Unit-2

Design for Flexure

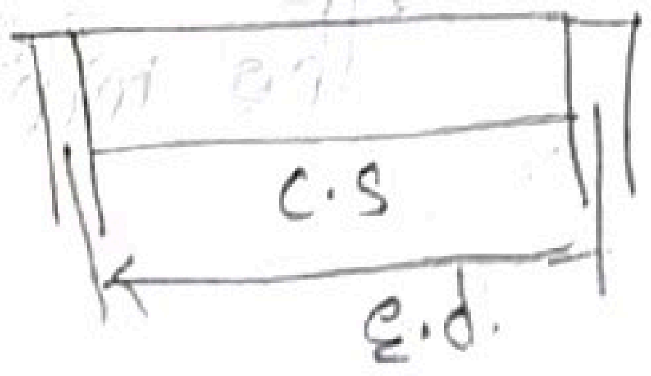
web → bvd

Pg No (37) 23.21. → 38 fig 4. → 46 25.5.) 47

Effective span   Pg No. 34,35.

a) Simply supported Beam = clear span + effective depth of beam.
(or)
c/c distance, b/w supports.
(center to center).

b) Continuous Beam:-



General design requirements for Beams:-

1) Effective span:-

a) Simply Supported Beam:-

SS = clear span + effective depth of Beam.
(or)
Center to center distance b/w supports

Taken which one is less.

b) Cantilever Beam:- = length of the place of support + $\frac{\text{effective depth}}{2}$

c) Continuous Where it forms a the end of a Continuous Beam the effective length = Center to Center distance of Support should be taken.

2) Limiting stiffness:- It is governed by l/d ratio.

Span ≯ 10m	l/d ≯
Cantilever	7
Simply Supported	20
Continuous	26.

If spans greater than 10m, the above values should be multiplied by 10/span in meters.

prefer clause No. 23.2 of IS-456.

· Depending on the amount and type of steel used the above Values Shall be multiply with the modification factors explained in fig 4 of IS 456 (pg.No. 38)

3) Minimum Reinforcement:- refer clause No 26.5.1 of IS456 (Pg no 46)

The minimum area of tension reinforcement

$$\not< \frac{A_s}{bd} = \frac{0.85}{f_y}$$

This works out only for i) 0.2% - fe 415

ii) 0.35% - fe 250

4) Maximum Reinforcement:- refer clause No. 26.5.1 of IS456 (PgNo 47)

Maximum tension Reinforcement:

Should $\not>$ 4% of gross ~~Se~~ Cross Sectional area.

$\not>$ 4% (bxD)

Where D→Over all depth (or) gross depth.

5) Spacing of bars:- The horizontal distance between two Parallel main reinforcement bars shall be usually not less than the greatest of the following.

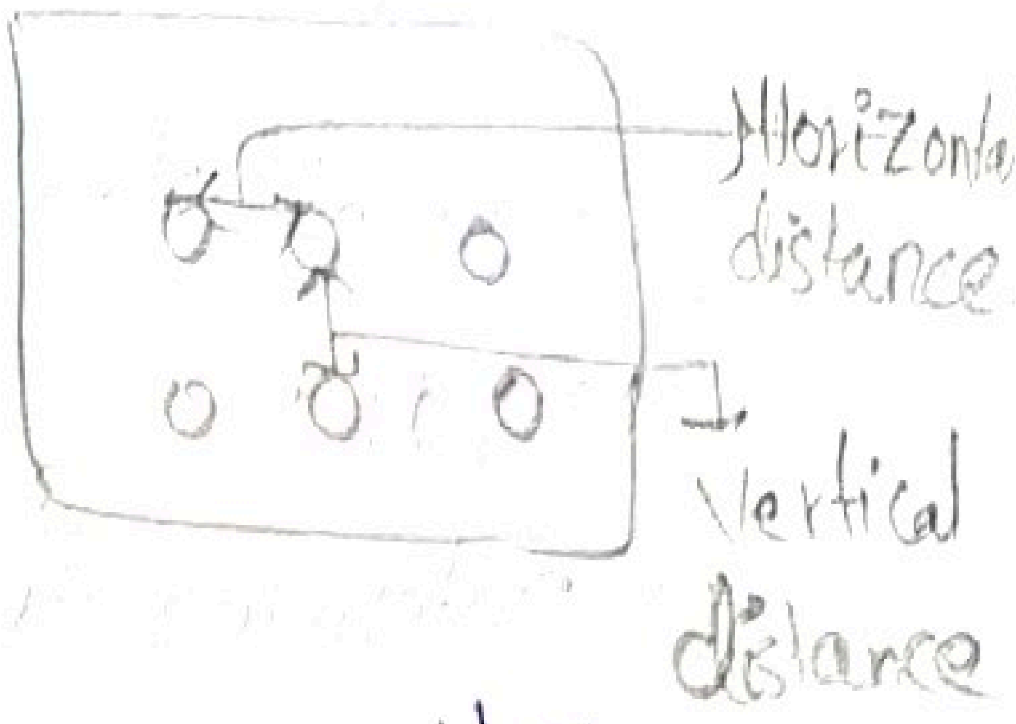
i) ~~A~~ diameter of the bar if Same diameter bars are used.

ii) Diameter of largest bar if diameters are not equal.

iii) 5mm greater than the nominal size of aggregate.

When there are 2 (or) more rows of bars the bars shall be vertically in line and the vertical distance b/w the bars shall be 15mm, 2/3rd of nominal maximum size of aggregate (or) the maximum size of bars which ever is greater

→ The Maximum spacing of bars in tension for Beams is taken from table No 15 of IS 456 depending on the amount of redistribution carried out by analysis.



6) Cover to reinforcement :-

Reinforcement shall have Concrete cover as thickness as follows.

a) At Each end of reinforcement bar should ≮ 25mm (or) 2 times of diameter of bar. [≮ 25mm (or) 2ϕ]

b.) For longitudinal reinforcing bar in beam should not less than 25mm (or) not less than diameter of bar. [≮ 25mm (or) ≮ ϕ].

7) Side face reinforcement:-

→ Depth of beam > 750mm the side face reinforcement should be provided along two faces.

→ The total area of such reinforcement should not less than 0.1% of the web area and shall be equally distributed ~~and~~ ~~on~~ two faces at a spacing of not exceeding 300mm (or) width of Beam. Which ever is less.

Use of SP-16 Curves:-

1) Tables 1 to 4. gives the percentage of steel required for various values of $\frac{M_u}{bd^2}$ and for fck = 15, 20, 25 and 30.

2) charts 1 to 18 gives the moment of resistance per meter depth 5 to 80cm.

3) Varying percentage of steel for fck = 15 and 20. Using fy = 250, 415 and 500.

Types of problems:-

Case i:- Determine the moment of resistance, given dimensions and area of steel.

Steps:-

1) Determine the depth of Neutral axis (x_u). [Compression force = Tension force

2) Calculation of x_u max.

3) If $x_u = x_u$ limit (or) $x_u$max [Balanced Section]

4) i)

$$M.O.R = \text{Comp} \times \text{lever arm}$$

$$= 0.36 f_{ck} \cdot b \cdot x_{umax} (d - 0.42 x_{umax})$$

$$M.O.R = \text{Tension} \times \text{lever arm}$$

$$= 0.87 f_y A_{st} (d - 0.42 x_{umax})$$

4) ii) If $x_u < x_u$max [under reinforced section]

$$M.O.R = \text{Tension} \times \text{lever arm}$$

$$= 0.87 f_y \cdot A_{st} [d - 0.42 \underset{\uparrow \text{less}}{x_u}]$$

5) iii) If $x_u > x_u$max [over reinforced Section]

$$M.O.R = \text{Comp} \times \text{lever arm}$$

$$= 0.36 f_{ck} \cdot b \cdot x_{umax} [d - 0.42 \underset{\downarrow \text{less}}{x_{umax}}]$$

Case ii:- Determine Area of Steel (Ast), given dimensions and Moment of resistance.

$$0.87 f_y \cdot A_{st} = 0.36 f_{ck} \cdot b \cdot x_u$$

$$M_u = 0.87 f_y \cdot A_{st} (d - 0.42 x_u)$$

$$0.87 f_y A_{st} \left(d - 0.42 \left[\frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b} \right] \right)$$

$$= 0.87 f_y A_{st} \left[d - \left[\frac{0.42 \times 0.87 = 0.36\, f_y A_{st}}{0.36 f_{ck} b} \right] \right].$$

$$= 0.87 f_y A_{st} \left[d - \frac{f_y A_{st}}{f_{ck} \cdot b} \right]$$

$$= 0.87 f_y A_{st}\, d \left[1 - \frac{f_y \cdot A_{st}}{f_{ck} \cdot b d} \right]$$

Quadratic equation in Ast^2. Calculate the roots. So, lesser value is Considered as Ast.

Case iii:- Design the Beam given Moment of resistance.

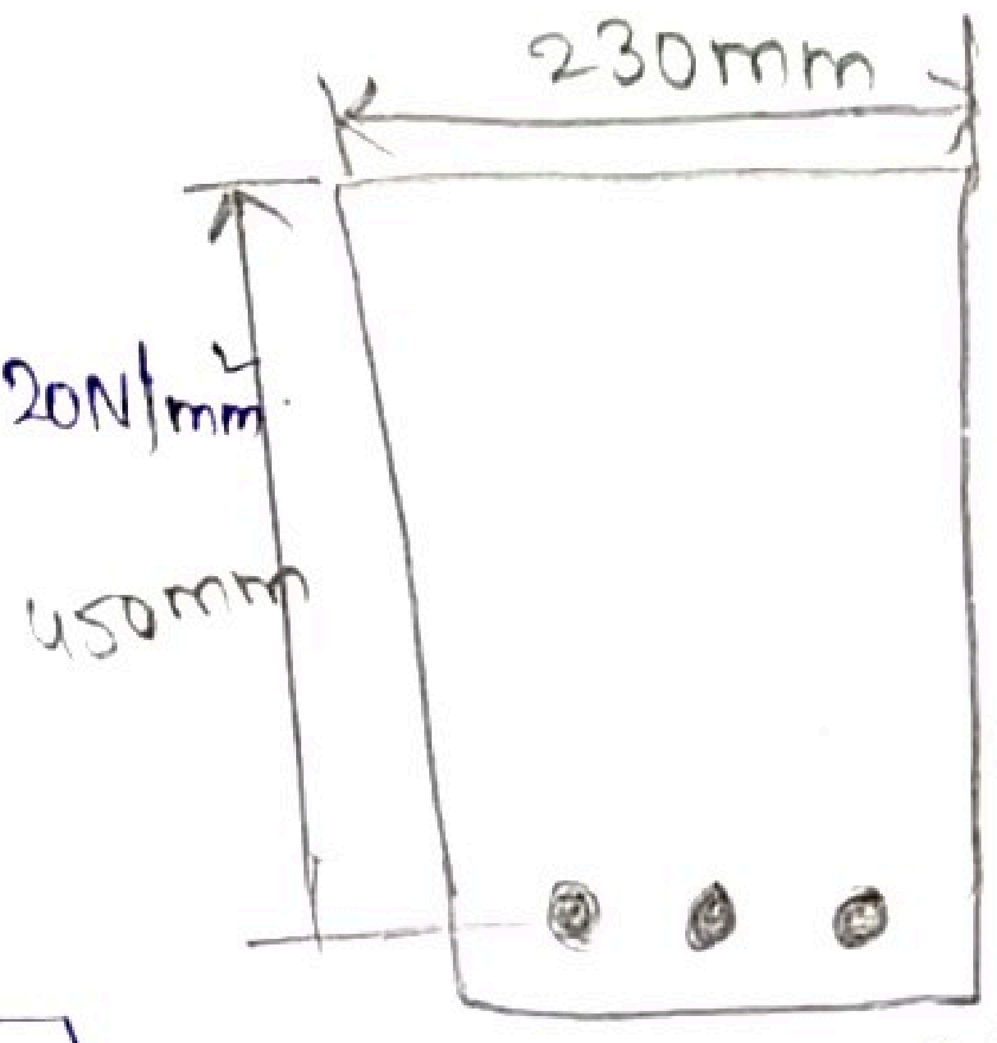
Steps:-

1) Assume b value and Calculate d by equating.

$$M_u = M_{ulimit}.$$

2) Calculate Ast using $M_u = 0.87 f_y Ast (d) \left[1 - \frac{f_y Ast}{f_{ck} \cdot b \cdot d}\right]$

✓ 1) The dimensions of a singly reinforced Beam and simply supported 230mm wide × 450mm effective depth. Use M_{20} grade of Concrete and Fe_{415} steel. Determine the Moment of resistance.



$\frac{225}{\cancel{450}}$ 2

$x = 225$
$x_{umax} = 216$
$x_u > x_{umax}$

Sol:- Given that:-

Fe415, $x_{umax} = 0.48d$.

$f_y = 415 N/mm^2$, $M_{20} \Rightarrow f_{ck} = 20 N/mm^2$.

$B = 230mm$

$d = 450mm$.

$x_{umax} = 0.48(450)$

$$\boxed{x_{umax} = 216mm}$$

x_u = center.

$$M_{ulimit} = 0.36 f_{ck} b\, x_{umax} [d - 0.42 x_{umax}].$$

for M_{20} & Fe_{415} $= 0.36 \times (\overset{20}{\cancel{0.96}}) \times 230 \times 216 [450 - 0.42 \times 216]$

= ~~6.16×10^6 N-mm~~

$M_{ulimit} = 128.51 \times 10^6$ N-mm

$$\boxed{M_{ulimit} = 128.51 KN\text{-}m.}$$

2) Find the depth of Neutral axis of singly reinforced rectangular beam 250×400mm effective depth reinforced with 4 bars of 16mm diameter. Use M_{20} grade of Concrete and Fe_{415} Steel.

Sol:- Given that:- Beam, width b = 250mm

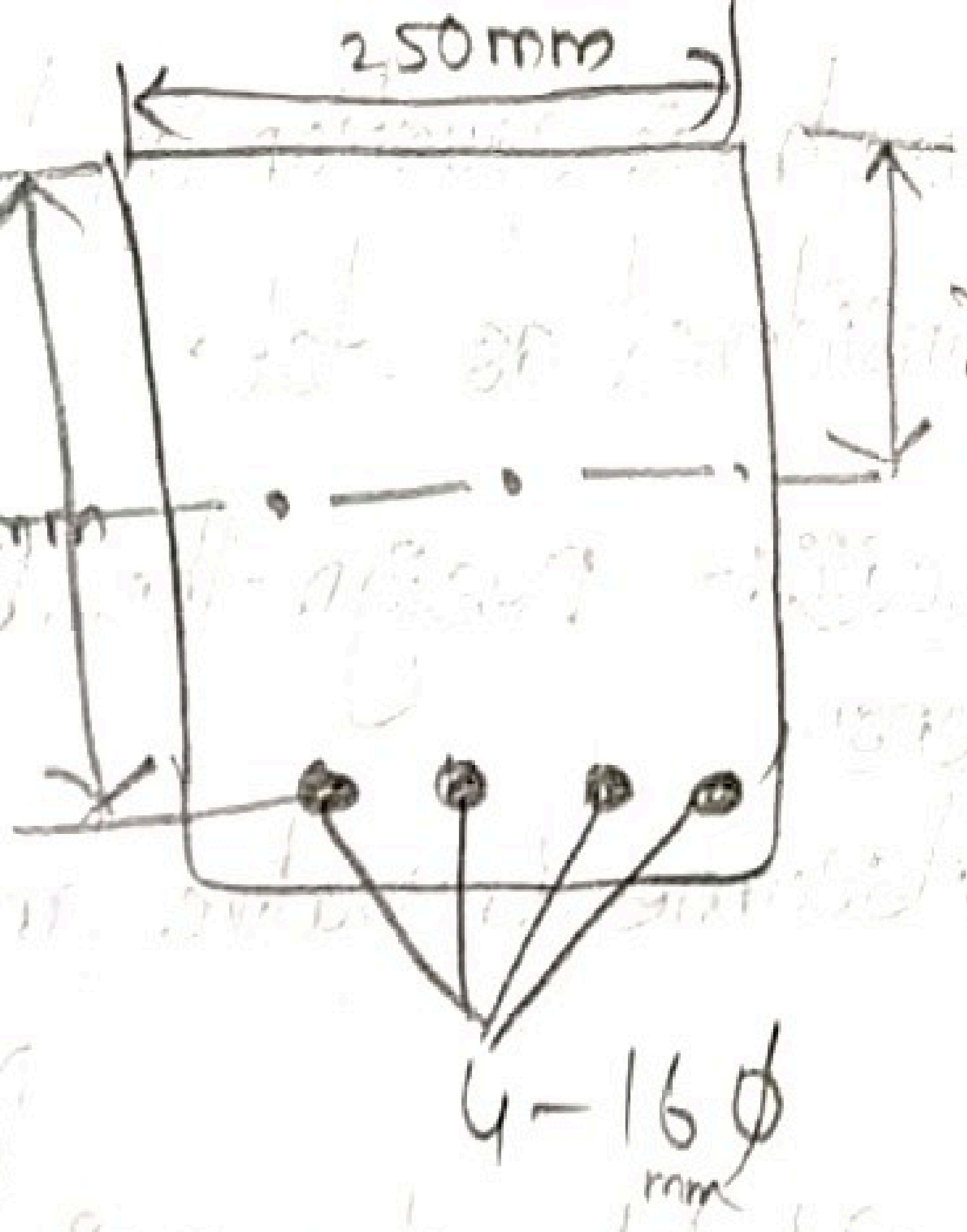
effective depth d = 400mm.

n = 4

diameter = 16mm

$M_{20} \Rightarrow f_{ck} = 20 N/mm^2$

$Fe_{415} \Rightarrow f_y = 415\,N/mm^2.$



$$A_{st} = n \times \pi/4\,(d)^2$$

$$= \cancel{4} \times \pi/\cancel{4}\,(16)^2$$

$$\boxed{A_{st} = 804.24\,mm^2}$$

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_y \cdot A_{st}.$$

$$x_u = \frac{0.87 f_y \times A_{st}}{0.36 f_{ck} \times b}$$

$$x_u = \frac{0.87 \times \overset{415}{\cancel{20}} \times 804.24}{0.36 \times 20 \times 250}$$

$$\boxed{x_u = 161.31\,mm}$$

$$x_{umax} = 0.48(d)$$

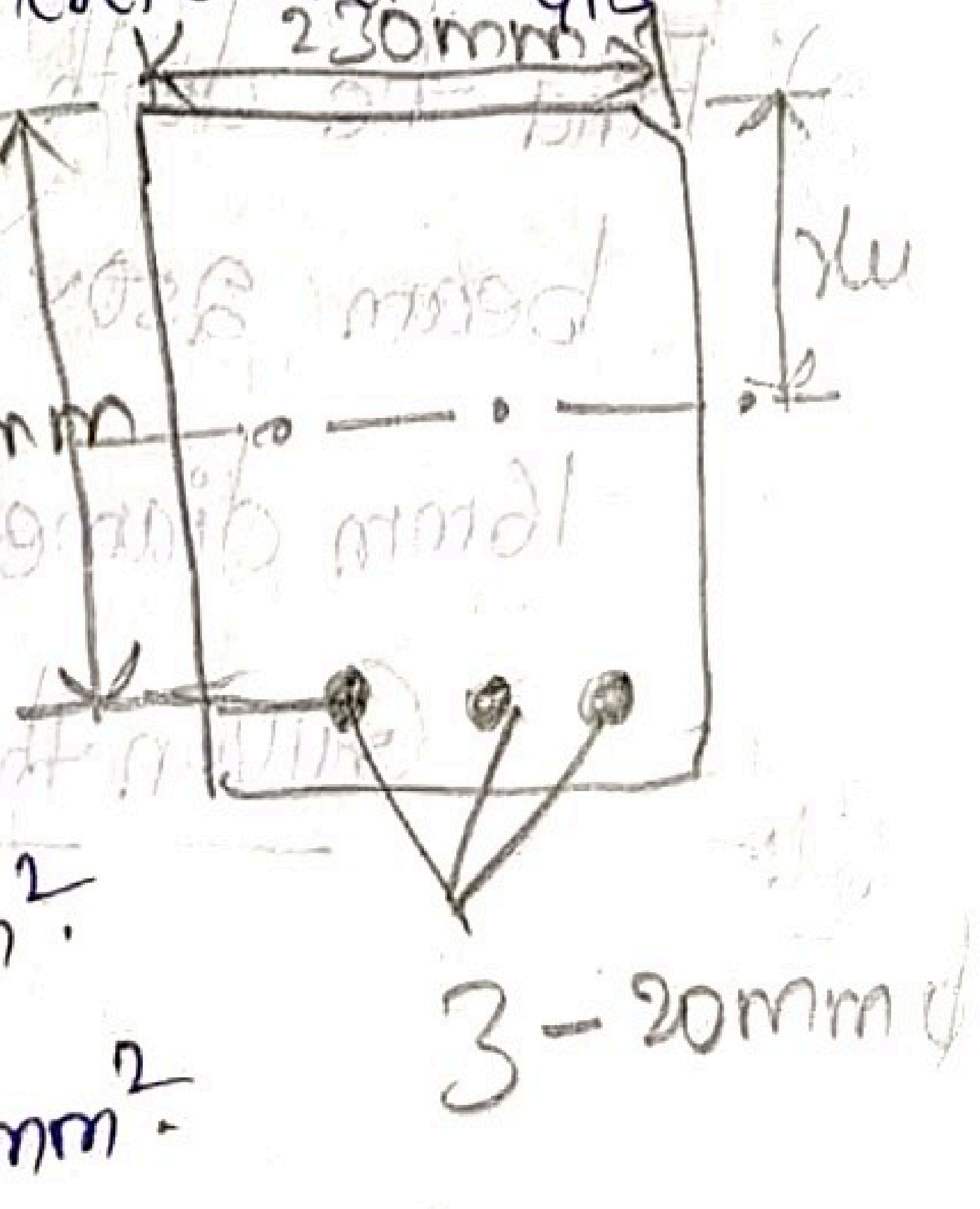
$$= 0.48(400)$$

$$\boxed{x_{umax} = 192\,mm.}$$

$$\boxed{x_u < x_{umax}}$$

It is an under reinforced Section.

3) Find the moment Carrying Capacity of a singly reinforced rectangular Section 230mm × 480mm effective depth reinforced with 3 bars of 20mm diameter use M_{20} grade of Concrete and Fe_{415} steel.



Sol: Given that:-

$B = 230mm$

$d = 480mm$

$n = 3$

diameter $(d) = 20mm.$

$M_{20} \Rightarrow f_{ck} = 20N/mm^2.$

$Fe_{415} \Rightarrow f_y = 415N/mm^2.$

$$A_{st} = n \times \pi/4\,(d^2)$$

$A_{st} = 3 \times \pi/4 (20)^2$

$A_{st} = 942.47 mm^2$

$x_u = \frac{0.87 f_y \times A_{st}}{0.36 f_{ck} \times b}$

$x_u = \frac{0.87 \times 415 \times 942.47}{0.36 \times 20 \times 230}$

$\boxed{x_u = 205.48 mm}$

$x_{umax} = 0.48 d$

$x_{umax} = 0.48(480)$

$\boxed{x_{umax} = 230.4 mm}$

$\boxed{x_u < x_{umax}}$, (under reinforced.)

[x_u is less.].

M.O.R = Tension x lever arm.

$= 0.87 f_y \times A_{st} (d - 0.42 x_u)$.

$= 0.87 \times 415 \times 942.47 (480 - 0.42 (205.48))$

M.O.R = 133.96×10^6 N-mm.

$\boxed{M.O.R = 133.96 kN\text{-}m}$

4.) A singly reinforced Concrete Beam 200mm x 450mm is reinforced with 4 bars of 20mm diameter having an effective cover of 40mm the Beam is simply supported over a span of 4m. Find the safe U.D.L of the Beam can carrying. Use M20 grade of concrete and Fe415 steel.

Sol:- Given that:-

B = 200mm

d = 450mm effectivedepth.

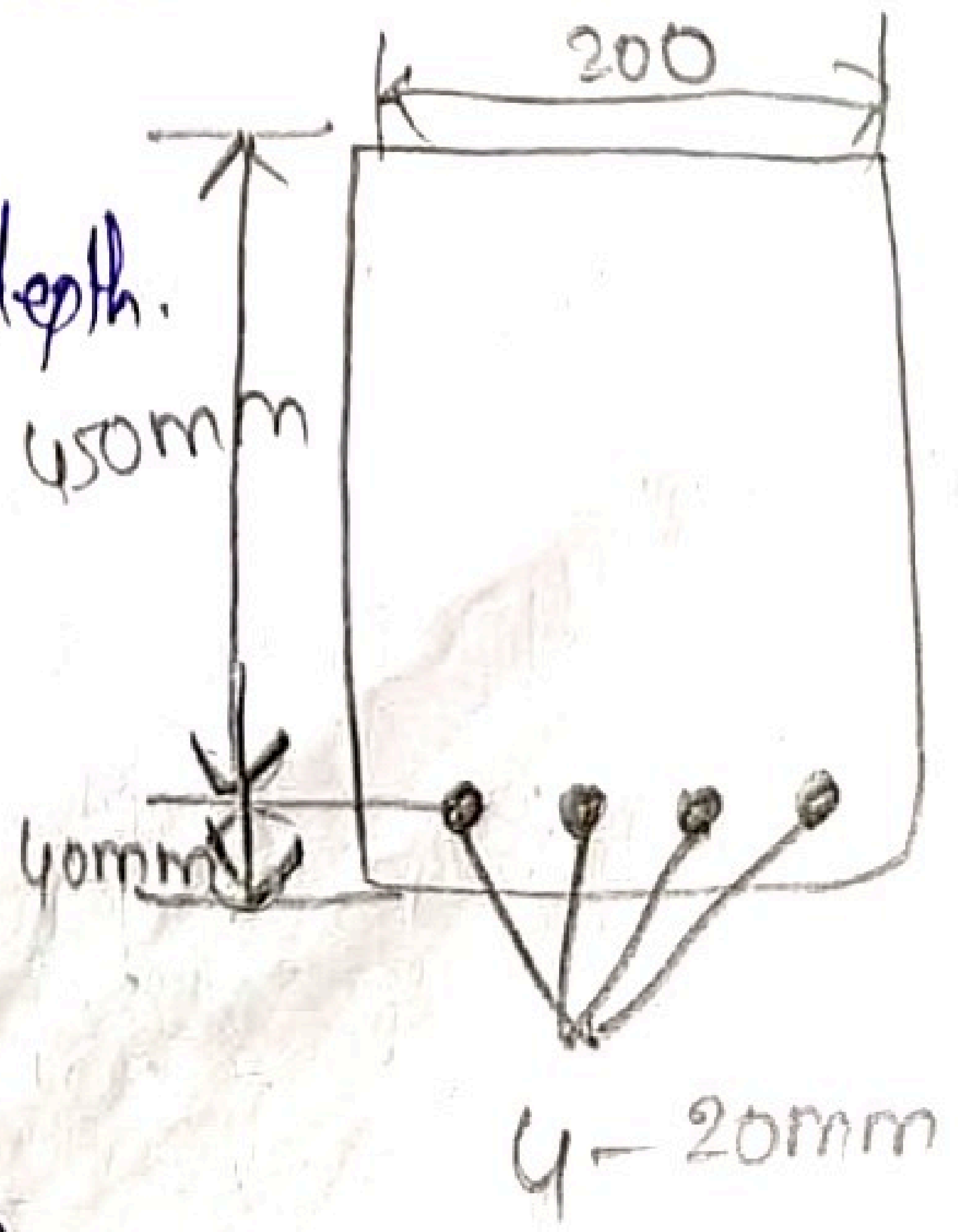
l = 4m

n = 4

diameter d = 20mm.

D = 490mm.

d=450mm



$A_{st} = n \times \pi/4 (d)^2$

$= 4 \times \pi/4 (20)^2$

$A_{st} = 1256.63 mm^2$.

$$x_u = \frac{0.87\, f_y \times A_{st}}{0.36 f_{ck} \cdot b}$$

$$x_u = \frac{0.87 \times 415 \times 1256.63}{0.36 \times 20 \times 200}$$

$$\boxed{x_u = 315.07 \text{mm}}$$

$$x_{umax} = 0.48 (d)$$
$$= 0.48 (450)$$

$$\boxed{x_{umax} = 216 \text{mm}.}$$

$\boxed{x_u > x_{umax}}$ over reinforced. (x_{umax} is less)

$$M.O.R = 0.36 f_{ck} \cdot b \left(d - 0.42 x_{u_{max}}\right) \times x_{umax}$$

$$= 0.36 \times 20 \times 200 \times 216 \left[450 - (0.42 \times 216) \right]$$

$$M.O.R = 111.75 \times 10^6 \text{ N-mm}.$$

$$\boxed{M.O.R = 111.75 \text{KN-m}}$$

$$M.O.R = Max.B.M$$

$$111.75 \times 10^{\cancel{6}3} = \frac{wl^2}{8} \quad 4m$$

$$111.75 \times 10^{\cancel{6}3} = \frac{w(4)^2}{8}$$

$$~~w = 223.5 \times 10^3~~ \quad \boxed{W = 55.87 \times 10^3}$$

$$~~w = 223.5 k~~ \quad \boxed{W = 55.87 \text{KN/m}}$$

U.D.L

Dead Load = Quantity × unit wt of R.C

(span × d × b) Area

$$= 4 \times (0.2)(0.45) \times 25.$$

200mm
= 0.2m
450mm
= 0.45m

$$= 2.25 \text{kN/m}$$

$$L.L = T.L - D.L$$

$$= 55.87 - 2.25$$

$$L.L = 53.62 \text{ KN./m}$$

5) · If the ultimate load moment is 80 KN/m and width of the beam is 230mm. Find the effective depth of the Beam. Use M_{20} grade of Concrete and Fe_{415} steel.

Sol:-

Given that

$M_u \text{ limit} = 80 \text{KN-m}.$

$b = 230 \text{mm}.$

$M_u \text{limit} = 2.76 b d^2$

$80 \times 10^6 = 2.76 \times (230) d^2$

$$\boxed{d = 354.99 \text{mm}}$$

(or)

$M_u \text{limit} = 0.36 f_{ck} \cdot b x_{umax} (d - 0.42 x_{umax})$

M_{20}, Fe_{415}.

$x_{umax} = 0.48d$

$80 \times 10^6 = 0.36 \times 20 \times 230 \times 0.48 d (d - 0.42 \times 0.48 d)$

$80 \times 10^6 / 634.63 = d^2$

$$\boxed{d = 355.0 \text{mm}.}$$

6) A singly reinforced rectangular Beam of 230mm width and 535 mm as effective depth subjected to a Bending Moment of 90KN-m at Working loads. Find the steel area required use M_{20} grade of Concrete and Fe_{415} Steel.

Sol:-

Given that:- width of beam, $B = 230 \text{mm}.$

depth of Beam, $d = 535 \text{mm}.$

$M = 90 \text{KN-m}.$

Grade of concrete, $f_{ck} = 20 \text{N/mm}^2$

Grade of steel, $f_y = 415 \text{N/mm}^2.$

factor B.M $= 90 \times 1.5$

$M_u = 135 \text{KN-m}.$

$Mu_{limit} = 2.76bd^2$

$= 2.76 \times 230 \times (535)^2$

$Mu_{limit} = 181.69 \times 10^6$ N-mm

$Mu_{limit} = 181.69$ KN-m

$M_u < Mu_{limit}$ (~~under reinforced section~~)

Mu is taken less value.

$$Mu = 0.87\, fy\, Ast \cdot d \left[1 - \frac{fy\ Ast}{fck \cdot b \cdot d}\right]$$

$$135 \times 10^6 = 0.87 \times 415 \times Ast\,(535)\left[1 - \frac{415 \times Ast}{20 \times 230 \times 535}\right]$$

$$135 \times 10^6 = 193.16\, Ast \times 10^3 \left(1 - 1.68 \times 10^{-4} Ast\right)$$

$$135 \times 10^6 = 193.16 \times 10^3 Ast - 193.16 \times 10^3 \times 1.68 \times 10^{-4} Ast^2$$

$$32.45 Ast^2 - 193.16 \times 10^3 Ast + 135 \times 10^6 = 0.$$

Taken less value

$$\boxed{Ast = 808.79\ mm^2}$$

~~tek value~~.

Choose $20mm\ \phi$, $n = 3$

$Ast = n \times \pi/4\,(20)^2 = 3 \times \pi/4\,(20)^2$

$Ast_{provided} = 942.47 mm^2.$

Design a singly reinforced rectangular beam of simply supported has a span of 4000mm and carries a load of 20kN/m and the support width is 300mm each. and width of the Beam 300mm ~~and width of the Beam 230mm~~.

Sol:- **Given that :-**

$l = 4000mm$
(or)
4m

$l/12$ to $l/15$ assumed

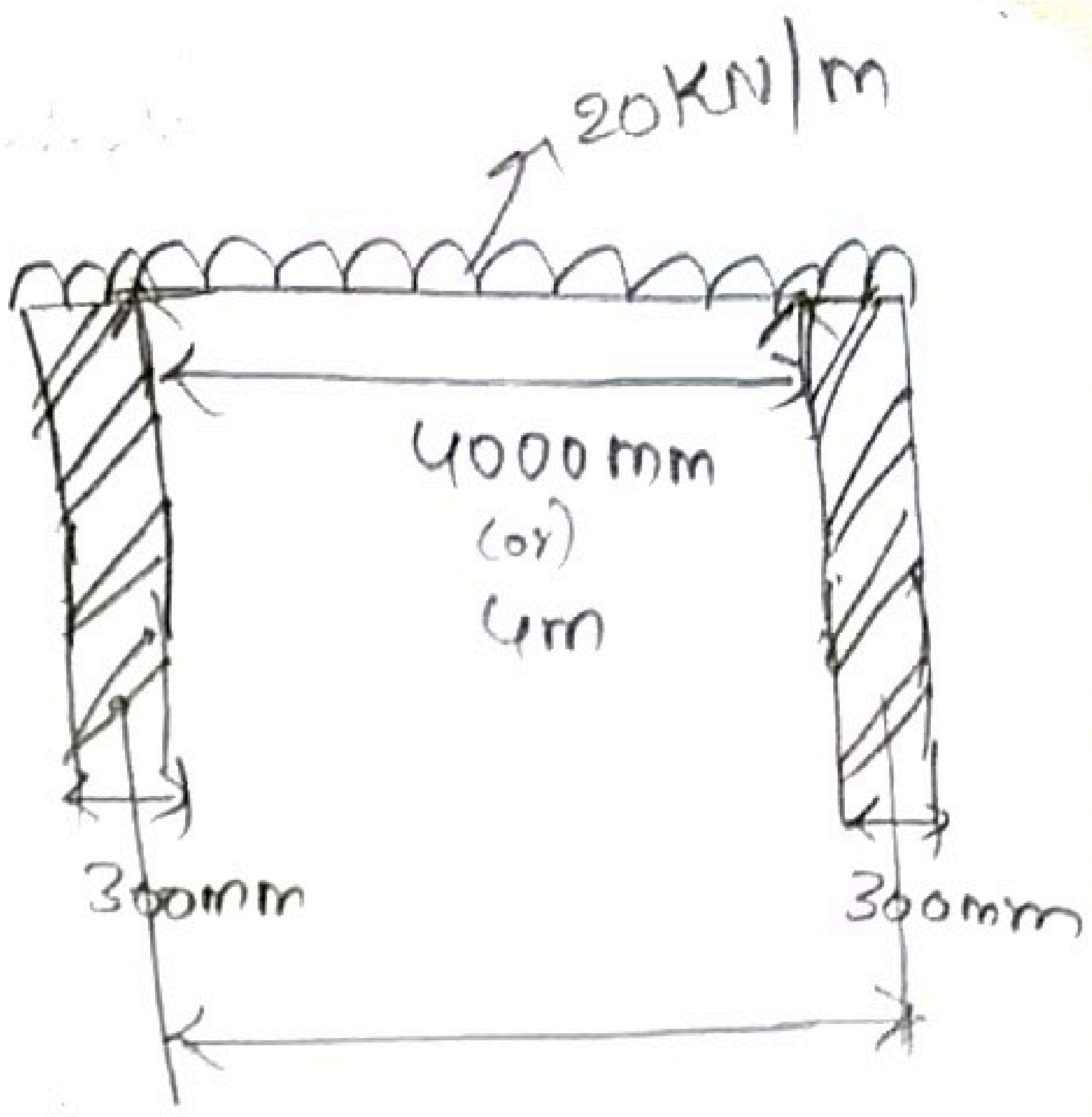
$Mu = Mu_{limit}$

$d_{assumed} > d_{actual}$

$l/12$ to $l/15$ (taken which one is higher.

assumed value d is must be greater, then the check is satisfied.

U.D.L = 20kN/m



Select the effective depth of the Beam in b/w l/12 to l/15.

Calculation of effective depth:-

1.) $\frac{4000}{12}$ (or) $\frac{4000}{15}$

333.33 (or) 266.66

= 350mm = 300mm

$\boxed{d = 350\text{mm}}$

Assume, c = 50mm.

$\boxed{D = 400\text{mm}}$ D = 0.4m

300mm×400mm

B = 0.3×0.4

$\boxed{B = 0.12\text{m}}$

2) effective span SSB = Clear span + effective depth of Beam.

= 4000+350

= 4350mm = 4.35m.

SS.B = Center to center distance b/w two Supports.

$= 4000 + \overset{\frac{300}{2}}{150} + \overset{\frac{300}{2}}{150} = 4300\text{mm} = 4.3\text{m}$

Take which one is less.

= 4300 mm

Effective span for SSB = 4.3m

3.) Calculation of loads:- Dead load = (b×D) unit wt of material.
(R.C) ↳ 25KN/m³.

= ~~0.12×400~~

= (0.3×0.4) × 25

= 3 KN/m

L.L = 20kN/m.

floor finish = 0.6kN/m.

TL = D.L+L.L+F.F = 3+20+0.6

$\boxed{TL = 23.6\ \text{KN/m}}$

factored load = T.L X 1.5

$= 23.6 \times 1.5$

$= 35.4$ KN/m.

$M_u = \frac{wl^2}{8}$ (l = Effective span)

$= \frac{35.4 \times (4.3)^2}{8}$

$$\boxed{M_u = 81.81 \text{ KN-m}}$$

4) Check for depth:-

To check the depth ~~for~~ maximum B.M We have to Compare.

$M_u = M_{ulimit}$

$81.81 \times 10^6 = 2.76 \times bd^2$ $b = 300mm$

$= 2.76 \times 300 d^2$

$d = 314.33$

Calculated Assumed

$d = 314.33 < 350mm.$

Assumed should be greater.

Check is Satisfied.

5) Calculation of Area of tension reinforcement:-

$$M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{f_y \cdot A_{st}}{f_{ck} \cdot b \cdot d}\right]$$

$$81.81 \times 10^6 = 0.87 \times 415 \times A_{st} \times 350 \left[1 - \frac{415 \times A_{st}}{20 \times 300 \times 350}\right]$$

$$81.81 \times 10^6 = 125367.5 A_{st} \left[1 - 1.976 \times 10^{-4} A_{st}\right]$$

$$24.97 A_{st}^2 - 126367.5 A_{st} + 81.81 \times 10^6 = 0.$$

$$\boxed{A_{st} = 762.18 \text{ mm}^2}$$

16mm

$A_{st} = n \times \pi/4 (d^2)$

$762.18 = n \times \pi/4 (16^2)$

$n = 3.79$

$\boxed{n = 4}$ bars.

$A_{st} = \pi/4 \times (16)^2 \times 4$

$\boxed{A_{st} = 804.24\ mm^2}$

6) Check for deflection:-

$l/d = 20$

% of Ast ⇒ $P_t = \frac{100 A_{st}}{bd}$

$= \frac{100 \times 804.24}{300 \times 350}$

$= 0.76\%$ [Pg No-38.]

$f_s = 0.58 f_y \times \frac{\text{area of c/s of steel required}}{\text{area of c/s of steel provided}}$

$= 0.58 \times 415 \times \frac{762.18}{804.24}$

$\boxed{f_s = 228.1}$

$l/d = 20 \times 1.15$ → By graph

$= 23$

$l/d = \frac{4300mm}{350mm}$

$= 12.28$

$12.28 < 23$

Check ok.

8) Effective span = 5.3m, liveload = 20kN/m. Use M_{20}, Fe_{415} & b = 300mm.

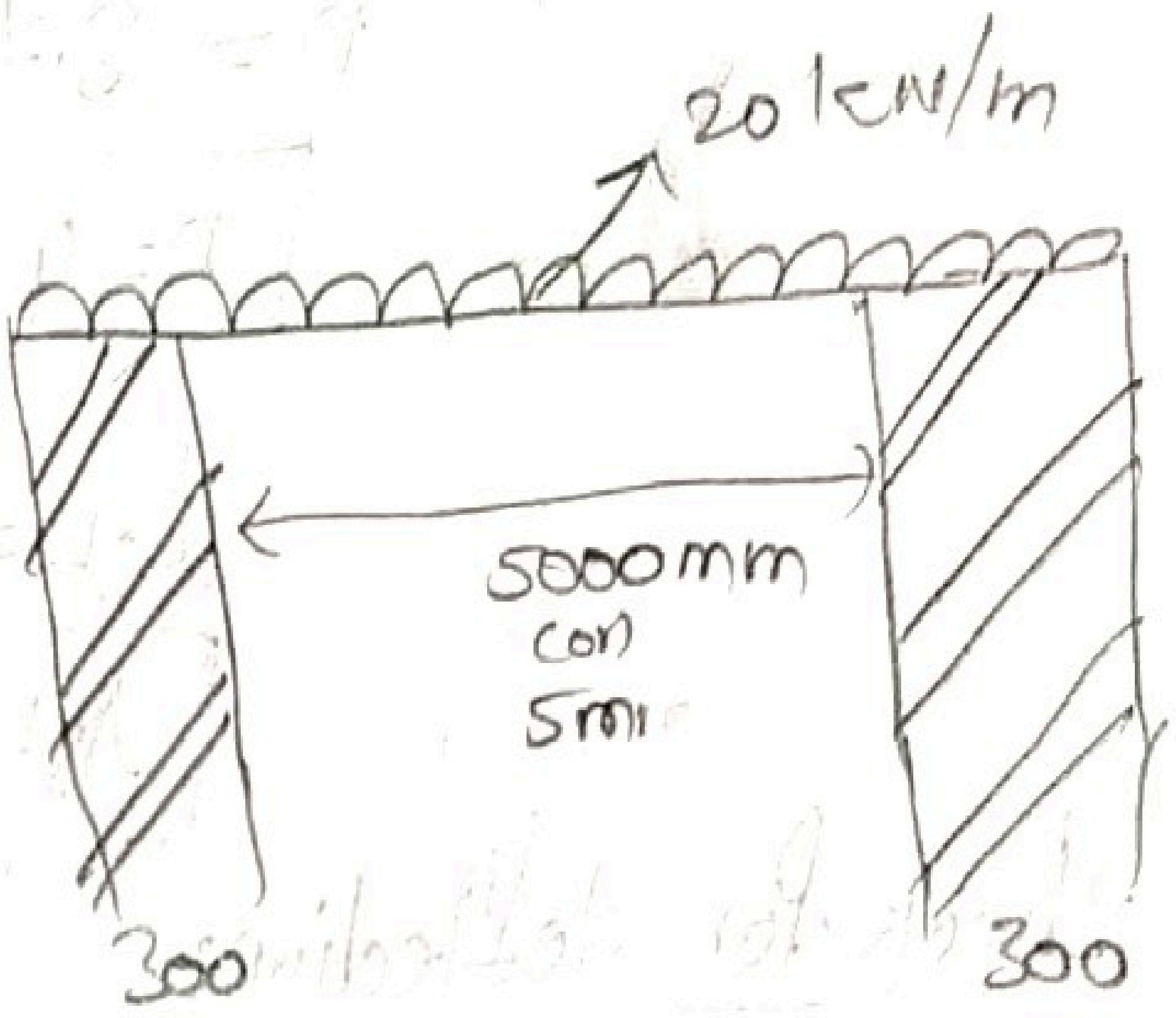
It is a simply supported.

Sol:- Step 1:- Assume e = 50 Calculation of effective depth

$\frac{l}{12}$ and $\frac{l}{15}$

Select the effective depth of beam.

$b = 300mm$



$l = 5m$

$\frac{5000}{12}$ (or) $\frac{5000}{15}$

416.66 (or) 333.33

450 (or) 350

$d = 450mm$

Assume, $e = 50mm$

$D = 500mm$.

$B = 300mm \times 500mm$

$B = 1\cancel{8}50000 \, mm^2$

$\boxed{B = \cancel{150m}}$

$\boxed{B = 0.15m}$

2) <u>Effective span</u> :-

SSB = clear span + effective depth of Beam

$= 5000 + 450$

$= 5450 = 5.45m$

SSB = Center to center distance b/w two supports

$= 5000 + \frac{300}{2} + \frac{300}{2}$

$= 5300mm$

$= 5.3m$

which one is less

effective span = 5.3m

3) <u>Calculation of loads</u>:-

Dead load = (bxD) unit wt of material (R.C) → 25 kN/m²

$= (300 \times 500) \times 25$

$= 150000 \times 25$

$= 0.15 \times 25$

$= 3.75 \, kN/m$

L.L = 20 kN/m

floor finish = 0.6 KN/m

$T.L = L.L + F.F + D.L$

$= 3.75 + 0.6 + 20$

$= 24.35 \text{ KN/m.}$

factored load $= T.L \times 1.5$

$= 24.35 \times 1.5$

$= 36.52 \text{ KN/m}$

$$M_u = \frac{Wl^2}{8}$$

$$= \frac{36.52 \times (5.3)^2}{8}$$

$$M_u = 128.23 \text{ KN-m}$$

4) check for depth:-

To check the depth for Maximum B.M we have to Compare.

$M_u = M_{u\,limit}$

$128.23 \times 10^6 = 2.76 bd^2$

$128.23 \times 10^6 = 2.76 \times 300 \times d^2$

$d = 393.53 \text{mm}$

$$d = \underset{\text{mm}}{\overset{\text{Calculated}}{393.53}} < \overset{\text{Assumed}}{450\text{mm}}$$

check is satisfied.

5) Calculation of Area of tension reinforcement:-

$$M_u = 0.87 f_y A_{st} \cdot d \left[1 - \frac{f_y \times A_{st}}{f_{ck} \cdot b \cdot d}\right]$$

$$128.23 \times 10^6 = 0.87 \times 415 \times A_{st} \times 450 \left[1 - \frac{415 \times A_{st}}{20 \times 300 \times 450}\right]$$

$$128.23 \times 10^6 = 162472.5 A_{st} \left[1 - 1.537 \times 10^{-4} A_{st}\right]$$

$$24.97 A_{st}^2 - 162472.5 A_{st} + 128.23 \times 10^6 = 0.$$

$A_{st} = 5587.6$ less value taken

$$\boxed{A_{st} = 919.05 \text{ mm}}$$

16mm $A_{st} = n \times \pi/4 \, (16)^2$

$$\frac{4 \times 919.05}{\pi \times 16^2} = n$$

$n = 4.57$

$n = 5$ bars

$$A_{st} = n \times \pi/4 (d^2)$$

$$A_{st} = 5 \times \pi/4 (16^2)$$

$$A_{st} = 1005.30 \text{ mm}^2$$

6.) Check for deflection:-

$$l/d = 20$$

$$\% \text{ of } A_{st} \Rightarrow P_t = \frac{100 A_{st}}{bd}$$

$$P_t = \frac{100 \times 1005.30}{300 \times 450}$$

$$P_t = 0.74\%$$

$$f_s = 0.58 \times f_y \times \frac{\text{area of c/s of steel required}}{\text{area of c/s of steel provided}}$$

$$= 0.58 \times 415 \times \frac{919.05}{1005.30}$$

$$f_s = 220.04 \text{ N/mm}^2$$

$$l/d = 20 \times (1.15)$$ By graph pg No-38, fig4

$$= 23$$

$$l/d = \frac{5300}{450}$$

$$l/d = 11.77 < 23$$

Check is ok.

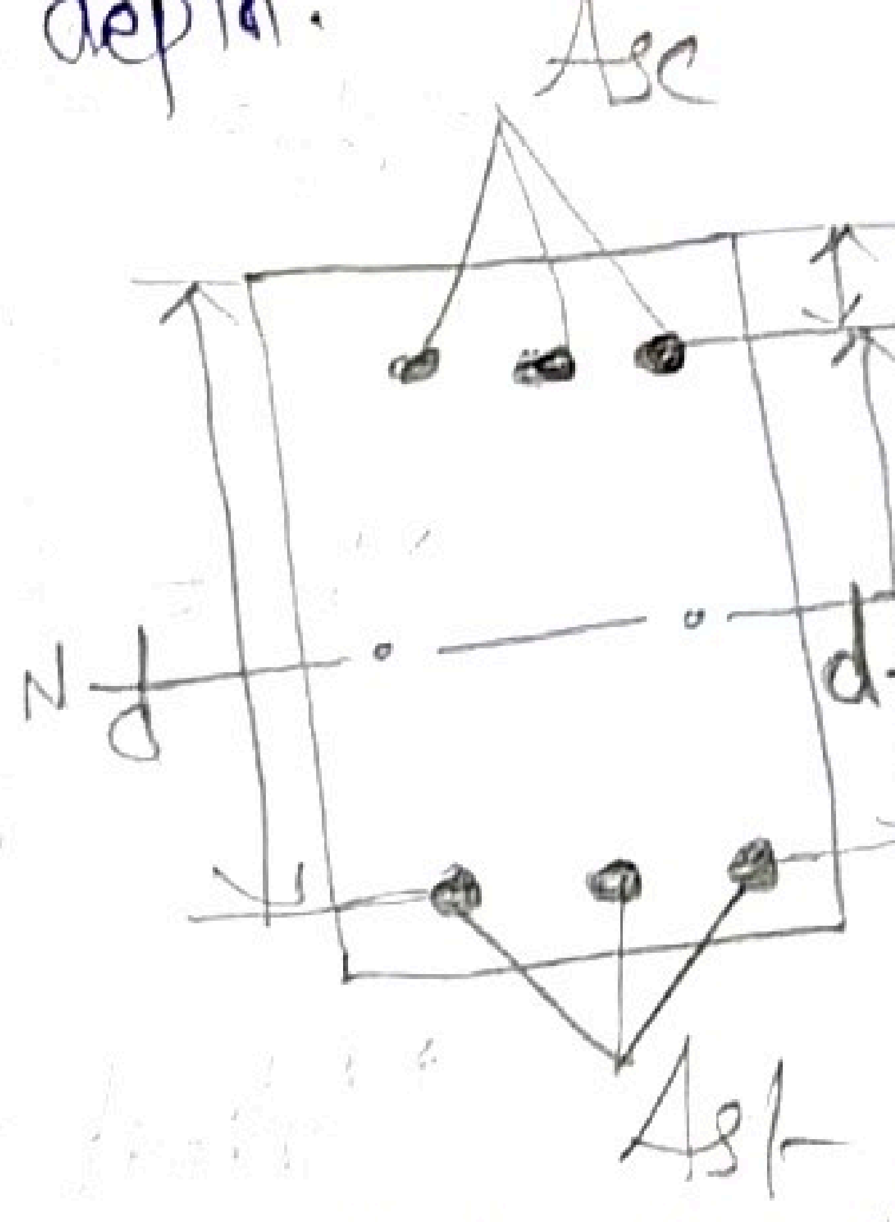
Doubly reinforced Beam:-

These beams are generally provided when the dimensions of the beam are ristricted. and it requires to resist the moment higher than the limiting moment of resistance occured in a singly reinforced beam.

The additional moment of resistance can be obtained by providing compression reinforcement additional to the tension reinforcement.

Analysis of doubly reinforced beam:-

Fig shows a typical doubly reinforced beam section and the variation of the section across the depth.



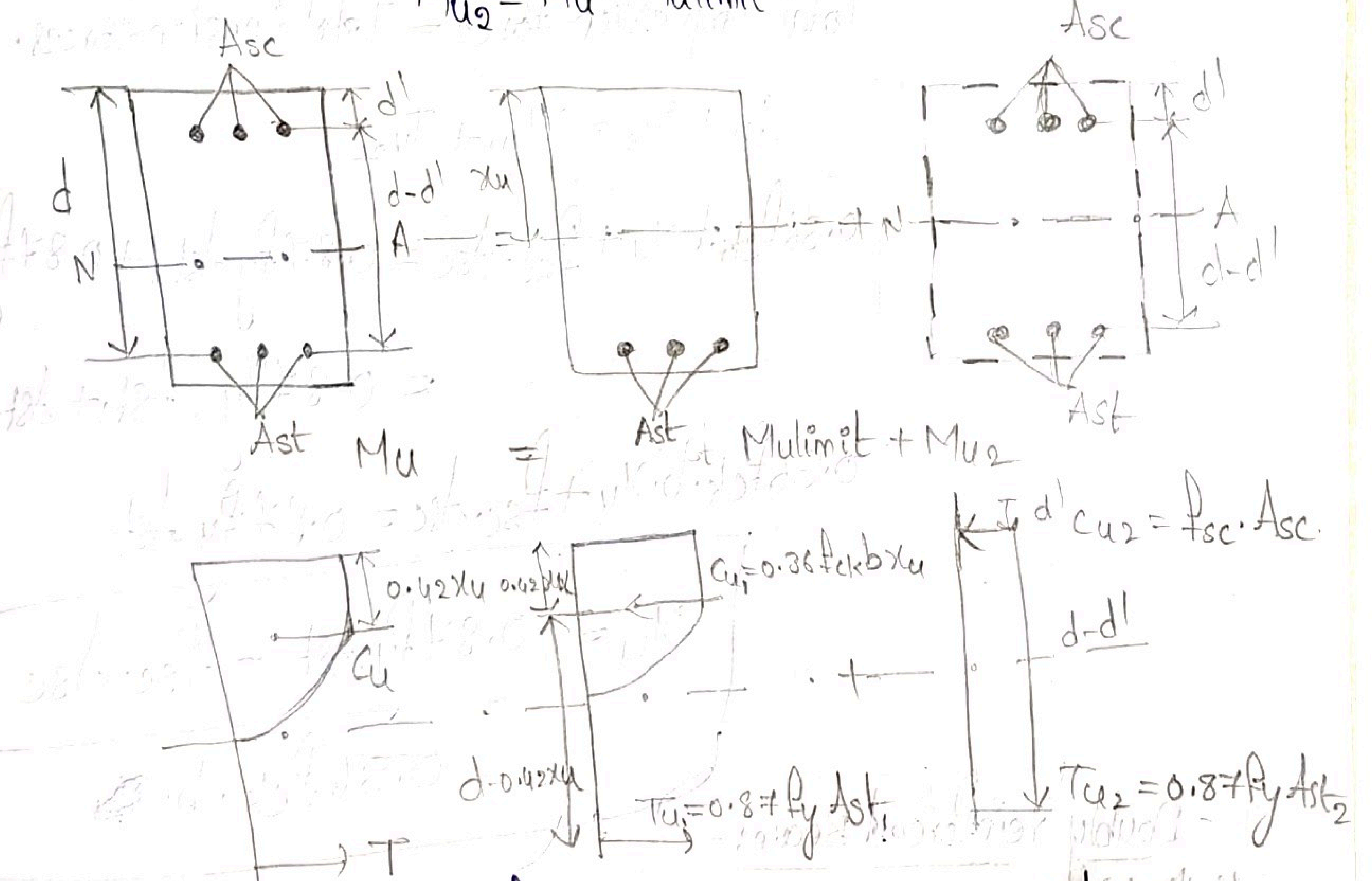
The doubly reinforced beam (D.R.B) section can be considered to be composite of two sections as given below.

a) A singly reinforced section with Mu limit.

b) A section with compression reinforcement and additional to the tension reinforcement to resist additional moment.

$$M_u = M_{ulimit} + M_{u2}.$$

$$M_{u2} = M_u - M_{ulimit}.$$



Where f_{sc} = stress in compression steel

d' = effective cover compression zone.
(or)
d' = distance of centroid of compression steel to the top most fiber of concrete.

Asc = (Area of steel in compression)

A_{sc} = Area of Compression reinforcement required to resist M_{u2}.

A_{st1} = Area of tension reinforcement for a balanced singly reinforced Section.

A_{st2} = Additional tension reinforcement to balance Compression Steel.

M_{u2} = Additional Moment of resistance to be resisted by Compression and tension Steel.

M_{ulimit} = limiting moment of resistance of singly reinforced Section.

① Neutral axis:-

Total Compression force = Total tension forces.

$$C_{u1} + C_{u2} = T_{u1} + T_{u2}$$

$$0.36 f_{ck} \cdot b x_u + f_{sc} \cdot A_{sc} = 0.87 f_y A_{st1} + 0.87 f_y A_{st2}$$

$$= 0.87 f_y (A_{st1} + A_{st2})$$

$$0.36 f_{ck} \cdot b \cdot x_u + f_{sc} \cdot A_{sc} = 0.87 f_y A_{st}$$

$$\boxed{x_u = \frac{0.87 f_y A_{st} - f_{sc} \cdot A_{sc}}{0.36 f_{ck} \cdot b}}$$

Doubly reinforced Beam:-

Limiting (or)

② Ultimate moment of Resistance:-

It is given by $M_u = M_{ulimit} + M_{u2}$.

Where $M_{ulimit} = 0.36 f_{ck} \cdot b \cdot x_{ulimit} (d - 0.42 x_{ulimit})$.

Where $M_{u2} = 0.87 f_y A_{st2} (d - d')$
(or)
$f_{sc} \cdot A_{sc} (d - d')$

By Considering Compressive force, then Mu

$$M_u = 0.36 f_{ck} \cdot b\, x_{umax} (d - 0.42 x_{umax}) + f_{sc} \cdot A_{sc} (d - d').$$

When $x_u > x_{umax}$.

$$M_u = 0.36 f_{ck} \cdot b \cdot x_{umax} (d - 0.42 x_{umax}) + f_{sc} \cdot A_{sc} (d - d').$$

③ **Area of Compression steel :-** $M_{u_2} = f_{sc} \cdot A_{sc} (d - d')$

$$A_{sc} = \frac{M_{u2}}{f_{sc} \cdot (d - d')}$$

The maximum area of compression reinforcement should not exceed 4% of gross cross sectional area.

