting the circle at opposite ends, say at points *F* and *C*. Mark the centre of circle as *O*.

(iii) Starting with either *F* or *C* as centre and side as length, go on marking the points on the circumference, *A*, *B*, *D* and *E*.

(iv) Join points *A-B-C-D-E-F*. You will get the required Hexagon (6 sided polygon).

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This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 11 for the following problem.

Problem 5-29. *To inscribe a regular octagon in a given square* (fig. 5-45).

(i) Draw the given square *ABCD*.

(ii) Draw diagonals *AC* and *BD* intersecting each other at *O*.

(iii) With centre *A* and radius *AO*, draw an arc cutting *AB* at 2 and *AD* at 7.

(iv) Similarly, with centres *B*, *C* and *D* and the same radius, draw arcs and obtain points 1, 3, 4 etc. as shown.

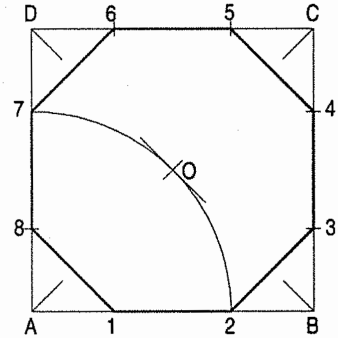


FIG. 5-45

Draw lines 2-3, 4-5, 6-7 and 8-1, thus completing the octagon.

5-14. REGULAR POLYGONS INSCRIBED IN CIRCLES

Problem 5-30. *To inscribe a regular polygon of any number of sides, say 5, in a given circle* (fig. 5-46).

(i) With centre *O*, draw the given circle.

(ii) Draw a diameter *AB* and divide it into five equal parts (same number of parts as the number of sides) and number them as shown.

(iii) With centres *A* and *B* and radius *AB*, draw arcs intersecting each other at *P*.

(iv) Draw a line *P*2 and produce it to meet the circle at *C*. Then *AC* is the length of the side of the pentagon.

(v) Starting from *C*, step-off on the circle, divisions *CD*, *DE* etc., equal to *AC*.

(vi) Draw lines *CD*, *DE* etc., thus completing the pentagon.

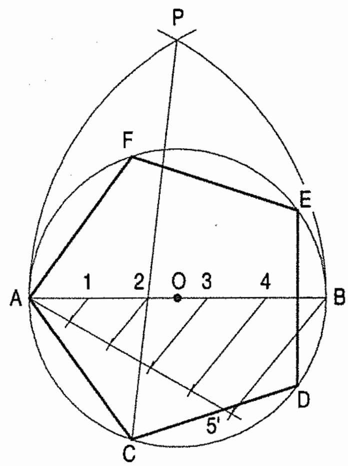


FIG. 5-46

Problem 5-31. *To inscribe a square in a given circle* (fig. 5-47).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* perpendicular to each other.

(iii) Draw lines *AC*, *CB*, *BD* and *DA*, thus completing the square.

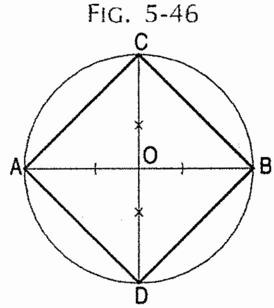


FIG. 5-47

Problem 5-32. *To inscribe a regular pentagon in a given circle* (fig. 5-48).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* perpendicular to each other.

(iii) Bisect *AO* in a point *P*. With centre *P* and radius *PC*, draw an arc cutting *OB* in *Q*.

(iv) With centre *C* and radius *CQ*, draw an arc cutting the circle in *E* and *F*.

(v) With centres *E* and *F* and the same radius, draw arcs cutting the circle in *G* and *H* respectively.

(vi) Draw lines *CE*, *EG*, *GH*, *HF* and *FC*, thus completing the required pentagon.

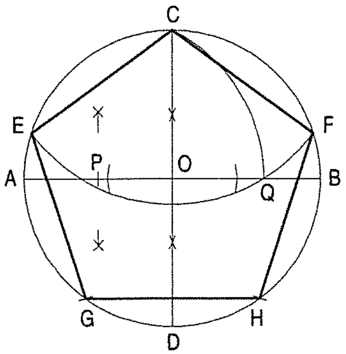


FIG. 5-48

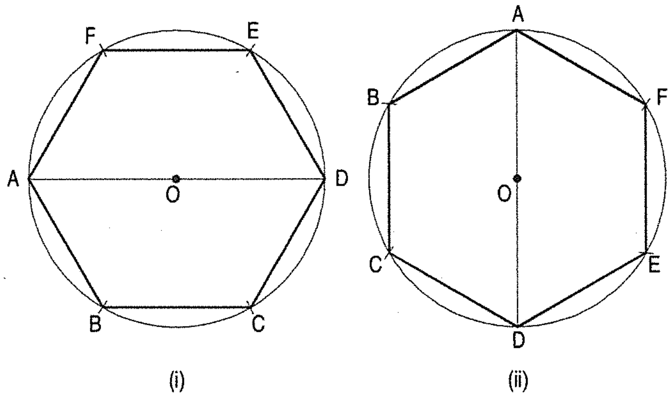


FIG. 5-49

Problem 5-33. *To inscribe a regular hexagon in a given circle* (fig. 5-49).

Apply the same method as shown in Problem 5-28(b).

Note: (a) When two sides of the hexagon are required to be horizontal the starting point for stepping-off equal divisions should be on an end of the horizontal diameter.

(b) If they are to be vertical, the starting point should be on an end of the vertical diameter.

In either case, to avoid inaccuracy, the points should be joined with the aid of T-square and 30°-60° set-square.

Problem 5-34. *To inscribe a regular heptagon in a given circle* (fig. 5-50).

(i) With centre *O*, draw the given circle.

(ii) Draw a diameter *AB*. With centre *A* and radius *AO*, draw an arc cutting the circle at *E* and *F*.

(iii) Draw a line *EF*, cutting *AO* in *G*.

Then *EG* or *FG* is the length of the side of the heptagon.

Therefore, from any point on the circle, say *A*, step-off divisions equal to *EG*, around the circle. Join the division-points and obtain the heptagon.

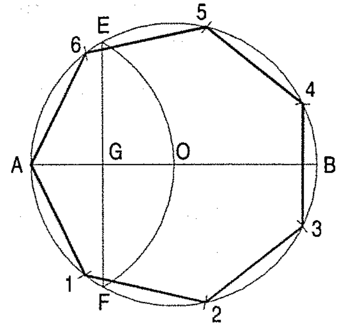


FIG. 5-50

Problem 5-35. *To inscribe a regular octagon in a given circle* (fig. 5-51).

(i) With centre *O*, draw the given circle.

(ii) Draw diameters *AB* and *CD* at right angles to each other.

(iii) Draw diameters *EF* and *GH* bisecting angles *AOC* and *COB*.

(iv) Draw lines *AE*, *EC* etc. and complete the octagon.

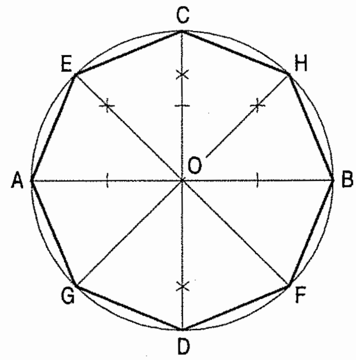


FIG. 5-51

5-15. TO DRAW REGULAR FIGURES USING T-SQUARE AND SET-SQUARES

Problem 5-36. *To describe an equilateral triangle about a given circle* (fig. 5-52).

(i) With centre *O*, draw the given circle.

(ii) Draw a vertical radius *OA*.

(iii) Draw radii *OB* and *OC* with a 30°–60° set-square, such that ∠*AOB* = ∠*AOC* = 120°.

(iv) At *A*, *B* and *C*, draw tangents to the circle, i.e. a horizontal line *EF* through *A*, and lines *FG* and *GE* through *B* and *C* respectively with a 30°–60° set-square.

Then *EFG* is the required triangle.

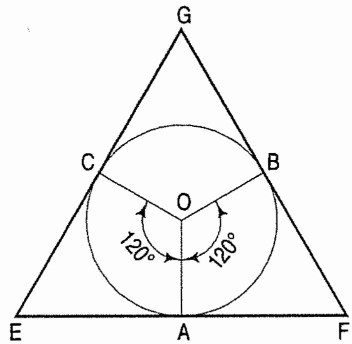


FIG. 5-52

Problem 5-37. *To draw a square about a given circle* (fig. 5-53).

(i) With centre *O*, describe the given circle.

(ii) Draw diameters *AB* and *CD* at right angles to each other as shown.

(iii) At *A* and *B*, draw vertical lines, and at *C* and *D*, draw horizontal lines intersecting at *E*, *F*, *G* and *H*.

EFGH is the required square.

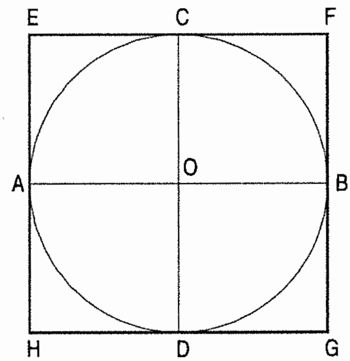


FIG. 5-53

Problem 5-38. *To describe a regular hexagon about a given circle* (fig. 5-54).

(i) With centre *O*, draw the given circle.

