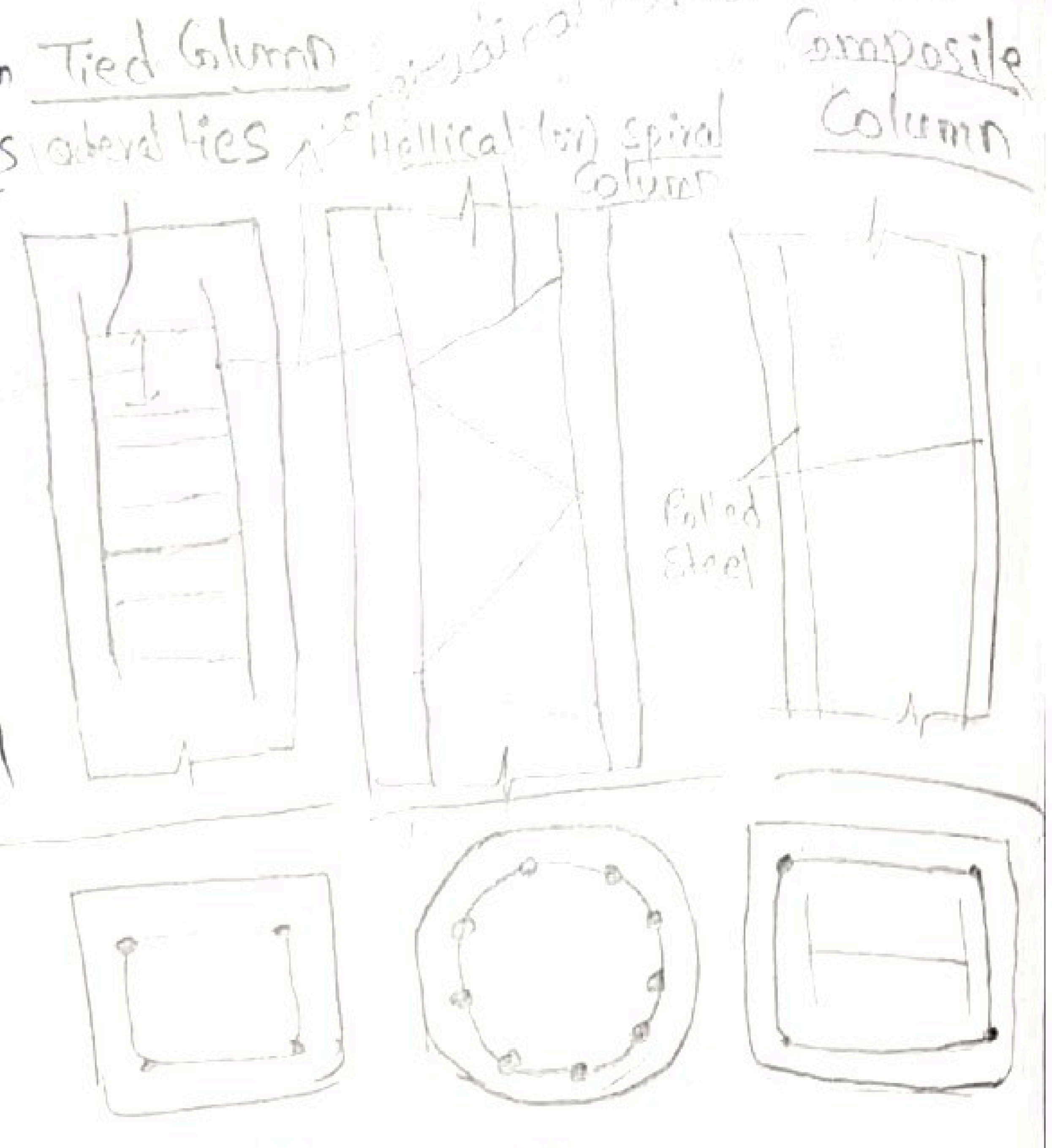
Unit-5

Design of Columns

Compression members



The vertical load is called column.

Columns are three types.

1. Based on Reinforcement.
2. Based on loading → axial, uniaxial, Biaxial.
3. Based on Slenderness Ratio.
   short column, long column.
   ratio <12, ratio >12.

Compression under Buckling failure.

<u>Column</u>:- A Column is a Compression member which transverse the loads to the ground and strut is a term applied to Compression members in any direction such as those in a truss.

(or)

As per IS 456:2000 difines the Column as a Compression member the effective length which exceeds 3 times of its least lateral dimension.

→ Axially loaded Columns may fail in the following modes (or) forms.

1) Pure Compression failure.
2) Combined compression and bending failure.
3) Failure by elastic instability.

The failure modes mainly depends on the

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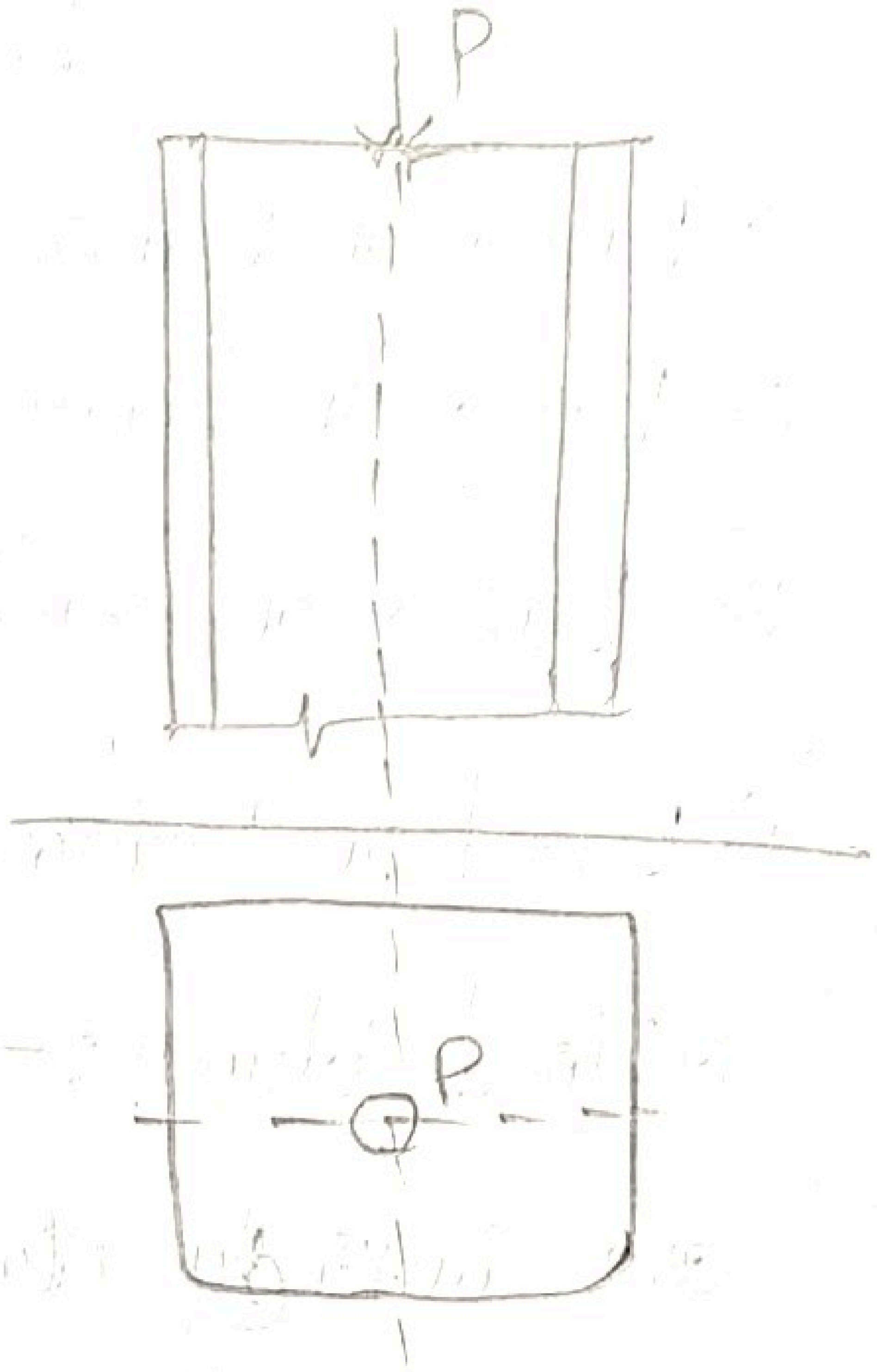
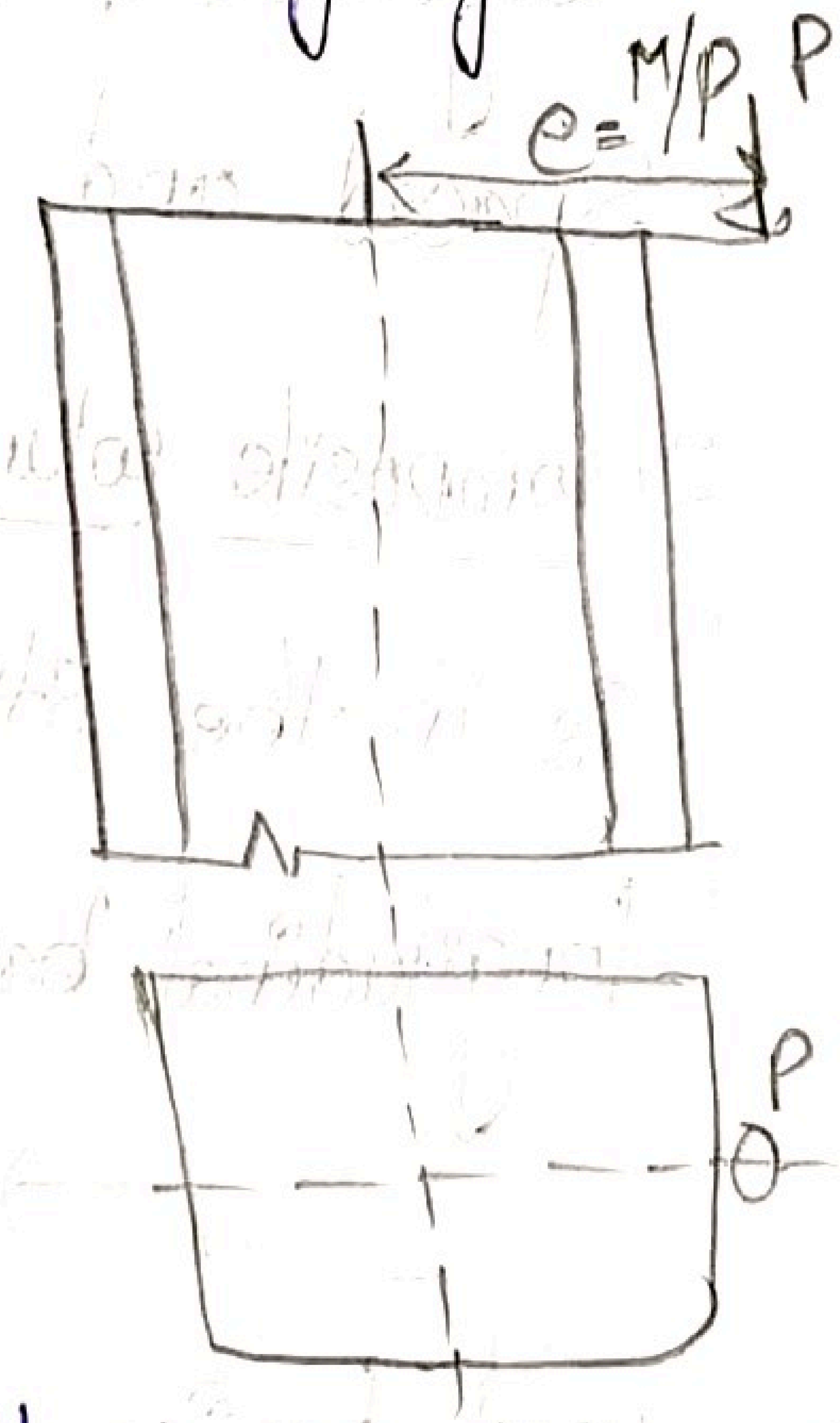
1.) <u>Axial Columns</u>:- The axially loaded Columns Supporting Concrete loads very rare. Interior Columns of multistore buildings with symmetrical loading from floor to slab from all sides.



2) <u>Uni-axial Column</u>:- Column with uni-axial eccentric loading are generally encountered in the Case of rigidly Connected to the beams from one side only.



3) <u>Bi-axial Columns</u>:- The Column with bi-axial eccentric loading or generally encountered in Corner Columns which are rigidly Connected to the top of the Column.

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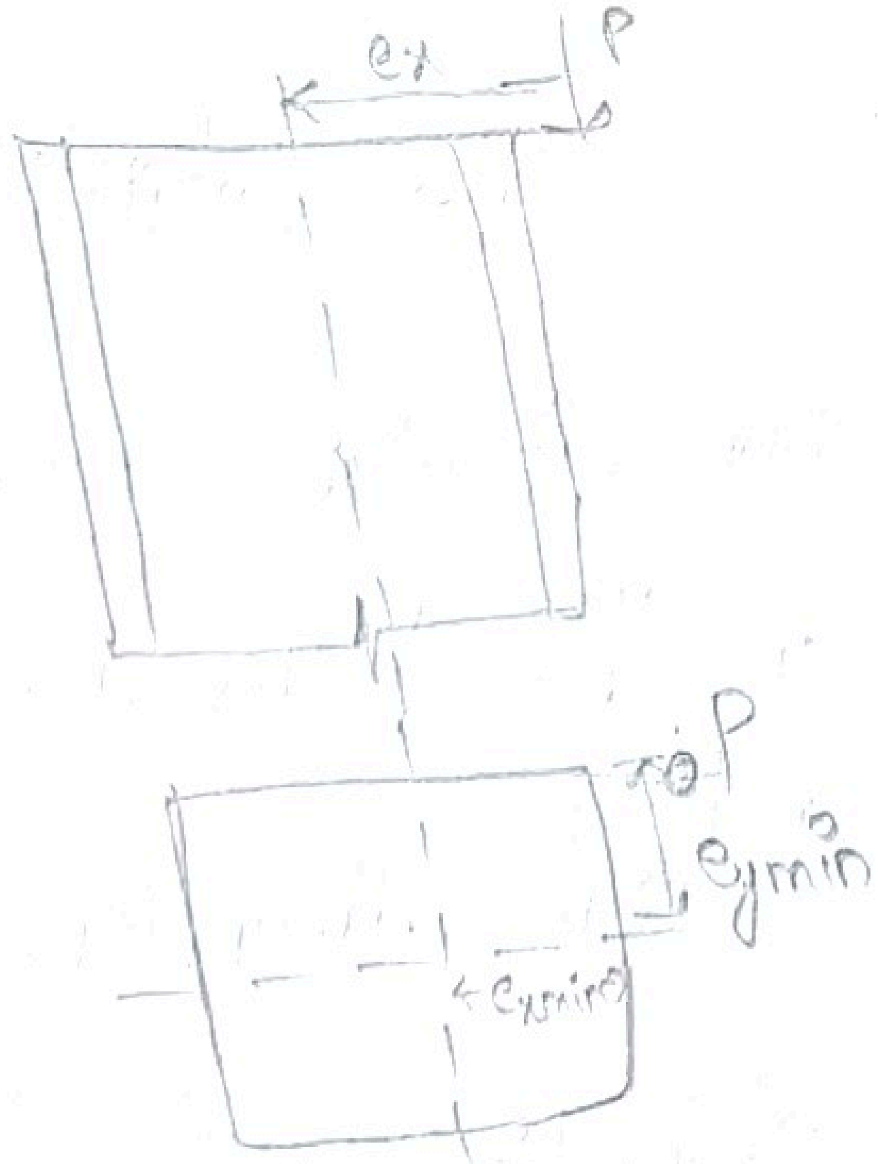

3) Based on slenderneous ratio:-

The Columns Can be classified as two types.

1) Short Column.
2) Long Column (or) Slender Column.

1) Short Column:- As per IS 456:2000 Code recomands the effective length to the least lateral dimension ratio less than 12 then refer to as short Column. i.e, $\frac{Lex}{D} \leq 12$

2) Long Column (or) Slender column:- As per IS 456:2000 Code recom-ands the effective length to the least lateral dimension ratio greater than 12 then refer to as long Column. i.e, $\frac{Lex}{D} > 12$.

Then it is also called as slender Column.

Where Lex = effective length along the major axis of a Column.

D = depth with respect to major axis.

Ley = effective length along the minor axis of a Column.

b = width of the member.

If any of these ratios is equal to (or) more than 12 then it may be treated as slender Column.

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The definition is not suitable for non-rectangular non-circular sections where the slenderness ratio is defined in the better terms of radius of gyration rather than the lateral dimension.

Effective length of Columns:-

The effective length of column is depends upon the unsupported length (distance between end restraints) and the boundary conditions at the end of the columns due to the conditions of the framing and other members. The effective length (lef) can be expressed in the form of

$L_{ef} = K \cdot L$

Where L = unsupported length (or) clear height of the column.

K = Effective length ratio (or) a constant depending upon the degrees of rational and translational and restraints at the ends of the columns.

For the design purpose the effective length l may be considered from table No 28 of IS 456: 2000.

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Slenderneous limits:-

The Column dimensions should be selected in such a way that it fails by material failure only and not by buckling.

For slenderneous limits can be considered clause No 25.3.2 of IS 456:2000. Pg No- 49. $\left(\frac{100b^2}{D}\right)$

Minimum Eccentricity:-

All Columns should be designed minimum Eccentricity as per Clause No 25.4, Pg No-42.

$$e_{min} = \frac{L}{500} + \frac{D}{30}$$

$\not< 20mm$.

Where L = unsupported length.

D = lateral dimensions in the plane of pure bending.

For Non-rectangular and non-circular Columns the

$e_{min} = \frac{Le}{300}$ (or) 20mm which ever is greater.

Design of Short Columns under axial Compression:-

Assumptions:-

*) The main assumptions made for limit state design of Columns falling under pure Compression as specified in clause No. 39.1

1) The maximum Compressive strain in Concrete in axial Compression is taken as 0.002.

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2) The plane Sections remains plane in Compression.

3) The design of Stress-Strain Curve of steel in Compression is taken to be the Same in tension.

According to the design strength of the short Columns are expressed as under pure axial loading Condition.

$$P_u = 0.4 f_{ck} A_c + \cancel{0.67 f_y} A_{sc} \cdot f_{sc}.$$

where, $f_{sc} = 0.87 f_y$.

The IS 456 : 2000 Code requires that all Columns Should be designed for minimum eccentricity of 0.05 times of lateral dimension. Hence the final expression for ultimate load 'P_u' by reducing 10% in the above equation ~~at~~ and formed as $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$.

$A_g = A_c + A_{sc}$

Gross Crosssectional Area.

$$0.4 f_{ck} (A_g - A_{sc})$$

$$0.4 f_{ck} \cdot A_g + [0.67 f_y - 0.4 f_{ck}] A_{sc}$$

$$0.4 f_{ck} A_g - 0.4 f_{ck} \cdot A_{sc} + 0.67 f_y A_{sc}$$

$$P_u = 0.4 f_{ck} A_g + [0.67 f_y - 0.4 f_{ck}] \cdot A_{sc}$$

Where P_u = axial ultimate load on the member.

f_{ck} = characteristic Compressive strength of Concrete.

A_c = Area of Concrete.

f_y = characteristic Compressive strength of Steel.

A_{sc} = Area of Steel in Concrete. (or) Area of longitudinal reinforcement.

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1) Design the reinforcement in the circular column of diameter 300mm with hellical reinforcement to support a factored load of 1500kN. The Columns has an unsupported length of 3m and is braced against side sway. Adopt M_{20} grade of concrete and Fe_{415} steel.

Sol:- Given that

a) $D = 300mm$.

$L = 3m$.

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$

$P_u = \text{Load} = 1500kN$.

b) $\frac{L_{ex}}{D} = \frac{\cancel{3000}^{10}}{\cancel{300}} = 10 < 12$.

$\frac{L_{ex}}{D} < 12$ [short column].

c) Minimum eccentricity:-

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right].$$

$$= \left[\frac{\cancel{3000}^{6}}{\cancel{500}} + \frac{\cancel{300}^{10}}{\cancel{30}}\right]$$

$= 16mm < 20mm$.

$0.05 \times D = 0.05 \times 300 = 15mm < 20mm$.

Hence it is an axially loaded compression member.

d) Longitudinal Reinforcement:-

$$P_u = 0.4 f_{ck} \cdot A_g + \left[0.67 f_y - 0.4 f_{ck}\right] \cdot A_{sc}.$$

$$A_g = \pi/4 \times D^2 = \pi/4 \times 300^2 = 70685.83 mm^2.$$

$$1500 \times 10^3 = 0.45 \times \cancel{415}^{20} \times 70685.83 + \left[0.67 \times 415 - 0.45 \times 20\right] \times A_{sc}.$$

$$1500 \times 10^3 = \cancel{3.55 \times 10^9 A_{sc}}\ 269.05 A_{sc} + 636.17 \times 10^3$$

$$A_{sc} = 3210.66 mm^2.$$

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choose 28ϕ bars.

$A_{sc} = n \times \pi/4\, d^2$

$3210.66 = n \times \pi/4 (28)^2$

$n = \cancel{16.88}\ 5.21$

$n = 6$

$A_{sc} = 6 \times \pi/4 (28)^2$

$A_{sc\,prov} = 3694.51 mm^2$

Logitudinal Reinforcement
Pg No-48
(clause - 26.5.3.1)
$\not< 0.8\%$ of G.C.A
$\not> 6\%$ of G.C.A

$A_{sc\,min} = 0.8\%$ Gross cross sectional area.

(or)

$= \frac{0.8}{100} \times \pi/4 \times (300)^2$

$A_{sc}\ min = 565.48 mm^2$

Hellical reinforcement design:- Design clause No. 39.4.1

$\frac{\text{Volume of helical reinforcement } V_{hs}}{\text{Volume of Core } V_C} \not< 0.36 \left(\frac{A_g}{A_c} - 1\right) f_{ck}/f_y$.

Assuming clear cover of 40mm over spirals then

Core diameter = 300 - 2 times of 40.