④ **Area of tension reinforcement :- (~~Ast~~).**

$$A_{st_1} = \frac{M_{ulimit}}{0.87 f_y \cancel{A} \cdot (d - 0.42 x_{ulimit})}$$

The additional area of tension steel A_{st_2} can be calculated by equating compression force in comp. steel and tension force in tension steel.

~~A_{st_2}~~

$$f_{sc} \cdot A_{sc} = 0.87 f_y A_{st_2}.$$

$$A_{st_2} = \frac{f_{sc} \cdot A_{sc}}{0.87 f_y}.$$

(or)

$$A_{st_2} = \frac{M_{u2}}{0.87 f_y (d - d')}$$

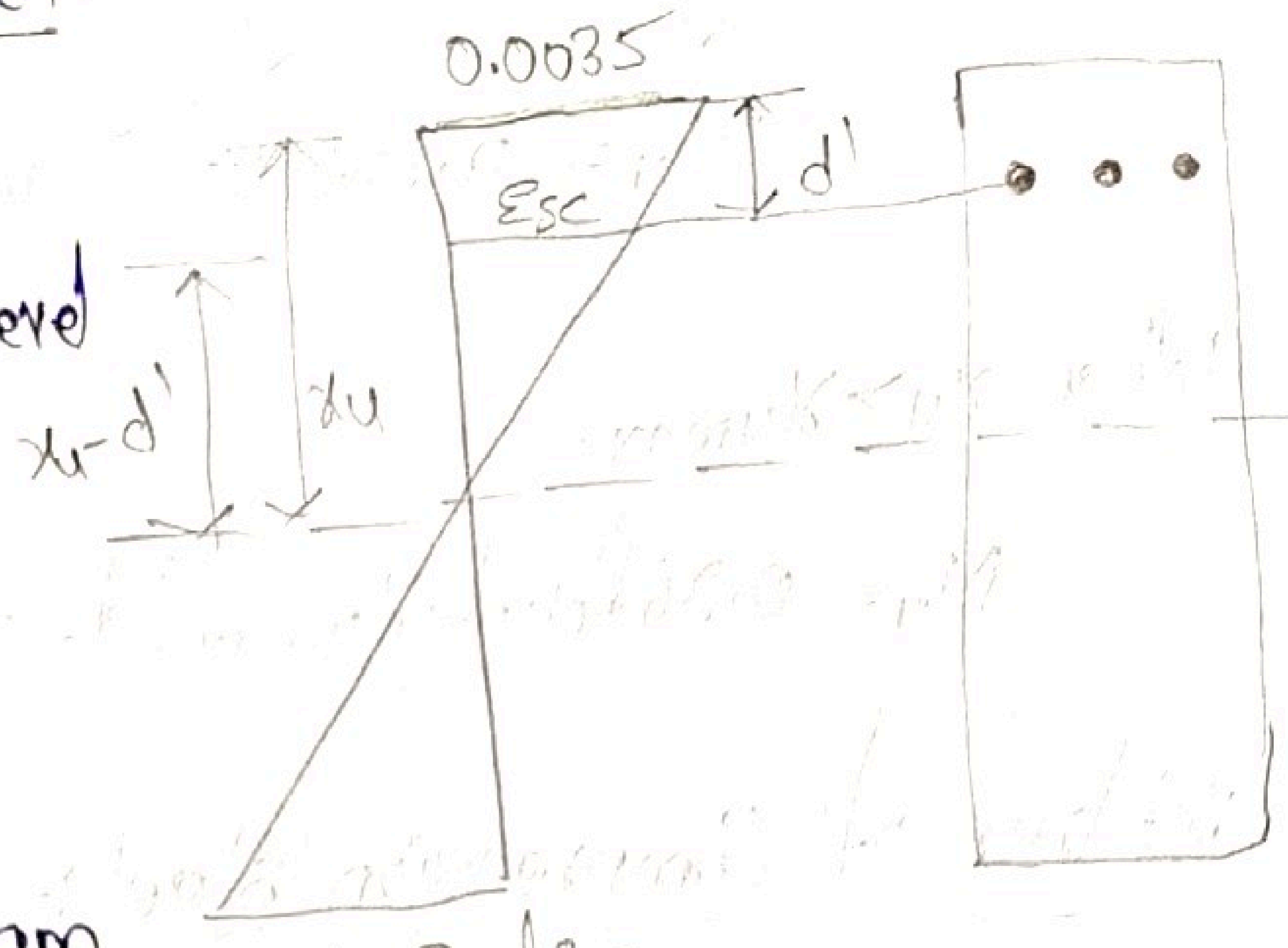
The total Area of tensile reinforcement $A_{st} = A_{st_1} + A_{st_2}$

$$A_{st} = \frac{M_{ulimit}}{0.87 f_y (d - 0.42 x_{ulimit})} + \frac{M_{u2}}{0.87 f_y (d - d')}.$$

Lever arm for doubly reinforcement beam :-

It can be taken from centroid of the compression reinforcement to the centroid of the tension reinforcement. $\boxed{Z = d - d'}$

5.) Stress in Compression Steel:-



strain diag

If ε_{sc} = strain at level of Compression Steel.

from the strain diagram at failure.

$$\frac{0.0035}{x_u} = \frac{\varepsilon_{sc}}{x_u - d'}$$

$$\boxed{\varepsilon_{sc} = \frac{(0.0035)(x_u - d')}{x_u}}$$

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{d'}{x_u}\right]$$

Knowing the strain, the stress in Compression steel Can be uptained from Stress-strain curve of the Corresponding Steel (or) table (A) of SP:16 Curves which are given below.

Stress level	Fe415		Fe 500	
	Strain	Stress	Strain	Stress
0.8fyd	0.00144	288.7	0.00174	347.8
0.85fyd	0.00163 ~~0.85fyd~~	306.7 ~~0.00163~~	0.00195	~~306.7~~ 369.6
0.9fyd	0.00192	324.8	0.00226	391.3
0.95fyd	0.00241	342.8	0.00277	413
0.975 fyd	0.00276	351.8	0.00312	423.9
1fyd	0.0038	360.9	0.00417	434.8

For intermediate Values linear interpolation, may ~~will~~ be done f_{yd} = design yield strength $= 0.87 f_y$.

So f_{sc} and x_u are the interrelated and Cannot be directly found.

By using trial and error method the values Can be adopted (or) assumed.

For mild Steel direct relation Can be establish between stress and strain. since the idealised S-S Curve is linear up to f_y and then it is Constant a value of f_y.

$$f_{sc} = \text{strain} \times E_s \qquad E_s = 2 \times 10^5 \, N/mm^2$$

$$= 0.0035\left(1 - \frac{d'}{x_u}\right) \times 2 \times 10^5$$

$$\boxed{f_{sc} = 700\left(1 - \frac{d'}{x_u}\right)}$$

Subjected to a maximum of $0.87 f_y$.

Stress in Compression Steel based on $\frac{d'}{d}$ ratio:-

As per SP-16 in designing doubly reinforced beam (assuming, $x_u = x_{umax}$) The following table gives the values of f_{sc} based on $\frac{d'}{d}$ ratio < 0.2.

	d'/d			
Grade of Steel f_y 250	0.05	0.1	0.15	0.2
415	355	353	342	329
500	424	412	395	370

Types of problems:-

Analysis of the Section:-

Find Out the ultimate moment of resistance for given dimensions, area of tension and Compression reinforcement.

Method 1:- (Strain Comparability method (or) excat method)

Steps:-

1) Assume $x_u = x_{umax}$.

2) Calculate Strain in Compression steel using the formula and the Corresponding stress f_{sc} from table (A) of Sp-16 Curves.

3) Calculate the depth of Neutral axis this value will be equal to the assumed value. otherwise, $x_u = x_u$ obtained in step(3). and

4) repeat the steps from 1 to 3.

4) Moment of resistance, $M_u = 0.36 f_{ck} \cdot b x_u (d - 0.42 x_u) + f_{sc} \cdot A_{sc} (d-d')$.

If $x_u > x_{umax}$.

$$M_u = 0.36 f_{ck} \cdot b\, x_{umax} (d - 0.42 x_{umax}) + f_{sc} \cdot A_{sc} (d-d').$$

Method 2:- (Approximate Method)

1) Determine the stress in compression steel depending upon the d'/d ratio values which are from Table-(F) of Sp-16 Curves.

2) Determine $x_u = \dfrac{0.87 f_y A_{st} - f_{sc} \cdot A_{sc}}{0.36 f_{ck} b}$

3) Ultimate Moment of resistance.

$$M_u = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u) + f_{sc} \cdot A_{sc} (d-d')$$

If $x_u > x_{umax}$.

$$M_u = 0.36 f_{ck} \cdot b\, x_{umax} (d - 0.42 x_{umax}) + f_{sc} \cdot A_{sc} (d-d')$$

Design of doubly reinforced beam for a given moment of resistance :-

steps:-

1) Find Mulimit as in the case of singly reinforced Beam.

$$M_{ulimit} = 0.36 f_{ck} \cdot b\, x_{umax}(d - 0.42 x_{umax}).$$

2) Calculation of Corresponding steel.

$$A_{st_1} = \frac{M_{ulimit}}{0.87 f_y \cdot (d - 0.42 x_{umax})}$$

(or)

$$A_{st_1} = \frac{P_{t\,limit} \times b \times d}{100}.$$

3) Find the excess moment can be resisted by the Compression steel.

$$M_u = M_{ulimit} + M_{u_2}$$

$$M_{u_2} = M_u - M_{ulimit}.$$

4) Find the stress in Compression steel based on d'/d Values.

5) Find the Area of Compression steel.

$$A_{sc} = \frac{M_{u2}}{f_{sc}(d-d')}$$

6) Find the additional tensile steel A_{st_2}

$$A_{st_2} = \frac{M_{u2}}{0.87 f_y (d-d')}.$$

7) Total area of tension steel $A_{st} = A_{st_1} + A_{st_2}$.

1) A doubly reinforced beam of 250mm wide and 550mm effective depth is reinforced with 2-bars of 16mm diameter as a Compressive reinforcement at an effective Cover of 50mm and 4 bars of 20mm-diameter as tension steel. Find the ultimate moment of resistance of the beam. Use F20 grade of Concrete and Fe415 steel.

Sol:- Given that :- Beam width ⇒ b = 250mm

Effective depth, d = 550

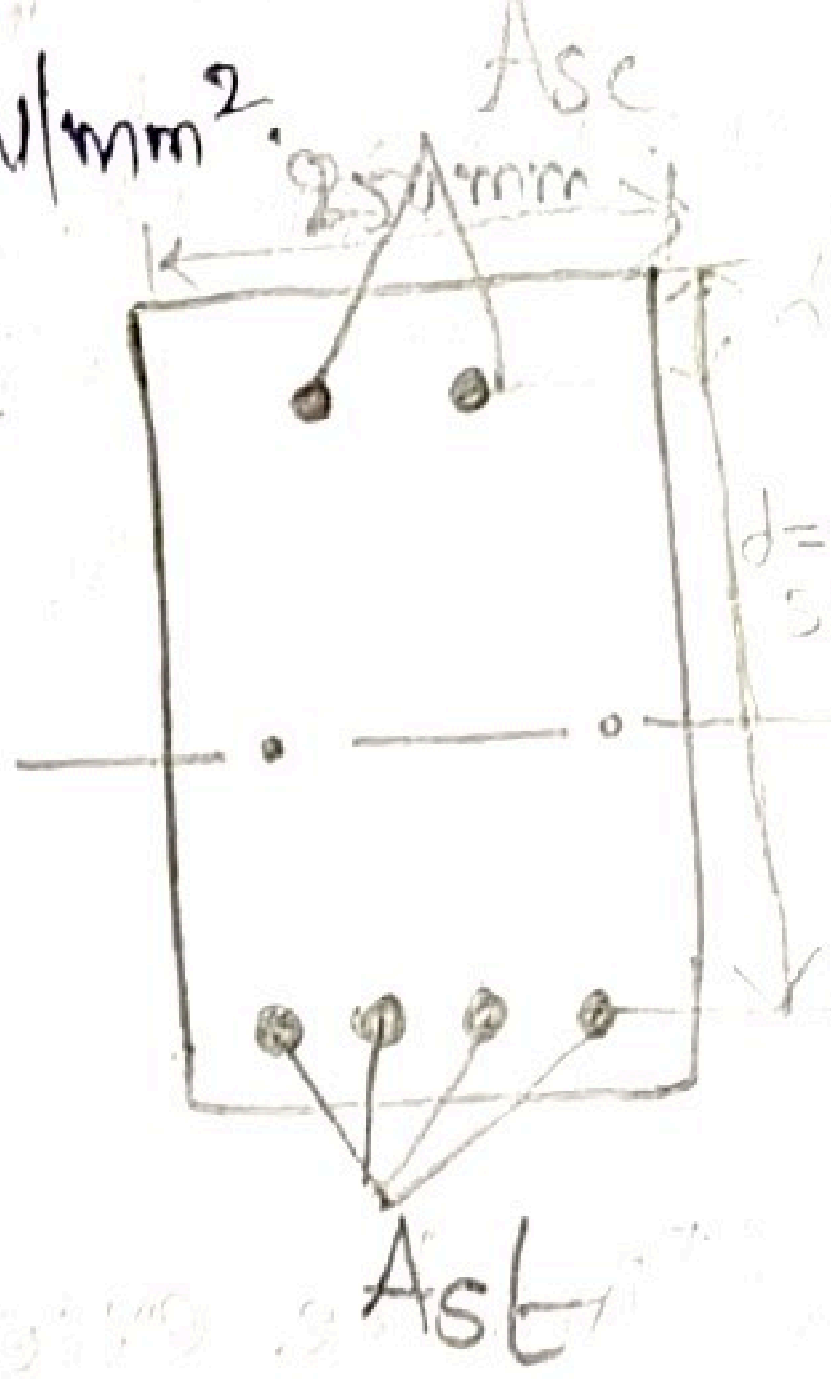
Compression, $\begin{cases} n = 2 \\ dia = 16mm. \end{cases}$

Effective Cover, $d' = 50mm$

Tension $\begin{cases} n = 4 \text{ bars} \\ dia = 20mm. \end{cases}$

Grade of Concrete, $f_{ck} = 20 N/mm^2$.

Grade of Steel, $f_y = 415 N/mm^2$.



$$A_{st} = n \times \pi/4 (d^2) = 4 \times \pi/4 (20)^2$$

$$A_{st} = 1256.63 mm^2.$$

$$A_{sc} = n \times \pi/4 (d^2)$$

$$= 2 \times \pi/4 (16)^2$$

$$A_{sc} = 402.12 mm^2$$

$$M_u = M_{ulimit} + M_{u2}$$

$$\varepsilon_{sc} = 0.0035 \left[1 - d'/x_u\right]$$

$$x_u = x_{umax},$$

$$x_u = 0.48 d$$

$$= 0.48 (550)$$

$$\boxed{x_u = 264 mm.}$$

$$\varepsilon_{sc} = 0.0035 \left[1 - \frac{50}{264}\right]$$

$$\varepsilon_{sc} = 2.8 \times 10^{-3}$$

$$\varepsilon_{sc} = 0.0028$$

From table (A) of SP-16 Curve.

0.00276	351.8
0.0028	?
0.0038	360.9

$$f_{sc} = 351.8 + \left[\frac{360.9 - 351.8}{0.0038 - 0.00276}\right] (0.0028 - 0.00276)$$

$$f_{sc} = 352.15 N/mm^2.$$

~~0.36fck.bxulim(d-0.42xu)+fsc.Asc(d-d')~~

$$x_u = \frac{0.87 f_y . A_{st} - f_{sc} . A_{sc}}{0.36 f_{ck} . b}$$

$$x_u = \frac{0.87(415)(1256.63) - (352.15)(402.12)}{0.36(20)(250)}$$

$x_u = 173.38$ (obtained).

$x_u \neq x_u$ obtained. Then repeat the steps.

$$\varepsilon_{sc} = 0.0035\left[1 - \frac{50}{173.38}\right].$$

$\varepsilon_{sc} = 0.0024$

$$\boxed{f_{sc} = 342.8 \text{ N/mm}^2.}$$

~~xu = 0.87fy.Ast - fsc.Asc.~~

$M_u = M_{ulimit} + M_{u2}$

$= 0.36 f_{ck}\, b\, x_{ulimit}(d - 0.42 x_{ulimit}) + f_{sc} . A_{sc}(d - d')$

$M_u = 0.36(20)(250)(550 - 0.42 \times 173.38) \times 173.38 + (\overset{342.8}{\cancel{352.15}})(402.12)(550 - 50)$

$= \overset{148.92 \times 10^6}{\cancel{85892}} + 68923368$

$= 217.84 \times 10^6$ N-mm.

$$\boxed{M_u = 217.84 \text{ kN-m}}$$

2) Calculate the moment of resistance 250mm wide and 500mm over all depth and 6 bars of 20mm dia in tension side and 2 bars of 20mm dia on compression side. Use M20 grade of concrete and Fe415. and effective cover 40mm each.

Given that

b = 250mm

D = 500mm

Tension, n = 6 bars
dia = 20mm

Compression ⇒ n = 2bars
dia = 20mm.

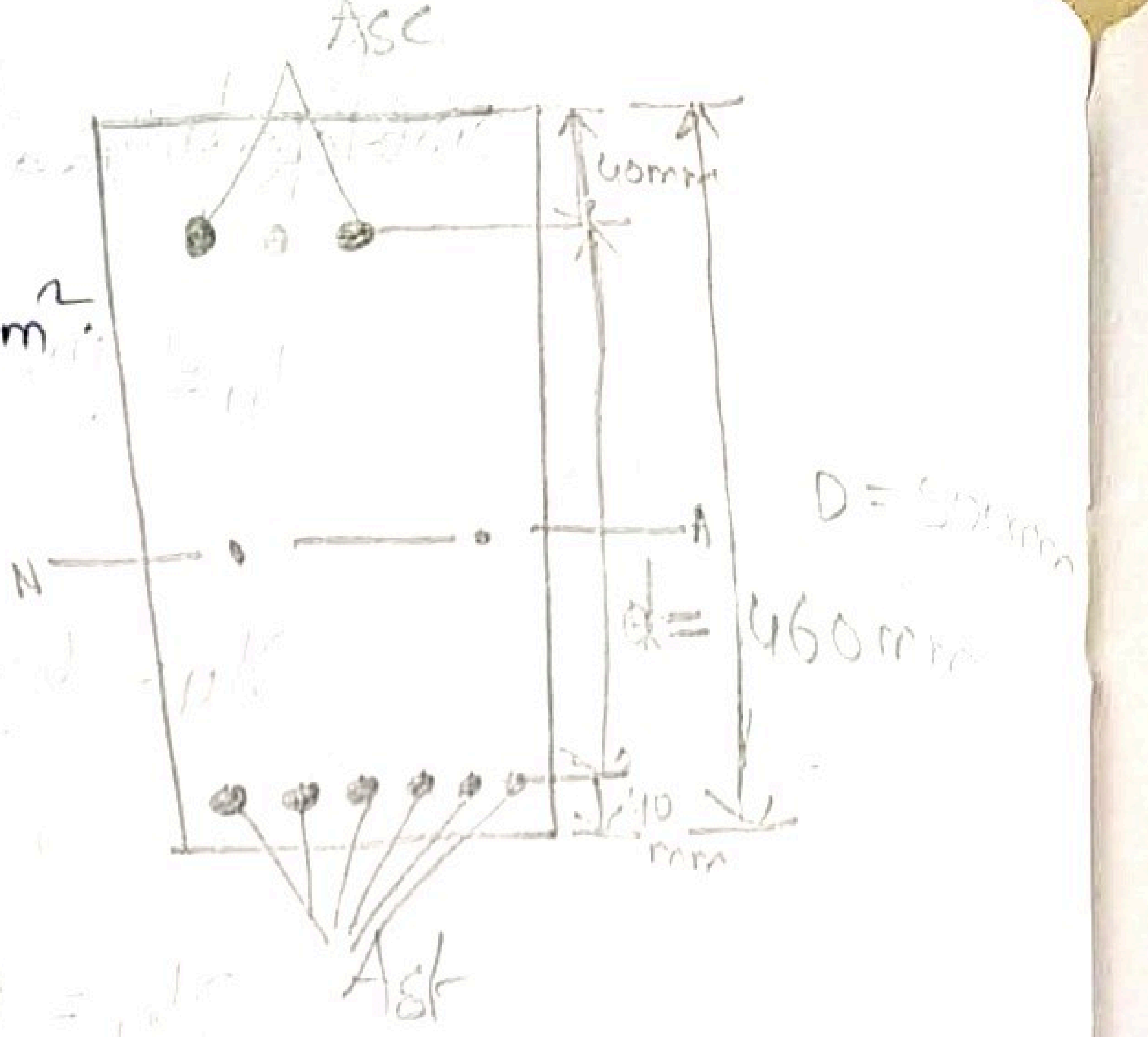
$A_{sc} = n \times \pi/4 (d^2)$

$A_{sc} = 2 \times \pi/4 \times (20)^2 = 628.31 mm^2.$

$A_{st} = n \times \pi/4 (d^2)$

$= 6 \times \pi/4 (20)^2$

$A_{st} = 1884.95 mm^2.$



① $\frac{d'}{d} = \frac{40}{460}$

$= 0.086$

$0.05 — 355$

$0.086 — ?$

$0.1 — 353$

②

$$\left[\cancel{355 + \left(\frac{353-355}{0.1-0.05}\right)(0.086-0.05)}\right]$$

$$\boxed{f_{sc} = 353.56 \; N/mm^2.}$$

(or)

$$353 + \frac{(353-355)}{(0.1-0.05)}(0.086-0.1)$$

③ $$x_u = \frac{0.87 f_y . A_{st} - f_{sc} . A_{sc}}{0.36 f_{ck} . b}$$

$$x_u = \frac{0.87 \times 415 \times 1884.95 - 353.56 \times 628.31}{0.36 \times 20 \times 250}$$

$$\boxed{x_u = 254.67 mm}$$

$x_{umax} = 0.48 \times d$

$x_{umax} = 0.48 \times 460$

$x_{umax} = 220.8 mm.$

$x_u > x_{umax}.$

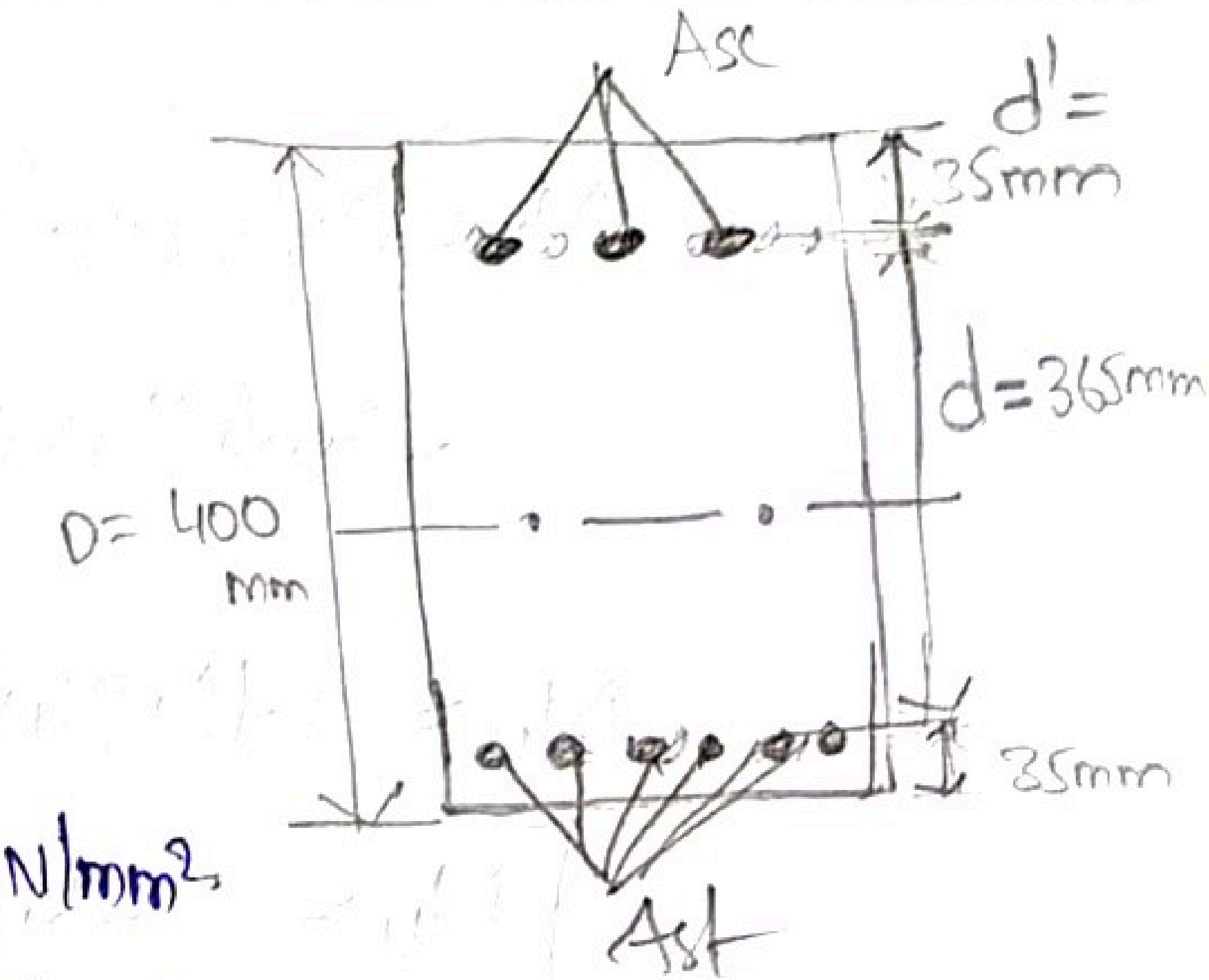
$$M_u = 0.36 f_{ck} \cdot b x_{umax} (d - 0.42 x_{umax}) + f_{sc} . A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 250 \times 220.8\left(460 - \overset{2.42}{220.8}\right) + 353.56 \times 628.31$$
$$(460 - 40)$$

$M_y = 239.26 \times 10^6$ N-mm

$$\boxed{M_u = 239.26 \text{ KN-m.}}$$

3) Calculate the ultimate moment of resistance of a reinforced Concrete beam of rectangular section 300mm x 400mm deep. A steel Consists of 6 bars of 18mm diameter in tension and 3 bars of 18mm diameter in Compression. Assume the grade of steel Fe_{415} and M_{20} grade of Concrete. we can effective cover of 35mm on both sides.



Sol:- Given that :- B = 300mm

D = 400mm

Tension, Ast = 6-18mm ϕ

Compression, Asc = 3-18mm ϕ

Grade of Concrete, $f_{ck} = 20 N/mm^2$.

Grade of steel, $f_y = 415 N/mm^2$.

effective Cover ⇒ 35mm.

$Ast = n \times \pi/4 (d^2) = 6 \times \pi/4 (18)^2 = 1526.81 mm^2$.

$Asc = n \times \pi/4 (d^2) = 3 \times \pi/4 (18)^2 = 763.40 mm^2$.

$\frac{d'}{d}$ ratio ⇒ $\frac{35}{365} = 0.095$ from table-F of sp-16.

0.05	355
0.095	
0.1	353

[illegible].

$$f_s = 353 + \left(\frac{353 - 355}{0.1 - 0.05}\right)(0.095 - 0.1)$$

$f_{sc} = 353.2 N/mm^2$.

$$x_u = \frac{0.87 f_y Ast - f_{sc} \cdot Asc}{0.36 f_{ck} \cdot b.}$$

$$x_u = \frac{0.87\times 415\times 1526.81 - 353.2\times 763.40}{0.36\times 20\times 300}$$

$$\boxed{x_u = 130.38\text{mm}}$$

$$x_{umax} = 0.48d$$
$$= 0.48(365)$$

$$\boxed{x_{umax} = 175.2\text{mm}}$$

$x_u < x_{umax}$ ~~⇒~~ (x_u less value)

$$M_u = 0.36 f_{ck}.b x_u(d-0.42x_u) + f_{sc}.A_{sc}(d-d')$$

$$= 0.36\times 20\times 300\times 130.38(365-0.42\times 130.38) + 353.2\times 763.40 \, (365-35)$$

$$M_u = 176.34\times 10^6 \text{ ~~K~~N-mm}$$

$$\boxed{M_u = 176.34 \text{ KN-m}}$$

4) Design a rectangular reinforced Concrete beam for a clear span of 4000mm the Super imposed load is 35KN/m at the size of the beam is restricted to 250mm × 400mm. Use M20 grade of Concrete and Fe415 Steel and the Support width is 300mm at each end and effective cover is 40mm.

Sol:- Given that:-

If Mu>Mulimit we can design doubly reinforced.

S.S.B = Clear span + effective depth.

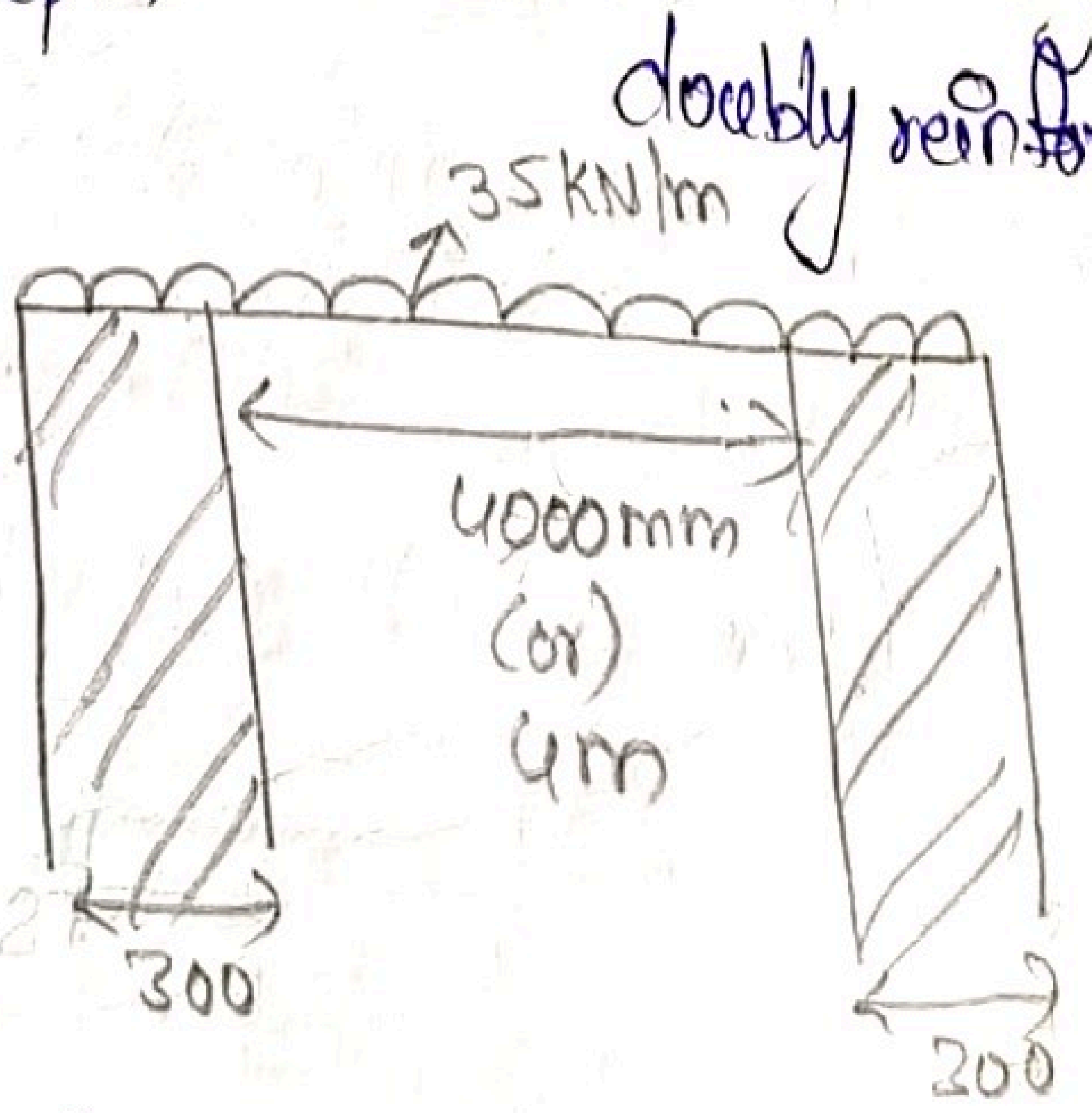
b = 250mm.

D = 400mm.

Live load = 35KN/m

Effective Cover, e = 40mm

Support width, = 300mm.



Clear span = 4000mm (or) 4m.

$d = D - e$

$d = 400 - 40$, $d = 360$mm.

① Calculation of effective span:-

S.S.B = 4000 + 300

= 4300mm

= 4.300m

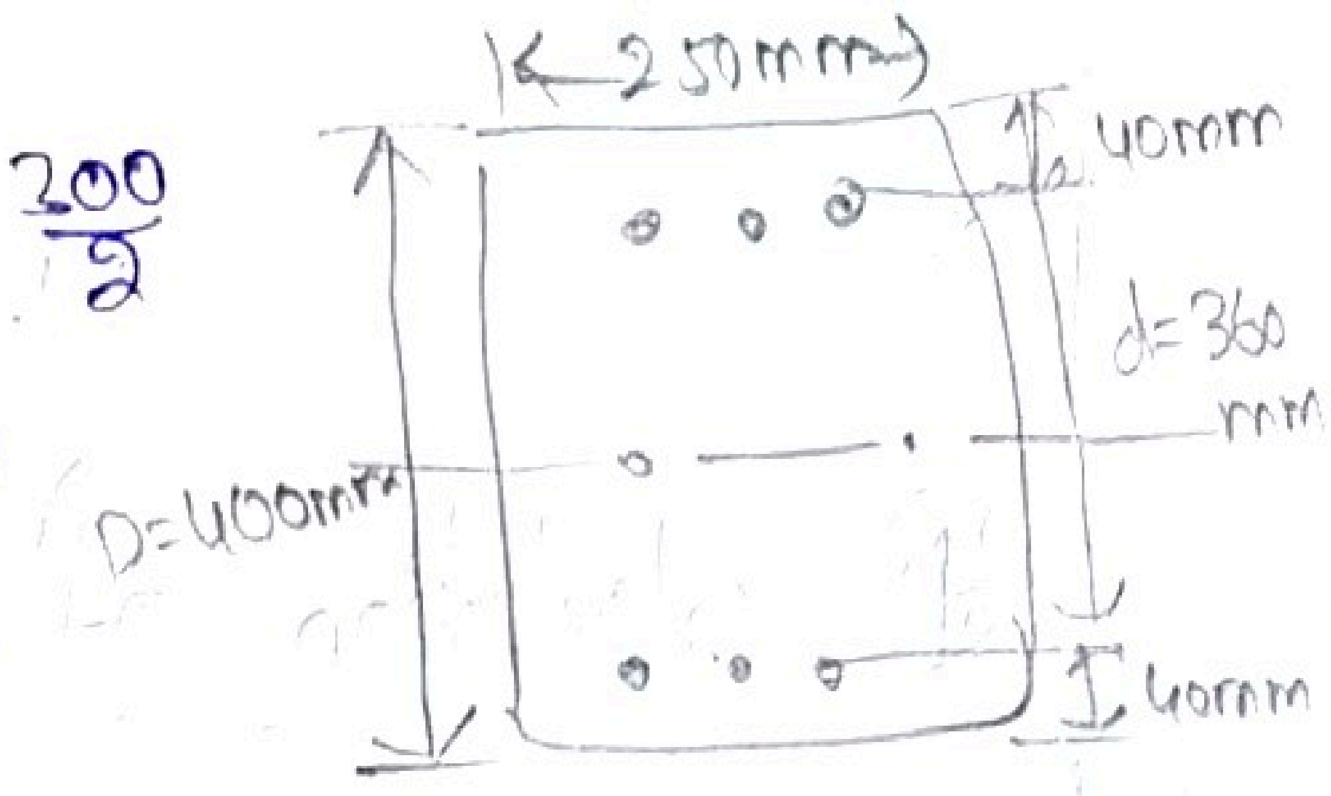
SS.B = center to center distance b/w supports.

$= \frac{300}{2} + 4000 + \frac{200}{2}$

= 4000 + 300

= 4300mm

= 4.30m



effective span for SSB = 4.30 m

② Calculation of load:-

Dead load = b×D× unit weight of Concrete.

= 250×400×25

= 0.25×0.4×25

= 2.5 KN/m.

Live load = 35 KN/m.

floor finish load = 0.6 KN/m.

T.L = D.L + L.L + F.F

= 2.5 + 35 + 0.6

T.L = 38.1 KN/m

factored load = T.L × 1.5

= 38.1 × 1.5

= 57.15 KN/m

$M_u = \frac{wl^2}{8}$

$= \frac{57.15 \times (4.3)^2}{8}$

$\boxed{M_u = 132.08 \text{ KN-m}}$

Mu limit = 2.76 bd²

= 2.76 × 250mm × 360mm

= 2.76 × 0.25 × 0.36

=

⑧

③ Check for M_u & M_{ulimit}.

$$M_{ulimit} = 2.76\, bd^2$$

$$M_{ulimit} = 2.76 \times 250 \times (360)^2$$

$$= 89.42 \times 10^6$$

$$M_{ulimit} = 89.42 \text{ KN-m.}$$

$M_u > M_{ulimit}$ (Doubly reinforced beam)

④ <u>Calculation of Area of Tension reinforcement</u> :-

~~$M_u = 0.87 f_y A_{st} d \left[1 - f_y\right.$~~

$$A_{st_1} = \frac{M_{ulimit}}{0.87 f_y (d - 0.42 x_{umax})}$$

$$x_{umax} = 0.48 d = 0.48(360)$$

$$x_{umax} = 172.8 \text{mm.}$$

$$A_{st_1} = \frac{89.42 \times 10^6}{0.87 \times 415 [360 - 0.42 \times 172.8]}$$

$$\boxed{A_{st_1} = 861.67 \text{mm}^2}$$

$$A_{st_2} = \frac{M_{u_2}}{0.87 f_y (d - d')}$$

$M_u = M_{ulimit} + M_{u_2}$.

$$M_{u_2} = M_u - M_{ulimit}$$

$$= 132.08 \times 10^6 - 89.42 \times 10^6$$

$$\boxed{M_{u_2} = 42.66 \times 10^6 \text{mm.}}$$

$$A_{st_2} = \frac{42.66 \times 10^6}{0.87 \times 415 \times (360 - 40)}$$

$$\boxed{A_{st_2} = 369.23 \text{mm}^2}$$

$$A_{sc} = \frac{M_{u_2}}{f_{sc}(d - d')} =$$

$\frac{d'}{d}$.

$$\begin{array}{r} 360 \\ 40 \\ \hline d\text{-}d' = 320 \end{array}$$

$\frac{d'}{d} = \frac{40}{368} = 0.11$

$\boxed{f_{sc} = 353 N/mm^2}$

$A_{sc} = \frac{49.66 \times 10^6}{353(320)}$

$\boxed{A_{sc} = 377.65 mm^2}$

$A_{st} = A_{st1} + A_{st2}$

$= 861.67 + 369.23$

$\boxed{A_{st} = 1230.9 mm^2}$

for Ast. Choose 20mm dia bars.

$A_{st} = n \times \pi/4 (d^2)$

$1230.9 = n \times \pi/4 (20)^2$

$n = 3.91$

$\boxed{n = 4 \text{ bars}}$

$A_{st} = 4 \times \pi/4 (20)^2$

$\boxed{A_{st} = 1256.63 mm^2}$

for Asc Choose 16mm dia

$377.65 = n \times \pi/4 (16)^2$

$n = 1.8$

$\boxed{n = 2 \text{ bars}}$

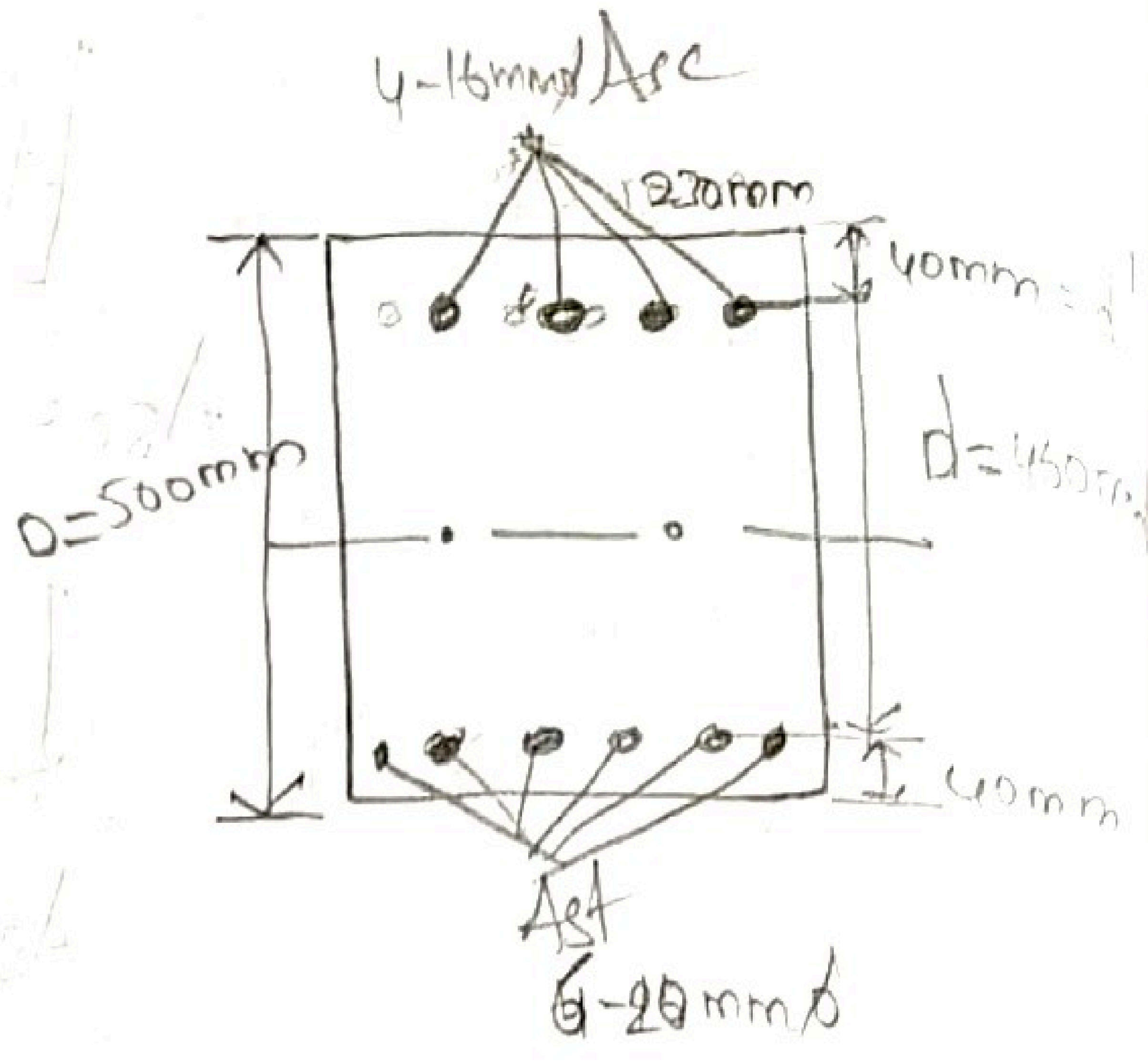
$A_{sc} = 2 \times \pi/4 (16^2)$

$\boxed{A_{sc} = 402.12 mm^2}$

~~5) The dimensions of 250~~

5) A doubly reinforced Beam of width 230mm and 500mm over all depth is reinforced with 4 bars of 16mm diameter as Compression reinforcement and 6 bars of 20mm diameter as tension reinforcement at an effective Cover of 40mm on both sides. Find the safe U.D.L on the Beam Can carry. If it is Simply supported over an

Effective span of 5m. Use M20 grade of concrete and Fe415 steel.



Sol:- Given that

$b = 220mm$

$D = 500mm$

$e = 40mm$

$d = D - e = 500 - 40$

$d = 460mm.$

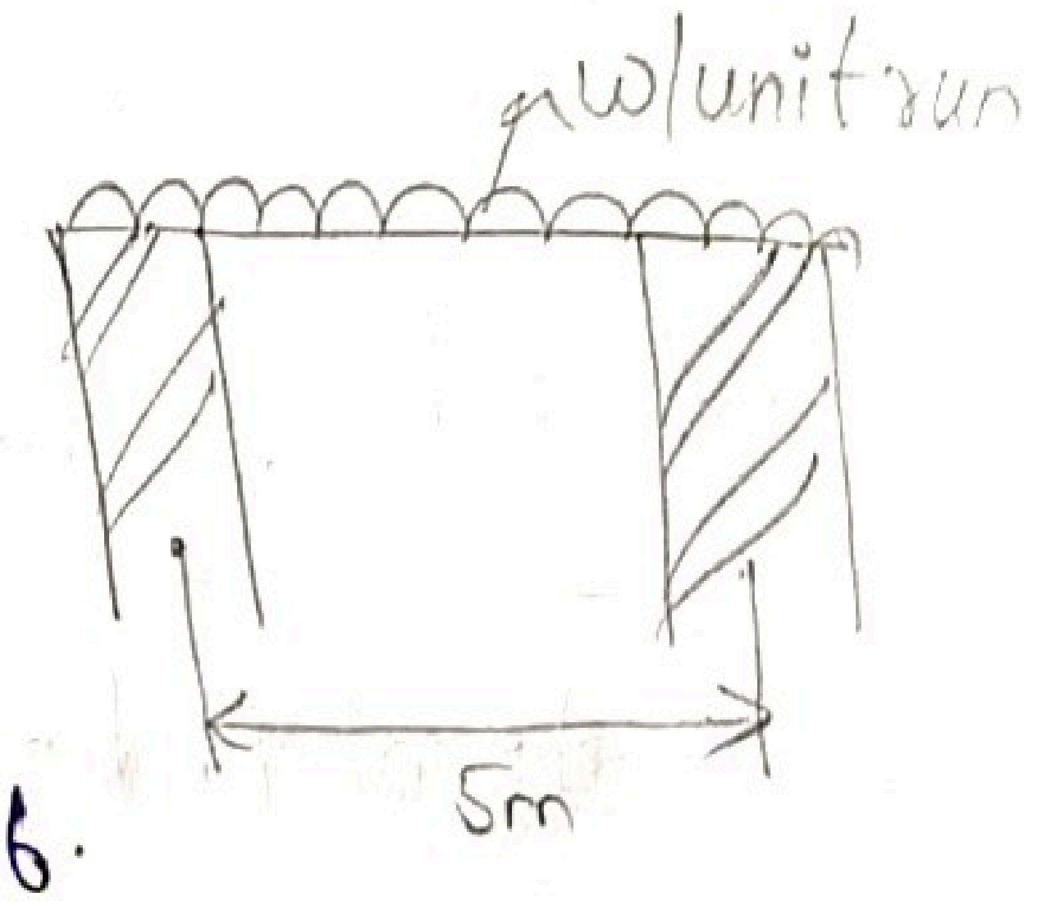
$Asc = 4 - 16mm\phi$

$Ast = 6 - 20mm\phi$

$A_{st} = n \times \pi/4 (d^2) = 6 \times \pi/4 (20)^2 = 1884.95mm^2.$

$A_{sc} = n \times \pi/4 (d^2) = 4 \times \pi/4 (16)^2 = 804.24mm^2.$

$l = 5m.$



① Calculate d'/d ratio:-

$d'/d = \frac{40}{460} = 0.086$

② from table '5' of sp-16.

$f_{sc} = 353.56 \ N/mm^2.$

0.05 355

0.086 ?

0.1 353

$$353 + \left[\frac{353 - 355}{0.1 - 0.05}\right] \times (0.086 - 0.1)$$

$$\boxed{f_{sc} = 353.56 N/mm^2.}$$

③ Calculation of x_u

$$x_u = \frac{0.87 f_y \ A_{st} - f_{sc} \cdot A_{sc}}{0.36 f_{ck} \cdot b}$$

$$x_u = \frac{0.87 \times 415 \times 1884.95 - 353.56 \times 804.24}{0.36 \times 20 \times 230}$$

$$\boxed{x_u = 239.25mm}$$

M20 /

$x_{umax} = 0.48d$

$= 0.48(460)$

$$\boxed{x_{umax} = 220.8mm}$$

$x_u > x_{umax}$. lex value.

③ $M_u = 0.36 f_{ck} \cdot b\, x_{umax}(d - \cancel{x}0.42 x_{umax}) + f_{sc} \cdot A_{sc}(d-d')$

$= 0.36 \times 20 \times 230 \times 220.8 (460 - 0.42(220.8)] + (353.56) \times ($
$(804.24)(460-40).$

$M_u = 253.71 \times 10^6$ N-mm

$$\boxed{M_u = 253.7 \text{ KN-m.}}$$

Max. B.M ⇒ $M_u = \frac{wl^2}{8}$

$253.7 \times 10^6 = \frac{w(5000)^2}{8}$

$W = 81.18$ N/mm.

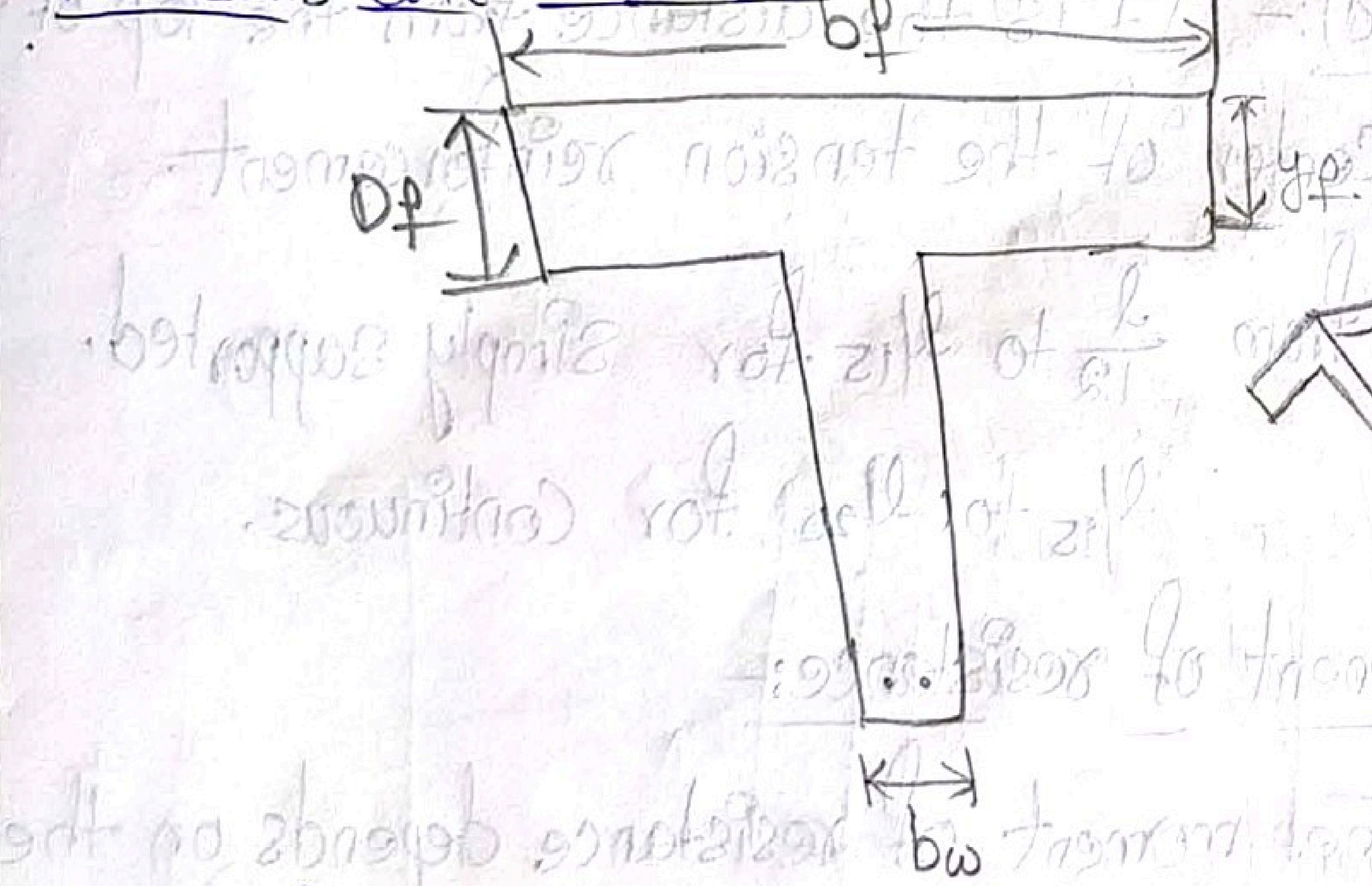
$$\boxed{W = 81.18 \text{ KN/m.}}$$

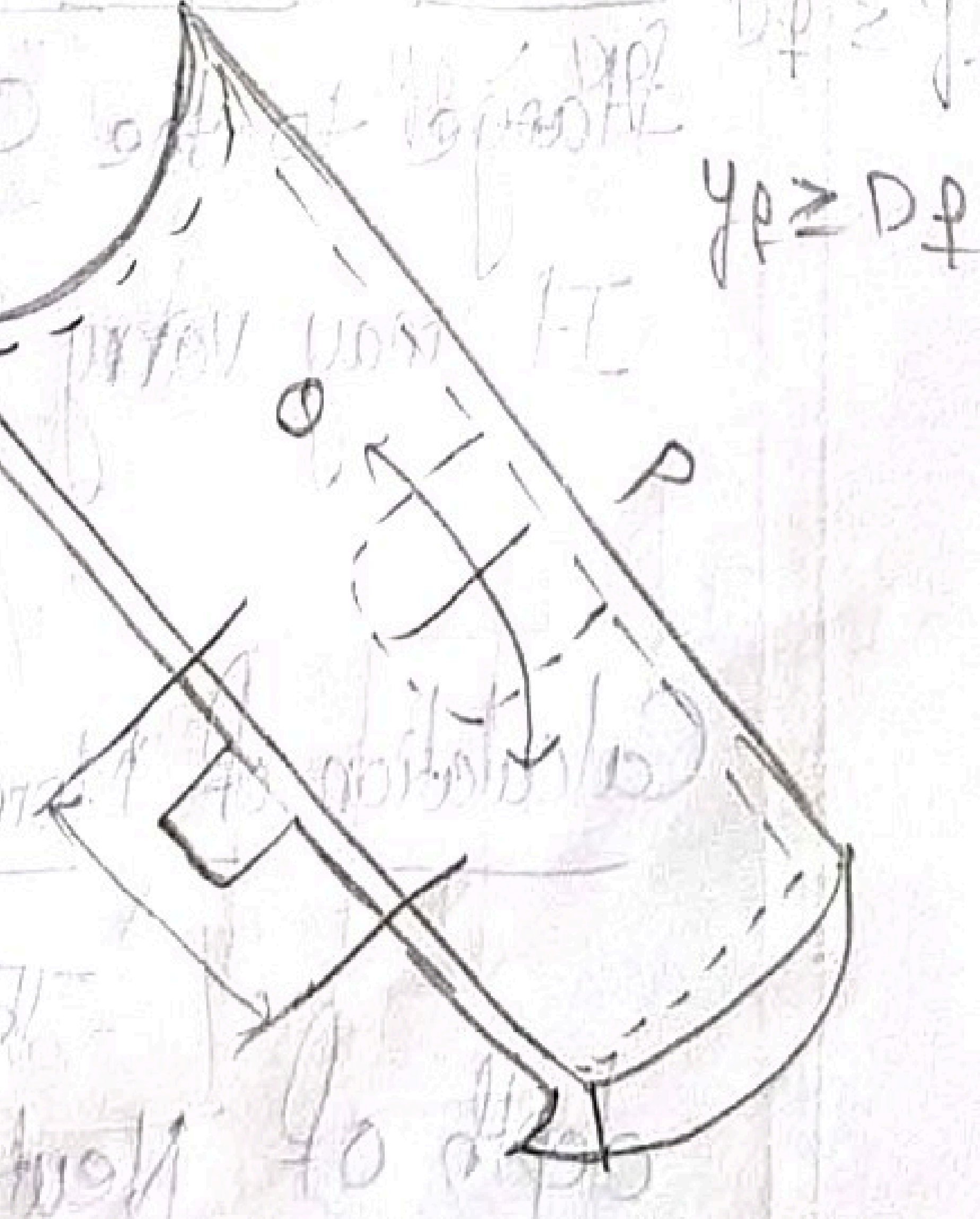
~~$w = 81.18$ kN-mm.~~

~~$w = 81.18$ N/mm. $= \frac{81.18 \times 10^3 N}{10^3 m}$,~~

~~$w = 81.18$ kN/m.~~

T-Beams and L-Beams



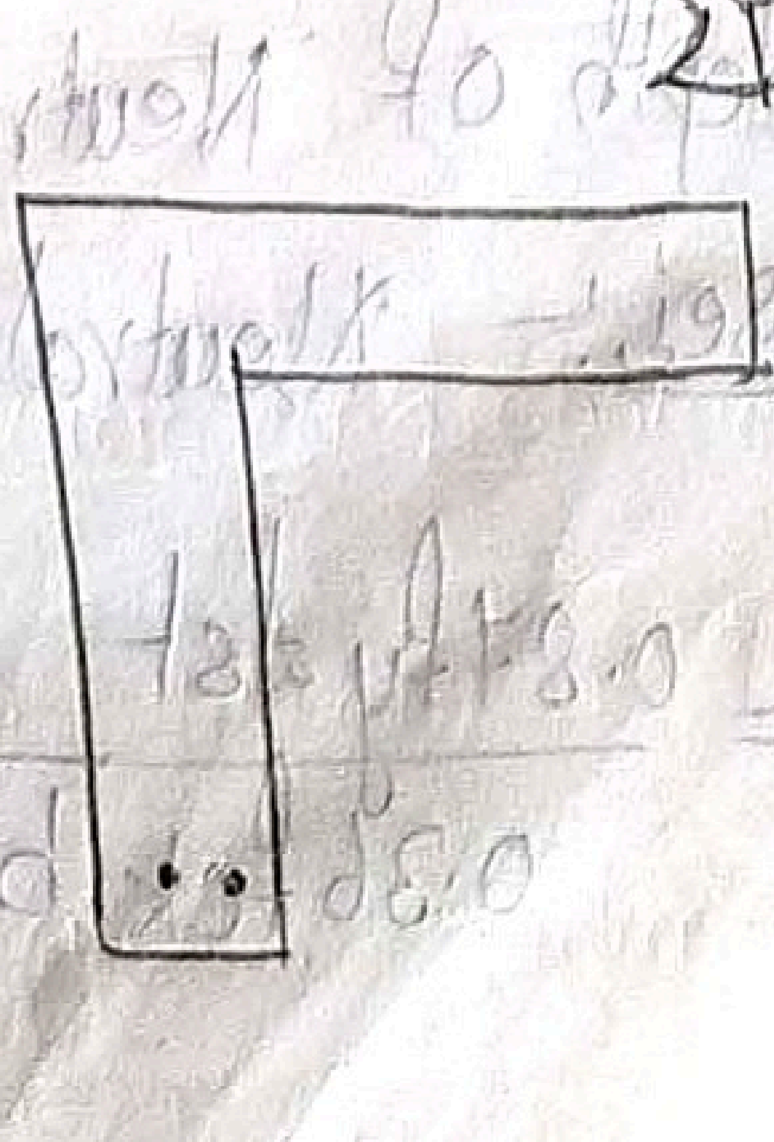


Df ≥ yf.