(ii) Draw horizontal diameter *AB*, and diameters *CD* and *EF* making 60° angle with *AB*.

(iii) Draw tangents at all the six ends, i.e. verticals at *A* and *B*, and lines with a 30°–60° set-square at the remaining points intersecting at 1, 2,.....6.

A hexagon with two sides horizontal can be drawn by drawing a vertical diameter *AB* and the other lines as shown in fig. 5-55.

Chapter 6

CURVES USED IN ENGINEERING PRACTICE

6-0. INTRODUCTION

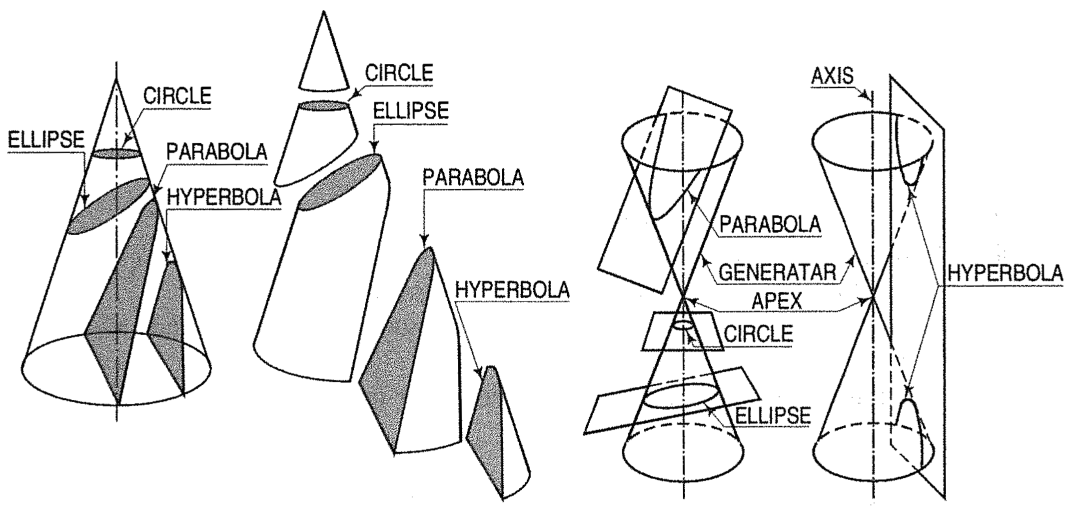
The profile of number of objects consists of various types of curves. This chapter deals with various types of curves which are commonly used in engineering practice as shown below:

1. Conic sections
2. Cycloidal curves
3. Involute
4. Evolutes
5. Spirals
6. Helix.

We shall now discuss the above in details with reference to their construction and applications.

6-1. CONIC SECTIONS

The sections obtained by the intersection of a right circular cone by a plane in different positions relative to the axis of the cone are called *conics*. Refer to fig. 6-1.



Conic sections

FIG. 6-1

(i) When the section plane is inclined to the axis and cuts all the generators on one side of the apex, the section is an *ellipse* [fig. 6-1].

(ii) When the section plane is inclined to the axis and is parallel to one of the generators, the section is a *parabola* [fig. 6-1].

(iii) A *hyperbola* is a plane curve having two separate parts or branches, formed when two cones that point towards one another are intersected by a plane that is parallel to the axes of the cones.

The conic may be defined as the locus of a point moving in a plane in such a way that the ratio of its distances from a fixed point and a fixed straight line is always constant. The fixed point is called the *focus* and the fixed line, the *directrix*.

The ratio $\dfrac{\text{distance of the point from the focus}}{\text{distance of the point from the directrix}}$ is called *eccentricity* and is denoted by e. It is always less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola i.e.

(i) ellipse : $e < 1$

(ii) parabola : $e = 1$

(iii) hyperbola : $e > 1$.

The line passing through the focus and perpendicular to the directrix is called the *axis*. The point at which the conic cuts its axis is called the *vertex*.

6-1-1. ELLIPSE

Use of elliptical curves is made in arches, bridges, dams, monuments, man-holes, glands and stuffing-boxes etc. Mathematically an ellipse can be described by equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Here '*a*' and '*b*' are half the length of major and minor axes of the ellipse and *x* and *y* co-ordinates.

(1) General method of construction of an ellipse:

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 12 for the following problem.

Problem 6-1. (fig. 6-2): *To construct an ellipse when the distance of the focus from the directrix is equal to 50 mm and eccentricity is* $\frac{2}{3}$.

(i) Draw any vertical line *AB* as directrix.

(ii) At any point *C* on it, draw the axis perpendicular to the *AB* (directrix).

(iii) Mark a focus *F* on the axis such that $CF = 50$ mm.

(iv) Divide *CF* into 5 equal divisions (sum of numerator and denometer of the eccentricity.).

(v) Mark the vertex *V* on the third division-point from *C*.

Thus, eccentricity, $e = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(vi) A scale may now be constructed on the axis (as explained below), which will directly give the distances in the required ratio.

(vii) At *V*, draw a perpendicular *VE* equal to *VF*. Draw a line joining *C* and *E*.

Thus, in triangle *CVE*, $\dfrac{VE}{VC} = \dfrac{VF}{VC} = \dfrac{2}{3}$.

(viii) Mark any point 1 on the axis and through it, draw a perpendicular to meet *CE*-produced at 1'.

(ix) With centre *F* and radius equal to 1-1', draw arcs to intersect the perpendicular through 1 at points P_1 and P'_1.

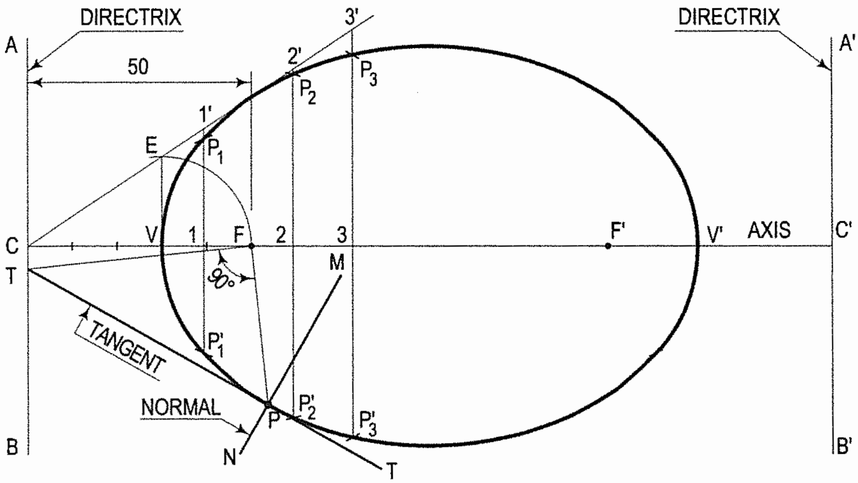
These are the points on the ellipse, because the distance of P_1 from *AB* is equal to *C*1,

$$P_1 F = 1\text{-}1'$$

and $$\frac{1-1'}{C1} = \frac{VF}{VC} = \frac{2}{3}.$$

Similarly, mark points 2, 3 etc. on the axis and obtain points P_2 and P'_2, P_3 and P'_3 etc.

(x) Draw the ellipse through these points. It is a closed curve having two foci and two directrices.



Directrix and focus

FIG. 6-2

(2) Construction of ellipse by other methods:

Ellipse is also defined as a curve traced out by a point, moving in the same plane as and in such a way that the sum of its distances from two fixed points is always the same.

(i) Each of the two fixed points is called the *focus*.

(ii) The line passing through the two foci and terminated by the curve, is called the *major axis*.

(iii) The line bisecting the major axis at right angles and terminated by the curve, is called the *minor axis*.

Conjugate axes: *Those axes are called conjugate axes when they are parallel to the tangents drawn at their extremities.*

In fig. 6-3, *AB* is the major axis, *CD* the minor axis and F_1 and F_2 are the foci. The foci are equidistant from the centre *O*.

The points A, P, C etc. are on the curve and hence, according to the definition,

$$(AF_1 + AF_2) = (PF_1 + PF_2) = (CF_1 + CF_2) \text{ etc.}$$

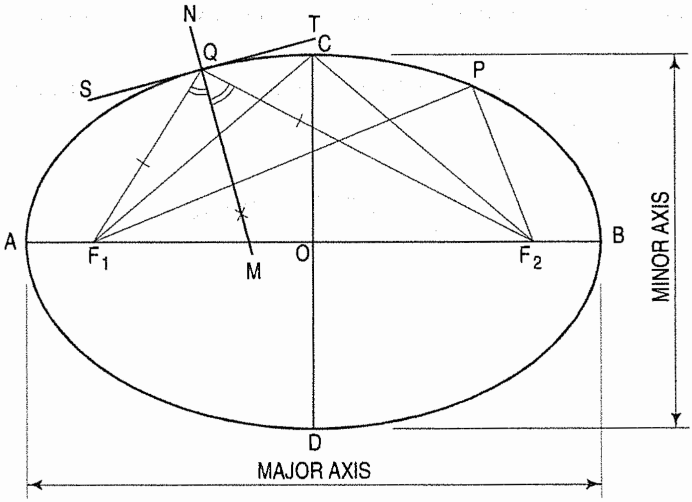
But $(AF_1 + AF_2) = AB$. $\therefore$ $(PF_1 + PF_2) = AB$, the major axis.