$= 300 - 2 \times 40 = 300 - 80 = 220 mm$

Then Area of Co~~ncrete~~re, $A_c = \left[\frac{\pi}{4} \times 220^2 - 3694.51\right]$

$\boxed{A_c = 34318.76 mm^2.}$

Volume of Core, $V_c = A_c \times 1000$

$= 34318.76 \times 1000$

$A_c \times L$
$L = 1m$
(or)
1000mm

$\boxed{V_c = 34.31 \times 10^6 mm^3.}$

The Gross Sectional area $A_g = \pi/4 \times 300^2$

$\boxed{A_g = 70685.8 mm^2}$

Using 8mm diameter for hellical reinforcement at a pitch(P) of 300mm the Volume of hellical per meter length is given by

$$V_{NS}/m = \frac{\text{circumference} \times \text{Area of helix}}{\text{pitch}} \times 1000\,mm$$

(or)

$$V_{NS} = \frac{\text{circumference} \times \text{Area of helix}}{\text{pitch.}} = \frac{\pi \times D \times \pi/4 (8)^2}{P} \times 1000$$

$$= \frac{\pi \times 220 \times \pi/4 (8)^2 \times 1000}{P}$$

$$V_{NS} = \frac{34.74 \times 10^6}{P}$$

$$\frac{V_{NS}}{V_C} \not< 0.36 \left[\frac{A_g}{A_c} - 1\right] f_{ck}/f_y$$

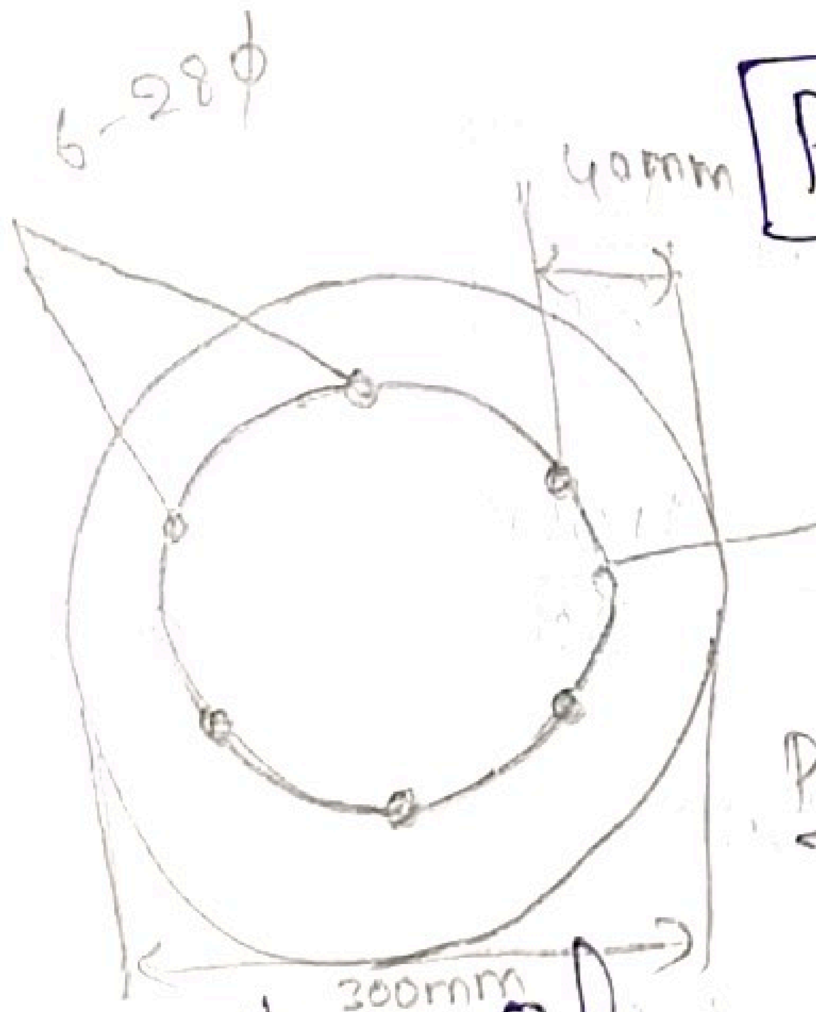
$$\frac{34.74 \times 10^6}{34.31 \times 10^6 \times P} \not< 0.36 \left(\frac{70685.8}{34318.76} - 1\right) 20/415$$

$$1.01/P = 0.018$$

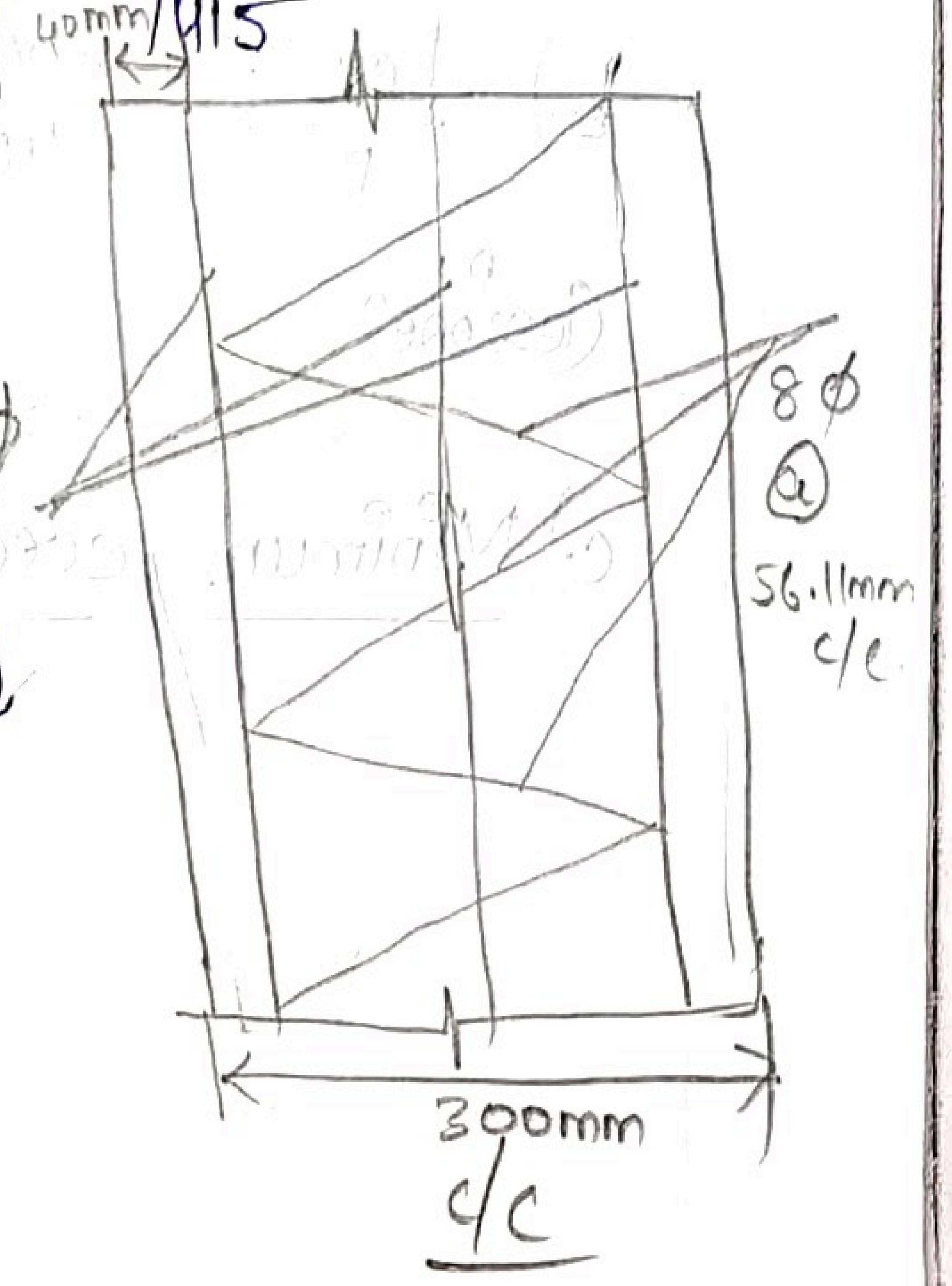
$$P = \frac{1.01}{0.018}$$

$$\boxed{P = 56.11\,mm\ c/c.}$$

6-28φ



Plan



2) Design the reinforcement of a column of size 400mm x 600mm Subjected to an axial load of 2000kN the column has an unsupport-ed length of 3m and is brassed against side sway in both the directions. Use M_{20} grade of Concrete and Fe_{415} steel

Sol:- Given that

$b = 400mm$

$d = 600mm.$

a) $L = 3m$

$f_{ck} = 20\,N/mm^2$

$f_y = 415\,N/mm^2$

$P_u = \text{Load} = 2000kN.$

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b.) $\frac{L_{ex}}{D} = \frac{\cancel{3000}}{\cancel{600}} = 5 < 12$ (short Column)

c) Minimum eccentricity :-

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right]$$

$$= \left[\frac{\cancel{3000}}{\cancel{500}} + \frac{\cancel{600}}{\cancel{30}}\right]$$

$$= 26mm$$

b.) $\frac{L_{ex}}{D} = \frac{3000}{400} = 7.5 < 12$ [Short Column].

(less one)

c.) Minimum eccentricity :-

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right]$$

$$= \frac{3000}{500} + \frac{400}{30}$$

$$e_{min} = 6 + 13.33$$

$$e_{min} = 19.33 < 20mm$$

(or)

$$0.05D = 0.05(400) = 20mm.$$

The given Column is to be designed as short column.

d) Longitudinal reinforcement :-

$$P_u = 0.4 f_{ck} A_g + [0.67 f_y - 0.4 f_{ck}] \cdot A_{sc}.$$

$$A_g = \pi/4 D^2 = \pi/4 \times (400)^2$$

$$A_g = b \times D = 400 \times 600 = 240,000 mm^2$$

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$P_u = 2000 \times 1.5 = 3000$

$3000 \times 10^3 = 0.45 \times 20 \times 24 \times 10^4 + [0.67 \times 415 - 0.45 \times 20] A_{sc}$

(P_u = factored load)

$840{,}000 = 269.05 A_{sc}$

$A_{sc} = 3122.09 \ mm^2$

choose 28mm ϕ bars.

$3122.09 = n \times \pi/4 (28)^2$

$n = 5.07$

$\boxed{n = 6 \text{ bars.}}$

$A_{sc} = 6 \times \pi/4 (28)^2$

$A_{sc} = 3694.51 \ mm^2$

$A_{sc\,min} \not< 0.8\%$ G.C.A

$= \frac{0.8}{100} \times \cancel{\pi/4} \times (400) \times 600$

$= 1920 \ mm^2$

Lateral ties :- pg No 49 c-2

$\not< 1/4$ (large longitudinal bar) $= \not< 1/4 (28)$

$= 7 \ mm.$

$\angle$ 16mm.

7m not their

choose 8mm dia.

(pg No-49, c-1) Pitch, L.L.D = 400mm.

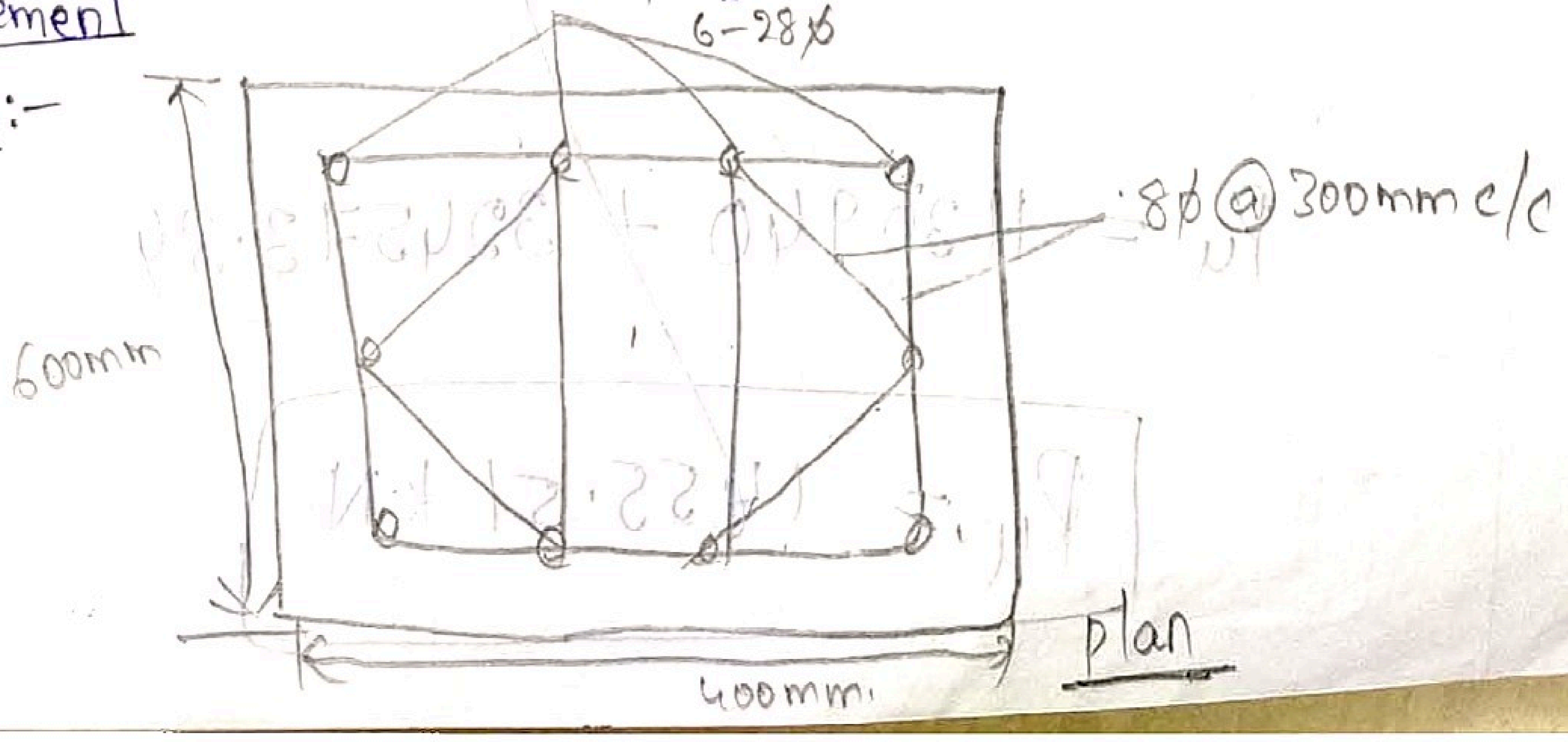
16 (smallest diameter) = 16 (28) = 448mm

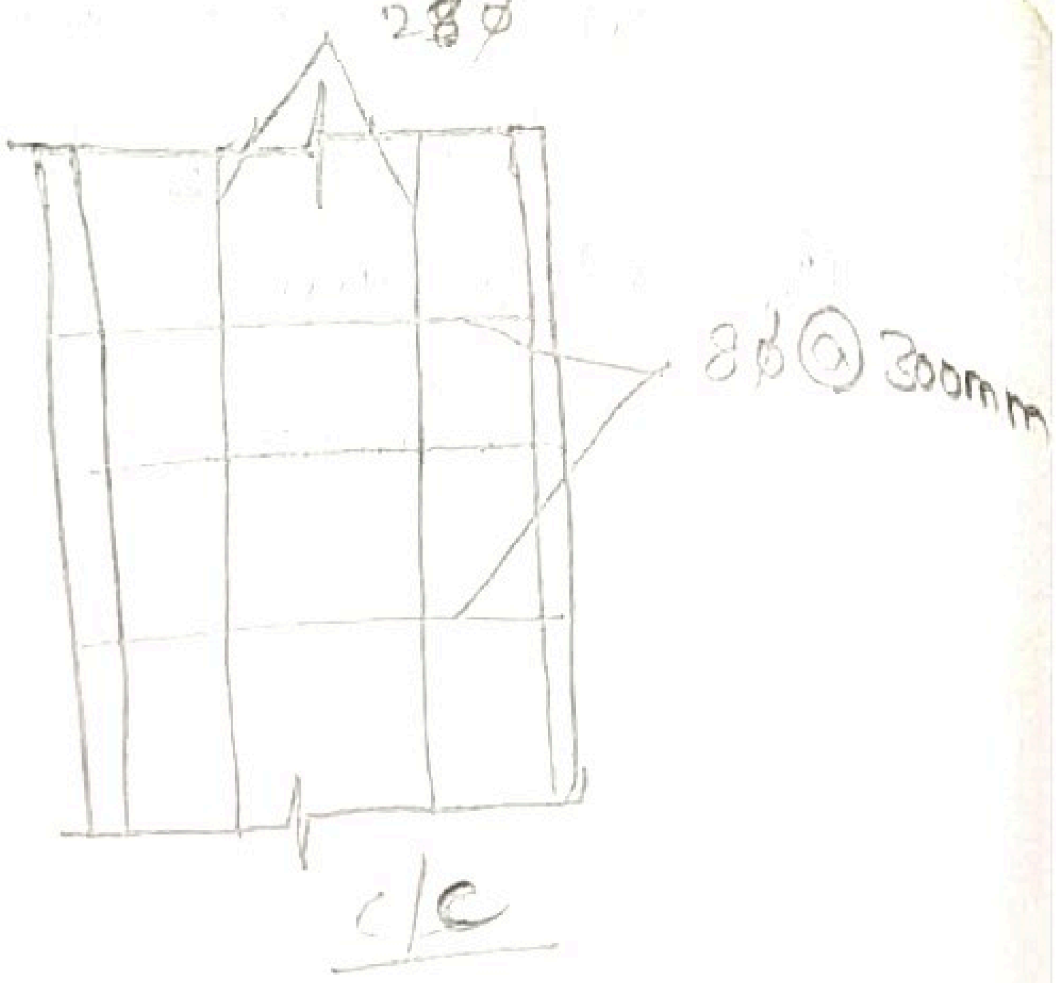
= 300mm

less.

$\therefore$ Provide 8ϕ @ 300mm c/c

Reinforcement Details:-





3.) A Short circular Column of diameter 400mm is reinforced with 6 bars of 16mm diameter. Find the axial factored load on the Column. Use M_{20} grade of Concrete and Fe_{415} Steel.

Sol:- Given that,

$d = 400mm$.

$n = 6 bars$.

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$.

$P_u = 0.45 f_{ck} \cdot A_g + [0.67 f_y - 0.45 f_{ck}] \cdot A_{sc}$.

$A_g = \pi/4 \times (400)^2$

$A_g = 125.66 \times 10^3 mm^2$.

$A_{sc} = n \times \pi/4 (d^2)$

$A_{sc} = 6 \times \pi/4 (16)^2$

$= 1206.37 mm^2$.

$P_u = 0.45 \times 20 \times 125.66 \times 10^3 + [0.67 \times 415 - 0.45 \times 20] \times 1206.37$

$P_u = 1130940 + 324573.84$

$\boxed{P_u = 1455.51 kN.}$

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$P_u = 0.4 f_{ck} \cdot A_c + A_{sc} \cdot f_{sc}$

(or)

$f_{sc} = 0.87 f_y$.

$P_u = 0.4 f_{ck} A_g + [0.67 f_y - 0.4 f_{ck}] \cdot A_{sc}$

$f_{sc} = 0.87 \times 415$

$P_u = 0.4 \times 20 \times 125.66 \times 10^3 + [0.67 \times 415 - 0.4 \times 20] \times 1206.37$

$f_{sc} = 361.05 N/mm^2$

$$\boxed{P_u = 1331.06 \, KN}$$

4.) Design the reinforcement for the short axially loaded square Columns of size 300mm x 300mm to support a load of 750KN. Use M_{20} and Fe415.

Sol:-

Given that:-

$b = 300mm$

$d = 300mm$

$P_u = 750KN$.

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$.

Longitudinal reinforcement:-

$P_u = 0.45 f_{ck} A_g + [0.67 f_y - \cancel{0.45 f_{ck}}] \cdot \cancel{A_{sc}}$.

$A_g = \overset{s \times s}{\cancel{b \times D}} = 300 \times 300 = 90 \times 10^3 mm^2$.

factored load:-

$1.5 \times 750 \times 10^3 \Rightarrow \cancel{0.45} = 1.125 \times 10^6 N$.

$P_u = 0.4 f_{ck} \cdot A_g + [0.67 f_y - 0.4 f_{ck}] \cdot A_{sc}$.

$1.125 \times 10^6 = 0.4 \times 20 \times 90 \times 10^3 + [0.67 \times 415 - 0.4 \times 20] \cdot A_{sc}$.

$405000 = 270.05 \, A_{sc}$

$$\boxed{A_{sc} = 1499.72 mm^2}$$

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$h = 4.7$

$n = 5$ bars

$A_{sc} = n \times \pi/4 \, d^2$

$A_{sc} = 5 \times \pi/4 (20)^2$

$A_{sc} = 1570.79 \, mm^2$

$A_{sc} \, min = 0.8\% \, G.C.A$

$= \frac{0.8}{100} \times (300)(300)$

$A_{sc} \, min = 720 \, mm^2$

Lateral ties:- [pg NO 49 c-2]

$\not< 1/4$ (large longitudinal bar) $\Rightarrow \not< 1/4(20)$

5.8mm

< 16mm.

~~Choose 8mm dia.~~

pitch:- [pg No 49 c-1]

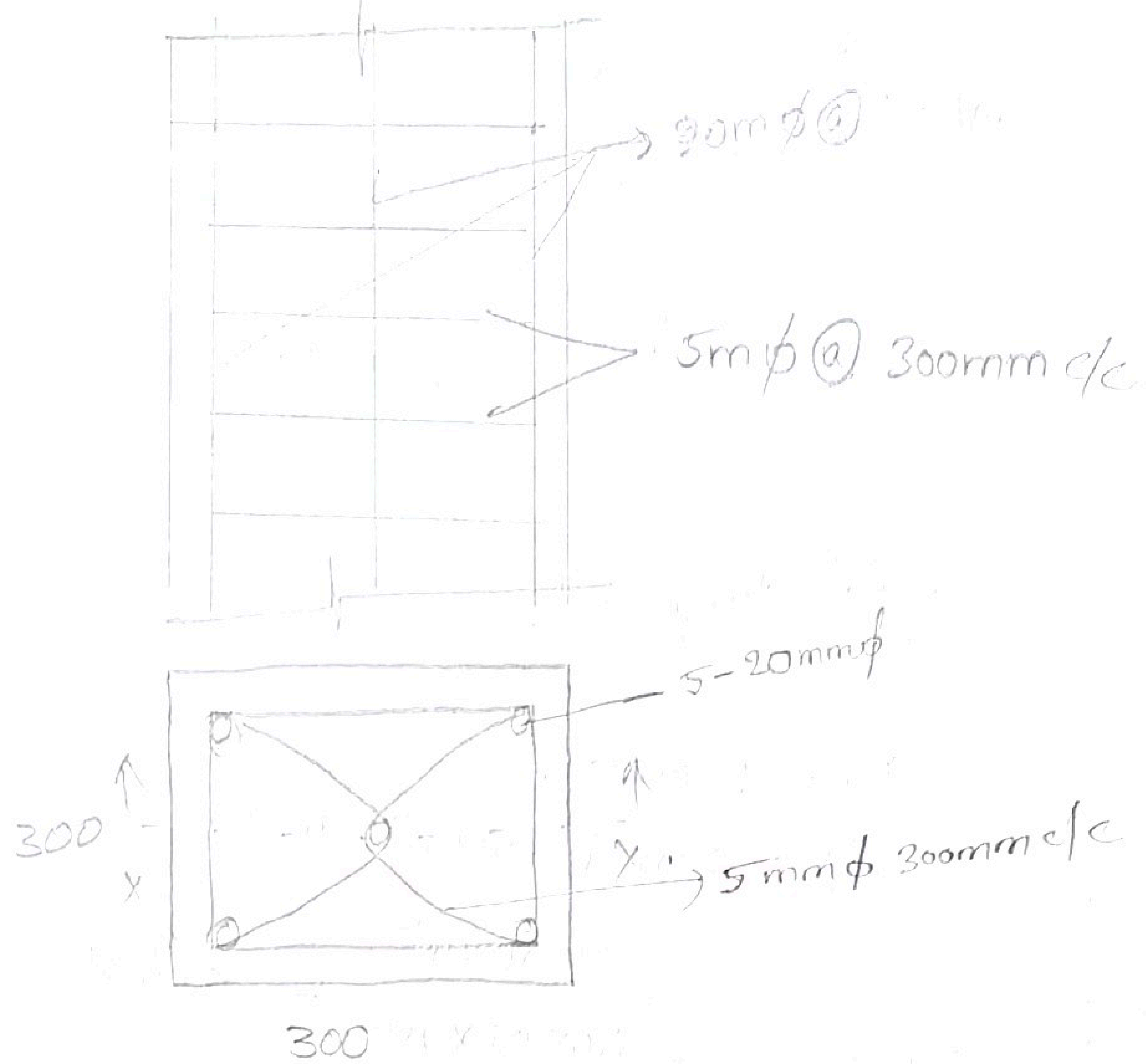
L.L.D = 300mm

16 (Smallest) = 16 (20) = 320 mm } less.

= 300mm

Provide 5m ø @ 300mm c/c.

Reinforcement Details: -



Columns with uni-axial Bending :-

1.) Design a square column with uni-axial Bending by the following data. size of the column 0.45m x 0.45m (or) 450mm x 450mm. The factored moment 200kN-m, factored load 2500kN. Use M_{20} grade of Concrete and Fe415 Steel.

factored moment is only one i.e, is uniaxial Bending

Arrangment of reinforcement

a.) on two sides b.) on four sides.

Given that :-

Size = 450 x 450mm

factored load = 2500kN

factored moment = 200kN-m

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$.

Assume the moment due to the minimum eccentricity is to be less than the actual moment.

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$P = 0.10 \times 20$

$P = 2$

$P = \frac{100 A_{st}}{bd}$

$2 = \frac{100 A_{st}}{450 \times 450}$

$\boxed{A_{st} = 4050 mm^2}$

$4050 = n \times \pi/4 (25)^2$

$n = 8.2$

$\boxed{n = 9 \text{ bars}}$

a) Lateral ties:-

$\not< 1/4$ (large longitudinal bar) $\rightarrow \not< 1/4 (25)$

$= 6.25 mm$

$< 16mm$

Choose 8mm dia.

Pitch → L.L.D = 450mm

16 (smallest) = 16(25) = 400mm

= 300mm

less.

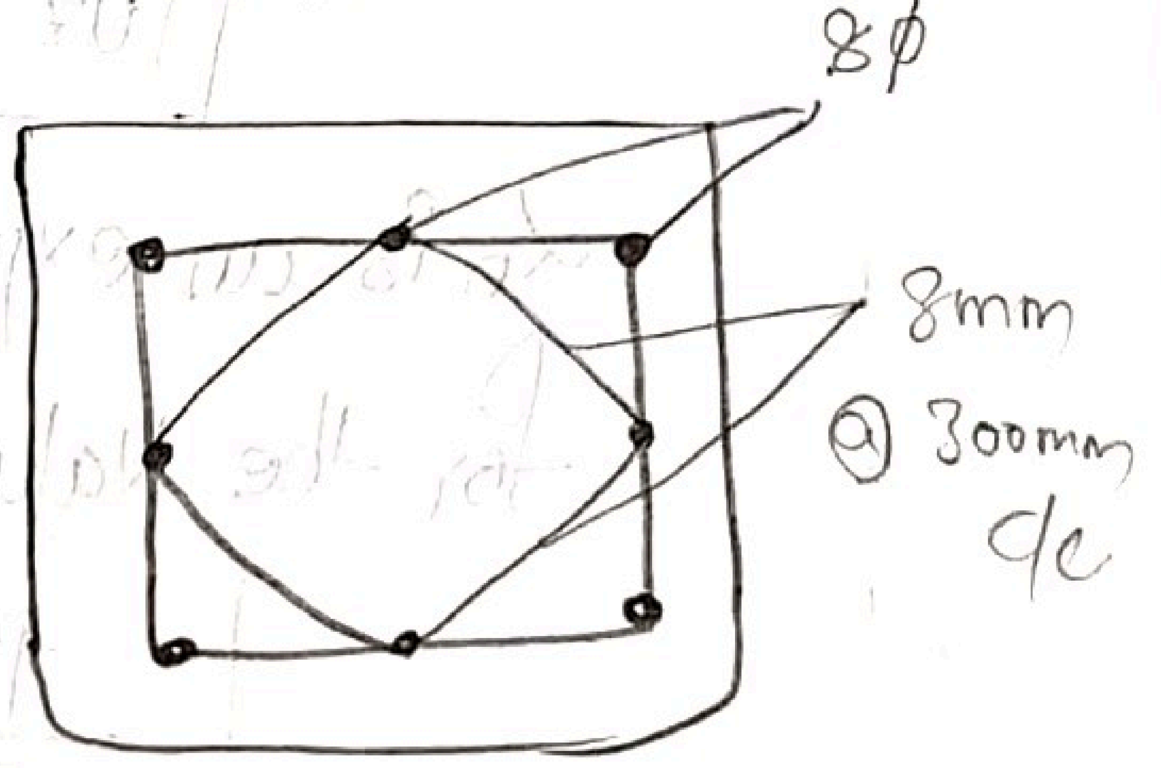
Provide 8mm ϕ @ 300mm c/c.