$y_f \geq D_f$.

Analysis of T-Beam:-

Steps:- Pg No - 37



1.) Effective width of the flange.

For T-beam, $b_f = \frac{l_o}{6} + b_w + 6D_f$

For L-beam, $b_f = \frac{l_o}{12} + b_w + 3D_f$.

and which is not greater than b_w + ~~sum~~ half of clear distances of adjacent beams.

l_o = distance b/w points of zero moments in the beam.

b_f = effective width of flange, b_w = breath of the web,

D_f = thickness of flange, b = actual width of flange

Isolated beams:-

Pg No-37.

For T-beam, $b_f = \frac{l_o}{\left(\frac{l_o}{b}\right)+4} + b_w$

For L-beam, $b_f = \frac{0.5\, l_o}{\left(\frac{l_o}{b}\right)+4} + b_w$

which is not greater than 'b' (actual width)

For Continuous beams l_o be taken as 0.7 times of the effective Span.

Thickness of the flange:-

Width of the beam:- It is the width of the web portion of the beam in tension zone(b_w). It is generally taken as half to 2/3 of b_f.

effective depth (d):- It is the distance from the top of the flange to the center of the tension reinforcement.

It may varry from $\frac{l}{12}$ to $l/15$ for simply supported.

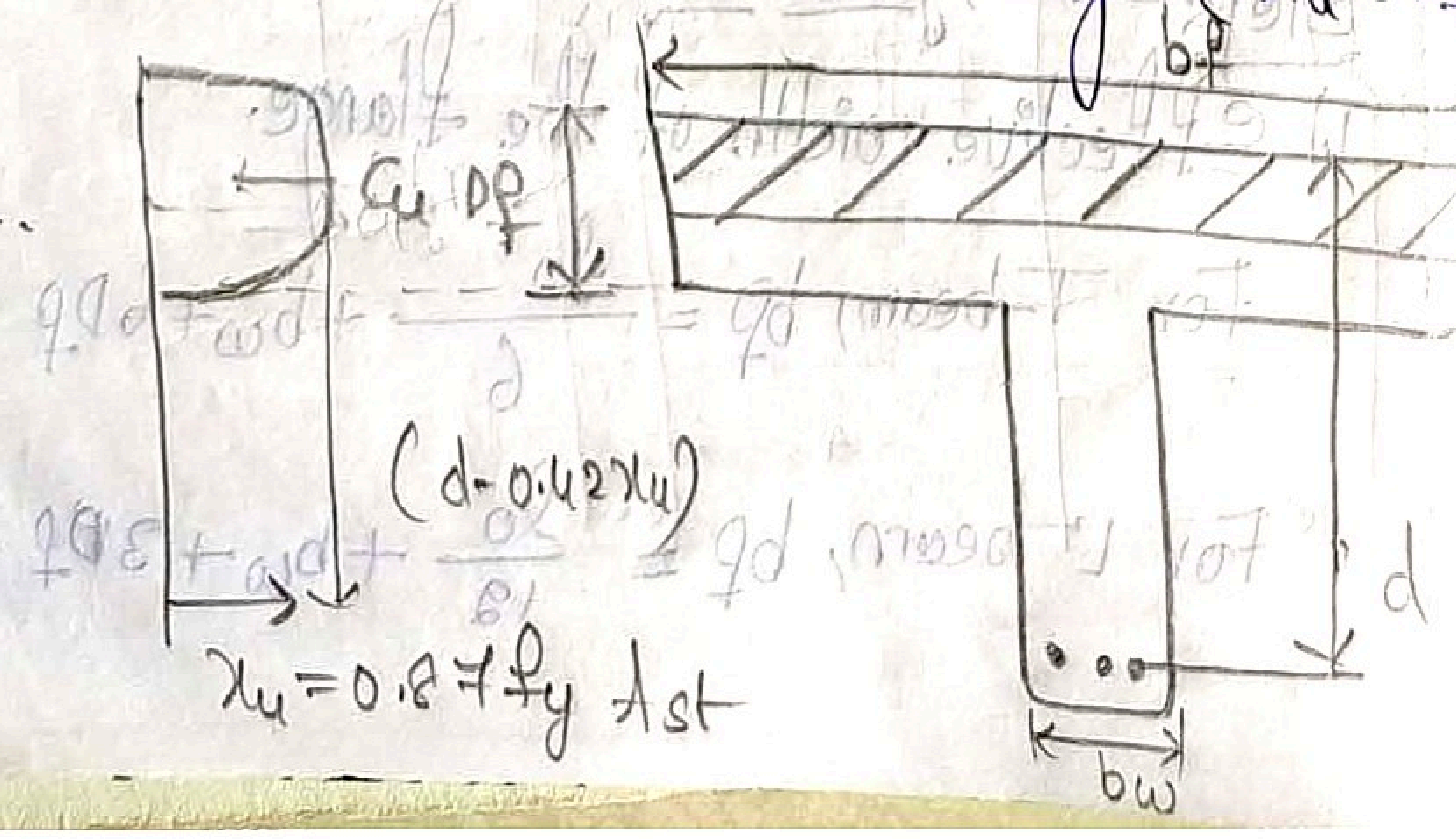
$l/15$ to $l/20$ for Continuous.

Calculation of Moment of resistance:-

The moment of resistance depends on the depth of Neutral axis.

Case i:- Neutral axis lies with in the flange. ($x_u < D_f$)

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}.$$



Moment of resistance, $M_u = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u)$

(or)

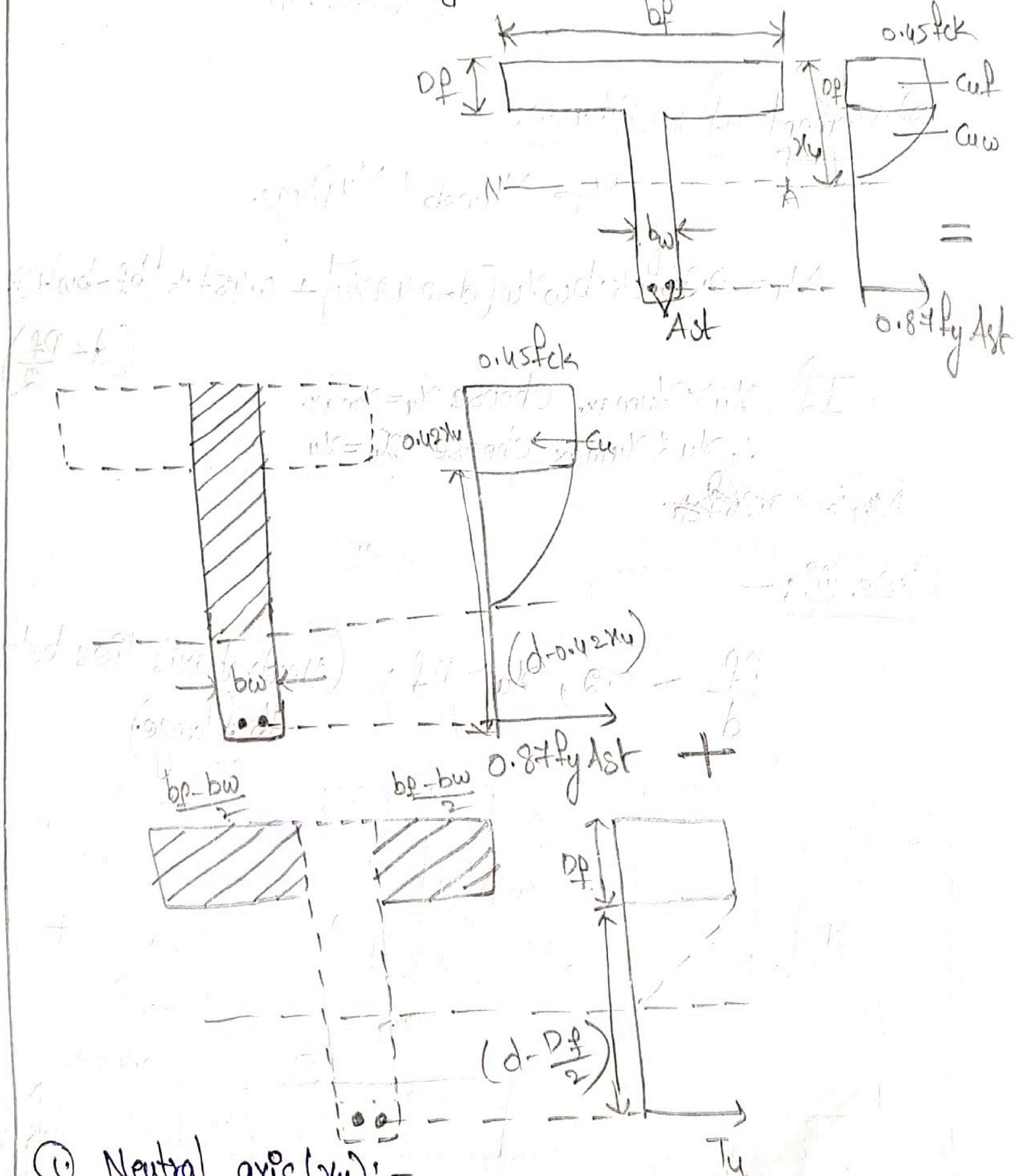
$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$

If $x_u > x_{umax}$ then take $x_u = x_{umax}$.

If $x_u < x_{umax}$ then take $x_u = x_u$.

Case ii :- When $x_u > D_f$

and $\frac{D_f}{d}$ (ratio) ≤ 0.2 [N.A lies in web position]



① Neutral axis (x_u):-

The Compression force in web, $C_{uw} = 0.36 f_{ck} b_w \times x_u$

Compression force in flange, $C_{uf} = 0.45 f_{ck} (b_f - b_w) \times D_f$.

Tensile force $T = 0.87 f_y A_{st}$.

Calculating x_u:-

$$C_{uf} + C_{uw} = T$$

$$0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) \cdot D_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w) \cdot D_f}{0.36 f_{ck} \cdot b_w}$$

2) Moment of resistance:-

$$M_u = M_{u\,web} + M_{u\,flange}.$$

$$M_u = 0.36 f_{ck} \cdot b_w x_u [d - 0.42 x_u] + 0.45 f_{ck} (b_f - b_w) \cdot D_f \left(d - \frac{D_f}{2}\right).$$

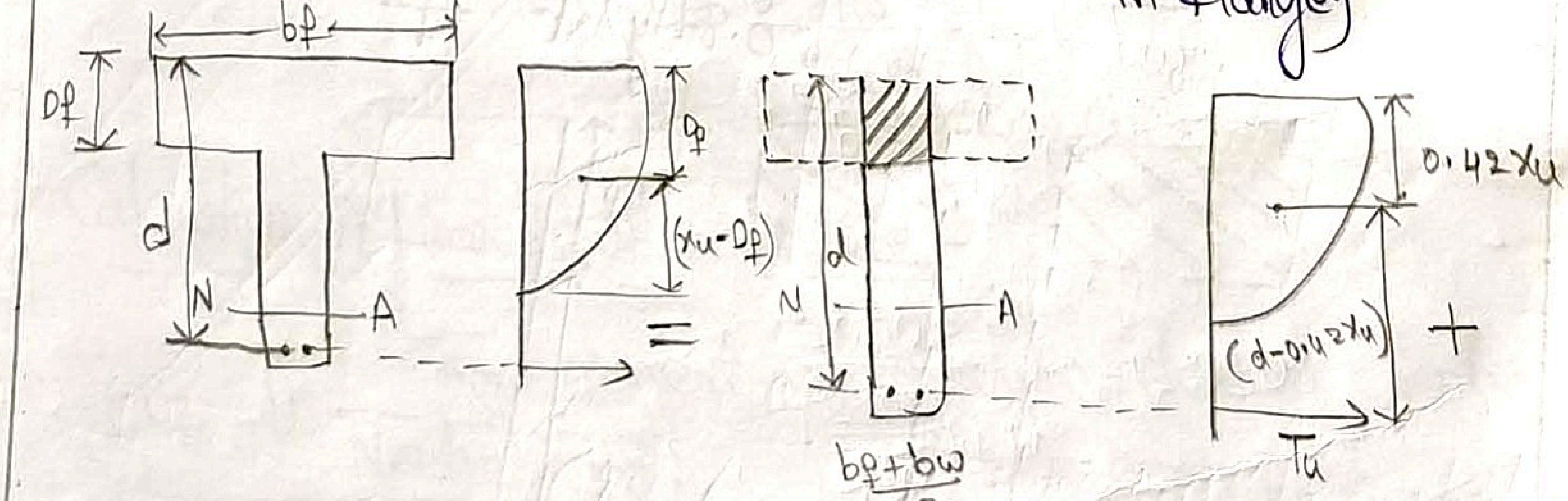
If $x_u > x_{umax}$. Choose $x_u = x_{max}$.

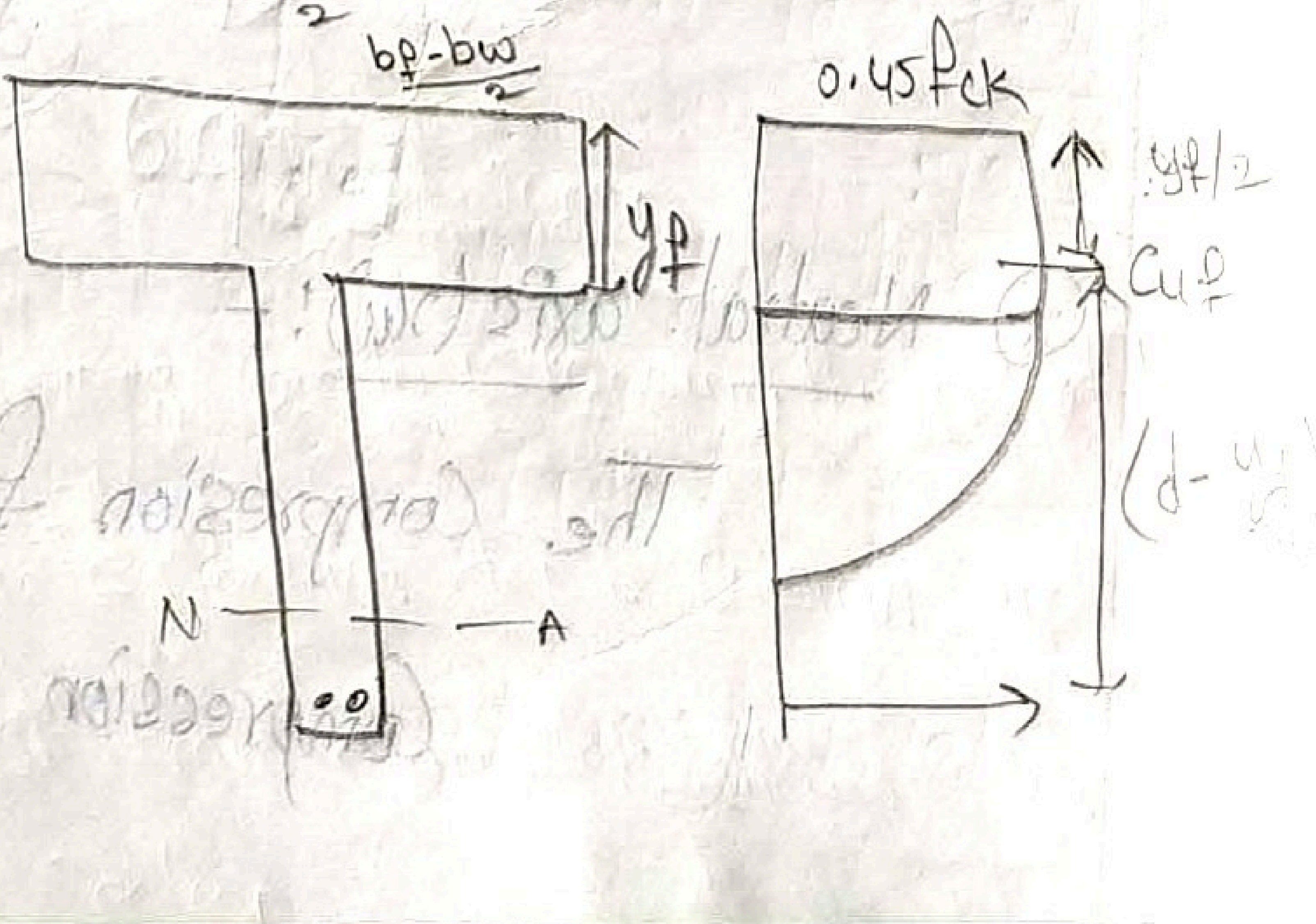
$\therefore x_u < x_{umax}$ choose $x_u = x_u$

~~$M_u = 0.36 f_{ck}$~~.

Case iii:-

$\frac{D_f}{d} > 0.2$, $x_u > D_f$. (Neutral axis lies below th flange)





① Modified thickness of the flange $y_f = (0.15x_u + 0.65D_f)$

$$0.36f_{ck} \cdot b_w x_u + 0.45f_{ck}(b_f - b_w)(y_f) = 0.87f_y A_{st}.$$

$$x_u = \frac{0.87f_y A_{st} - 0.45f_{ck}(b_f - b_w)y_f}{0.36f_{ck} \cdot b_w}$$

② Moment of resistance:-

$$M_u = M_{u\,web} + M_{u\,flange}.$$

$$= 0.$$

$$M_u = 0.36f_{ck} b_w x_u [d - 0.42x_u] + 0.45f_{ck}(b_f - b_w)y_f \left(d - \frac{y_f}{2}\right).$$

$x_u > x_{umax}$ choose $x_u = x_{max}$

$x_u < x_{umax}$ choose $x_u = x_u$

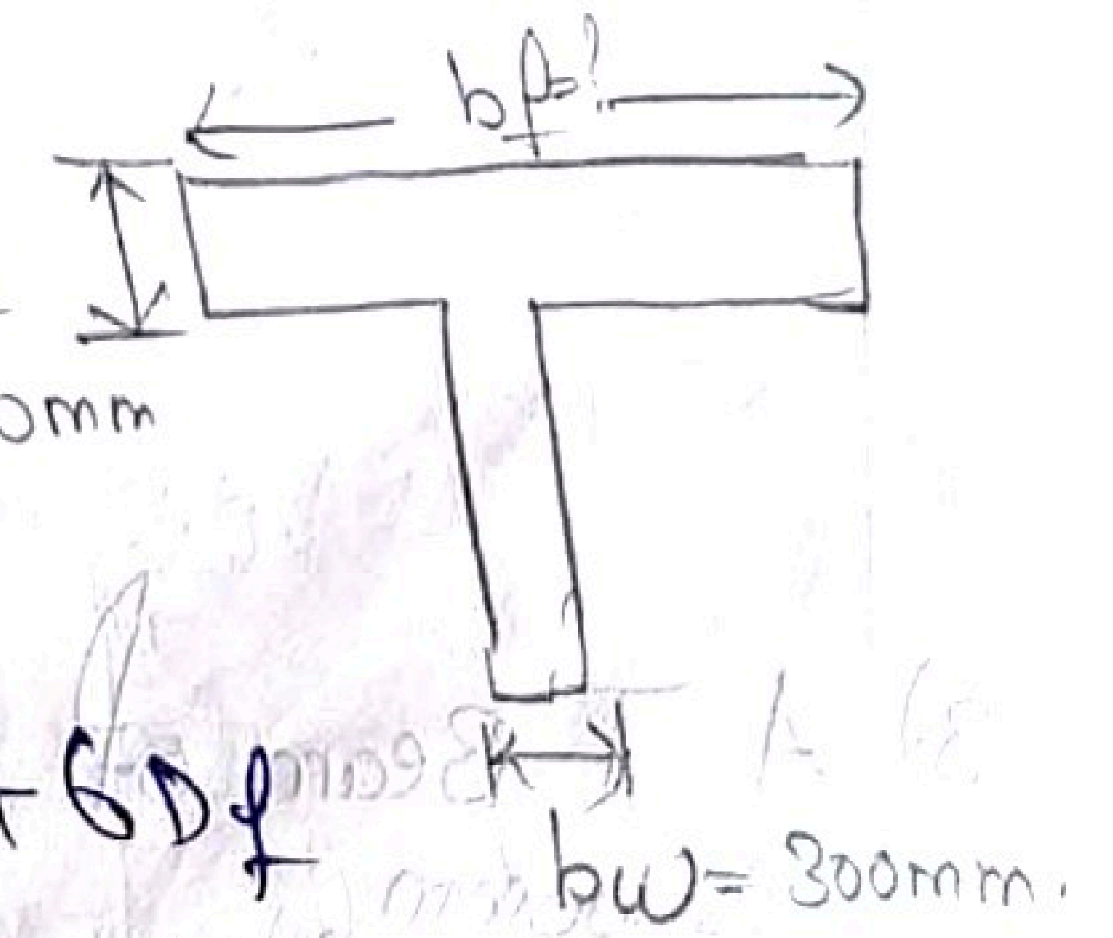
Use of SP-16 Curves:- Refer table 57 to 59. Calculate the M_{ulimit}/ $b_w d^2 f_{ck}$. For f_y = 250, 415 and 500 for f_{ck} = 20, 25, 30 and 35 and $\frac{d_f}{d}$ ratio varying from 0.6 to 0.45 and $\frac{b_f}{b_w}$ ratio varying from 1 to 10.

① Find the effective width of the flange of the following simply Supported T-Beam whose effective span is 5m and center to Center distance of the adjacent pannels is 4m width of the web is 300mm. Thickness of the slab is 110mm.

Sol:- Given that l_o = 5m ; c/c of adjacent Panels = 4m.

b_w = 300mm

D_f = 110mm



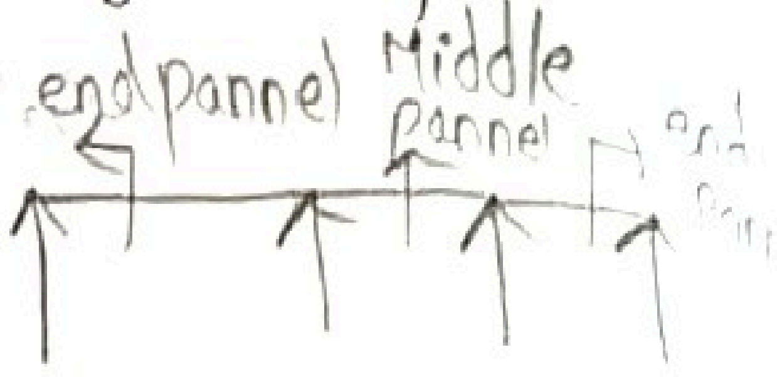
(Pg No 37 IS456) $b_f = \frac{l_o}{6} + b_w + 6D_f$

$$= \frac{5000}{6} + 300 + 110 \times 6$$

$$b_f = 1793.33 \text{ mm}$$

$$b_f = 1.79 \text{m}.$$

clearspan of the slab [To the left (or) right of the Beam.] $\not>$ bw + half ~~sum~~ of the clear distance of adjacent pannel.



~~bwt~~

= c/c distance b/w pannel - bw.

$= 4000 - 300$

$= 3700\text{mm}$

$$b_w + \left[\frac{3700}{2} + \frac{3700}{2}\right]$$

$$= 300 + \left[1850 + 1850\right]$$

$$= 4000\text{mm}$$

$$b_f \not> 4000\text{mm}.$$

2) Find the effective flange width of the following simply supported Isolated T-Beam $l_o = 5\text{m}$, width of web (b_w) = 230mm, depth of flange (D_f) = 110mm, Support width (b) = 750mm. Actual width of flange 450mm. Calculate the b_f value.

Sol:-

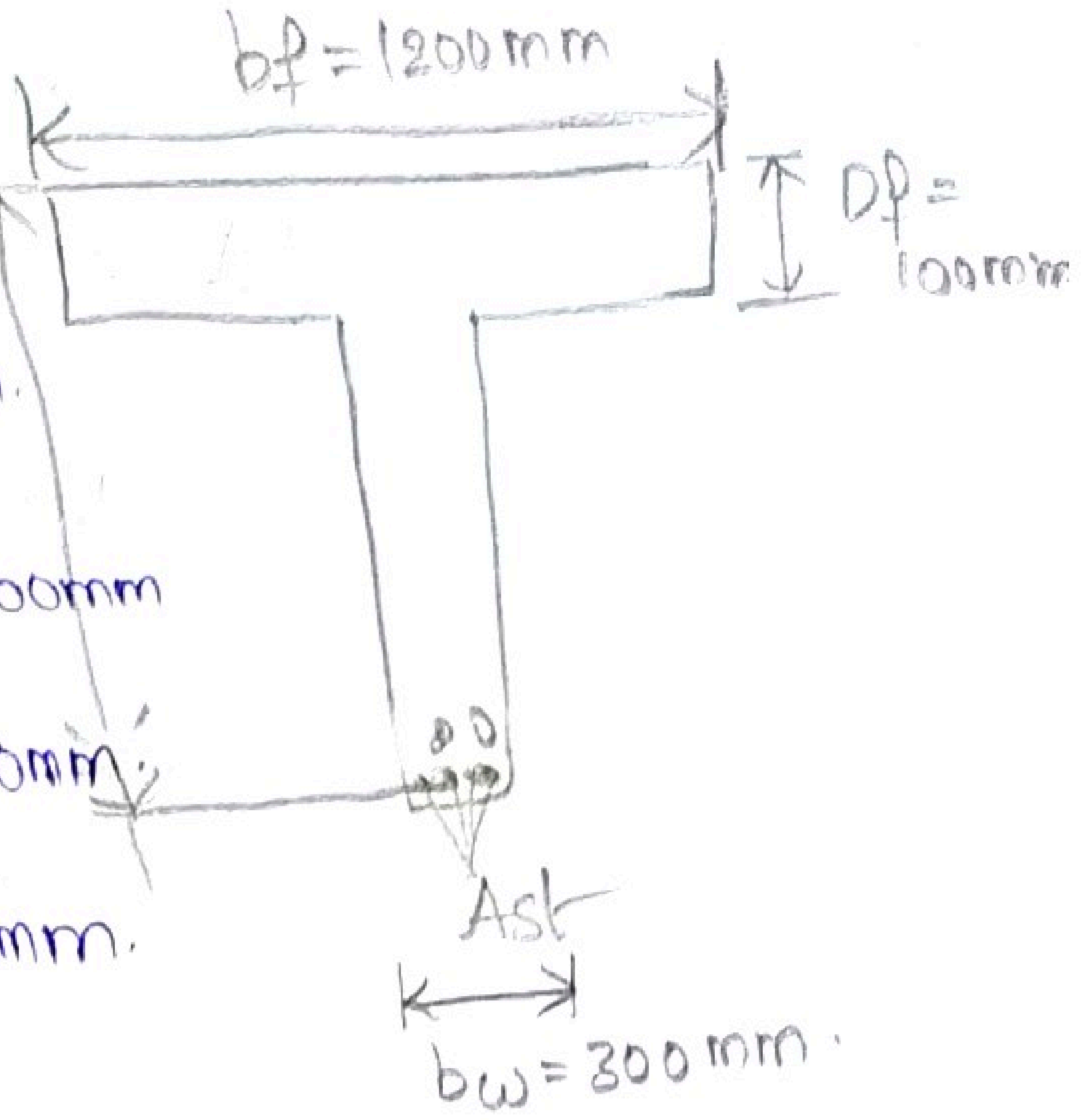
$$b_f = \frac{l_o}{\left(\frac{l_o}{b}\right) + 4} + b_w.$$

$$= \frac{5000}{\left(\frac{5000}{750}\right) + 4} + 230$$

$$b_f = 698.75\text{mm}.$$

3) A T-Beam of effective flange width is 1200mm, thickness of the slab is 100mm width of the web is 300mm effective depth 460mm and is reinforced with 4 bars of 12mm diameter. Calculate the moment of resistance for the Section.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b}$$



Given that

width of flange, b_f = 1200mm.

Thickness of flange D_f = 100mm

effective depth, d = 460mm

width of web, b_w = 300mm.

A_{st} = 4 – 12mm ϕ.

$A_{st} = n \times \pi/4 (d^2) = 4 \times \pi/4 (12)^2$

$A_{st} = 452.38\ mm^2$.

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} \cdot b_w}$$

Fe415

M_{20}

$$= \frac{0.87 \times 415 \times 452.38}{0.36 \times 20 \times 300}$$

$$\boxed{x_u = 75.61\ mm.}$$

$x_{umax} = 0.48 d$

$= 0.48 \times 460mm$

$= 220.8$

$\boxed{x_u < x_{umax}}$

$\boxed{x_u < D_f.}$

M.O.R = $0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u)$

$= 0.36 \times 20 \times 75.61 [460 - 0.42 (75.61)]$.
$\times 300$

M.O.R = 69.93×10^6 N-mm

$\boxed{M.R = 69.93\ KN-m}$

M.O.R = $0.87 f_y A_{st} (d - 0.42 x_u)$

$= 0.87 \times 415 \times 452.38 (460 - 0.42 \times 75.61)$

$\boxed{M.O.R = 69.94\ KN-m}$

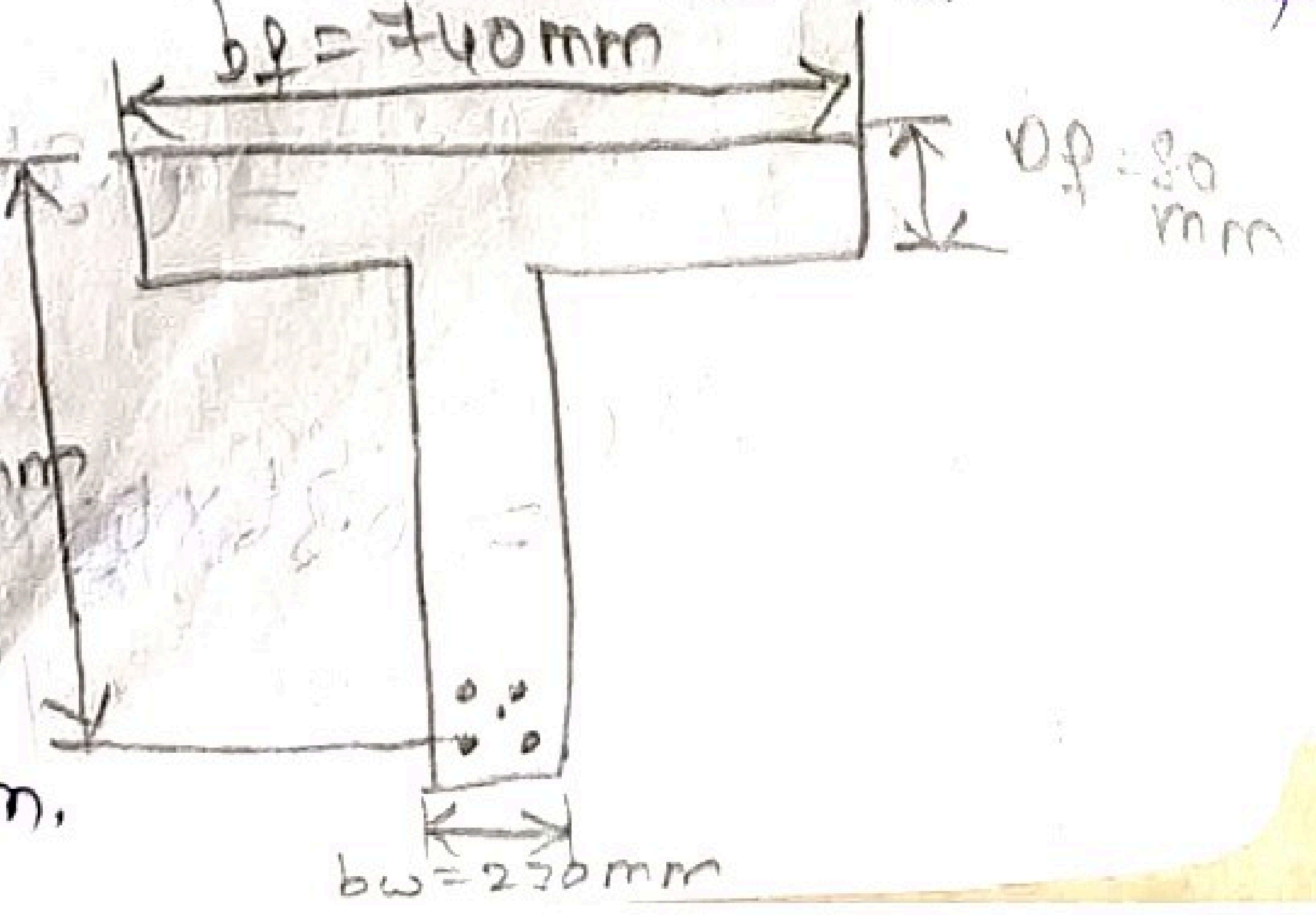
4) Find the Moment of Resistance of the T-Beam b_f = 740mm, b_w = 230mm, D_f = 80mm, d = 400mm. Area of Steel A_{st} = 5-20mm ϕ use M_{20} and Fe415.

Sol:-

Given that

width of flange, b_f = 740mm

Thickness of flange, D_f = 80mm.



width of web, ⇒ $b_w = 230mm$.

$A_{st} = n \times \pi/4 \, d^2$

$= 5 \times \pi/4 (20)^2$

$A_{st} = 1570.79 mm^2$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_w \cdot f_{ck}}$$

$$= \frac{0.87 \times 415 \times 1570.79}{0.36 \times 230 \times 20}$$

$x_u = 342.47.$

$\boxed{x_u > D_f.}$

$$\frac{D_f}{d} = \frac{\cancel{80}}{\cancel{400}} = 0.2.$$

Case ii

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck} (b_f - b_w) \cdot D_f}{0.36 f_{ck} \cdot b_w}$$

$$x_u = \frac{0.87 \times 415 \times 1570.79 - 0.45 \times 20 (740 - 230) \cdot 80}{0.36 \times 20 \times 230}$$

$x_u = 120.73 mm.$

$x_{umax} = 0.48 d$

$= 0.48 (400)$

$x_{umax} = 192 mm$

$x_u < x_{umax}.$

M.O.R = Muweb + Muflange.

$= 0.36 f_{ck} b_w x_u [d - 0.42 x_u] + 0.45 f_{ck} (b_f - b_w) D_f \times [d - \frac{D_f}{2}]$

$= 0.36 \times 20 \times 230 \times 120.73 [400 - 0.42 \times 120.73] + 0.45 \times 20 (740 - 230) \times 80 [400 - \frac{80}{2}]$

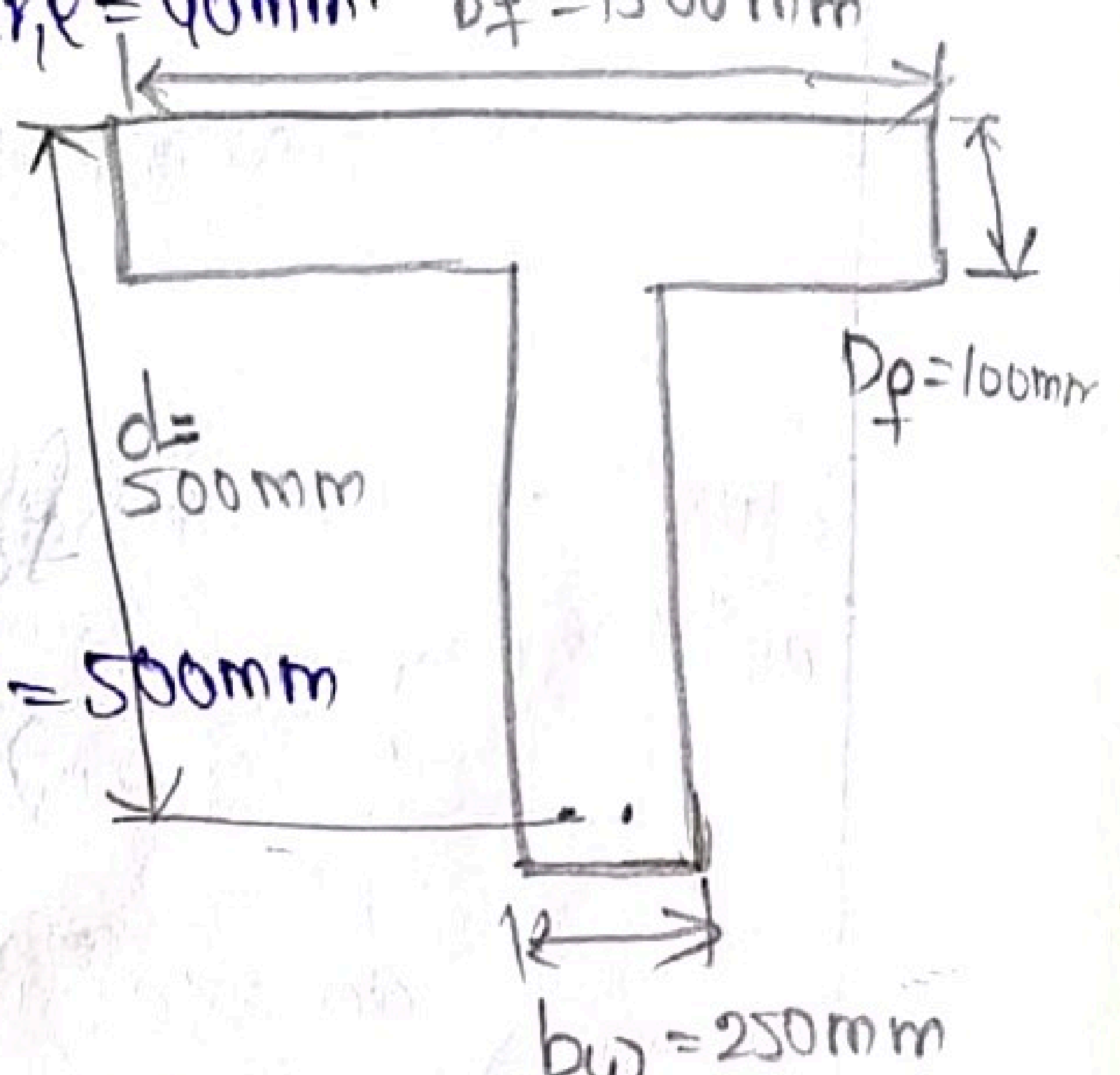
M.O.R = $69.83\times10^6 + 132.19\times10^6$

M.O.R = 202.02×10^6 N-mm

$\boxed{\text{M.O.R} = 202.02 \text{ KN-m.}}$

4) Calculate the Max. U.D.L and limit state of the T-Beam can carry including its own weight on a simply ~~singly~~ supported span of 5m width of the flange is 1500mm thickness of the flange is 100mm depth of the tensile steel from the top flange is 500mm $b_w = 250$mm effective cover = 40mm. $A_{st} = 804 \text{ mm}^2$. Use M20 and Fe415.

Sol:- Given that :- Effective cover, e = 40mm.



Span (l) = 5m.

width of flange. b_f = 1500mm

b_w = 250mm.

Thickness of flange, D_f = 100mm.

depth of flange to steel, d = 500mm

$f_{ck} = 20$, $f_y = 415$, $A_{st} = 804 \text{mm}^2$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 b_w \cdot f_{ck}} = \frac{0.87\times415\times804}{0.36\times250\times20}$$

x_u = 161.26mm

~~Max UDL = $\frac{wl^2}{8}$~~

$\boxed{x_u > D_f.}$

Case ii, $\frac{D_f}{d} = \frac{100}{500} = 0.2$

$$x_u = \frac{0.87 f_y A_{st} - 0.45 f_{ck}(b_f - b_w)D_f}{0.36 f_{ck} \cdot b_w}$$

$$= \frac{0.87\times415\times804 - 0.45\times20(1500-250)\times100}{0.36\times20\times250}$$

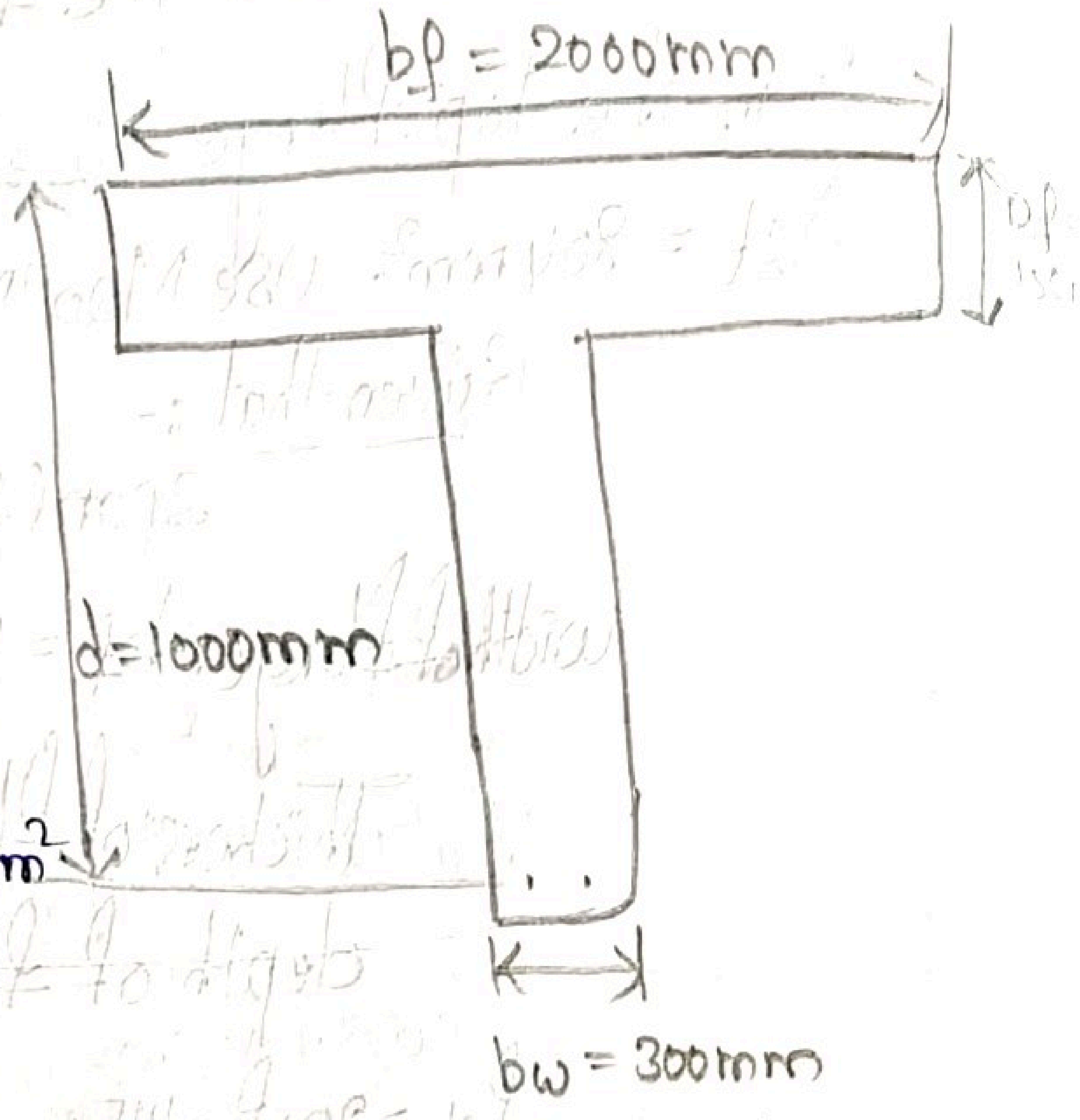
$= -463.73$

The given wrong section is adequated.

⑤) Determine the area of tension reinforcement for a T-Beam having the ~~sec~~following sectional dimensions effective width of the flange $b_f = 2000mm$, thickness of the flange ~~T~~$D_f = 150mm$, width of the web, $b_w = 300mm$, effective depth $d = 1000mm$. Use M_{20} grade of Concrete and Fe_{415} steel.

Sol:-

Given that:-



$b_f = 2000mm$

$D_f = 150mm$

$b_w = 300mm$

$d = 1000mm.$

$f_{ck} = 20\,N/mm^2,\ f_y = 415\,N/mm^2.$

$x_u = x_{umax}.$

$x_u = 0.48(d)$

$= 0.48(1000)$

$$\boxed{x_u = 480mm.}$$

$f_{tc_{comp}} = 0.36 f_{ck} b_f D_f$

$f_{ts} = 0.87 f_y A_{st}$

$f_{tc} > f_{ts}$ (flange N.A)

$f_{tc} < f_{ts}$ (N.A lies in web)

~~[illegible]~~

~~[illegible]~~

$x_u > D_f.$

$$\frac{D_f}{d} = \frac{150}{1000} = 0.15$$

$$\boxed{\frac{D_f}{d} < 0.2}$$

[~~N.A lies in web portion~~].

Case ii, The N.A lies in web portion.

~~$x_u = 0.87 f_y A_{st} - 0.45 f_{ck}$~~

$$M_u = 0.36 f_{ck} \cdot b_w x_u \left[d - 0.42 x_u\right] + 0.45 f_{ck} (b_f - b_w) D_f \left[d - \frac{D_f}{2}\right].$$

$$= 0.36(20) \times 200 \times 480 \left[1000 - 0.42(480)\right] + 0.45(20)(2000 - 300) \times 150 \left[1000 - \frac{150}{2}\right.$$

$$M_u = 827.7 \times 10^6 + 2.12 \times 10^9$$

$$M_u = 2950.5 \times 10^6 \text{ N-mm}$$

$$M_u = 2950.5 \text{ KN-m}$$

$$A_{st} = A_{st\,web} + A_{st\,flange}.$$

$$A_{st} = \frac{0.36 f_{ck} \cdot b_w x_u}{0.87 f_y} + \frac{0.45 f_{ck} (b_f - b_w) \cdot D_f}{0.87 f_y}.$$

$$= \frac{0.36 \times 20 \times 300 \times 480}{0.87 \times 415} + \frac{0.45 \times 20 (2000-300) \times \overset{150}{2}}{0.87 \times 415}$$

$$= 2871.62 + \cancel{84752} \; 6356.46$$

$$\boxed{A_{st} = 9228.08 \text{ mm}^2.}$$

16mm

$$A_{st} = n \times \pi/4 (d^2)$$

$$9228.08 = n \times \pi/4 (16^2).$$

$$n = 45.8$$

$$n = 46.$$

$$A_{st} = 46 \times \pi/4 (16)^2$$

$$\boxed{A_{st} = 9248.84 \text{mm}^2}$$

7) A T-Beam floor Consists of 150mm thick R.C.C slab Monolithic with 300mm wide Beams. The beams are spaced at 3.5m center to center and their effective span is 6m. If the Super imposed load on ~~beam~~ slab is 5KN/m². Design an Isolated T-Beam. Use M_{20} grade and Fe_{415} steel.

Sol:- Given that

c/c distance beam, $l_0 = 3.5$m

$D = 150$mm.

$b = 300$mm.

l = Effective span = 6m.

$f_{ck} = 20$ N/mm², $f_y = 415$ N/mm²

①

~~$\frac{l}{12}$ to $l/15$.~~

~~$\frac{6000}{12}$ to $\frac{6000}{15}$~~

~~500 to 400.~~

~~d = 500.~~

$\frac{l}{12}$ to $l/15$

$\frac{6000}{12}$ to $\frac{6000}{15}$

500 to 400

d = 500

Assume, c = 50.

D = 550.

② Calculation of load:-

Dead load ⇒ b×D×unit wg of material
of slab
(b=1)

= 1×0.15×25

= 3.75 kN/m².

Live load = 5 kN/m².
slab.

5+3.75 * c/c

Liveload ~~[illegible]~~ on Beam = 8.75×3.5

= 30.625 ~~k~~N/m².

b × D × 25

Dead load = 0.3×0.55×25

= 4.125 kN/m².

floor finish = 0.6 kN/m².

Factored load = ~~0.6 kN/m~~².

= T.L×1.5.

T.L = L.L + D.L + F.F

= ~~8~~ 30.625 + 4.125 + 0.6

T.L = 35.35 kN/m²

Factored load = T.L×1.5 = 35.35×1.5

= 53.02 kN/m².

$$M_u = \frac{wl^2}{8}$$

$$M_u = \frac{53.02(6)^2}{8} = 238.5 \text{ kN-m}.$$

③ Calculation of depth:-

$$M_u = M_{u\,limit}$$

$$= 2.76\,bd^2$$

$$238.5 \times 10^6 = 2.76\,(0.3)(0.5)^2\; d^2$$

$$d = 536.69$$

$$A_{st} = A_{st\,web} + A_{st\,flange}$$

$$= \frac{0.36 f_{ck} \cdot b_w x_u}{0.87 f_y} + \frac{0.45 f_{ck}(b_f - b_w) \cdot D_f}{0.87 f_y}.$$

$x_u = x_{umax}$, $x_u = 0.48d = 0.48(500)$

$x_u = 240\text{mm}.$

$x_u > D_f$

$D_f = 150\text{mm}$

$b_w = 300\text{mm}.$

$\frac{150}{150} = 1 < 2$

$$b_f = \frac{l_0}{6} + b_w + 6D_f = \frac{\overset{\overset{1000}{\cancel{6000}}}{\cancel{3500}}}{\cancel{6}} + 300 + 6(150)$$

$\frac{D_f}{d} < 2$

~~$b_f = 1783.33$~~

$b_f = 2200\text{mm}.$

$$A_{st} = \frac{0.36 \times 20 \times 300 \times 240}{0.87 \times 415} + \frac{0.45 \times 20 \times (2200 - 300)150}{0.87 \times 415}$$

$$A_{st} = 1435.81 + 7104.27$$

$$\boxed{A_{st} = 8540.08\,\text{mm}^2}$$

Assume, $d = 16\text{mm}$ ~~As~~ $8540.08 = n \times \pi/4\,(16)^2$

$$n = 42.47$$

$$\boxed{n = 43.}$$

$$A_{st} = \pi/4 \times (43)(16)^2$$

$$\boxed{\underset{\text{Provided}}{A_{st}} = 8645.66\,\text{mm}^2}$$

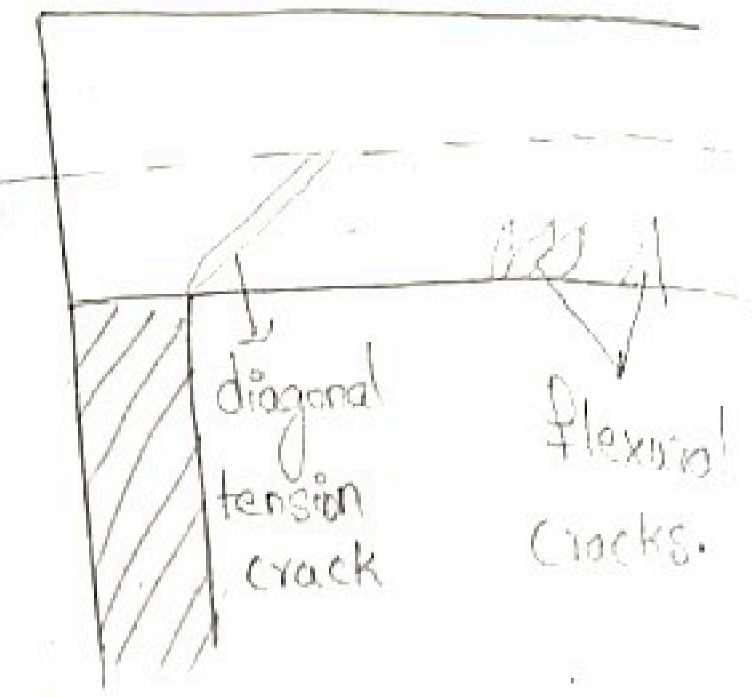
Unit-3

Design for Shear, Torsion and Bond

Flexure and shear in homogeneous beam:-

$$\tau = \frac{VA\bar{y}}{Ib}$$

Where $\tau_{max} = 3/2\, \tau_{avg}$



Elastic theory:-

Shear stress, $\tau = \frac{V}{bJd}$

$$\tau = \frac{V}{bZ} \qquad Z = Jd.$$

Diagonal tension and Compression:-

Principle stresses σ_1 (or) $\sigma_2 = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$

① If $B.M = 0$, $\sigma = 0$, $\sigma_1 = \tau$, $\sigma_2 = -\tau$, $\tan 2\theta = \infty$ and $\theta = 45°$ (or) $135°$.

② If $B.M = max$, $\tau = 0$, $\theta = 0$

Limit State:- $\tau_v = \frac{V_u}{bd}$ (uniform depth)

Beams of Varrying depth

$$\tau_v = \frac{V_u \pm \frac{M_u}{d}\tan\theta}{bd}$$

If $\tau_v < \tau_c$, minimum (or) No shear reinforcement provided.

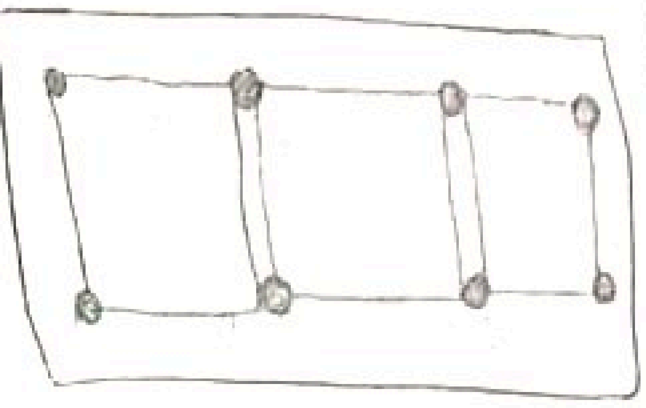
If $\tau_v > \tau_c$, design for shear reinforcement

Design for Shear Reinforcement:-

Shear reinforcement in beams can be provided as in the following forms.

a) Vertical stirrups.

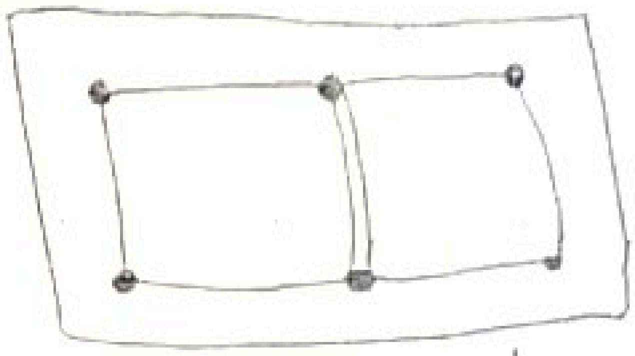
$$V_{us} = \frac{0.87 f_y A_{sv} d}{S_v}$$



Multilegged

1-legged

2-legged



4-legged.

b) Bent-up bars. → $V_{us} = 0.87 f_y A_{sv} \sin\alpha$.

c) Inclined Stirrups. → $V_{us} = \frac{0.87 f_y A_{sv} d}{S_v} (\sin\alpha + \cos\alpha)$.

procedure for design for Shear Reinforcement:-

1) Calculate the factored (or) ultimate shear force (V_u)

2) Calculate the Nominal Shear Stress $\tau_v = \frac{V_u}{bd}$.

3) Calculate the shear strength of Concrete 'τ_c', based on the percentage of Tension reinforcement and grade of Concrete. (Refer Table No. 19 of IS 456:2000).

4) Calculate $\tau_{c\,max}$ (refer 20 Table No, ~~of~~ IS 456:2000)

5) Check $\tau_v < \tau_c$ (or) $\tau_v > \tau_c$

6) Calculate the shear to be resisted by shear reinforcement $V_{us} = V_u - \tau_c \cdot bd$

7) Calculate the area of vertical stirrups.

8) Calculate the Spacing of vertical stirrups.

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$ (or) $S_v = 300mm$ (or) $S_v = 0.75d$

(or)

$S_v = 0.87 f_y A_{sv} / 0.4 b$. which one is less.

1) The ultimate shear force at a Section of reinforced Concrete beam is 200kN. The effective depth is 500mm and width is 250mm and one percent of Steel is provided. what is the shear for which the Shear reinforcement is required. use M_{20} grade of Concrete and Fe415 Steel.

$V_u = 200kN.$

Sol:- Given that:- $f_{ck} = 20N/mm^2.$

$f_y = 415 N/mm^2.$

$d = 500mm$

$b = 250mm.$

$P_t = 1\%.$

$$\tau_v = \frac{V_u}{bd}$$

$$= \frac{200\times10^3}{500\times250}$$

$$\boxed{\tau_v = 1.6 \ N/mm^2}$$

Table 19 IS 456.

for 1% & M_{20} = 0.62 τ_c value taken.

$$\boxed{\tau_c = 0.62N/mm^2}$$

$$\boxed{\tau_v > \tau_c}$$

Provide Shear reinforcement.

$$V_{us} = V_u - \tau_c \cdot bd$$

$$V_{us} = 200\times10^3 - 0.62\times250\times500$$

$$V_{us} = 122.5\times10^3$$

$$\boxed{V_{us} = 122.5 \ kN}$$

2) Determine the Spacing of 8mm dia 2legged vertical stirrups R.C.C. Beam of width 230mm and effective depth 450mm to resists the factored Shear force of 85kN. use M_{20} grade of Concrete and Fe415 Steel.