Therefore, *the sum of the distances of any point on the curve from the two foci is equal to the major axis.*

Again, $(CF_1 + CF_2) = AB$.

But $CF_1 = CF_2$ $\therefore$ $CF_1 = CF_2 = \frac{1}{2} AB$.

Hence, *the distance of the ends of the minor axis from the foci is equal to half the major axis.*



Conjugate axes
FIG. 6-3

Problem 6-2. *To construct an ellipse, given the major and minor axes.*

The ellipse is drawn by, first determining a number of points through which it is known to pass and then, drawing a smooth curve through them, either freehand or with a french curve. Larger the number of points, more accurate the curve will be.

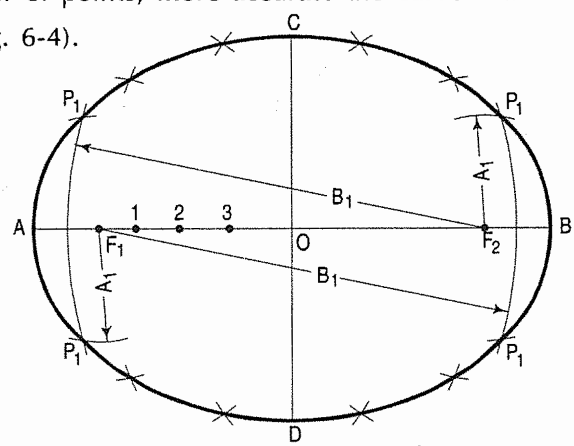
Method I: *Arcs of circles method* (fig. 6-4).

(i) Draw a line AB equal to the major axis and a line CD equal to the minor axis, bisecting each other at right angles at O.

(ii) With centre C and radius equal to half AB (i.e. AO) draw arcs cutting AB at F_1 and F_2, the foci of the ellipse.

(iii) Mark a number of points 1, 2, 3 etc. on AB.

(iv) With centres F_1 and F_2 and radius equal to $A1$, draw arcs on both sides of AB.



Arc of circle method
FIG. 6-4

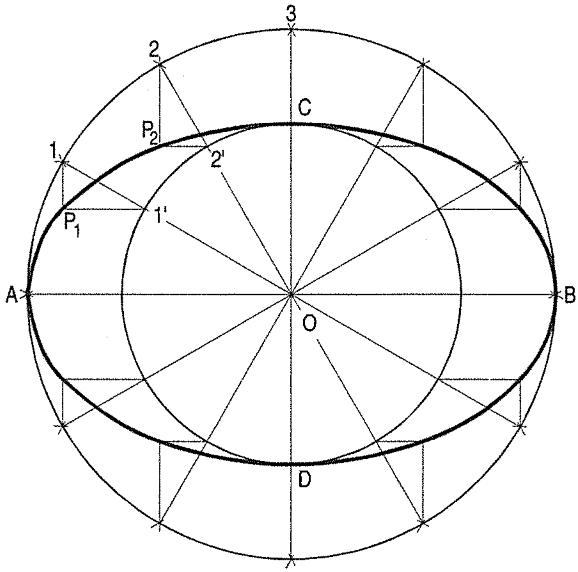
(v) With same centres and radius equal to $B1$, draw arcs intersecting the previous arcs at four points marked P_1.

(vi) Similarly, with radii $A2$ and $B2$, $A3$ and $B3$ etc. obtain more points.

(vii) Draw a smooth curve through these points. This curve is the required ellipse.

CHAROTAR COGNIFRONT This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 13 for the following method II.

Method II: *Concentric circles method* (fig. 6-5).



Concentric circle method

FIG. 6-5

(i) Draw the major axis AB and the minor axis CD intersecting each other at O.

(ii) With centre O and diameters AB and CD respectively, draw two circles.

(iii) Divide the major-axis-circle into a number of equal divisions, say 12 and mark points 1, 2 etc. as shown.

(iv) Draw lines joining these points with the centre O and cutting the minor-axis-circle at points 1', 2' etc.

(v) Through point 1 on the major-axis-circle, draw a line parallel to CD, the minor axis.

(vi) Through point 1' on the minor-axis-circle, draw a line parallel to AB, the major axis. The point P_1, where these two lines intersect is on the required ellipse.

(vii) Repeat the construction through all the points. Draw the ellipse through A, P_1, P_2... etc.

Method III: *Loop of the thread method* (fig. 6-6).

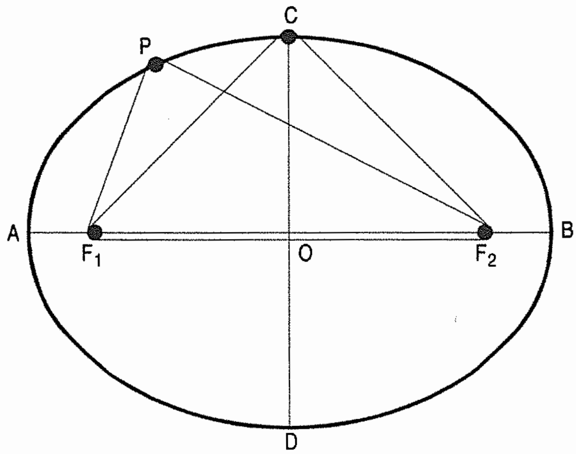
This is practical application of the first method.

(i) Draw the two axes AB and CD intersecting at O. Locate the foci F_1 and F_2.

(ii) Insert a pin at each focus-point and tie a piece of thread in the form of a loop around the pins, in such a way that the pencil point when placed in the loop (keeping the thread tight), is just on the end C of the minor axis.

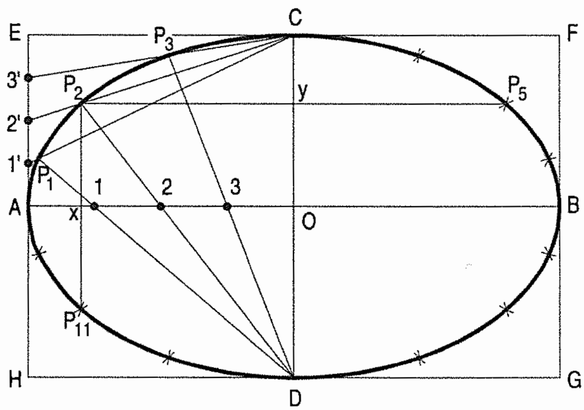
(iii) Move the pencil around the foci, maintaining an even tension in the thread throughout and obtain the ellipse.

It is evident that $PF_1 + PF_2 = CF_1 + CF_2$ etc.



Loop of the thread method
FIG. 6-6

Method IV: *Oblong method* (fig. 6-7).



Oblong method
FIG. 6-7

(i) Draw the two axes *AB* and *CD* intersecting each other at *O*.

(ii) Construct the oblong *EFGH* having its sides equal to the two axes.

(iii) Divide the semi-major-axis *AO* into a number of equal parts, say 4, and *AE* into the same number of equal parts, numbering them from *A* as shown.

(iv) Draw lines joining 1', 2' and 3' with *C*.

(v) From *D*, draw lines through 1, 2 and 3 intersecting C'_1, C'_2 and C'_3 at points P_1, P_2 and P_3 respectively.

(vi) Draw the curve through A, P_1.....C. It will be one quarter of the ellipse.

(vii) Complete the curve by the same construction in each of the three remaining quadrants.

As the curve is symmetrical about the two axes, points in the remaining quadrants may be located by drawing perpendiculars and horizontals from P_1, P_2 etc. and making each of them of equal length on both the sides of the two axes.

For example, $P_2x = x\,P_{11}$ and $P_2y = yP_5$.

An ellipse can be inscribed within a parallelogram by using the above method as shown in fig. 6-8.

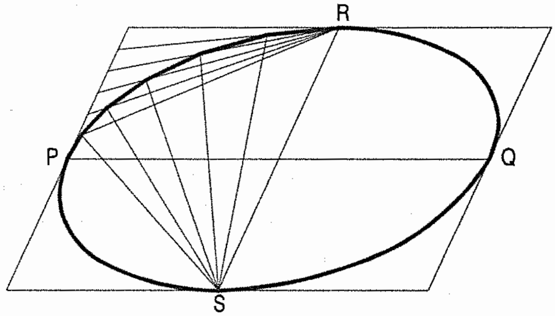
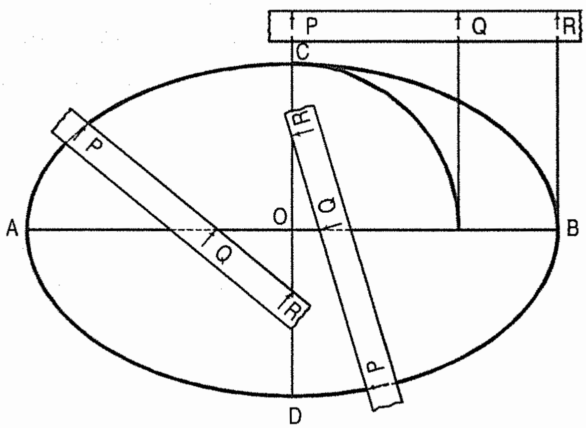


FIG. 6-8

Lines *PQ* and *RS*, joining the mid-points of the opposite sides of the parallelogram are called conjugate axes.

Method V: *Trammel method* (fig. 6-9).



Trammel method

FIG. 6-9

(i) Draw the two axes *AB* and *CD* intersecting each other at *O*. Along the edge of a strip of paper which may be used as a trammel, mark *PQ* equal to half the minor axis and *PR* equal to half the major axis.