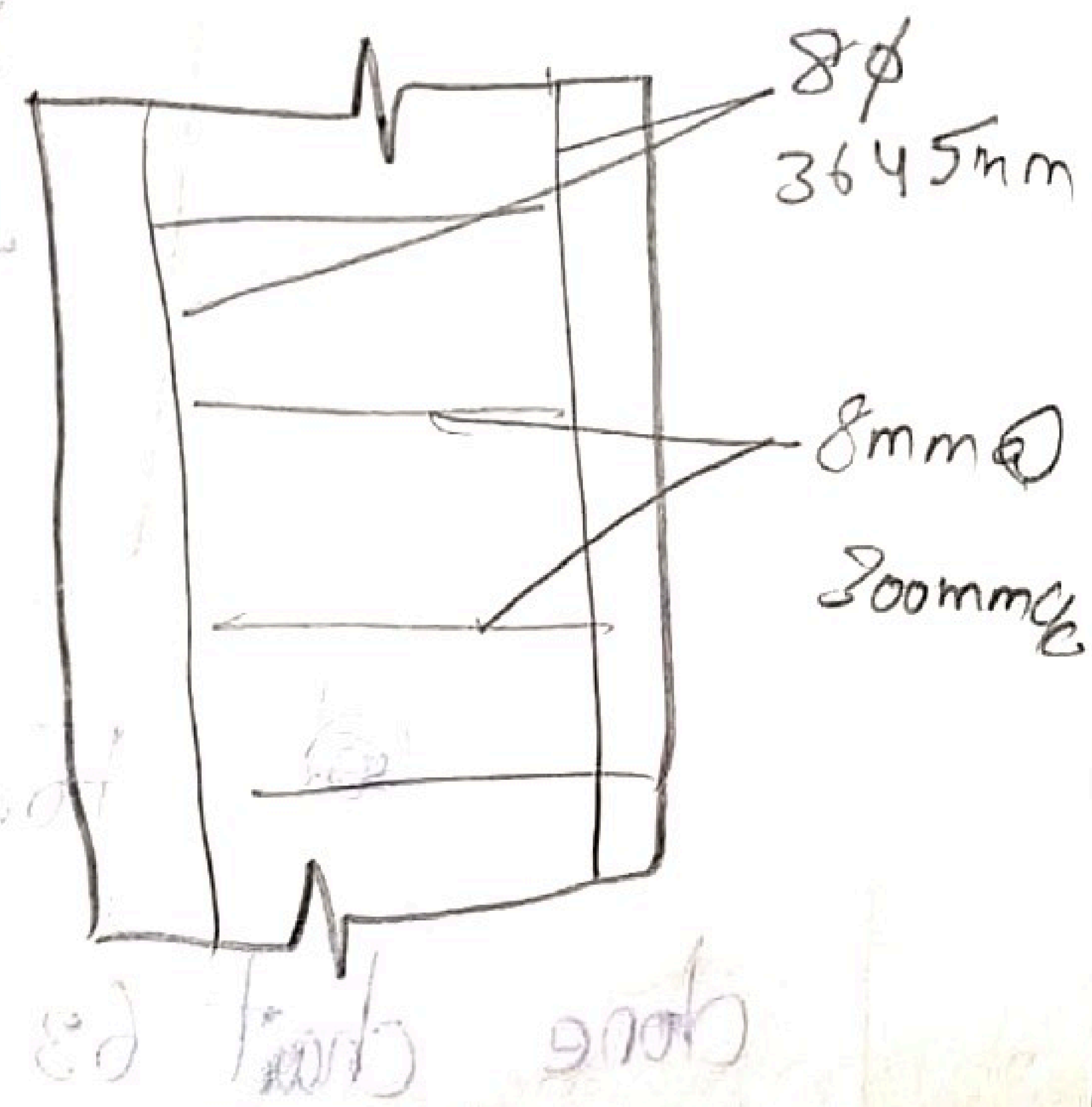
b) Lateral ties:- $\not< 1/4 (25)$

$= 6.25mm$

$< 16mm.$

p.





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Columns with Biaxial Bending:-

Exact design of members subjected to an axial load and biaxial bending is extremely laborious. There fore the code permits the design of such members by the following equation.

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1.0$$

M_{ux}, M_{uy} = moments about x and y axes due to design loads.

M_{ux1}, M_{uy1} = maximum uniaxial moment capacity for an axial load of P_u, bending about x and y axes respectively, and

α_n is related to P_u/P_{uz}.

Where $\boxed{P_{uz} = 0.45 f_{ck} \cdot A_c + 0.75 f_y A_{sc}}$

α_n is an exponent whose value depends upon $\frac{P_u}{P_{uz}}$.
for the values of α_n.

$\frac{P_u}{P_{uz}}$	α_n
< 0.2	1.0
> 0.8	2.0

For intermediate values linear interpolation may be done chart 63 can be use for evaluating P_{uz}.

→ For different values of P_u/P_{uz} the appropriate values

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of α_n has to be taken from Chart No-64 for the equation

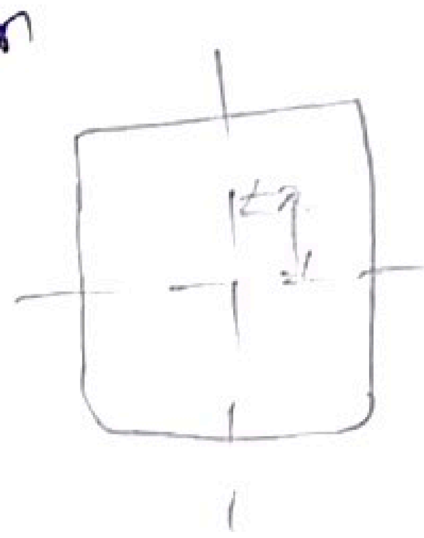
$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left\{\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} = 1$$

1.) Determine the reinforcement of a short column subjected to biaxial Bending with the following data.

Size of the column = 400mm x 600mm, factored load = 1600KN, factored moment acting parallel to the larger dimensions 120KN-m, factored moment acting parallel to the shorter dimensions 90KN-m, M15 grade of concrete and Fe415 steel, equally distributed on four sides.

Sol:- Given that:-

Size of Column = 400mm x 600mm



Factored load, P_u = 1600KN.

$f_{ck} = 15 N/mm^2$.

$f_y = 415 N/mm^2$.

$M_{ux} = 120 KN-m$

$M_{uy} = 90 KN-m$

As a first trial, Assume the reinforcement percentage

$p = 1.2$ [p = 1, 1.2, 1.3, 1.4 ...]

$\Rightarrow p/f_{ck} = \frac{1.2}{15} = \frac{12}{150} = 0.08$

Uniaxial moment capacity of the Section x-x axis.

$\frac{d'}{D}$ ratio wheter it is square, rectangular

$$\frac{P_u}{f_{ck} \cdot bd} = \frac{1600 \times 10^3}{400 \times 600 \times 15}$$

$= ~~6.66~~\ 0.44$

Assume cover 40mm and bar dia as 25mm.

$$d' = 40 + \frac{25}{2} = 52.5mm$$

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$$d'/D = \frac{52.5}{600}$$

$$\simeq 0.08$$

$$d'/D \simeq 0.1$$

Refer chart-44 of sp-16 curves

$x = 600$ $\frac{P_u}{f_{ck} \cdot bd}$ 0.44 $\frac{P}{f_{ck}} = 0.08$

$\frac{M_{ux}}{f_{ck} \cdot bd^2} = ?$

$p = 1.2$

$\frac{P_u}{f_{ck} bd} = 0.44$

$\frac{P}{f_{ck}} = 0.08$

$$\frac{M_{ux_1}}{f_{ck} \cdot bd^2} = 0.09 \text{ (from chart)}$$

$$M_{ux_1} = 0.09 \times 15 \times 400 \times 600^2$$

$$\boxed{M_{ux_1} = 194.4 \text{ KN-m}}$$

uni-axial bending capacity of moment of about Y-Y axis

$y = 400$.

$$d'/D = \frac{52.5}{400}$$

$$= 0.13$$

$$\simeq 0.15$$

Refer chart-45 of sp-16 curves.

$$\frac{P_u}{f_{ck} \cdot bd} = \frac{1600 \times 10^3}{15 \times 600 \times 400}$$

$$P_u = 0.44$$

$$\frac{M_{uy_1}}{f_{ck} \cdot bd^2} = 0.083$$

$$M_{uy_1} = 0.083 \times 15 \times 400^2 \times 600$$

$$\boxed{\cancel{M_{uy_1} = 179.28 \text{ KN-m}}}$$

$$\boxed{M_{uy_1} = 119.52 \text{ KN-m}}$$

Calculation of P_{uz}.

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{sc}.$$

Refer chart-63 corresponding to $\boxed{p \to 1.2}$ (percentage of steel)

$f_y = 415$ and $f_{ck} = 15$.

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$$\frac{P_{uz}}{A_g} = 10.3 \text{ (from chart)}$$

$$P_{uz} = 10.3 \times A_g$$

$$P_{uz} = 10.3 \times b \times d$$

$$P_{uz} = 10.3 \times 400 \times 600$$

$$P_{uz} = 2472 \text{ kN.}$$

$$\frac{P_u}{P_{uz}} = \frac{1600}{2472} = 0.64$$

$$\frac{M_{ux}}{M_{ux1}} = \frac{120}{194.4} = 0.617$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{90}{119.52} = 0.753$$

Refering to chart 64 the permissible value of $\frac{P_u}{P_{uz}}$, $\frac{M_{ux}}{M_{ux1}}$, $\frac{M_{uy}}{M_{uy1}}$ and $\frac{P_u}{P_{uz}} = 0.7$

[$\frac{P_u}{P_{uz}} = 0.67$ chart 64 graph 0.7 graph value is less & Assuming P_u is wrong Again P_u is assumed.]

$$\frac{P_u}{P_{uz}} = 0.7 \simeq 0.8$$

then $\alpha_n = 0.2$

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy1}}\right]^{\alpha_n} \leq 1$$

$$[0.617]^{2} + [0.753]^{2} \leq 1$$

$$= \cancel{0.882}\ 0.94 \leq 1$$

Hence OK.

Lateral ties
$\frac{1}{4}$ ϕ (large longitudinal bar)
$\Rightarrow \frac{1}{4} \times \phi(25)$
$= 6.25\text{mm}$
$< 16\text{mm}$

Pitch:- L.L.D = 400mm
16(smallest) = 16(25)
= 400mm
= 300mm

Provide 6mm ϕ @ 300mm c/c

Find Ast:- $P_{uz} = 0.45 f_{ck} \cdot A_g + [0.75 f_y - 0.45 f_{ck}] A_{sc}$

$$2472 \times 10^3 = 0.45 \times 15 \times (400 \times 600) + [0.75 \times 415 - 0.45 \times 15] A_{sc}$$

$$A_{sc} = 2798.02 \text{ mm}^2$$

Slender Compression members:- (Pg No-71, IS 456).

$$M_{ax} = \frac{P_u D}{2000}\left[\frac{l_{ex}}{D}\right]^2$$

$$M_{ay} = \frac{P_u b}{2000}\left[\frac{l_{ey}}{b}\right]^2$$

P_u → axial load

l_{ex} → effective length in respect of the major axis.

l_{ey} → effective length in respect of the minor axis.

D → depth of the cross-section at right angles to the major axis.

b → width

The following table gives the values of $\frac{e_{ax}}{D}$ & $\frac{e_{ay}}{b}$ for different values of slenderness ratio.

Additional eccentricity for slender compression Members.

l_{ex}/D (or) l_{ey}/b	e_{ax}/D (or) e_{ay}/b	l_{ex}/D (or) l_{ey}/b	e_{ax}/D (or) e_{ay}/b
12	0.072	25	0.312
13	0.085	30	0.450
14	0.098	35	0.612
15	0.113	40	0.800
16	0.128	45	1.012
17	0.145	50	1.250
18	0.162	55	1.512
19	0.181	60	1.800
20	0.200		

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1) Determine the reinforcement required for a column which is restrained against sway with the following data.

Size of Column = 400x300mm

effective length for bending parallel to larger dimension 6m,

effective length for bending parallel to shorter dimension 5m.

~~up~~ unsupported length = 7m.

Factored load = 1500KN

Factored moment in the direction of larger dimension 40KNm at top and 22.5KN-m at bottom.

Factored moment in the direction of shorter dimension 30KN-m at top and 20KN-m at bottom.

The column is bending double curvature (biaxial). The reinforcement will be distributed equally on four sides., Grade of concrete M30 and Fe415 steel.

Sol:- Given that,

Size of Column = 400x300mm.

$l_{ex} = 6m$, $l_{ey} = 5m.$, $P_u = 1500KN.$

(1) $\frac{l_{ex}}{D} = \frac{6000}{400} = 15 > 12$, $\frac{l_{ey}}{b} = \frac{5000}{300} = 16.6 > 12$

This is a long column [slender column.]

(2) Cal of e_{ax}:-

from table 1 of SP-16 curves.

$\frac{l_{ex}}{D} = 15 \rightarrow e_{ax}/D = 0.113$

$\frac{l_{ey}}{D} = 16.67$, $e_{ay}/b = \cancel{0.34}\ 0.139.$

16	0.128
16.67	?
17	0.145

$0.145 + \left[\frac{0.145 - 0.128}{17 - 16}\right] \times [16.67 - 17].$

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③ Cal. of additional moments :-

$M_{ax} = P_u \frac{e_{ax}}{D} \times D = 1500 \times 0.113 \times \cancel{400} \times (0.4)$

$= 67.8 \text{ KN-m}$

$M_{ay} = P_u \frac{e_{ay}}{b} \times b = 1500 \times 0.139 \times \cancel{300} \times 0.3$

$= 62.5 \text{ KN-m}$

$M_{ax} = \frac{P_u D}{2000}\left(\frac{l_{ex}}{D}\right)^2$

$M_{ay} = \frac{P_u b}{2000}\left(\frac{l_{ex}}{b}\right)^2$

The above moments will be has to reduce accordance with clause No-39.7.1.1 of IS.456 but multiplication factors can be evaluated only if the reinforcement is known.

For that assume (% of steel) $p = 3.0$

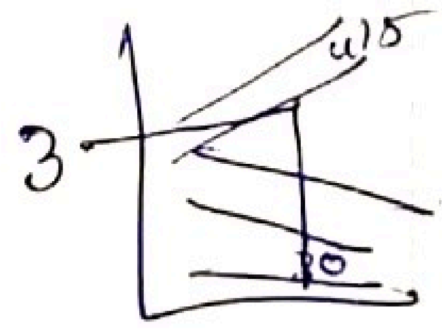
[with reinforcement equally provided on four sides]

$\frac{p}{f_{ck}} = \frac{3.0}{30} = 0.1$

Area of a gross section, $A_g = 300 \times 400$

$A_g = 120 \times 10^3 \text{ mm}^2$.

From chat - 63



$\frac{P_{uz}}{A_g} = 22.5 \text{ N/mm}^2$

$P_{uz} = 22.5 \times 120 \times 10^3$

$P_{uz} = 2700 \text{ KN}$.

④ Calculation of P_b:-

Assume 25mmφ & 40mm cover.

$d' = 25/2 + 40$

$d' = 52.5 \text{ mm}$

d'/D @ x-xaxis $= \frac{52.5}{400} = 0.131$

$\simeq 0.15$

d'/b @ y-yaxis $= \frac{52.5}{300} = 0.175$

$\simeq 0.2$

From table No-60, Pg No-171

P_b (@ x-xaxis)

$$P_b = \left(k_1 + k_2 \frac{P}{f_{ck}}\right) f_{ck} \cdot b \cdot D$$

$d'/D = 0.15 <$ $k_1 = 0.196$, $k_2 = 0.203$

$$P_b(x-x) = \left[0.196 + 0.203 \times \frac{3}{30}\right] 30 \times 300 \times 400$$

$$P_b(x-x) = 778.68 \, kN.$$

$P_b(y-y) \rightarrow d'/b = 0.2$

$k_1 - 0.184$

$k_2 - 0.028$

$$P_b(y-y) = \left[0.184 + 0.028 \times \frac{3}{30}\right] \times 30 \times 300 \times 400$$

$$P_b(y-y) = 672.48 \, kN.$$

Find K value:-

$$K_x = \frac{P_{uz} - P_u}{P_{uz} - P_{bx}}, \quad K_y = \frac{P_{uz} - P_u}{P_{uz} - P_{by}}$$

$$K_x = \frac{2700 \times 10^3 - 1500 \times 10^3}{2700 \times 10^3 - 778.68 \times 10^3}, \quad K_y = \frac{27000 - 1500}{2700 - 672.48}$$

$$K_x = 0.62, \quad K_y = 0.59.$$

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$M_{ax} = K_x \times 67.8$
$= 0.625 \times 67.8$
$= 42.37$ KN-m.

$M_{ay} = K_y \times 62.5$
$= 0.59 \times 62.5$
$= 36.87$ KN-m.

The additional moments due to slenderneous effects should be added to the initial moments as follows. (See Note 1 in clause No 38.7.1 of ~~IS 456 code~~ SP 16 curves.)

$M_{ux_i} = 0.6 \times 40 - 0.4 \times 22.5$ (large)
$= 15$ KN-m. (smaller)

$M_{uy_i} = (0.6 \times 30 - 0.4 \times 20) = 10$ KN-m

The above actual moments should be compared with the values which are calculated from minimum eccentricity and greater values can be taken as the initial moment and adding to the additional moments.

$e_{min_x} = \frac{L}{500} + \frac{D}{30}$
$= \frac{7000}{500} + \frac{400}{30}$
$= 27.33 > 20mm$

$e_{min_y} = \frac{L}{500} + \frac{b}{30}$
$= \frac{7000}{500} + \frac{200}{30}$
$= 24 > 20mm.$

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$M_{uxi} = P_u \times e_{max} = 1500 \times 27.33 = 40.9\ KN\text{-}m$

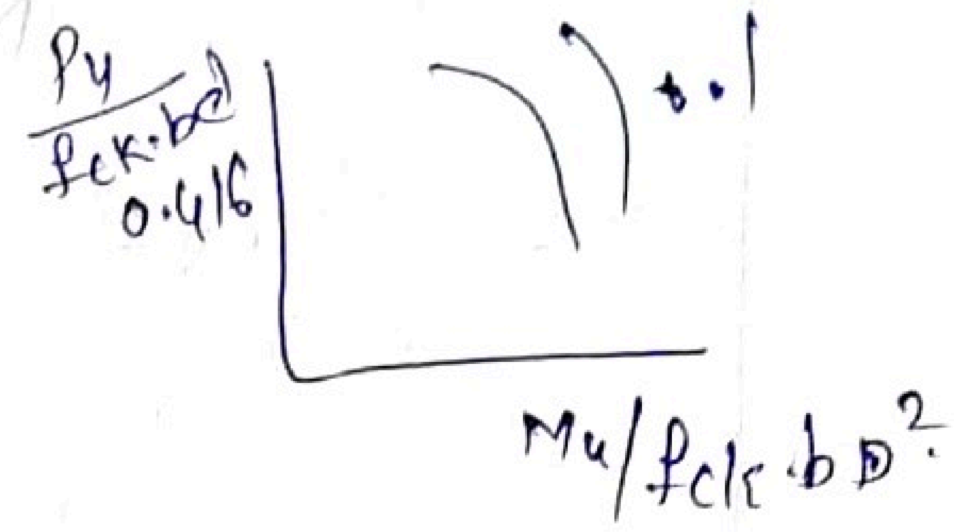
$M_{uyi} = P_u \times e_{min} = 1500 \times 24 = 36\ KN\text{-}m.$

The total moment in $M_{ux} = M_{ax} + M_{uxi} = 42.37 + 40.9$
$= 83.27\ KN\text{-}m.$

The total moment in $M_{uy} = M_{ay} + M_{uyi} = 36.87 + 36$
$= 72.87\ KN\text{-}m.$

Check for Bi-axial Bending:-

$d'/D = \frac{52.5}{400} \Rightarrow d'/D = 0.15$



$\frac{P}{f_{ck}} = \frac{3}{30} = 0.1$

$\frac{P_u}{f_{ck} \cdot bd} = \frac{1500 \times 10^3}{30 \times 300 \times 400} = 0.416$

uni-axial Bending in x-x

$\frac{M_{ux1}}{f_{ck} \cdot b \cdot D^2} = 0.104$

$M_{ux1} = 0.104 \times 30 \times 300 \times 400^2$

$M_{ux1} = 149.76\ KN\text{-}m$

$d'/D = \frac{52.5}{300} = 0.17$
$\simeq 0.2$

$\frac{P_u}{f_{ck} \cdot b \cdot D} = \frac{1500 \times 10^3}{30 \times 300 \times 400} = 0.416$

$P/f_{ck} = 3/30 = 0.1$

uni-axial Bending in Y-Y

$\frac{M_{uy1}}{f_{ck} \cdot D \cdot b^2} = 0.096$

$M_{uy1} = 0.096 \times 30 \times 400 \times 300^2$

$M_{uy1} = 103.68\ KN\text{-}m$

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$\frac{P_u}{P_{uz}} =$

$f_{ck} = 30$, $f_e = 415$ (chart 64) $P = 3.0$

$$\frac{M_{ux}}{M_{ux1}} = \frac{83.27}{149.76} = 0.55$$

$$\frac{M_{uy}}{M_{uy1}} = \frac{72.87}{103.68} = 0.70$$

from chart 64

$$\frac{P_u}{P_{uz}} = 0.5$$

$$P_u = 0.5 \times 2700$$

$$P_u = 1350\ KN.$$

$$\left[\frac{M_{ux}}{M_{ux1}}\right]^{\alpha_n} + \left[\frac{M_{uy}}{M_{uy}}\right]^{\alpha_n}$$

(from table).

0.2	1
0.5	.
0.8	2

$$2 + \left[\frac{2-1}{0.8-0.2}\right] \times \left[0.5-0.8\right]$$

$$= 2 + \left(\frac{2-1}{0.8-0.2}\right) \times \left[0.5-0.8\right]$$

$$\alpha_n = 1.5$$

$$(0.55)^{1.5} + (0.70)^{1.5} \leq 1$$

$$0.99 \leq 1$$

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Find Asc

$P_{uz} = 0.45 f_{ck} \cdot A_g + [0.75 f_y - 0.45 f_{ck}] \cdot A_{sc}$

$A_g = 300 \times 400$
$= 120000$

$P_{uz} = 0.45 \times 30 \times 120 \times 10^3 + [0.75 \times 415 - 0.45 \times 30] \cdot A_{sc}$

$2700 \times 10^3 = 0.45 \times 30 \times 120 \times 10^3 + [0.75 \times 415 - 0.45 \times 30) A_{sc}$

$A_{sc} = 3627.2 mm^2$

(or)

$A_{sc} = \frac{PbD}{100} = \frac{\cancel{1500 \times 10^3} \times 300 \times 400 \times 3}{100}$

$A_{sc} = 3600 mm^2$

Assume 25mm ϕ bars.

$A_{sc} = n \times \pi/4 (25)^2$

$3600 = n \times \pi/4 (25)^2$

$n = 7.3$

$n = 8$ bars.

$A_{sc} = 8 \times \pi/4 (25)^2$

$A_{sc} = 3926.9 mm^2$

$A_{sc\,min} = 0.8\,\%\,G.C.A$

$= \frac{0.8}{100} \times 300 \times 400$

$A_{sc\,min} = 960 mm^2$

Lateral ties (Pg No - 49 c-2)

$\not< 1/4$ (large longitudinal bar) $= \not< 1/4 (25)$
$= 6.25mm$

$< 16mm$

Pitch (Pg no 49, c-1)

L.L.D = 300mm
16 (smallest) = 16 (25) = 400 } less
= 300mm

Provide 6mm ϕ @ 300mm c/c.

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Pressure distribution below the footing:-

The pressure intensity depends upon.

1.) rigidity of the footing
2) Type of soil
3) Depth of foundation.

Variation of pressure ↓ with ↑ in depth of the foundation.

$\frac{P_u}{P_{uz}}$	α_n
≤ 0.02	1
0.25	1.084
0.3	1.167
0.35	1.250
0.4	1.333
0.45	1.417
0.5	1.500
0.55	1.584
0.6	1.667
0.65	1.750
0.7	1.833
0.75	1.917
≥ 0.8	2.

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CHAPTER 10

Limit State Design of Columns and Footings

10.1 INTRODUCTION

Structural concrete members in compression are generally referred to as columns and struts. The term 'Column' is associated with members transferring loads to the ground and the term 'strut' is applied to compression members is any direction such as those in a truss.The IS:456-2000 code clause 25.1.1 defines the column as a 'compression member' the effective length of which exceeds three times the least lateral dimension.The term 'pedestal' is used to describe a vertical compression member whose effective length is less than three times to least lateral dimension.

Axially loaded columns may fail in any of the following three modes :

1) Pure compression failure
2) Combined compression and bending failure
3) Failure by elastic instability.

The failure modes depend primarily on the slenderness ratio of the member which is turn depends on the cross sectional dimensions, effective length, and support conditions of the member.

10.2 Classification of Columns

a) Based on Type of Reinforcement

Depending on the type of reinforcement used, reinforced concrete columns are classified into the following three groups.