$d = 8mm$

2legged. $2-8mm\phi$.

$b = 230mm$

$d = 450mm$

factored shear force $V_{us} = 85KN$

$$A_{sv} = n(\pi/4 d^2) = 2\times\pi/4(8)^2$$

$$A_{sv} = 100.52mm^2.$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87\times415\times100.53\times450}{85\times10^3}$$

$$= 192.15mm$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87\times415\times100.53}{0.4(230)}$$

$$= 394.52mm$$

$$S_v = 0.75d = 0.75\times450 = 337.5mm$$

$$S_v = 300mm$$

S_v is taken which one is less

$$S_v = 192.15mm.$$

3) A R.C.C Beam width 230mm overall depth 450mm is reinforced with 4 bars of 16mm diameter and Fe415 on tension side. If design Shear force is 60KN. Design shear reinforcement is consisting of only shear reinforcement (only vertical stirrups). Use M20 grade of Concrete. Assume effective Cover, e = 50mm

Sol:- Given that:- $b = 230mm$

$D = 450mm$

$n = 4 - 16mm\phi$.

$f_{ck} = 20N/mm^2$.

$f_y = 415N/mm^2$.

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$V_u = 60\times10^3 N.$, $e = 50mm$, $D = d + e.$

$d = D - e = 450 - 50$

$d = 400mm.$

$$\tau_v = \frac{V_u}{bd}$$

$$\tau_v = \frac{60\times10^3}{230\times400}$$

$$\boxed{\tau_v = 0.65 N/mm^2.}$$

$$A_{st} = n\times\pi/4(d^2)$$

$$= 4\times\pi/4(16)^2$$

$$\boxed{A_{st} = 804.24mm^2}$$

Percentage of steel, $P_t = \frac{100\times A_{st}}{bd}$

$$P_t = \frac{100\times804.24}{230\times400}$$

$$\boxed{P_t = 0.87\%}$$

interpolation (Table No 19. Pg No-73)

0.75 0.56

0.87

1 0.62

$$0.62 + \left[\frac{0.62-0.56}{1-0.75}\right]\times(0.87-1)$$

$\tau_c = 0.588 N/mm^2.$

$$\boxed{\tau_c = 0.59 N/mm^2}$$

$$\boxed{\tau_v > \tau_c.}$$

we have

$V_{us} = V_u - \tau_c.bd$ → code book formula. IS456:2000 pg No-73

$$= 60\times10^3 - 0.588\times230\times400$$

$$= 5.72\times10^3$$

$$\boxed{V_{us} = 5.72 KN}$$

$$A_{sv} = n \times \pi/4 (d^2)$$

Choose → 2legged 6ϕ.

$$A_{sv} = 2 \times \pi/4 (6)^2$$

$$A_{sv} = 56.54 mm^2$$

$$\leftarrow S_v = \frac{0.87 f_y A_{sv} \cdot d}{V_{us}} = \frac{0.87 \times 415 \times 56.54 \times 400}{5.72 \times 10^3} = 1427.53 mm$$

Pg No. 73
IS456:2000 code

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 415 \times 56.54}{0.4 \times 230} = 221.88 mm$$

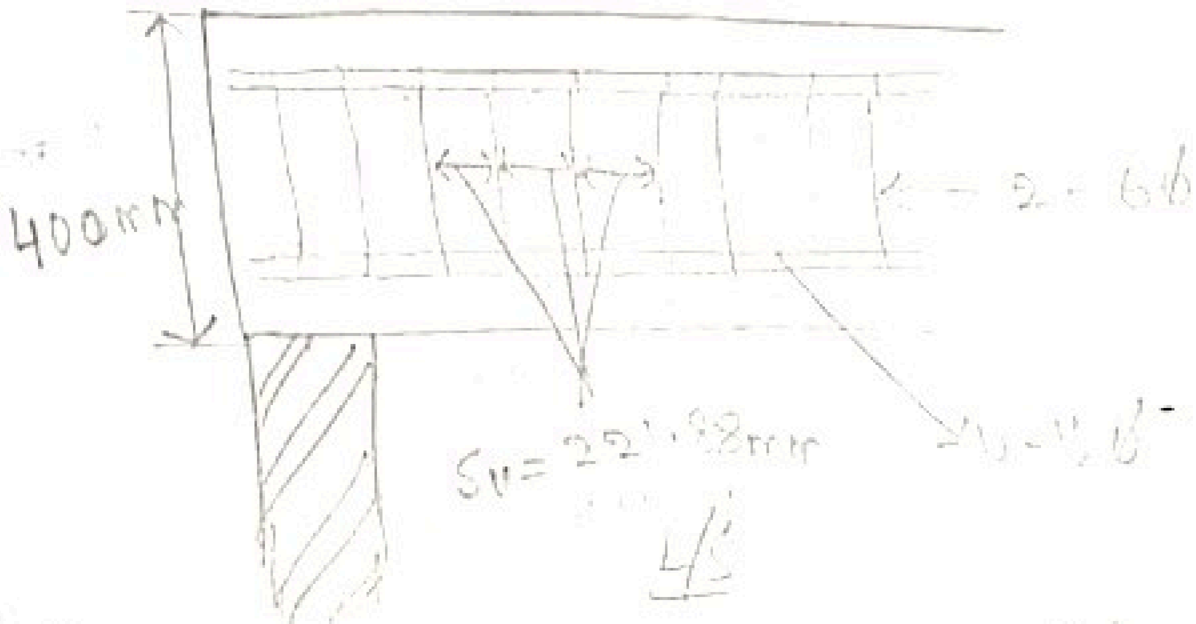
$$S_v = 0.75d = 0.75 \times 400 = 300mm$$

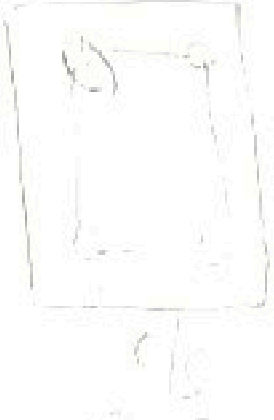
$$S_v = 300mm = 200mm$$

S_v is taken which one is less.

~~$S_v = 300mm.$~~

$$\boxed{S_v = 221.88 mm.}$$





4) A rectangular Beam 300mm X600mm, effective depth carries a UDL of 40kN/m including its self weight over a simply supported span of 6m and is reinforced with 5bars of 25mm diameter of which 2bars are cranked up near the support. Use M20 grade of Concrete and Fe415 steel. Design the beams for shear reinforcement at the support.

Sol:-

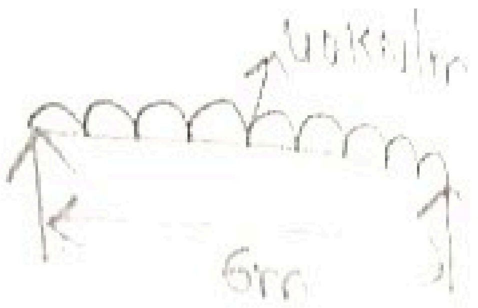
$b = 300mm$

$D = 600mm$

U.D.L = 40kN/m.

Simply Supported span, $l = 6m$.

5 - 25mm dia



~~$V_u = \frac{Wl}{2}$~~

~~$= \frac{40 \times 6 \times 10^3}{2} = 120 \times 10^3 kN$~~

~~$V_u = 120 \times 10^3 kN.$~~

~~$\tau_v = \frac{V_u}{bd} = \frac{120 \times 10^3}{300 \times 600}$~~

~~$\tau_v = 0.66 N/mm^2.$~~

factored load $= 40 \times 1.5$

$= 60 kN/m.$

$V_u = \frac{wl}{2} = \frac{60 \times 6}{2} = 180 kN.$

$\boxed{\tau_v = \frac{V_u}{bd}}$ → Code book IS456:2000 clause 40 pg No-72

$\tau_v = \frac{180 \times 10^3}{300 \times 600}$

$\tau_v = 1 N/mm^2.$

$\tau_c = ?$ → pg No-73, IS456:2000 Table 19.

$A_{st} = n \times \pi/4 (d^2)$

$= 5 \times \pi/4 (25)^2$

$A_{st} = 2454.36 mm^2.$

$P_t = \frac{100 A_{st}}{bd} = \frac{100 \times 2454.36}{300 \times 600}$

$P_t = 1.36 \%$

1.25   0.67

1.36

1.50   0.72

$$0.72+\left(\frac{0.72-0.67}{1.50-1.25}\right)\times(1.36-1.50)$$

$$\boxed{\tau_c = 0.69\ N/mm^2}$$

$$\boxed{\tau_v > \tau_c}$$

$$\left.\begin{array}{l}\boxed{V_{us_2} = 0.87 f_y A_{sv} \sin\alpha.}\\ \boxed{V_{us_1} = V_u - \tau_c bd}\end{array}\right\} \rightarrow \text{IS 456:2000 Code book pg No-73.}$$

$$= 180\times10^3 - 0.69\times300\times600$$

$$\boxed{V_{us_1} = 55.8KN.}$$

$$V_{us_2} = 0.87 f_y A_{sv} \sin\alpha.$$

Choose 2-legged 6ϕ.

$$A_{sv} = 2\times\pi/4(6)^2$$

$$\boxed{A_{sv} = 56.54mm^2}$$

$$V_{us_2} = 0.87\times415\times56.54\times\sin(45°)$$

$$\boxed{V_{us_2} = 14.43\times10^3 N.}$$

$$V_{us} = V_{us_1} + V_{us_2}$$

$$= 55.8\times10^3 + 14.43\times10^3$$

$$\boxed{V_{us} = 70.23KN}$$

Code book formula

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87\times415\times56.54\times600}{70.23\times10^3}$$

$$= 174.40\ mm$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87\times415\times56.54}{0.4\times300}$$

$$= 170.11\ mm$$

$$S_v = 0.75d = 0.75\times600 = 450mm$$

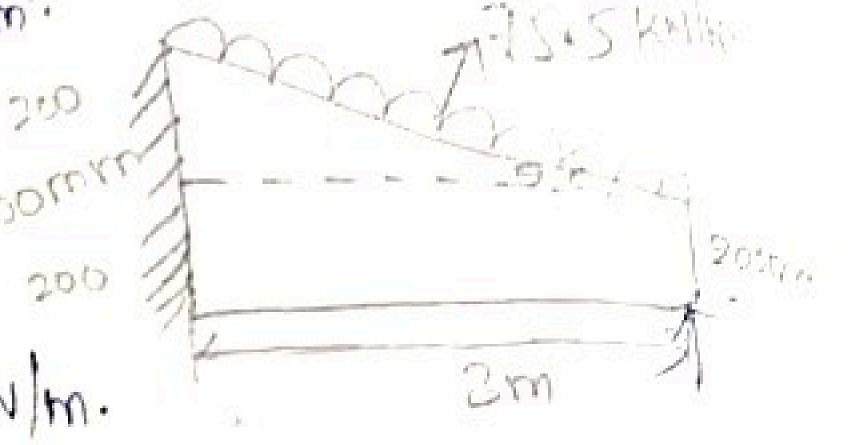
$$S_v = 300mm = 300mm.$$

S_v taken which one is less $\boxed{S_v = 170.11mm}$

5.) In a Cantilever Beam with span 3m has an effective depth 400mm at the support and 200mm at the free end and a constant width of 250mm. It carries a load of 75KN/m including its self weight. It is reinforced with 4 bars of 20mm diameter. Use M_{20} grade of concrete and Fe_{415} Steel. Design the Shear Reinforcement.

Sol:- Given that:- $f_{ck} = 20 N/mm^2$

$f_y = 415 N/mm^2$



load = 75KN/m.

factored load = 75×1.5

$W = 112.5 KN/m$.

$V_u = w \times l$

$= 112.5 \times 3$

$= 337.5 KN$

$M_u = \frac{wl}{2} = \frac{337.5}{2}$

$M_u = 168.75 KN\text{-}m$

$\tau_v = \frac{V_u}{bd} \approx$ ~~337.5~~

$\tan\theta = \frac{200mm}{3m} = \frac{200}{3000} = 0.066$

$\tan\theta = 0.066$.

b = 250mm = 0.25m

d = 400mm = 0.4m

Code book clause 40.1.1 IS 456:2000 Pg No-72 ←

$$\tau_v = \frac{V_u \pm \frac{M_u}{d} \tan\theta}{bd}$$

$$\tau_v = \frac{337.5 - \frac{168.75}{0.4}(0.066)}{0.25 \times 0.4}$$

$\tau_v = 3096.56 KN/m^2$

~~$\tau_v =$~~

$$\tau_v = \frac{337.5 \times 10^3 - \frac{168.75 \times 10^6}{400} \times 0.066}{250 \times 400}$$

Scanned with CamScanner

$\tau_v = 3.09\ N/mm^2$.

Area of steel, 4-20mm φ

$A_{st} = n \times \pi/4 (d^2)$

$= \cancel{4} \times \pi/\cancel{4} (20)^2$

$A_{st} = 1256.63 mm^2$

$p_t = \frac{100 \times A_{st}}{bd} = \frac{100 \times 1256.63}{250 \times 400}$

$p_t = 1.25\%$.

(pg No - 73)

$\therefore \boxed{\tau_c = 0.67}\ N/mm^2 \rightarrow$ Table 19 :- IS 456:2000

$\boxed{\tau_v > \tau_c}$

$V_{us} = V_u - \tau_c b d$

$= 337.5 \times 10^3 - 0.67 \times 250 \times 400$

$V_{us} = 270.5 \times 10^3 N$

$V_{us} = 270.5 KN.$

choose 2-legged 10mm dia.

$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 415 \times 157.07 \times 400}{270.5 \times 10^3} = 83.85$ mm

$A_{sv} = 2 \times \pi/4 (10)^2$

$= 157.07 mm^2$ $S_v = \frac{0.87 f_y A_{sv}}{0.4 b} = \frac{0.87 \times 415 \times 157.07}{0.4 \times 250} = 567.10$ mm

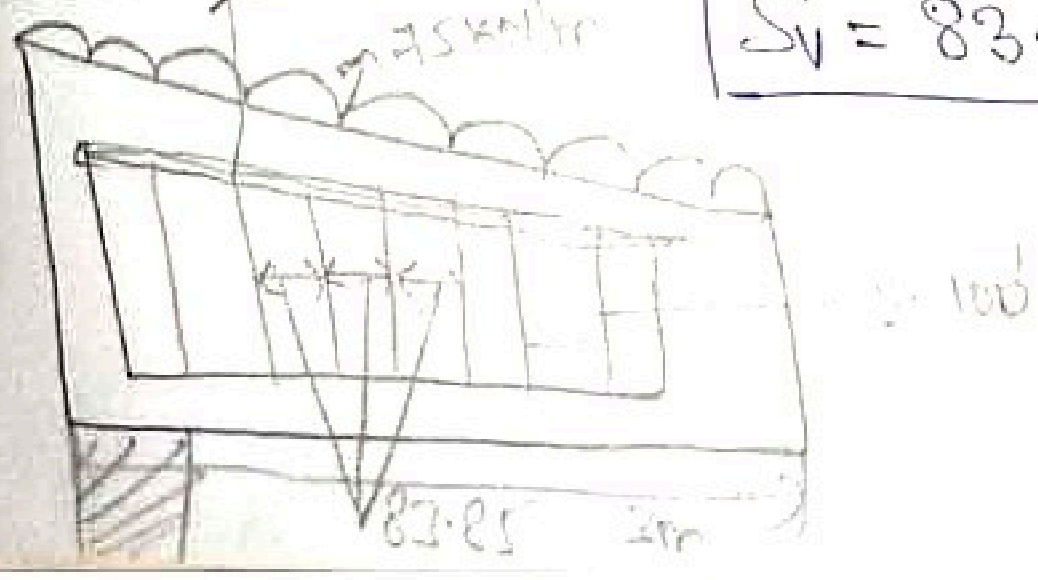
$S_v = 0.75 d = 0.75 (400) = 300mm$

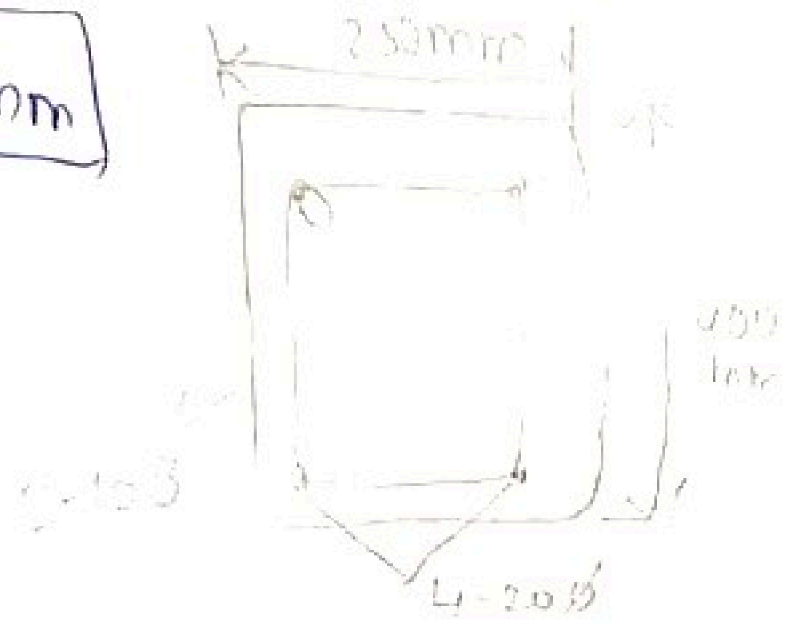
$S_v = 300mm$

S_v taken which one is less

$\boxed{S_v = 83.85mm}$

Reinforcement details





adhesion b/w Concrete & steel which resist the sliping of steel bar from the Concrete.

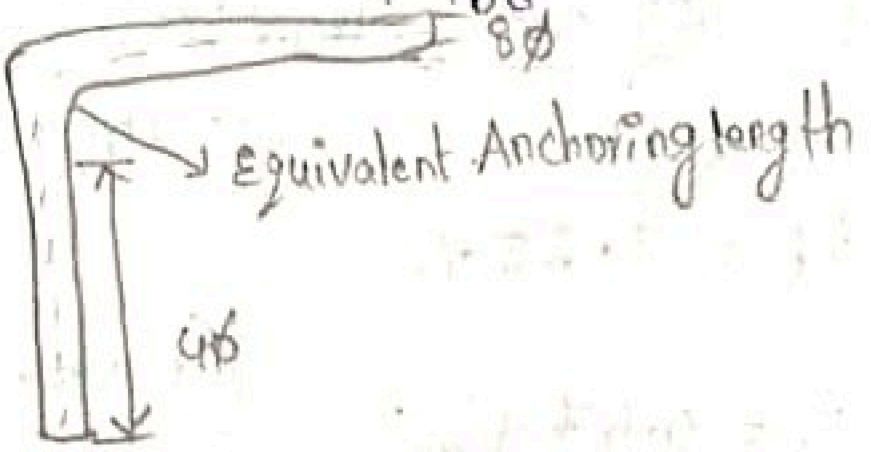
Bond:- Bond is also called development length.

Column-Beam joint

Design for Bond:-

development length.

Bond length $L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$



τ_{bd} = Design bond stress.

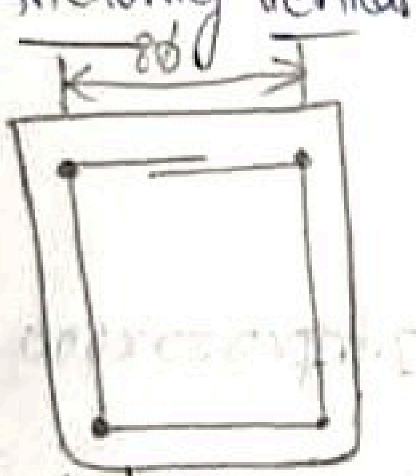
Table 26.2.1

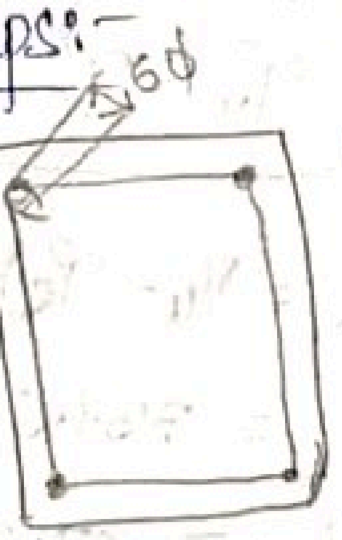
Pg No-43.

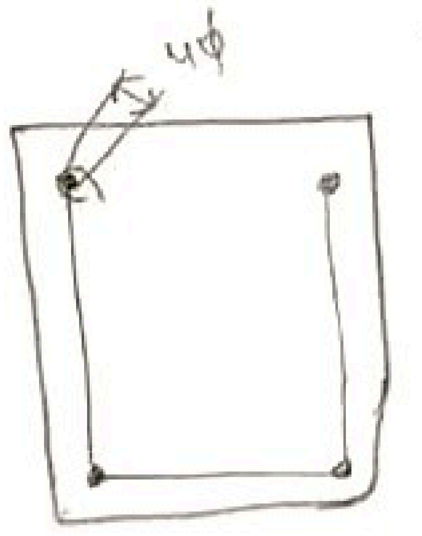
deformed → 60% increased → 100+60
160%
= 1.6

For bars in Compression, the value of bond stress for bars in tension shall be increased 25%
= 100+25 = 125%
= 1.25

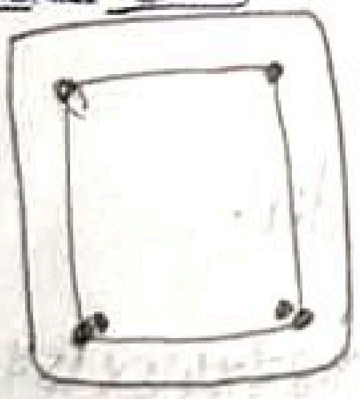
Anchoring Vertical stirrups:-



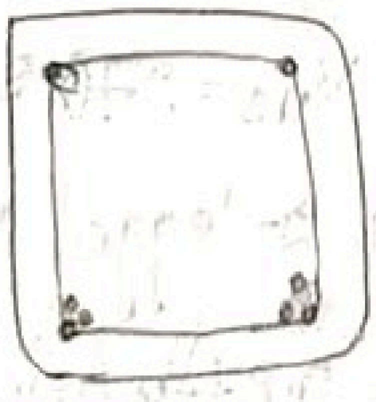




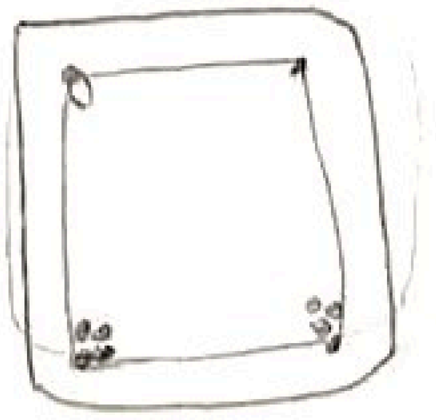
Bonded Bars:-



2 bars in Contact

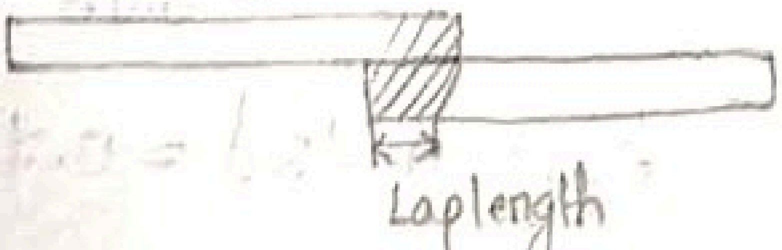


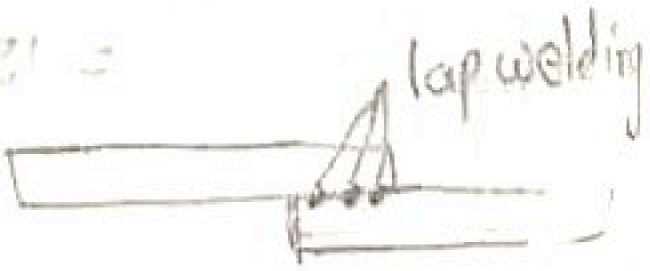
3 bars in Contact

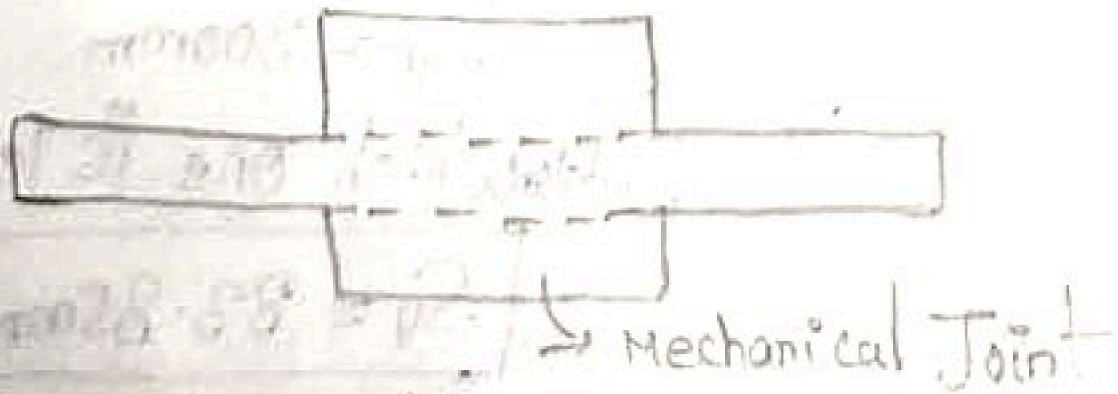


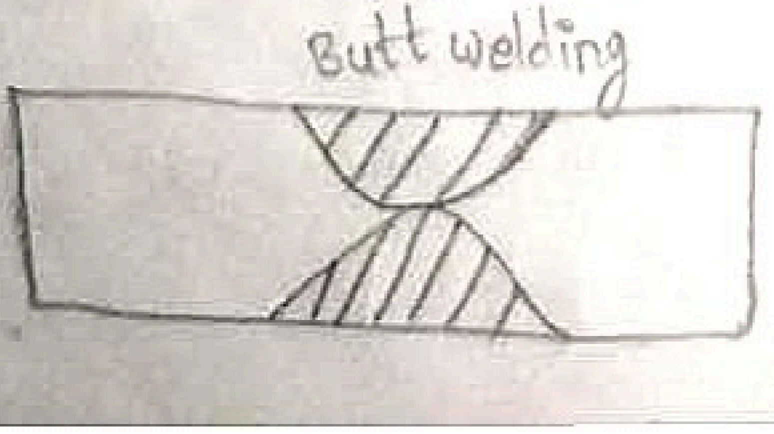
4-bars in Contact

Splicing:-









1) Determine the development length of 20mm, Fe415 in tension and Compression. Use M20 grade of Concrete.

Sol:- Development length, $L_d = \dfrac{0.87 f_y \phi}{4 \tau_{bd}}$.

$f_{ck} = 20 N/mm^2$, $f_y = 415 N/mm^2$.

Tension (M20) and Fe415 table No - 26.2.1, Pg No-43.

$\tau_{bd} \Rightarrow 1.2$

$= 1.2 \times 1.6$

$= 1.92$

$$L_d = \frac{0.87 \times 415 \times (20)}{4 \times (1.92)} = 940.23 mm.$$

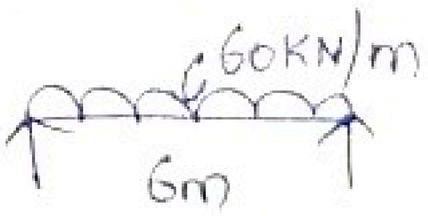
When bar in Compression.

$$\tau_{bd} = 1.92 \times 1.25 = 2.4 N/mm^2.$$

$$L_d = \frac{0.87 \times 415 \times 20}{4 \times (2.4)} = 752.18 mm.$$

2) A simply supported beam of span 6m and carries a u.d.l of 60kN/m if 6 bars of 20mm diameter are provided at the center of span and 4 no. of these bars are continued in to the support. Check the development length at the support. Use M20 grade of Concrete and Fe415 steel.

Sol:- Given that Span $l = 6m$



6 - 20mm ϕ.

$f_{ck} = 20 N/mm^2$.

$f_y = 415 N/mm^2$.

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$\tau_{bd} = 1.2 \times 1.6$

$= 1.2 \times 1.6$

$= 1.92 N/mm^2$.

$$L_d = \frac{0.87 (415)(20)}{4 \times (1.92)}$$

$L_d = 940.23 mm$.

IS456:2000
Pg No-44

$$\boxed{\frac{M_1}{V} + L_0}$$

factored load = 60×1.5
= 90kN/m

$$M = \frac{wl}{2} = \frac{90 \times 10^3 \times 6}{2} = 27[illegible]0 KN$$

$$M_1 = \frac{wl^2}{8} = \frac{90\times10^3\times6^2}{8} = 405\text{kN-m}.$$

$$\frac{M_1}{V} + L_0$$

$$= \frac{405\times10^3}{270\times10^3} + 6$$

$$= 1.5+6$$

$$= 7.5\text{m}$$

$$= 7500\text{mm}.$$

$$L_d \ngtr \frac{M_1}{V} + L_0.$$

$$\underset{mm}{940.23} \ngtr 7500\text{mm}$$

Check ok.

~~L~~3) Design a regtangular simply Supported R.C.C Beam, over a clear Span of 6m if the live load is 12 kN/m and the Support width is 230mm. use M_{20} grade of Concrete and Fe415 Steel. The beam has a width of 300mm. Design the shear reinforcement do the necessary checks.

Sol:- Given that $l = 6\text{m}$, Support width $b = 230\text{mm}$, $f_{ck} = 20\text{N/mm}^2$.

Live Load = 12KN/m

$f_y = 415\text{N/mm}^2$.

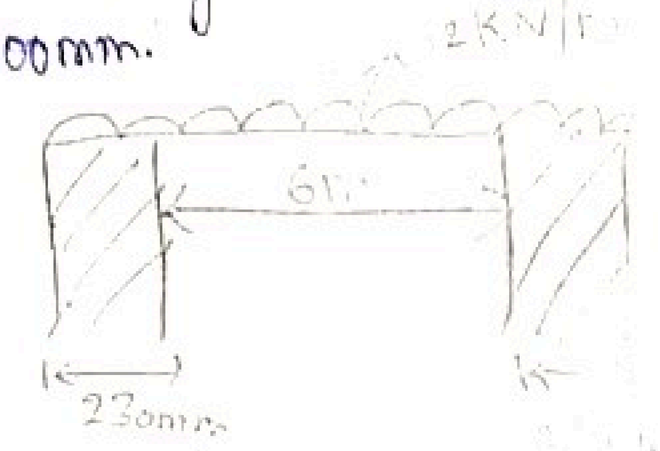
① Calculation of effective depth:-

Beam width $b = 300\text{mm}$.

$\frac{l}{12}$ to $l/15$.

$$\frac{6000}{12} \text{ to } \frac{6000}{15}$$

500 to 400.



$d = 500\text{mm}$

Assume, $c = 50\text{mm}$

$D = 550\text{mm}$

$B = b\times D$

$B = 230\times550$

$B = 126.5\text{m}.$

② Effective Span :-

SSB = Clear Span + effective depth

= 6000 + 500

SSB = 6500mm

SSB = c/c to distance.

b = 230 mm

$= \frac{230}{2} + 6000 + \frac{220}{2}$

$= 6230$ mm.

Which one is less.

Effective span = 6230 mm = 6.23 m

③ Calculation of load:–

b = 300 mm

Dead load = (b×D) × unit wt of material

$= (300 \times 550)\, 25$

$= (0.30 \times 0.55) \times 25$

$= 4.12$ kN/m

Live load = 12 kN/m

floor finish = 0.6 kN/m.

T.L = L.L + F.F + D.L = ~~3.16~~ 4.125 + 12 + 0.6

= 16.725 kN/m

factored load = T.L × 1.5 = 16.725 × 1.5

= 25.08 kN/m.

Shear force

$V = \frac{wl}{2}$

$= \frac{25.08(6.23)}{2}$

V = 78.12 KN.

$M_u = \frac{wl^2}{8} = \frac{25.08(6.23)^2}{8}$

$M_u =$ ~~114.69 kN-m~~ 121.67 kN-m

4) Check for depth:–

$M_u = M_{u\,limit}$.

$M_u = 2.76\, b d^2$

$121.67 \times 10^6 = 2.76 \times (300)(d^2)$

d = 383.33 mm.

d = 383.33 mm < 500 mm

Check is satisfied.

[Assumed should be greater]

⑤ Calculation of Ast:–

$M_u = 0.87 f_y A_{st} d \left[1 - \frac{f_y A_{st}}{f_{ck} \cdot b d}\right]$

$121.67 \times 10^6 = 0.87 \times 415 \times A_{st}(500)\left[1 - \frac{415 \times A_{st}}{20 \times 300 \times 500}\right]$

$$121.67 \times 10^6 = 180525 A_{st} - 24.97 A_{st}^2$$

$$24.97 A_{st}^2 - 180525 A_{st} + 121.67 \times 10^6 = 0.$$

$$\boxed{A_{st} = 752.25 \text{ mm}^2}$$ less value

$$\boxed{A_{st} = 6477.42 \text{ mm}^2.}$$

$d = 16$mm

5)

$$752.25 = n \times \pi/4 (16)^2$$

$$n = ~~4.75~~\ 3.74$$

$$\boxed{n = 4 \text{ bars}}$$

$$A_{st} = \pi/4 (n) d^2$$

$$A_{st} = \pi/4 (4)(16)^2$$

$$\boxed{A_{st} = 804.24 \text{ mm}^2}$$

6) check for stiffness:-

$$l/d = 20$$

$$\% A_{st} \Rightarrow P_t = \frac{100 A_{st}}{bd}$$

$$P_t = \frac{100 \times 804.24}{300 \times 500}$$

$$P_t = 0.53\%$$

$$f_s = 0.58 f_y \times \frac{\text{area of c/s of steel required}}{\text{area of c/s of steel provided}}$$

$$= 0.58 \times 415 \times \frac{752.25}{804.24}$$

$$f_s = 225.13 \text{ N/mm}^2$$

$$l/d = 20 \times 1.5 = 30$$

$$l/d = \frac{6000}{500} = 12$$

$$12 < 30$$

Hence ok.

7) Check for shear reinforcement:-

$$\tau_v = \frac{V_u}{bd}$$

$$V_u = \frac{wl}{2} = \frac{25.08(6.23)}{2}$$

$$V_u = 78.12KN.$$

$$\tau_v = \frac{78.12\times10^3}{300\times500}$$

$$\tau_v = 0.52\ N/mm^2.$$

M20, Fe415

$$\tau_c\ P_t = \frac{100\times A_{st}}{bd} = \frac{100\times 804.24}{300\times500}$$

$$P_t = 0.53\%$$

0.50	0.48
0.53	
0.75	0.56

$$0.56 + \left[\frac{0.56-0.48}{0.75-0.50}\right]\times(0.53-0.75)$$

$$\tau_c = 0.48\ N/mm^2$$

$$\boxed{\tau_v > \tau_c}$$

$$V_{us} = V_u - \tau_c\cdot bd$$

$$= 78.12\times10^3 - 0.48\times300\times500$$

$$V_{us} = 6120$$

$$V_{us} = 6.12\times10^3 N$$

$$\boxed{V_{us} = 6.12KN}$$

Choose 2legged 6φ

$$A_{sv} = 2\times\pi/4(6)^2$$

A_{sv} = ~~402.12mm²~~ $56.54mm^2$.

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}} = \frac{0.87 \times 415 \times (56.54)(500)}{\cancel{78.12 \times 10^3}\ 6.12 \times 10^3} = \cancel{130.65\text{mm}}\ 1667.79\text{mm}$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87 \times 415 \times 56.54}{0.4 \times 300} = 170.11\text{mm}$$

$$S_v = 0.75d = 0.75(500) = 375\text{mm}$$

$$S_v = 300\text{mm} = 300\text{mm}.$$

less value.

~~$S_v = 130.65$mm~~

$$\boxed{S_v = 170.11\text{mm}}$$

provide 2-6ϕ at 170.11mm c/c.

8) Check for Ld :-

$$L_d = \frac{0.87 f_y \phi}{4 \boxed{\tau_{bd}}} \rightarrow$$ M20 pg NO-43) Table 26.2.1.1

~~ϕ = 16mm dia~~ ~~M20~~

$$L_d \not> \frac{M_1}{V} + L_0.$$

$$\tau_{bd} = 1.2 \times 1.6$$
$$= 1.92\ \text{N/mm}^2.$$

ϕ = 16mm dia.

$$L_d = \frac{0.87 \times 415 \times 16}{4 \times 1.92}$$

~~Ld = 940 752.18mm 282.07mm~~

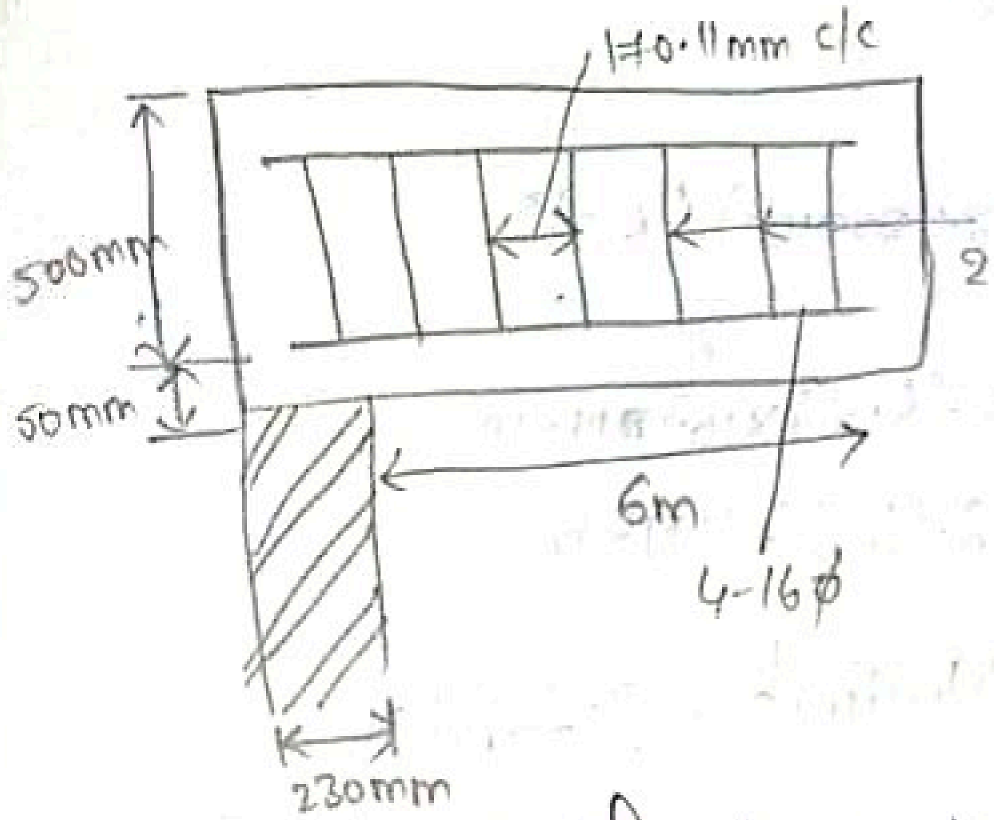
$$L_d = 752.18\text{mm}$$

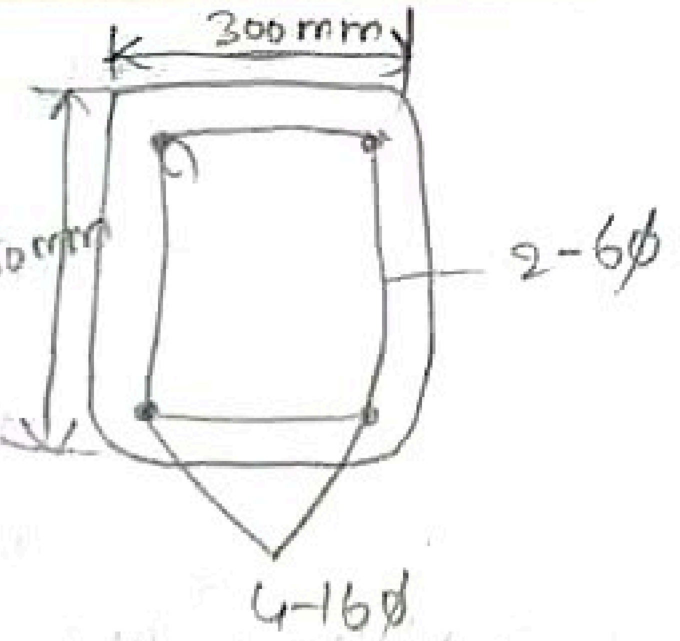
$$L_d \not> \frac{M_1}{V} + L_0$$

$$L_d \not> \frac{121.66 \times 10^6}{78.12 \times 10^3} + (6.23)$$

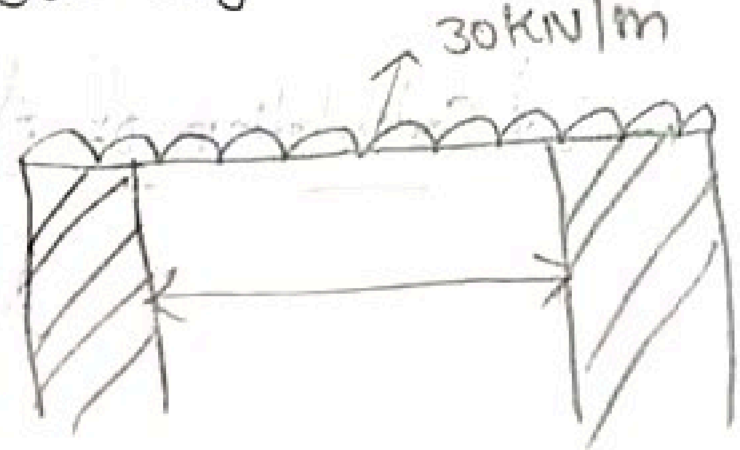
$$L_d \not> 1563.57\text{mm}$$

$$752.18\text{mm} \not> 1563.57\text{mm}$$





4.) Design a rectangular reinforced Concrete Beam carries a UDL of 30kN/m excluding its Self weight over a effective span of 6·5m. The over all Size of Beam 300mm×580mm are ~~restricted~~ Doubly. Design a mid span section of Beam for flexure. check the shear reinforcement and check for stiffness. Take effective Cover = 40mm. Use M_{20} grade of Concrete and Fe_{415} Steel.



Live load = 30kN/m

Sol:- Given that :- effective span = 6·5m.

width, b = 300mm

D = 580mm

e = 40mm

d = D−e = 580−40 = 540mm.

f_{ck} = 20N/mm², f_y = 415 N/mm².

① Calculation of loads:-

D.L = b×D×25

= 300×580×25

= 4·35kN/m

Live load = 30kN/m

floor ~~fish factored~~ finish load = 0·6kN/m.

Total load = D.L + L.L + F.L = 4·35+30+0·6

= 34·95kN/m

Factored load = T.L×1·5

= 34·95×1·5

= 52·425kN/m.

$$M_u = \frac{wl^2}{8}$$

$$M_u = \frac{(52.425\times10^3)(6.5)^2}{8}$$

$$M_u = 276.86\times10^3 \text{ N-m}$$

$$M_u = 276.86 \text{ KN-m.}$$

2) Calculation of Mulimit :- ~~Mu~~ $Mulimit = 2.76bd^2.$

$$Mulimit = 2.76(300)(540)^2$$

$$= 241.44\times10^6 \text{ N-mm}$$

$$Mulimit = 241.44 \text{KN-m.}$$

$$M_u > Mulimit \text{ (doubly)}.$$

3.) Calculation of Area of steel :-

$$Ast_1 = \frac{Mulimit}{0.87 fy(d-0.42x_{umax})}$$

$$x_{umax} = 0.48(d) = 0.48(540) = 259.2\text{mm.}$$

$$Ast_1 = \frac{241.44\times10^6}{0.87\times415[540-0.42(259.2)]}$$

$$Ast_1 = 1551.05\text{mm}^2.$$

④ Cal of Ast_2 :- $Ast_2 = \frac{Mu_2}{0.87fy(d-d')}$

$$Mu_2 = Mu. - Mulimit.$$

$$= 276.86\times10^6 - 241.44\times10^6$$

$$= 35.42\times10^6$$

$$Ast_2 = \frac{35.42\times10^6}{0.87\times415[540-40)}$$

$$Ast_2 = 196.20\text{mm}^2.$$

⑤ Cal of Asc:- $Asc = \frac{Mu_2}{fsc(d-d')}$.

$fsc \rightarrow$ d'/d ratio.

$\frac{40}{540}$

$= 0.074$

$0.05 \quad 355$

0.07

$0.1 \quad 353$

$$353 + \left[\frac{353-355}{0.1-0.05}\right](0.07-0.1)$$

⑤ Ca.

$$\boxed{fsc = 354.2 \text{ N/mm}^2}$$

$$Asc = \frac{35.42 \times 10^6}{354.2(540-50)}$$

$Asc = 200 N/mm^2$.

$Ast = Ast_1 + Ast_2 = 1551.05 + 196.20$

$Ast = 1747.25 mm^2$.

choose 20 ϕ bars.

$1747.25 = n \times \pi/4 (20)^2$

$n = 5.56$

$n = 6$ bars.

$Ast_{prov} = 6 \times \pi/4 (20)^2$

$Ast_{prov} = 1884.95 mm^2$.

6.) Check for Stiffness:-

$l/d = 20$

$\frac{6500}{540} = 12$

$12 < 22$

Hence ok.

$\% P_t = \frac{100 Ast}{bd} = \frac{100 \times 1884.95}{300 \times 500}$

$= 1.16\%$.

$F_s = 0.58 \times 415 \times \frac{1747.25}{1884.95}$

$F_s = 223.11$

$M.F = 1.1$

$l/d = 20 \times 1.1 = 22$

7) Check for Shear Reinforcement:-

$$\tau_v = \frac{V_u}{bd}$$

$$V_u = \frac{wl}{2} = \frac{(52.425 \times 10^3)(6.5)}{2} = 170.38 kN$$

Scanned with CamScanner

$$\tau_v = \frac{170.38 \times 10^3}{300 \times 540}$$

$$\tau_v = 1.05\,N/mm^2.$$

M_{20}, Fe_{415}.

$$p_t = \frac{100\,A_{st}}{bd}$$

$$p_t = \frac{100 \times 1884.95}{300 \times 540}$$

$$p_t = 1.16\,\%.$$

1	0.62
1.16	
1.25	0.67

$$0.67 + \left(\frac{0.67 - 0.62}{1.25 - 1}\right) \times (1.16 - 1.25)$$

$$\tau_c = 0.652\,N/mm^2.$$

$$\tau_v > \tau_c$$

provide shear reinforcement.

$$V_{us} = V_u - \tau_c \cdot bd$$

$$= 170.38 \times 10^3 - 0.652 \times 300 \times 540$$

$$V_{us} = 64.756\,KN$$

Choose 2legged 8 ϕ.

$$A_{sv} = 2 \times \pi/4 (8)^2$$

$$A_{sv} = \cancel{56.54\,mm^2}.\ 100.53\,mm^2.$$

$$S_v = \frac{0.87 f_y A_{sv} \cdot d}{V_{us}} = \frac{0.87 \times 415 \times 100.53 \times 540}{64.756 \cancel{170.38} \times 10^3} = \cancel{115.03\,mm}\ 302.67\,mm$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4 b} = \frac{0.87 \times 415 \times 100.53}{0.4 \times 300} = 302.46\,mm$$

$$S_v = 0.75d = 0.75(540) = 405\,mm$$

$$S_v = 300 = 300\,mm$$

$$\boxed{S_v = 300\,mm}$$

provide - 2-6ϕ at 300mm c/c.

8.) Check for Ld:-

$$Ld = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

bar in Tension

$$Ld \ngtr \frac{M_1}{V} + L_0$$

$$Ld = \frac{0.87\times415\times20}{4\times1.92}$$

$\tau_{bd} = 1.2\times1.6$

$= 1.92 N/mm^2$

$\phi = 20mm$

$$Ld = \frac{0.87\times415\times20}{7.68}$$

$$\boxed{Ld = 940.23mm}$$

bar in Compresion.

$$Ld = \frac{0.87 f_y \phi}{4 \tau_{bd}}$$

$\tau_{bd} = 1.92\times1.25$

$\tau_{bd} = 2.4 N/mm^2$

$$= \frac{0.87\times415\times20}{4\times2.4}$$

$$\boxed{Ld = 752.98mm.}$$

$$Ld \ngtr \frac{M_1}{V} + L_0$$

~~Tension~~

$$= \frac{276.86\times10^6}{170.38\times10^3} + 6500$$

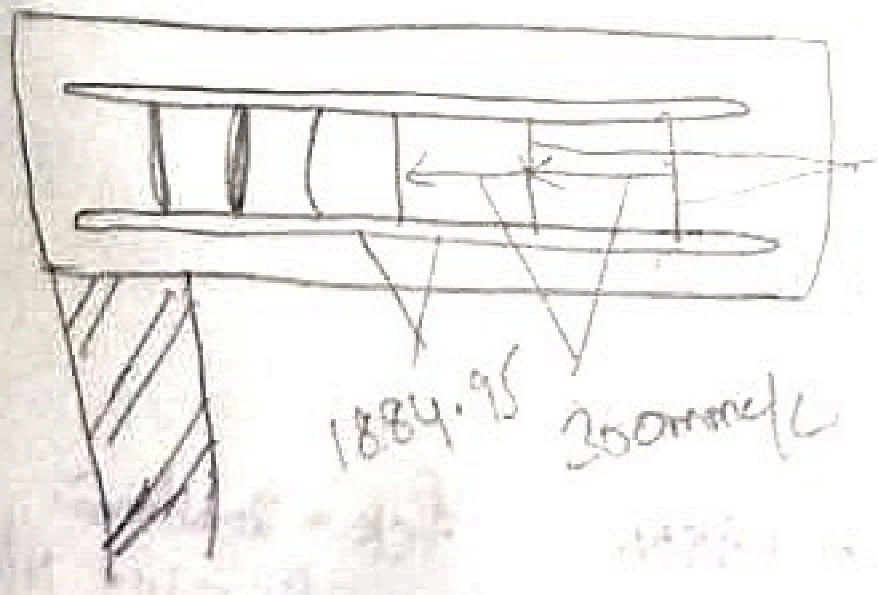
$$\boxed{\cancel{Ld} = 8124.95mm.}$$

$$Ld \ngtr \frac{M_1}{V} + L_0.$$

~~8124.95~~ for Tension & Compression zone.

$$Ld \ngtr \frac{M_1}{V} + L_0$$

Check ok.



Torsion:-

Equivalent shear,

$$V_e = V_u + 1.6 \frac{T_u}{b}$$

V_e = Equivalent shear.
V_u = Shear
T_u = torsional moment
b = width of beam.

$$\tau_{ve} = \frac{V_e}{bd} \qquad \tau_{ve} \not> \tau_c.$$

Where τ_{ve} = Equivalent nominal shear stress.

If $\tau_{ve} < \tau_c$ provide minimum reinforcement.

If $\tau_{ve} > \tau_c$ design for torsional reinforcement. (Both Longitudinal & transverse).

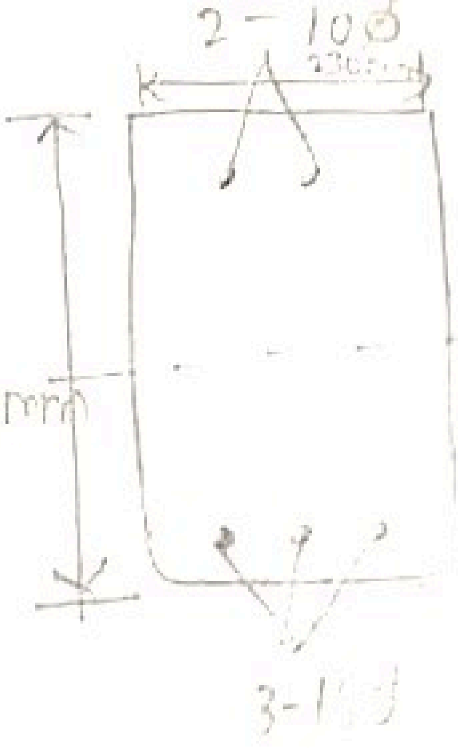
Longitudinal Reinforcement:- The longitudinal Reinforcement should be design to resist an equivalent moment M_{e1}.

$$M_{e1} = M_u + M_t.$$

M_u = Bending moment at cross-section.

$$M_t = T_u \left[\frac{1 + D/b}{1.7} \right].$$

1.) A rectangular Beam Section of size 230mmx400mm overall depth is reinforced with 3 bars of 16mm ϕ of bars at the bottom being tension zone and two bars of 10mm ϕ at top. It is subjected to characteristic loads, shear force of 18KN and torsinal moment of 1.2KN-m and a B.M of 18KNm. check for the torsion reinforcement. Use M_{20} grade of Concrete and Fe_{415} steel.



Assume effective Cover = 40mm = e.

$D = 400mm$
$b = 230mm$

$D - e = d$
$d = 400 - 40$
$d = 360mm.$

torsion, 3 bars 16ϕ

$T_u = 1.2 KN\text{-}m$
$V_u = 18KN.$

$f_{ck} = 20N/mm^2$
$f_y = 415\ N/mm^2$

$$V_e = V_u + 1.6\frac{T_u}{b}$$

$$= 18\times10^3 + 1.6\left[\frac{1.2\times10^6}{230}\right]$$

$$V_e = 26.34\times10^3\ N$$

$$\boxed{V_e = 26.34\ kN.}$$

$$\tau_{ve} = \frac{V_e}{bd} = \frac{26.34\times10^3}{230\times360}$$

$$\boxed{\tau_{ve} = 0.318\ N/mm^2.}$$

M20 & Pt steel.

$$P_t = \frac{100A_{st}}{bd} \quad \frac{100\times 3\times\pi/4(16)^2}{230\times360}$$

$$P_t = 0.72\%$$

0.50 0.48

0.72

0.75 0.56

$$0.56 + \left[\frac{0.56-0.48}{0.75-0.50}\right]\times\left[0.72-0.75\right]$$

$$\boxed{\tau_c = 0.55\ N/mm^2.}$$

$$\boxed{\tau_{ve} < \tau_c.}$$

provide minimum Reinforcement.

choose 2-6mmϕ bars.

$$A_{sv} = n\pi/4(d^2)$$

$$= 2\times\pi/4(6)^2$$

$$= 56.54\ mm^2.$$

$$V_{us} = V_u - \tau_c \cdot bd$$

$$= 18\times10^3 - 0.55\times230\times360$$

~~V_{us} =~~

$$S_v = \frac{0.87 f_y A_{sv}\cdot d}{V_{us}} =$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4b} = \frac{0.87\times415\times56.54}{0.4\times230} = 221.88\ mm$$

$$S_v = 0.75d = 0.75(360) = 225\ mm$$

$$S_v = 300\ mm$$

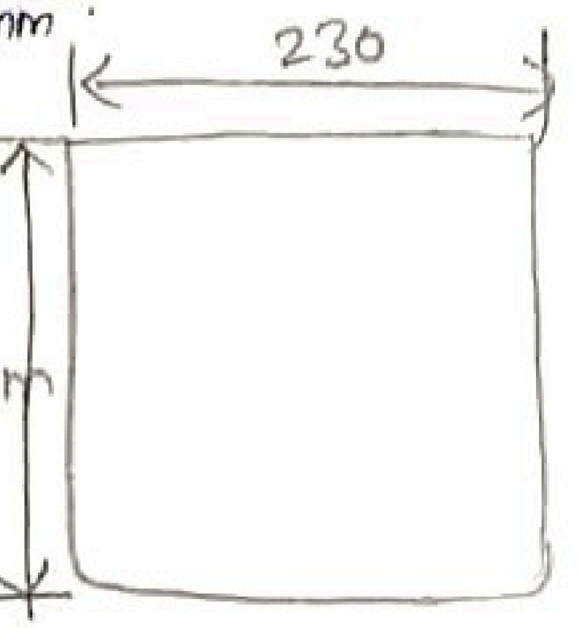
$$\boxed{S_v = 221.88\ mm}$$

provide 2-6mm∅ @ 221.88mm c/c.

8) A rectangular Beam Section of 230mm x 600mm overall depth is Subjected to a factored B.M of 48KN-m, Factored Shear force of 48kN, factored torsional moment of 18KN-m. Design the reinforcement at the Section. Use M_{20} grade of Concrete and Fe_{415} steel.

Given that $f_{ck}=20 N/mm^2$, $f_y=415 N/mm^2$.

Sol:-



$b = 230mm$

$D = 600mm$

$T_u = 18KN\text{-}m.$

$V_u = 48KN.$

M_u, $B.M = 48KN\text{-}m.$

$e = 40mm$

$D - e = d$

$d = 600 - 40$

$d = 560mm.$

$$\tau_{ve} = \frac{V_e}{bd}$$

$$V_e = V_u + 1.6\left[\frac{T_u}{b}\right].$$

$$= 48\times10^3 + 1.6\left(\frac{18\times10^6}{230}\right)$$

$$\boxed{V_e = 173.21KN}$$

$$\tau_{ve} = \frac{173.21\times10^3}{230\times560}$$

$$\boxed{\tau_{ve} = 1.34\ N/mm^2}$$

$$P_t = \frac{100\,A_{st}}{bd}$$

$$M_u = 0.87 f_y A_{st}\, d\left[1 - \frac{f_y \cdot A_{st}}{f_{ck}\cdot bd}\right]$$

M_u

$$48\times10^6 = 0.87\times415\times A_{st}\times560\left[1 - \frac{415\times A_{st}}{20\times230\times560}\right]$$

CASIO Eq1 degree2 Ast less value

$$48\times10^6 = 202188 A_{st} - 32.57 A_{st}^2$$

$$32.57 A_{st}^2 - 202188 A_{st} + 48\times10^6 = 0.$$

$$\boxed{A_{st} = 247.25 mm^2}$$ less value

$$P_t = \frac{100 \times 247.25}{230 \times 560}$$

$$P_t = 0.19\,\%$$

Assume 20 ϕ bars.

$$A_{st} = n \times \pi/4\,(20)^2$$

$$247.25 = n \times \pi/4\,(20)^2$$

$$n = 0.78$$

Assume 16ϕ bars.

$$A_{st} = n \times \pi/4\,(d^2)$$

$$247.25 = n \times \pi/4\,(16)^2$$

$$n' = 1.23$$

$$\boxed{n = 2 \text{ bars}}$$

$$A_{st} = 2 \times \pi/4\,(16)^2$$

$$\boxed{A_{st} = 402.12\,mm^2}$$

$$\%\,p_t = \frac{100 \times A_{st}}{bd} = \frac{100 \times 402.12}{230 \times 560} = 0.312\,\%$$

0.25 0.36

0.31

0.50 0.48

$$0.48 + \left[\frac{0.48 - 0.36}{0.50 - 0.25}\right] \times (0.31 - 0.50)$$

$$\tau_c = 0.38\,N/mm^2$$

$\boxed{\tau_{ve} > \tau_c}$ design for torsional reinforcement.