(ii) Place the trammel so that *R* is on the minor axis *CD* and *Q* on the major axis *AB*. Then *P* will be on the required ellipse. By moving the trammel to new positions, always keeping *R* on *CD* and *Q* on *AB*, obtain other points. Draw the ellipse through these points.

Problem 6-3. (fig. 6-10): *ABC is a triangle such that AB = 100 mm, AC = 80 mm and BC = 60 mm. Draw an ellipse passing through points A, B and C.*

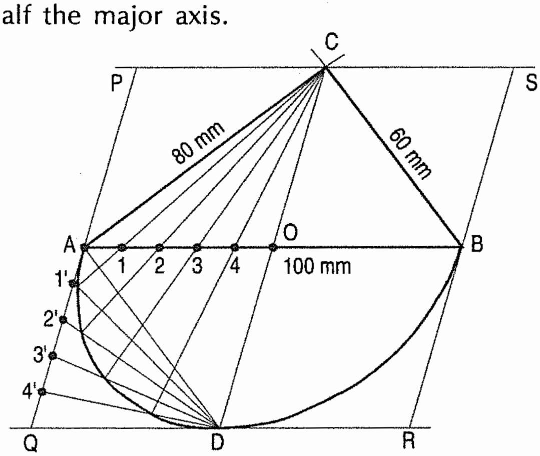


Fig. 6-10

(i) Draw horizontal line AB = 100 mm. Take A as centre draw an arc of 80 mm. Similarly B as centre and the radius equal to 60 mm, draw the arc such that it intersects previously drawn arc at the point C. Join ABC to complete triangle.

(ii) Mark the mid point of AB such that $OA = OB$ = 50 mm. Join OC and extend CO such that $CO = OD$.

(iii) Draw parallel lines from C and D to the line AB. Similarly draw parallel lines from A and B to the line CD and complete the rhombus ($PQRS$).

(iv) Divide AO into convenient number of equal parts $A1 = 12 = 23 = 34 = 4O$ and AQ to same number of equal parts $A1' = 1'2' = 2'3' = 3'4' = 4'Q$. Join A, 1', 2', 3', 4' with D. Join $C1$ and extend it to intersect line $D1'$. Similarly join $C2$, $C3$, $C4$ and extend it to intersect $D2'$, $D3'$, $D4'$ respective. Draw smooth curve passing through all intersection.

(v) Complete the ellipse by above method for the remaining part.

(3) **Normal and tangent to an ellipse:** The normal to an ellipse at any point on it bisects the angle made by lines joining that point with the foci.

The tangent to the ellipse at any point is perpendicular to the normal at that point.

Problem 6-4. (fig. 6-3): *To draw a normal and a tangent to the ellipse at a point Q on it.*

Join Q with the foci F_1 and F_2.

(i) Draw a line NM bisecting $\angle F_1 QF_2$. NM is the normal to the ellipse.

(ii) Draw a line ST through Q and perpendicular to NM. ST is the tangent to the ellipse at the point Q.

Problem 6-5. (fig. 6-11): *To draw a curve parallel to an ellipse and at distance R from it.*

This may be drawn by *two* methods:

(a) A large number of arcs of radius equal to the required distance R, with centres on the ellipse, may be described. The curve drawn touching these arcs will be parallel to the ellipse.

(b) It may also be obtained by drawing a number of normals to the ellipse, making them equal to the required distance R and then drawing a smooth curve through their ends.

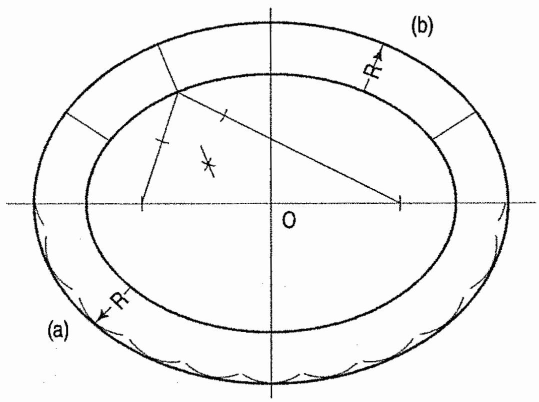


FIG. 6-11

Problem 6-6. (fig. 6-12): *To find the major axis and minor axis of an ellipse whose conjugate axes and angle between them are given.*

Conjugate axes PQ and RS, and the angle α between them are given.

(i) Draw the two axes intersecting each other at O.

(ii) Complete the parallelogram and inscribe the ellipse in it as described in problem 6-2, method (iv).

(iii) With *O* as centre and *OR* as radius, draw the semi-circle cutting the ellipse at a point *E*.

(iv) Draw the line *RE*.

(v) Through *O* draw a line parallel to *RE* and cutting the ellipse at points *C* and *D*. *CD* is the minor axis.

(vi) Through *O*, draw a line perpendicular to *CD* and cutting the ellipse at points *A* and *B*. *AB* is the major axis.

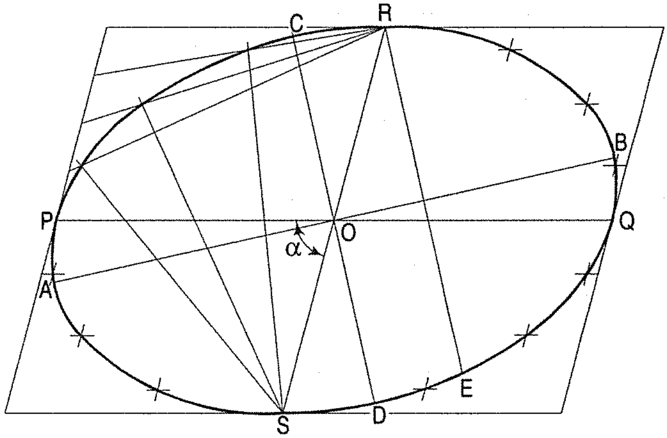


FIG. 6-12

Problem 6-7. (fig. 6-13): *To find the centre, major axis and minor axis of a given ellipse.*

(i) Draw any two chords 1-2 and 3-4 parallel to each other.

(ii) Find their mid-points *P* and *Q*, and draw a line passing through them, cutting the ellipse at points *R* and *S*. Bisect the line *RS* in the point *O* which is the centre of the ellipse.

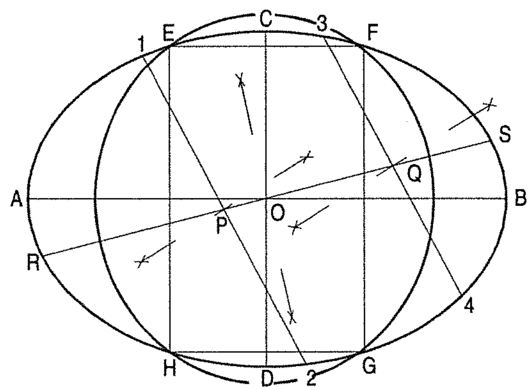


FIG. 6-13

With *O* as centre and any convenient radius, draw a circle cutting the ellipse in points *E*, *F*, *G* and *H*. Complete the rectangle *EFGH*. Through *O*, draw a line parallel to *EF* cutting the ellipse in points *A* and *B*. Again through *O*, draw a line parallel to *FG* cutting the ellipse at points *C* and *D*. *AB* and *CD* are respectively the major axis and the minor axis.

6-1-2. PARABOLA

CHAROTAR COGNIFRONT

This book is accompanied by a computer CD, which contains an audiovisual animation presented for better visualization and understanding of the subject. Readers are requested to refer Presentation module 14 for the following problem.

Chapter 4

SCALES

4-1. INTRODUCTION

Drawings of small objects can be prepared of the same size as the objects they represent. A 150 mm long pencil may be shown by a drawing of 150 mm length. Drawings drawn of the same size as the objects, are called full-size drawings. The ordinary full-size scales are used for such drawings.

A scale is defined as the ratio of the linear dimensions of element of the object as represented in a drawing to the actual dimensions of the same element of the object itself.

4-2. SCALES

The scales generally used for general engineering drawings are shown in table 4-1 [SP : 46].

TABLE 4-1

(i)	Reducing scales	1 : 2	1 : 5	1 : 10
		1 : 20	1 : 50	1 : 100
		1 : 200	1 : 500	1 : 1000
		1 : 2000	1 : 5000	1 : 10000
(ii)	Enlarging scales	50 : 1	20 : 1	10 : 1
		5 : 1	2 : 1	
(iii)	Full size scales			1 : 1

All these scales are usually 300 mm long and sub-divided throughout their lengths. The scale is indicated on the drawing at a suitable place near the title. The complete designation of a scale consists of word scale followed by the ratio, i.e. scale 1 : 1 or scale, full size.

It may not be always possible to prepare full-size drawings. They are, therefore, drawn proportionately smaller or larger. When drawings are drawn smaller than the actual size of the objects (as in case of buildings, bridges, large machines etc.) the scale used is said to be a *reducing scale* (1 : 5). Drawings of small machine parts, mathematical instruments, watches etc. are made larger than their real size. These are said to be drawn on an *enlarging scale* (5 : 1).

The scales can be expressed in the following *three* ways:

(1) Engineer's scale: In this case, the relation between the dimension on the drawing and the actual dimension of the object is mentioned numerically in the style as 10 mm = 5 m etc.

(2) Graphical scale: The scale is drawn on the drawing itself. As the drawing becomes old, the engineer's scale may shrink and may not give accurate results.