1) 'Tied Columns' in which the main longitudinal bars are confined within closely spaced lateral ties [Fig. 10.1 (a)]
2) 'Spiral Columns' having main longitudinal reinforcements enclosed within closely spaced and continuously wound spiral reinforcement [Fig. 10.1 (b)]
3) 'Composite Columns' in which the longitudinal reinforcement is in the form of structural steel section or pipes with or without longitudinal bars [Fig. 10.1 (c)]

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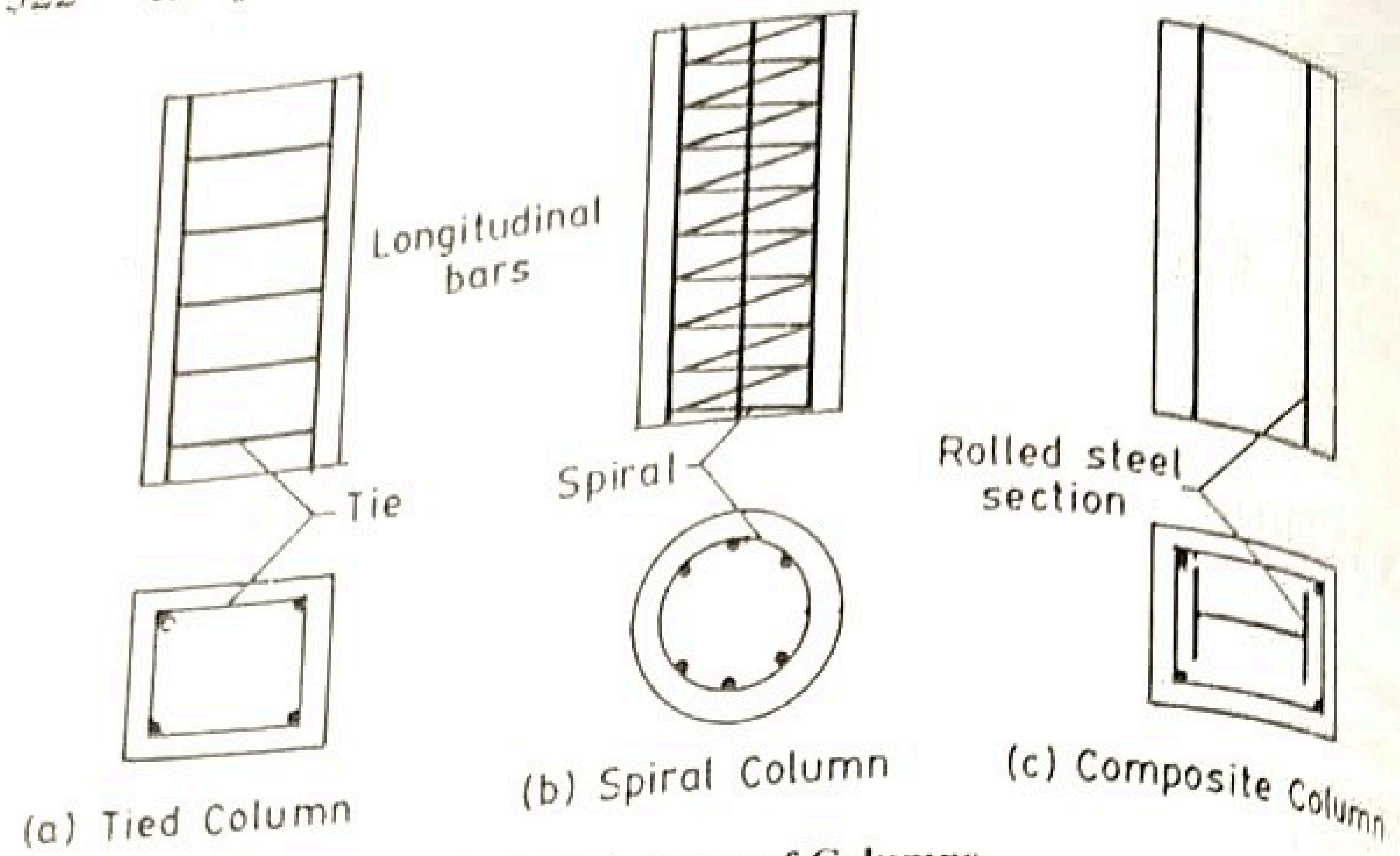

Fig. 10.1 Types of Columns

In general tied columns are the most commonly used having different shapes (Square, rectangular, T, L, circular etc).

Spiral columns are adopted with circular cross sections and also for square and octagonal sections.

b) Based on type of loading

Depending upon the type of loading columns may be classified into the following three types.

i) Axially loaded columns supporting concrete loads are relatively rare. Interior columns of multistoried buildings with symmetrical loads from floor slabs from all sides are common examples of this type [Fig. 10.2 (a)].

ii) Column with uniaxial eccentric loading are generally encountered in the case of columns rigidly connected to beams from one side only such as the edge columns [Fig. 10.2 (b)].

iii) Columns with biaxial eccentric loading is common in corner columns with beams rigidly connected at right angles on the top of the column [Fig. 10.2 (c)].

Eccentrically loaded columns have to be designed for combined axial force and bending moments.

c) Based on Slenderness Ratio

Depending on the slenderness ratio, (Effective length/Least lateral dimension)

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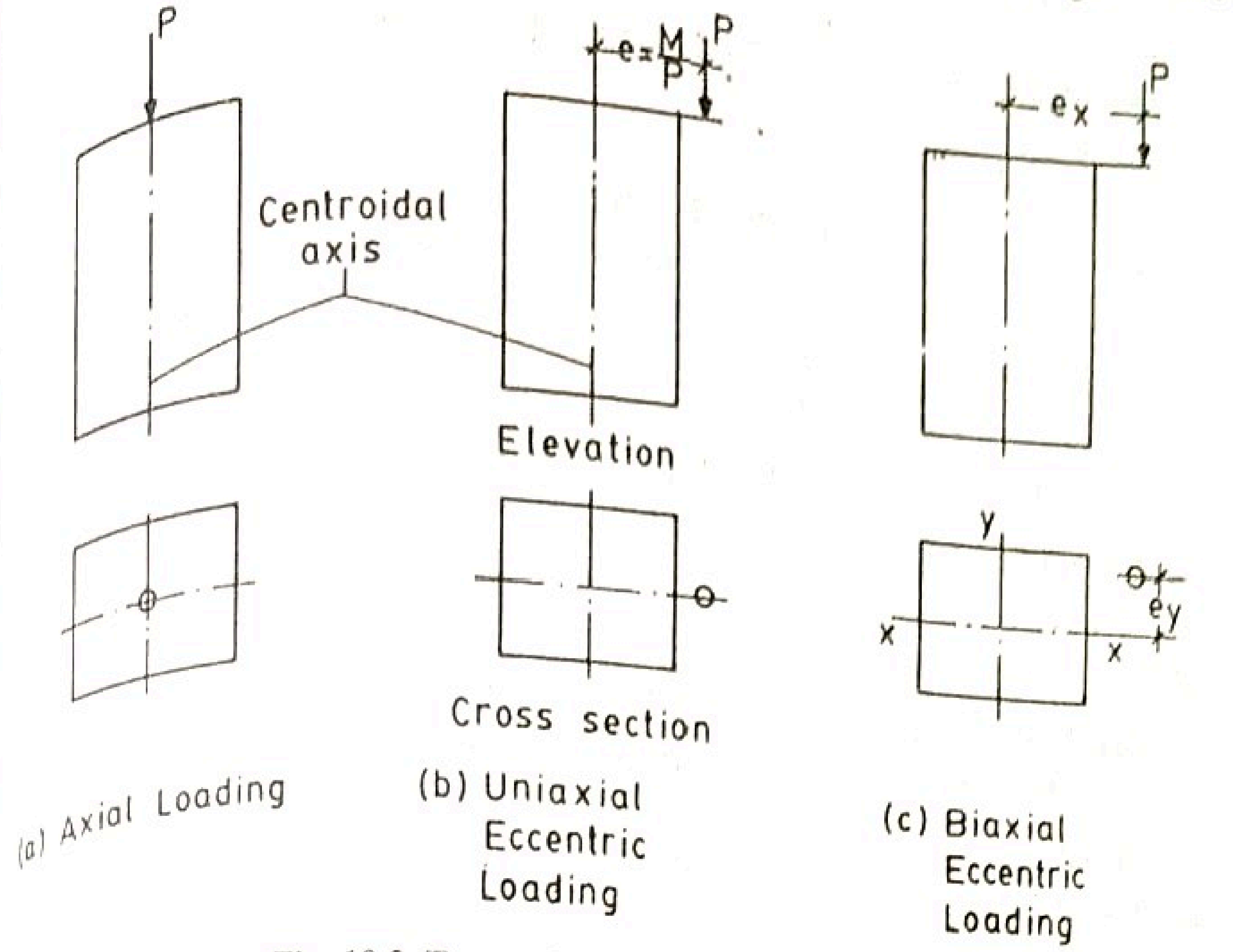

Fig. 10.2 Types of Loading on Columns

Columns may be classified as.

i) Short Columns
ii) Slender or Long columns

IS: 456-2000 code clause 25.1.2 classifies a rectangular compression member as short when both the slenderness ratio's (L_{ex}/D) and (L_{ey}/b) are less than 12,

Where L_{ex} = effective length in respect of major axis
D = depth in respect of major axis
L_{ey} = Effective length in respect of minor axis and
b = width of the member.

If any of these ratios is equal to or more than 12, then it is termed as slender or long column. This definition is not suitable for non-rectangular and non-circular sections where the slenderness ratio is better defined in terms of the radius of gyration rather than the lateral dimensions.

10.3 EFFECTIVE LENGTH OF COLUMNS

10.3.1 Computation of Effective Length

The effective length of a column depends upon the unsupported length (distance between lateral connections) and the boundary conditions at the ends of column due to the conditions of the framing beams and other members.

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The effective length 'L_{ef}' can be expressed in the form.

$$L_{ef} = kL$$

Where L = Unsupported length or clear height of columns

k = Effective length ratio or a constant depending upon the degrees of rotational and translational restraints at the ends of column.

The effective length of compression members depends upon the bracing and end conditions. For braced (Laterally restrained at ends) columns, the effective length is less than the clear height between the restrains, whereas for unbraced and partially braced columns, the effective length is greater than the clear length between the restraints.

For design purposes, assuming idealized conditions, the effective length Le may be assessed for different types of end conditions using the Table-10.1 (Table 28 of IS: 456-2000).

Table 10.1 Effective Length of Compression members (Table-28 of IS: 456-2000)

Degree of End Restraint of Compression Member	*Theoretical Value of Effective Length*	*Recommended Value of Effective length*
1	2	3
Effectively held in position and restrained against rotation at both ends	0.5 *L*	0.65 *L*
Effectively held in position at both ends, restrained, against rotation at one end	0.7 *L*	0.80 *L*
Effectively held in position at both ends, but no restrained rotation	1.00 *L*	1.00 *L*
Effectively held in position and restrained against rotation at one end, and at the other restrained against rotation but not held in position	1.00 *L*	1.20 *L*
Effectively held in position and restrained against rotation at one end, and at the other partially restrained against rotation but not held in position	–	1.50 *L*
Effectively held in position at one end but not restrained against rotation, and at the other end restrained against rotation but not held in position	2.00 *L*	2.00 *L*
Effectively held in position and restrained against rotation at one end but not held in position nor restrained against rotation at the other end	2.00 *L*	2.00 *L*

10.3.2 Slenderness Limits

The columns dimensions should be selected in such a way that it fails by material failure only and not by buckling. To ensure this criterion, the code

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recommends that the clear distance between restraints (un supported length) should never exceed 60 times the least lateral dimensions of the column (clause 25.3.1). For unbraced columns, it is recommended that this value is limited to 30. In cantilever columns, in addition to the above restriction ($L \leq 60b$), the clear height should also not exceed the value of $L = (100\ b^2/D)$, where D is the depth of cross section measured in the plane under consideration and 'b' is the width of cross section (clause 25.3.2)

10.3.3 Minimum Eccentricities

All columns should be designed for minimum eccentricity (Clause 25.4), which may arise due to imperfections in constructions and inaccuracy in loading given by the relation,

$$e_{\min} = \left[\frac{L}{500} + \frac{D}{30}\right] \qquad \ldots(10.1)$$

but not less than 20 mm

Where L = Unsupported length

D = Lateral dimensions in the plane of bending

For non-rectangular and non-circular cross sectional shapes, SP: 24[14] recommends the minimum eccentricity as

$$e_{\min} = (L_e/300) \text{ or } 20 \text{ mm (whichever is greater)}$$

10.3.4 Braced and Unbraced Columns

In a framed structure, an approximate method of deciding whether a column is 'braced' or 'unbraced' is specified in the ACI code commentary[85] and is reproduced in the revised IS: 456-2000 code. For this purpose, the 'stability index' (Q) of a storey in a framed multistorey structure is defined as

$$Q = \left[\frac{\Sigma P_u}{h_s} \times \frac{\Delta_u}{H_u}\right] \qquad \ldots(10.2)$$

Where ΣP_u = sum of axial loads on all columns in the storey

h_s = height of the storey

Δ_u = elastically computed first order lateral deflection of the storey

H_u = total lateral force acting within the storey.

In the absence of bracing elements, Taranath[86] has shown that the lateral

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flexibility measure of the storey (Δ_u/H_u) (storey drift per unit storey shear) can be expressed by the relation.

$$\left(\frac{\Delta_u}{H_u}\right)=\left[\frac{h_s^2}{12E_{c,col}\,\Sigma(I_c/h_s)}+\frac{h_s^2}{12E_{c,beam}\,\Sigma(I_b/L_b)}\right] \quad \ldots(10.3)$$

Where ΣI_c = sum of second moment of areas of all columns in the storey in the plane under consideration.

$\Sigma(I_b/L_b)$ = sum of the ratios of second moment of area to span of all floor members in the storey in the plane under consideration.

E_c = modulus of elasticity of concrete

The equation for the stability index 'Q' is based on the assumption that the points of contra flexure occurs at the mid heights of all columns and mid span points of all beams and by applying unit load method to an isolated store[86]. If Bracing elements such as trusses, shear walls and infill walls are used then their beneficial effect will be to reduce the ratio (Δ_u/H_u) significantly.

If the value of $Q \le 0.04$, then the column may be considered as no sway column (braced), otherwise the column may be treated as sway column (unbraced).

IS: 456-2000 codal charts (Fig. 10.3 & 10.4) are very useful in determining the effective length ratios of braced and unbraced columns respectively; in terms of β_1 & β_2 which represent the degree of rotational freedom at the top and bottom ends of the column. The values of β_1 and β_2 for braced and unbraced columns are given by the relations,

$$\beta_1=\left[\frac{\Sigma I_c/h_s}{\Sigma I_c/h_s+\Sigma 0.5(I_b/L_b)}\right] \quad \text{(For braced columns)} \quad \ldots(10.4)$$

$$\beta_2=\left[\frac{\Sigma I_c/h_s}{\Sigma I_c/h_s+\Sigma 1.5(I_b/L_b)}\right] \quad \text{(For unbraced columns)} \quad \ldots(10.5)$$

The limiting values $\beta = 0$ and $\beta = 1$, represent the 'fully fixed' 'fully hinged' conditions respectively.

The following example illustrates the checking of braced and unbraced columns and the computation of effective length.

10.3.5 Example

A multistoreyed building plan shown in Fig. 10.5 (a) has 16 columns of size 300 × 300 mm interconnected by floor beams of size 250 mm by 500 mm in the longitudinal & transverse directions. The storey height is 3.5 m.

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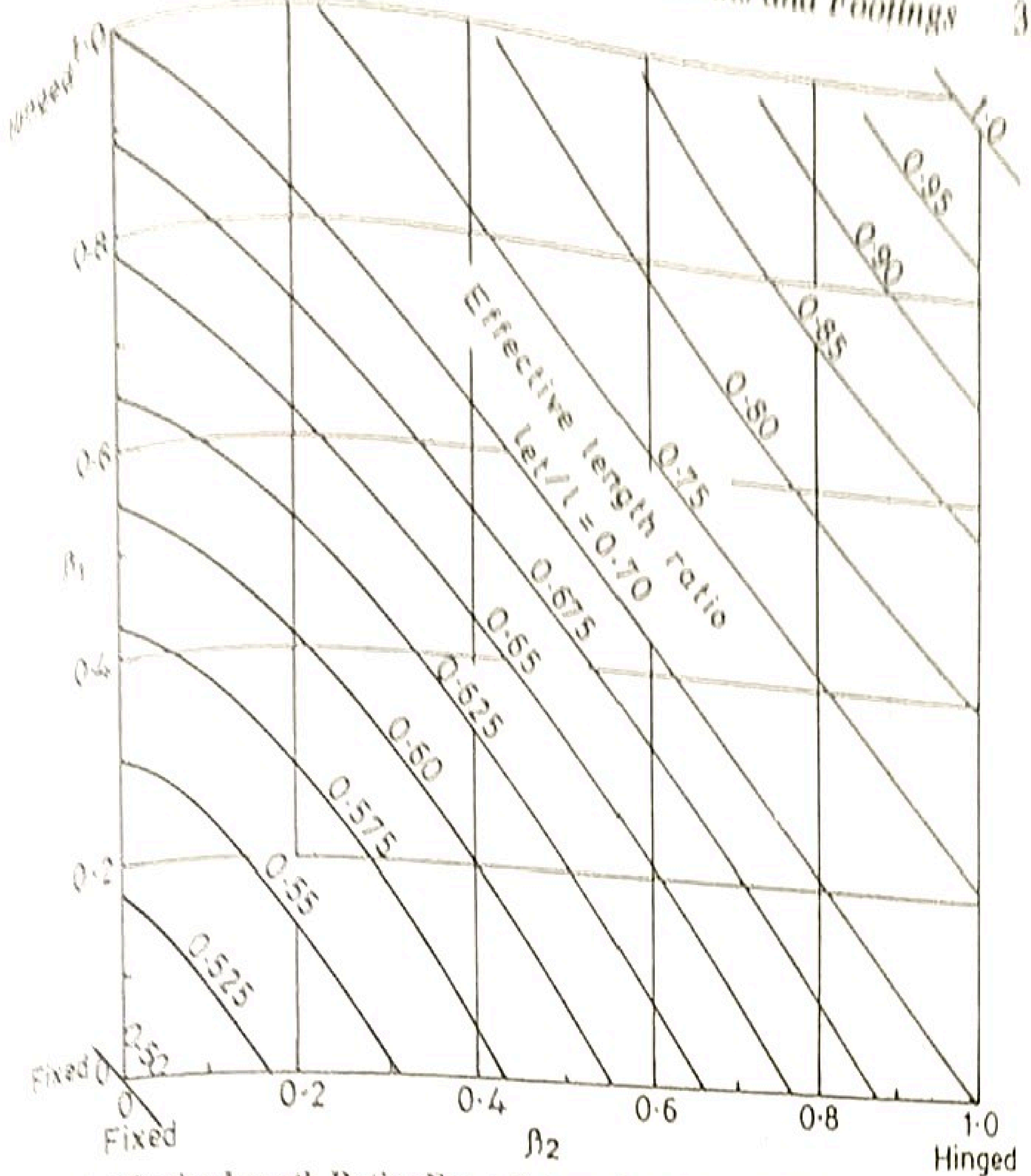

Fig. 10.3 **Effective Length Ratios For a Column in a Frame With no Sway (Braced Columns) (IS: 456:200 Fig. 26)**

Calculate the effective length of the typical lower storey columns assuming a total distributed load 30 kN/m^2 from all the floors above & the grade of concrete as M-20. Adopt IS: 456-2000 codal method for computations.

a) Data

Size of columns = 300 × 300 mm
Height of storey = h_s = 3.5 m
Width of beam = 250mm
Depth of beam = 500 mm
Length of beam = 4 m
Total distributed load = 30 kN/m^2
No. of Columns = 16
No. of Beams in XX or YY–directions = 12
Grade of concrete= M-20

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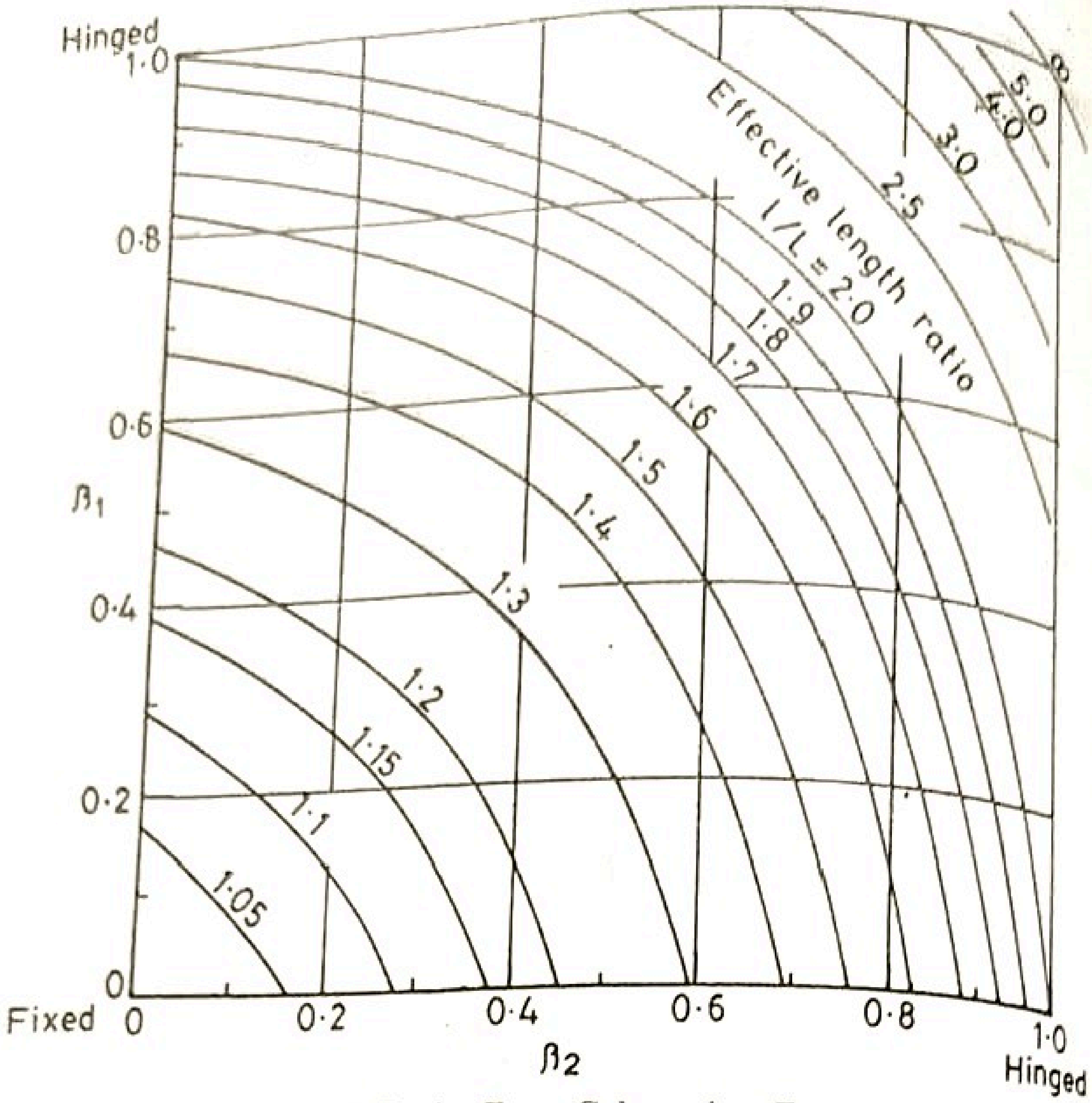

Fig. 10.4 Effective Length Ratios For a Column in a Frame Without Restraint Against Sway (Unbraced Columns) (IS: 456:200 Fig. 27)

b) Relative stiffness of Columns and Beams

Referring to Fig. 10.5(b)

Un-supported length of column = L = (3500–500) = 300 mm

i) Columns : 16 Nos, (300 × 300mm) and h_s = 3500 m

$$\Sigma\left(\frac{I_c}{h_s}\right) = \left[\frac{16 \times (300)^4/12}{3500}\right] = (3086 \times 10^3)\ \text{mm}^3$$

ii) Beams in each direction XX or YY

$$\Sigma\left(\frac{I_b}{L_b}\right) = \left[\frac{12 \times 250 \times (500)^3/12}{4000}\right] = (7812 \times 10^3)\ \text{mm}^3$$

c) Check for braced or Unbraced Columns

$$\left(\frac{\Delta_u}{H_u}\right) = \frac{h_s^2}{12E_c}\left[\frac{1}{\Sigma(I_c/h_s)} + \frac{1}{\Sigma(I_b/L_b)}\right]$$

$E_c = 5000\sqrt{f_{ck}}$ according to clause 6.2.3.1 of IS: 456 - 2000

Therefore $E_c = 5000\sqrt{20} = 22360\ \text{N/mm}^2$

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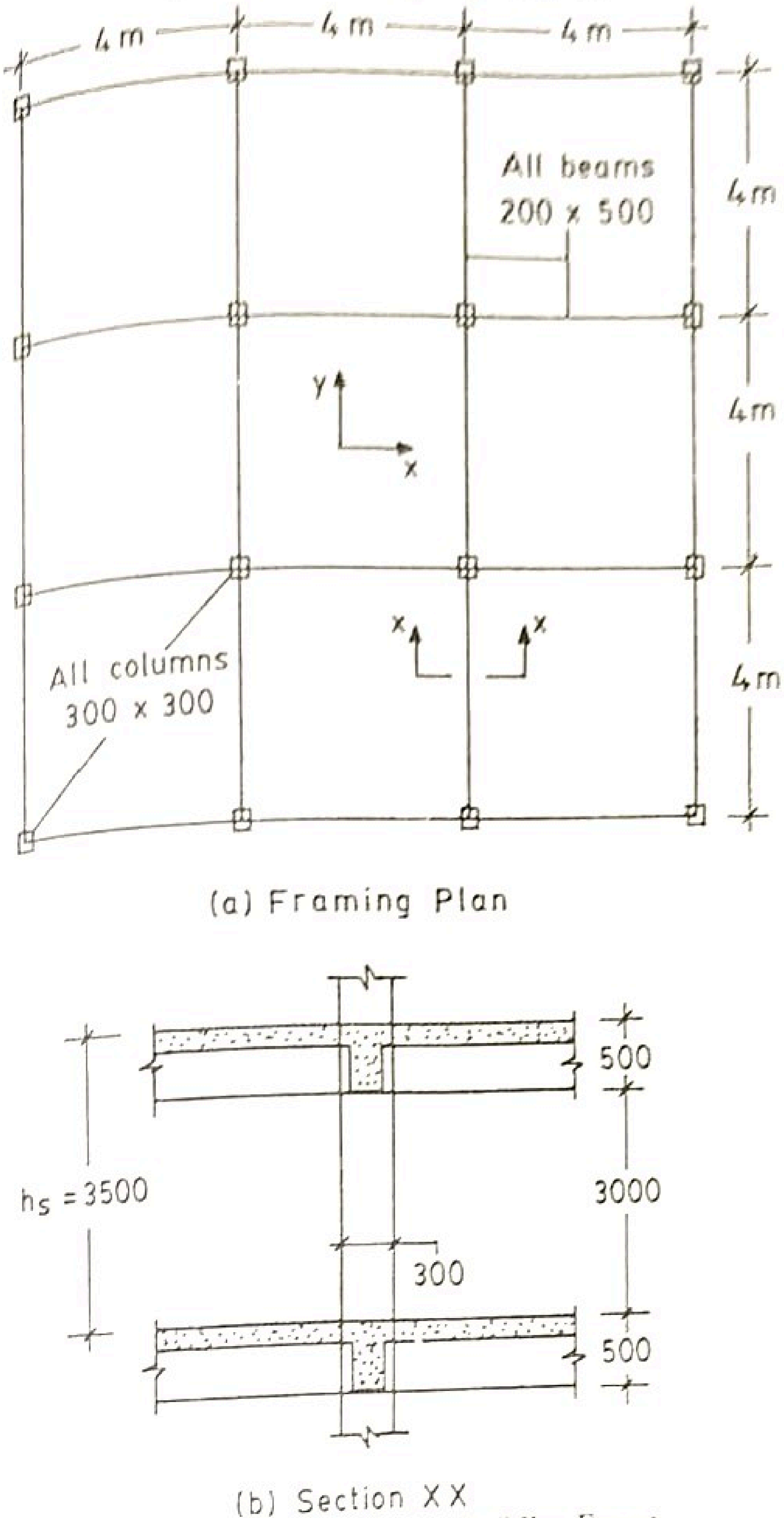

Fig. 10.5 Multi-Storey Building Frame

$$\left(\frac{\Delta_u}{H_u}\right) = \frac{3500^2}{(12 \times 22360)}\left[\frac{1}{(3086 \times 10^3)} + \frac{1}{(7812 \times 10^3)}\right] = (5.991 \times 10^{-6}) \text{ mm/N}$$

Total axial Load on all columns = $(12 \times 12 \times 30) = P_u = 4320$ kN

$$\text{Stability Index} = Q = \left[\frac{P_u}{h_s} \times \frac{\Delta_u}{H_u}\right] = \left[\frac{4320 \times 10^3}{3500}(5.991 \times 10^{-6})\right] = 0.00739 < 0.04$$

Hence, the columns in the storey can be considered as braced in XX and YY directions.