Longitudinal reinforcement :-

$$M_{e1} = M_u + M_t.$$

$$M_t = T_u\left(\frac{1 + D/b}{1.7}\right) = 18 \times 10^6\left(\frac{1 + \frac{600}{230}}{1.7}\right)$$

$$M_t = 38.2 \times 10^6\ N\text{-}mm.$$

$$M_t = 38.2\ KN\text{-}m$$

$Me_1 = M_u + M_t$

$= 48\times10^6 + 38.2\times10^6$

$\boxed{Me_1 = 86.2 \text{ kN.m}}$

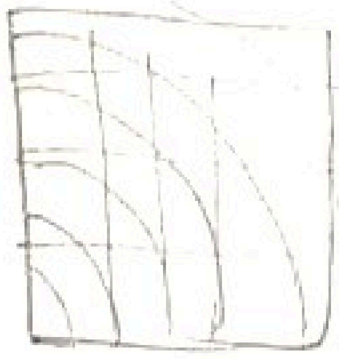
from SP-16.

$\frac{Me_1}{bd^2} = \frac{86.2\times10^6}{230\times(560)^2} = 1.195$

$= 1.12$

X $\quad \% P_t = \frac{100 A_{st}}{bd} = 0.359$

Pt



Me_1/bd^2

from SP-16

$\% P_t = 0.359$

other wise

$A_{st} = ?$ $\quad M_u = 0.87 f_y A_{st} d(1-\ldots)$

$\% P_t = \frac{100\times A_{st}}{230\times 560}$

$\boxed{A_{st} = 462.39 \text{ mm}^2}$

$86.2\times10^6 = 0.87\times415\times A_{st} \times 560\left[1 - \frac{415\times A_{st}}{20\times230\times560}\right]$

$32.57 A_{st}^2 - 202183 A_{st} + 86.2\times10^6 = 0$

$\boxed{A_{st} = 462.39 \text{ mm}^2}$

Assume, 20ϕ bars. $\quad P_t = \frac{100\times462.39}{230\times560}$

$\boxed{P_t = 0.357}$

$462.39 = n\times\pi/4(20)^2$

$n = 2$ bars.

$A_{st} = 2\times\pi/4(20)^2$

$\boxed{A_{st} = 628.31 \text{ mm}^2}$

When the depth of Beam > 450mm. side face reinforcement Should be provided as per Clau No - 26.5.1.3.

~~560 >450mm.~~

Pg No - 48

26.5.1.7

560 > 450mm

↓

Pg No - 47

26.5.1.3.

✓

$\nless \frac{0.1}{100}$ (c/c area)

for two faces $\nless \frac{0.1}{100}\left(\frac{b\times d}{230\times560}\right)$

$\nless 128.8$mm

for each face $= \frac{1}{2}(128.8)$

$= 64$mm.

Spacing of side face reinforcement.

Transverse Reinforcement:- (73 Pg No IS 456).

Assume 2legged and 8ϕ bars.

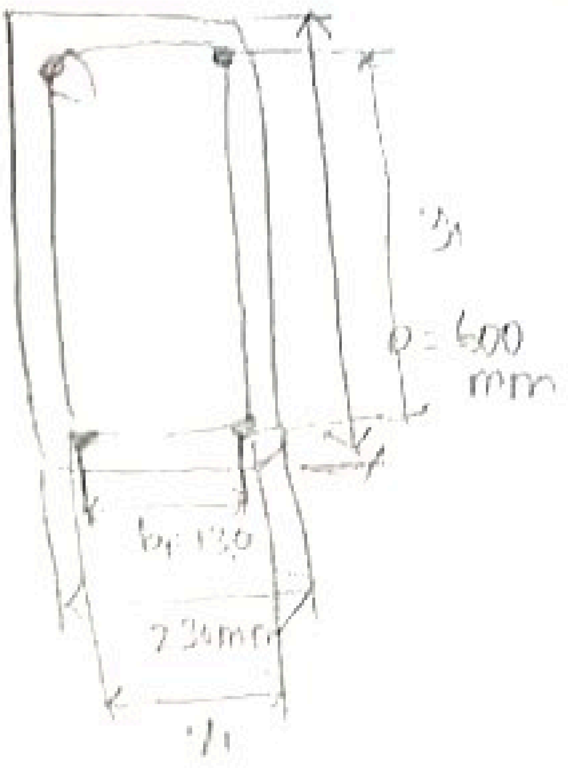
$A_{sv} = n\times\pi/4(d^2) = 2\times\pi/4(8)^2$

$A_{sv} = 100.53 \text{ mm}^2$

$b_1 = 230 - e - e - d/2 - d/2$ $\quad d = 20$

$= 230-40-40-10-10$

$b_1 = 130mm$



$d_1 = D - e - e - d/2 - d/2$

$= 600-40-40-10-10$

$d_1 = 600 - 100$

$d_1 = 500mm.$

$$A_{sv} = \frac{T_u S_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u S_v}{2.5 d_1 (0.87 f_y)} \rightarrow Pg\ NO\ 75$$

~~$= 18 \times 10^6 x.$~~

~~$S_v = \frac{0.87 f_y A_{sv}}{0.4 b} = \frac{0.87 \times 415 \times 100.53}{0.4 \times 230} = 394.52mm$~~

~~$S_v = 0.87 f_y \cdot 0.75 d = 0.75 \times 560 = 172.5mm.$~~

~~$S_v = 300mm$~~

$$100.53 = \frac{18 \times 10^6 \times S_v}{130 \times 500 (0.87 \times 415)} + \frac{48 \times 10^3 \times S_v}{2.5(500)(0.87 \times 415)}$$

$100.53 = 0.76 S_v + 0.106 S_v$

$100.53 = 0.866 S_v$

$\boxed{S_v = 116.08mm}$

$x_1 = 230 - e - e = 230 - 80 = 150mm$

$y_1 = 600 - e - e = 600 - 80 = 520mm$

$x_1 = 150mm, \quad \frac{x_1 + y_1}{4} = \frac{150 + 520}{4} = 167.5mm.$

$\boxed{S_v \ngtr x_1}$

Pg No-48
26.5.1.7
Clause
IS456:2000

∴ provide 2-8mm φ @ 116.08mm.

CHAPTER

3

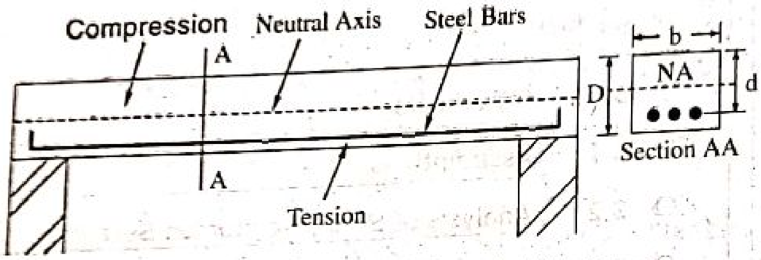
ANALYSIS AND DESIGN OF RECTANGULAR BEAMS

Chapter Outline

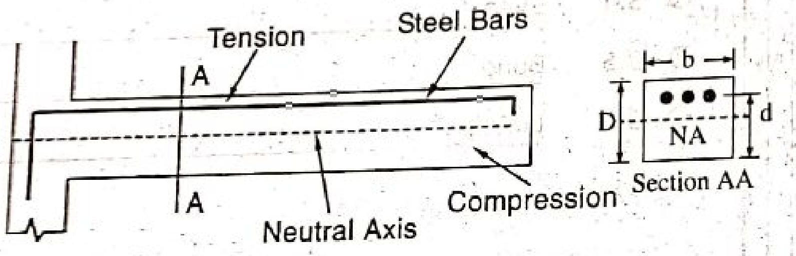
3.0 INTRODUCTION

Concrete is fairly strong in compression but very weak in tension. Hence Plain concrete can not be used in situations where considerable tensile stresses develop. If flexural members like beams and slabs are made of plain concrete their load carrying capacity is very low due to its low tensile strength. Since steel is very strong in tension, steel bars are provided to resist tensile stresses at a place where the maximum tensile stresses are developed.

In case of simply supported beam, tensile stresses are induced in bottom layers because of positive bending moment (sagging bending moment) and hence steel bars are provided near the bottom of the beam. In cantilever beams steel bars are placed near the top of the beam to resist the tensile stresses developed in top layers due the negative bending moment (hogging bending moment) as shown in the Fig.3.1.



(a) Reinforcement in Simply Supported Beam



(b) Reinforcement in Cantilever Beam

FIG 3.1 :

3.1 ASSUMPTIONS

The analysis and design of a reinforced concrete section for flexure is based on the following assumptions. (IS : 456 - 2000, Clause 38.1)

(i) Plane sections normal to the axis remains plane after bending.

(ii) The maximum strain in concrete at the outermost compression fiber is taken as 0.0035 in bending regardless of strength of concrete.

(iii) The tensile strength of concrete is ignored.

(iv) The relationship between stress-strain distribution in concrete is assumed to be parabolic as shown in Fig.3.2. Compressive strength of concrete in the structure (size effect) is assumed to be 0.67 times the characteristic strength of concrete.

The partial safety factor γ_m equal to 1.5 is applied to the strength of concrete in addition to it. Therefore, the design compressive strength of concrete is $0.67\ f_{ck}/1.5 = 0.446\ f_{ck}$.

(v) The stress in reinforcement is derived from the representative stress-strain curve for the type of steel used as shown in the Fig. 3.3. The partial safety factor γ_m equal to 1.15 is applied to the strength of reinforcement. Therefore, the design strength of steel is $f_y/1.15 = 0.87\ f_y$.

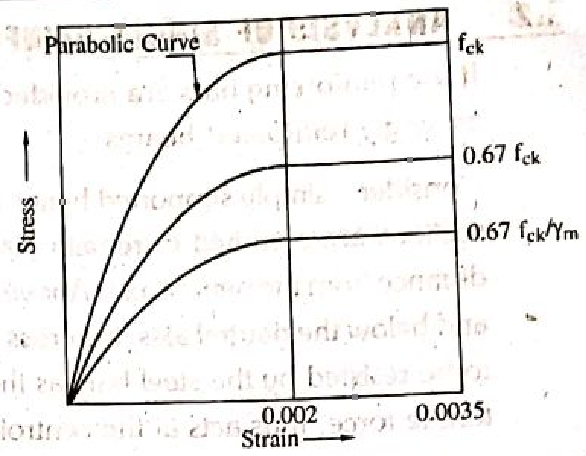


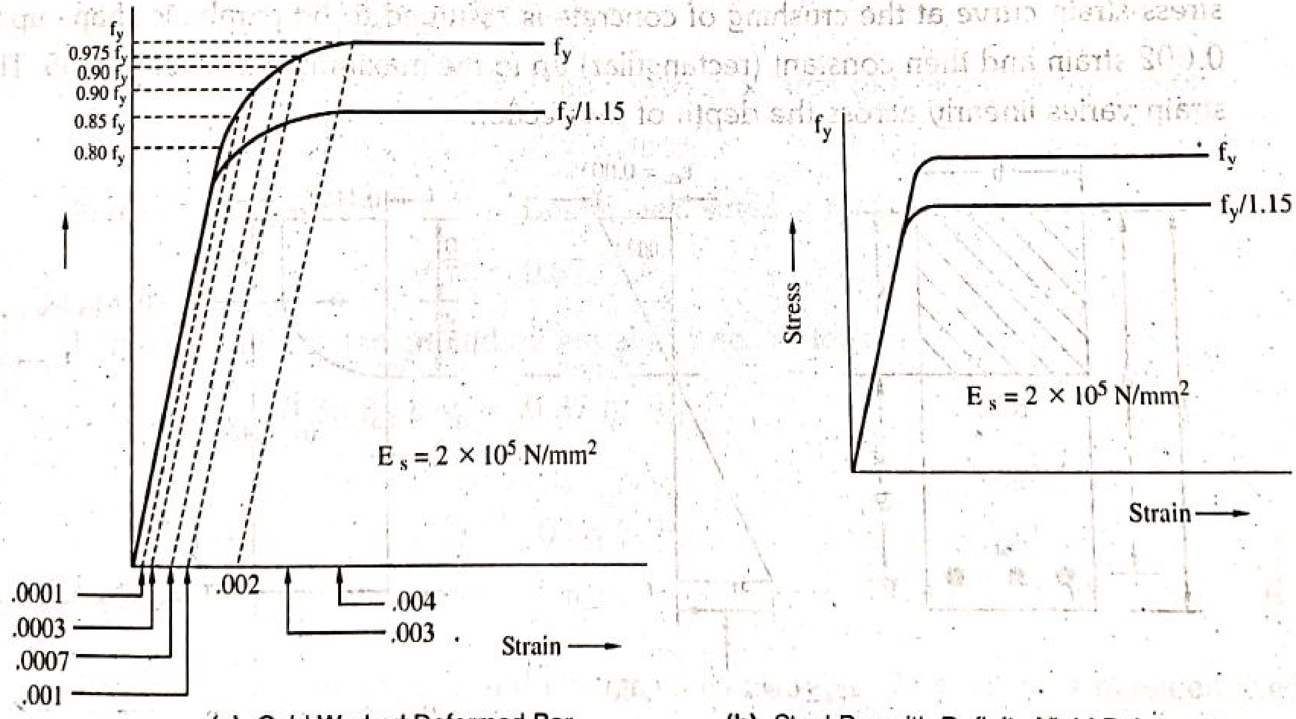
FIG 3.2 : Stress-Strain Curve for Concrete

(vi) The maximum strain in tension reinforcement in the section at failure should not be less than the

$$\frac{f_y}{1.15\ E_s} + 0.002$$

Where f_y = Characteristic strength of steel

E_s = Modulus of elasticity of steel



(a) Cold Worked Deformed Bar

(b) Steel Bar with Definite Yield Point

FIG 3.3 : Representative Stress-Strain Curves for Reinforcement

3.2 ANALYSIS OF SINGLY REINFORCED SECTIONS

If the reinforcing bars are provided only on tension side in the beam section, it is called as singly reinforced beams.

Consider a simply supported beam subjected to bending under factored loads. Since plane sections are assumed to remain plane before and after bending, strain is proportional to distance from the neutral axis. Above the neutral axis the entire cross section is in compression and below the neutral axis, the cross section is in tension. All the tensile stresses are assumed to be resisted by the steel bars as the tensile strength of concrete is ignored. The resultant tensile force, thus acts at the centroid of reinforcing bars.

3.2.1 EFFECTIVE DEPTH

Effective depth of a beam is the distance between the centroid of tension reinforcement and the maximum compression fibre, excluding the thickness of finishing material not placed monolithically with the member.

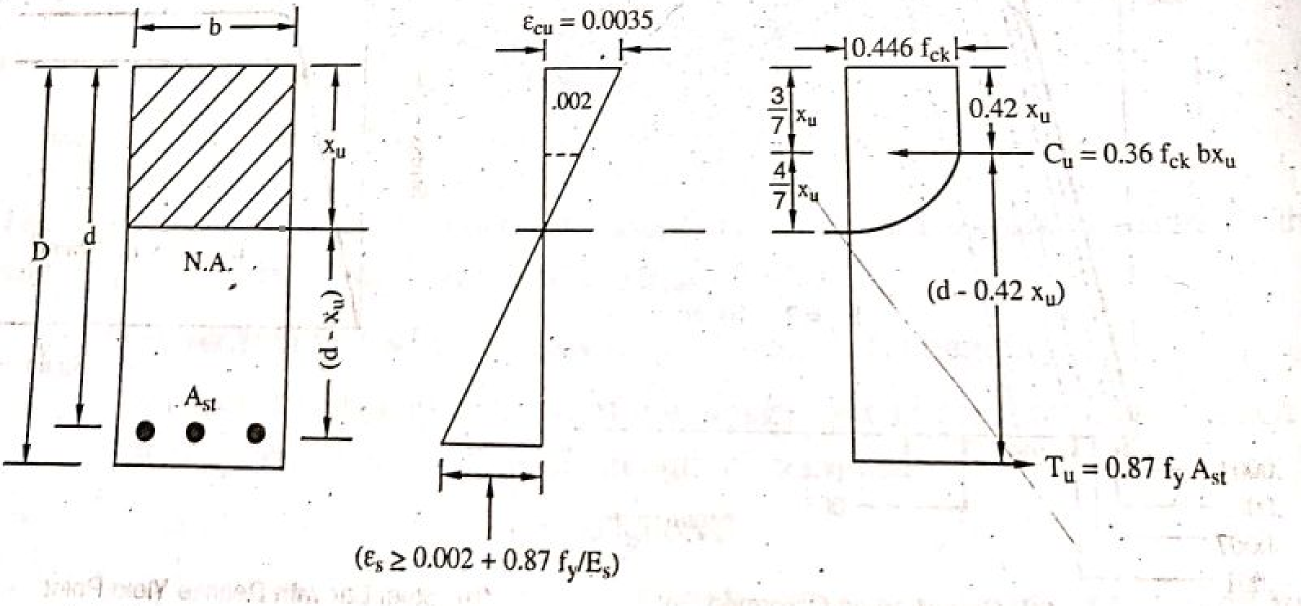
Effective depth, $d = D - \text{clear cover} - \phi/2$

Where D = Gross depth or overall depth

ϕ = Diameter of the bar

3.2.2 STRESS BLOCK PARAMETERS

Fig. 3.4 shows stress block and strain diagram for a singly reinforced section. The stress-strain curve at the crushing of concrete is assumed to be parabolic shape up to 0.002 strain and then constant (rectangular) up to the maximum strain of 0.0035. The strain varies linearly across the depth of the section.



(a) Cross section (b) Strain diagram (c) Stress diagram

FIG 3.4 : Stress Strain Diagram for Singly Reinforced Rectangular Section

The depth of parabolic part of the stress block $= \left(\frac{0.002}{0.0035}\right) x_u = \frac{4}{7} x_u$

The depth of rectangular part $= x_u - \frac{4}{7} x_u = \frac{3}{7} x_u$

Area of the stress block = area of rectangular portion + area of parabolic portion

$$= \frac{3}{7} \times x_u \times 0.446\, f_{ck} + \frac{2}{3} \times \frac{4}{7} x_u \times 0.446\, f_{ck}$$

$$= 0.36\, f_{ck}\, x_u$$

The distance of centroid of stress block from the top fiber

$$x_c = \frac{\left(\frac{3}{7} x_u \times 0.446\, f_{ck}\right)\left(\frac{1}{2} \times \frac{3}{7} x_u\right) + \left(\frac{2}{3} \times \frac{4}{7} x_u \times 0.466\, f_{ck}\right)\left(\frac{3}{8} \times \frac{4}{7} x_u + \frac{3}{7} x_u\right)}{0.36 f_{ck}\, x_u}$$

$$x_c = 0.42\, x_u$$

Hence, the total compressive force in concrete is $0.36\, f_{ck}\, b\, x_u$ and it acts at a distance $0.42\, x_u$ from the top fiber.

3.2.3 DEPTH OF NEUTRAL AXIS (x_u)

The depth of neutral axis can be obtained by considering the equilibrium of internal forces of compression and tension.

Force of compression C = Average stress × area of beam in compression

$= 0.36\, f_{ck}\, b\, x_u$

Force of tension T = Design yield stress × area of steel

$= 0.87\, f_y\, A_{st}$

Force of compression should be equal to force of tension

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y \cdot A_{st}$$

$$x_u = \frac{0.87 f_y\, A_{st}}{0.36\, f_{ck}\, b}$$

3.2.4 LEVER ARM (Z)

The forces of compression and tension form a couple. The distance between the lines of action of Compression and Tension forces is called as lever arm.

Lever arm, $z = d - 0.42\, x_u$

3.2.5 MODES OF FAILURES / TYPE OF SECTIONS

A reinforced concrete member is considered to have failed when the strain in concrete in extreme compression fiber reaches its ultimate value equal to 0.0035.

1. **Balanced Section :** When the maximum strains in steel and concrete reach their maximum values simultaneously, the section is known as a balanced section. The percentage of steel provided for balanced section is called as limiting percentage of steel.

$$x_u = x_{u.max}$$

2. **Under Reinforced Section (Tension failure or Ductile Failure) :** When the amount of steel in a section is less than that required for a balanced section, the section is called as under reinforced section.

In under reinforced sections, the strain in concrete does not reach its maximum value while the strain in steel reaches its maximum value. The position of neutral axis will shift upwards to maintain equilibrium between force of compression and tension.

$$x_u < x_{u,max}$$

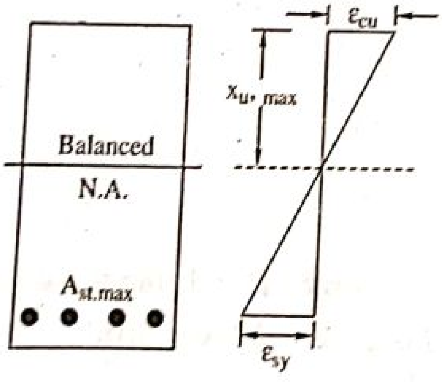
So failure of the section is initiated by steel reaching its yield value. Before failure, beam undergoes substantial deflection excessive cracking of concrete giving sufficient warning of impending failure. For this reason and from economy point of view the under reinforced sections are designed. IS code prefers design of under reinforced sections and at the most it can be a balanced section ($x_u \leq x_{u,max}$).

3. **Over Reinforced Section (Compression Failure or Brittle Failure) :** When the amount of steel in a section is more than that required for balanced section, the section is called over reinforced section.

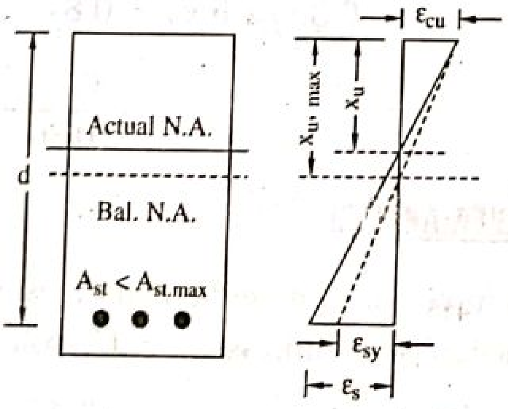
In over reinforced sections, the strain in concrete reaches its ultimate value before steel reaches its yield value. Neutral axis shift downwards to maintain equilibrium.

$$x_u > x_{u,max}$$

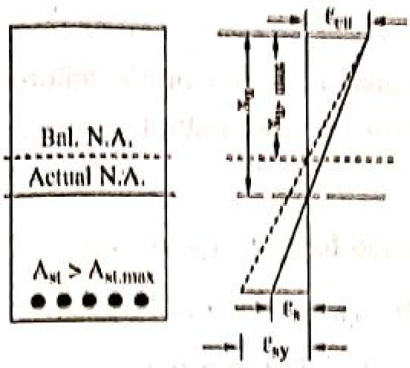
Hence, in over reinforced sections sudden failure occurs by crushing of concrete with out giving any warning. So this type sections should be avoided. IS code also recommends avoid of over reinforced sections.



(a) Balanced



(b) Under Reinforced



ε_{cu} = Ultimate compressive strain in concrete = 0.0035

ε_{sy} = Yield strain in steel = $\frac{0.87\, f_y}{E_s} + 0.002$

ε_c & ε_s = Actual strain in concrete and steel respectively

(c) Over Reinforced

FIG 3.5 : Balanced, Under Reinforced and Over Reinforced Sections

3.2.6 MAXIMUM DEPTH OF NEUTRAL AXIS ($x_{u,max}$)

The maximum depth of neutral axis is limited to ensure that tensile steel will reach its yield stress before concrete fails in compression, thus brittle failure (sudden failure with less alarming deflection) is avoided

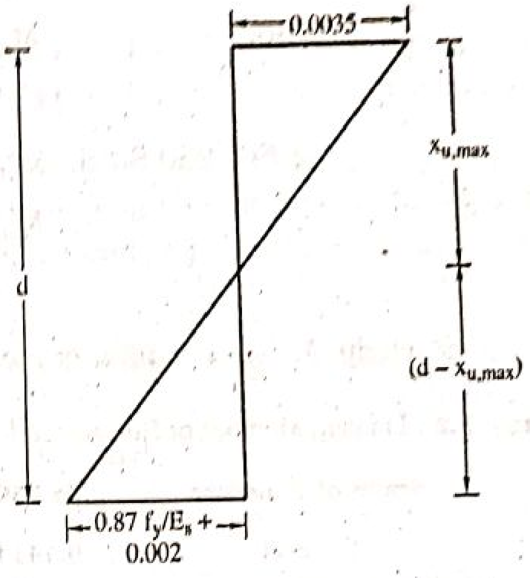


FIG 3.6 : Strain Diagram

From the strain diagram

$$\frac{x_{u,max}}{0.0035} = \frac{(d - x_{u,max})}{\left(\frac{0.87\, f_y}{E_s}\right) + 0.002}$$

$$\frac{x_{u,max}}{d} = \frac{0.0035}{\frac{0.87\, f_y}{E_s} + 0.0055}$$

It may be noted that $x_{u,max}$ is dependent on grade of steel only.

Table 3.1 : Values of $x_{u,max}/d$ for Different Grades of Steel

f_y (N/mm^2)	$\frac{x_{u,max}}{d}$
250	0.53
415	0.48
500	0.46

3.2.7 LIMITING VALUE OF MOMENT OF RESISTANCE

Since the maximum depth of neutral axis is limited to avoid brittle failure (sudden failure), the maximum value of moment of resistance is also limited.

For a singly reinforced section

$M_{u,lim}$ with respect to concrete = Compressive force × Lever arm

$$M_{u,lim} = 0.36\, f_{ck}\, b\, x_{u,max}\, (d - 0.42\, x_{u,max})$$

$M_{u,lim}$ with respect to steel = Tensile force × Lever arm

$$M_{u,lim} = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_{u,max})$$

For Fe-415 steel, $x_{u,max} = 0.48\, d$

So, $M_{u,lim} = 0.36\, f_{ck}\, b \times 0.48\, d\, (d - 0.42 \times 0.48\, d)$

$$= 0.138\, f_{ck}\, bd^2$$

For Fe - 250 Steel, $x_{u,max} = 0.53\, d$

$\therefore$ $M_{u,lim} = 0.36\, f_{ck} \times 0.53\, d\, (d - 0.42 \times 0.53\, d)$

$$= 0.148\, f_{ck}\, bd^2$$

Similarly, $M_{u,lim}$ for other grades of steels are shown in Table 3.2.

Table 3.2 : Limiting Moment of Resistance for Singly Reinforced Rectangular Sections

Grade of Concrete	Fe 250 Steel	Fe 415 Steel	Fe 500 Steel
General	$0.148\, f_{ck}\, bd^2$	$0.138\, f_{ck}\, bd^2$	$0.133\, f_{ck}\, bd^2$
M 20	$2.96\, bd^2$	$3.45\, bd^2$	$3.33\, bd^2$
M 25	$3.7\, bd^2$	$3.45\, bd^2$	$3.33\, bd^2$

3.2.8 LIMITING PERCENTAGE OF STEEL

The percentage of tensile reinforcement corresponding to the limiting moment of resistance is known as limiting percentage of steel. It can be obtained by equating force of tension and compression.

$$0.87\, f_y\, A_{st,lim} = 0.36\, f_{ck}\, b\, x_{u,max}$$

$$A_{st,lim} = \frac{0.36\, f_{ck}\, b\, x_{u,max}}{0.87\, f_y}$$

$$\text{Limiting percentage of steel } P_{t,lim} = \frac{A_{st,lim}}{bd} \times 100$$

$$= \frac{0.36\, f_{ck}}{0.87\, f_y} \times \frac{x_{u,max}}{d} \times 100$$

The limiting values of tensile reinforcement percentage corresponding to different grades of concrete and steel in a singly reinforced rectangular beam are given below.

Table 3.3 : Limiting Percentage of Steel for Singly Reinforced Rectangular Sections

Grade of Concrete	Limiting Percentage of Tensile Steel		
	Fe-250	Fe-415	Fe-500
M15	1.32	0.72	0.57
M20	1.76	0.96	0.76
M25	2.20	1.19	0.94

3.2.9 GENERAL DESIGN REQUIREMENTS FOR BEAMS

1. **Effective Span :** The effective span of a simply supported beam shall be taken as clear span plus effective depth of the beam of center to center distance between the supports which ever is less.

   The effective span of a cantilever shall be taken as its length to the face of the support plus half the effective depth except where it forms the end of a continuous beam where the length to the centre of support shall be taken.

2. **Limiting Stiffness :** The stiffness of beams is governed by the span to depth ratio. As per Clause 23.2 of IS : 456 for spans not exceeding 10 m, the span to effective depth ratio should not exceed the limits (Basic values) given below.

   Cantilevers – 7
   Simply supported – 20
   Continuous – 26

   For spans above 10 m, the above values may be multiplied by 10/$span$ in m.

   Depending on the amount and type of steel, the above values shall be modified by multiplying with the modification factors obtained from Fig 4 & 5 of IS : 456.

3. **Minimum Reinforcement :** The minimum area of tension reinforcement should not be less than the following (Clause 26.5.1 of IS : 456)

$$\frac{A_{st}}{bd} = \frac{0.85}{f_y}$$

   This works out only 0.2% for Fe 415 steel and 0.34% for Fe 250 steel.

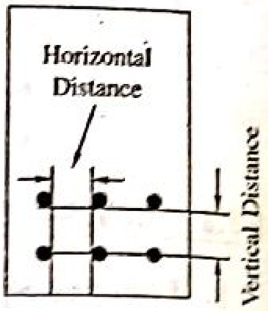
4. **Maximum Reinforcement :** The maximum area of tension reinforcement should not exceed 4 % of the gross cross sectional area (Clause 26.5.1 of IS : 456)

$$P_{tmax} < 0.04\ bD$$

Where D = gross depth of the beam

5. **Spacing of Bars :** The horizontal distance between two parallel main reinforcing bars shall usually be not less than the greatest of the following.
   - (a) Diameter of the bar if the diameters are equal
   - (b) Diameter of the largest bar if the bars are unequal
   - (c) 5 *mm* more than the nominal maximum size of the aggregate.

   When there are two or more rows of bars, the bars shall be vertically in line and the minimum vertical distance between the bars shall be 15 *mm*, two-thirds of nominal maximum size of aggregate or the maximum size of the bars which ever is greater.

   The maximum spacing of bars in tension for beams is taken from Table-15 of IS : 456-2000 depending on the amount of redistribution carried out in analysis and f_y.

6. **Cover to Reinforcement :** Reinforcement shall have concrete cover of thickness as follows.
   - (a) At each end of reinforcement bar not less than 25 *mm* nor less than twice the diameter of such bar
   - (b) For longitudinal reinforcing bar in beam, not less than 25 *mm* nor less than the diameter of such bar.

7. **Side Face Reinforcement :** Where the depth of the beam exceeds 750 *mm*, side face reinforcement shall be provided along the two faces. The total area of such reinforcement shall be not less than 0.1 % of the beam area and shall be distributed equally on two faces at a spacing not exceeding 300 *mm* or width of the beam which ever is less.

3.2.10 USE OF SP16 FOR DESIGN AND ANALYSIS OF SINGLY REINFORCED BEAMS

The Indian Standards Institution's special publication SP16, Design aids for Reinforced concrete to IS : 456, contains a number of charts and tables for design of reinforced concrete members.

The following are the data presented in SP16 for design and analysis singly reinforced beams.

- (i) Tables 1 to 4 gives the percentage steel required for various values of (M_u/bd^2) and f_y for concrete grades f_{ck} = 15, 20, 25 and 30.
- (ii) Charts 1 to 18 gives the moment of resistance per meter width for varying depths (5 to 80 *cm*) and varying percentage of steel, for various values of f_{ck} = 15 & 20 using steel grades of f_y = 250,415,500.

3.2.11 TYPES OF PROBLEMS

1. ***To determine the moment of resistance of the section, given dimensions and area of steel (Annexure G.1.1 of IS456).***

   (i) Determine the depth of neutral section by equating compressive force to tensile force (C = T)

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}$$

   (ii) Determine the limiting depth of neutral section $x_{u,max}$ as explained in 3.2.6

   (iii) Determine the moment of resistance of the section as the case may be.

   (a) *Under Reinforced Section* ($x_u < x_{u,max}$)

$$M_u = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_u)$$

   (b) *Balanced Section* ($x_u = x_{u,max}$)

$$M_{u,lim} = 0.36\, f_{ck}\, b\, x_{u,max}\,(d - 0.42\, x_{u,max})$$

(or)

$$M_{u,lim} = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_{u,max})$$

   (c) *Over Reinforced Section* ($x_u > x_{u,max}$)

   x_u is limited to $x_{u,max}$

$$M_u = 0.36\, f_{ck}\, b\, x_{u,max}\,(d - 0.42\, x_{u,max})$$

2. ***To determine the area of steel, given dimensions and M_u***

   Area of steel can be calculated by equating the bending moment to the moment of resistance with respect to tension steel

$$M_u = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_u)$$

$$= 0.87\, f_y\, A_{st}\left(d - \frac{0.42 \times 0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}\right)$$

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1 - \frac{f_y\, A_{st}}{f_{ck}\, b\, d}\right)$$

   A_{st} can found by solving the above equation and taking the lower root.

3. ***To design the beam given M_u***

   (i) Assuming breadth, find the minimum depth of the section by equating M_u with $M_{u,lim}$

$$M_u = M_{u,lim} = k\, f_{ck}\, bd^2$$

Where k is a factor which depends on grade of steel

For Fe 415, $k = 0.138$, for Fe 250, $k = 0.148$

2. Calculate the A_{st} using the following equation.

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

EXAMPLE - 1

The dimensions of a singly reinforced, simply supported rectangular beam are 300 mm wide and 450 mm deep effective, provided with Fe-415 steel and M-20 grade concrete. Determine the Limiting moment of resistance of the beam.

(OCT/NOV-2016 [TS] ; 2013)

Solution:

Breadth, $b = 300$ mm

Effective depth, $d = 450$ mm

Characteristic strength of concrete, $f_{ck} = 20$ N/mm^2

Characteristic strength of steel, $f_y = 415$ N/mm^2

For Fe415 steel, $x_{u,max} = 0.48\ d$

$$\begin{aligned} M_{u,lim} &= 0.36\ f_{ck}\ b\ x_{u,max}\ (d - 0.42\ x_{u,max}) \\ &= 0.36\ f_{ck}\ b\ (0.48d)\ (d - 0.42 \times 0.48d) \\ &= 0.138 .\ f_{ck} . bd^2 \\ &= 0.138 \times 20 \times 300 \times 450^2 \\ &= 167.67 \times 10^6 \text{ N-mm} \\ &= 167.67 \text{ kN-m} \end{aligned}$$

EXAMPLE - 2

Find the depth of neutral axis of singly reinforced rectangular beam 230 mm x 400 mm effective depth, reinforced with 4 bars of 16 mm diameter. Concrete is of M 20 grade and steel Fe 415.

(OCT/NOV-2015)

Solution :

$b = 230$ mm

$d = 450$ mm

$f_{ck} = 20$ N/mm^2

$$f_y = 415 \text{ N/mm}^2$$

Area of steel, $A_{st} = 4 \times \frac{\pi}{4} \times 16^2 = 804.2 \text{ mm}^2$

Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

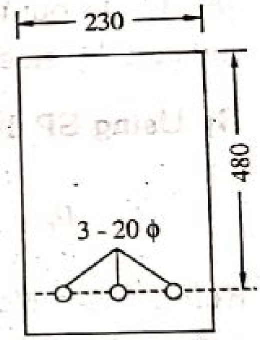
$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b} = \frac{0.87 \times 415 \times 804.2}{0.36 \times 20 \times 230} = 175.3 \text{ mm}$$

EXAMPLE - 3

Find the moment carrying capacity of singly reinforced rectangular beam 230× 480 mm effective depth reinforced with 3 bars of 20 mm diameter. Concrete is of M 20 grade and steel Fe 415.

(MARCH/APRIL. 2009)

Solution :



$b = 230$ mm

$d = 480$ mm

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.5 \text{ mm}^2$$

1. **Depth of Neutral Axis :** Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$x_u = \frac{0.87 f_y\, A_{st}}{0.36 f_{ck}\, b}$$

$$x_u = \frac{0.87 \times 415 \times 942.5}{0.36 \times 20 \times 230} = 205.5 \text{ mm}$$

The limiting depth of neutral axis, for Fe-415 steel

$$x_{u,max} = 0.48\, d = 0.48 \times 480 = 230.4 \text{ mm}$$

$$x_u < x_{u,max}$$

Therefore, the section is Under reinforced section.

2. Moment of Resistance :

$$Mu = \text{Tensile force} \times \text{Lever arm}$$

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

$$= 0.87 \times 415 \times 942.5\, (480 - 0.42 \times 205.5)$$

$$= 133.97 \times 10^6 \text{ N-mm}$$

$$= 133.97 \text{ kN-m}$$

M_u can also be calculated from

$$M_u = 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u)$$

$$= 0.36 \times 20 \times 230 \times 205.5\, (480 - 0.42 \times 205.5)$$

$$= 133.97 \times 10^6 \text{ N-mm}$$

$$= 133.97 \text{ kN-m}$$

(M_u works out to be same. This is because, in under reinforced section failure may be initiated by steel but finally failure occurs by crushing of concrete.)

By Using SP 16 : Percentage of reinforcement in the section

$$P_t = \frac{942.5}{230 \times 480} \times 100 = 0.854\ \%$$

Refer, Table No.2 of *SP-16* and read out the value of M_u/bd^2 corresponding to f_y = 415 N/mm² and f_{ck} = 20 N/mm²

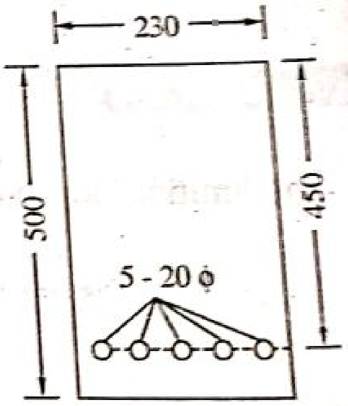
$$M_u/bd^2 = 2.53$$

$$M_u = 2.53 \times bd^2 = 2.53 \times 230 \times 480^2 = 134.07 \times 10^6 \text{ N-mm} = 134.07 \text{ KN-m}$$

EXAMPLE - 4

Find the Ultimate moment of resistance of singly reinforced rectangular beam 230×500 mm reinforced with 5 bars of 20 mm diameter with an effective cover of 50 mm. Concrete is of M 20 grade and steel Fe 415.

Solution :



Breadth, $b = 230$ mm

Overall depth, $D = 500$ mm

Effective depth, $d = 500 - 50 = 450$ mm

Characteristic strength of concrete,

$$f_{ck} = 20\ N/mm^2$$

Characteristic strength of steel,

$$f_y = 415\ N/mm^2$$

Area of steel, $A_{st} = 5 \times \frac{\pi}{4} \times 20^2 = 1571\ mm^2$

1. **Depth of Neutral Axis :** Equating Compressive force in concrete to tensile force in steel.

$$C = T$$

$$0.36\ f_{ck}\ b\ x_u = 0.87\ f_y\ A_{st}$$

$$x_u = \frac{0.87 f_y\ A_{st}}{0.36\ f_{ck}\ b} = \frac{0.87 \times 415 \times 1571}{0.36 \times 20 \times 230} = 342.5\ mm$$

The limiting depth of neutral axis, for Fe-415 steel

$$x_{u,max} = 0.48\ d = 0.48 \times 450 = 216\ mm$$

$$x_u > x_{u,max}$$

The section is over reinforced.

For over reinforced sections, as per IS 456, x_u is limited to $x_{u,max}$

$$x_u = x_{u,max} = 216\ mm$$

2. **Moment of Resistance :** M_u should be calculated with respect to concrete

$$M_u = 0.36\ f_{ck}\ b\ x_{u,max}\ (d - 0.42\ x_{u,max})$$

$$= 0.36 \times 20 \times 230 \times 216\ (450 - 0.42 \times 216)$$

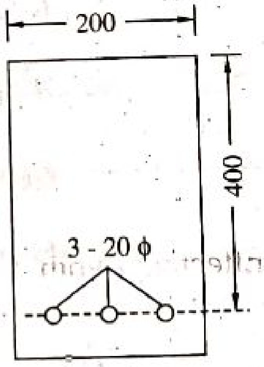
$$= 128.5 \times 10^6\ N\text{-}mm$$

$$= 128.5\ kN\text{-}m$$

EXAMPLE - 5

Find the ultimate moment of resistance of singly reinforced rectangular beam 200 × 400 mm effective depth reinforced with 3 bars of 20 mm diameter. Concrete is of M 20 grade and steel Fe 250.

Solution :



$b = 200\ mm$

$d = 400\ mm$

$f_{ck} = 20\ N/mm^2$

$f_y = 250\ N/mm^2$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.5\ mm^2$$

1. **Depth of Neutral Axis :** Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$x_u = \frac{0.87 f_y\, A_{st}}{0.36 f_{ck}\, b}$$

$$x_u = \frac{0.87 \times 250 \times 942.5}{0.36 \times 20 \times 200} = 142.4\ mm$$

The limiting depth of neutral axis, for Fe-250 steel

$$x_{u,max} = 0.53\, d$$

$$= 0.53 \times 400 = 212\ mm$$

$$x_u < x_{u,max}$$

Therefore, the section is Under reinforced section.

2. **Moment of Resistance :** The moment of resistance is governed by steel

$$M_u = 0.87\, f_y\, A_{st}\, (d - 0.42\, x_u)$$

$$= 0.87 \times 250 \times 942.5\, (400 - 0.42 \times 142.4)$$

$$= 69.74 \times 10^6\ N\text{-}mm$$

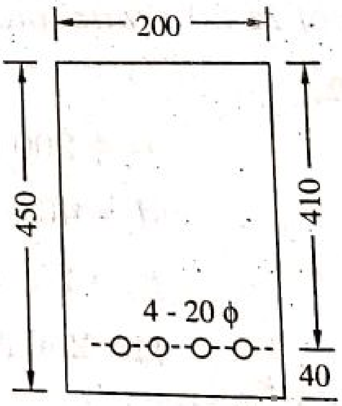
$$= 69.74\ kN\text{-}m$$

EXAMPLE - 6

A singly reinforced concrete beam section 200× 450 mm is reinforced with 4 bars of 20 mm diameter with an effective cover of 40 mm. The beam is simply supported over a span of 4 m. Find the safe uniformly distributed load the beam can carry. Use M 20 grade concrete and Fe-415 steel.

(APRIL, 2008)

Solution :



$b = 200\ mm$

$d = 450 - 40 = 410\ mm$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$$A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.6\ mm^2$$

1. **Depth of Neutral Axis :** Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$x_u = \frac{0.87 f_y\, A_{st}}{0.36 f_{ck}\, b}$$

$$x_u = \frac{0.87 \times 415 \times 1256.6}{0.36 \times 20 \times 200} = 315.1 \text{ mm}$$

The limiting depth of neutral axis, for Fe-415 steel

$$x_{u,max} = 0.48\, d$$

$$= 0.48 \times 410 = 196.8 \text{ mm}$$

$$x_u > x_{u,max}$$

Therefore, the section is over reinforced section.

For over reinforced sections, as per IS: 456, x_u is limited to $x_{u,max}$

$$x_u = x_{u,max} = 196.8 \text{ mm}$$

2. **Moment of Resistance :** The moment of resistance should be calculated with respect to concrete.

$$M_u = 0.36\, f_{ck}.b.x_{u,max}\,(d - 0.42\, x_{u,max})$$

$$= 0.36 \times 20 \times 200 \times 196.8\,(410 - 0.42 \times 196.8)$$

$$= 92.77 \times 10^6 \text{ N-mm}$$

$$= 92.77 \text{ kN-m}$$

3. **Safe Load :**

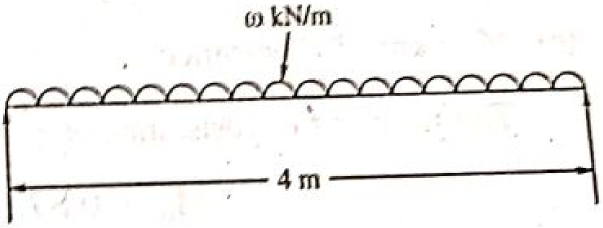
Let w_u kN/m be the safe ultimate load the beam can carry

Factored bending moment $= \dfrac{w_u\, l^2}{8} = \dfrac{w_u\, 4^2}{8} = 2\, w_u$

Equating factored bending moment to the Moment of resistance of the section

$$2\, w_u = 92.77$$

$$w_u = \frac{92.77}{2} = 46.39 \text{ kN/m}$$



Safe working load $\quad w = \dfrac{w_u}{\text{load factor}} = \dfrac{w_u}{1.5}$

$$= \frac{46.39}{1.5} = 30.92 \text{ kN/m (including self weight)}$$

Self weight of the beam $= 0.2 \times 0.45 \times 25 = 2.25$ kN/m

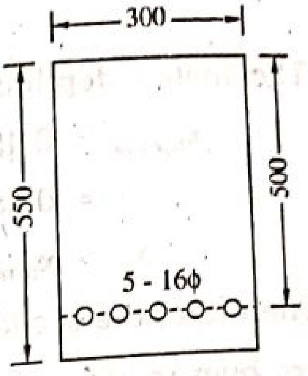
Net super imposed load the beam can carry $= 30.92 - 2.25 = 28.67$ kN/m

EXAMPLE-7

A singly reinforced concrete beam section 300 mm × 550 mm is reinforced with 5 bars of 16 mm diameter with an effective cover of 50 mm. The beam is simply supported over a span of 5 m. Find the safe uniformly distributed load the beam can carry. Use M-20 grade concrete and Fe-415 steel.

(APRIL/MAY-2015, OCT/NOV-2011)

Solution :



$$b = 300 \text{ mm}$$

$$d = 550 - 50 = 500 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 5 \times \frac{\pi}{4} \times 16^2 = 1005.3 \text{ mm}^2$$

(i) Depth of neutral axis

Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36 f_{ck}\, b\, x_u = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck}\, b}$$

$$x_u = \frac{0.87 \times 415 \times 1005.3}{0.36 \times 20 \times 300} = 168 \text{ mm}$$

The limiting depth of neutral axis, for Fe-415 steel

$$x_{u,max} = 0.48\, d = 0.48 \times 500 = 240 \text{ mm}$$

$$x_u < x_{u,max}$$

Therefore, the section is Under reinforced section.

(ii) Moment of Resistance

The moment of resistance is governed by steel

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 1005.3 (500 - 0.42 \times 168)$$

$$= 155.87 \times 10^6 \text{ N-mm}$$

$$= 155.87 \text{ kN-m}$$

(iii) Safe load

Let w_u KN/m be the safe ultimate load the beam can carry

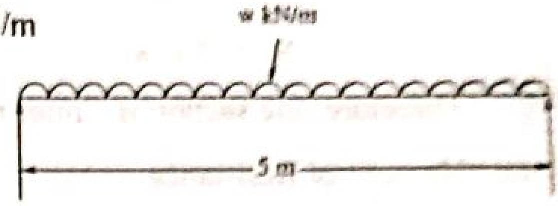
$$\text{Factored bending moment} = \frac{w_u l^2}{8} = \frac{w_u 5^2}{8} = 3.125\, w_u$$

Equating factored bending moment to the Moment of resistance of the section

$$3.125\, w_u = 155.87$$

$$w_u = \frac{155.87}{3.125} = 49.88 \text{ kN/m}$$

Safe working load



$$w = \frac{w_u}{\text{load factor}} = \frac{w_u}{1.5}$$

$$= \frac{49.88}{1.5} = 33.25 \text{ kN/m (including self weight)}$$

Self weight of the beam = 0.3 × 0.55 × 1 × 25 = 4.125 kN/m

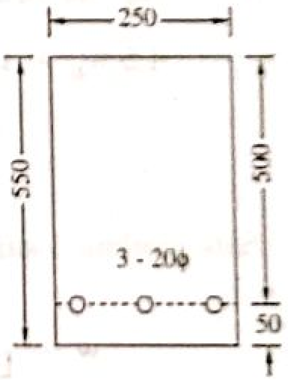
Net super imposed load the beam can carry = 33.25 – 4.125 = 29.125 kN/m

EXAMPLE-8

A singly reinforced concrete beam section 250 mm × 550 mm overall is reinforced with 3 bars of 20 mm diameter with an effective cover of 50 mm. The beam is a cantilever over a span of 3 m. Find the uniformly distributed load the beam can carry. Use M 20 grade concrete and Fe 415 steel.

(APRIL/MAY-2012)

Solution :



$$b = 250 \text{ mm}$$

$$d = 550\text{-}50 = 500 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 20^2 = 942.5 \text{ mm}^2$$

(i) Depth of neutral axis

Equating Compressive force in concrete to Tensile force in steel

$$C = T$$

$$0.36\, f_{ck}\, b\, x_u = 0.87\, f_y\, A_{st}$$

$$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}$$

$$x_u = \frac{0.87 \times 415 \times 942.5}{0.36 \times 20 \times 250} = 189.1 \text{ mm}$$

The limiting depth of neutral axis, for Fe-415 steel

$$x_{u,max} = 0.48\, d = 0.48 \times 500 = 240 \text{ mm}$$

$$x_u < x_{u,max}$$

Therefore, the section is Under reinforced section.

(ii) Moment of Resistance

The moment of resistance is governed by steel

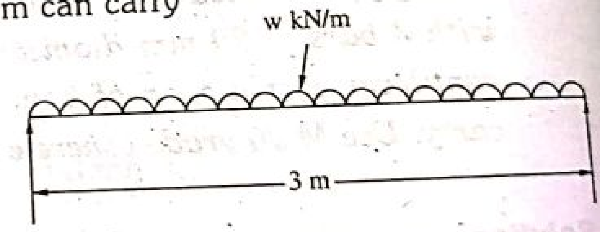
$$M_u = 0.87\, f_y\, A_{st}\,(d - 0.42\, x_u)$$

$$= 0.87 \times 415 \times 942.5\,(500 - 0.42 \times 189.1)$$

$$= 143.12 \times 10^6 \text{ N-mm} = 143.12 \text{ kN-m}$$

(iii) Safe load

Let w_u KN/m be the safe ultimate load the beam can carry



Factored bending moment

$$= \frac{w_u l^2}{2} = \frac{w_u 3^2}{2} = 4.5\, w_u$$

Equating factored bending moment to the Moment of resistance of the section

$$4.5\, w_u = 143.12$$

$$w_u = \frac{143.12}{4.5} = 31.8 \text{ kN/m}$$

Safe working load

$$w = \frac{w_u}{\text{Load factor}} = \frac{w_u}{1.5} = \frac{31.8}{1.5}$$

$$= 21.2 \text{ kN/m (including self weight)}$$

Self weight of the beam = 0.25 × 0.55 × 1 × 25 = 3.44 kN/m

Net super imposed load the beam can carry = 21.2 – 3.44 = 17.76 kN/m

EXAMPLE - 9

If the ultimate load moment is 80 kN-m, what is the effective depth of singly reinforced concrete section, if the width of the beam is 230 mm and concrete grade is M 20 and type of steel is Fe-415.

(MARCH/APRIL. 2003)

Solution :

$$b = 230 \text{ mm}$$

$$M_u = 80 \text{ kN-m} = 80\times 10^6 \text{ N-mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

For Fe-415 steel, $M_{u,lim} = 0.138.\, f_{ck}.bd^2$

$$80 \times 10^6 = 0.138\times20\times230\times d^2$$

$$d = \sqrt{\frac{80\times10^6}{0.138\times20\times230}} = 355 \text{ mm}$$

EXAMPLE - 10

A singly reinforced rectangular beam of width 230 mm and 535 mm effective depth is subjected to a bending moment of 90 kN-m at working loads. Find the steel area required. The materials used are M20 grade concrete and HYSD reinforcement of grade Fe415 steel.

(OCTOBER. 2006)

Solution :

$$b = 230 \text{ mm}$$

$$d = 535 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

Factored Bending moment $M_u = 1.5 \times 90 = 135$ kN-m

For Fe 415 steel, limiting moment of resistance is given by

$$M_{u,lim} = 0.138.\, f_{ck}.bd^2 = 0.138\times20\times230\times535^2$$

$$= 181.7\times10^6 \text{ N-mm}$$

$$= 181.7 \text{ kN-m}$$

$$M_u < M_{u,lim}$$

As the Ultimate moment to be resisted is less than the limiting moment, the section can be under reinforced section.

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1-\frac{f_y\, A_{st}}{f_{ck}\, bd}\right)$$

$$135\times10^6 = 0.87\times415\times A_{st}\times535\left(1-\frac{415\times A_{st}}{20\times230\times535}\right)$$

$$698.9 = A_{st}\times\left(1-\frac{A_{st}}{5930.1}\right)$$

$$A_{st}^2 - 5930.1\, A_{st} + 698.9\times5930.1 = 0$$

By solving the above quadratic equation and taking the lower root

$$A_{st} = \frac{5930.1-\sqrt{5930.1^2-4\times698.9\times5930.1}}{2} = 809.4\ mm^2$$

Provide 3-20 *mm* bars, A_{st} provided = 942.5 mm^2

By Using SP16 :

$$\frac{M_u}{bd^2} = \frac{135\times10^6}{230\times535^2} = 2.05$$

Refer, Table No.2 of SP-16 and read out the value of Percentage of reinforcement corresponding to f_y = 415 N/mm^2 and f_{ck} = 20 N/mm^2

For $\frac{M_u}{bd^2} = 2.05,\ P_t = 0.6585\ \%$

$$A_{st} = \frac{P_t\times bd}{100} = \frac{0.6585}{100}\times230\times535 = 810\ mm^2$$

Provide 3 - 20 *mm* bars, A_{st} provided = 942.5 mm^2

EXAMPLE - 11

Calculate the area of reinforcement required for a simply supported reinforced concrete beam 230 mm wide and 400 mm effective depth to resist an ultimate moment of 50 kN-m. Assume M 20 and Fe 415 combination of concrete and steel.

(MARCH/APRIL. 2007)

Solution :

b = 230 *mm*

d = 400 *mm*

f_{ck} = 20 N/mm^2

$$f_y = 415 \text{ N/mm}^2$$

Ultimate bending moment $M_u = 50$ kN-m

For Fe 415 steel, limiting moment of resistance is given by

$$M_{u,lim} = 0.138 . f_{ck} . bd^2$$

$$= 0.138 \times 20 \times 230 \times 400^2 = 101.57 \times 10^6 \text{ N-mm}$$

$$= 101.57 \text{ kN-m}$$

$$M_u < M_{u,lim}$$

As the ultimate moment to be resisted is less than the limiting moment, the section can be under reinforced section.

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$50 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400 \left(1 - \frac{415 \times A_{st}}{20 \times 230 \times 400}\right)$$

$$346.2 = A_{st} \left(1 - \frac{A_{st}}{4433.7}\right)$$

$$A_{st}^2 - 4433.7 A_{st} + 346.2 \times 4433.7 - 0$$

By solving the above quadratic equation and taking the lower root.

$$A_{st} = \frac{4433.7 - \sqrt{4433.7^2 - 4 \times 4433.7 \times 346.2}}{2}$$

$$A_{st} = 378.5 \text{ mm}^2$$

Provide 4-12 mm bars, A_{st} provided = 452.4 mm^2

EXAMPLE - 12

A reinforced concrete beam has a section of 200×500 mm overall. It is subjected to a factored moment of 80 kN-m. Design the reinforcement using Fe-250 steel and M 20 grade concrete. Use effective cover of 50 mm.

(SEPTEMBER/OCTOBER, 2004)

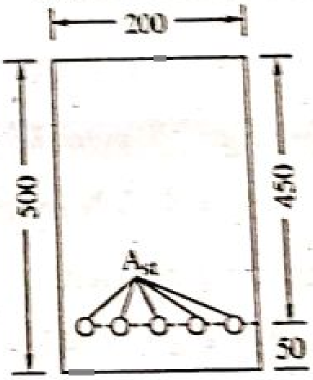
Solution :

$$b = 200 \text{ mm}.$$

$$d = 500 - 50 = 450 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$



Factored Bending moment $M_u = 80\ kN\text{-}m$

For Fe 250 steel, limiting moment of resistance is given by

$$M_{u,lim} = 0.148.\ f_{ck}.bd^2$$
$$= 0.148 \times 20 \times 200 \times 450^2$$
$$= 119.88 \times 10^6\ N\text{-}mm$$
$$= 119.88\ kN\text{-}m$$

$$M_u < M_{u,lim}$$

As the Ultimate moment to be resisted is less than the limiting moment; the section can be under reinforced section

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1 - \frac{f_y\ A_{st}}{f_{ck}\ bd}\right)$$

$$80 \times 10^6 = 0.87 \times 250 \times A_{st} 450\left(1 - \frac{250 \times A_{st}}{20 \times 200 \times 450}\right)$$

$$817.4 = A_{st}\left(1 - \frac{A_{st}}{7200}\right)$$

$$A_{st}^2 - 7200\ A_{st} + 817.4 \times 7200 = 0$$

$$A_{st} = \frac{7200 - \sqrt{7200^2 - 4 \times 817.4 \times 7200}}{2}$$

$$A_{st} = 940.2\ mm^2$$

Provide 3 - 20 mm bars, A_{st} provided = $942.5\ mm^2$

EXAMPLE - 13

A singly reinforced rectangular beam is subjected to a bending moment of 45 kN-m at working loads. Design the beam for flexure. The materials used are M 20 grade concrete and Fe 415 steel. Provide effective depth 1.5 times the breadth.