However, such is not the case with graphical scale because if the drawing shrinks, the scale will also shrink. Hence, the graphical scale is commonly used in survey maps.

(3) Representative fraction: The ratio of the length of the object represented on drawing to the actual length of the object represented is called the Representative Fraction (i.e. R.F.).

$$\text{R.F.} = \frac{\text{Length of the drawing}}{\text{Actual length of object}}$$

When a 1 cm long line in a drawing represents 1 metre length of the object, the R.F. is equal to $\frac{1 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ cm}}{1 \times 100 \text{ cm}} = \frac{1}{100}$ and the scale of the drawing will be 1 : 100 or $\frac{1}{100}$ full size. *The R.F. of a drawing is greater than unity when it is drawn on an enlarging scale*. For example, when a 2 mm long edge of an object is shown in a drawing by a line 1 cm long, the R.F. is $\frac{1 \text{ cm}}{2 \text{ mm}} = \frac{10 \text{ mm}}{2 \text{ mm}} = 5$. Such a drawing is said to be drawn on scale 5 : 1 or *five times full-size*.

4-3. SCALES ON DRAWINGS

When an unusual scale is used, it is constructed on the drawing sheet. To construct a scale the following information is essential:

(1) The R.F. of the scale.

(2) The units which it must represent, for example, millimetres and centimetres, or feet and inches etc.

(3) The maximum length which it must show.

The length of the scale is determined by the formula:

Length of the scale = R.F. × maximum length required to be measured.

It may not be always possible to draw as long a scale as to measure the longest length in the drawing. The scale is therefore drawn 15 cm to 30 cm long, longer lengths being measured by marking them off in parts.

4-4. TYPES OF SCALES

The scales used in practice are classified as under:

(1) Plain scales

(2) Diagonal scales

(3) Comparative scales

(4) Vernier scales

(5) Scale of chords.

(1) Plain scales: A plain scale consists of a line divided into suitable number of equal parts or units, the first of which is sub-divided into smaller parts. Plain scales represent either two units or a unit and its sub-division.

In every scale,

(i) The zero should be placed at the end of the first main division, i.e. between the unit and its sub-divisions.

(ii) From the zero mark, the units should be numbered to the right and its sub-divisions to the left.

(iii) The names of the units and the sub-divisions should be stated clearly below or at the respective ends.

(iv) The name of the scale (e.g. scale, 1 : 10) or its R.F. should be mentioned below the scale.

Problem 4-1. (fig. 4-1): *Construct a scale of 1 : 4 to show centimetres and long enough to measure upto 5 decimetres.*

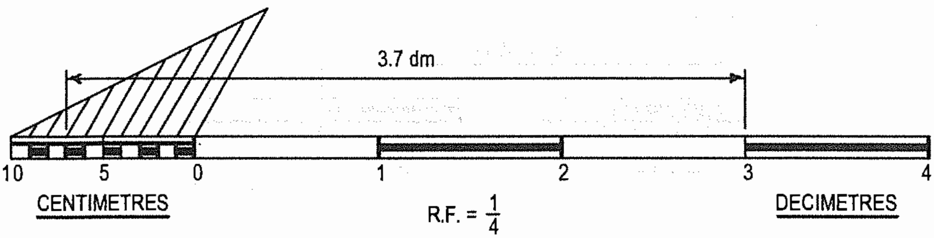


FIG. 4-1

(i) Determine R.F. of the scale. Here it is $\frac{1}{4}$.

(ii) Determine length of the scale.

Length of the scale = R.F. × maximum length = $\frac{1}{4}$ × 5 dm = 12.5 cm.

(iii) Draw a line 12.5 cm long and divide it into 5 equal divisions, each representing 1 dm.

(iv) Mark 0 at the end of the first division and 1, 2, 3 and 4 at the end of each subsequent division to its right.

(v) Divide the first division into 10 equal sub-divisions, each representing 1 cm.

(vi) Mark cms to the left of 0 as shown in the figure.

To distinguish the divisions clearly, *show the scale as a rectangle of small width (about 3 mm) instead of only a line*. Draw the division-lines showing decimetres throughout the width of the scale. Draw the lines for the sub-divisions slightly shorter as shown. Draw thick and dark horizontal lines in the middle of all alternate divisions and sub-divisions. This helps in taking measurements. Below the scale, print DECIMETRES on the right-hand side, CENTIMETRES on the left-hand side, and the R.F. in the middle.

To set-off any distance, say 3.7 dm, place one leg of the divider on 3 dm mark and the other on 7 cm mark. The distance between the ends of the two legs will represent 3.7 dm.

Problem 4-2. (fig. 4-2): *Draw a scale of 1 : 60 to show metres and decimetres and long enough to measure upto 6 metres.*

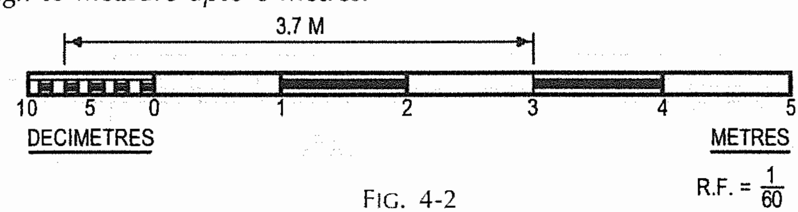


FIG. 4-2

(i) Determine R.F. of the scale, here R.F. $= \frac{1}{60}$.

(ii) Determine length of the scale.

Length of the scale $= \frac{1}{60} \times 6$ m $= \frac{1}{10}$ metre $= 10$ cm.

(iii) Draw a line 10 cm long and divide it into 6 equal parts.

(iv) Divide the first part into 10 equal divisions and complete the scale as shown. The length 3.7 metres is shown on the scale.

Problem 4-3. (fig. 4-3): *Construct a scale of 1.5 inches = 1 foot to show inches and long enough to measure upto 4 feet.*

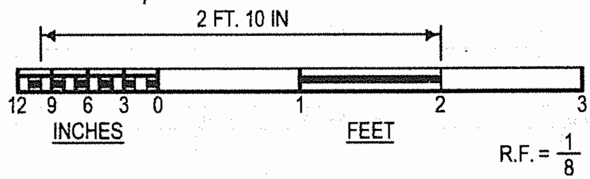


FIG. 4-3

(i) Determine R.F. of the scale. R.F. $= \frac{1.5 \text{ inches}}{1 \times 12 \text{ inches}} = \frac{1}{8}$.

(ii) Draw a line, $1.5 \times 4 = 6$ inches long.

(iii) Divide it into four equal parts, each part representing one foot.

(iv) Divide the first division into 12 equal parts, each representing 1". Complete the scale as explained in problem 4-1. The distance 2'-10" is shown measured in the figure.

Problem 4-4. (fig. 4-4): *Construct a scale of R.F.* $= \frac{1}{60}$ *to read yards and feet, and long enough to measure upto 5 yards.*

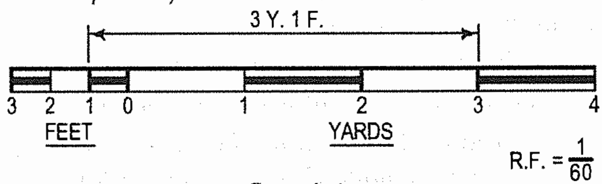


FIG. 4-4

(i) Length of the scale = R.F. × max. length $= \frac{1}{60} \times 5$ yd

$= \frac{1}{12}$ yd $= 3$ inches.

(ii) Draw a line 3 inches long and divide it into 5 equal parts.

(iii) Divide the first part into 3 equal divisions.

(iv) Mark the scale as shown in the figure.

Problem 4-5. (fig. 4-5): *Construct a scale of R.F.* $= \frac{1}{84480}$ *to show miles and furlongs and long enough to measure upto 6 miles.*

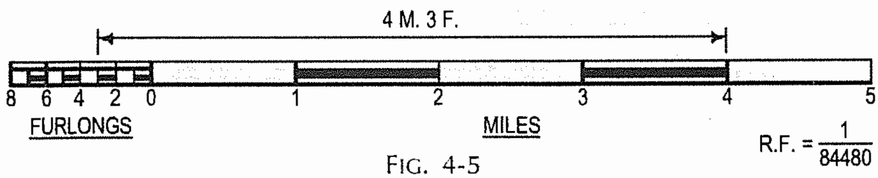


FIG. 4-5

(i) Length of the scale $= \frac{1}{84480} \times 6 = \frac{1}{14080}$ miles $= 4\frac{1}{2}$".

(ii) Draw a line $4\frac{1}{2}$" long and divide it into 6 equal parts. Divide the first part into 8 equal divisions and complete the scale as shown.

The distance 4 miles and 3 furlongs is shown measured in the figure.

(2) Diagonal scales: A diagonal scale is used when very minute distances such as 0.1 mm etc. are to be accurately measured or when measurements are required in three units; for example, dm, cm and mm, or yard, foot and inch.

Small divisions of short lines are obtained by the principle of diagonal division, as explained below.

Principle of diagonal scale: To obtain divisions of a given short line *AB* in multiples of $\frac{1}{10}$ its length, e.g. 0.1 *AB*, 0.2 *AB*, 0.3 *AB* etc. (fig. 4-6).

(i) At one end, say *B*, draw a line perpendicular to *AB* and along it, step-off ten equal divisions of any length, starting from *B* and ending at *C*.