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d) Effective length of columns using IS: 456 code charts

$$\beta_1 = \beta_2 = \left[\frac{\Sigma(I_c/h_s)}{\Sigma(I_c/h_s) + \Sigma\, 0.5(I_b/L_b)}\right]$$

$$\Sigma(I_c/h_s) = \left[\frac{(300)^4/12}{3500} \times 2\right] = (385 \times 10^3)\ \text{mm}^3$$

$$\Sigma(I_b/L_b) = \left[\frac{250 \times 500^3/12}{4000} \times 2\right] = (1302 \times 10^3)\ \text{mm}^3$$

$$\therefore \quad \beta_1 = \beta_2 = \left[\frac{385 \times 10^3}{(385 \times 10^3) + (0.5 \times 1302 \times 10^3)}\right] = 0.371$$

Referring to Fig. 10.3 [Fig. 26 of IS: 456-2000] and interpolating the effective length ratio as,

$$k = \left(\frac{L_e}{L}\right) = 0.630$$

$$\therefore \quad L_e = (0.630 \times 3000) = 1890\ \text{mm}$$

Slenderness ratio of the column is $= \left(\frac{L_e}{D}\right) = \left(\frac{1890}{300}\right) = 6.3 < 12$

Hence, the column should be designed as short column.

10.4 DESIGN OF SHORT COLUMNS UNDER AXIAL COMPRESSION

10.4.1 Assumptions

The main assumptions made for limit state design of columns failing under pure compression as specified in clause 39.1 are as follows:

a) The maximum compressive strain in concrete in axial compression is 0.002.
b) Plane sections remain plane in compression
c) The design stress-strain curve of steel in compression is taken to be the same as in tension

The design stress in steel is $0.87 f_y$ in Fe-250, 415 and Fe-500 grade steels.

Accordingly, under pure axial loading conditions the design strength of short columns is expressed as

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$$P_u = [0.45\, f_{ck}.A_c + f_{sc}.A_{sc}] \quad \ldots(10.6)$$

$$f_{sc} = 0.87\, f_y$$

Where

The IS: 456-2000 code requires that all columns are to be designed for minimum eccentricity of 0.05 times the lateral dimension. Hence the final expression for the ultimate load is obtained by reducing the value of P_u by 10 percent in the equation (10.6) specified above as

$$P_u = 0.4\, f_{ck}A_c + 0.67\, f_y.A_{sc} \quad \ldots(10.7)$$

$$= [0.4\, f_{ck}.A_g + (0.67\, f_y - 0.4 f_{ck})A_{sc}] \quad \ldots(10.8)$$

Where
P_u = axial ultimate load on the member
f_{ck} = characteristic compressive strength of concrete
A_c = area of concrete
f_y = characteristic strength of the compression reinforcement
A_{sc} = area of longitudinal reinforcement.

Short columns with helical reinforcement (spiral columns) have increased ductility prior to collapse and hence the code permits 5 percent increase in the load carrying capacity of spiral columns. However the ratio of the volume of helical reinforcement to the volume of the core shall be not less than,

$$0.36\left(\frac{A_g}{A_c} - 1\right)\left(\frac{f_{ck}}{f_y}\right)$$

according to clause 39.4.1 of IS:456-2000.

10.4.2 Design Example

Design the reinforcements in a column of size 400 mm by 600 mm subjected to an axial working load of 2000 kN. The column has an unsupported length of 3m and is braced against side sway in both directions. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

Column Dimensions 400 mm by 600mm
Axial service load = 2000 kN
Un supported length L = 3 m
Column Braced against side sway
$f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2
D_y = 400 mm and D_x = 600 mm

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b) Slenderness Ratio

$$k_x = \left(\frac{L_{ex}}{L}\right) \quad \text{and} \quad k_y = \left(\frac{L_{ey}}{L}\right)$$

As the column is braced against side sway in both directions, effective length ratio k_x and k_y are both less than unity.

And
$$\left(\frac{L}{D_y}\right) = \left(\frac{3000}{400}\right) = 7.5 < 12$$

Hence, the columns is designed as a short column.

c) Minimum Eccentricity

$$e_{x,\min} = \left[\frac{3000}{500} + \frac{600}{30}\right] = 26 > 20 \text{ mm}$$

$$e_{y,\min} = \left[\frac{3000}{500} + \frac{400}{30}\right] = 19.33 < 20 \text{ mm}$$

Also $\quad 0.05\,D_x = (0.05 \times 600) = 30 > e_{x,\min}$

$\quad 0.05\,D_y = (0.05 \times 400) = 20 > e_{y,\min}$

Hence, the codal formula (Eq: 10.6) for short columns is applicable

d) Factored (Ultimate) Load

$$P_u = (1.5 \times 2000) = 3000 \text{ kN}$$

a) Longitudinal Reinforcements

$$P_u = [0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}]$$

$$(3000 \times 10^3) = (0.4 \times 20 \times 400 \times 600) + [(0.67 \times 415) - (0.4 \times 20)]A_{sc}$$

Solving $A_{sc} = 4000 \text{ mm}^2$

Provide 6-25 mm diameter bars: $(6 \times 491) = 2946 \text{ mm}^2$

4-20 mm diameter bars: $(4 \times 314) = 1256 \text{ mm}^2$

Total $A_{sc} = 4202 \text{ mm}^2 > 4000 \text{ mm}^2$

The area of reinforcement provided is greater than the minimum steel requirement of 0.8 percent = $(0.008 \times 400 \times 600) = 1920 \text{ mm}^2$

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f) **Lateral Ties**

Tie diameter: < (1/4)(25) = 6.25 mm
> 16 mm

Hence, provide 8 mm diameter ties

Tie spacing: > 400 mm
> (16 × 20) = 320 mm

∴ Provide 8 mm diameter ties at 300 mm c/c

g) The detailing of reinforcements in the column section is shown in Fig. 10.6.

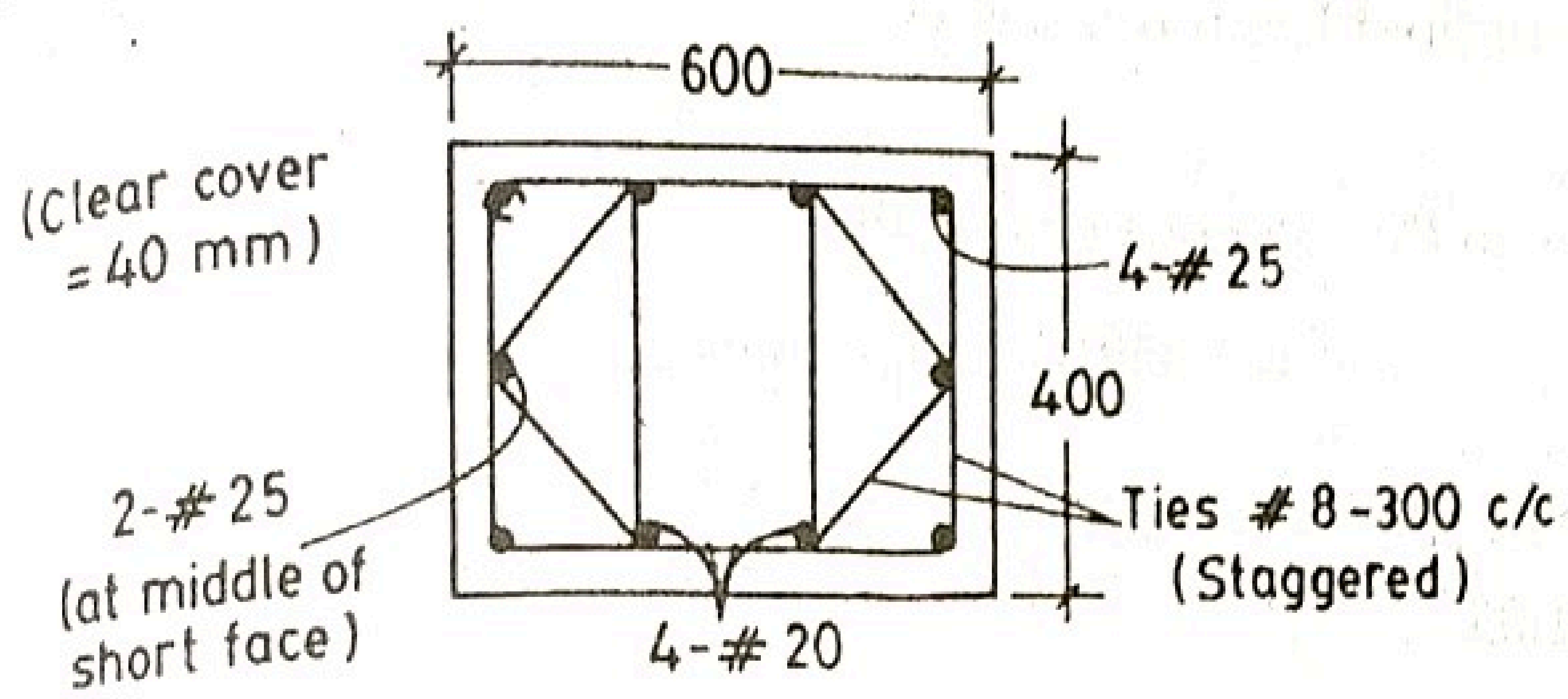

Fig. 10.6 Reinforcements in Short Column

10.4.3 Design Example

Design the reinforcements in a circular column of diameter 300 mm with helical reinforcement to support a factored load of 1500 KN. The columns has an unsupported length of 3 m and is braced against sidesway. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) **Data**

Diameter of column $= D = 300$ mm
Unsupported length $= L = 3000$ mm
Column braced against sidesway.
Factored Load $= P_u = 1500$ kN
$f_{ck} = 20$ N/mm^2
$f_y = 415$ N/mm^2

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b) Slenderness Ratio

$$(L_e/D) = (3000/300) = 10 < 12$$

Hence, the column designed as short column.

c) Minimum Eccentricity

$$e_{min} = \left[\frac{L}{500} + \frac{D}{30}\right] = \left[\frac{3000}{500} + \frac{300}{30}\right] = 16 \text{ mm}(20 \text{ mm})$$

Also $0.05D = (0.05 \times 300) = 15 \text{ mm} < 20 \text{ mm}$

Hence, the codal formula for axially compressed column can be used.

d) Longitudinal Reinforcements

According to IS: 456-code clause 39.4

$$P_u = 1.05[0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{sc}]$$

$$\left(\frac{1500 \times 10^3}{1.05}\right) = \left[\frac{0.4 \times 20 \times \pi \times 300^2}{4} + \{(0.67 \times 415) - (0.4 \times 20)\} A_{sc}\right]$$

Solving $A_{sc} = 3197 \text{ mm}^2$

$A_{sc,min} = 0.8\%$ of gross cross section $= (0.008 \times \pi \times 300^2/4) = 565 \text{ mm}^2$

Provide 6 bars of 28 mm diameter ($A_{sc} = 3696 \text{ mm}^2$)

e) Helical Reinforcement (spirals)

Assuming clear cover of 40 mm over spirals

Core diameter $= [300 - (2 \times 40)] = 220$ mm

$$\text{Area of core} = A_c = \left[\left(\frac{\pi \times 220^2}{4}\right) - 3696\right] = 34317 \text{ mm}^2 = \frac{\pi}{4}(d)^2 - ASC$$

Volume of core/m $= V_c = (34317 \times 10^3) \text{ mm}^3$

$$\text{Gross Area of section} = A_g = \left(\frac{\pi \times 300^2}{4}\right) = 70685 \text{ mm}^2$$

Using 8mm diameter helical spirals at a pitch 'p' mm, the volume of helical spiral per metre length is given by

$$V_{ns} = \pi(300 - 80 - 8)50 \times (1000/p) \text{ mm}^3/\text{m}$$

$$= (33301 \times 10^3)/p \text{ mm}^3/\text{m}$$

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According to code clause 39.4.1 (IS:456)

$$\left(\frac{V_{ns}}{V_c}\right) < 0.36[(A_g/A_c) - 1]\ (f_{ck}/f_y)$$

$$\left(\frac{33301 \times 10^3}{p(34317 \times 10^3)}\right) < 0.36\left[\left(\frac{70685}{34317}\right) - 1\right]\left(\frac{20}{415}\right)$$

Solving pitch 'p' 52.78 mm

Codal restriction on pitch [Clause 26.5.3.2 (d)]

$p <$ 75 mm or (core diameter/6) = (220/6) = 36.6 mm

$p >$ 25 mm or (3 times the diameter of helix) = (3 × 8) = 24 mm

Hence, provide 8 mm diameter spirals at a pitch of 36 mm.

f) Reinforcement Details

The details of reinforcements in the helically reinforced column are shown in Fig. 10.7.

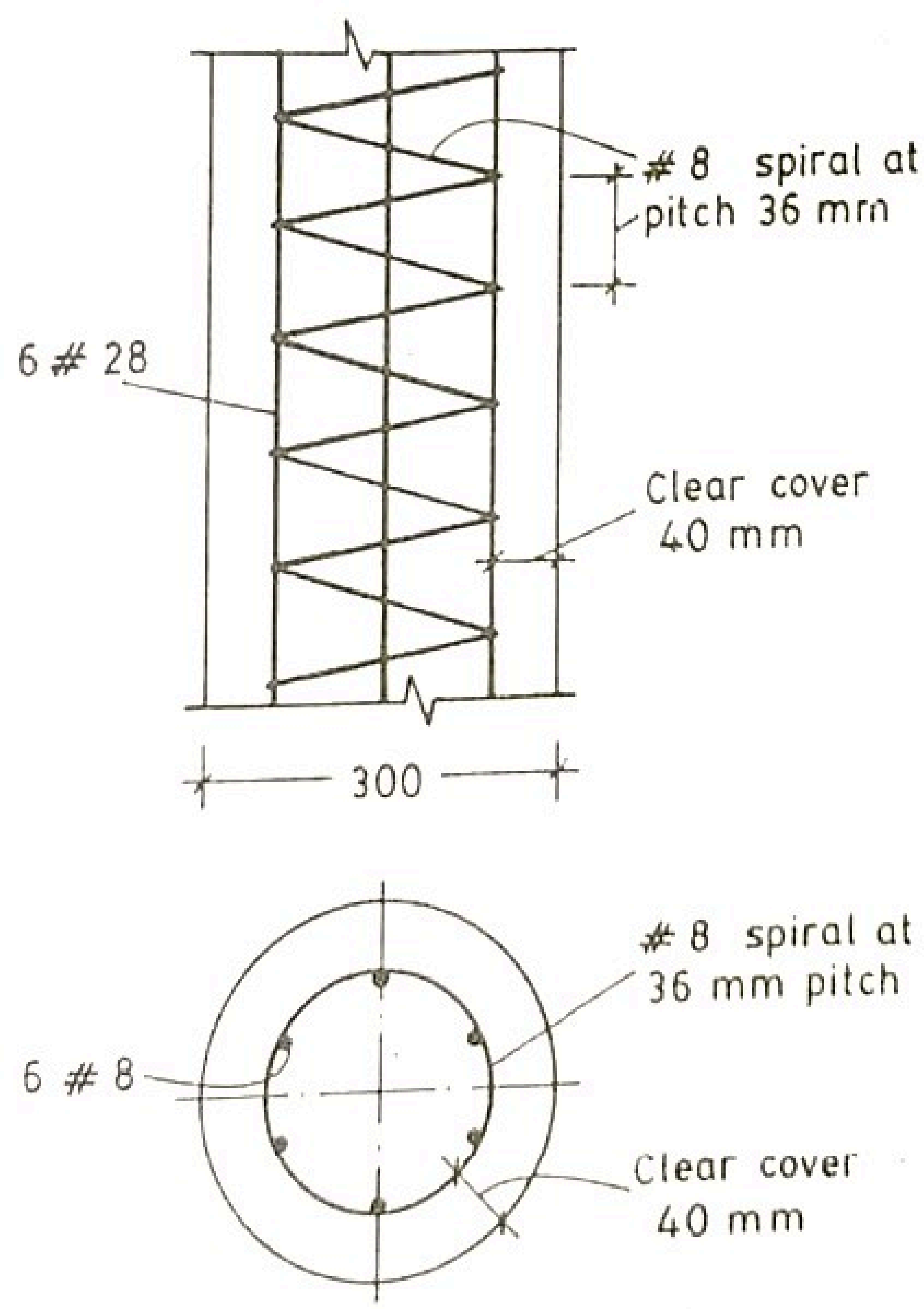

Fig. 10.7 Reinforcement in Helically Reinforced Column

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10.5 DESIGN OF SHORT COLUMNS UNDER COMPRESSION WITH UNIAXIAL BENDING

10.5.1 Introduction

The external columns of multistoreyed buildings and columns supporting crane loads through corbels are subjected to direct loads and bending moments. The compression members should be designed for axial load and bending moment based on the assumptions prescribed in IS:456-2000 code clauses 39.1 and 39.2

The analytical design of members subjected to combined axial load and uniaxial bending involves lengthy calculation by trial and error and the method uses equilibrium equal to determine the area of reinforcement required to resist direct loads and uniaxial moment. In order to overcome these difficulties, I.S code recommends the use of interaction diagrams involving non-dimensional parameters presented in SP: 16 design aids for reinforced concrete.

10.5.2 Interaction Diagrams

The interaction diagram represents the design strength of eccentrically loaded column of known section properties. The salient points on the interaction curve corresponds to the design strength values of axial load P_u and the moment M_u associated with an eccentricity '*e*'. Fig. 10.8 shows a typical interaction curve with P_u on Y-axis and M_u on X-axis along with strain profiles.

The interaction curve defines the different load-moment (P_u & M_u) combinations for all possible eccentricities of loading. For design purposes, the calculations of M_u and P_u are based on the design stress-strain curves (including partial safety factors).The design interaction curve represents the failure envelope and the point given by the co-ordinates (M_u and P_u) falling within the interaction curve indicates the safe values of the combination of load and moments.

The salient points on the interaction curve are note worthy.

1) Point-1, on the load axis corresponds to the axial loading with zero moment (P_{uo}) and $e = 0$.
2) Point-1'corresponds to the condition of axial load with the minimum eccentricity prescribed in IS:456 code clause-25.4. The corresponding ultimate load is represented as P'_{uo}.
3) As the eccentricity increases, the moment, increases with the neutral axis x_u moving from outside towards the extreme fibre.

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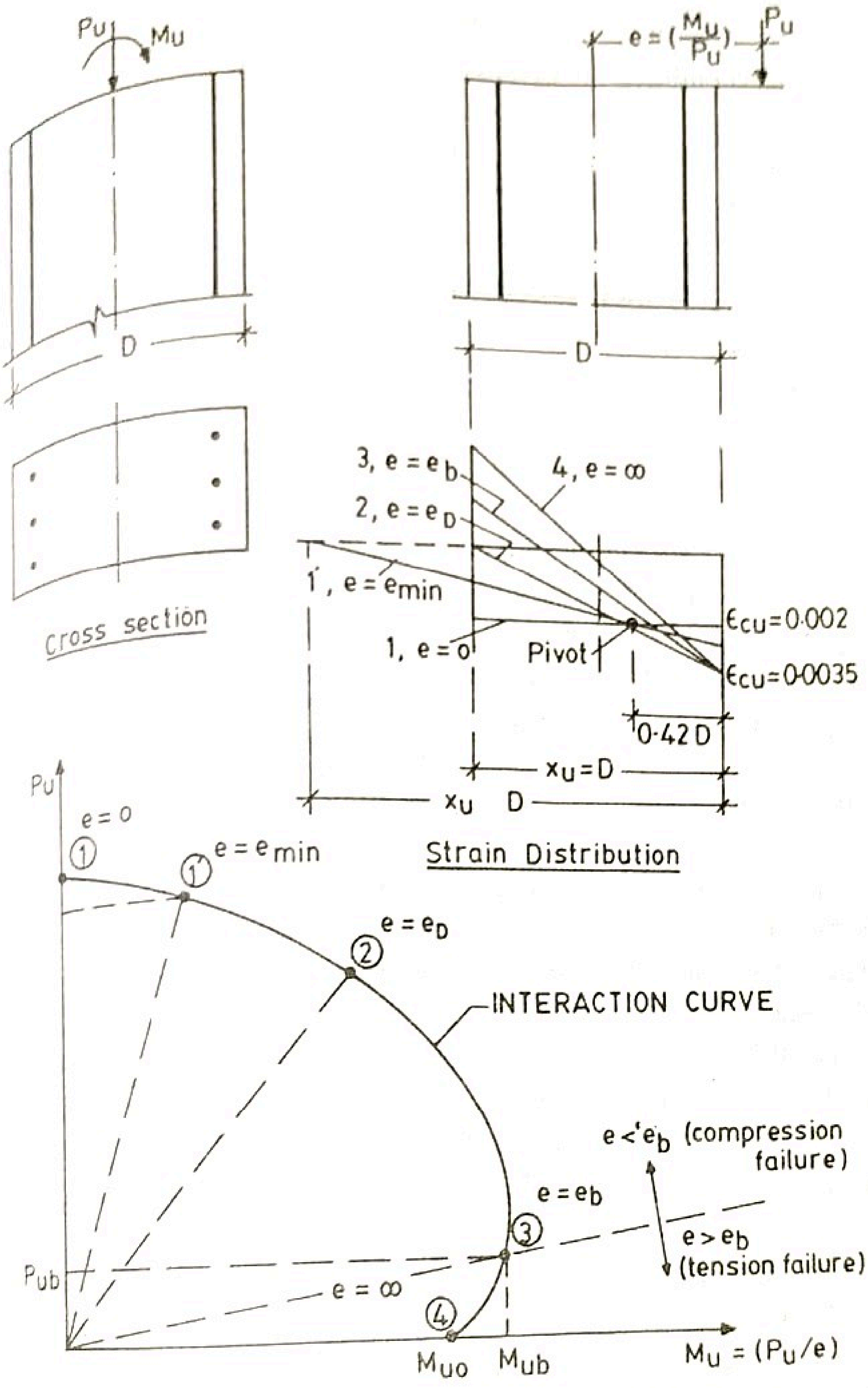

Fig. 10.8 Column Under Compression With Uniaxial Bending

Point-2 corresponds to the condition,
$x_u = D$ and $e = e_D$. For $e < e_D$, the entire cross section is under compression and the neutral axis is located outside the section ($x_u > D$) and the extreme fibre strain in concrete lies between 0.002 & 0.0035. With a further increase in the moment and eccentricity ($e > e_D$), the neutral axis lies within the section (xu < D) and the extreme concrete fibre strain ecu = 0.0035.

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4) Point-3 on the interaction diagram represents the balanced failure state with $e = e_b$ and $x_u = x_{u,max}$. The design strength values for the balanced failure condition are denoted as Pub and Mub. For values of $e > e_b$, P_u < P_{ub} and the failure mode is termed as tension failure, similar to that of beams. It is important to note that M_{ub} is only marginally less than the ultimate moment of resistance of the section M_{uo} under pure flexural condition.

5) Point-4 on the interaction curve refers to the pure flexural state ($e = \infty$ and $P_{uR} = 0$) with the ultimate moment of resistance M_{uo} associated with the minimum neutral axis depth $x_{u,min}$.