(OCT. 2005)

Solution :

$$f_{ck} = 20\ N/mm^2$$
$$f_y = 415\ N/mm^2$$

Factored Bending moment

$$M_u = 1.5 \times 45 = 67.5\ kN\text{-}m$$

1. Dimensions of the Beam :

Given, depth = 1.5 breadth

$$d = 1.5\ b$$

Equating factored moment to the limiting moment of resistance

$$M_u = M_{u,lim}$$

$$M_u = 0.138.\ f_{ck}.bd^2$$

$$67.5\times10^6 = 0.138 \times 20 \times b\ (1.5\ b)^2 = 6.21\ b^3$$

$$b = 221.5\ mm$$

$$d = 1.5\ b = 1.5\times221.5 = 332.3\ mm$$

Adopt 230×350 *mm* effective section

2. Tension Reinforcement :

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1-\frac{f_y\ A_{st}}{f_{ck}\ bd}\right)$$

$$67.5\times10^6 = 0.87 \times 415 \times A_{st} \times350\left(1-\frac{415\times A_{st}}{20\times230\times350}\right)$$

$$534.2 = A_{st}\left(1-\frac{A_{st}}{3879.5}\right)$$

$$A_{st}^2 - 3879.5\ A_{st} + 534.2\times3879.5 = 0$$

$$A_{st} = \frac{3879.5-\sqrt{3879.5^2-4\times534.2\times3879.5}}{2}$$

$$A_{st} = 639.7\ mm^2$$

Provide 4 bars of 16 *mm* diameter.

A_{st} provided = 804 mm^2

EXAMPLE - 14

Design a rectangular simply supported reinforced concrete beam over a clear span of 4000 mm. The superimposed load is 20 kN/m and support width is 300 mm each. Use M 20 grade concrete and Fe 415 grade steel. Check the design for deflection.

(OCT/NOV-2015, 2008, MARCH/APRIL-2005)

Solution :

$$f_{ck} = 20\ N/mm^2$$

$$f_y = 415\ N/mm^2$$

$$l = 4\ m$$

1. **Depth of the Beam :**

Selecting the depth in range of $\frac{l}{12}$ to $\frac{l}{15}$ based on stiffness

$$d = \frac{4000}{12} = 333.3\ mm$$

$$\text{Adopt } d = 350\ mm$$

$$D = 400\ mm$$

Assuming width $b = 300\ mm$

2. **Effective Span :** It is the least of the following

Centre to centre of supports = $4 + 0.3 = 4.3\ m$

Clear span + $d = 4 + 0.35 = 4.35\ m$

Hence, Effective span = $4.3\ m$

2. **Loads Per Meter Length of the Beam :**

Self weight of the beam = Area × length × Unit *wt.* of RCC

$= 0.3 \times 0.40 \times 1 \times 25 = 3\ kN/m$

Imposed load = $20\ kN/m$

Total load = $23\ kN/m$

Factored load $w_u = 1.5 \times 23 = 34.5\ kN/m$

Factored Bending moment

$$M_u = \frac{w_u\, l^2}{8} = 34.5 \times \frac{(4.3)^2}{8} = 79.74\ kN\text{-}m$$

4. **Depth Required :**

The minimum depth required to resist the Bending Moment

$$M_u = M_{u,lim}$$

$$M_u = 0.138.\ f_{ck}.bd^2$$

$$79.74 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{79.74 \times 10^6}{0.138 \times 20 \times 300}} = 310.3\ mm < 350\ mm, \text{ effective depth provided}$$

Hence provided depth is adequate

5. Tension Reinforcement :

$$M_u = 0.87\, f_y\, A_{st}\, d\left(1-\frac{f_y\, A_{st}}{f_{ck}\, bd}\right)$$

$$79.74\times10^6 = 0.87 \times 415 \times A_{st} \times 350\left(1-\frac{415\times A_{st}}{20\times300\times350}\right)$$

$$631 = A_{st}\left(1-\frac{A_{st}}{5060.2}\right)$$

$$A_{st}^2 - 5060.2\, A_{st} + 5060.2\times631 = 0$$

$$A_{st} = \frac{5060.2-\sqrt{5060.2^2-4\times631\times5060.2}}{2}$$

$$A_{st} = 738.9\ mm^2$$

Provide 4 bars of 16 *mm* diameter.

A_{st} Provided = 804 mm^2

6. Check for Deflection (Stiffness) :

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

$$\%\text{ of steel} = \frac{804}{300\times350}\times100 = 0.766$$

Stress in steel under service or working loads

$$f_s = 0.58 \times f_y \times \left(\frac{\textit{area of steel required}}{A_{st}\ \textit{provided}}\right)$$

$$= 0.58 \times 415 \times \frac{738.9}{804} = 221.2\ N/mm^2$$

From Fig.4 of IS : 456, modification factor = 1.15

Maximum permitte $\frac{l}{d}$ ratio = 1.15 × 20 = 23

$$\frac{l}{d}\ \text{provided} = \frac{4300}{350} = 12.29 < 23$$

Hence deflection control is safe.

EXAMPLE - 15

Design a rectangular simply supported reinforced concrete beam over a clear span of 6 m. The superimposed load is 30 kN/m and support width is 230mm each. Use M 20 grade concrete and Fe 415 grade steel. Check the design for deflection.

(MARCH/APRIL. 2018, APRIL/MAY. 2015, 2014)

Solution :

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$l = 6 \text{ m}$$

(i) Depth of the beam : Selecting the depth in range of $\frac{l}{12}$ to $\frac{l}{15}$ based on stiffness

$$d = \frac{6000}{12} = 500 \text{ mm}$$

Adopt $d = 500$ mm

$D = 550$ mm

Assuming width $b = 300$ mm

(ii) Effective span : It is the least of the following

Centre to centre of supports

$$= 6 + 0.\ 23 = 6.23 \text{ m}$$

Clear span + d = 6 + 0.5 = 6.5 m

Hence, Effective span = 6.23 m

(iii) Loads per meter length of the beam

Self weight of the beam

= Area × length × Unit wt. of RCC

$= 0.3 \times 0.55 \times 1 \times 25$

= 4.125 kN/m

Imposed load = 30 kN/m

Total load = 34.125 kN/m

Factored load $w_u = 1.5 \times 34.125$

= 51.19 kN/m

Factored Bending moment

$$M_u = \frac{w_u.l^2}{8} = \frac{51.19 \times (6.23)^2}{8}$$

$$= 248.35 \text{ kN-m}$$

$$= 248.35 \times 10^6 \text{ N-mm}$$

(iv) Depth required

The minimum depth required to resist the Bending Moment

$$M_u = M_{u,lim}$$

$$M_u = 0.138.\, f_{ck}.bd^2$$

$$248.35 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{248.35 \times 10^6}{0.138 \times 20 \times 300}}$$

$$= 547.66 \text{ mm} > 500 \text{ mm, effective depth provided}$$

Hence provided depth is not adequate. Provide an effective depth of 550 mm.

(v) Tension reinforcement

$$M_u = 0.87\, f_y\, A_{st}\, d\left(\frac{f_y\, A_{st}}{f_{ck}.\, bd}\right)$$

$$248.35 \times 10^6 = 0.87 \times 415 \times A_{st} \times 550 \left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 550}\right)$$

$$1250.6 = A_{st}\left(1 - \frac{A_{st}}{7951.8}\right)$$

$$A_{st}^2 - 7951.8A_{st} + 1250.6 \times 7951.82 = 0$$

$$A_{st} = \frac{7951.8 - \sqrt{7951.8^2 - 4 \times 1250.6 \times 7951.8}}{2}$$

$$A_{st} = 1554.5 \text{ mm}^2$$

Providing 20 mm diameter bars

Area of one bar, $a_{st} = \frac{\pi\phi^2}{4} = \frac{\pi \times 20^2}{4} = 314.16 \text{ mm}^2$

No. of bars required, $n = \frac{A_{st}}{a_{st}} = \frac{1554.5}{314.16} = 4.95$

Provide 5 bars of 20 mm diameter.

$$A_{st} \text{ provided} = 5 \times 314.16$$

$$= 1570.8 \text{ mm}^2$$

(vi) Check for deflection (Stiffness)

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

$$\% \text{ of steel} = \frac{1570.8 \times 100}{300 \times 550} = 0.952$$

Stress in steel under service or working loads

$$f_s = 0.58 \times f_y \times \left(\frac{\text{area of steel required}}{A_{st} \text{ provided}}\right)$$

$$= 0.58 \times 415 \times \left(\frac{1554.5}{1570.8}\right)$$

$$= 238.2 \text{ N/mm}^2$$

From Fig.4 of IS 456, modification factor = 1.2

Maximum permitted $\frac{l}{d}$ ratio = 1.2 × 20 = 24

$$\frac{l}{d} \text{ provided} = \frac{6230}{550}$$

$$= 11.32 < 24$$

Hence deflection control is safe.

EXAMPLE - 16

A beam simply supported over an effective span of 5.3 m carries a LL of 20 kN / m. Design the singly reinforced beam for flexure. M 20 concrete and Fe 415 steel are used. Breadth of the beam is 300 mm.

(MARCH/APRIL. 2003)

Solution :

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Effective span $l = 5.3\ m$

Breadth $b = 300\ mm$

1. **Depth of the Beam :** Selecting the depth in range of $\frac{l}{12}$ to $\frac{l}{15}$ based on stiffness

$$d = \frac{5300}{12} = 441.7\ mm$$

Adopt $d = 450\ mm$

$D = 500\ mm$

2. **Loads Per Meter Length of the Beam :**

Self weight of the beam $= 0.3 \times 0.5 \times 1 \times 25 = 3.75\ kN/m$

Imposed load $= 20\ kN/m$

Total load $= 23.75\ kN/m$

Factored load $w_u = 1.5 \times 23.75 = 35.625\ KN/m$

Factored Bending moment $w_u = \frac{w_u L^2}{8} = 35.625 \times \frac{(5.3)^2}{8} = 125.09\ kN\text{-}m$

3. **Depth Required :** The minimum depth required to resist the Bending Moment

$$M_u = M_{u,lim}$$

$$M_u = 0.138. f_{ck}.bd^2$$

$$125.09 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{125.09 \times 10^6}{0.138 \times 20 \times 300}} = 388.7\ mm < 450\ mm\text{, provided depth}$$

As the assumed depth is much higher, adopt

$d = 400\ mm$

4. Tension Reinforcement :

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right)$$

$$125.09 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400 \left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 400}\right)$$

$$866.2 = A_{st} \left(1 - \frac{A_{st}}{5783.1}\right)$$

$$A_{st}^2 - 5783.1 A_{st} + 5783.1 \times 866.2 = 0$$

$$A_{st} = \frac{5783.1 - \sqrt{5783.1^2 - 4 \times 5783.1 \times 866.2}}{2}$$

$$A_{st} = 1060.8 \ mm^2$$

Provide 6 bars of 16 *mm* diameter.

Ast provided = 1206.4 mm^2

5. Check for Deflection (Stiffness) :

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

$$\% \text{ of steel} = \frac{1206.4}{300 \times 400} \times 100 = 1\%$$

Stress in steel under service or working loads

$$f_s = 0.58 \times f_y \times \left(\frac{\textit{area of steel required}}{A_{st} \textit{ provided}}\right)$$

$$= 0.58 \times 415 \times \frac{1060.8}{1206.4} = 211.7 \ N/mm^2$$

From Fig.4 of IS : 456, modification factor = 1.15

Maximum permitte $\frac{l}{d}$ ratio = 1.15 × 20 = 23

$$\frac{l}{d} \text{ provided} = \frac{5300}{400} = 13.25 < 23$$

Hence deflection control is safe.

3.3 DOUBLY REINFORCED BEAMS

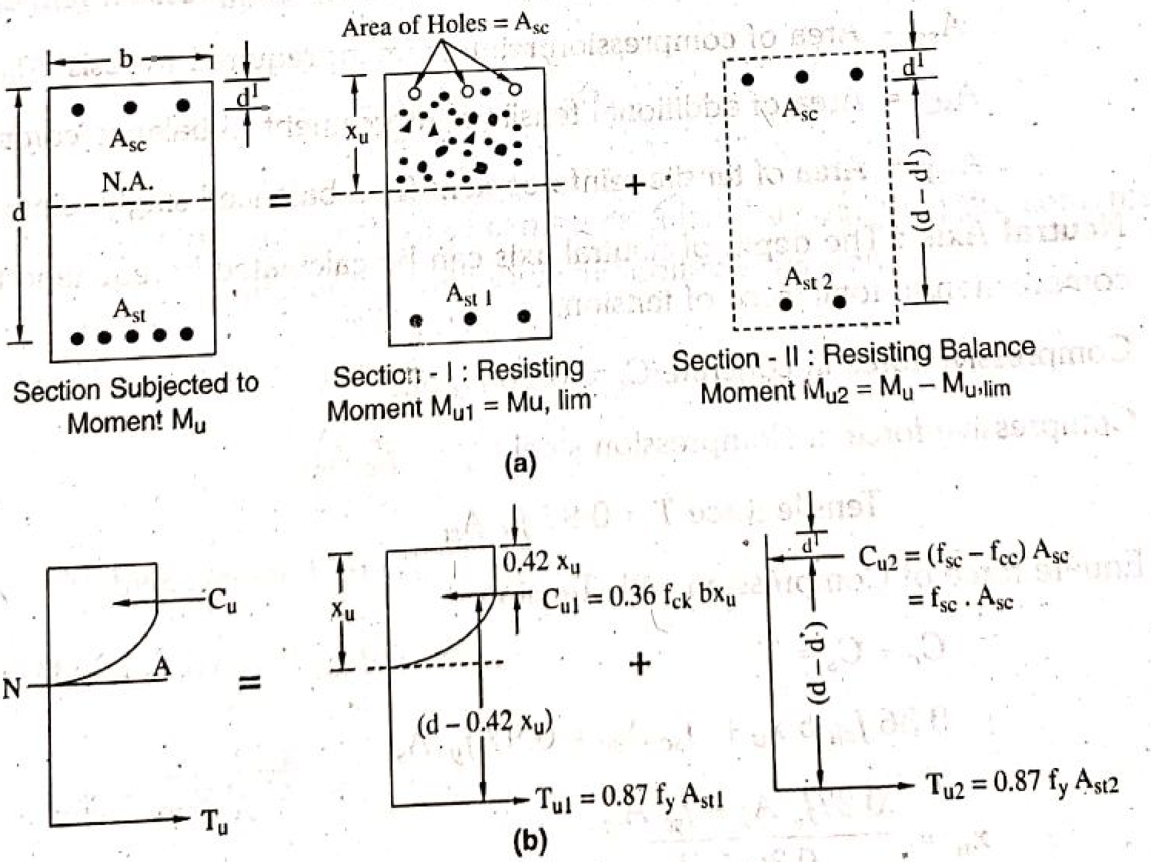
Beams which are reinforced in both compression and tension sides are called as doubly reinforced beam. These beams are generally provided when the dimensions of the beam are restricted and it is required to resist moment higher than the limiting moment of resistance of a singly reinforced section. The additional moment of resistance required can be obtained by providing compression reinforcement and additional tension reinforcement.

3.3.1 SITUATIONS UNDER WHICH DOUBLY REINFORCED BEAMS ARE USED

1. When the depth of the beam is restricted due to architectural or any construction problems
2. At the supports of a continuous beam where bending moment changes its sign
3. In precast members (during handling bending moment changes its sign)
4. In bracing members of a frame due to changes in the direction of wind loads
5. To improve the ductility of the beams in earth quack regions
6. To reduce long term deflections or to increase stiffness of the beam

3.3.2 ANALYSIS OF DOUBLY REINFORCED BEAMS

Fig. 3.7. shows a typical doubly reinforced section and the variation of stress across the depth.



Section Subjected to Moment M_u

Section - I : Resisting Moment M_{u1} = Mu, lim

Section - II : Resisting Balance Moment $M_{u2} = M_u - M_{u,lim}$

FIG 3.7 : Doubly Reinforced Section

This doubly reinforced section can be considered to be composed of two sections as given below.

(a) A singly reinforced section with $M_{u,lim}$

(b) A section with compression steel and additional tension steel to resist additional moment M_{u2} (= $M_u - M_{u,lim}$), i.e., a steel beam with out concrete.

Hence, moment of resistance of doubly reinforced beam

$$M_u = M_{u,lim} + M_{u2}$$

Where $M_{u,lim}$ = Limiting moment of resistance of singly reinforced section

M_{u2} = Additional moment of resistance to be resisted by compression steel and additional tension steel

The lever arm for the additional moment of resistance M_{u2} is equal to the distance between the centroid of the tension and compression reinforcements, i.e. d-d' . Hence, the additional moment of resistance is given by

$$M_{u2} = f_{sc} A_{sc} (d - d') = 0.87 f_y A_{st2} (d - d')$$

Where f_{sc} = Stress in compression steel

d' = Distance of centroid of compression reinforcement from the maximum compression fiber (effective cover to compression reinforcement)

A_{sc} = Area of compression reinforcement required to resist M_{u2}

A_{st2} = Area of additional tensile reinforcement to balance compression steel

A_{st1} = Area of tensile reinforcement for a balanced singly reinforced section.

1. **Neutral Axis :** The depth of neutral axis can be calculated by equating total force of compression to total force of tension.

Compressive force in concrete $C_c = 0.36 f_{ck} b x_u$

Compressive force in Compression steel $C_s = f_{sc} A_{sc}$

Tensile force $T = 0.87 f_y A_{st}$

Equate force of Compression with Tension

$$C_c + C_s = T$$

$$0.36 f_{ck} b x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st} - f_{sc} A_{sc}}{0.36 f_{ck} b}$$

2. **Ultimate Moment of Resistance :** The Ultimate Moment of resistance of doubly reinforced section is given by

$$M_u = M_{u1} + M_{u2}$$

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42\, x_u) + f_{sc} A_{sc} (d - d')$$

When $x_u > x_{u,max}$, x_u is limited to $x_{u,max}$

$$M_u = 0.36 f_{ck}\, b\, x_{u,max} (d - 0.42\, x_{u,max}) + f_{sc} A_{sc} (d - d')$$

3. **Area of Compression Steel :**

Additional moment of resistance $M_{u2} = f_{sc} A_{sc} (d - d')$

$$\therefore \quad A_{sc} = \frac{M_{u2}}{f_{sc}(d-d')}$$

The maximum area of compression reinforcement shall not exceed 0.04 bD i.e. 4% of gross cross sectional area.

4. **Area of Tension Steel :** The limiting moment of resistance of singly reinforced section is given by

$$M_{u,lim} = 0.87 f_y\, A_{st1} (d - 0.42\, x_{u,max})$$

$$\therefore \quad A_{st1} = \frac{M_{u,lim}}{0.87 f_y (d - 0.42 x_{u,max})}$$

Additional area of tensile steel (A_{st2}) can be calculated by equating the compressive force in compression steel and tensile force in additional tension steel.

$$0.87 f_y\, A_{st2} = f_{sc} A_{sc}$$

$$\therefore \quad A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y}$$

A_{st2} can also be calculated by using

$$M_{u2} = 0.87 f_y\, A_{st2}\, (d - d')$$

$$A_{st2} = \frac{M_{u2}}{0.87 f_y (d-d')}$$

Total area of tension steel $A_{st} = A_{st1} + A_{st2}$

3.3.3 STRESS IN COMPRESSION STEEL (f_{sc})

If ε_{sc} is the strain at the level of compression steel, from the strain diagram at failure

$$\frac{\varepsilon_{sc}}{x_u - d'} = \frac{0.0035}{x_u}$$

$$\varepsilon_{sc} = 0.0035\left(\frac{x_u - d'}{x_u}\right)$$

$$= 0.0035\left(1 - \frac{d'}{x_u}\right)$$

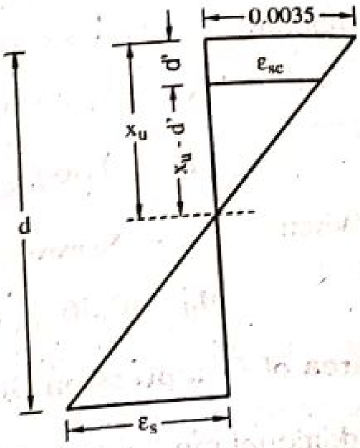


FIG 3.8 : Strain Diagram

Knowing the strain, the stress in compression steel can be obtained from stress strain curve of corresponding steel or from the Table-A of SP16 which is given below.

TABLE 3.4 : Salient Points on the Design Stress Strain Curve for Cold Worked Bars **(Table-A of SP16)**

Stress Level	Fe 415		Fe 500	
	Strain	Stress N/mm^2	Strain	Stress N/mm^2
0.80 f_{yd}	0.00144	288.7	0.00174	347.8
0.85 f_{yd}	0.00163	306.7	0.00195	369.6
0.90 f_{yd}	0.00192	324.8	0.00226	391.3
0.95 f_{yd}	0.00241	342.8	0.00277	413.0
0.975 f_{yd}	0.00276	351.8	0.00312	423.9
1.0 f_{yd}	0.00380	360.9	0.00417	434.8

Note : Linear interpolation may be done for intermediate values

$$f_{yd} = \text{Design yield strength} = 0.87\ f_y$$

So f_{sc} and x_u are interrelated and can not be found directly. Trail and error procedure should be adopted.

For mild steel direct relation can be established between stress and strain since the idealized stress strain curve is linear up to f_y and then it is constant equal to f_y.

$$f_{sc} = \text{strain} \times E_s$$

Substituting the value of strain and E for steel = 2×10^5 N/mm^2

$$= 0.0035\ (1 - d'/x_u)\ 2 \times 10^5$$

$$= 700\ (1 - d'/x_u),\ \text{subjected to a maximum of } 0.87\ f_y$$

Stress in Compression Steel (f_{sc}) based on d'/d :

As per SP16, in **designing** doubly reinforced beam (by assuming $x_u = x_{u,max}$) the following table gives the values of f_{sc} for different values of d'/d.

TABLE 3.5 : Stress in Compression Steel f_{sc}, N/mm^2 in Doubly Reinforced beams with Cold Worked Bars

(Table-F In SP16) when $d'/d < 0.2$

Grade of Steel	d'/d			
	0.05	0.10	0.15	0.20
Fe 415	355	353	342	329
Fe 500	424	412	395	370

For $d'/d < 0.2$, f_{sc} for mild steel is $0.87\ f_y$

3.3.4 USE OF DESIGN AIDS SP16

Sp16 design tables 45 to 56 gives the percentage of tension and compression reinforcement (P_t and P_c) for different ratios of (d'/d) varying from 0.05 to 0.20 and for various grades of concrete (f_{ck} = 15 to 30 N/mm^2) and different grades of steel (f_y = 250, 415 and 500 N/mm^2) covering the moment of resistance factor (M_u/bd^2) varying from 2.24 to 8.30.

3.3.5 TYPES OF PROBLEMS

1. **Analysis of the Section**, i.e. to find the ultimate moment of resistance, given the section dimensions and area of tension & compression reinforcement.

Method-1 (Strain Compatibility Method-Exact Method)

(a) Assume $x_u = x_{u,max}$

(b) Calculate the strain in compression steel, $\varepsilon_{sc} = 0.0035\ (1 - d'/x_u)$ and the corresponding stress f_{sc} from the stress strain curve of steel or from table 3.4 (Table A of SP16).

(c) Determine the depth of neutral axis x_u

$$x_u = \frac{0.87\ f_y\ A_{st} - f_{sc} \cdot A_{sc}}{0.36\ f_{ck} \cdot b}$$

This value should be equal to the assumed value. Otherwise, assume x_u equal to the value obtained in step C and repeat the steps (a) to (c).

(d) Moment of resistance of the section is given by

$$M_u = 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u) + f_{sc}\, A_{sc}\, (d - d')$$

If $x_u > x_{u,max}$, x_u is limited to $x_{u,max}$

$$M_u = 0.36\, f_{ck}\, b\, x_{u,max}\, (d - 0.42\, x_{u,max}) + f_{sc}\, A_{sc}\, (d - d')$$

Method-2 (Approximate Method) :

(a) Determine the stress in compression steel depending on d'/d ratio from Table 3.5. (Table-F of SP16)

(b) Determine the depth of neutral axis x_u

$$x_u = \frac{0.87 f_y\, A_{st} - f_{sc}\, A_{sc}}{0.36 f_{ck}\, b}$$

(c) Moment of resistance of the section is given by

$$M_u = 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u) + f_{sc}\, A_{sc}\, (d - d')$$

If $x_u > x_{u,max}$, x_u is limited to $x_{u,max}$

$$M_u = 0.36\, f_{ck}\, b\, x_{u,max}\, (d - 0.42\, x_{u,max}) + f_{sc}\, A_{sc}\, (d - d')$$

2. Design of Doubly Reinforced Section for a Given Moment of Resistance :

(a) Find $M_{u,lim}$ as a singly reinforced section

$$M_{u,lim} = 0.36\, f_{ck}\, b\, x_{u,max}\, (d - 0.42\, x_{u,max})$$

(b) Calculate corresponding steel

$$A_{st1} = \frac{M_{u,lim}}{0.87\, f_y\, (d - 0.42\, x_{u,max})}$$

or $$A_{st1} = P_{t,lim} \times \frac{bd}{100}$$

(c) Find the excess moment to be resisted by compression steel

$$M_{u2} = M_u - M_{u,lim}$$

(d) Find the stress in compression steel f_{sc} depending on d'/d ratio from Table 2.5. (Table-F of SP16)

(e) Find the area of compression steel

$$M_{u2} = f_{sc}\, A_{sc}\, (d - d')$$

$$A_{sc} = \frac{M_{u2}}{f_{sc}\, (d - d')}$$

(f) Find the additional tensile steel, A_{st2}

$$0.87\, f_y\, A_{st2} = f_{sc}\, A_{sc}$$

$$A_{st2} = \frac{f_{sc} \cdot A_{sc}}{0.87\, f_y}$$

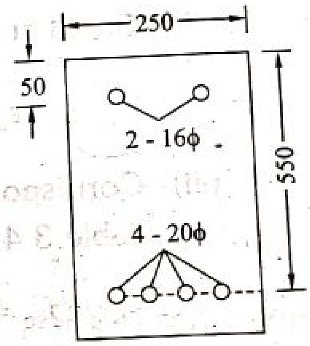
Total area of tension steel = $A_{st1} + A_{st2}$

EXAMPLE 17

A doubly reinforced beam of width 250 mm and 550 mm effective depth is reinforced with 2 bars of 16mm diameter as compression reinforcement at an effective cover of 50 mm and 4 bars 20 mm diameter as tension steel. Find the ultimate moment of resistance of the beam section. Use M20 grade concrete and Fe 415 steel.

(MARCH/APRIL. 2018, 2013)

Solution :



Breadth, $b = 250$ mm

Effective depth $d = 550$ mm

Effective cover to compression steel $d' = 50$ mm

Area of tensile steel, $A_{st} = 4 \times \frac{\pi}{4} \times 20^2 = 1256\ mm^2$

Area of compression steel, $A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402\ mm^2$

$f_{ck} = 25\ N/mm^2$

$f_y = 415\ N/mm^2$

Method - I (Exact Method) :

$$x_{u,max} = 0.48\, d = 0.48 \times 550 = 264\ mm$$

First Trail :

(i) Assume $x_u = x_{u,max} = 264$ mm

(ii) Strain at the level of compression steel

$$\varepsilon_{sc} = 0.0035\, \frac{(x_u - d')}{x_u}$$

$$= 0.0035\, \frac{(264-50)}{264} = 0.00284$$

(iii) Corresponding to this strain, read the stress from the Table 3.4 (Table-A of SP16).

$$f_{sc} = 352\ N/mm^2$$

4. **Depth of Neutral Axis :** To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36 f_{ck}\, b\, x_u + f_{sc} A_{sc} = 0.87 f_y\, A_{st}$$

$$0.36 \times 25 \times 250 \times x_u + 352 \times 402 = 0.87 \times 415 \times 1256$$

$$2250\, x_u + 141504 = 453478.8$$

$$x_u = 138.66\ mm$$

This is much less than the assumed value.

Second Trail :

(i) Reassume $x_u = 139\ mm$.

(ii) Strain at the level of compression steel

$$\varepsilon_{sc} = 0.0035\, \frac{(x_u - d')}{x_u} = 0.0035\, \frac{(139-50)}{139} = 0.00224$$

(iii) Corresponding to this strain, read the stress in compression steel from the Table 3.4. (Table-A of SP16)

$$f_{sc} = 336.56\ N/mm^2$$

(iv) Depth of neutral axis

To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36 f_{ck}\, b\, x_u + f_{sc} A_{sc} = 0.87 f_y\, A_{st}$$

$$0.36 \times 25 \times 250 \times x_u + 336.56 \times 402 = 0.87 \times 415 \times 1256$$

$$1800\, x_u + 138609.6 = 453478.8$$

$x_u = 141.41\ mm$, this is nearly equal to the assumed value.

So; $x_u = 141\ mm$

(v) The moment of resistance of the section is

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42\, x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 250 \times 141\, (550 - 0.42 \times 141) + 336.56 \times 402\, (550 - 50)$$

$$= 223.35 \times 10^6\ N\text{-}mm$$

$$= 223.35\ kN\text{-}m$$

Method - II (f_{sc} based on d/d' Ratio)

1. **Stress in Compression Steel :**

$$\frac{d'}{d} = \frac{50}{550} = 0.09$$

$$f_{sc} = 353.4\ N/mm^2 \text{ from the Table 3.5 (Table-F of SP16)}$$

2. **Depth of Neutral Axis :** To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st}$$

$$0.36 \times 25 \times 250 \times x_u + 353.4 \times 402 = 0.87 \times 415 \times 1256$$

$$2250\, x_u + 142066.8 = 453478.8$$

$$x_u = 138.4\ mm$$

Limiting depth of neutral axis

$$x_{u,max} = 0.48\, d = 0.48 \times 550 = 264\ mm$$

$$x_u < x_{u,max}$$

3. **Moment of Resistance :**

$$M_u = 0.36\, f_{ck}\, b\, x_u\, (d - 0.42\, x_u) + f_{sc}\, A_{sc}\, (d - d')$$

$$= 0.36 \times 25 \times 250 \times 138.4\, (550\text{-}0.42 \times 138.4) + 402 \times 353.4\, (550 - 50)$$

$$= 219.68 \times 10^6\ N\text{-}mm$$

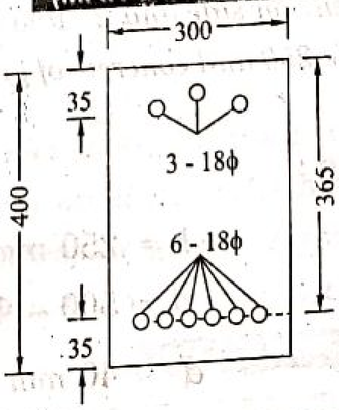
$$= 224.20\ kN\text{-}m$$

It is nearly equal to the value obtained in method 1.

EXAMPLE - 18

Calculate the ultimate moment of resistance of an R.C. beam of rectangular section 300 mm wide and 400 mm deep. Area of steel consists of 6 Nos 18 ϕ in tension side and 3 Nos 18 ϕ in compression side. Assume steel of grade Fe 415 and concrete of grade M 20 and an effective cover 35 mm on both sides.

(MARCH/APRIL. 2018, 2007)



Solution :

$b = 300\ mm$

$d = 400 - 35 = 365\ mm$

$d' = 35\ mm$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$$A_{st} = 6\times\frac{\pi}{4}\times18^2 = 1526.8\ mm^2$$

$$A_{sc} = 3\times\frac{\pi}{4}\times18^2 = 763.4\ mm^2$$

1. Stress in Compression Steel :

$$\frac{d'}{d} = \frac{35}{365} = 0.096$$

f_{sc} = 353.4 N/mm^2 from the Table 3.5 (Table-F of SP 16)

2. Depth of Neutral Axis :

To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36\ f_{ck}\ b\ x_u + f_{sc}\ A_{sc} = 0.87\ f_y\ A_{st}$$

$$0.36\times20\times300\times x_u + 353.4\times763.4 = 0.87\times415\times1526.8$$

$$2160\ x_u + 269785.6 = 551251.14$$

$$x_u = 130.3\ mm$$

Limiting depth of neutral axis

$$x_{u,max} = 0.48\ d = 0.48\times365 = 175.2\ mm$$

$$x_u < x_{u,max}$$

3. Moment of Resistance :

$$M_u = 0.36\ f_{ck}\ b\ x_u\ (d-0.42\ x_u) + f_{sc}\ A_{sc}\ (d-d')$$

$$= 0.36\times20\times300\times130.3\ (365-0.42\times130.3) + 353.4\times763.4\ (365-35)$$

$$= 176.36\times10^6\ N\text{-}mm = 176.36\ kN\text{-}m$$

EXAMPLE - 19

Calculate the moment of resistance of a reinforced concrete beam of rectangular section 250 mm wide and 500 mm deep, if it is reinforced with 6 numbers of 20 mm bars on tension side and 2 numbers of 20 mm bars on compression side. Assume steel of grade Fe 250 and concrete of grade M 20. Effective cover provided is 40 mm on both the sides.

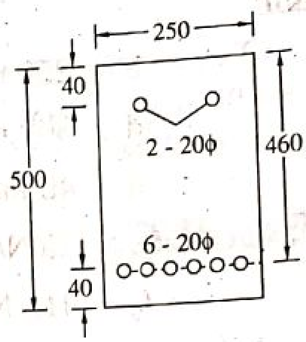
OCT/NOV. 2016 [TS]

Solution :

$$b = 250\ mm$$

$$d = 500-40 = 460\ mm$$

$$d' = 40\ mm$$



$f_{ck} = 20\ N/mm^2$

$f_y = 250\ N/mm^2$

$$A_{st} = 6 \times \frac{\pi}{4} \times 20^2 = 1885\ mm^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2 = 628.3\ mm^2$$

1. **Stress in Compression Steel :**

$$\frac{d'}{d} = \frac{40}{460} = 0.09 < 0.2, \text{ Hence for mild steel}$$

$f_{sc} = 0.87\ f_y = 0.87 \times 250 = 217.5\ N/mm^2$ from the Table 3.5 (Table-F of SP16)

2. **Depth of Neutral Axis :** To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36\ f_{ck}\ b\ x_u + f_{sc}\ A_{sc} = 0.87\ f_y\ A_{st}$$

$$0.36 \times 20 \times 250 \times x_u + 217.5 \times 628.3 = 0.87 \times 250 \times 1885$$

$$1800\ x_u + 136655.3 = 409987.5$$

$$x_u = 151.85\ mm$$

Limiting depth of neutral axis

$$x_{u,max} = 0.53\ d = 0.53 \times 460 = 243.8\ mm$$

$$x_u < x_{u,max}$$

3. **Moment of Resistance :**

$$M_u = 0.36\ f_{ck}\ b\ x_u\ (d - 0.42\ x_u) + f_{sc}\ A_{sc}\ (d - d')$$

$$= 0.36 \times 20 \times 250 \times 151.85\ (460 - 0.42 \times 151.85) + 217.5 \times 628.3\ (460 - 40)$$

$$= 165.69 \times 10^6\ N\text{-}mm$$

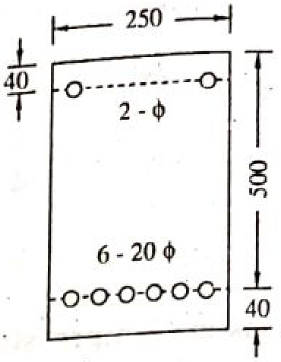
$$= 165.69\ kN\text{-}mm$$

EXAMPLE - 20

A doubly reinforced beam of width 250 mm and 500 mm effective depth is reinforced with 6 numbers of 20 mm bars on tension side and 2 numbers of 20 mm bars on compression side. Find the ultimate moment of resistance of the section.. Effective cover provided is 40 mm on both the sides. Concrete of grade is M 25 and steel is Fe 415

(MARCH/APRIL-2014)

Solution:



$$b = 250 \text{ mm}$$

$$d = 500 \text{ mm}$$

$$d = 40 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{st} = 6\times\frac{\pi}{4}\times 20^2 = 1885 \text{ mm}^2$$

$$A_{sc} = 2\times\frac{\pi}{4}\times 20^2 = 628.3 \text{ mm}^2$$

(i) Stress in compression steel

$$\frac{d}{d} = \frac{40}{500} = 0.08 < 0.2$$

f_{sc} = 354.2 N/mm^2 from the table 3.5 (Table-F of SP16)

(ii) Depth of neutral axis

To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36\, f_{ck}\, b\, x_u + f_{sc}\, A_{sc} = 0.87\, f_y\, A_{st}$$

$$0.36 \times 20 \times 250 \times x_u + 354.2 \times 628.3 = 0.87 \times 415 \times 1885$$

$$1800\, x_u + 222543.9 = 680579.3$$

$$x_u = 254.5 \text{ mm}$$

Limiting depth of neutral axis

$$x_{u,max} = 0.48\, d = 0.48 \times 500 = 240 \text{ mm}$$

$$x_u > x_{u,max}$$

But as per code, x_u is limited to $x_{u,max}$

(iii) Moment of Resistance

$$M_u = 0.36\, f_{ck}\, b\, x_{u,max}\, (d - 0.42\, x_{u,max}) + f_{sc}\, A_{sc}\, (d - d')$$

$$= 0.36 \times 20 \times 415 \times 240\, (500 - 0.42\times 240) + 354.2 \times 628.3\, (500-40)$$

$$= 388.64 \times 10^6 \text{ N-mm}$$

$$= 388.64 \text{ kN-m}$$

EXAMPLE - 21

Design a rectangular reinforced concrete beam for a clear span of 4000 mm. The super imposed load is 35 kN / m and the size of the beam is limited to 250 mm × 400 mm. Use M 20 grade concrete and Fe 415 steel. Support width is 300 mm each and effective cover is 40 mm.

(MARCH/APRIL. 2006)

Solution :

$b = 250$ mm

$d = 400 - 40 = 360$ mm

$d' = 40$ mm

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

1. **Effective Span :** Least of

Centre to centre of supports = 4 + 0. 3 = 4.3 m

Clear span + d = 4 + 0.36 = 4.36 m

Hence, Effective span = 4.3 m

2. **Loads**

Self weight of the beam= 0.25 × 0.4 × 25 = 2.5 kN/m

Super imposed load = 35 kN/m

Total load = 35 + 2.5 = 37.5 KN/m

Factored load $w_u = 1.5 \times 37.5 = 56.25$ kN/m

Factored Bending moment $M_u = \frac{w_u\, l^2}{8} = \frac{56.25 \times 4.3^2}{8} = 130$ kN-m

3. **Limiting moment of resistance of the given section as a singly reinforced section**

$$M_{u,lim} = 0.138.\, f_{ck}.\, bd^2$$

$$= 0.138 \times 20 \times 250 \times 360^2$$

$$= 89.42 \times 10^6 \text{ N-mm}$$

$$= 89.42 \text{ kN-m}$$

As $M_u > M_{u,lim}$, the section should be designed as a doubly reinforced section.

4. **Area of Tension Steel Corresponding to $M_{u,lim}$ (A_{st1})**

$$0.87\, f_y\, A_{st1} = 0.36\, f_{ck}\, b\, x_{u,max}$$

$$A_{st1} = \frac{0.36 f_{ck}\, b\, x_{u,max}}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 250 \times 0.48 \times 360}{0.87 \times 415} = 861.5 \ mm^2$$

5. Compression Reinforcement (A_{sc}) :

For $\frac{d'}{d} = \frac{40}{360} = 0.11$, stress in compression steel f_{sc} from table F of SP16

$f_{sc} = 351 \ N/mm^2$

[For design problems take f_{sc} from Table-F of SP16 for different values of $\frac{d'}{d}$.

For analysis problems (to find M_u) take f_{sc} from Table-A of SP16 or from stress strain curve of corresponding steel]

The remaining bending moment has to be resisted by couple consisting of compression steel and the corresponding tension steel.

$$M_{u2} = M_u - M_{u,lim}$$

$$= 130 - 89.42 = 40.58 \ KN\text{-}m$$

$$M_{u2} = f_{sc} A_{sc} (d - d')$$

$$40.58 \times 10^6 = 351 \times A_{sc} (360 - 40)$$

$$A_{sc} = \frac{40.58 \times 10^6}{351 (360 - 40)} = 361.29 \ mm^2$$

6. Additional Tensile Steel (A_{st2}) :

$$0.87 f_y A_{st2} = f_{sc} A_{sc}$$

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{351 \times 361.29}{(0.87 \times 415)} = 351.23 \ mm^2$$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$= 861.5 + 351.2$

$= 1212.7 \ mm^2$

Provide 4 - 20 *mm* bars in tension ($A_{st} = 1256 \ mm^2$) and

2 - 16 *mm* bars in compression ($A_{sc} = 402 \ mm^2$)

USING SP-16 :

$$M_u/bd^2 = \frac{130\times10^6}{250\times360^2} = 4.01$$

$$\frac{d'}{d} = \frac{40}{360} = 0.11$$

Refer Table 50 of SP-16 corresponding to $f_y = 415\ N/mm^2$ and $f_{ck} = 20\ N/mm^2$, read the corresponding values of percentage of reinforcements P_t and P_c

$$P_t = 1.342 \text{ and } P_c = 0.408$$

$$A_{st} = P_t\frac{bd}{100} = \frac{1.342}{100}\times 250\times 360 = 1208\ mm^2$$

$$A_{sc} = P_c\frac{bd}{100} = \frac{0.408}{100}\times 250\times 360 = 367\ mm^2$$

EXAMPLE - 22

Determine the main tensile and compressive reinforcement required for a rectangular beam with the following data.

Overall size of the beam = 250 × 550 mm

Concrete grade = M 20

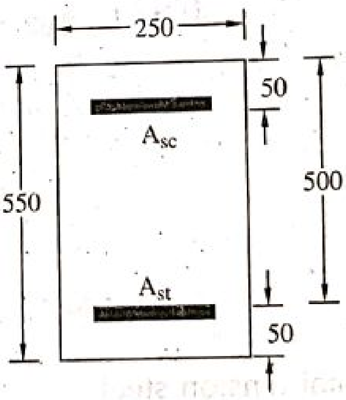
Characteristic strength of steel = 415 N/mm²

Factored moment = 200 kN-m

Effective cover = 50 mm

(APRIL/MAY. 2011 ; SEPTEMBER/OCTOBER. 2004)

Solution :



$$b = 250\ mm$$

$$d = 550 - 50 = 500\ mm$$

$$d' = 50\ mm$$

$$f_{ck} = 20\ N/mm^2$$

$$f_y = 415\ N/mm^2$$

$$M_u = 200\ kN\text{-}m$$

1. Limiting Moment of Resistance of the given Section as a Singly Reinforced Section :

$$M_{u,lim} = 0.138.\ f_{ck}.bd^2 = 0.138\times 20\times 250\times 500^2$$

$$= 172.5\times 10^6\ N\text{-}mm$$

$$= 172.5\ kN\text{-}m$$

As $M_u > M_{u,lim}$, the section should be designed as a doubly reinforced section

2. Area of Tension Steel Corresponding to $M_{u,lim}$ (A_{st1}) :

$$0.87\, f_y\, A_{st1} = 0.36\, f_{ck}\, b\, x_{u,max}$$

$$A_{st1} = \frac{0.36 f_{ck} b\, x_{u,max}}{0.87 f_y} = \frac{0.36\times20\times250\times0.48\times500}{0.87\times415} = 1196.5\ mm^2$$

3. Compression Reinforcement (A_{sc}) :

For $\frac{d'}{d} = \frac{50}{500} = 0.1$, stress in compression steel f_{sc} from table F of SP16

$$f_{sc} = 353\ N/mm^2$$

$$M_{u2} = M_u - M_{u,lim}$$

$$= 200 - 172.5 = 27.5\ KN\text{-}m$$

$$M_{u2} = f_{sc}\, A_{sc}\, (d - d')$$

$$27.5\times10^6 = 353 \times A_{sc}\, (500 - 50)$$

$$A_{sc} = \frac{27.5\times10^6}{353\,(500-50)}$$

$$= 173.1\ mm^2$$

4. Additional Tensile Steel (A_{st2}) :

$$0.87\, f_y\, A_{st2} = f_{sc}\, A_{sc}$$

$$A_{st2} = \frac{f_{sc}\, A_{sc}}{0.87\, f_y}$$

$$= \frac{353\times173.1}{(0.87\times415)}$$

$$= 169.3\ mm$$

Total tension steel

$$A_{st} = A_{st1} + A_{st2}$$

$$= 1196.5 + 169.3$$

$$= 1365.8\ mm^2$$

Provide 5 - 20 *mm* bars in tension (A_{st} = 1570.8 mm^2) and

2 - 12 *mm* bars in compression (A_{sc} =226 mm^2)

EXAMPLE - 23

Design the reinforcement for a rectangular beam with the following data.

Overall size of the beam = 250 × 450 mm.

Factored moment = 170 kN-m

Effective cover on both sides = 50 mm

Use M 20 grade concrete and Fe 250 steel.

Solution :

Breadth, $b = 250$ mm

Effective depth, $d = 450 - 50 = 400$ mm

Effective cover to compression steel $d' = 50$ mm

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 250 \text{ N/mm}^2$

1. Limiting Moment of Resistance of the given Section as a Singly Reinforced Section :

$$M_{u,lim} = 0.148.\ f_{ck}.bd^2$$

$$= 0.148 \times 20 \times 250 \times 400^2$$

$$= 118.4 \times 10^6 \text{ N-mm}$$

$$= 118.4 \text{ KN-m}$$

As $M_u > M_{u,lim}$, the section should be designed as a doubly reinforced section.

2. Area of Tension Steel Corresponding to $M_{u,lim}$ (A_{st1}) :

$$0.87\ f_y\ A_{st1} = 0.36\ f_{ck}\ b\ x_{u,max}$$

$$A_{st1} = \frac{0.36\ f_{ck}\ b\ x_{u,max}}{0.87\ f_y}$$

$$= \frac{0.36 \times 20 \times 250 \times 0.53 \times 400}{0.87 \times 250} = 1754.5 \text{ mm}^2$$

3. Compression Reinforcement (A_{sc}) :

$$\frac{d'}{d} = \frac{50}{400} = 0.125 < 0.2, \text{ Hence for mild steel}$$

$$f_{sc} = 0.87\ f_y = 0.87 \times 250 = 217.5 \text{ N/mm}^2$$

$$M_{u2} = M_u - M_{u,lim}$$

$$= 170 - 118.4 = 51.6 \text{ kN-m}$$

$$M_{u2} = f_{sc} A_{sc} (d - d')$$

$$51.6 \times 10^6 = 217.5 \times A_{sc} (400 - 50)$$

$$A_{sc} = \frac{51.6 \times 10^6}{217.5\,(400-50)} = 677.8 \; mm^2$$

4. **Additional Tensile Steel (A_{st2}) :**

$$0.87 \, f_y \, A_{st2} = f_{sc} \, A_{sc}$$

$$A_{st2} = \frac{f_{sc} \, A_{sc}}{0.87 \, f_y} = \frac{217.5 \times 677.8}{(0.87 \times 250)} = 677.8 \; mm^2$$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$$= 1754.5 + 677.8 = 2432.3 \; mm^2$$

Provide 8 - 20 mm bars in tension (A_{st} = 2513.3 mm^2) and

4 - 16 mm bars in compression (A_{sc} = 804.3 mm^2)

[In case of design of doubly reinforced beam with mild steel, A_{st2} will be equal to A_{sc} as the design stress is same (= 0.87 f_y) both in compression and tension when $d'/d < 0.2$]

EXAMPLE - 24

A doubly reinforced beam of width 230 mm and 500 mm total depth is reinforced with 4 bars of 16 mm diameter as compression reinforcement and 6 bars 20 mm diameter as tension steel at an effective cover of 40 mm on both the sides. Find the safe uniformly distributed load the beam can carry if it is simply supported over an effective span of 5 m. Use M 20 grade concrete and Fe 415 steel.

(OCT/NOV. 2016 [AP] ; 2009)

Solution :

b = 230 mm

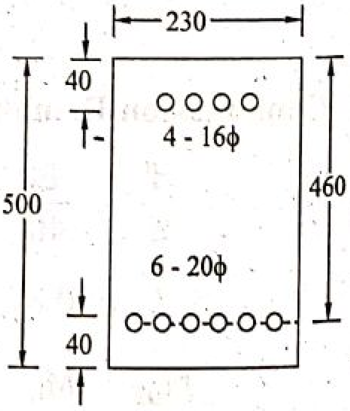
d = 500 − 40 = 460 mm

d' = 40 mm

f_{ck} = 20 N/mm²

f_y = 415 N/mm²

l = 5 m



$$A_{st} = 6 \times \frac{\pi}{4} \times 20^2 = 1885 \; mm^2$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 16^2 = 804.2 \; mm^2$$

1. **Stress in Compression Steel :**

$$\frac{d'}{d} = \frac{40}{460} = 0.087$$

$$f_{sc} = 353.5 \text{ N/mm}^2 \text{ from the Table 3.5 (Table-F of SP16)}$$

2. **Depth of Neutral Axis :** To find the depth of neutral axis, equate total compressive force to tensile force

$$0.36 f_{ck}\, b\, x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 20 \times 230 \times x_u + 353.5 \times 804.2 = 0.87 \times 415 \times 1885$$

$$1656\, x_u + 284284.7 = 680579.25$$

$$x_u = 239.3 \text{ mm}$$

Limiting depth of neutral axis

$$x_{u,max} = 0.48\, d$$

$$= 0.48 \times 460 = 220.8 \text{ mm}$$

$$x_u > x_{u,max}, \quad x_u \text{ is limited to } x_{u,max}$$

$$x_u = x_{u,max} = 220.8 \text{ mm}$$

3. **Moment of Resistance :**

$$M_u = 0.36 f_{ck}\, b\, x_u (d - 0.42\, x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 20 \times 230 \times 220.8 (460 - 0.42 \times 220.8) + 353.5 \times 804.2 (460 - 40)$$

$$= 253.69 \times 10^6 \text{ N-mm} = 253.69 \text{ kN-m}$$

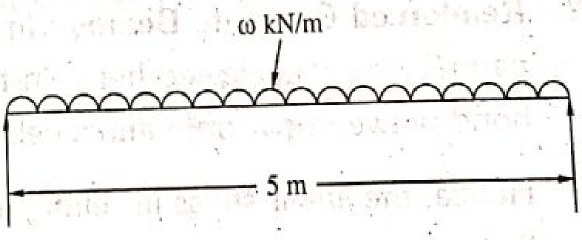
4. **Safe Load :** Let w_u kN/m be the safe ultimate load the beam can carry

Factored bending moment $= w_u l^2 / 8 = w_u 5^2/8 = 3.125\, w_u$

Equating factored bending moment to the Moment of resistance of the section

$$3.125\, w_u = 253.69$$

$$w_u = \frac{253.69}{3.125} = 81.18 \text{ kN/m}$$



Safe working load $w = \dfrac{w_u}{\text{load factor}}$

$$= \frac{w_u}{1.5} = \frac{81.18}{1.5} = 54.12 \text{ kN/m (including self weight)}$$

3.4 SHEAR

Bending is usually accompanied by shear. The combination of shear and bending stresses produces the principle stress which causes diagonal tension in the beam section. The diagonal tensile stress caused by the shear and combination of shear and bending is likely to cause failure of the section by producing cracks as shown in the Fig. 3.8 This should be resisted by providing shear reinforcement in the form of vertical stirrups or bent up bars along with stirrups.

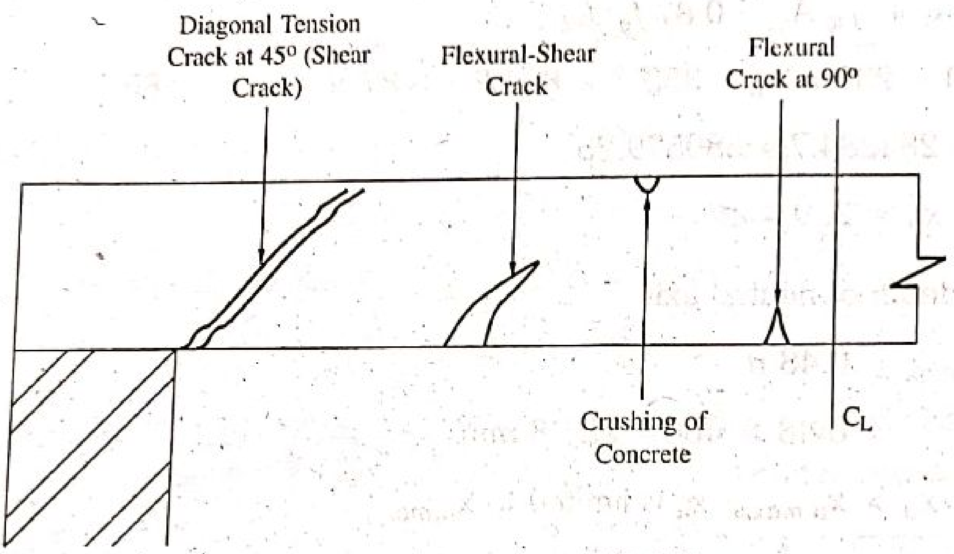


FIG 3.8 : Crack Pattern in Beams (Simply Supported Beam)

3.4.1 SHEAR STRESS IN BEAMS

1. **Homogeneous Beams :** The distribution of shear stress across the homogeneous beam of rectangular section is shown in Fig. 3.9 (a). The variation of shear stress is parabolic. It is zero at the top and bottom and is maximum at the neutral axis.

The maximum shear stress = 1.5 × Average shear stress

$$\tau_{max} = 1.5\ \tau_{ave} = 1.5 \times \frac{V}{bd}$$

Where V = Shear force

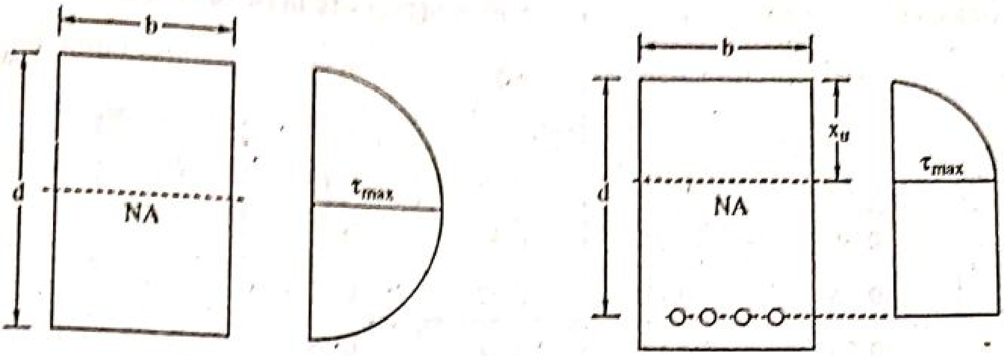
2. **Reinforced Concrete Beams :** In reinforced concrete beams, the concrete below the neutral axis is neglected being in tension. The shear is assumed to be resisted by the bond between concrete and steel.

Hence, the shear stress in reinforced concrete beams varies parabolically with zero at the top compression face, reaching maximum value at the neutral axis and constant from neutral axis up to the centre of gravity of steel bars as shown in Fig. 3.9 (b).

The maximum shear stress is given by

$$T_{max} = \frac{V}{b.z}$$

Where z = lever arm



(a) Homogeneous Beams (b) Reinforced Concrete Beam

FIG 3.9 : Shear Stress Distribution in Beams

However the procedure presented by IS: 456-2000 is based on average shear stress across the section. The average shear stress also called as nominal shear stress is given by

Nominal Shear stress $\tau_v = \frac{V_u}{b.d}$

Where V_u = Ultimate shear force

d = Effective depth

b = Width of the member

In case of beams of varying depth

Nominal Shear stress $\tau_v = \frac{V_u \pm \left(\frac{M_u}{d}\right) \tan\beta}{bd}$

Where M_u = Factored bending moment at the section

β = Angle between top and bottom edges of the beam

(Negative sign when bending moment increases with the increase in effective depth & positive when BM decreases with the increases in effective depth)

3.4.2 DESIGN SHEAR STRENGTH OF CONCRETE

Concrete is capable of resisting shear force to some extent. This value depends mainly on grade of concrete and the area of steel provided for resisting bending moment.

Table 19 of IS : 456-2000 and Table 61 of SP:16 gives the design shear strength of concrete in beams as a function of grade of concrete and percentage of tension steel.

However in finding the percentage of steel in any section, the steel bars should extend at least one effective depth beyond the section considered.

Table 3.6 : Design Shear Strength of Concrete τ_c, N/mm^2 **(Table 19 in IS : 456-2000)**

$100\ A_{st}/bd$	M15	M20	M25	M30	M35	M40 and above
≤ 0.15	0.28	0.28	0.29	0.29	0.29	0.30
0.25	0.35	0.36	0.36	0.37	0.37	0.38
0.50	0.46	0.48	0.49	0.50	0.50	0.51
0.75	0.54	0.56	0.57	0.59	0.59	0.60
1.00	0.60	0.62	0.64	0.66	0.67	0.68
1.25	0.64	0.67	0.70	0.71	0.73	0.74
1.50	0.68	0.72	0.74	0.76	0.78	0.79
1.75	0.71	0.75	0.78	0.80	0.82	0.84
2.00	0.71	0.79	0.82	0.84	0.86	0.88
2.25	0.71	0.81	0.85	0.88	0.90	0.92
2.50	0.71	0.82	0.88	0.91	0.93	0.95
2.75	0.71	0.82	0.90	0.94	0.96	0.98
3.00 and above	0.71	0.82	0.92	0.96	0.99	1.01

3.4.3 MAXIMUM SHEAR STRESS IN CONCRETE

To avoid compression failure of the section in shear, the nominal shear stress τ_v should not exceed the maximum shear stress in concrete $\tau_{c,max}$ values given in table 20 of IS: 456-2000.

