CHAPTER 1

INTRODUCTION TO R.C.C. AND PRINCIPLES OF WORKING STRESS METHOD

Chapter Outline

1.0 INTRODUCTION

Concrete structures have become very common in Civil engineering construction. Concrete has established as a universal building material because of its high compressive strength, its adoptability to take any form and shape and resistant to fire and corrosion with negligible maintenance cost. Concrete is very strong in compression but very weak in tension. Its low tensile strength is compensated by introducing steel reinforcement in the tension zone. Thus, the concrete is strengthened by steel and the resultant composite mass is known as Reinforced Cement Concrete. Thus RCC is used extensively in construction of buildings, bridges, tanks, dams etc. The design of these modern reinforced concrete structures may appear to be complex. However, most of these structures are the assembly of several basic structural elements such as beams, columns, slabs, walls and foundations. Accordingly, the designer has to learn the design of these basic reinforced concrete elements. The joints and connections are then carefully developed.

1.1 CONCRETE

Concrete is a composite material consisting of cement, aggregate and water in suitable proportions. The chemical interaction between cement and water binds the aggregates in to a solid mass. Fresh concrete will be plastic, so that it can be moulded to any desired shape in the moulds and compacted to form a dense mass. Water has to be applied for few days over the concrete surface soon after its setting because the hydration reactions between cement and water continue for a longer period due to which hardening of concrete takes place. This period when concrete is kept moist during which concrete gains strength is called curing period. Hence, the strength of concrete increases with age. The process of solidification of concrete from plastic stage is called setting while gaining of strength after setting is called hardening. Usually, setting completes within a maximum duration of 10 hours, while about 90% of hardening is completed by 28 days.

The properties and quality of cement concrete are influenced by the properties of its ingredients and quality control maintained during its making and curing.

1.2 INGREDIENTS OF CONCRETE

The main ingredient materials in concrete are:

(a) Cement

(b) Aggregates

(c) Water

1.2.1 CEMENT

Cement is the binding material which is obtained by burning calcareous, siliceous and argillaceous materials together in definite proportions at high temperature and grinding the resultant clinker in to a fine powder. Various types of cements have been developed for the use in different types of structures under different situations. According to IS : 456-2000, the types of cements and their suitability for a specific situations are given in Table 1.1.

TABLE 1.1: Types of Cements and their Suitability

S.No.	Type of Cement	IS Code	Where Used
1.	Ordinary Portland cement		
	33 Grade	IS: 269	All general concreting works
	43 Grade	IS: 8112	Multi story structures
	53 Grade	IS: 12269	Bridges, Tall structures, Prestressed concrete work
	(Compressive strength of cement at 28 days in N/mm^2 is called as grade of cement)		
2.	Rapid hardening cement	IS: 8041	Road works and repairs
3.	Low heat Portland cement	IS: 12600	Mass concreting-Dams
4.	Port land slag cement	IS: 455	Marine structures
5.	Portland pozzolana cement	IS: 1489	General building works, mass concrete, Marine structures
6.	Sulphate resisting Portland cement	IS: 12330	Marine structure foundations in sulphate bearing soils
7.	Hydrophobic cement	IS: 8043	Swimming pools, floors of food processing plants
8.	High alumina cement	IS: 6452	Marine structures
9.	Supersulphated cement	IS: 6909	Marine structures, construction of sewers

1.2.2 AGGREGATES

Around 75% volume of concrete is occupied by the aggregates. Hence, the structural behaviour of concrete is significantly influenced by the type of aggregates used. The aggregates used for the concrete should be durable, strong, hard, chemically inert and well graded.

Aggregates whose particle size varies from 0.075 *mm* to 4.75 *mm* are called as fine aggregate. Aggregates with particle sizes more than 4.75 *mm* are called as coarse aggregates. Usually sand is used as fine aggregate where as crushed rock and gravel is used as coarse aggregate.

The nominal maximum size of the coarse aggregate shall be as large as possible but it should be limited to ¼ th of the minimum thickness of the member.

The various properties of aggregates like specific gravity, strength, toughness, hardness, soundness, particle size distribution and grading should comply with the IS code IS:383-1979.

1.2.3 WATER

Water plays an active role in the chemical process of hydration of cement and in curing of concrete. Hence, the water used for mixing and curing of concrete should be clean and free from injurious amount of oils, acids, alkalies, salts, organic matter etc. that may be deleterious to concrete and steel. Drinking water is generally considered satisfactory for mixing of concrete. Sea water should not be used for mixing and curing because of presence of harmful salts in it. The *PH* value of water should not be less than 6. The physical and chemical tests for water should be done as per IS: 3025.

1.2.4 ADMIXTURES

Admixtures are added to the concrete before or during mixing, to modify one or more of the specific properties of concrete in the fresh or hardened states. IS:9103-1979 lays down the procedures for evolution of admixtures for concrete. The different types of admixtures used are given below.