10.5.3 Design charts (Uniaxial eccentric compression) in SP: 16

The design of structural concrete members subjected to combined axial load and uniaxial bending moment involves lengthy theoretical computations by trial and error procedure. To overcome these difficulties, interaction diagrams involving non dimensional parameters are useful in the rapid design of reinforcements in eccentrically loaded columns. SP:16 presents the design charts covering the following three different cases of symmetrically reinforced column sections, covering rectangular and circular cross sections.

The non dimensional parameters used for the construction of design charts are $(P_u/b.d.f_{ck})$ and $(M_u/b.d^2 f_{ck})$ plotted along the Y and X-axis respectively.These parameters are plotted for different values of the ratio $(p/f_{ck})$where 'p' is the percentage reinforcement in the section.

The following cases are covered in the SP: 16 Design charts: -

1) Rectangular section reinforced with equal number of bars on opposite sides parallel to that axis of bending (Charts 27 to 38)
2) Rectangular sections reinforced with equal number of bars on all the four sides (Charts 39 to 50)
3) Circular sections reinforced with 8 bars symmetrically spaced (charts 51 to 62) and these charts can also be used for bars not less than 6.

The charts for each of these types have been given for three grades of steel (Fe-250, Fe-415 and Fe-500) and four values of the ratio (d'/D).

The dotted lines in these charts indicate the stress in the bars nearest to the tension face of the member. It is pertinent to note that all these stress values are at the failure condition corresponding to the limit state of collapse and not at working loads.

The construction of these design charts are based on the equilibrium equations at the limit state collapse as outlined in SP:16.

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Typical design charts covering the parameters f_y = 415 N/ $(d'/D) = 0.10$ are reproduced in Figs. 10.9, 10.10 & 10.11 for the t ferent arrangements of reinforcements in the cross section.

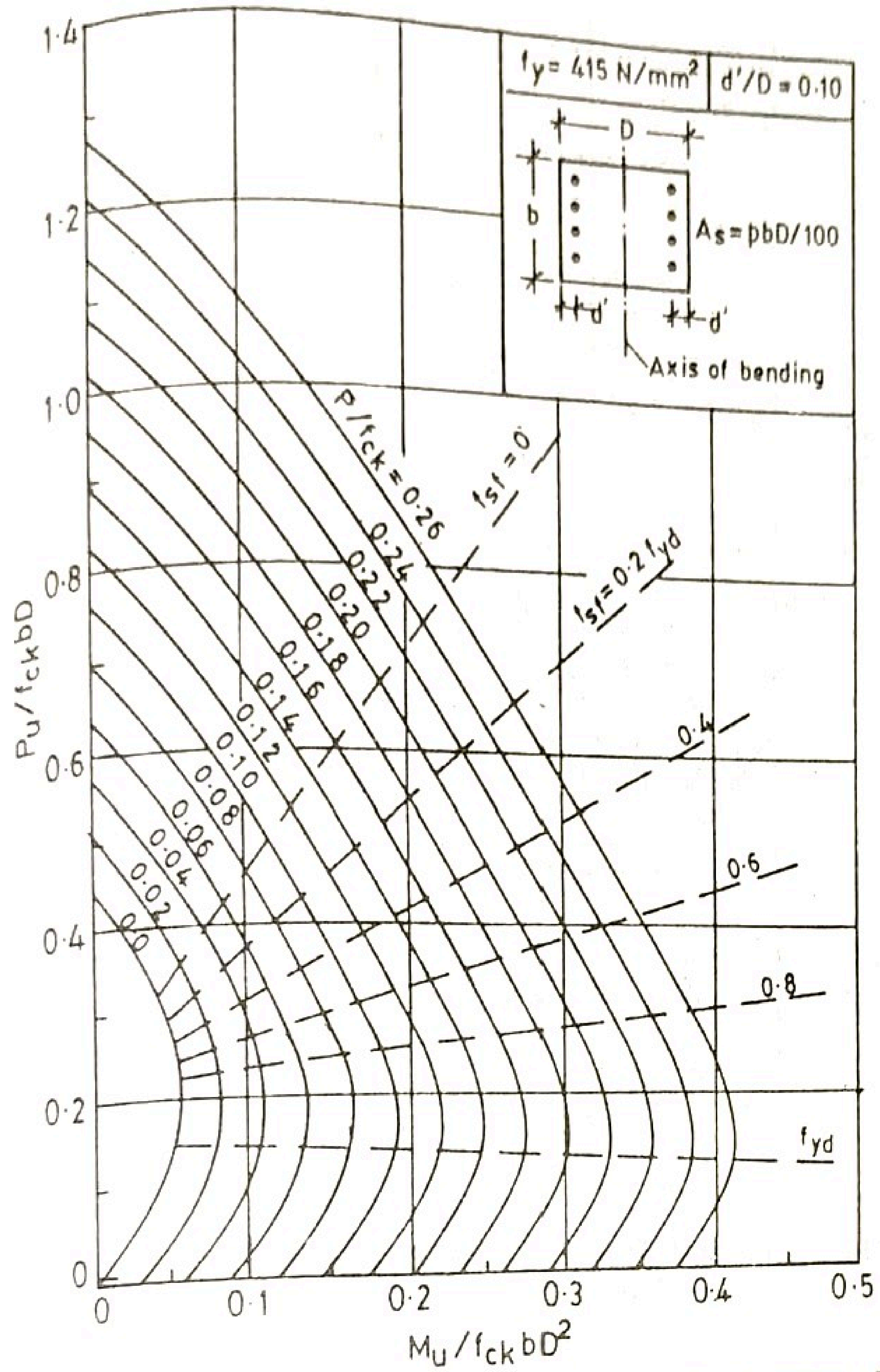

Fig. 10.9 Compression With Bending-Rectangular Section-Reinforcement Distributed Equally on Two Sides (SP: 16 Chart 32)

The following examples demonstrate the use of the design charts to design reinforcements in the columns subjected to combined axial load and uniaxial bending moment.

10.5.4 Design Example

Design the longitudinal and lateral reinforcement in a rectangular reinforced concrete column of size 300mm by 400 mm subjected to a design

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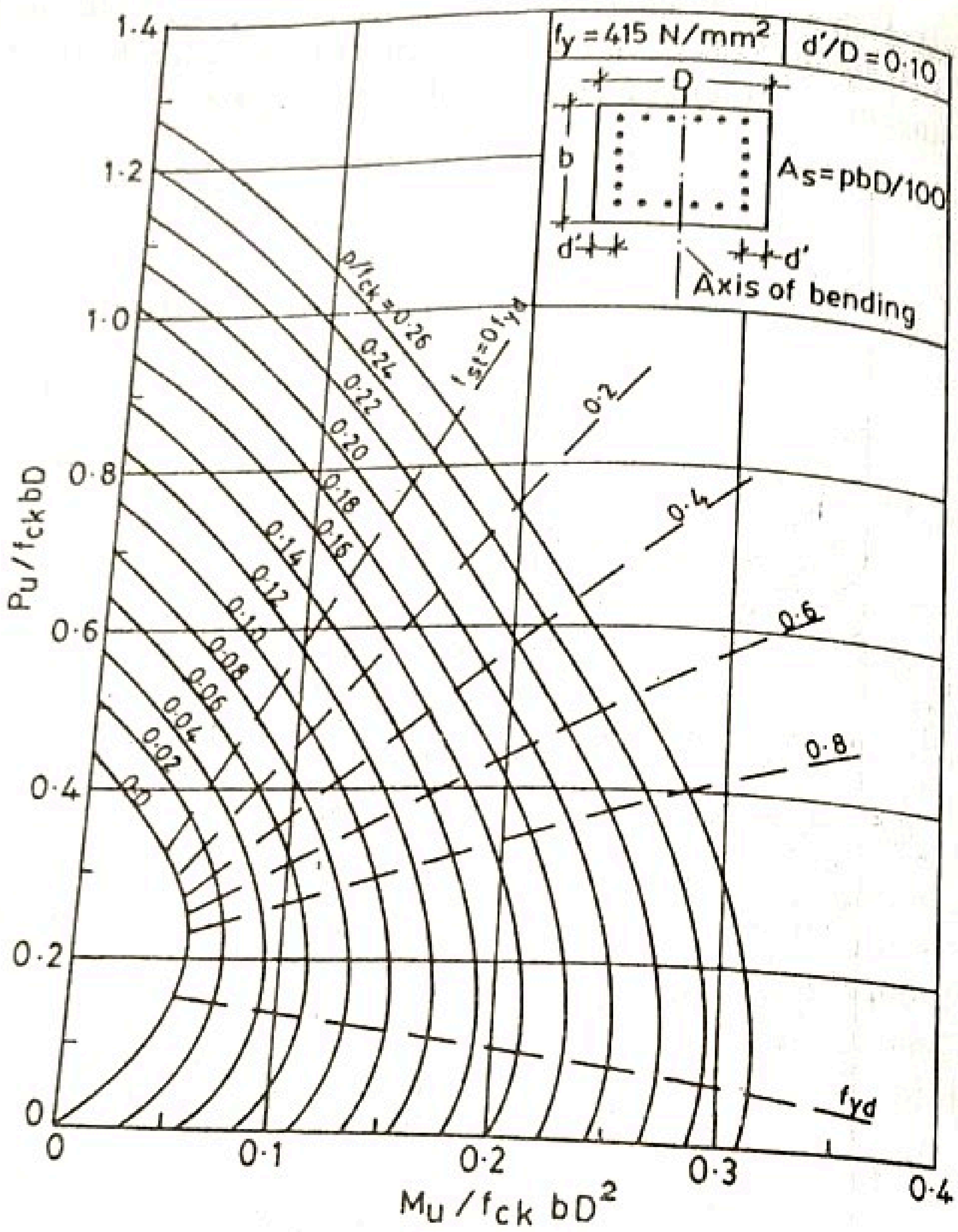

Fig. 10.10 Compression With Bending-Rectangular Section-Reinforcement Distributed Equally on Four Sides (SP: 16 Chart-44)

ultimate load of 1200 kN and an ultimate moment of 200 kN.m with respect to the major axis. Adopt M-20 grade concrete and Fe-415 grade HYSD bars.

a) Data

$b = 300$ mm $f_{ck} = 20$ N/mm²

$D = 400$ mm $f_y = 415$ N/mm²

$P_u = 1200$ kN

$M_u = 200$ kN.m

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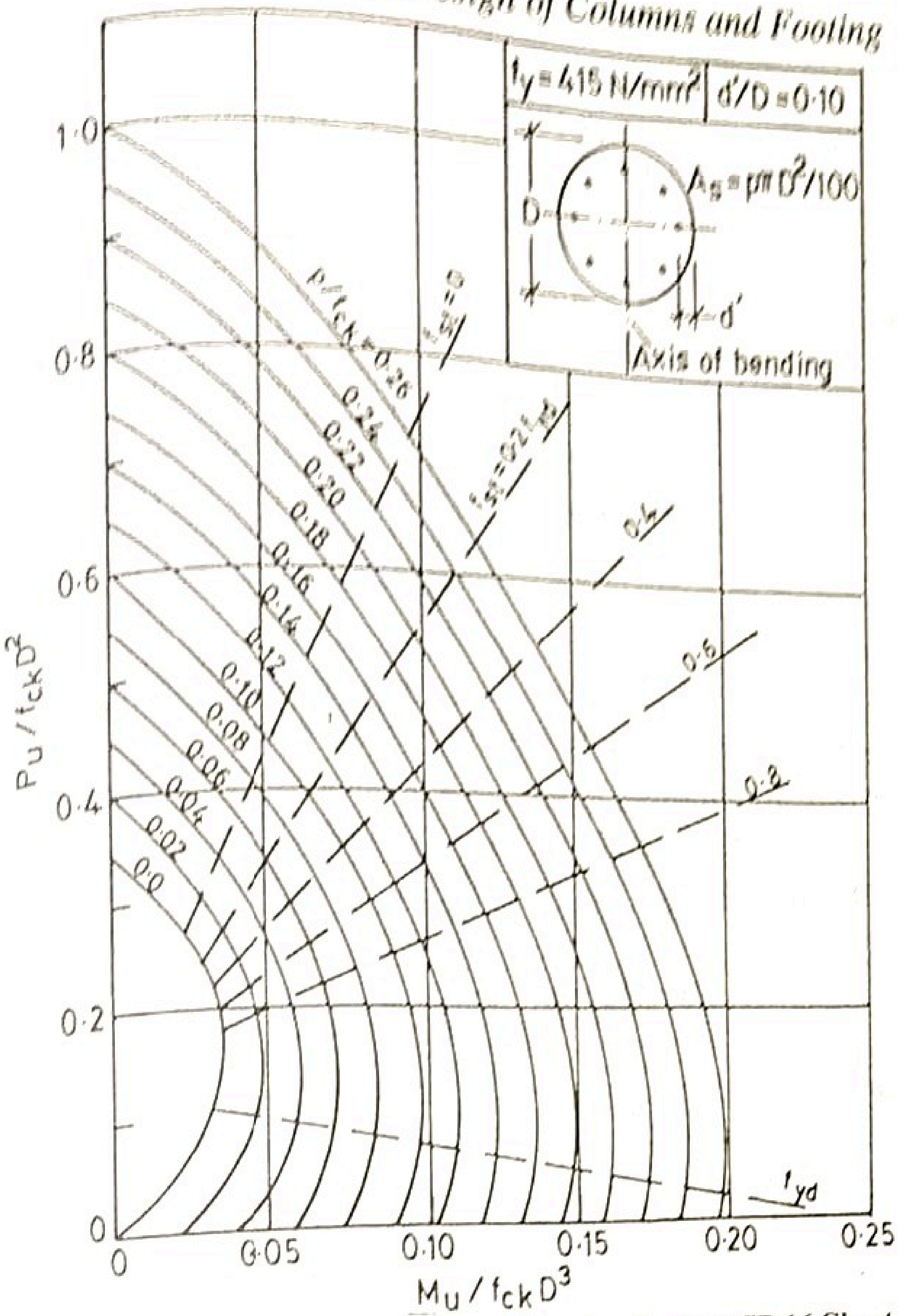

Fig. 10.11 Compression With Bending-Circular Section (SP:16 Chart-56)

b) Non Dimensional Parameters

$$\left(\frac{P_u}{f_{ck} b\, D}\right) = \left(\frac{1200 \times 10^3}{20 \times 300 \times 400}\right) = 0.5$$

$$\left(\frac{M_u}{f_{ck} b D^2}\right) = \left(\frac{200 \times 10^6}{20 \times 300 \times 400^2}\right) = 0.208$$

c) Longitudinal Reinforcement

Adopting an effective cover of 50 mm = d'

(d'/D) = (50/400) = 0.124 nearly equal to 0.15

Refer Chart 33 of SP: 16 and read out the ratio $(p/f_{ck}) = 0.20$

$$p = (20 \times 0.20) = 4$$

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$$\therefore \quad A_s = \left(\frac{pbD}{100}\right) = \left(\frac{4 \times 300 \times 400}{100}\right) = 4800 \text{ mm}^2$$

Adopt 8 bars of 28 mm diameter
Area provided = $(8 \times 616) = 4928 \text{ mm}^2$
Provide 4 bars on each of the short faces at an effective cover of 50 mm.

According to IS: 456-2000 clause 26.5.3.1, the spacing of longitudinal bars measured along the periphery of the column shall not exceed 300 mm.
Hence distance between centre lines of reinforcement = $[400 - (2 \times 50)] =$ 300 mm

d) Lateral Ties

Tie diameter $< (28/4) = 7$ mm and not greater than 16 mm
Hence, provide 8 mm ties.
Tie spacing > 300 mm and $> (16 \times 28) = 448$ mm
Provide 300 mm spacing.

e) Detailing of Reinforcements

The detailing of reinforcement is shown in Fig. 10.12

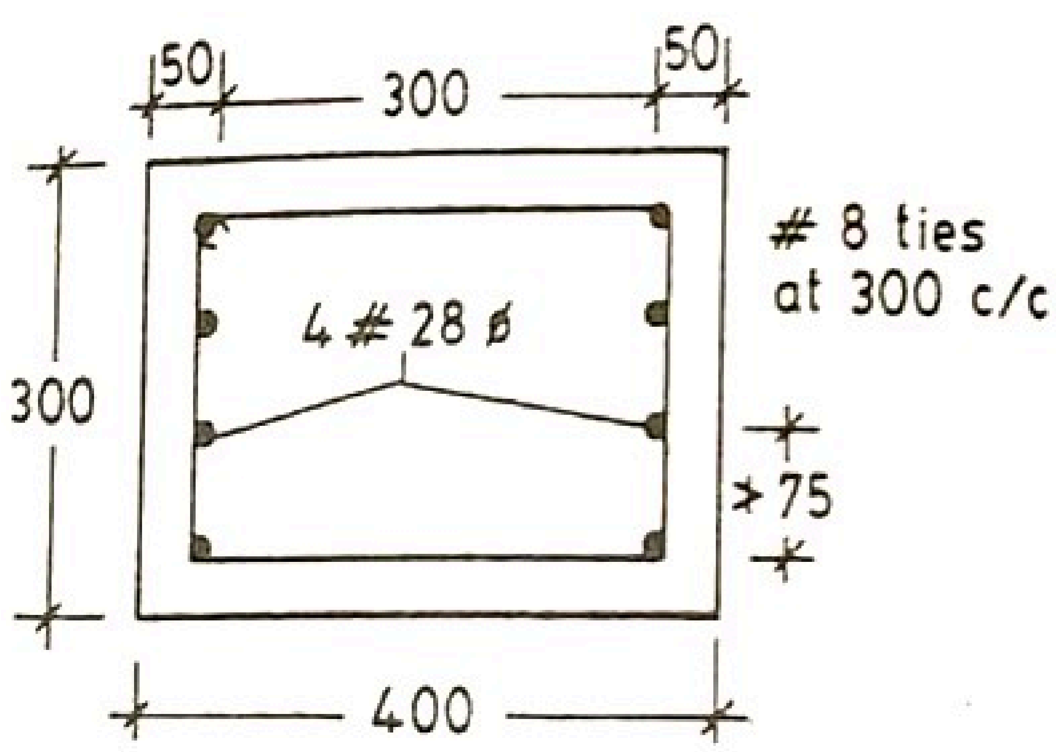

Fig. 10.12 Reinforcements in Columns With Uniaxial Bending

10.5.5 A short reinforced concrete rectangular column of size 300 x 500 mm is subjected to an axial compressive factored load of 200 kN and a factored moment of 250 kN.m about the major axis. Adopting M-25 grade concrete and Fe-415 HYSD bars, determine the reinforcement in the column section.

a) Data

$b = 300$ mm $\qquad f_{ck} = 25 \text{ N/mm}^2$
$D = 500$ mm $\qquad f_y = 415 \text{ N/mm}^2$

$P_u = 1000$ kN Adopt $d' = 50$ mm
$M_u = 250$ kN.m Ratio of $(d'/D) = 0.1$

b) **Non Dimensional Parameters**

$$\left(\frac{P_u}{f_{ck} b\, D}\right) = \left(\frac{1000 \times 10^3}{25 \times 300 \times 500}\right) = 0.266$$

$$\left(\frac{M_u}{f_{ck} b D^2}\right) = \left(\frac{250 \times 10^6}{25 \times 300 \times 500^2}\right) = 0.133$$

c) **Longitudinal Reinforcements**

Refer chart - 44 (SP:16) with equal steel on all the sides and read out

$$\left(\frac{P}{f_{ck}}\right) = 0.09 \qquad \therefore \quad p = (0.09 \times 25) = 2.25$$

$\therefore$

$$A_s = \left(\frac{pbd}{100}\right) = \left(\frac{2.25 \times 300 \times 500}{100}\right) = 3375 \text{ mm}^2$$

$\therefore$

Provide 8 bars of 25 mm diameter ($A_{sc} = 3927$ mm^2).
The bars are arranged equally on all the four sides (3 bars on each face)

d) **Ties**

Tie diameter $\not<$ (25/4) = 6.25 mm. Hence, provide 8 mm ties.
$\not<$ 6 mm

Tie spacing $\not>$ 300 mm and $\not>$ $(16 \times 25) = 400$ mm

Hence provide 8 mm diameter ties at 300 mm c/c (staggered)

e) **Details of Reinforcements**

Fig. 10.13 shows the detailing of reinforcements in the column section

10.5.6 Design Example

Design a short circular column of diameter 400 mm to support a factored axial load of 900 kN, together with a factored moment of 100 kN.m. Adopt M-20 grade concrete and Fe-415 grade reinforcements.

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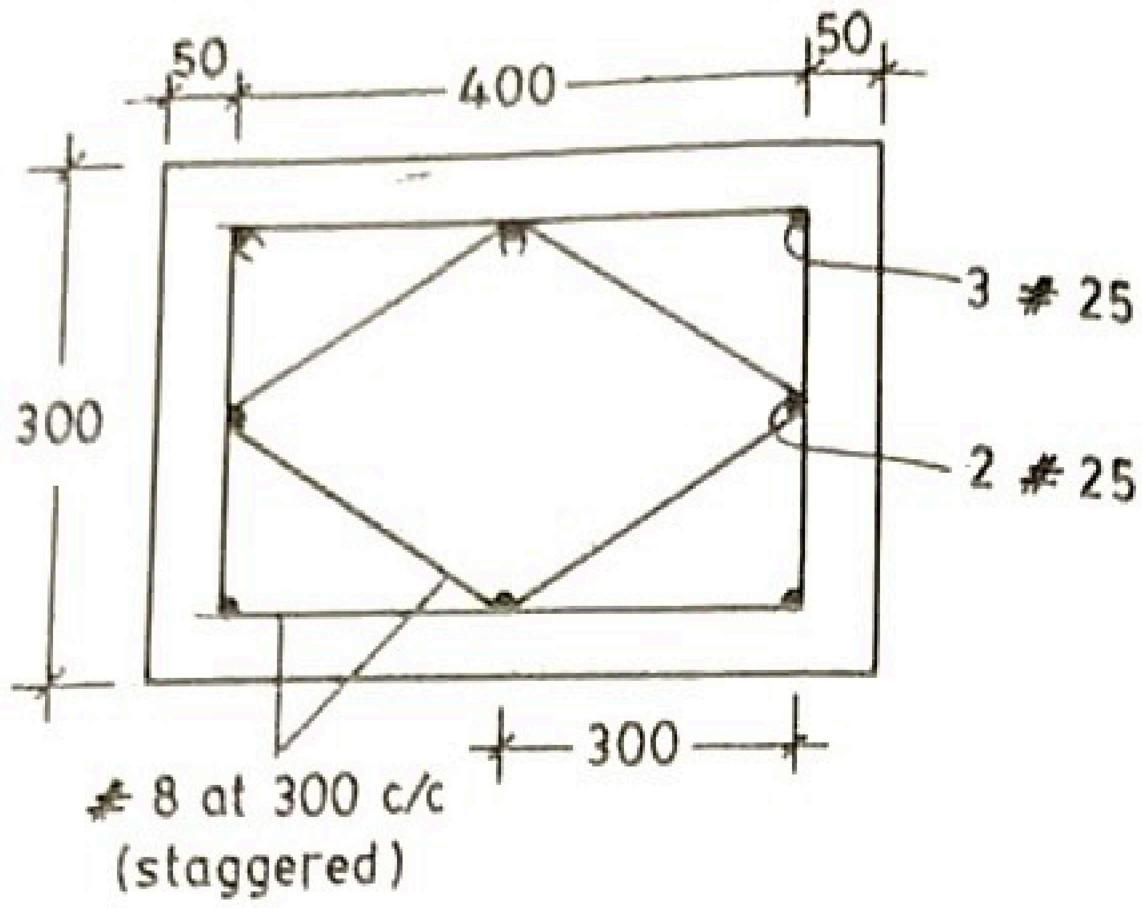

Fig. 10.13 Reinforcements in Columns With Uniaxial Bending

a) **Data**

$D = b = 400$ mm Assume $d' = 40$ mm

$P_u = 900$ kN $\therefore (d'/D) = 0.10$

$M_u = 100$ kN.m

$f_{ck} = 20$ N/mm^2

$f_y = 415$ N/mm^2

b) **Non Dimensional Parameters**

$$\left(\frac{P_u}{f_{ck}D}\right) = \left(\frac{900\times10^3}{20\times400^2}\right) = 0.28$$

$$\left(\frac{M_u}{f_{ck}D^3}\right) = \left(\frac{100\times10^6}{20\times400^3}\right) = 0.078$$

c) **Longitudinal Reinforcements**

Refer Chart-56 of SP:16 and read out the values of the parameter

$$\therefore \quad \left(\frac{p}{f_{ck}}\right) = 0.10 \qquad \therefore \quad p = (20\times0.10) = 2$$

$$\therefore \quad A_s = \left(\frac{p\pi D^2}{400}\right) = \left(\frac{2\times\pi\times400^2}{400}\right) = 2512 \text{ mm}^2$$

Provide 6 bars of 25 mm diameter ($A_s = 2945$ mm^2)

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d) **Lateral ties**

Tie diameter ≮ (25/4) = 6.25 mm
≯ 16 mm (Hence select 8 mm diameter ties)

Tie spacing ≯ 400 mm
≯ (16 × 25) = 400 mm
≯ 300 mm

Provide 8 mm diameter ties at 300 mm centers.

e) **Reinforcements**

Fig. 10.14 shows the details of reinforcements in the column section.