Table 3.7 : Maximum Shear Stress $\tau_{c,max}$, N/ mm^2 **(Table 20 in IS 456-2000)**

Concrete Grade	M15	M20	M25	M30	M35	M40 on Words
$t_{c,max}$	2.5	2.8	3.1	3.5	3.7	4.0

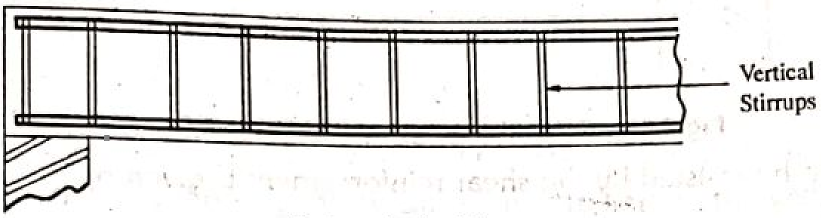
3.4.4 DESIGN OF SHEAR REINFORCEMENT

Shear reinforcement has to be provided against diagonal tensile stresses caused by the shear force. The longitudinal bars do not prevent the diagonal tension failure. The inclined shear crack starts at the bottom near the support and extend towards compression zone. The shear reinforcement can be provided in any of the following forms as shown in Fig 3.10.

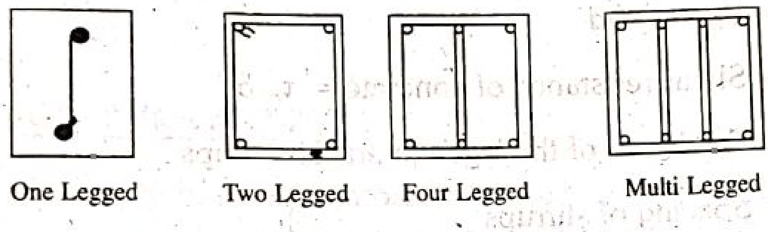
(a) Vertical stirrups

(b) Bent up bars along with stirrups

(c) Inclined stirrups



(a) Longitudinal View



(b) Cross Sectional View

Vertical Stirrups

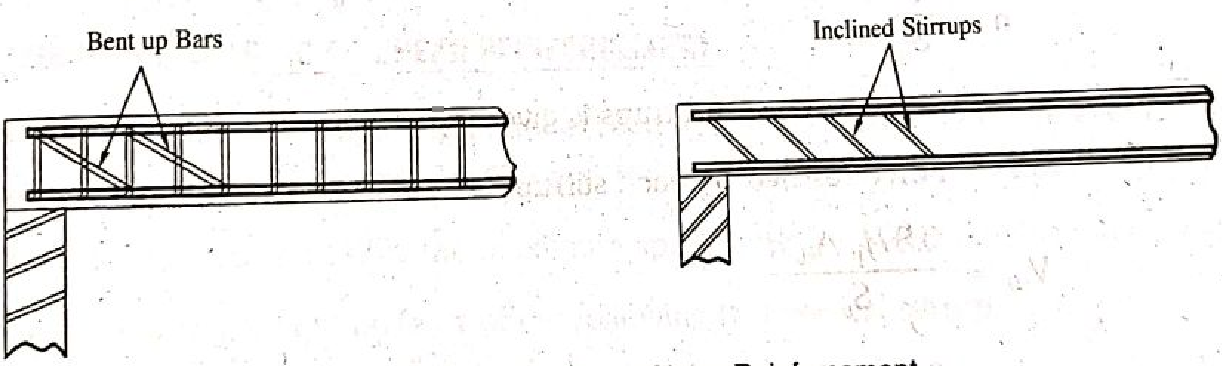


Fig 3.10 : Different Forms of Shear Reinforcement

VERTICAL STIRRUPS :

Fig. 3.11. shows the general arrangement of vertical stirrups. Generally the vertical stirrups are provided as two legged or four legged stirrups bend round the tensile reinforcement and taken to the compression zone and anchored to the hanger bars. Hanger bars are provided to keep vertical stirrups in position otherwise they may get displaced while concreting.

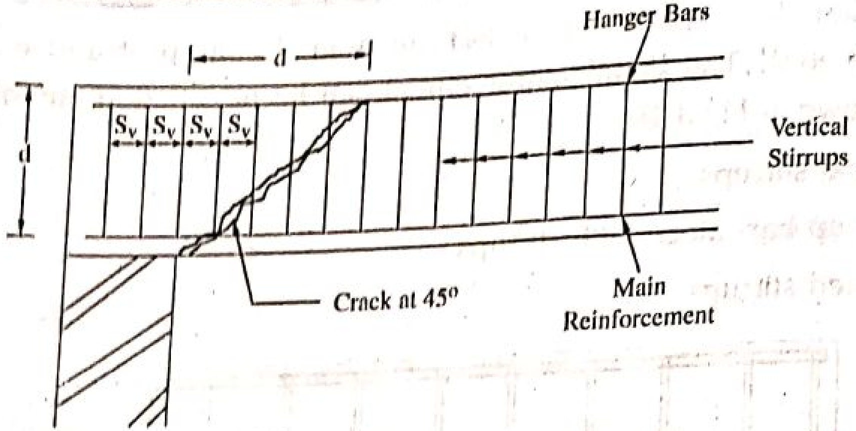


Fig 3.11 : General Arrangement of Vertical Stirrups

The shear to be resisted by the shear reinforcement is given by

$$V_{us} = V_u - V_{uc}$$

$$= V_u - \tau_c bd$$

Where V_{uc} = Shear resistance of concrete = $\tau_c bd$

Let A_{sv} = Total area of the legs of vertical stirrups

S_v = Spacing of stirrups

d = Effective depth of the section

The number of stirrups cut by the 45° crack line is

$$n = \frac{d}{S_v}$$

Total shear resistance of vertical stirrups is given by

S.F = Force resisted by each stirrup x Number of stirrups

$$V_{us} = \frac{0.87 f_y A_{sv}.d}{S_v}$$

$$S_v = \frac{0.87 f_y A_{sv}.d}{V_{us}}$$

BENT-UP BARS :

Some of longitudinal bars can be bent up near the supports as the bending moment to be resisted near the supports is very little. Such bent up bars resist diagonal tension. If all the bars are bent up at the same cross section at an angle of α , the shear resistance of bent up bars is given by

$$V_{usb} = 0.87\, f_y\, A_{sb} \sin\alpha$$

Where V_{usb} = Shear resistance of bent up bars

A_{sb} = Total area of bent up bars

α = Angle between the bent up bars and the axis of the member (> 45°)

If the bent up bars or inclined stirrups are provided at a spacing of S_v, the shear resistance of bent up bars

$$V_{usb} = 0.87\, f_y\, A_{sb} (\sin\alpha + \cos\alpha)\frac{d}{S_v}$$

The shear resistance of bent up bars shall not exceeds 50 % of the total shear to be resisted by the shear reinforcement. Because bent up bars alone (with out stirrups) are not effective in preventing shear failure.

$$V_{usb} < \frac{V_{us}}{2}$$

3.4.5 MINIMUM SHEAR REINFORCEMENT

The minimum quantity of shear reinforcement that should be provided for all beams except those of minor importance like lintels is given IS : 456, Clause 26.5.1.6 by the revised equation

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{0.87 f_y}$$

3.4.6 MAXIMUM SPACING OF SHEAR REINFORCEMENT

Spacing of vertical stirrups should not exceed **0.75 *d* (or) 300 *mm*** which ever is less as given in Clause 26.5.1.5 of IS : 456 and the diameter should not less then 6 *mm*.

For inclined stirrups at 45°, the maximum spacing is *d* or 300 *mm* which ever is less.

Hence, spacing should be least of the following (for vertical stirrups).

(a) Spacing calculated to resist V_{us}

$$S_v = \frac{0.87 f_y\, A_{sv}\, d}{V_{us}}$$

(b) Spacing calculated from minimum shear reinforcement consideration

$$S_v = \frac{0.87\, f_y\, A_{sv}}{0.4\, b}$$

(c) 0.75 *d*

(d) 300 *mm*

3.4.7 PROCEDURE FOR DESIGN OF SHEAR REINFORCEMENT

1. Calculate the Factored Shear force V_u in the beam
2. Calculate the nominal shear stress

$$\tau_v = \frac{V_u}{bd}$$

3. Calculate the % of tension reinforcement at the section P_t and obtain the design shear strength of concrete τ_c from table 19 of IS : 456.

Case (a) : When $\tau_v < \tau_c$, provide minimum shear reinforcement as explained in 2.4.5.

Case (b). When $\tau_v > \tau_c$, (and less than $\tau_{c,max}$ given in table 20 of IS : 456), design the shear reinforcement as explained in 3.4.4.

Case (c) : When $\tau_v > \tau_{c,max}$, the section must be redesigned such that the nominal shear stress falls with in the maximum limit.

EXAMPLE - 25

The ultimate shear force at a section of a RCC beam is 200 KN. The effective depth and width of the beam is 500 mm and 250 mm and 1 % steel is provided. What is the shear for which shear reinforcement is required.

(NOVEMBER. 2003)

Solution :

$$b = 250\ mm$$

$$d = 500\ mm$$

Ultimate shear force,

$$V_u = 200\ kN$$

Percentage of tension steel

$$P_t = 1\ \%$$

Referring to the Table. 19 of IS : 456, Shear strength of concrete is (assuming M20 grade concrete)

$$\tau_c = 0.62\ N/mm^2$$

Shear resistance of concrete

$$V_{uc} = \tau_c\ bd = 0.6 \times 250 \times 500 = 77500\ N = 77.5\ kN$$

Shear to be resisted by shear reinforcement

$$V_{us} = V_u - V_{uc} = 200 - 77.5 = 122.5\ kN$$

EXAMPLE - 26

A singly reinforced rectangular beam 230 mm × 450 mm effective depth is subjected to a shear force of 40 kN under working loads. Calculate the nominal shear stress in concrete.

(MARCH/APRIL-2016, 2013, OCT/NOV-2015)

Solution :

$$b = 230\text{mm}$$

$$d = 450\text{mm}$$

Shear force, $V = 40$ kN

Factored Shear force $V_u = 1.5 \times 40 = 60$ kN

Nominal Shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{60\times10^3}{230\times450} = 0.58 \text{ N/mm}^2$

EXAMPLE - 27

Calculate the spacing of two legged 8 mm diameter vertical stirrups as per minimum shear reinforcement for a beam of 350 mm wide and 500 mm overall depth if Fe 415 bars are used. Effective cover is 35 mm.

(OCT/NOV-2013, APRIL/MAY-2012)

Solution :

$$b = 350 \text{ mm}$$

$$D = 500 \text{ mm}$$

Effective depth $d = 500 - 35 = 465$ mm

For 2-legged 8mm stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.5 \text{ mm}^2$$

Spacing from minimum shear reinforcement consideration as per IS 456

$$\frac{A_{sv}}{b\,S_v} = \frac{0.4}{0.87\,f_y}$$

$$S_v = \frac{0.87\,f_y\,A_{sv}}{0.4\times b} = \frac{0.87\times415\times100.5}{0.4\times350} = 259.2 \text{ mm}$$

Maximum allowed spacing

$$= 0.75d = 0.75 \times 465 = 348.75 \text{ mm}$$

Or 300 mm which ever is less.

Spacing should be least of the above.

Hence provide 2-legged 8 mm stirrups @ 250 mm c/c.

EXAMPLE - 28

Determine the spacing of 8 mm 2 legged stirrups for R.C.C beam of 230 mm wide and 450 mm effective depth to resist a factored shear force of 85 kN. Use M20 concrete and Fe 250 steel.

(OCT/NOV. 2006)

Solution :

$$b = 230 \text{ mm}$$

$$d = 450 \text{ mm}$$

$$V_{us} = 85 \text{ kN} = 85000 \text{ N}$$

For 2-legged 8 mm stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5 \text{ mm}^2$$

Spacing of stirrups to resist V_{us}

$$S_v = \frac{0.87 f_y . A_{sv} \, d}{V_{us}}$$

$$= \frac{0.87 \times 250 \times 100.5 \times 450}{85000} = 115.7 \text{ mm}$$

Spacing from minimum shear reinforcement consideration as per IS : 456

$$\frac{As_v}{b \, S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y \, A_{sv}}{0.4 b} = \frac{0.87 \times 250 \times 100.5}{0.4 \times 230} = 237.6 \text{ mm}$$

Maximum allowed spacing = $0.75 \, d = 0.75 \times 450 = 337.5$ mm

or 300 mm which ever is less.

Spacing should be least of the above.

Hence provide 2-legged 8 mm stirrups @ 115 mm c/c

EXAMPLE - 29

An R.C.C. beam 230 mm wide and 450 mm deep is reinforced with 4 bars of 16 mm Φ and of grade Fe 415 on tension side. If design shear force is 60 kN. Design the shear reinforcement consisting only of vertical stirrups. The grade of concrete used is M20.

(APRIL/MAY. 2011 ; MARCH/APRIL. 2007)

Solution :

Breadth $b = 230$ mm

Gross depth, $D = 450$ mm

Design shear force, $V_u = 60 \text{ kN} = 60000 \text{ N}$

Area of steel, $A_{st} = 4\times\frac{\pi}{4}\times 16^2 = 804.3 \text{ mm}^2$

Assuming an effective cover of 50 mm, effective depth, $d = 450 - 50 = 400$ mm

1. **Nominal Shear Stress :**

Nominal shear stress

$$\tau_v = \frac{V_u}{bd} = \frac{60\times10^3}{230\times400} = 0.65 \text{ N/mm}^2$$

2. **Shear Resistance of Concrete :**

Percentage of tension steel at support

$$P_t = \frac{A_{st}\times100}{bd} = \frac{804.3\times100}{230\times400} = 0.874\ \%$$

Referring to the Table - 19 of IS : 456, shear strength of concrete is

$$\tau_c = 0.59 \text{ N/mm}^2$$

(Interpolation has to be done for intermediate values of P_t as given below)

P_t, %	τ_c N/mm^2
0.75	0.56
1.0	0.62
0.874	$0.56 + \frac{(0.62-0.56)}{(1.0-0.75)}\times(0.874-0.75) = 0.59$

Maximum shear stress in concrete

$$\tau_{c,max} = 2.8 \text{ N/mm}^2$$

As $\tau_v < \tau_{c,max}$ and $\tau_v > \tau_c$, Shear reinforcement has to be designed.

3. **Design of Vertical Stirrups :** Shear to be resisted by shear reinforcement (Vertical stirrups)

$$V_{us} = V_u - \tau_c\, bd$$
$$= 60\times10^3 - 0.59\times230\times400 = 5720 \text{ N}$$

Spacing of 2-legged 6 mm stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times6^2 = 56.5 \text{ mm}^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 56.5 \times 400}{5720} = 1426.5 \text{ mm}$$

Spacing from minimum shear reinforcement consideration as per IS : 456.

$$\frac{A_{sv}}{b S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 56.66}{0.4 \times 230} = 221.7 \text{ mm}$$

Maximum allowed spacing = 0.75 d = 0.75 × 400 = 300 mm

(or) 300 mm which ever is less

Spacing should be least of above.

Hence provide 2-legged 6 mm stirrups @ 220 mm c/c through out the span of the beam.

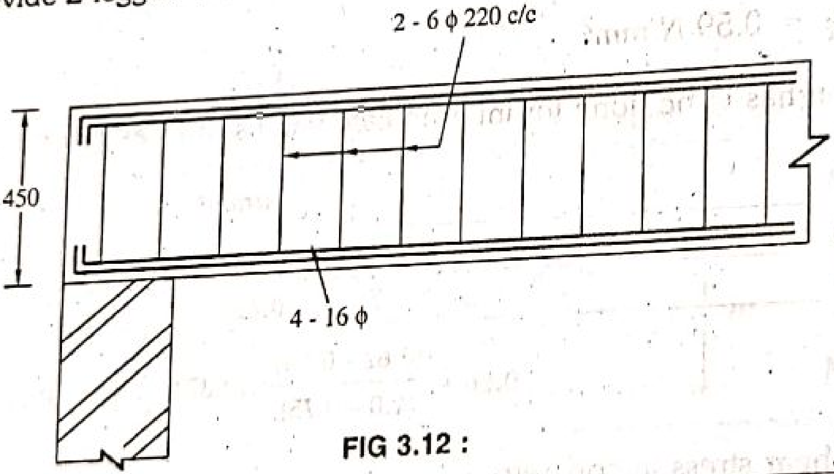


FIG 3.12 :

EXAMPLE - 30

A singly reinforced rectangular beam 300 × 600 mm effective depth carries a uniformly distributed load of 40 kN/m including its self weight over a simply supported span of 6 m and is reinforced with 5 bars of 25 mm diameter of which 2 bars are cranked up near the support. Fe 415 grade steel and M 20 grade concrete are used. Design the beam for shear reinforcement at support.

(OCT. 2005)

Solution :

b = 300 mm

d = 600 mm

f_{ck} = 20 N/mm^2

f_y = 415 N/mm^2

1. Nominal Shear Stress :

Factored load $w_u = 1.5 \times 40 = 60$ kN/m

Factored Shear force $V_u = \frac{w_u\, l}{2} = \frac{60\times6}{2} = 180$ kN $= 180 \times 10^3$ N

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{180\times10^3}{300\times600} = 1$ N/mm²

2. Shear Resistance of Concrete :

Percentage of tension steel at support

$$P_t = \frac{A_{st}\times100}{bd} = \frac{3\times\frac{\pi}{4}\times25^2\times100}{300\times600} = 0.818\ \%$$

Referring to the Table. 19 of IS 456, Shear strength of concrete is

$$\tau_c = 0.58\ N/mm^2$$

P_t ,%	t_c N/mm²
0.75	0.56
1.0	0.62
0.818	$0.56 + \frac{(0.62-0.56)}{(1.0-0.75)} \times (0.818\text{-}0.75) = 0.58$

Maximum shear stress in concrete $\tau_{c,max}$ from Table. 20 of IS456

$$\tau_{c,max} = 2.8\ N/mm^2$$

As $\tau_v < \tau_{c,max}$ and $\tau_v > \tau_c$, Shear reinforcement has to be designed

Shear to be resisted by shear reinforcement (bent up bars & vertical stirrups)

$$V_{us} = V_u - \tau_c\, bd$$

$$= 180\times10^3 - 0.58 \times 300 \times 600 = 75600\ N$$

3. Shear Resistance of Bent up Bars :

Shear resistance of bent up bars

$$V_{usb} = 0.87\, f_y\, A_{sb} \sin\alpha$$

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 25^2 \times \sin 45^\circ$$

$$= 250641\ N$$

Theoretically the two inclined bars can carry shear force completely. But IS456 specifies half of the design shear must be resisted by vertical stirrups.

4. Design of Vertical Stirrups :

Shear resistance to be provided by the vertical stirrups $V_{usv} = \frac{V_{us}}{2} = \frac{75600}{2} = 37800\ N$

Spacing of 2-legged 6 *mm* stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.5\ mm^2$$

$$S_v = \frac{0.87 f_y\, A_{sv}.d}{V_{usv}} = \frac{0.87 \times 415 \times 56.5 \times 600}{37800}$$

$$= 323.8\ mm$$

Spacing from minimum shear reinforcement consideration as per IS 456

$$\frac{A_{sv}}{b \cdot S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87\, f_y\, A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 56.5}{0.4 \times 300} = 170\ mm$$

Maximum allowed spacing

$$= 0.75\ d = 0.75 \times 600 = 450\ mm$$

(or) 300 *mm* which ever is less.

Spacing should be least of the above.

Hence provide 2-legged 6 *mm* stirrups @ 170 *mm c/c* through out the span of the beam.

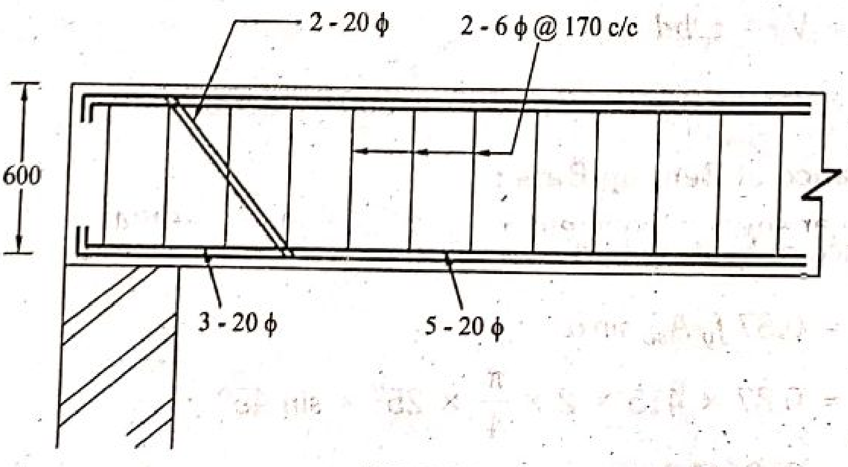


FIG 3.13 :

EXAMPLE - 31

An RCC beam 250 mm wide and 450 mm deep effective is reinforced with 6 bars of 16 mm diameter on tension side of which two bars are cranked up near the support. If the design shear force is 120 kN, design the shear reinforcement considering bent-up bars. Concrete is M-20 grade and steel Fe-415.

MARCH/APRIL. 2016 ; OCT/NOV. 2015

Solution :

$$b = 250 \text{ mm}$$
$$d = 450 \text{ mm}$$
$$f_{ck} = 20 \text{ N/mm}^2$$
$$f_y = 415 \text{ N/mm}^2$$

(i) Nominal Shear Stress :

Factored or Design Shear force, $V_u = 120 \text{ kN} = 120 \times 10^3 \text{ N}$

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{120 \times 10^3}{250 \times 450} = 1.067 \text{ N/mm}^2$

(ii) Shear Resistance of Concrete :

Percentage of tension steel at support

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{\pi/4 \times 16^2 \times 4 \times 100}{250 \times 450} = 0.715 \%$$

Referring to the table-19 of IS456, Shear strength of concrete is

$$\tau_c = 0.55 \text{ N/mm}^2$$

Pt, %	τ_c, N/mm²
0.5	0.48
0.75	0.56
0.715	$= 0.48 + \frac{(0.56 - 0.48)}{(0.75 - 0.5)} \times (0.715 - 0.5) = 0.55$

Maximum shear stress in concrete $\tau_{c,max}$ from table 20 of IS456

$$\tau_{c,max} = 2.8 \text{ N/mm}^2$$

As $\tau_c < \tau_{c,max}$ and $\tau_v > \tau_c$, Shear reinforcement has to be designed.

Shear to be resisted by shear reinforcement (bent up bars and vertical stirrups)

$$V_{us} = V_u - \tau_c\, bd$$
$$= 120 \times 10^3 - 0.55 \times 250 \times 450 = 58125\ N$$

(iii) Shear Resistance of Bent up Bars :

Shear resistance of bent up bars

$$V_{usb} = 0.87\, f_y\, A_{sb} \sin \alpha$$
$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^0$$
$$= 102662.6\ N$$

Theoretically the two inclined bars can carry shear force completely. But IS-456 specifies half of the design shear must be resisted by vertical stirrups.

(iv) Design of Vertical Stirrups :

Shear resistance to be provided by the vertical stirrups

$$V_{usv} = \frac{V_{us}}{2} = \frac{58125}{2} = 29062.5\ N$$

Spacing of 2-legged 6 *mm* stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.6\, mm^2$$

$$S_v = \frac{0.87\, f_y\, A_{sv} d}{V_{usv}} = \frac{0.87 \times 415 \times 56.5 \times 450}{29062.5}$$
$$= 316\ mm$$

Spacing from minimum shear reinforcement consideration as per IS 456

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{8.87\, f_y}$$

$$S_v = \frac{0.87\, f_y\, A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 56.5}{0.4 \times 250}$$
$$= 204\ mm$$

Maximum allowed spacing = 0.75 *d* = 0.75 × 450 = 337.5 *mm*

(or) 300 *mm* whichever is less.

Spacing should be least of the above.

Hence provide 2-legged 6 *mm* stirrups @ 200 *mm* c/c throughout the span of the beam.

EXAMPLE - 32

A simply supported beam with span 5.5 m has an effective depth of 575 mm and width of 275 mm. It carries a service load of 100 kN/m including self weight. It is reinforced with 4 bars of 20 mm diameter. Use M20 grade concrete and Fe415 grade steel. Determine whether shear reinforcement is required. If required design the shear reinforcement.

(OCT/NOV. 2010 ; OCTOBER, 2004)

Solution :

$$b = 275 \text{ mm}$$

$$d = 575 \text{ mm}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

2. Nominal Shear Stress :

Factored load $w_u = 1.5 \times 100 = 150 \text{ kN/m}$

Factored Shear force $V_u = \frac{\omega_u l}{2} = \frac{150 \times 5.5}{2} = 412.5 \text{ kN} = 412.5 \times 10^3 \text{ N}$

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{412.5 \times 10^3}{275 \times 575} = 2.61 \text{ N/mm}^2$

3. Shear Resistance of Concrete : Percentage of tension steel at support

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{\frac{\pi}{4} \times 20^2 \times 4 \times 100}{275 \times 575} = 0.8\%$$

Referring to the Table. 19 of IS : 456, Shear strength of concrete is

$$\tau_c = 0.56 + \frac{(0.62 - 0.56)}{(1.0 - 0.75)} \times (0.8\text{-}0.75) = 0.57 \text{ N/mm}^2$$

Maximum shear stress in concrete $\tau_{c,max}$ from Table. 20 of IS456

$$\tau_{c,max} = 2.8 \text{ N/mm}^2$$

As $\tau_v < \tau_{c,max}$ and $\tau_v > \tau_c$, Shear reinforcement has to be designed.

4. Design of Vertical Stirrups : Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - \tau_c bd$$

$$= 412.5 \times 10^3 - 0.57 \times 275 \times 575 = 322369 \text{ N}$$

Spacing of 2-legged 12 *mm* stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 12^2 = 226.2\ mm^2$$

$$S_v = \frac{0.87\, f_y . A_{sv} . d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 226.2 \times 575}{322369} = 145.7\ mm$$

Spacing from minimum shear reinforcement consideration as per IS 456

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87\, f_y . A_{sv}}{0.4\, b}$$

$$= \frac{0.87 \times 415 \times 226.}{0.4 \times 275} = 742\ mm$$

Maximum allowed spacing = 0.75 *d* = 0.75 × 575 = 431 *mm*

(or) 300 *mm* which ever is less.

Spacing should be least of the above, i.e. 145 *mm c/c*.

The above designed spacing has to be adopted in the regions where $\tau_v > \tau_c$ and minimum shear reinforcement has to be provided in the remaining region where $\tau_v < \tau_c$.

If *x* is the distance from the centre of the beam to the section where the nominal shear stress is equal to the shear strength of concrete, then from the principles of triangles

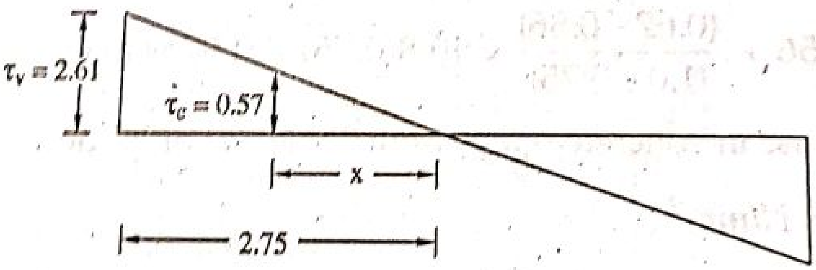


FIG 3.14 :

$$\frac{x}{0.57} = \frac{2.75}{2.61}$$

$$x = 0.57 \times \frac{2.75}{2.61} = 0.6\ m$$

Therefore for a distance of 2.75 – 0.6 = 2.15 *m* from the supports, the designed shear reinforcement is to be provided and for the remaining distance (0.6 × 2 = 1.2 *m*) in mid span only nominal shear reinforcement is to be provided.

Hence provide 2-legged 12 *mm* stirrups @ 145 *mm c/c* for a distance of 2.15 *m* from the supports and @ 300 *mm c/c* in the mid span for a length of 1.2 *m* where minimum shear reinforcement has to be provided as shown in the Fig. 3.15.

Using Sp 16 :

From Table 61, corresponding to

$$P_t = 0.8\%, \ \tau_c = 0.57\ N/mm^2$$

$$\frac{V_{us}}{d} = \frac{332.34}{57.5} = 5.78\ KN/cm$$

From Table 62, corresponding to $\frac{V_{us}}{d}$ (= 5.78) and the diameter of the stirrup (12 *mm*), spacing of 2-legged 12 *mm* stirrups = 14.5 *cm* = 145 *mm*.

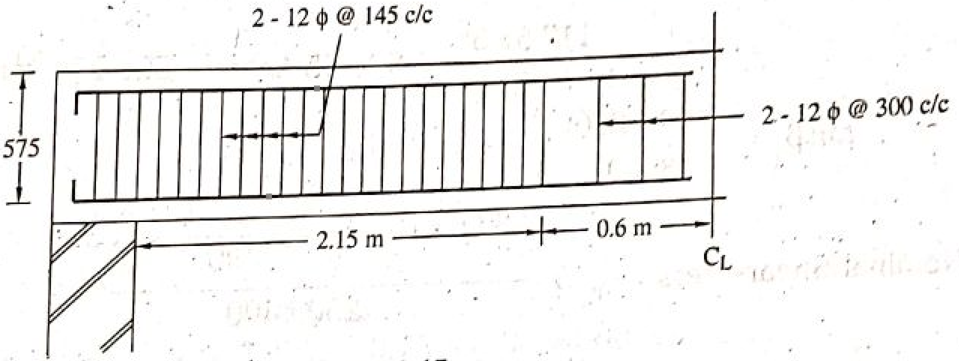


FIG 3.15 :

EXAMPLE - 33

A cantilever beam with span 3 m has an effective depth of 400 mm at the support and 200 mm at the free end and a constant width of 250 mm. It carries a load of 75 kN/m including self weight. It is reinforced with 4 bars of 20 mm diameter. Use M20 grade concrete and Fe415 grade steel. Design the shear reinforcement

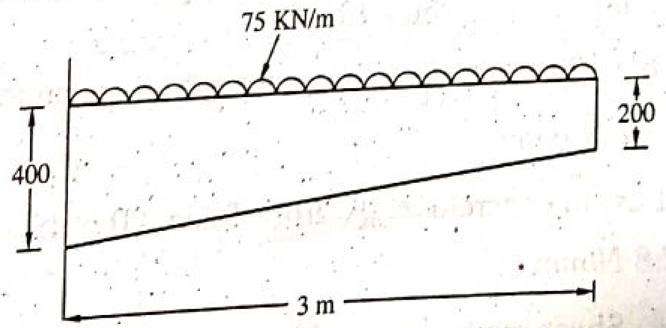


FIG 3.16 :

Solution :

$b = 250\ mm$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

Factored load $w_u = 1.5 \times 75 = 112.5\ kN/m$

Factored Shear force $V_u = w_u l = 112.5 \times 3 = 337.5\ kN = 337.5 \times 10^3\ N$

1. Nominal Shear Stress :

For beams of varying depth

Nominal Shear stress

$$\tau_v = \frac{V_u \pm (M_u/d)\tan\beta}{bd}$$

Adopt negative sign as BM increases with the increase of depth

M_u = Factored bending moment at the section

$$= \frac{w_u l^2}{2} = \frac{112.5 \times 3^2}{2} = 506.25\ kN\text{-}m = 506.25 \times 10^6\ N\text{-}mm$$

$$\tan\beta = \frac{(400 - 200)}{3000} = 0.067$$

Nominal Shear stress
$$\tau_v = \frac{337.5 \times 10^3 - \left(\frac{506.25 \times 10^6}{400}\right) 0.67}{250 \times 400}$$

$$= 2.53\ N/mm^2$$

2. Shear Resistance of Concrete :

Percentage of tension steel at support

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{\frac{\pi}{4} \times 20^2 \times 4 \times 100}{250 \times 400} = 1.25\%$$

Referring to the table-19 of IS : 456, Shear strength of concrete is

$\tau_c = 0.67\ N/mm^2$

Maximum shear stress in concrete $\tau_{c,max}$ from Table. 20 of IS : 456

$\tau_{c,max} = 2.8\ N/mm^2$

As $\tau_v > \tau_c$, Shear reinforcement has to be designed.

3. Design of Vertical Stirrups :

Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - \tau_c bd$$

$$= 337.5 \times 10^3 - 0.67 \times 250 \times 400 = 270.5 \times 10^3 N$$

Spacing of 2-legged 12 mm stirrups

$$A_s = 2 \times \frac{\pi}{4} \times 12^2 = 226.2 \ mm^2$$

$$S_v = \frac{0.87 f_y A_{sv} d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 226.2 \times 400}{270 \times 5 \times 10^3}$$

$$= 120.8 \ mm$$

Spacing from minimum shear reinforcement consideration as per IS : 456.

$$\frac{A_{st}}{b S_v} = \frac{0.4}{0.87 f_y}$$

$$S_v = \frac{0.87 f_y . A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 226.2}{0.4 \times 250}$$

$$= 816.7 \ mm$$

Maximum allowed spacing = $0.75 d = 0.75 \times 400 = 300 \ mm$

(or) 300 mm which ever is less.

Hence, provide 2-legged 12 mm vertical stirrups @ 120 mm c/c through out the length of the beam.

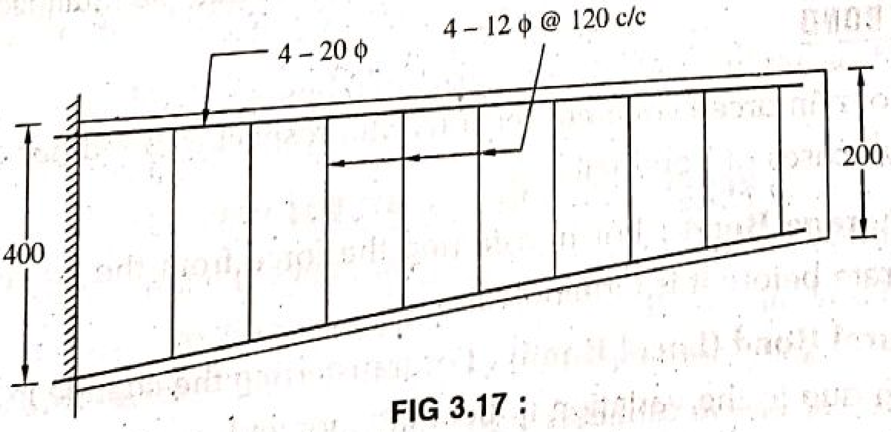


FIG 3.17 :

3.5 BOND

The important assumption made in the theory of reinforced concrete is that there is a perfect bond between steel and concrete. They have to act together without any slip. Due to bond only the force will be transferred to the steel from the surrounding concrete and to concrete from steel.

3.5.1 BOND STRESS

Bond stress is the shear stress acting parallel to the bar on the interface between the reinforcing bar and the surrounding concrete. Hence, it is the stress developed between the contact surface of steel and concrete to keep them together. It resists any force that tries to pull out the rods from the concrete.

Bond is developed due to combined effect of

(a) Adhesion between concrete and steel (provided by concrete during setting)

(b) Friction (provided by gripping of bars due to shrinkage of concrete)

(c) Interlocking of ribs in bars with concrete (deformed bars with transverse ribs)

Bond stress depends on grade of concrete, diameter of the bar, bar profile condition, nature of force in the bar, grouping of bars, bends and hooks in the bar. The values of design bond stress prescribed by IS 456-2000 are given in Table 3.8 for plain round bars in tension.

Table 3.8 : Design Bond Stress in Plain Bars in Tension (Clause 26.2.1.1 in IS 456-2000)

Grade of Concrete	M20	M25	M30	M35	M40 and Above
Design Bond Stress $\tau_{bd,}$ N/mm^2	1.2	1.4	1.5	1.7	1.9

Note : For deformed bars these values may be increased by 60%

For bars in compression, the above values may be increased by 25%

For deformed bars in compression, the above values may be multiplied by 1.25 × 1.6.

3.5.2 TYPES OF BOND

The design of reinforced concrete sections with respect to bond has to take care of the following two cases of bond failures.

(a) **Anchorage Bond :** For transferring the force from the bar to the surrounding concrete before it is terminated.

(b) **Flexural Bond (Local Bond) :** For transferring the change in bar force along its length due to the variation in bending moment.

3.5.3 ANCHORAGE BOND AND DEVELOPMENT LENGTH

Anchorage bond arises when a bar carrying certain force is terminated. In such cases, it is necessary to transfer this force in the bar to the surrounding concrete over a certain length.

The length of the bar L_d required to transfer the force in the bar to the surrounding concrete through bond is called development length.

The development length can be easily determined by pull out test.

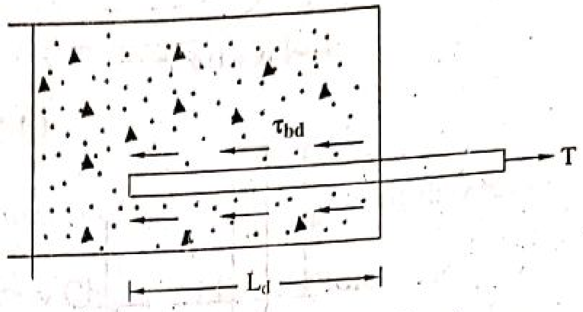


FIG 3.18 : Anchorage Bond

Fig. 3.18 shows a steel reinforcing bar embedded in concrete and subjected to a pull T.

$$T = \text{Design stress x area of bar} = 0.87\, f_y\, (\frac{\pi}{4}\phi^2)$$

This force must be transferred from steel to concrete through bond acting over the perimeter of the bar over a length L_d

If τ_{bd} is the average design bond stress, for equilibrium

Ultimate bond force = Pull out force

$$\tau_{bd}\ (\pi\ \phi)\ L_d\ =\ 0.87\ f_y \left(\pi\ \frac{\phi^2}{4}\right)$$

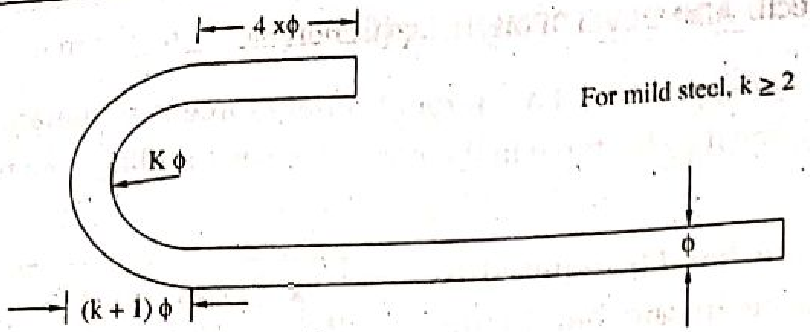
$$L_d = \frac{0.87\ f_y\ \phi}{4\ \tau_{bd}}$$

Where ϕ is the diameter of the bar

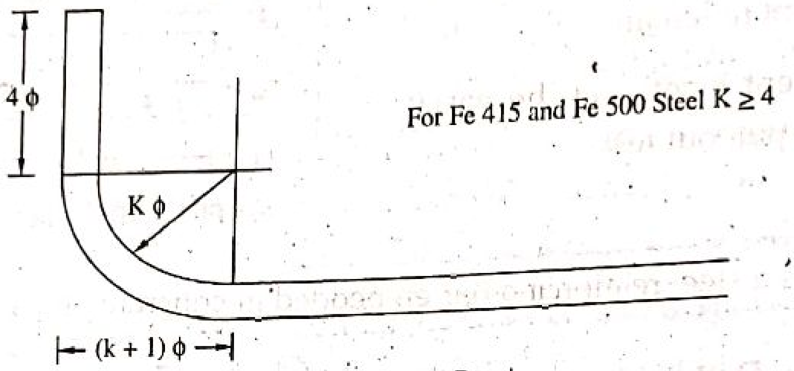
Hence all the bars should extend to a distance of L_d beyond the section where they are required to take full design force.

3.5.4 EQUIVALENT DEVELOPMENT LENGTH OF HOOKS AND BENDS

In situations where straight anchorage length can not be provided due to lack of space (at supports), to improve the anchorage of bars many times standard hooks are provided in plain bars and standard bends are provided in deformed bars as shown in Fig 3.19. The equivalent development lengths for these standard hooks and bends may be taken as given below (IS : 456 -2000, clause 26.2.2)



(a) Standard Hook



(b) Standard Bend

FIG 3.19 : Standard Hook and Bend

(a) The anchorage value of a standard U-type hook shall be equal to 16 times the diameter of the bar.

(b) The anchorage value of a standard bend shell be taken as 4 times the diameter of the bar for each 45° bend subjected to a maximum of 16 times the dia.

Table 3.9 : Anchorage Values for Hooks and Bends

Type of Hook/Bend	U-Hook	45° Bend	90° Bend	135° Bend	180° Bend & More
Anchorage value	16 ϕ	4 ϕ	8 Φ	12 ϕ	16 ϕ

3.5.5 FLEXURAL BOND

Flexural bond at a point is the rate of change of tension in the steel at a given location in a reinforced concrete member due to variation of bending moment.

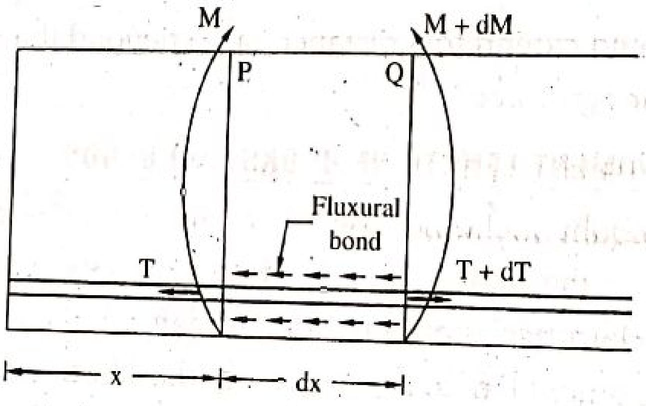


FIG 3.20 : Flexural Bond

Consider two sections P & Q at a distance dx apart along the beam as shown in Fig. 2.20.

Tension in bar at $P = T = M/Z$ Where Z is lever arm

Tension in bar at $Q = T + dT = (M + dM)/Z$

Change in the bar force $dT = dM/Z$

This force has to be transferred through flexural bond

If τ_{bd} is the bond stress acting along the surface of the bar, then for equilibrium

Bond force along the periphery of the bar = Change in the bar force dT

$$\tau_{bd}\,\pi\,\phi\,dx = \frac{dM}{Z} \qquad (\phi = \text{dia of the bar})$$

$$\tau_{bd}\,\pi\,\phi\,Z = \frac{dM}{dx} = V \qquad (\text{Shear force } V = \frac{dM}{dx})$$

$$\tau_{bd} = \frac{V}{\pi\phi Z} = \frac{V}{Z.O} \qquad (O \text{ is the perimeter of the bar})$$

If there are N bars of equal size

$$\tau_{bd} = \frac{V}{Z\,(\Sigma.O)}$$

Where $\Sigma.O$ = Sum of perimeters of all the bars at a given section

V = Shear force at the section

3.5.6 CHECK FOR DEVELOPMENT LENGTH FOR FLEXURAL BOND AT SIMPLE SUPPORTS

The flexural bond is maximum at the section where the shear force is large. Therefore, the check for flexural bond is necessary at the sections where shear force is maximum and bending moment is zero. The locations of such sections are

(a) Simple supports

(b) Point of contraflecture

(c) Point of curtailment of mid span reinforcement

Development Length at Simple Support :

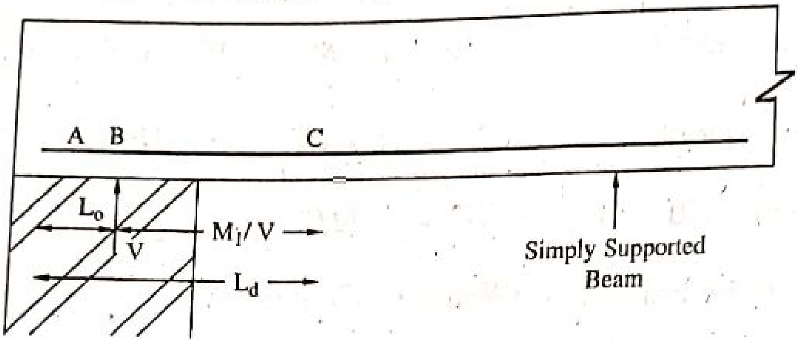


FIG 3.21 : Development Length at Simple Support

Consider a simply supported beam as shown in Fig. 2.21. The bending moment at B is zero and hence tension in the bar at *B* is zero. The bending moment increases towards the mid span.

Let T_1 be the tension at C corresponding to the bending moment M_1

$$T_1 = \frac{M_1}{Z}$$

This force has to transmit over a length *BC* which is called as development length L_d. If it falls short of L_d then the bar can extended up to *A* so that $AC = L_d$

$$T_1 = \frac{M_1}{Z} = \tau_{bd}\ BC.\sum.O$$

$$M_1 = (\tau_{bd}.Z.\sum.O).BC$$

$$M_1 = V.BC$$

$$BC = \frac{M_1}{V}$$

As per the requirements of the development length

$$AC \geq L_d$$

$$\frac{M_1}{V} + L_o \geq L_d$$

$$L_d \leq \frac{M_1}{V} + L_o$$

Where M_1 = Moment of resistance of section corresponding to the area of steel Continued in to the support.

V = Shear force at the support due to the design loads

Lo = Extended length of the bar beyond point of zero bending.

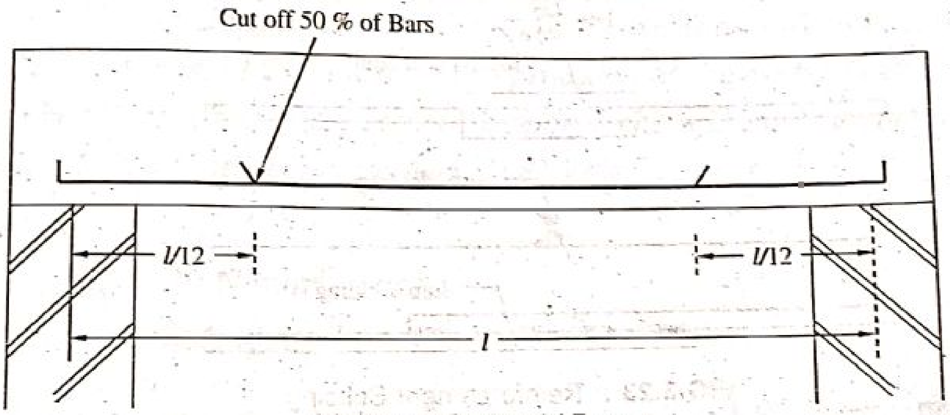
The value of $\frac{M_1}{V}$ in the above expression may be increased by 30 % when the ends of the reinforcements are confined by a compressive reaction.

3.5.7 CURTAILMENT OF TENSION REINFORCEMENT

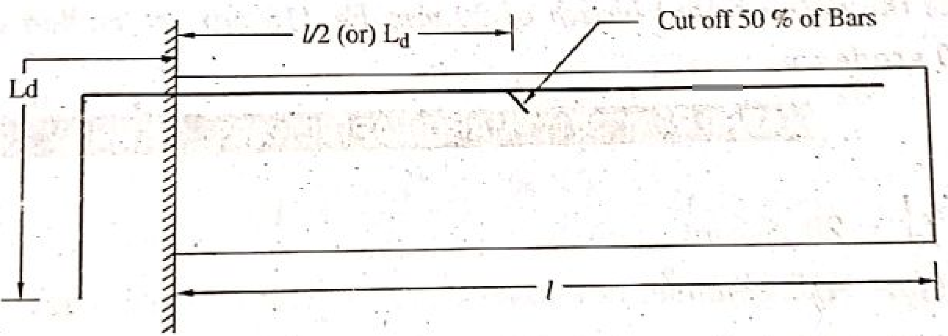
Tension reinforcement is designed for sections where bending moment is maximum. Bending moment varies along the span of a beam depending on the loading and support conditions. For economy it is general practice to curtail the bars where the bending moment is less.

The actual point of curtailment shell extend beyond the theoretical point of curtailment towards the support for a distance equal to 12 ϕ or the effective depth of the beam which ever is more.

Simplified rules for curtailment of bars in simply supported and cantilever beams are shown in Fig. 3.22.



(a) Simply Supported Beams



(b) Cantilever Beams

FIG 3.22 : Simplified Curtailment Rules

In case of cantilever beams, 50 % of bars may be curtailed at 0.5 l or L_d which ever is more from the face of the support.

In simply supported beams at least 50% of bars shall extend in to the support for a length of $L_d/3$ from the face of the support and the remaining 50% can be curtailed at a distance of $l/12$ from the centre of the supports.

3.5.9 SPLICING OF TENSION REINFORCEMENT

Splices are provided when the length of the bar available is less than that required. The splicing of reinforcement is provided either by lapped joint or mechanical joint or welded joint as shown in Fig. 3.23. Where splices are provided in the reinforcing bars, they shall be as far as possible be away from the sections of maximum stresses and should be staggered. In flexural members splices should not be at sections where bending moment is more than 50 % of the moment of resistance and not more than half the bars shall be spliced at a section.

Lap splices should not be used for bars larger than 36 mm. For larger diameters, bars may be welded. Lap length for bars in flexural tension shall be L_d or $30\,\phi$ which ever is greater and for direct tension shall be $2\,L_d$ or $30\,\phi$ which ever is greater. The lap length in compression shall be equal to the L_d or $24\,\phi$ which ever is greater.

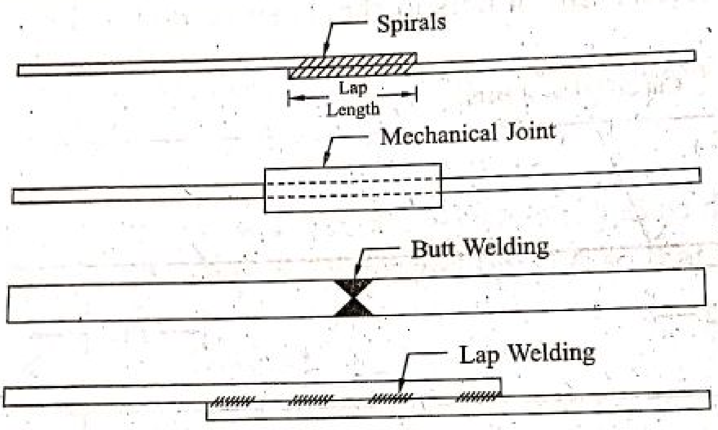


FIG 3.23 : Reinforcement Splicing

EXAMPLE - 34

Determine the development length of 20 mm Fe 415 bar in tension & compression with M20 grade concrete.

(APRIL/MAY. 2011 ; MARCH/APRIL. 2009 ; OCT/NOV. 2016 [TS] , 2007)

Solution :

$$f_{ck} = 20\ N/mm^2$$

$$f_y = 415\ N/mm^2$$

Bond stress for plain bars $\tau_{bd} = 1.2\ N/mm^2$ (from Table 3.8)

For deformed bars bond stress should be increased by 60 %

Hence $\tau_{bd} = 1.2 \times 1.6 = 1.92\ N/mm^2$

1. Bars in Tension :

Development length $L_d = \dfrac{0.87\, f_y\, \phi}{4.\tau_{bd}} = \dfrac{0.87 \times 415 \times 20}{4 \times 1.92} = 940.2\ mm$

2. **Bars in Compression :**

Bond stress for bars in compression shall be increased by 25 %

$$\tau_{bd} = 1.25 \times 1.92$$

Development length $L_d = \dfrac{0.87\, f_y\, \phi}{4 . \tau_{bd}}$

$$= \frac{0.87 \times 415 \times 20}{4 \times 1.25 \times 1.92} = 752.2 \text{ mm}$$

Using SP 16 : From Table 65, read out the development length corresponding to M 20 concrete and 20 mm diameter bars.

L_d for Tension bars = 940 mm

L_d for Compression bars = 752 mm

EXAMPLE - 35

A simply supported beam is 6 m in span and carries a uniformly distributed load of 60 kN/m. If 6 Nos. of 20 mm bars are provided at the centre of the span and 4 Nos. of these bars are continued in to the supports, check the development at the supports assuming M 20 grade concrete and Fe 415 steel.

Solution :

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 415 \text{ N/mm}^2$

Bond stress for deformed bars, $\tau_{bd} = 1.6 \times 1.2 = 1.92 \text{ N/mm}^2$ (from table 2.8)

Factored load $w_u = 1.5 \times 60 = 90 \text{ kN/m}$

Factored Shear force $V_u = \dfrac{w_u\, l}{2} = \dfrac{90 \times 6}{2} = 270 \text{ kN}$

Factored bending moment $\dfrac{w_u\, l^2}{8} = \dfrac{90 \times 6^2}{8} = 405 \text{ kN-m}$

Moment of resistance of the bars continued in to the support (4 bars)

$$M_1 = 405 \times \frac{4}{6} = 270 \text{ kN-m}$$

Development length of 20 mm diameter bar with M 20 concrete and Fe 415 steel

$$L_d = \frac{0.87\, f_y\, \phi}{4 . \tau_{bd}}$$

$$= \frac{0.87 \times 415 \times 20}{4 \times 1.92} = 940 \text{ mm}$$

According to clause 26.2.3.3 of IS : 456, the condition to be satisfied

$$L_d \leq \frac{1.3\,M_1}{V} + L_o$$

$$\frac{1.3\,M_1}{V} = \frac{1.3 \times 270 \times 10^6}{270 \times 10^3}$$

$$= 1300 \; mm$$

$$Lo + 1300 > L_d$$

Hence, Development length is satisfied without any anchorage value.

EXAMPLES ON COMPLETE DESIGN OF BEAMS

EXAMPLE - 36

Design a rectangular simply supported R.C. Beam over a clear span of 6 m, if the superimposed load is 12 kN/m and the support width is 230 mm. Use M 20 grade concrete and Fe 415 steel. The beam is to have width of 300 mm. Design the shear reinforcement and do the check for deflection.

Solution :

$b = 300 \; mm$

$f_{ck} = 20 \; N/mm^2$

$f_y = 415 \; N/mm^2$

$l = 6 \; m$

1. **Depth of the Beam :** Selecting the depth in range of $\frac{l}{12}$ to $\frac{l}{15}$ based on stiffness

$$d = \frac{6000}{15} = 400 \; mm$$

Adopt $d = 400 \; mm$

$D = 450 \; mm$

2. **Effective Span :** Least of

Centre to centre of supports = $6 + 0.23 = 6.23 \; m$

Clear span + $d = 6 + 0.4 = 6.4 \; m$

Hence, Effective span = $6.23 \; m$

3. **Loads :**

Self weight of the beam = $0.3 \times 0.45 \times 1 \times 25 = 3.375 \; kN/m$

Imposed load = $12 \; kN/m$

$$\text{Total load} = 15.375 \; kN/m$$

$$\text{Factored load } w_u = 1.5 \times 15.375 = 23.06 \; kN/m$$

$$\text{Factored Bending moment } M_u = \frac{w_u l^2}{8}$$

$$= \frac{23.06 \times 6.23^2}{8} = 111.9 \; kN\text{-}m$$

$$\text{Factored shear force } V_u = \frac{w_u l}{2} = 23.06 \times \frac{6.23}{2} = 71.83 \; kN$$

4. Depth Required :

The minimum depth required to resist Bending Moment

$$M_u = 0.138 . f_{ck} . bd^2$$

$$111.9 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{111.9 \times 10^6}{0.138 \times 20 \times 300}}$$

$$= 367.6 \; mm < 400 \; mm, \text{ provided depth}$$

Hence provided depth is adequate

5. Tension Reinforcement :

$$M_u = 0.87 \, f_y \, A_{st} \, d \left(1 - \frac{f_y \, A_{st}}{f_{ck} . b.d}\right)$$

$$111.9 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400 \left(1 - \frac{415 \times A_{st}}{20 \times 300 \times 400}\right)$$

$$774.8 = A_{st} \left(1 - \frac{A_{st}}{5783.1}\right)$$

$$A_{st}^2 - 5783.1 \, A_{st} + 774.8 \times 5783.1 = 0$$

$$A_{st} = \frac{5783.1 - \sqrt{5783.1^2 - 4 \times 774.8 \times 5783.1}}{2} = 921.7 \; mm$$

Provide 3-20 mm bars, A_{st} provided = 942.5 mm^2

6. Design of Shear Reinforcement :

$$\text{Nominal Shear stress} \quad \tau_v = \frac{V_u}{bd} = \frac{71.83 \times 10^3}{300 \times 400} = 0.598 \; N/mm^2$$

Percentage of tension steel at support

$$P_t = \frac{A_{st}\times 100}{bd} = \frac{942.5\times 100}{300\times 400} = 0.785\ \%$$

Referring to the Table-19 of IS : 456. Shear strength of concrete is

$$\tau_c = 0.57\ N/mm^2$$

Maximum shear stress in concrete $\tau_{c.max}$ from table 20 of IS456

$$\tau_{c.max} = 2.8\ N/mm^2$$

As $\tau_v > \tau_c$, Shear reinforcement has to be designed.

Shear resistance of concrete

$$V_{uc} = \tau_c\ bd = 0.57\times 300\times 400$$

$$= 68400\ N = 68.4\ kN$$

Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - V_{uc}$$

$$= 71.83 - 68.4 = 3.43\ kN$$

Using 8 *mm*, 2 legged Fe 250 steel stirrups

$$A_{sv} = 2 \times \frac{\Pi}{4} \times 8^2 = 100.5\ mm^2$$

$$\text{Spacing,}\quad S_v = \frac{0.87\, f_y\, A_{sv}\, d}{V_{us}}$$

$$= \frac{0.87\times 250\times 100.5\times 400}{3430} = 2549.1\ mm$$

Spacing from minimum shear reinforcement consideration as per IS : 456

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{0.87\, f_y}$$

$$S_v = \frac{0.87\, f_y . A_{sv}}{0.4\times b} = \frac{0.87\times 250\times 100.5}{0.4\times 300} = 182.2\ mm$$

Maximum allowed spacing = 0.75 *d* = 0.75 × 400 = 300 *mm*

(or) 300 *mm* which ever is less.

Hence, provide 2 legged 8 *mm* stirrups @ 180 *mm c/c* throughout.

7. **Check for deflection (Stiffness) :**

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

% of steel = 0.785

$$f_s = 0.58 \times f_y \times (\text{area of steel required}/A_{st}\ \text{provided})$$

$$= 0.58 \times 415 \times \left(\frac{922}{942}\right) = 235.6\ N/mm^2$$

From Fig.4 of IS 456, modification factor = 1.1

Maximum permitted $\frac{l}{d}$ ratio = 1.1 × 20 = 22

$\frac{l}{d}$ provided = $\frac{6230}{400}$ = 15.575 < 22

Hence deflection control is safe.