(ii) Number the division-points, 9, 8, 7,.....1 as shown.

(iii) Join *A* with *C*.

(iv) Through the points 1, 2 etc. draw lines parallel to *AB* and cutting *AC* at 1', 2' etc. It is evident that triangles 1'1*C*, 2'2*C* ... *ABC* are similar.

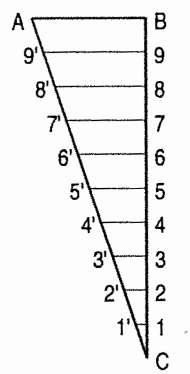


FIG. 4-6

Since *C*5 = 0.5*BC*, the line 5'5 = 0.5*AB*.

Similarly, 1'1 = 0.1*AB*, 2'2 = 0.2*AB* etc.

Thus, each horizontal line below *AB* becomes progressively shorter in length by $\frac{1}{10}$ *AB* giving lengths in multiples of 0.1*AB*.

Problem 4-6. (fig. 4-7): *Construct a diagonal scale of 3 : 200 i.e. 1 : 66 $\frac{2}{3}$ showing metres, decimetres and centimetres and to measure upto 6 metres.*

Length of the scale $= \frac{3}{200} \times 6$ m $= 9$ cm.

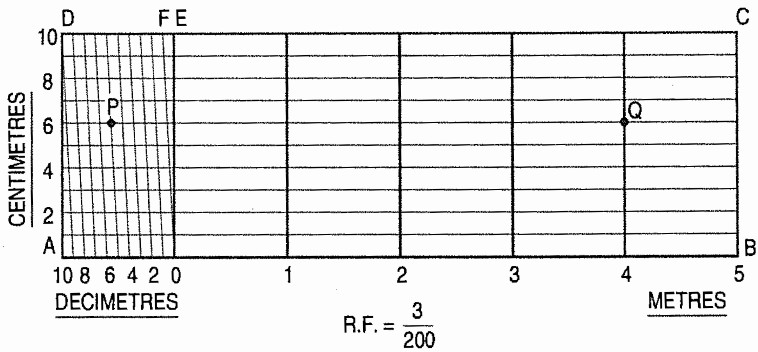


FIG. 4-7

(i) Draw a line *AB* 9 cm long and divide it into 6 equal parts. Each part will show a metre.

(ii) Divide the first part *A*0 into 10 equal divisions, each showing a decimetre or 0.1 m.

(iii) At *A* erect a perpendicular and step-off along it, 10 equal divisions of any length, ending at *D*. Complete the rectangle *ABCD*.

(iv) Erect perpendiculars at metre-divisions 0, 1, 2, 3 and 4.

(v) Draw horizontal lines through the division-points on *AD*.

(vi) Join *D* with the end of the first division along *A*0, viz. the point 9.

(vii) Through the remaining points i.e. 8, 7, 6 etc. draw lines parallel to *D*9.

In △ *OFE*, *FE* represents 1 dm or 0.1 m. Each horizontal line below *FE* progressively diminishes in length by 0.1*FE*. Thus, the next line below *FE* is equal to 0.9*FE* and represents 0.9 A 1 dm = 0.9 dm or 0.09 m or 9 cm.

Any length between 1 cm or 0.01 m and 6 m can be measured from this scale. To show a distance of 4.56 metres, i.e. 4 m, 5 dm and 6 cm, place one leg of the divider at *Q* where the vertical through 4 m meets the horizontal through 6 cm and the other leg at *P* where the diagonal through 5 dm meets the same horizontal.

Problem 4-7. (fig. 4-8): *Construct a diagonal scale of R.F. =* $\frac{1}{4000}$ *to show metres and long enough to measure upto 500 metres.*

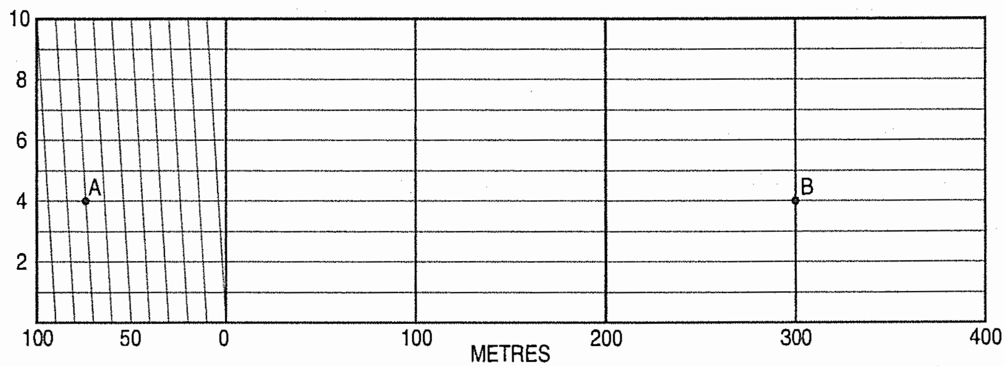


FIG. 4-8

Length of the scale = $\frac{1}{4000} \times 500$ m = $\frac{1}{8}$ metre = 12.5 cm.

(i) Draw a line 12.5 cm long and divide it into 5 equal parts. Each part will show 100 metres.

(ii) Divide the first part into ten equal divisions. Each division will show 10 metres.

(iii) At the left-hand end, erect a perpendicular and on it, step-off 10 equal divisions of any length.

(iv) Draw the rectangle and complete the scale as explained in problem 4-6.

The distance between points *A* and *B* shows 374 metres.

Problem 4-8. (fig. 4-9): *Draw a diagonal scale of 1 : 2.5, showing centimetres and millimetres and long enough to measure upto 20 centimetres.*

Length of the scale $= \frac{1}{2.5} \times 20$ cm $= 8$ cm.

(i) Draw a line 8 cm long and divide it into 4 equal parts. Each part will represent a length of 5 cm.

(ii) Divide the first part into 5 equal divisions. Each division will show 1 cm.

(iii) At the left-hand end of the line, draw a vertical line and on it step-off 10 equal divisions of any length.

Complete the scale as explained in problem 4-6. The distance between points *C* and *D* shows 13.4 cm.

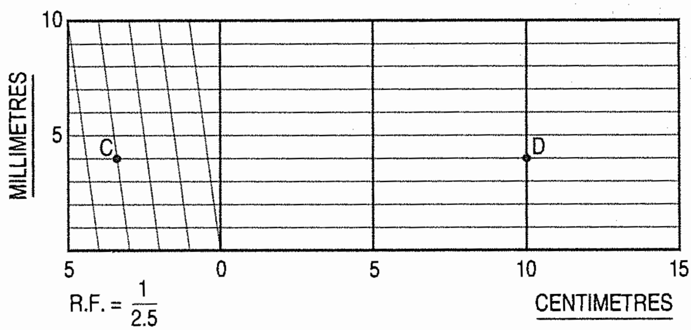


FIG. 4-9

Problem 4-9. (fig. 4-10): *Construct a diagonal scale of R.F. $= \frac{1}{32}$ showing yards, feet and inches and to measure upto 4 yards.*

Length of the scale $= \frac{1}{32} \times 4$ yd $= 18$ yd $= 4\frac{1}{2}''$.

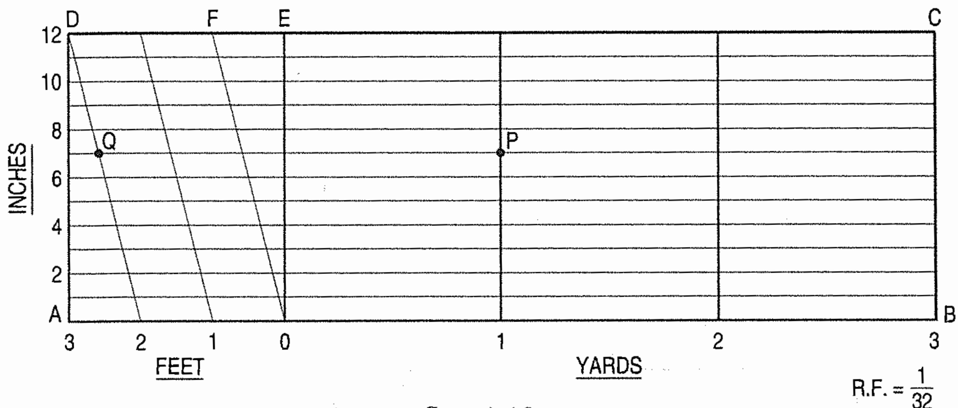


FIG. 4-10

(i) Draw a line *AB* $4\frac{1}{2}''$ long.

(ii) Divide it into 4 equal parts to show yards. Divide the first part *A*0 into 3 equal divisions showing feet.

(iii) At *A*, erect a perpendicular and step-off along it, 12 equal divisions of any length, ending at *D*. Complete the scale as explained in problem 4-6.

To show a distance of 1 yard, 2 feet and 7 inches, place one leg of the divider at *P*, where the horizontal through 7" meets the vertical from 1 yard and the other leg at *Q* where the diagonal through 2' meets the same horizontal.

Problem 4-10. (fig. 4-11): *Draw a scale of full-size, showing $\frac{1}{100}$ inch and to measure upto 5 inches.*

(i) Draw a line *AB* 5" long and divide it into five equal parts. Each part will show one inch.

(ii) Sub-divide the first part into 10 equal divisions. Each division will measure $\frac{1}{10}$ inch.