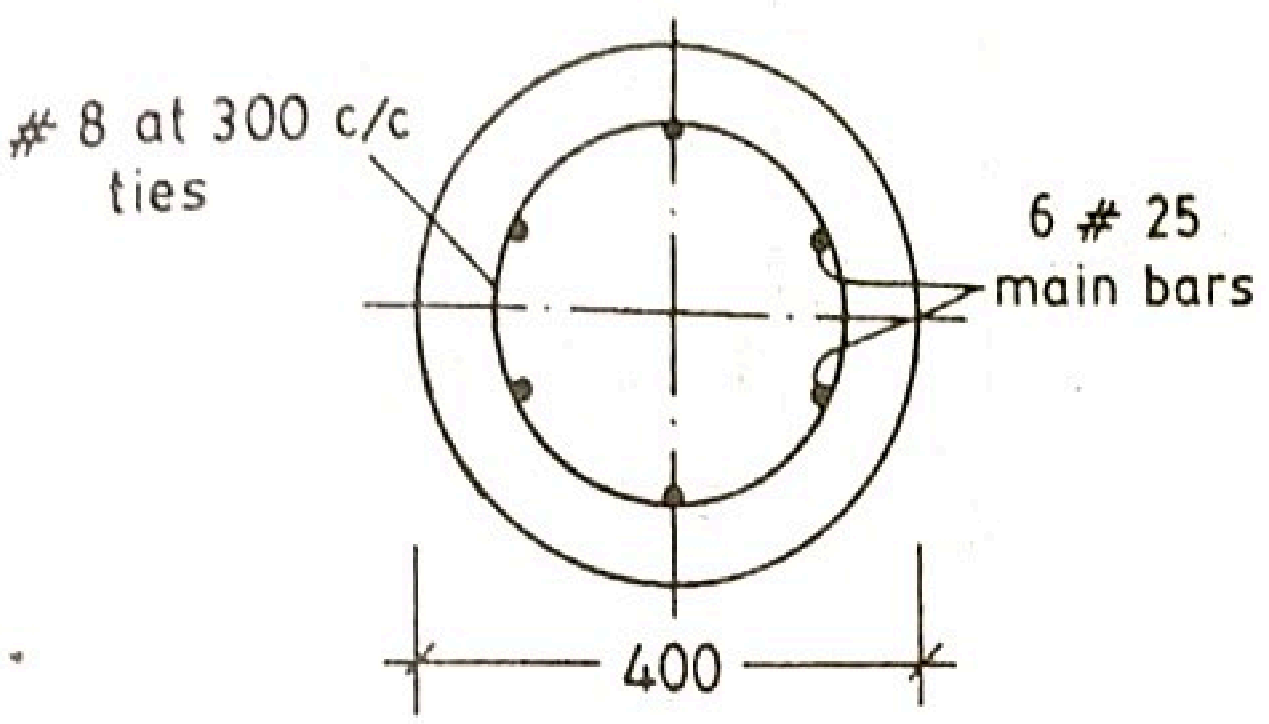

Fig. 10.14 Reinforcements in Circular Column

10.6 DESIGN OF SHORT COLUMNS UNDER COMPRESSION AND BIAXIAL BENDING

10.6.1 Introduction

Columns located at the corners of a multistoreyed building with rigidly connected beams at right angles, develop biaxial moments together with the axial compressive load transmitted from beams. Fig. 10.15(a) shows the column section subjected to the axial compressive load P_u and the moments M_{ux} and M_{uy} about the major and minor axis respectively. Fig. 10.15(b) shows the axis of bending and the resultant moment M_u acts about this axis inclined to the two principal axes. The resultant eccentricity is computed ass $e = (M_u/P_u)$ and this can also be expressed as,

$$e = \sqrt{e_x^2 + e_y^2} \quad \text{where} \quad e_x = (M_{ux}/P_u) \quad \text{and} \quad e_y = (M_{uy}/P_u)$$

The possible neutral axis lies in the X-Y plane as shown in Fig. 10.15(c).

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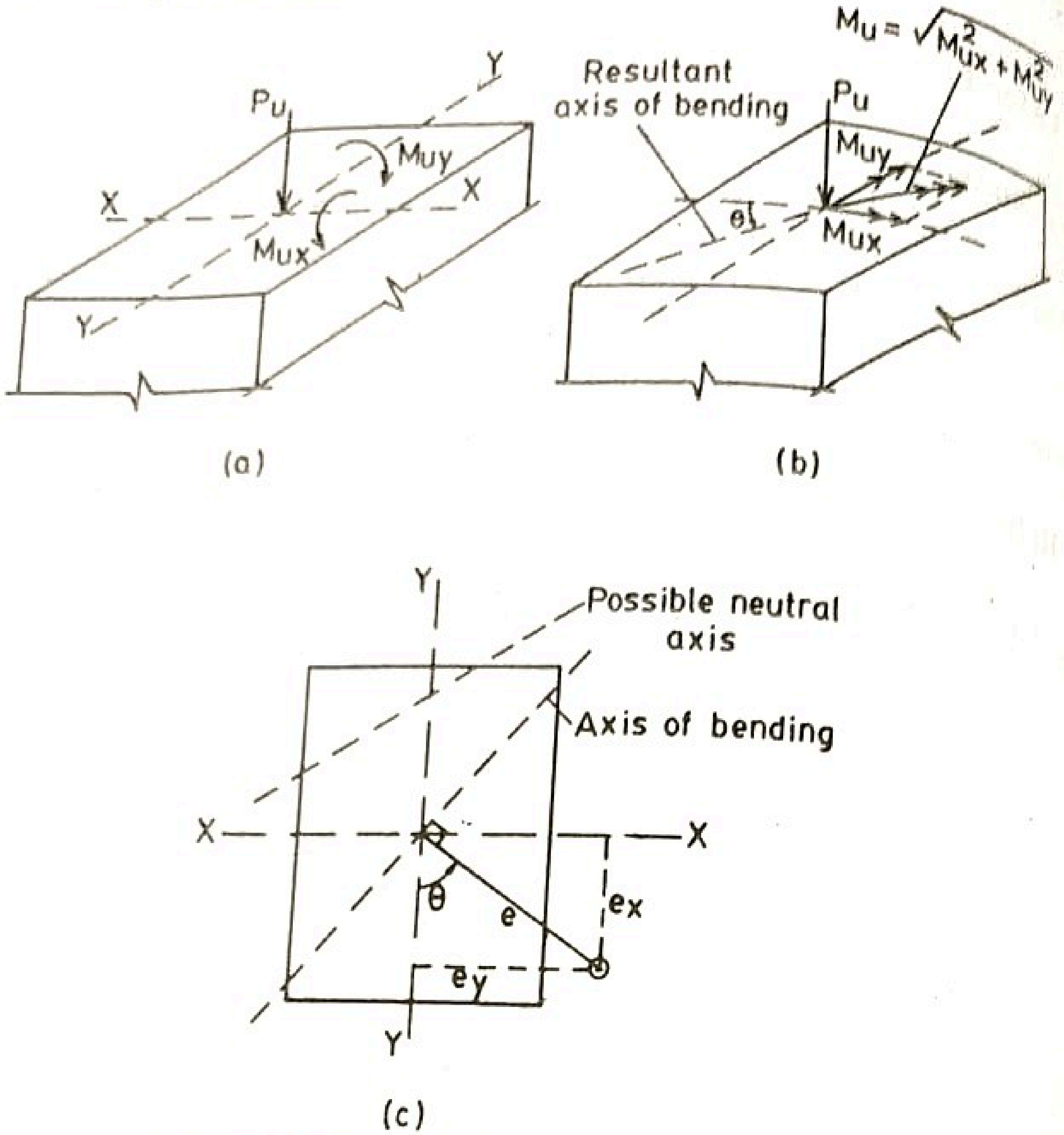

Fig. 10.15 Biaxial Bending of Short Columns

By choosing the neutral axis which is in the X-Y plane, calculations are made from fundamentals to satisfy the equilibrium of load and moments about both the axes. This procedure is tedious and is not generally recommended for routine design.

To overcome the difficulties of trial and error procedure in the design of columns subjected to biaxial moments, The Indian standardcode IS:456-2000 recommends a simplified procedure based on Bresler's[87] formulation which facilitates faster design of reinforcements in the columns. This method is outlined in the following section.

10.6.2 Codal Method for Design of Compression members subject to Biaxial Bending

The simplified procedure adopted by the code (clause 39.6) based on Bresler's empirical formulation is expressed by the relation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leq 1.0$$

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Where M_{ux}, M_{uy} are the moments about X and Y axes respectively due to design loads. M_{ux1} and M_{uy1} are the maximum uniaxial moment capacities with an axial load P_u, bending about X and Y axes respectively.
α_n is an exponent whose value depends on the ratio (P_u/P_{uz}) where

$$P_{uz} = [0.45\, f_{ck}.A_{sc} + 0.75 f_y.A_{sc}] \quad \text{i.e., Value of } P_u \text{ when } M = 0$$

The range of values of the ratio (P_u/P_{uz}) and the corresponding value of an are shown in Table-10.2 as well as in Fig. 10.16.

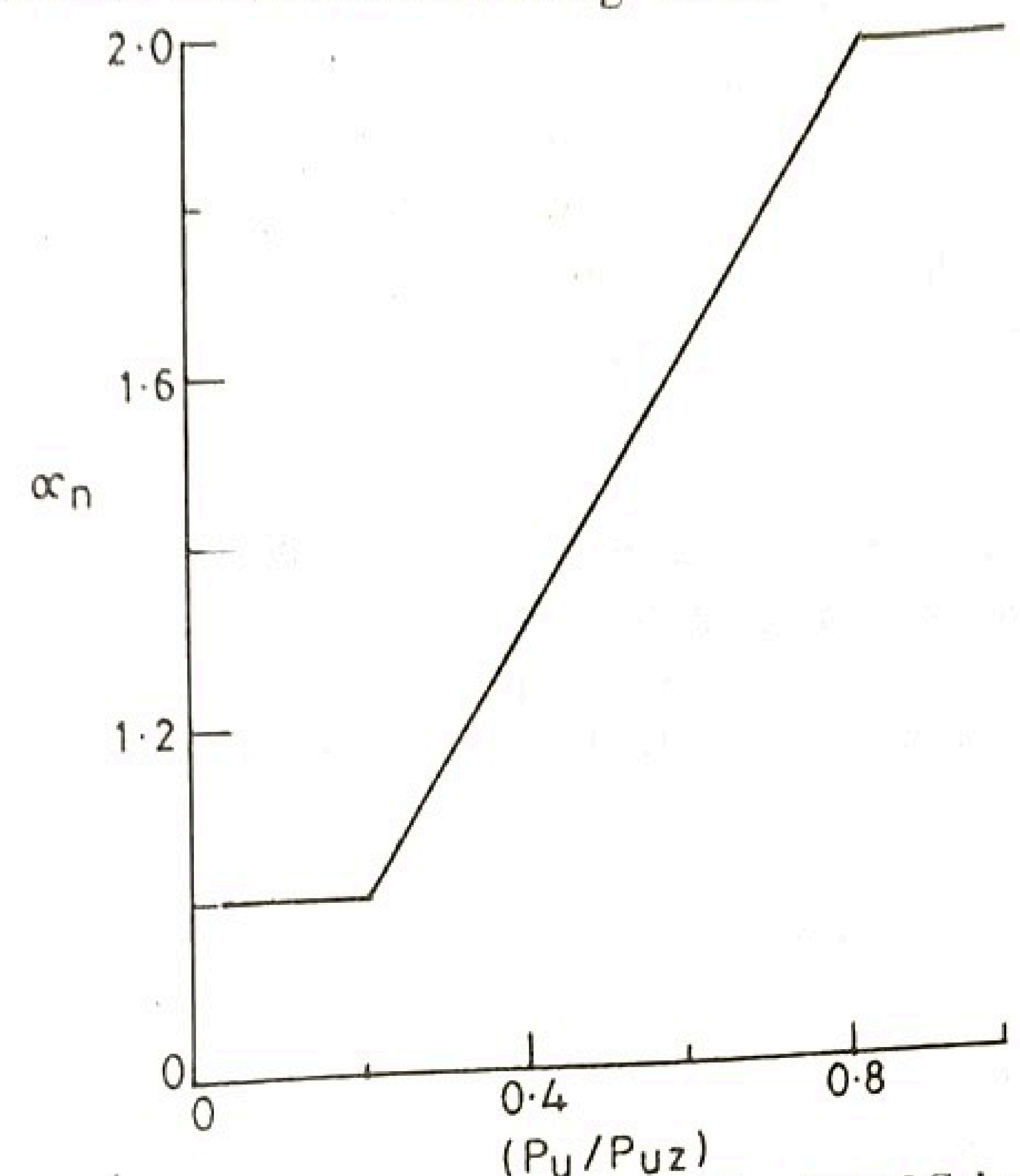

Fig. 10.16 Coefficient α_n For Biaxial Bending of Columns

For intermediate values, linear interpolation may be done. Chart-63 of SP-16 can be used for evaluating P_{uz} for different grades of concrete and steel and the percentage of reinforcement in the section.

Table 10.2 Values of α_n

(P_u/P_{uz})	α_n
≤ 0.2	1.0
≥ 0.8	2.0

Chart-64 of SP: 16 shows the relation,

$$\left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} = 1.0 \text{ for different values of } (P_u/P_{uz})$$

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These curves are very useful in the design of columns subjected to biaxial bending.

The use of SP:16 charts for the design of columns subjected to axial compression and biaxial bending is illustrated in the example 10.6.4.

10.6.3 Selection of Trial Section and reinforcements

In practice, the cross sectional dimensions of the column are selected before the structural analysis is performed and the biaxial moments are derived from the frame analysis. Hence, only reinforcements need to be suitably assumed for the design. However, Devdas Menon[88] has suggested a simpler approach for the selection of reinforcements based on the resultant moment given by the relation,

$$M_u = 1.15\sqrt{M_{ux}^2 + M_{uy}^2}$$

This bending moment is considered to act in association with the axial compressive load Pu and using the design charts, the reinforcement percentage in the cross section is determined. Thereafter the procedure is the same as specified in section 10.6.2 for checking the adequacy of the designed section.

10.6.4 Design Example

Design the reinforcements in a short column 400 mm by 600 mm subjected to an ultimate axial load of 1600 kN together with ultimate moments of 120 kN.m and 90 kN.m about the major and minor axis respectively. Adopt M-20 grade concrete and Fe-415 HYSD bars.

a) Data

b	= 400 mm	f_{ck}	= 20 N/mm^2
D	= 600 mm	f_y	= 415 N/mm^2
P_u	= 1600 kN	d'	= 60 mm
M_{ux}	= 120 kN.m	(d'/D)	= 0.1
M_{uy}	= 90 kN.m		

b) Reinforcements

Reinforcements are distributed equally on all the four sides.

As a first trial, adopt percentage of reinforcement in the cross section as p = 1 percent

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$$A_s = \left(\frac{p\,b\,D}{100}\right) = \left(\frac{1\times 400\times 600}{100}\right) = 2400 \text{ mm}^2$$

$\therefore$

Use 8 bars of 20 mm diameter distributed 3 on each face ($A_s = 2512$ mm^2).

$$p = \left(\frac{100\times 2512}{400\times 600}\right) = 1.04 \quad \text{and} \quad \left(\frac{p}{f_{ck}}\right) = \left(\frac{1.04}{20}\right) = 0.052$$

$$\left(\frac{P_u}{f_{ck} b\,D}\right) = \left(\frac{1600\times 10^3}{20\times 400\times 600}\right) = 0.333$$

Refer Chart-44 of SP:16 and read out the ratio $[M_{ux1}/f_{ck}\,b\,D^2]$ corresponding to the ratio $[P_u/f_{ck}\,b\,D] = 0.333$ and $(d'/D) = 0.10$ and $(p/f_{ck}) = 0.052$.

$$\therefore \quad \left(\frac{M_{ux1}}{f_{ck} b D^2}\right) = 0.085$$

$$\therefore \quad M_{ux1} = (0.085\times 20\times 400\times 600^2)10^{-6} = 245 \text{ kN.m}$$

For moments about the minor axis YY, $b = 600$ mm, $D = 400$ mm and $d' =$ 60 mm

$$\therefore \quad \left(\frac{d'}{D}\right) = \left(\frac{60}{400}\right) = 0.15$$

Refer chart-45 of SP:16 and read out the ratio $[M_{uy1}/\,f_{ck}\,b\,D^2]$ corresponding to the ratio $[P_u/\,f_{ck}\,b.D] = 0.333$ and $(p/\,f_{ck}) = 0.052$

$$\therefore \quad \left(\frac{M_{uy1}}{f_{ck} b D^2}\right) = 0.08$$

$$\therefore \quad M_{uy1} = (0.08\times 20\times 600\times 400^2)10^{-6} = 153 \text{ kN.m}$$

$$P_{uz} = [0.45\,f_{ck}A_c + 0.75\,f_y A_s]$$

$$= [0.45\times 20\{(600\times 400) - 2512\} + (0.75\times 415\times 2512)]10^{-3} \text{ kN} = 2919 \text{ kN}$$

$$\therefore \quad \text{Ratio} \quad \left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1600}{2929}\right) = 0.548$$

Refer Fig. 10.16 and read out the coefficient an corresponding to the ratio $(P_u/P_{uz}) = 0.548$. The value of $\alpha_n = 1.58$.

$$\therefore \quad \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \le 1$$

$$\left(\frac{120}{245}\right)^{1.58} + \left(\frac{90}{153}\right)^{1.58} = 0.756 < 1$$

Hence, the design is safe. Provide suitable lateral ties as per codal provisions.

Provide 8 mm diameter lateral ties at 300 mm centers will conform to the codal requirements.

The above problem can be solved by using Charts-63 and 64 of SP:16 as shown below:

From Chart-63, for $p = 1$ percent, $f_{ck} = 20$ N/mm^2 and $f_y = 415$ N/mm^2, read out the

Corresponding ratio,

$$\left(\frac{P_{uz}}{A_g}\right) = 12 \text{ and hence } P_{uz} = (12 \times 600 \times 400)10^{-3} = 2880 \text{ kN}$$

Also $$\left(\frac{M_{ux}}{M_{ux1}}\right) = \left(\frac{120}{245}\right) = 0.49$$

And $$\left(\frac{M_{uy}}{M_{uy1}}\right) = \left(\frac{90}{153}\right) = 0.59$$

Ratio $$\left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1600}{2880}\right) = 0.55$$

From Chart-64 of SP: 16 for $[M_{ux}/M_{ux1}] = 0.49$ and $[P_u/P_{uz}] = 0.55$, read out the ratio

$$\left(\frac{M_{uy}}{M_{uy1}}\right) = 0.8 > \text{ calculated value of } 0.59$$

Hence, the design is safe. However for economical design, a second trial is made with lower value of reinforcement and the various steps repeated such that the ratio of $[M_{uy}/M_{uy1}]$ obtained from Chart-64 is slightly greater than the calculated value.

10.6.5 Design Example

A short column located at the corner of a storied building is subjected to an axial factored load of 2000 kN together with factored moments of 75 and 60 kN.m acting in perpendicular planes. The size of the column is fixed as 450 by 450 mm. Adopting concrete of M-20 grade and Fe-415 HYSD bars, design suitable reinforcements in the column section.

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a) **Data**

$$b = 450 \text{ mm} \qquad f_{ck} = 20 \text{ N/mm}^2$$
$$D = 450 \text{ mm} \qquad f_y = 415 \text{ N/mm}^2$$
$$M_{ux} = 75 \text{ kN.m} \qquad d' = 50 \text{ mm}$$
$$M_{uy} = 60 \text{ kN.m} \qquad (d'/D) = 0.10$$

b) **Equivalent Moment**

The reinforcement in section is designed for the axial compressive load P_u and the equivalent moment given by the relation,

$$M_u = 1.15\sqrt{M_{ux}^2 + M_{uy}^2}$$
$$= 1.15\sqrt{75^2 + 60^2} = 110 \text{ kN.m}$$

c) **Non Dimensional Parameters**

$$\left(\frac{P_u}{f_{ck} b d}\right) = \left(\frac{2000 \times 10^3}{20 \times 4505450}\right) = 0.49$$
$$\left(\frac{M_u}{f_{ck} b\, D^2}\right) = \left(\frac{110 \times 10^6}{20 \times 450 \times 450^2}\right) = 0.06$$

d) **Reinforcements**

Refer chart-44 of SP: 16 (equal reinforcement on all faces) with $(d'/D) =$ 0.10 and read out the value of $(p/f_{ck}) = 0.06$.

$$\therefore \quad p = (20 \times 0.06) = 1.2$$

$$\therefore \quad A_s = \left(\frac{pbD}{100}\right) = \left(\frac{1.2 \times 450 \times 450}{100}\right) = 2430 \text{ mm}^2$$

Provide 8 bars of 20 mm diameter ($A_s = 2512$ mm^2) with 3 bars in each face.

$$p = \left(\frac{100 \times 2512}{450 \times 450}\right) = 1.24 \text{ and the ratio} \left(\frac{p}{f_{ck}}\right) = \left(\frac{1.24}{20}\right) = 0.062$$

Refer Chart-44 (SP:16) and read out the value of the ratio $[M_{ux1}/(f_{ck}\, b\, D^2)]$ corresponding to the value of ratio $[P_u/f_{ck}\, b\, D] = 0.49$ and $(p/f_{ck}) = 0.062$.

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$$\left(\frac{M_{ux1}}{f_{ck}bD^2}\right) = 0.06$$

$$\therefore \quad M_{ux1} = (0.06 \times 20 \times 450 \times 450^2)10^{-6} = 109 \text{ kN.m}$$

Due to symmetry, $M_{ux1} = M_{uy1} = 109$ kN.m

$$P_{uz} = [0.45\, f_{ck}.A_c + 0.75\, f_y A_s]$$

$$= (0.45 \times 20)[(450 \times 450) - 2512] + (0.75 \times 415 \times 2512)$$

$$= (2581 \times 10^3)\text{N}$$

$$= 2581 \text{ kN}$$

$$\therefore \quad \left(\frac{P_u}{P_{uz}}\right) = \left(\frac{2000}{2581}\right) = 0.77$$

Refer Fig. 10.16 and read out the coefficient an = 1.95

e) Check for Safety under Biaxial Loading

$$\therefore \quad \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \leq 1$$

$$\left(\frac{75}{109}\right)^{1.95} + \left(\frac{60}{109}\right)^{1.95} = 0.79 \leq 1$$

Hence, the section is safe under specified loading.

f) Reinforcements

Provide 8 bars of 20 mm diameter as main reinforcement and 8 mm lateral ties at 300 mm centres.

10.7 DESIGN OF SLENDER COLUMNS

10.7.1 Introduction

Compression members having the ratio of effective length to its least lateral dimension (slenderness ratio) exceeding 12 are categorized as slender or long columns according to IS:456-2000 code. The deformation characteristics of slender columns are significantly different from that of short columns. When slender columns are loaded even with axial loads, the lateral deflection is significantly greater in comparison with short columns as shown in Fig. 10.17. Consequently, in slender columns, the moment produced by the deflection is large and should be considered in design.

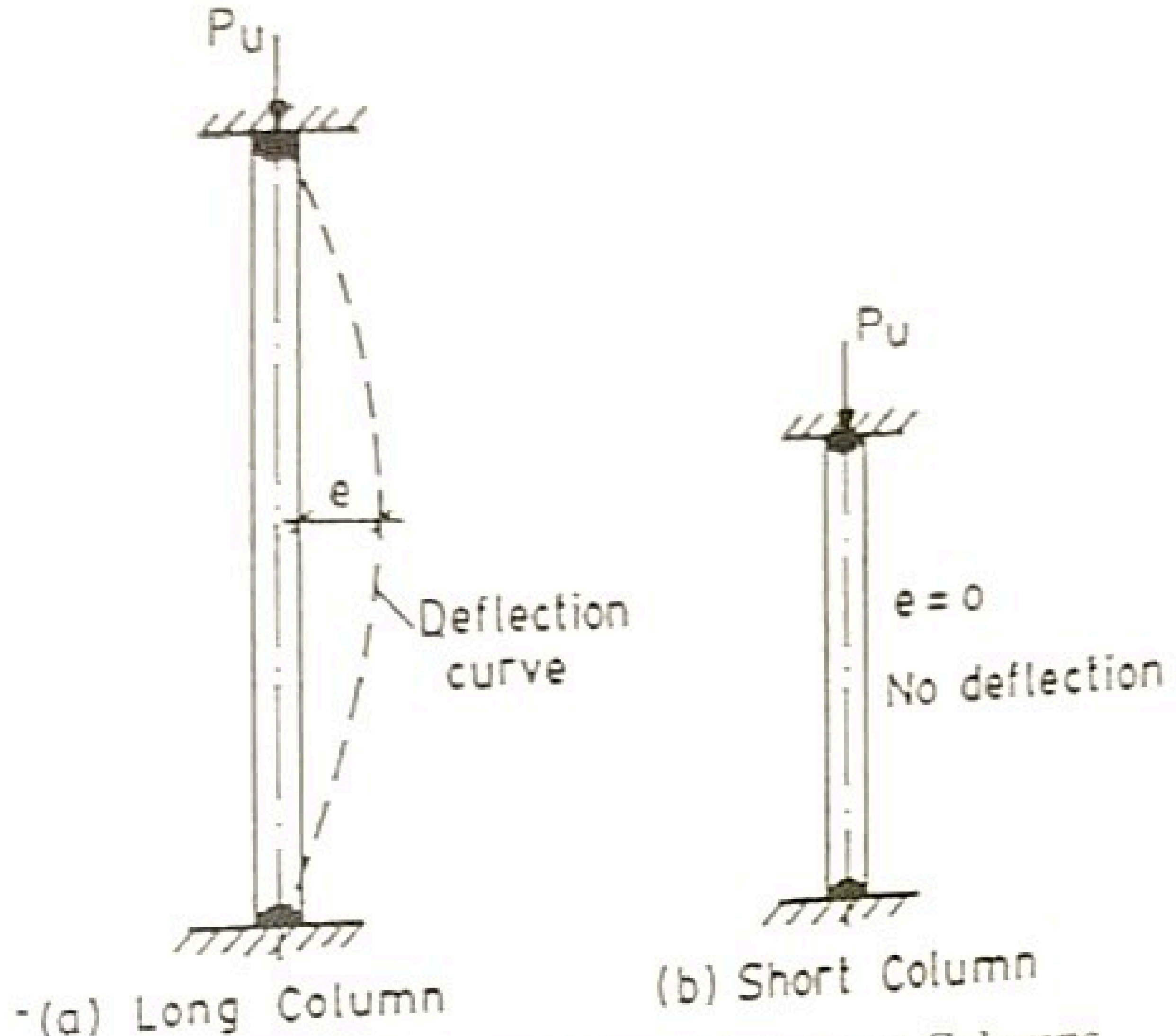

Fig. 10.17 Behaviour of Long And Short Columns

In the case of eccentrically loaded long columns, the effect of secondary moments developed due to the lateral deflection together with the primary moments significantly influences the load carrying capacity of the compression member.