8. **Details of Reinforcement are shown in Figure 2.24.**

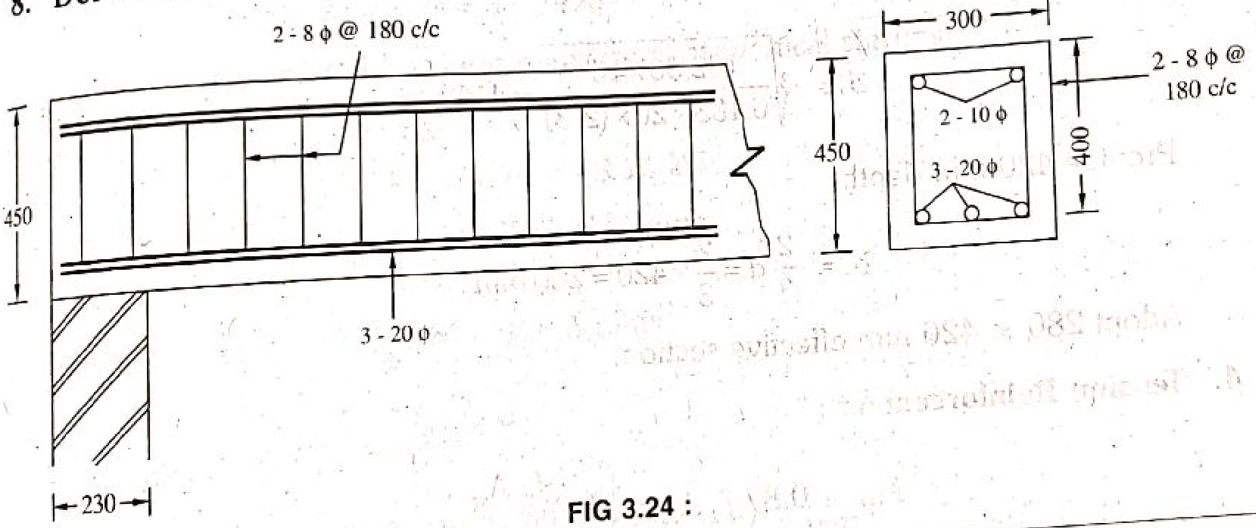


FIG 3.24 :

EXAMPLE - 37

Design a simply reinforced beam using M 20 concrete and Fe 415 steel to carry a total working load of 25.13 kN/m. The effective span of the beam is 5.3 m. Also design the shear reinforcement. Assume $b = \frac{2}{3}d$

(MARCH/APRIL, 2010)

Solution :

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

$l_{eff} = 5.3\ m$

1. **Dimensions of the Beam :**

Given $b = \frac{2}{3}d$

2. **Loads :**

Total load $= 25.13\ kN/m$

Factored load $w_u = 1.5 \times 25.13 = 37.695\ kN/m$

Factored Bending moment $M_u = \frac{w_u l^2}{8}$

$$= \frac{37.695 \times 5.3^2}{8} = 132.36\ kN\text{-}m$$

Factored shear force $V_u = \frac{w_u l}{2} = 37.695 \times \frac{5.3}{2} = 99.89\ kN$

3. **Depth Required :** The minimum depth required to resist Bending Moment

$$M_u = 0.138.\ f_{ck}.bd^2$$

$$132.36 \times 10^6 = 0.138 \times 20 \times \left(\frac{2}{3}d\right) \times d^2$$

$$d = \sqrt[3]{\frac{132.36 \times 10^6}{0.138 \times 20 \times (2/3)}} = 416\ mm$$

Provide 420 *mm* depth

$$b = \frac{2}{3}d = \frac{2}{3} \times 420 = 280\ mm$$

Adopt 280 × 420 *mm* effective section.

4. **Tension Reinforcement :**

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1 - \frac{f_y\ A_{st}}{f_{ck}.b.d}\right)$$

$$132.36 \times 10^6 = 0.87 \times 415 \times A_{st} \times 240\left(1 - \frac{415 \times A_{st}}{20 \times 280 \times 240}\right)$$

$$872.9 = A_{st}\left(1 - \frac{A_{st}}{5667.5}\right)$$

$$A_{st}^2 - 5667.5\ A_{st} + 872.9 \times 5667.5 = 0$$

$$A_{st} = \frac{5667.5 - \sqrt{5667.5^2 - 4 \times 872.9 \times 5667.5}}{2} = 1077.9\ mm^2$$

Provide 3-22 *mm* bars, A_{st} provided = 1140.4 mm^2

5. Design of Shear Reinforcement :

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{99.89\times10^3}{280\times420} = 0.849\ N/mm^2$

Percentage of tension steel at support

$$P_t = \frac{A_{st}\times100}{bd} = \frac{1140.4\times100}{280\times420} = 0.969\ \%$$

Referring to the Table-19 of IS : 456, Shear strength of concrete is

$$\tau_c = 0.61\ N/mm^2$$

Maximum shear stress in concrete $\tau_{c,max}$ from table 20 of IS456

$$\tau_{c,max} = 2.8\ N/mm^2$$

As $\tau_v > \tau_c$, Shear reinforcement has to be designed.

Shear resistance of concrete

$$V_{uc} = \tau_c\ bd = 0.61\times280\times420$$
$$= 71736\ N = 71.74\ kN$$

Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - V_{uc}$$
$$= 99.89 - 71.74 = 28.15\ kN$$

Using 8 mm, 2 legged Fe 250 steel stirrups

$$A_{sv} = 2\times\frac{\pi}{4}\times8^2 = 100.5\ mm^2$$

Spacing, $S_v = \frac{0.87\ f_y\ A_{sv}\ d}{V_{us}}$

$$= \frac{0.87\times250\times100.5\times420}{28150} = 326.1\ mm$$

Spacing from minimum shear reinforcement consideration as per IS : 456

$$\frac{A_{sv}}{b\ S_v} = \frac{0.4}{0.87\ f_y}$$

$$S_v = \frac{0.87\ f_y.A_{sv}}{0.4\times b} = \frac{0.87\times250\times100.5}{0.4\times280} = 195.2\ mm$$

Maximum allowed spacing = 0.75 d = 0.75 × 420 = 315 mm

(or) 300 mm which ever is less.

Hence, provide 2 legged 8 mm stirrups @ 190 mm c/c throughout.

6. **Details of Reinforcement are shown in Figure below**

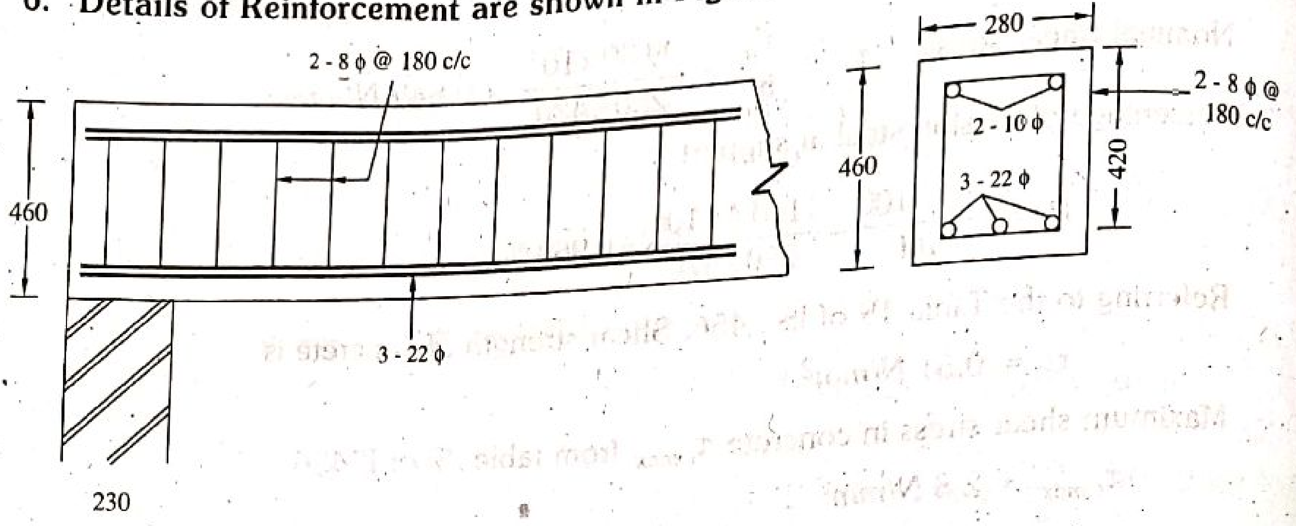


FIG 3.24 :

EXAMPLE - 38

A simply supported R.C.C beam carries an udl of 30 kN / m (exclusive of self weight) over an effective span of 6.5 m. The overall size of the beam has to be restricted to 300 mm × 580 mm. Design the mid span section of the beam for flexure and check the beam for stiffness. Use M 20 concrete and Fe 415 steel. Effective cover is 40 mm.

(MARCH/APRIL. 2003)

Solution :

$b = 300\ mm$

$D = 580\ mm$

$d = 580 - 40 = 540\ mm$

$d' = 40\ mm$

$f_{ck} = 20\ N/mm^2$

$f_y = 415\ N/mm^2$

1. **Loads :**

Self weight of the beam = $0.3 \times 0.58 \times 1 \times 25 = 4.35\ kN/m$

Imposed load = $30\ kN/m$

Total load = $34.35\ kN/m$

Factored load w_u = $1.5 \times 34.35 = 51.525\ kN/m$

$$\text{Factored Bending moment } M_u = \frac{w_u\, l^2}{8} = 51.525 \times \frac{(6.5)^2}{8} = 272.12\ KN\text{-}m$$

$$\text{Factored shear force } V_u = \frac{w_u\, l}{2} = 51.525 \times \frac{6.5}{2} = 167.46\ KN$$

2. **Limiting moment of resistance of the given section as a singly reinforced section**

$$M_{u,lim} = 0.138 . f_{ck} . bd^2$$
$$= 0.138 \times 20 \times 300 \times 540^2$$
$$= 241.45 \times 10^6 \text{ N-mm}$$
$$= 241.45 \text{ kN-m}$$

As $M_u > M_{u,lim}$, the section should be designed as a doubly reinforced section.

3. **Area of Tension Steel Corresponding to $M_{u,lim}$ (A_{st1}) :**

$$0.87 f_y A_{st1} = 0.36 f_{ck} b x_{u,max}$$

$$A_{st1} = \frac{0.36 f_{ck} b x_{u,max}}{0.87 f_y} = \frac{0.36 \times 20 \times 300 \times 0.48 \times 540}{0.87 \times 415} = 1550.7 \text{ mm}^2$$

4. **Compression Reinforcement (A_{sc}) :**

$$\frac{d'}{d} = \frac{40}{540} = 0.07$$

From table F of SP16, stress in compression steel

$$f_{sc} = 354 \text{ N/mm}^2$$

The remaining bending moment has to be resisted by couple consisting of compression steel and the additional tension steel.

$$M_{u2} = M_u - M_{u,lim}$$
$$= 272.12 - 241.45 = 30.67 \text{ kN-m}$$

$$M_{u2} = f_{sc} A_{sc} (d - d')$$

$$30.67 \times 10^6 = 354 \times A_{sc} (540 - 40)$$

$$A_{sc} = \frac{30.67 \times 10^6}{354 (540 - 40)} = 173.3 \text{ mm}^2$$

5. **Additional Tensile Steel (A_{st2}) :**

$$0.87 f_y A_{st2} = f_{sc} A_{sc}$$

$$A_{st2} = \frac{f_{sc} A_{sc}}{0.87 f_y} = \frac{354 \times 173.3}{0.87 \times 415} = 169.9 \text{ mm}^2$$

Total tension steel $A_{st} = A_{st1} + A_{st2}$

$$= 1550.7 + 169.9$$

$$= 1720.6\ mm^2$$

Provide 6-20 *mm* bars in tension (A_{st} = 1885 mm^2) and

2-12 *mm* bars in compression (A_{sc} = 226.2 mm^2)

6. Design of Shear Reinforcement :

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{167.46 \times 10^3}{300 \times 540} = 1.034\ N/mm^2$

Percentage of tension steel at support (assuming only 50 % of bars are extending in to support)

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{\frac{1885}{2} \times 100}{300 \times 540} = 0.582\%$$

Referring to the Table-19 of IS : 456, Shear strength of concrete is

$$\tau_c = 0.51\ N/mm^2$$

Maximum shear stress in concrete $\tau_{c,max}$ from Table -20 of IS : 456

$$\tau_{c,max} = 2.8\ N/mm^2$$

As $\tau_v > \tau_c$, Shear reinforcement has to be designed.

Shear resistance of concrete

$$V_{uc} = \tau_c bd = 0.51 \times 300 \times 540$$

$$= 82620\ N = 82.62\ KN$$

Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - V_{uc}$$

$$= 167.46 - 82.62 = 84.84\ KN$$

Using 8 *mm*, 2 legged Fe 415 steel stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.5\ mm^2$$

$$S_v = \frac{0.87\, f_y\, Asv\, d}{V_{us}}$$

$$= \frac{0.87 \times 415 \times 100.5 \times 540}{82620} = 237.2\ mm$$

Spacing from minimum shear reinforcement consideration as per IS :456

$$\frac{A_{sv}}{b\,S_v} \doteq \frac{0.4}{0.87\,f_y}$$

$$S_v = \frac{0.87 f_y \cdot A_{sv}}{0.4 \times b}$$

$$= \frac{0.87 \times 415 \times 100.5}{0.4 \times 300} = 302.4\text{ mm}$$

Maximum allowed spacing = 0.75 d = 0.75 × 540 = 405 mm

(or) 300 mm which ever is less.

Hence, provide 2 legged 8 mm stirrups @ 230 mm c/c throughout.

7. **Check for Deflection :**

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

% of steel @ mid span = 1.164

$$f_s = 0.58 \times f_y \times \frac{\text{area of steel required}}{A_{st}\text{ provided}}$$

$$= 0.58 \times 415 \times \frac{1720.6}{1885}$$

$$= 219.7\ N/mm^2$$

From Fig.4 of IS : 456, modification factor F_1= 1.05

Modification factor for compression steel F_2

$$\%\text{ of compression steel} = \frac{A_{sc}}{bd} \times 100 = \frac{226.2}{300 \times 540} \times 100 = 0.14$$

From Fig.5 of IS 456, modification factor F_2 = 1.05

Maximum permitted $\frac{l}{d}$ ratio = 1.05 × 1.05 × 20 = 22.05

$$\frac{l}{d}\text{ provided} = \frac{6500}{540} = 12.04 < 22.05$$

Hence deflection control is safe.

8. **Details of Reinforcement are shown in Figure 3.25.**

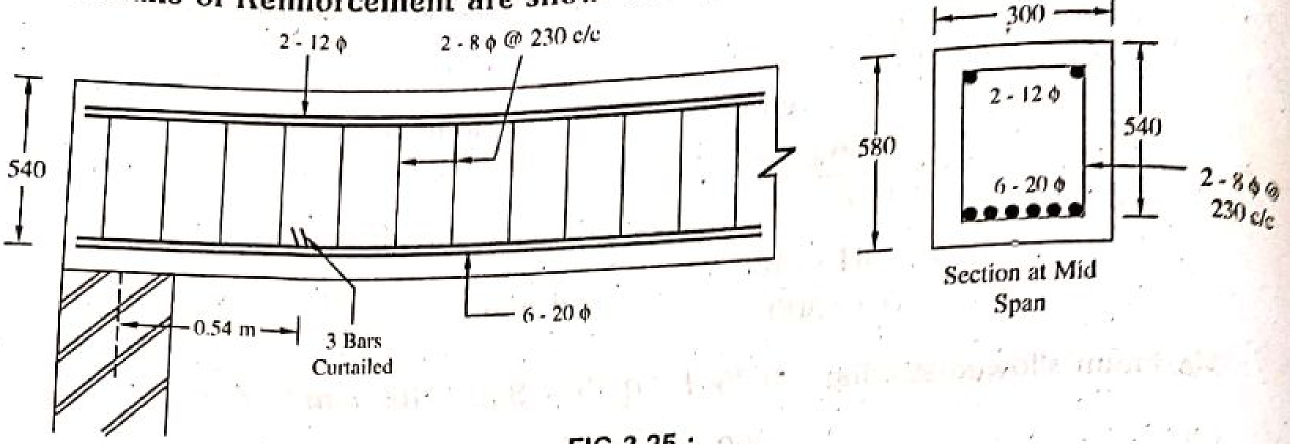


FIG 3.25 :

EXAMPLE - 39

Design a cantilever beam of uniform depth for a span of 2.5 m, if the superimposed load is 20 kN/m and it is monolithic with R.C column of 300 mm width and 400 mm deep. Use M20 grade concrete and Fe 415 steel.

(OCT/NOV 2010)

Solution :

$$f_{ck} = 20\ N/mm^2$$
$$f_y = 415\ N/mm^2$$
$$l = 2.5\ m$$

1. **Depth of the Beam :**

Selecting the depth as $\frac{l}{7}$ based on stiffness

$$d = \frac{2500}{7} = 357\ mm$$

Adopt $d = 400\ mm$

$D = 450\ mm$

Width $b = 300\ mm$

2. **Loads :**

Self weight of the beam = $0.3 \times 0.45 \times 1 \times 25 = 3.375\ kN/m$

Imposed load = $20\ kN/m$

Finishes = $0.625\ kN/m$

Total load = $24\ kN/m$

Factored load $w_u = 1.5 \times 24 = 36\ kN/m$

Factored Bending moment $M_u = \frac{\omega_u . l^2}{2} = \frac{36 \times (2.5)^2}{2} = 112.5\ kN\text{-}m$

Factored shear force $V_u = w_u \cdot l = 36 \times 2.5 = 90\ kN$

3. Depth Required :

The minimum depth required to resist Bending Moment

$$M_u = 0.138.\ f_{ck}.bd^2$$

$$112.5 \times 10^6 = 0.138 \times 20 \times 300 \times d^2$$

$$d = \sqrt{\frac{112.5\times10^6}{0.138\times20\times300}}$$

$$= 368.6\ mm < 400\ mm,\ \text{provided depth}$$

Hence provided depth is adequate

4. Tension Reinforcement :

$$M_u = 0.87\ f_y\ A_{st}\ d\left(1-\frac{f_y\ A_{st}}{f_{ck}.b.d}\right)$$

$$112.5\times10^6 = 0.87 \times 415 \times A_{st} \times 400\left(1-\frac{415\times A_{st}}{20\times300\times400}\right)$$

$$779 = A_{st}\left(1-\frac{A_{st}}{5783.1}\right)$$

$$A_{st}^2 - 5783.1\ A_{st} + 779 \times 5783.1 = 0$$

$$A_{st} = \frac{5783.1-\sqrt{5783.1^2-4\times779\times5783.1}}{2} = 927.9\ mm^2$$

Provide 3-20 mm bars, $A_{s.}$ provided = 942.5 mm^2

5. Design of Shear Reinforcement :

Nominal Shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{90\times10^3}{300\times400} = 0.75\ N/mm^2$

Percentage of tension steel at support

$$P_t = \frac{A_{st}\times100}{bd} = \frac{942.5\times100}{300\times400} = 0.785\ \%$$

Referring to the Table-19 of IS : 456, Shear strength of concrete is

$$\tau_c = 0.57\ N/mm^2.$$

Maximum shear stress in concrete $\tau_{c,max}$ from Table - 20 of IS : 456

$$\tau_{c,max} = 2.8\ N/mm^2$$

As $\tau_v > \tau_c$, Shear reinforcement has to be designed.

Shear resistance of concrete

$$V_{uc} = \tau_c bd = 0.57 \times 300 \times 400$$

$$= 68400\ N = 68.4\ kN$$

Shear to be resisted by shear reinforcement (vertical stirrups)

$$V_{us} = V_u - V_{uc}$$

$$= 90 - 68.4 = 21.6\ kN$$

Using 6 *mm*, 2 legged Fe 415 steel stirrups

$$A_{sv} = 2 \times \frac{\Pi}{4} \times 6^2 = 56.55\ mm^2$$

$$S_v = \frac{0.87\, f_y\, A_{sv}\, d}{V_{us}} = \frac{0.87 \times 415 \times 56.55 \times 400}{21.6 \times 10^3} = 378\ mm$$

Spacing from minimum shear reinforcement consideration as per IS : 456

$$\frac{A_{sv}}{b\, S_v} = \frac{0.4}{0.87\, f_y}$$

$$S_v = \frac{0.87\, f_y . A_{sv}}{0.4 \times b} = \frac{0.87 \times 415 \times 56.55}{0.4 \times 300} = 170\ mm$$

Maximum allowed spacing = 0.75 d = 0.75 × 400 = 300 *mm*

(or) 300 *mm* which ever is less.

Hence, provide 2 legged 6 *mm* stirrups @ 170 *mm* c/c throughout.

6. Check for Deflection :

For cantilever beams basic value of $\frac{l}{d}$ ratio = 7

Modification factor for tension steel F_1

% of steel = 0.785

$$f_s = 0.58 \times f_y \times \frac{\text{area of steel required}}{A_{st}\ \text{provided}}$$

$$= 0.58 \times 415 \times \frac{922}{942} = 235.6\ N/mm^2$$

From Fig.4 of IS : 456, modification factor F_1 = 1.05

Maximum permitted $\frac{l}{d}$ ratio = 1.1 × 7 = 7.7

$$\frac{l}{d} \text{ provided} = \frac{2500}{400} = 6.25 < 7.7$$

Hence deflection control is safe.

7. **Anchorage Length at Support :**

The anchorage length required is given by

$$L_d = \frac{0.87\, f_y\, \Phi}{4.\tau_{bd}} = \frac{0.87 \times 415 \times 20}{4 \times 1.2 \times 1.6} = 940\ mm$$

The main tension bars extended in to column to a length of 350 *mm* and bent at 90° and extended up to 600 *mm* as shown in Fig.3.26.

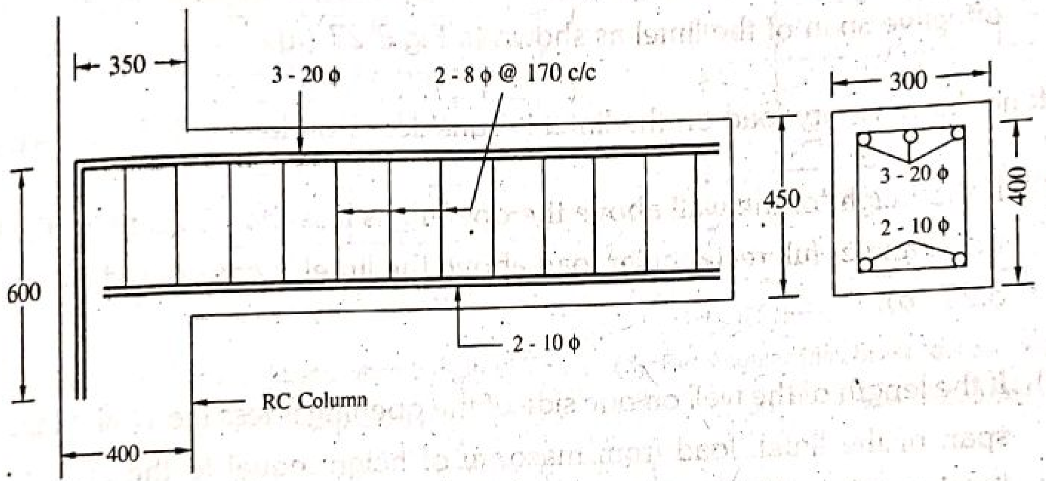


FIG 3.26 :

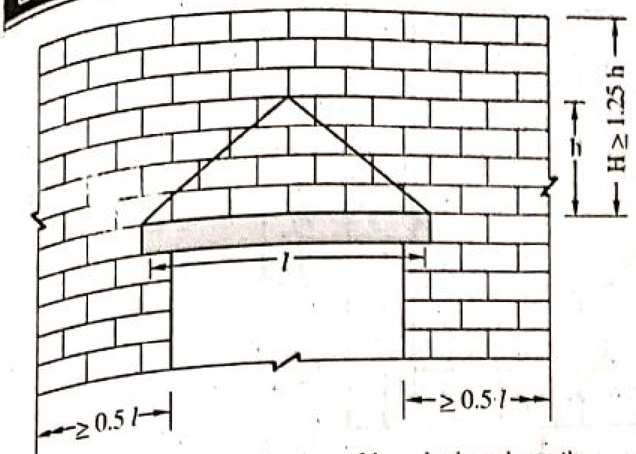
3.6 LINTELS

Beams provided over the openings in walls for doors, windows, cupboards etc.; are called lintels. They support the load of the wall above the openings. Due to the good bond in brick masonry, only a portion of the load of the wall above the opening will be acting on the lintel and the remaining will be transferred to the sides of the opening by arch action. Therefore, it is assumed that a load of traingular portion of the masonry is considered to act on the lintel. The base angle of the triangle depends upon the quality of brick masonry. It may be taken as 45° for good masonry and 60° for poor masonry. The arch action is possible provided height of the wall above the top of the lintel is more than or equal to 1.25 times the height of the triangle.

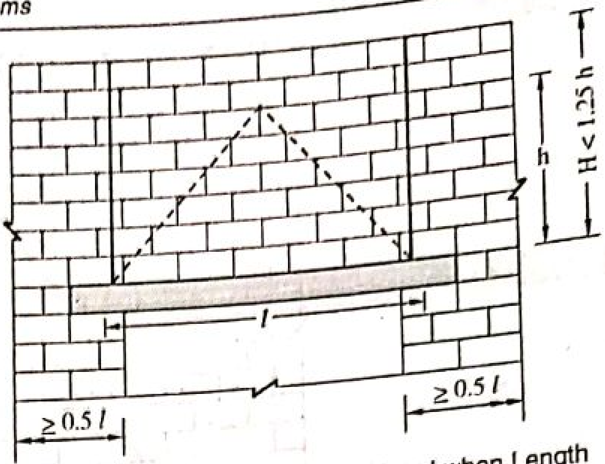
3.6.1 LOADS ON THE LINTEL

In addition to the self weight, weight of masonry above the lintel is to be considered in designing the lintel. The portion of masonry (triangular or rectangular) which acts on the lintel depends on the height of masonry above the lintel and length of wall on each side of the opening. The loading on the lintel for different situations shall be taken as given below.

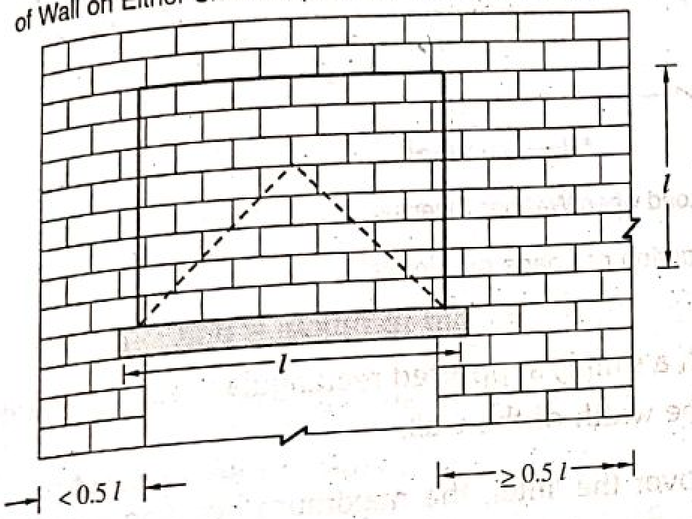
1. Triangular masonry load on the lintel is considered only when the following conditions are satisfied.
   (a) The height of the wall above the opening is more than 1.25 times the height of the triangle and
   (b) Length of the wall on each side of the opening is at least equal to half of the effective span of the lintel as shown in Fig 3.27 (a).
2. Rectangular masonry load on the lintel is considered under the following situations.
   (a) If the height of the wall above the opening is less than 1.25 times the height of the triangle, full rectangular load above the lintel is considered as shown in Fig 3.27 (b).
   (b) If the length of the wall on one side of the opening is less than half of the effective span of the lintel, load from masonry of height equal to the effective span of lintel is to be taken as shown in Fig. 3.27 (c).
   (c) If the length of the wall on both sides of the opening is less than half of the effective span, the loading must be computed for the full height of the wall above the lintel as shown in Fig. 3.27 (d).
3. If a roof slab comes with in the triangular portion, the rectangular load of masonry above the lintel, load from the slab and the triangular portion of masonry above the slab is taken to act on the lintel as shown in Fig 3.27 (e).
4. When there are openings above the lintel in the triangular portion, the loading is computed allowing dispersion lines at 60° as shown in Fig. 3.27 (f).



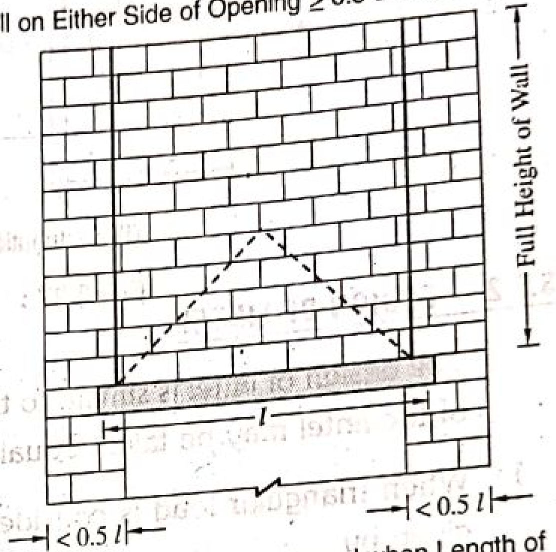
(a) Triangular Distribution of Load when Length of Wall on Either Side of Opening ≥ 0.5 Effective Span



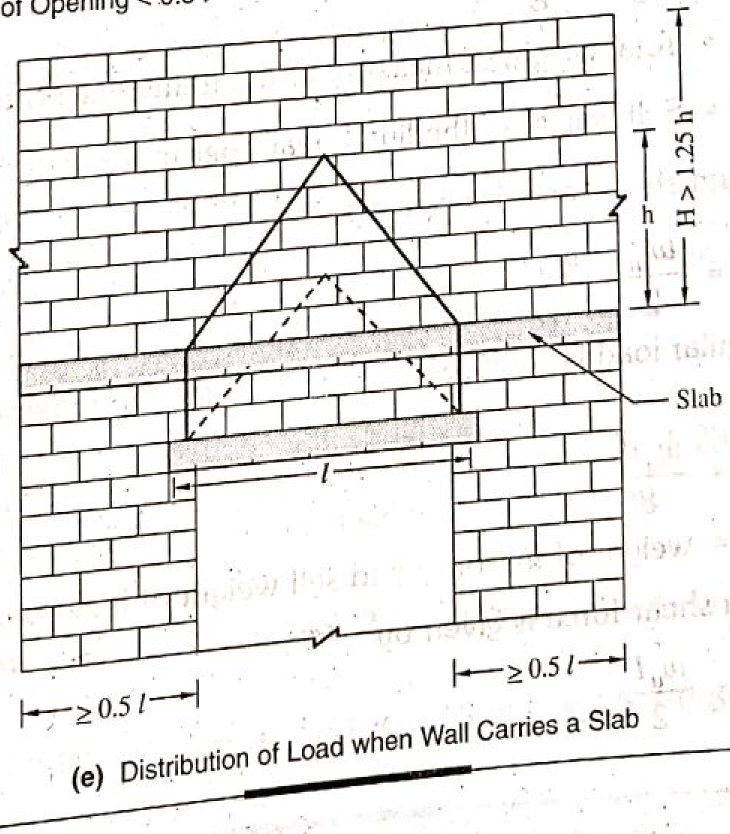
(b) Rectangular Distribution of Load when Length of Wall on Either Side of Opening ≥ 0.5 Effective Span



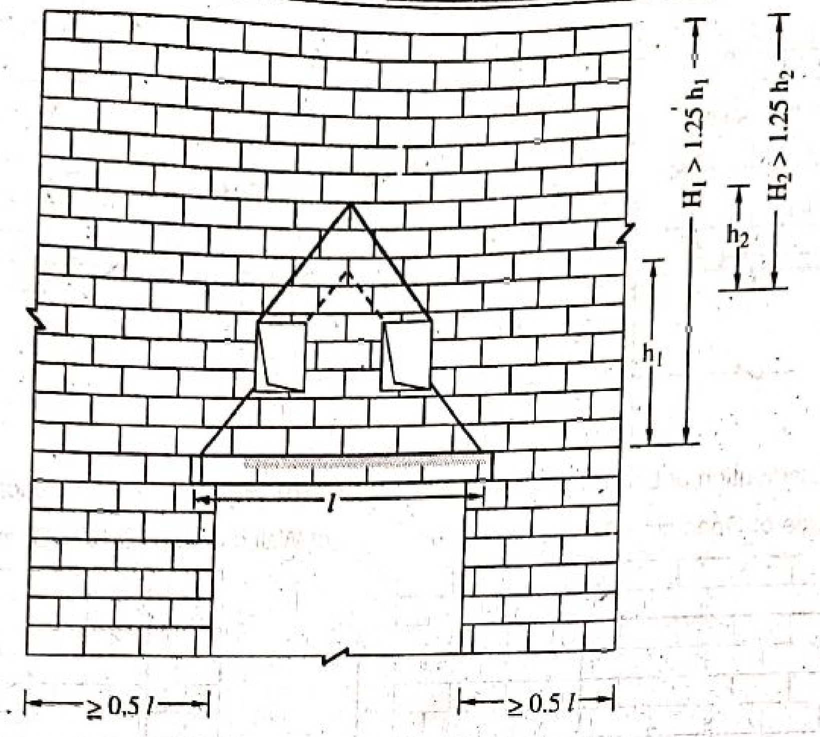
(c) Rectangular Distribution of Load when Length of Wall on One Side of Opening < 0.5 *l*



(d) Rectangular Distribution of Load when Length of Wall on Either Side of Opening < 0.5 *l*



(e) Distribution of Load when Wall Carries a Slab



(f) Distribution of Load when Wall has Opening

FIG 3.27 : Distribution of Loads on Lintels

3.6.2 DESIGN OF LINTEL

The design of lintel is similar to that of a simply supported rectangular beam. The width of the lintel may be taken equal to the width of the wall.

1. When triangular load is considered over the lintel, the maximum bending moment is given by

$$M_u = \frac{w_m\, l}{6} + \frac{w_d\, l^2}{8}$$

Where W_m = Total weight of masonry in the triangular portion

w_d = Self weight of the lintel beam per meter length

The maximum shear force is given by

$$V_u = \frac{w_m}{2} + \frac{w_d\, l}{2}$$

2. When rectangular load is considered over the lintel, the maximum bending moment is given by

$$M_u = \frac{w_u l^2}{8}$$

Where w_u = weight of masonry and self weight of lintel per meter length.

The maximum shear force is given by

$$V_u = \frac{w_u\, l}{2}$$

EXAMPLE - 40

Design a lintel over a door of 2.4m wide. The height of brick work above the opening is 3m. Masonry weighs 19 kN/m³. The brick walls are 230mm thick. Use M-20 grade concrete and Fe-415 grade steel.

(OCT/NOV-2013, 2010, 2006, APRIL/MAY-2012)

Solution :

$$f_{ck} = 20\ N/mm^2$$

$$f_y = 415\ N/mm^2$$

1. **Depth of the Beam :** Selecting the depth in range of $\frac{l}{15}$ to $\frac{l}{20}$ based on stiffness

$$d = \frac{2400}{20} = 120\ mm$$

Adopt $d = 120\ mm$

$D = 150\ mm$

Width b = width of the wall = 230 mm

Assume 150 mm bearing on walls

2. **Effective Span :** Least of

Centre to centre of supports = 2.4 + 0.15 = 2.55 m

Clear span + d = 2.4 + 0.12 = 2.52 m

Hence, Effective span = 2.52 m

3. **Loads :**

Assuming 60° dispersion, height of equilateral triangle

$$h = l \sin 60^\circ$$

$$= 2.52 \times \sin 60^\circ = 2.18\ m$$

$$1.25\ h = 1.25 \times 2.18 = 2.73 < 3\ m$$

Since the height of the masonry above the lintel is more than 1.25 times the height of the triangle, triangular load has to be considered on the lintel.

Masonry load (triangular) $w_m = \frac{1}{2} \times 2.52 \times 2.18 \times 0.23 \times 19 = 12\ KN$

Self weight of the Lintel $w_d = 0.23 \times 0.15 \times 1 \times 25 = 0.86\ KN/m$

Factored Bending moment $M_u = \frac{w_{um}.l}{6} + \frac{w_{ud}.l^2}{8}$

$$= \frac{1.5 \times 12 \times 2.52}{6} + \frac{1.5 \times 0.86 \times 2.52^2}{8}$$

$$= 8.58 \ KN\text{-}m$$

Factored shear force $V_u = \frac{w_{um}}{2} + \frac{w_{ud}.l}{2}$

$$= 1.5 \times \frac{12}{2} + 1.5 \times 0.86 \times \frac{2.52}{2} = 10.63 \ KN$$

4. **Depth Required :**

The minimum depth required to resist Bending Moment

$$M_u = 0.138.\ f_{ck}.bd^2$$

$$8.58 \times 10^6 = 0.138 \times 20 \times 230 \times d^2$$

$$d = \sqrt{\frac{8.58 \times 10^6}{0.138 \times 20 \times 230}}$$

$$= 116.3 \ mm < 120 \ mm$$

Hence provided depth is adequate

5. **Tension Reinforcement :**

Area of steel is calculated by equating C & T (as a balanced section)

$$0.87 \ f_y \ A_{st} = 0.36 \ f_{ck} \ b.\ x_{u,max}$$

$$A_{st} = \frac{0.36 \ f_{ck} \ x_{u,max}}{0.87 \ f_y}$$

$$= \frac{0.36 \times 20 \times 230 \times 0.48 \times 120}{0.87 \times 415} = 264.2 \ mm^2$$

Provide 3 – 12 mm bars, A_{st} provided = 339.3 mm^2

6. **Design of Shear Reinforcement :**

Nominal Shear stress $\tau_v = \frac{V_u}{bd} = \frac{10.63 \times 10^3}{230 \times 120} = 0.39 \ N/mm^2$

Percentage of tension steel at support

$$P_t = \frac{A_{st} \times 100}{bd} = \frac{339.3 \times 100}{230 \times 120} = 1.23\%$$

Referring to the table-19 of IS : 456, Shear strength of concrete is

$$\tau_c = 0.67 \ N/mm^2$$

Maximum shear stress in concrete $\tau_{c,max}$ from Table-20 of IS : 456

$$\tau_{c,max} = 2.8 \ N/mm^2$$

As $\tau_v < \tau_c$, Minimum Shear reinforcement has to be provided.

Using 6 mm, 2 legged Fe250 steel stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.55 \ mm^2$$

Spacing from minimum shear reinforcement consideration as per IS : 456

$$\frac{A_{sv}}{b\,S_v} = \frac{0.4}{0.87\,f_y}$$

$$S_v = \frac{0.87\,f_y\,A_{sv}}{0.4 \times b}$$

$$= \frac{0.87 \times 250 \times 56.55}{0.4 \times 230} = 133.7 \ mm$$

Maximum allowed spacing = 0.75 d = 0.75 × 120 = 90 mm

(or) 300 mm which ever is less.

Hence, provide 2 legged 6 mm stirrups @ 90 mm c/c throughout.

7. Check for Deflection (Stiffness) :

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

% of steel = 1.23

$$f_s = 0.58 \times f_y \times \frac{\text{area of steel required}}{A_{st} \text{ provided}}$$

$$= 0.58 \times 415 \times \left(\frac{264.2}{339.3}\right) = 187.4 \ N/mm^2$$

From Fig.4 of IS 456, modification factor = 1.1

Maximum permitted $\frac{l}{d}$ ratio = 1.1 × 20 = 22

$$\frac{l}{d} \text{ provided} = \frac{2520}{120} = 21 < 22$$

Hence deflection control is safe.

8. Details of Reinforcement are Shown in Fig. 3.28

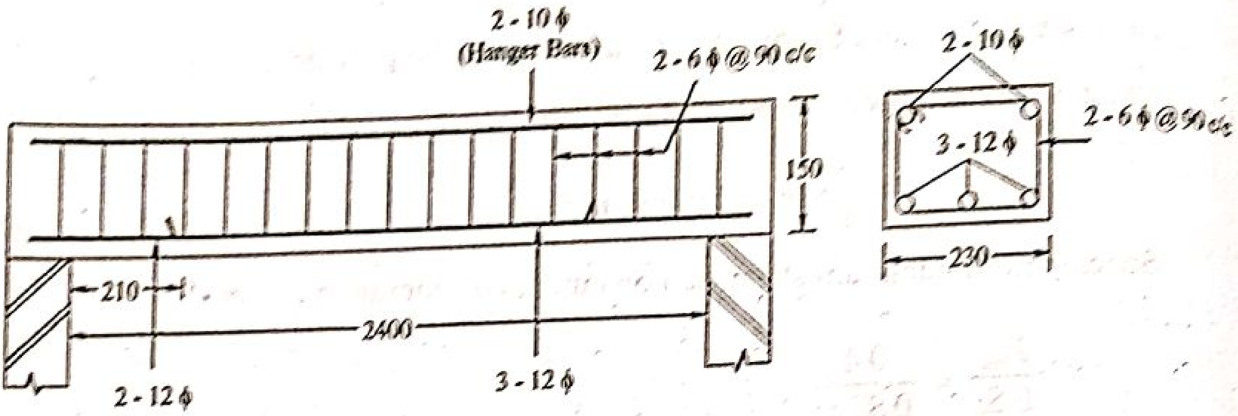


FIG 3. 28 :

EXAMPLE - 41

Design an R.C.lintel for an opening of 1.2 m width on a masonry wall of 230 mm width using M 20 grade concrete and mild steel grade I. The height of masonry wall above the opening is 2.0 m. The lintel has a bearing of 150 mm on the walls. The unit weight of masonry may be taken as 19 kN/m³. No shear reinforcement design is required.

OCT/NOV. 2016 [TS & AP] ; 2007, 2005 ; MARCH/APRIL. 2013 ; 2009

Solution :

$f_{ck} = 20 \text{ N/mm}^2$

$f_y = 250 \text{ N/mm}^2$

1. Depth of the Beam : Selecting the depth in range of $\frac{l}{15}$ to $\frac{l}{20}$ based on stiffness

$$d = \frac{1200}{20} = 60 \text{ mm (too small)}$$

Adopt $d = 90 \text{ mm}$

$D = 120 \text{ mm}$

Width b = width of the wall = 230 mm

Bearing on walls = 150 mm

2. Effective Span : Least of

Centre to centre of supports = 1.2 + 0.15 = 1.35 m

Clear span + d = 1.2 + 0.09 = 1.29 m

Hence, Effective span = 1.29 m

3. **Loads :**

Assuming 60° dispersion, height of equilateral triangle,

$$h = l \text{ Sin } 60^\circ$$

$$= 1.29 \times \text{Sin } 60^\circ = 1.117 \text{ m}$$

$1.25\,h = 1.25 \times 1.117 = 1.4 < 2.0$ m, height of masonry above the lintel

Since the height of the masonry above the lintel is more than 1.25 times the height of the triangle, triangular load has to be considered on the lintel.

Masonry load (triangular) $w_m = \frac{1}{2} \times 1.29 \times 1.117 \times 0.23 \times 19 = 3.15$ kN

Self weight of the Lintel $w_d = 0.23 \times 0.12 \times 1 \times 25 = 0.69$ kN/m

Factored Bending moment $M_u = \frac{w_{um}.l}{6} + \frac{w_{ud}.l^2}{8}$

$$= \frac{1.5 \times 3.15 \times 1.29}{6} + \frac{1.5 \times 0.69 \times 1.29^2}{8}$$

$$= 1.23 \text{ kN-m}$$

4. **Depth Required :**

The minimum depth required to resist Bending Moment

$$M_u = 0.148.\, f_{ck}.bd^2$$

$$1.23 \times 10^6 = 0.148 \times 20 \times 230 \times d^2$$

$$d = \sqrt{\frac{123 \times 10^6}{0.148 \times 20 \times 230}}$$

$$= 42.5 \text{ mm} < 90 \text{ mm}$$

Hence provided depth is adequate

5. **Tension Reinforcement :**

Area of steel is calculated by equating C & T (as a balanced section)

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b\, x_{u,max}$$

$$A_{st} = \frac{0.36\, f_{ck}\, b\, x_{u,max}}{0.87\, f_y}$$

$$= \frac{0.36 \times 20 \times 230 \times 0.53 \times 90}{0.87 \times 250} = 363.2\ mm^2$$

Provide 4-12 mm bars, A_{st} provided = 452.4 mm^2

6. **Check for Deflection (Stiffness) :**

For simply supported beams basic value of $\frac{l}{d}$ ratio = 20

Modification factor for tension steel F_1

$$\%\ \text{of steel} = \frac{A_{st} \times 100}{bxd} = \frac{452.4 \times 100}{230 \times 90} = 2.19\ \%$$

$$f_s = 0.58 \times f_y \times \frac{\text{area of steel required}}{A_{st}\ \text{provided}}$$

$$= 0.58 \times 250 \times \left(\frac{363.2}{452.4}\right) = 116.4\ N/mm^2$$

From Fig.4 of IS 456, modification factor = 1.2

Maximum permitted $\frac{l}{d}$ ratio = 1.2 × 20 = 24

$\frac{l}{d}$ provided = $\frac{1290}{90}$ = 14.33 < 24

Hence deflection control is safe.

7. **Details of Reinforcement are Shown in Fig. 3.29.**

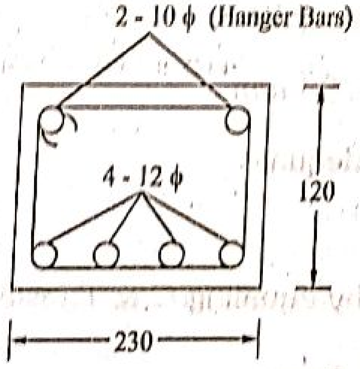


Fig 3.29 :

REVIEW QUESTIONS

Short Answer Questions :

1. List the four assumptions made in the design of flexural members by limit state method. **(March/April. 2017 ; 2007, 2003 ; April/May. 2011 ; Oct/Nov. 2004)**

2. Explain : (a) Depth of neutral axis and (b) Lever arm **(Oct/Nov. 2005)**

3. Name the types of sections of a singly reinforced rectangular beam. How do you identify each? **(April/May. 2011 ; Oct/Nov. 2005)**

4. What is a balanced design ? **(March/April. 2006)**

5. State the equations for minium and maximum area of tension reinforcement in beams and hence calculate minimum and maximum area of steel for a beam 300 mm × 450 mm effective dimensions, effective cover 40 mm, Fe-415 grade steel is used. **(Oct/Nov. 2004)**
[Ans : $A_{st,min}$ = 276.5 mm^2, $A_{st,max}$ = 5880 mm^2]

6. If the ultimate moment is 100 kN-m, what is the effective depth of singly reinforced concrete section, if the width of the beam is 230 mm and concrete grade is M 20 and type of steel is Fe 415 ? **[Ans : 396.9 mm]**

7. Find the depth of neutral axis of singly reinforced rectangular beam 230 mm x 400 mm effective depth, reinforced with 4 bars of 12 mm diameter. Grade of concrete is M-20 and grade of steel is Fe-415. Use Limit state method **(March/April. 2015)**

8. A singly reinforced concrete beam 230 mm wide and having effective depth 400 mm is of concrete grade M 20. The steel is of grade Fe 415. What will be the moment of resistance is the depth of neutral axis is restricted to 0.3 d? **(March/April. 2007)**
[Ans : 55.64 kN-m]

9. The dimensions of a simply supported rectangular beam are 230 mm wide and 450 mm deep effective and provided with Fe 415 steel and M 20 grade concrete. Determine the limiting moment of resistance of beam. **(Oct. 2005)**
[Ans : 128.55 kN-m]

10. The dimensions of a singly reinforced, simply supported rectangular beam are 300 mm wide and 450 mm deep effective, provided with Fe-415 grade steel and M20 grade concrete. Determine the limiting moment of resistance of the beam. **(March/April. 2019 ; Oct/Nov. 2013)**

11. State four situations in which doubly reinforced sections are used.
(March/April. 2017, 2003 ; April/May. 2011 ; Oct/Nov. 2006, March/April. 2008)

12. What is meant by doubly reinforced beam and mention two situations where it is used.
(April/May. 2011 ; Oct/Nov. 2003)

13. For what purpose shear reinforcement is provided in beams. (Oct/Nov. 2003)

14. What are the different forms of shear reinforcement in beams?
(March/April. 2016 ; Oct/Nov. 2009, 2003)

15. What are the different forms of shear reinforcement in beams? (Oct/Nov. 2009, 2003)

16. Give steps involved in shear design of a simply supported beam (March/April. 2007)

17. State various modes of shear failure. (March/April. 2005)

18. A singly reinforced rectangular beam of size 230 × 459 mm (effective size) is reinforced with 4 16 mm dia. HYSD bars as tension steel. The ultimate shear force at the section is 120 kN. State whether shear reinforcement is required or not. (Oct/Nov. 2004)

19. A singly reinforced rectangular beam of size 230 x 450 mm (effective size) is reinforced with 4-16 mm dia. HYSD bars as tension steel. The ultimate shear force at the section is 120 kN. State whether shear reinforcement is required or not.
(March/April. 2014 ; Oct/Nov-2004)

20. Calculate the spacing of two legged 8 mm diameter stirrups as per minimum shear reinforcement for a beam 250 mm wide and 400 mm overall depth, if Fe250 bars are used.
(April/May. 2011)

21. Calculate the spacing of two legged 8 mm diameter stirrups as per minimum shear reinforcement for a beam 350 mm wide and 500 mm overall depth of Fe415 bars are used. (April/May. 2012)

22. Calculate the spacing of two legged 8 mm diameter stirrups as per minimum shear reinforcement for a beam 350 mm wide and 500 mm overall depth. Fe-415 bars are used.
(Oct/Nov. 2013 ; April/May. 2012)

23. What is bond ? Define development length. (March/April. 2007)

24. Define development length and State factors affecting bond strength
(March/April. 2004)

25. What is development length? State its importance in reinforced concrete structures. *(March/April. 2005)*

26. Calculate the development length for a single Fe 415 grade steel bar of 12 *mm* diameter in concrete of M20 grade under (a) Tension (b) Compression *(Oct/Nov. 2005)*

27. Calculate the development length in tension for Fe-250 bar of 25 mm diameter and M-20 concrete. **(April/may-2015)**

28. What is the necessity of anchorage of reinforcement.

29. Calculate the anchorage length in tension for a single HYSD bar of diameter ϕ in concrete grade M 20. *(March/April. 2006)*

Write the anchorage value of a standard hook and 90° bend. *(Oct/Nov. 2006)*

31. Draw the sketch of a standard bend. What is its anchorage value ? *(April/May. 2012)*

32. Draw a sketch of a standard bend. What is its anchorage value? *(March/April-2014, 2012)*

33. State simplified rules for curtailment of bars in

(a) Simply supported beams (b) Cantilevers *(April/May. 2011)*

34. Define a lintel and explain any one case, how the load on a lintel can be found *(Oct/Nov. 2005)*

35. What are the functions of a lintel. *(March/April. 2008)*

36. Explain with neat sketch how the loading on lintel is taken for different situations. *(Oct/Nov. 2009)*

37. Explain with a neat sketch, when do you consider the triangular masonry load on the lintel. *(Oct/Nov. 2010 ; April/May. 2011)*

38. Define lintel and draw a neat sketch of lintel cum sunshade with structural detailing *(March/April. 2010)*

Essay Type Questions :

1. Find the limiting moment of resistance of a singly reinforced beam of size 250 *mm* × 400 *mm* effective depth. Concrete used is of M 20 grade ans steel Fe 415. Also determine the limiting percentage of steel. *(Oct/Nov. 2009)*

[Ans : $M_{u.lim}$ = 110.4 *kN-m* & $P_{t,lim}$ = 0.96 %]

2. Find the ultimate moment or resistance of singly reinforced rectangular beam 230 mm × 500 mm effective depth reinforced with 4 bars of 16 mm diameter. Concrete is of M 20 grade and steel Fe 415. **[Ans : 123.8 kN-m]**

3. Find the ultimate moment or resistance of singly reinforced rectangular beams 230 mm × 400 mm effective depth reinforced with 4 bars of 20 mm diameter. Concrete is of M 20 grade ans steel Fe 415. **[Ans : 101.54 kN-m]**

   [Hint : Section is over reinforced and hence limit x_u to $x_{u,max}$]

4. A simply supported rectangular beam of width 300 mm and effective width is subjected to a factored moment of 185 kN-m. Find the area of steel required if M20 grade concrete and Fe 415 grade steel is used. [Ans : A_{st} = 830 mm^2]

5. Calculate the area of reinforcement required for as simply supported reinforced concrete beam 230 mm wide and 400 mm effective depth to resist an ultimate moment of 50 kN-m. Assume M 20 and Fe 415 combination of concrete and steel. *(Apr. 2007)*

   [Ans. 378.5 mm^2]

6. A reinforced concrete beam of rectangular section 300 mm wide and 550 mm effective depth is reinforced with 4 bars and 25 mm diameter. The beam is simply supported over an effective span of 7 m. Find the uniformly distributed ultimate load the beam can carry if M 20 grade concrete and Fe 415 grade steel is used. [Ans : w_u = 40.81 kN/m]

7. A reinforced concrete beam 300 mm x 600mm overall depth is reinforced with 4 bars of 20 mm diameter at an effective cover of 40 mm. What uniformly distributed load this beam can carry exclusing self weight, over a simply supported span of 5m. Assume M20 concrete and Fe415 steel. ***(April/May. 2011)***

8. A singly reinforced concrete beam of rectangular section 250 mm x 550 mm overall depth is reinforced with 5 bars of 16 mm diameter with an effective cover of 50 mm. The beam is simply supported over an effective span of 6 m. Find the uniformly distributed ultimate load the beam can carry excluding its self weight. Use M 20 grade concrete and Fe 415 grade steel. Use Limit state method.

   (March/April-2016)

9. Design a singly reinforced beam of width 230 mm simply supported over a clear span of 3.5 m. The width of the support is 230 mm and it carries a live load of 22 kN/m. Use M 20 grade concrete and Fe 415 grade steel. Also check for development length.

   (Oct/Nov. 2009)

10. Calculate the ultimate moment of resistance of an R.C. beam of rectangular section 300 *mm* wide and 400 *mm* deep. Area of steel consists of 6 Nos 18 ϕ in tension and 2 Nos 18 ϕ in compression. Assume steel of grade Fe 415 and concrete of grade M 20 and an effective cover 35 mm on both sides. ***(March/April 2007)***

[Ans : 176.36 kN-m]

11. Design a rectangular beam for an effecitve span of 6 *m*. The superimposed load is 80 *kN/m* and size of rectangular section is limited to 300 *mm* wide and 700 *mm* deep with an effective cover of 70 *mm* for both tension and compression. Assume M 20 and Fe 415. ***(Oct/Nov. 2007)***

12. A doubly reinforced beam of width 250 mm and 500 mm effective depth is reinforced with 2 bars of 20 mm diameter in compression and 6 bars of 20 mm diameter in tension zones. Find the ultimate moment of resistance of the beam section. Effective cover is 40 mm for both the steels. Concrete grade is of M 25 and steel is Fe 415. **(March/April-2014)**

13. A rectangular doubly reinforced beam of size 230 *mm* wide and 500 *mm* effective depth is subjected to a factored moment of 175 *kN-m*. Find the reinforcement for flexure. The materials used are M 20 concrete and HYSD bars. ***(March/April. 2010)***

14. An R.C.C. beam 230 *mm* wide and 450 *mm* deep is reinforced with 4 bars of 16 *mm* ϕ and grade Fe 415 on tension side. If design shear force is 60 kN. Design the shear reinforcement consisting only of vertical stirrups. The grade of concrete used is M20. ***(March/April. 2007)***

15. A simply supported R.C. beam 380 *mm* wide and 750 *mm* effective depth carries a UDL of 80 *kN/m* (including self weight) over a span of 6 m. The beam is reinforced with 6 Nos. 22 *mm* diameter bars of grade Fe 415 on tension face. Design the shear reinforcement using vertical stirrups and bent up bars. Assume 2 Nos. of 22 *mm* dia. are bent up. ***(March/April. 2004)***

[Ans : 2- 8 ϕ @ 240 *mm c/c*]

16. An RCC beam, 250 mm wide and 450 mm effective is reinforced with 6 bars of 16 mm diameter on tension side of which two bars are cranked up near the support. if the design shear force is 150 kN, design the shear reinforcement considering bent up bars. Assume 2 Nos. of 22 mm dia. are bent up. Concrete grade is of M 20 and steel is Fe 415. **(Oct/Nov-2015)**

17. An R.C. beam of breadth 300 *mm* and overall depth of 550 *mm* is provided with an effective cover of 50 *mm* and reinforced with 4 nos of 25 *mm*. ϕ bars as main reinforcement. Design shear is 150 *kN*. Use M 20 grade concrete and Fe 415 steel.
    - (i) Value of nominal shear stress
    - (ii) Shear resistance of beam
    - (iii) Spacing of 2 legged 8 *mm* dia vertical stirrups.
    - (iv) Check for maximum spacing of stirrups **(March/April. 2008)**

[Ans : τ_v 1 N/mm^2, τ_e = 0.68 N/mm^2]

18. Design a lintel over a door of 2.0 *m* wide. The height of brick work above the opening is 2.3 *m*. Masonry weighs 19 kN/m^3. The brick walls are 230 *mm* thick. Use M20 grade concrete and Fe 415 grade steel. **(March/April. 2008)**

19. A lintel is to be provided to support a masonry wall of 230 *mm* thick over an opening of 2 *m*. Concrete grade M 20 and steel grade Fe 415 are used. Assuming perfect arch action, design the lintel. **(March/April. 2010)**

20. Design a lintel over a door of 1.2 m wide. The height of brick work above the opening is 2 m. Masonry weighs 19 kN/m^3. The brick walls are 230mm thick. Use M-20 grade concrete and Fe-250 grade steel. **(March/April-2013)**