(iii) At *A*, draw a perpendicular to *AB* and on it, step-off ten equal divisions of any length, ending at *D*.

(iv) Draw the rectangle *ABCD* and complete the scale as explained in problem 4-6. The line *QP* shows 2.68 inches.

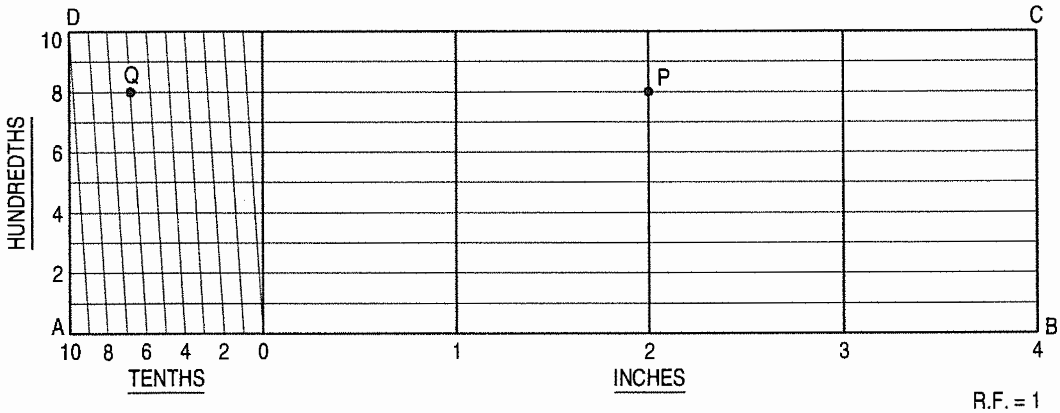


FIG. 4-11

Problem 4-11. (fig. 4-12): *The area of a field is 50,000 sq m. The length and the breadth of the field, on the map is 10 cm and 8 cm respectively. Construct a diagonal scale which can read upto one metre. Mark the length of 235 metre on the scale. What is the R.F. of the scale?*

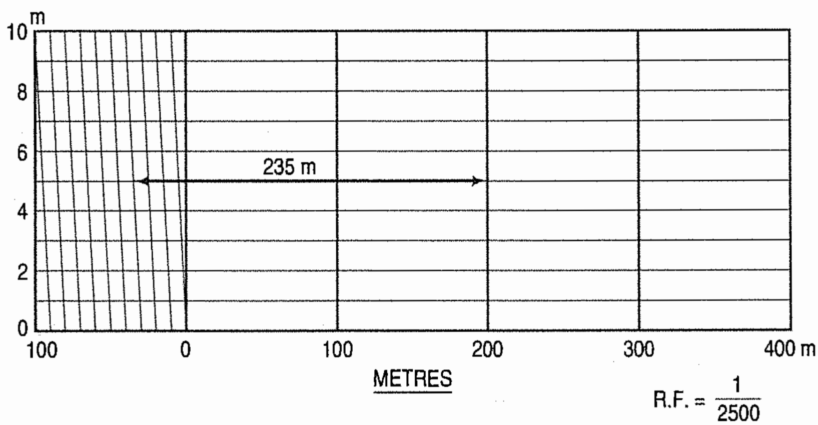


FIG. 4-12

The area of the field = 50,000 sq m.

The area of the field on the map = 10 cm × 8 cm = 80 cm².

$$\therefore \quad 1 \text{ sq cm} = \frac{50000}{80} = 625 \text{ sq m.}$$

$$\therefore \quad 1 \text{ cm} = 25 \text{ m.}$$

$$\text{Now representative fraction} = \frac{1 \text{ cm}}{25 \text{ m}} = \frac{1}{2500}.$$

$$\text{Length of the scale} = \frac{1}{2500} \times \frac{500 \times 100}{1} = \frac{50000}{2500} = 20 \text{ cm.}$$

Take 20 cm length and divide it into 5 equal parts. Complete the scale as shown in fig. 4-12.

(3) Comparative scales: Scales having same representative fraction but graduated to read different units are called *comparative scales*. A drawing drawn with a scale reading inch units can be read in metric units by means of a metric comparative scale, constructed with the same representative fraction. Comparative scales may be plain scales or diagonal scales and may be constructed separately or one above the other.

Problem 4-12. [fig. 4-13(i) and fig. 4-13(ii)]: *A drawing is drawn in inch units to a scale $\frac{3}{8}$ full size. Draw the scale showing $\frac{1}{8}$ inch divisions and to measure upto 15 inches. Construct a comparative scale showing centimetres and millimetres, and to read upto 40 centimetres.*

(i) *Inch scale:*

Length of the scale

$$= \frac{3}{8} \times 15 = \frac{45}{8}$$

$$= 5\frac{5}{8} \text{ inches.}$$

Construct the diagonal scale as shown in fig. 4-13(i).

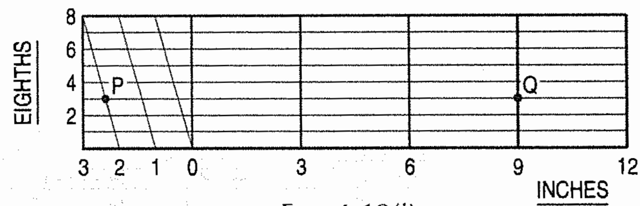


FIG. 4-13(i)

(ii) *Comparative scale:*

Length of the scale

$$= \frac{3}{8} \times 40$$

$$= 15 \text{ cm.}$$

Construct the diagonal scale as shown in fig. 4-13(ii).

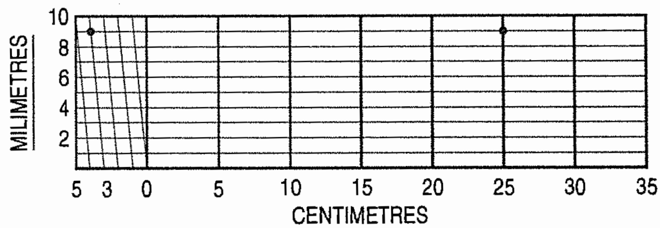


FIG. 4-13(ii)

The line *PQ* on the inch scale shows a length equal to $11\frac{3}{8}''$. Its equivalent, when measured on the comparative scale is 28.9 cm.

Problem 4-13. (fig. 4-14): *Draw comparative scales of R.F. = $\frac{1}{485000}$ to read upto 80 kilometres and 80 versts. 1 verst = 1.067 km.*

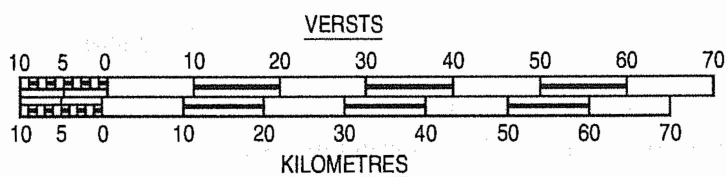


FIG. 4-14

$$\text{Length of kilometre scale} = \frac{1}{485000} \times 80 \times 1000 \times 100 = 16.5 \text{ cm.}$$

$$\text{Length of verst scale} = \frac{1}{485000} \times 80 \times 1.067 \times 1000 \times 100 = 17.6 \text{ cm.}$$

Draw the two scales one above the other as shown in the figure.

Problem 4-14. (fig. 4-15): *On a road map, a scale of miles is shown. On measuring from this scale, a distance of 25 miles is shown by a line 10 cm long. Construct this scale to read miles and to measure upto 40 miles. Construct a comparative scale, attached to this scale, to read kilometres upto 60 kilometres. 1 mile = 1.609 km.*

(i) *Scale of miles:*

$$\text{Length of the scale} = \frac{10 \times 40}{25} = 16 \text{ cm.}$$

Draw a line 16 cm long and construct a plain scale to show miles.

(ii) *Scale of kilometres:*

$$\text{R.F.} = \frac{10}{25 \times 1.609 \times 1000 \times 100} = \frac{1}{402250}$$

$$\text{Length of the scale} = \frac{1}{402250} \times 60 \times 1000 \times 100 = 14.9 \text{ cm.}$$

Construct the plain scale 14.9 cm long, above the scale of miles and attached to it, to read kilometres.

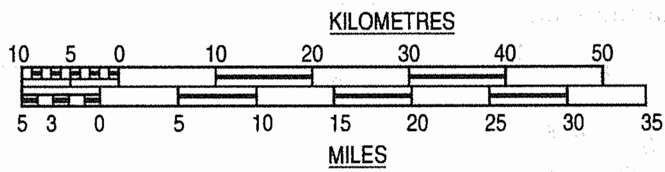


FIG. 4-15

Problem 4-15. (fig. 4-16): *The distance between Bombay and Poona is 180 km. A passenger train covers this distance in 6 hours. Construct a plain scale to measure time upto a single minute. The R.F. of the scale is $\frac{1}{200000}$. Find the distance covered by the train in 36 minutes.*

$$\text{Speed of the train} = \frac{180}{6} = 30 \text{ km/hour.}$$

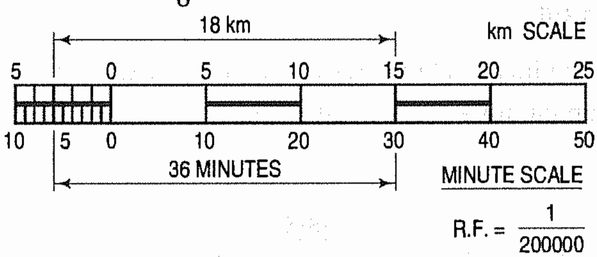


FIG. 4-16

(i) Distance scale (kilometres scale):

Length of the scale = R.F. × maximum distance

$$= \frac{1}{200000} \times 30 \times 1000 \times 100 = 15 \text{ cm.}$$

(ii) Time scale (minute scale):

Speed of the train = 30 km/hour.

i.e. 30 km is covered in 60 minutes.