10.7.2 Behaviour of Slender Columns

The structural behaviour of slender columns is significantly different from that of short columns with increasing slenderness ratios. Consider a column hinged at supports subjected to an eccentric load '*P*' at an eccentricity 'e' as shown in Fig. 10.18(a).

As the load is increased, the lateral deflection of the column increases.

If Δ = lateral deflection of the longitudinal axis,

$(e + \Delta)$ = total eccentricity

The moment at any section is expressed as,

$$M = P\,(e + \Delta)$$

$$M = M_{pr} + \Delta$$

Where M_{pr} = primary moment due to eccentricity of the load.

$P.\Delta$ = secondary moment which varies along the length of the column.

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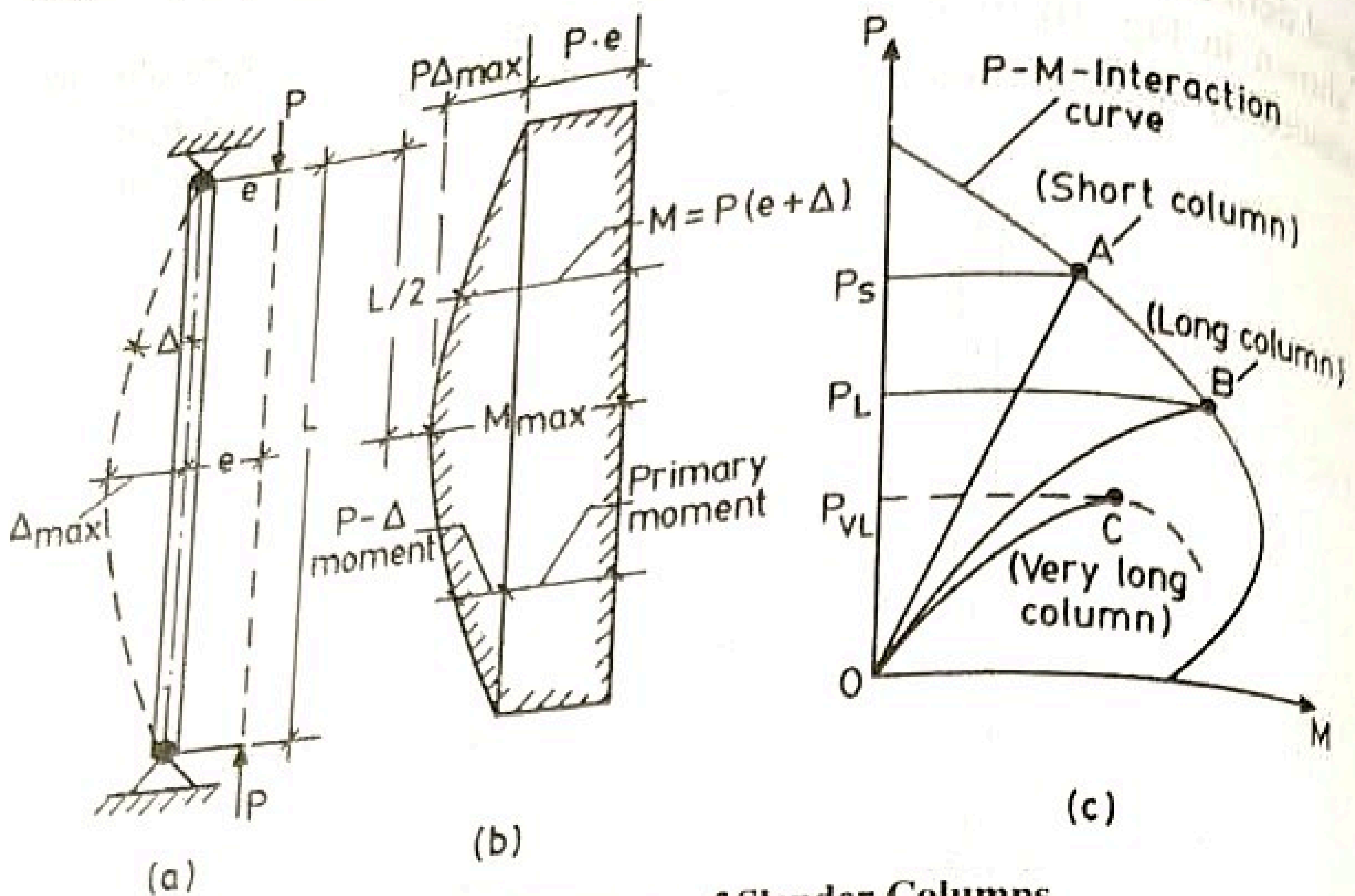

Fig. 10.18 Behaviour of Slender Columns

The maximum moment occurs at the mid height of the column and is expressed as

$$M_{max} = P\,(e + \Delta_{max})$$

The variation of maximum moment is non linear with the flexural stiffness reducing with increasing values of the load P [Refer Fig. 10.18(b)].

In the case of very short column, the flexural stiffness being very high, the lateral deflection ? is very small and the primary moment controls the behaviour of the column.

In the case of very slender column, it is possible that the flexural stiffness is effectively reduced to zero resulting in buckling or instability failure.

Fig. 10.18(c) shows the load-moment interaction diagram at the limit state of collapse representing the strength of the column with varying slenderness ratios.

In the case of short column, $\Delta_{max} = 0$ and hence the failure is due to the primary moment and axial load. Point A represents the behaviour of short column with material failure. Point B indicates the long column behaviour with primary and secondary moments with material failure.

In the case of very long columns, the failure is due to buckling or instability. The curve OC represents the behaviour of very long columns.

In the case of braced slender columns which is not subjected to sidesway, there is no significant relative lateral displacement between the top and bottom ends of the column. The ends of a braced column are partially

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restrained against rotation due to the floor level beams and moments M_1 and M_2 may develop at the ends. The column may be bent in single or double curvature, depending upon the nature of moments. The effect of these moments are taken into account in the design of such columns.

Unbraced slender columns are subjected to sidesway or lateral drift due to the action of lateral loads or gravity loads inducing additional moments at the supports. The moment amplification due to the lateral drift effect which is significantly greater than that of braced columns should be considered in the design of such columns.

The design of slender columns is similar to that of columns subjected to a given factored axial compression P_u and factored moments M_{ux} and M_{uy}, the only difference being that the moments should include the secondary moment components in slender column design, where as these are ignored being negligible in short column design.

10.7.3 Codal method for design of Slender columns

The IS:456-2000 code (clause 39.7) prescribes that the design of slender compression members should include the forces and moments determined from structural analysis and also the effects of deflections on moments and forces. The second order analysis involving deflections and their effect on moments and forces being computationally difficult and laborious, the code recommends simplified procedures for the design of slender columns, which involves the process of increasing the moments or reducing the strength to take care of slenderness effects.

The IS:456 code clause 39.7.1 recommends additional moments M_{ux} and M_{uy} expressed in terms of the factored axial load P_u, overall depth of the member (D) and the slenderness ratios (L_{ex}/D) and (L_{ey}/D) derived from the deformation characteristics of a pin ended braced slender column shown in Fig. 10.19.

The additional eccentricity Δ_{max} is a function of curvature. Denoting the maximum curvature at mid height as ϕ_{max}, it can be shown that Δ_{max} lies between (ϕ_{max}, $L^2/12$) and (ϕ_{max}, $L^2/8$). From Fig. 10.19, case(a) & (b), considering an average value for eccentricity as,

$$e_a = \Delta_{max} = (\phi_{max} L^2/10)$$

Referring to Fig. 10.20 showing the relation between curvature and failure strain profile and assuming,

$\varepsilon_{cu} = 0.0035$ and $\varepsilon_{st} = 0.002$, $d' = 0.1D$ and $(D - d') = 0.9D$

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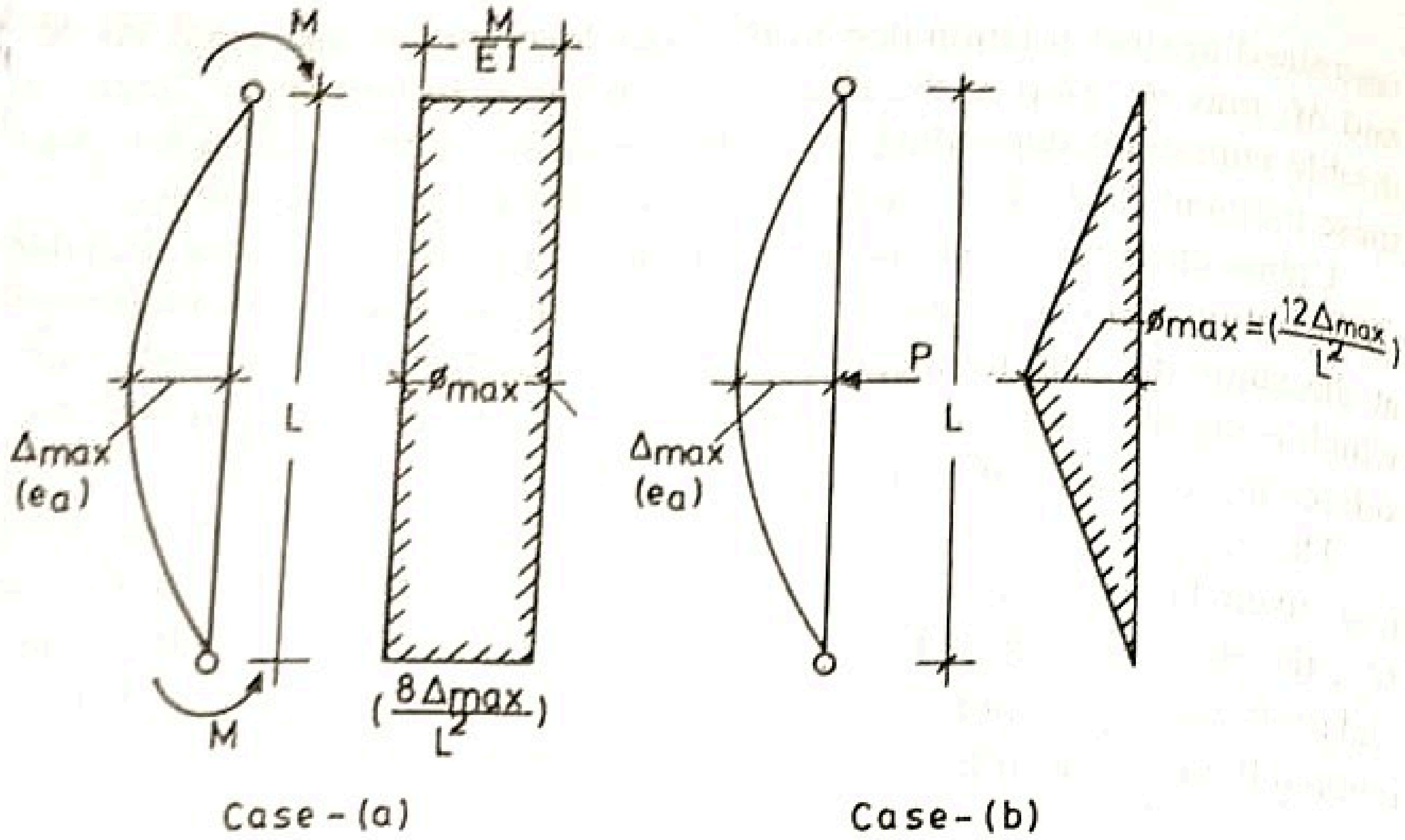

Fig. 10.19 Relation Between Deflection And Curvature in Pin Ended Slender Column

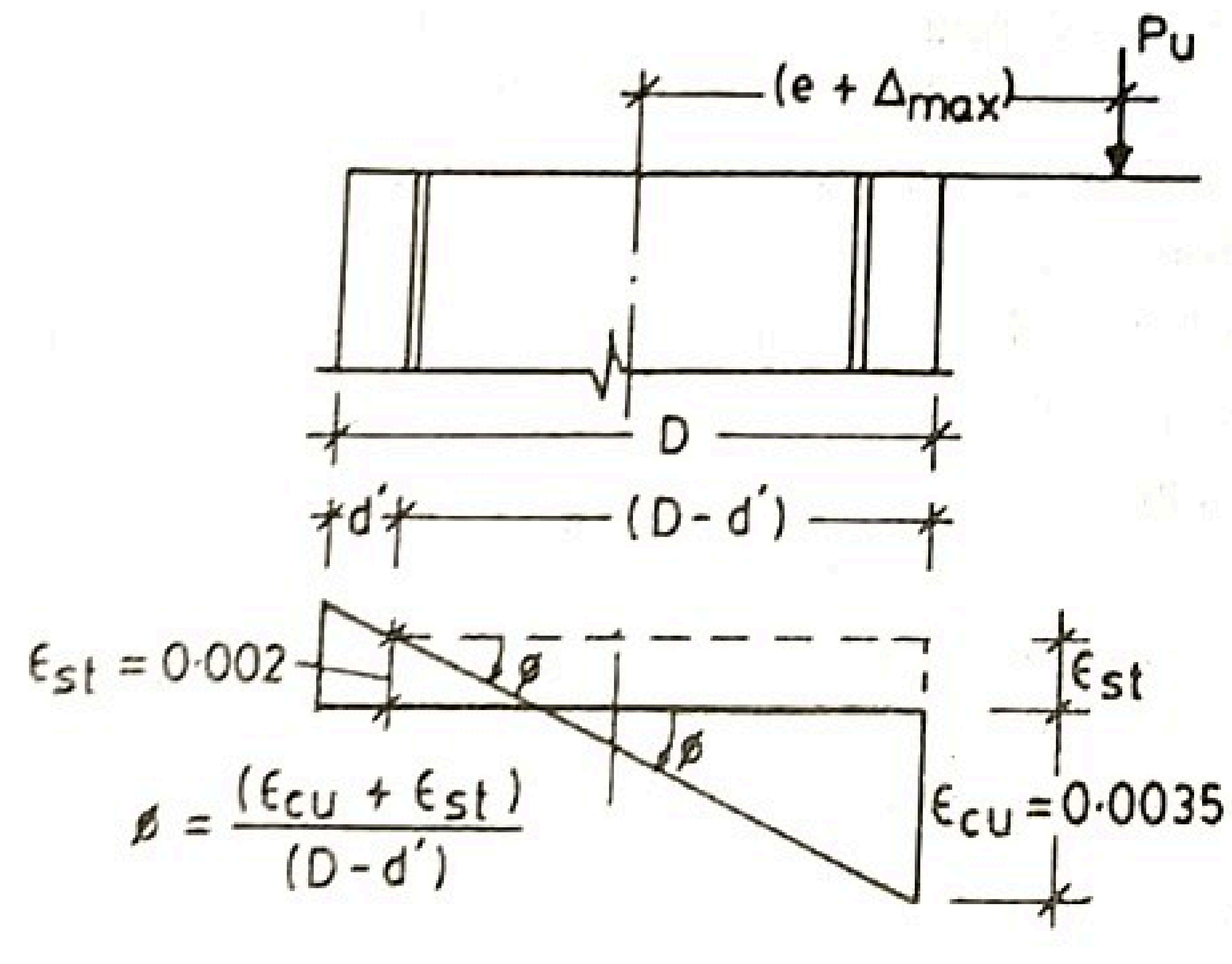

Fig. 10.20 Curvature-Strain Relationship

the additional moment comprises about 80 percent of the total moment. We can express the maximum curvature as,

$$\phi_{max} = \left[\frac{(0.0035 + 0.002)}{0.9D} \times 0.8\right] = \left(\frac{1}{200D}\right)$$

Substituting the value of ϕ_{max} in the expression for deflection e_a or Δ_{max} we have

$$\left(\frac{e_a}{D}\right) = \left(\frac{(L/D)^2}{2000}\right)$$

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Hence, the expressions recommended in IS: 456-2000 code for additional moments are

$$M_{ax} = P_u e_{ax} = \frac{P_u D_x}{2000}\left[\frac{L_{ex}}{D_x}\right]^2$$

$$M_{ay} = P_u e_{ay} = \frac{P_u b}{2000}\left[\frac{L_{ey}}{b}\right]^2$$

Where

P_u = axial load on the member
L_{ex} = effective length in respect of major axis
L_{ey} = effective length in respect of minor axis
D = depth of cross section at right angles to the major axis
b = width of member
e_{ax} and e_{ay} are additional eccentricities (Refer Table-I of SP:16).

It is important to note that the additional moments to be considered are in addition to the factored primary moments M_{ux} and M_{uy} in the design of columns. The additional moments specified in the code are derived on the assumption that the column is braced and bent symmetrically in single curvature. Also the axial load corresponds nearly to the balanced failure condition ie $P_u = P_b$ .If these conditions are not satisfied, the code recommends the following modifications.

For $P_u > P_b$, the additional moments may be reduced by the multiplying factor 'k' given by the relation,

$$k = \left[\frac{P_{uz} - P_u}{P_{uz} - P_b}\right] \leq 1$$

Where $P_{uz} = [0.45 f_{ck} A_c + 0.75 f_y A_s]$ and this value can be read out from chart-63 of SP:16 and P_b is the axial load corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in the outer most layer of tension steel.

The modification suggested in the code is optional and it should always be taken advantage of since the value of 'k' could be substantially less than unity.

The value of P_b depends on the arrangement of reinforcement and the cover ratio (d'/D) and the grades of concrete and steel. The values of P_b can be computed for rectangular and circular sections using the constants k_1 and k_2 given in Table-60 of SP: 16 and the relation expressed as,

$$\left(\frac{P_b}{f_{ck} b\, d}\right) = k_1 + k_2\left(\frac{p}{f_{ck}}\right)$$

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The value of the reduction factor 'k' can be read out from Chart-65 of SP: 16 after evaluating the ratios (P_u/P_{uz}) and (P_b/P_{uz})

For braced columns subjected to unequal primary moments M_1 and M_2 at the two ends, the value of M_u to be considered in computations of the total moment may be taken as (clause 39.7.1),

$$M_u = (0.4\, M_1 + 0.6\, M_2) \geq 0.4\, M_2$$

For un-braced columns, the lateral drift effect has to be included. Hence, an approximate method of including this effect is to assume the additional moment M_a to act at the column end where the maximum primary moment M_1 is operational. For design purposes, the total moment is computed as,

$$M_u = (M_a + M_1)$$

The use of these design principles is illustrated in the following example.

10.7.4 Design Example

Design the reinforcements required for a column which is restrained against sway using the following data:

a) Data

Size of column = 530 mm by 450 mm
Effective length = 6.6 m
Un supported length = 7.7 m
Factored load = 1600 kN
Factored moment about major axis = 45 kN.m at top and 30 kN.m at bottom
Factored moment about minor axis = 35 kN.m at top and 20 kN.m at bottom.
Concrete grade = M-25
Steel grade = Fe-500 HYSD bars

Column is bent in double curvature and reinforcement is distributed equally on all the four sides of the section.

b) Slenderness ratio

$$\left(\frac{L_e}{D}\right) = \left(\frac{6600}{530}\right) = 12.45 > 12$$

$$\left(\frac{L_e}{b}\right) = \left(\frac{6600}{450}\right) = 14.67 > 12$$

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Hence, the column is slender about both axes

c) Additional Eccentricities

From Table-I of SP: 16, for $(L_e/D) = 12.45$, $(e_{ax}/D) = 0.078$
For $(L_e/b) = 14.67$, $(e_{ay}/b) = 0.108$
$\therefore\ e_{ax} = (0.078 \times 530) = 41.34$ mm
$e_{ay} = (0.108 \times 450) = 48.60$ mm

d) Additional Moments

$M_{ax} = [1600\ (41.34/1000)] = 66.14$ kN.m
$M_{ay} = [1600\ (48.60/1000)] = 77.76$ kN.m

The above moments have to be multiplied by modification factor (k) as per clause 39.7.1.1 of IS:456-2000.

$$k = \left[\frac{P_{uz} - P_u}{P_{uz} - P_b}\right] \leq 1$$

Assuming 3.28 percent reinforcement for the first trial, the ratio,

$$(p/f_{ck}) = (3.28/25) = 0.131$$

From Chart-63 of SP:16, read out the ratio of $[P_{uz}/A_g] = 21$

$$\therefore \quad P_{uz} = \left[\frac{21 \times 530 \times 450}{100}\right] = 5008 \text{ kN}$$

Assuming 25 mm diameter bars with 50 mm cover,
$(d'/D) = (50/530) = 0.1$ and $(d'/b) = (50/450) = 0.1$
From Table-60 of SP: 16, read out the values of k_1 and k_2 as
$k_1 = 0.207$ and $k_2 = 0.425$

$$P_{bx} = P_{by} = [k_1 + k_2(p/f_{ck})]f_{ck}.b.D$$
$$= [0.207 + 0.425(0.131)]\ [(25 \times 450 \times 530)/1000] = 1566 \text{ kN}$$
$$k_x = k_y = [(5008 - 1600)/(5008 - 1566)] = 0.99$$

Additional moments are modified as,

$$M_{ax} = (66.14 \times 0.99) = 65.48 \text{ kN.m}$$
$$M_{ay} = (77.16 \times 0.99) = 76.39 \text{ kN.m}$$

As per clause 39.7.1 of IS: 456-2000 code, the initial moment acting on the column should be modified as follows:

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$$M_{ux} = [(0.6 \times 45) - (0.4 \times 30)] = 15 < (0.4 \times 45) = 18$$
$$M_{uy} = [(0.6 \times 35) - (0.4 \times 20)] = 13 < (0.4 \times 35) = 14$$

As the above values are less than 0.4 times the larger end moment, we have to consider for design the modified initial moments as,

$$M_{ux} = 18 \text{ kN.m and } M_{uy} = 14 \text{ kN.m}$$

These moments are to be compared with the moment due to minimum eccentricity and greater of the two values is to be taken as the initial moment.

From clause 25.4 of IS: 456-2000, the minimum eccentricities are computed as,

$$e_x = \left[\frac{7700}{500} + \frac{530}{30}\right] = 33.07 \text{ mm} > 20 \text{ mm}$$

$$e_y = \left[\frac{7700}{500} + \frac{450}{30}\right] = 30.4 \text{ mm} > 20 \text{ mm}$$

$$M_{ux,min} = 1600\,(33.07/1000) = 52.92 \text{ kN.m} > 18.0 \text{ kN.m}$$
$$M_{uy,min} = 1600\,(30.4/1000) = 48.64 \text{ kN.m} > 14.0 \text{ kN.m}$$

Therefore the total moment for which the column is to be designed are,

$$M_{ux} = (52.92 + 65.48) = 118.40 \text{ kN.m}$$
$$M_{uy} = (48.64 + 76.39 = 125.03 \text{ kN.m}$$

$$\left(\frac{P_u}{f_{ck}bd}\right) = \left(\frac{1600 \times 10^3}{25 \times 530 \times 450}\right) = 0.27$$

From Chart-48 of SP:16, for the ratio (p/f_{ck}) read out the moments as,

$$M_{ux1} = (0.19 f_{ck}\, b\, d^2) = (0.19 \times 25 \times 450 \times 530^2)\, 10^{-6} = 600 \text{ kN.m}$$
$$M_{uy1} = (0.19 f_{ck}\, d\, b^2) = (0.19 \times 25 \times 530 \times 450^2)\, 10^{-6} = 510 \text{ kN.m}$$

$$\left(\frac{M_{ux}}{M_{ux1}}\right) = \left(\frac{118.40}{600}\right) = 0.20$$

$$\left(\frac{M_{uy}}{M_{uy1}}\right) = \left(\frac{125.03}{510}\right) = 0.25$$

$$\left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1600}{5008.5}\right) = 0.32$$

From Chart-64, for $(P_u/P_{uz}) = 0.32$ and $(M_{ux}/M_{ux1}) = 0.2$ read out the value of $(M_{uy}/M_{uy1}) = 0.92 > 0.25$.

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Hence, the section is safe but not economical. In the second trial, the area of reinforcement may be reduced in the section and the various design steps are repeated until an economical section is obtained.

Provide 12 bars of 25 mm diameter equally spaced on each face and lateral ties as per codal specifications.

10.8 DESIGN OF FOOTINGS

10.8.1 Introduction

Reinforced concrete columns are generally supported by the footings which are located below the ground level and is referred to as the foundation structure. The main purpose of the footing is to effectively support the super structure like columns by transmitting the applied loads, moments and other forces to the soil without exceeding the safe bearing capacity and also the settlement of the structure should be within tolerable limits and as nearly uniform as possible.

The footings are generally designed to resist the bending moments and shear forces developed due to soil reaction as specified in the Indian standard code IS:456-2000. This chapter deals with the design principles of different types of footings outlined in the following section.

10.8.2 Types of Footings

Footings are grouped under shallow foundations (in contrast to deep foundations like piles and caissons) which are adopted when the soil of adequate bearing capacity is available at a relatively short depth below the ground level. Column footing has a large plan area in comparison with the cross sectional area of the column. The loads on the columns are resisted by concrete and steel and these load effects are transmitted by the footing to the relatively weak supporting soil by bearing pressure.

Generally, the safe bearing capacity of the soil is very low in the range of 100 to 400 kN/m^2, whereas the permissible compressive stress in concrete is around 5 to 15 N/mm^2 and in steel, it is in the range of 130 to 190 N/mm^2 in reinforced concrete columns under working loads.

a) Isolated Column Footing

In the case of framed buildings with columns located on reasonably firm soil, it is generally sufficient to provide separate independent footings for each of the columns. Such a footing is referred to as isolated footing which is square, rectangular or circular in shape depending upon the shape of