As length of the scale of 15 cm represents 30 km, 60 minutes which is the time required to cover 30 km, can be represented on the same length of the scale.

(iii) Draw a line 15 cm long and divide it into 6 equal parts. Each part represents 5 km for the distance scale and 10 minutes for the time scale.

(iv) Divide the first part of the distance scale and the time scale into 5 and 10 equal parts respectively. Complete the scales as shown. The distance covered in 36 minutes is shown on the scale.

Problem 4-16. (fig. 4-17): *On a Russian map, a scale of versts is shown. On measuring it with a metric scale, 150 versts are found to measure 15 cm. Construct comparative scales for the two units to measure upto 200 versts and 200 km respectively. 1 verst = 1.067 km.*

(i) *Scale of verst:*

$$\text{Length of the scale} = \frac{15 \times 200}{150} = 20 \text{ cm.}$$

Draw a line 20 cm long and construct a plain scale to show versts.

(ii) *Scale of kilometres:*

$$\text{R.F.} = \frac{15}{150 \times 1.609 \times 1000 \times 10} = \frac{1}{160900}$$

$$\text{Length of the scale} = \frac{1}{160900} \times 200 \times 1000 \times 10 = 12.4 \text{ cm.}$$

Construct the plain scale 12.4 cm long, above the scale of versts and attached to it, to read kilometres (fig. 4-17).

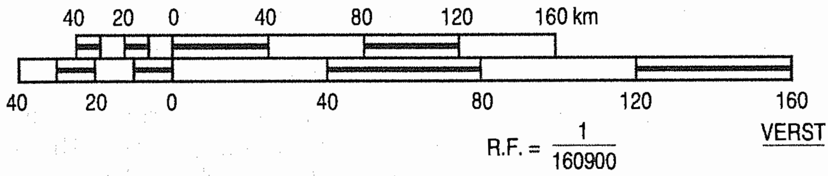


FIG. 4-17

(4) Vernier scales: Vernier scales, like diagonal scales, are used to read to a very small unit with great accuracy. A vernier scale consists of two parts — a primary scale and a vernier. The primary scale is a plain scale fully divided into minor divisions.

As it would be difficult to sub-divide the minor divisions in the ordinary way, it is done with the help of the vernier. The graduations on the vernier are derived from those on the primary scale.

(a) ***Principle of vernier:*** Fig. 4-18 shows a part of a plain scale in which the length $A0$ represents 10 cm. If we divide $A0$ into ten equal parts, each part will represent 1 cm. It would not be easy to divide each of these parts into ten equal divisions to get measurements in millimetres.

Now, if we take a length $B0$ equal to $10 + 1 = 11$ such equal parts, thus representing 11 cm, and divide it into ten equal divisions, each of these divisions will represent $\frac{11}{10} = 1.1$ cm or 11 mm.

The difference between one part of $A0$ and one division of $B0$ will be equal $1.1 - 1.0 = 0.1$ cm or 1 mm.

Similarly, the difference between two parts of each will be 0.2 cm or 2 mm.

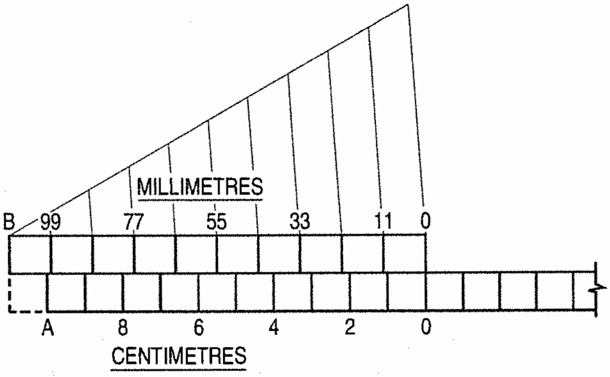


FIG. 4-18

The *upper scale* $B0$ is the vernier. The combination of the plain scale and the vernier is the vernier scale.

In general, if a line representing n units is divided into n equal parts, each part will show $\frac{n}{n} = 1$ unit. But, if a line equal to $n + 1$ of these units is taken and then divided into n equal parts, each of these parts will be equal to $\frac{n+1}{n} = 1 + \frac{1}{n}$ units.

The difference between one such part and one former part will be equal to $\frac{n+1}{n} - \frac{n}{n} = \frac{1}{n}$ unit.

Similarly, the difference between two parts from each will be $\frac{2}{n}$ unit.

(b) ***Least count of a vernier:*** It is the difference of 1 primary scale division and 1 vernier scale division. It is denoted by $\angle C$.

$\angle C$ = 1 primary scale division – 1 vernier scale division.

The vernier scales are classified as under:

(i) *Forward vernier:* In this case, the length of one division of the vernier scale is smaller than the length of one division of the primary scale. The vernier divisions are marked in the same direction as that of the main scale.

(ii) *Backward vernier:* The length of each division of vernier scale is greater than the length of each division of the primary scale. The numbering is done in the opposite direction as that of the primary scale.

Problem 4-17. (fig. 4-19): *Draw a vernier scale of R.F.* $= \frac{1}{25}$ *to read centimetres upto 4 metres and on it, show lengths representing 2.39 m and 0.91 m.*

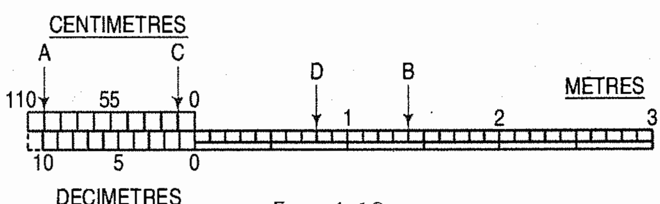


FIG. 4-19

Length of the scale $= \frac{1}{25} \times 4 \times 100 = 16$ cm.

(i) Draw a line 16 cm long and divide it into 4 equal parts to show metres. Divide each of these parts into 10 equal parts to show decimetres.

(ii) To construct a vernier, take 11 parts of dm length and divide it into 10 equal parts. Each of these parts will show a length of 1.1 dm or 11 cm.

To measure a length representing 2.39 m, place one leg of the divider at *A* on 99 cm mark and the other leg at *B* on 1.4 m mark. The length *AB* will show 2.39 metres (0.99 + 1.4 = 2.39).

Similarly, the length, *CD* shows 0.91 metre (0.8 + 0.11 = 0.91).

The necessity of dividing the plain scale into minor divisions throughout its length is quite evident from the above measurements.

Problem 4-18. (fig. 4-20): *Construct a full-size vernier scale of inches and show on it lengths 3.67", 1.54" and 0.48".*

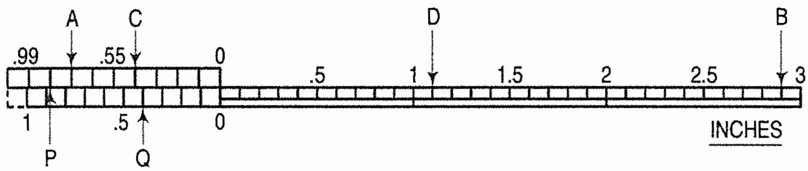


FIG. 4-20

(i) Draw a plain full-size scale 4" long and divide it fully to show 0.1" lengths.

(ii) Construct a vernier of length equal to 10 + 1 = 11 parts and divide it into 10 equal parts. Each of these parts will $\frac{11 \times 0.1}{10} = 0.11''$.

The line *AB* shows a length of 3.67" (0.77" + 2.9" = 3.67"). Similarly, lines *CD* and *PQ* show lengths of 1.54" (0.44" + 1.1" = 1.54") and 0.48" (0.88" – 0.4" = 0.48") respectively.

Problem 4-19. (fig. 4-21): *Construct a vernier scale of R.F.* = $\frac{1}{80}$ *to read inches and to measure upto 15 yards.*

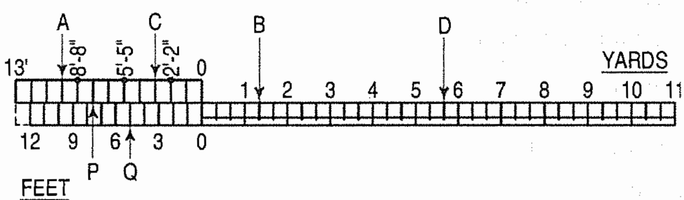


FIG. 4-21

Length of the scale = $\frac{1}{80} \times 15$ yd = $\frac{3}{16}$ yd = $6\frac{3}{4}''$.

(i) Draw the plain scale $6\frac{3}{4}''$ long and divide it fully to show yards and feet.

(ii) To construct the vernier, take a length of 12 + 1 = 13 feet-divisions and divide it into 12 equal parts. Each part will represent $\frac{13}{12}$ ft or 1'-1".

Lines *AB*, *CD* and *PQ* show respectively lengths representing 4 yd 1 ft 9 in (9' – 9" + 4'), 6 yd 2 ft 3 in (3' – 3" + 17') and 0 yd 2 ft 7 in (7' – 7" – 5").