1. **Accelerating Admixtures:** These are added to concrete to increase the rate of early strength development, which in turn facilitates earlier removal of form work. Common accelerators are calcium chloride, fluosilicates and trietanlamine.
2. **Retarding Admixtures:** These are added to slow down the rate of setting of cement. They are useful in hot weather concreting. Common types of retarders are starches and cellulose products, sugar and hydroxyl-carboxylic acids.
3. **Water Reducing or Plasticizing Admixtures:** The addition of plasticizer allows greater workability for given water cement ratio or alternatively retains the workability while reducing the water content. The basic ingredients of water reducing agents are either lignosulphonate salts or polyhydroxy compounds.
4. **Air-Entraining Admixtures:** These are used to incorporate air in the form of minute bubbles in concrete usually to increase workability and resistance to freezing and thawing. Commonly used air-entraining agents are animal and

vegetable oils, natural wood resins and their sodium salts, alkali salts of sulphated and sulphonated organic compounds.

1.3 IMPORTANT PROPERTIES OF CONCRETE

1.3.1 COMPRESSIVE STRENGTH/GRADE OF CONCRETE

The compressive strength of concrete is the characteristic compressive strength (f_{ck}) of 150 *mm* concrete cubes at the age of 28 days in N/mm^2. The characteristic strength is defined as the strength of material below which not more than 5% of the test results are expected to fall.

IS 456-2000 specifies the characteristic compressive strength of 150 *mm* cube at the age of 28 days in N/mm^2, as the Grades of concrete varying from 15 to 80 N/mm^2 designated as M 15 to M 80. Based on the characteristic compressive strength (f_{ck}), concrete is graded as given below :

TABLE 1.2 : Grades of Concrete (Table 2 of IS 456-2000)

Group	Grade Designation	Characteristic Compressive Strength of 150 mm Cubes at 28 Days in N/mm^2, f_{ck}
Ordinary concrete	M 10	10
	M 15	15
	M 20	20
Standard concrete	M 25	25
	M 30	30
	M 35	35
	M 40	40
	M 45	45
	M 50	50
	M 55	55
High strength concrete	M 60	60
	M 65	65
	M 70	70
	M 75	75
	M 80	80

In the designation of concrete mix, *M* refers to the mix and the number to the characteristic compressive strength of 150 *mm* cubes at 28 days in N/mm^2.

For reinforced concrete, the minimum grade of concrete to be used is M 20. For various weather conditions, the minimum grade of concrete to be used for reinforced concrete is given in Table 1.3.

TABLE 1.3 : Minimum Grade of Concrete for Different Exposure Conditions

Exposure	Minimum Grade of Concrete for RCC
Mild	M 20
Moderate	M 25
Severe	M 30
Very severe	M 35
Extreme	M 40

1.3.2 TENSILE STRENGTH

The tensile strength of concrete is very low and hence it is not taken in to account in the design of reinforced concrete. But it is an important property which affects the extent and width of cracks in the structure. According to IS 456-2000, the tensile strength of concrete can be calculated from the compressive strength using the following relation.

$$f_{cr} = 0.7\sqrt{f_{ck}}\ N/mm^2$$

Where f_{ck} is the characteristic cube compressive strength of concrete.

1.3.3 MODULUS OF ELASTICITY

Modulus of elasticity of concrete is an important property required for computation of deflections of structural concrete members. In the absence of test data, the modules of elasticity of concrete is related to compressive strength by the following relation as per IS 456 - 2000.

$$E_c = 5000\sqrt{f_{ck}}\ N/mm^2$$

Where E_c is the short term static modules of elasticity in N/mm^2

1.3.4 POISSON RATIO

Poisson ratio varies between 0.1 for high strength concrete and 0.2 for weak concrete. Usually it is taken as 0.15 for strength design and 0.2 for serviceability calculations.

1.3.5 SHRINKAGE

The property of decreasing in volume during the process of drying and hardening of concrete is called shrinkage. Shrinkage in concrete may results in surface cracks. Shrinkage of concrete also influences the deflections of reinforced concrete members.

It depends up on the ingredients of concrete and environmental conditions like temperature and humidity. The IS : 456-2000 recommends the total shrinkage strain of 0.0003 for design purpose.

1.3.6 CREEP

Creep is defined as plastic deformation under constant load or stress. The creep coefficient which is defined as the ratio of ultimate creep strain to the elastic strain at various ages of loadings as recommended by IS 456-2000 is given below.

Age at Loading	Creep Coefficient
7 days	2.2
28 days	1.6
1 year	1.1

Creep of concrete considerably increases the deflections of reinforced concrete flexural members. Higher the creep coefficient more will be the deflections. The value of creep coefficient is useful in the calculation of time dependent deflections in reinforced concrete members.

1.3.7 WORKABILITY

The workability of freshly mixed concrete is that property which determines the ease and homogeneity with which it can be mixed, placed, compacted and finished. A workable concrete should not show any segregation (separation of constituents of concrete in its plastic stage) or bleeding (rise of water in the mix to the surface when it is compacted).

A workable concrete possesses required lubrication for handling concrete with out segregation, can be placed in the forms with out loss of homogeneity, compacted with minimum effort and shall be finished easily.

The factors which influence the workability are

(i) Water-Cement ratio

(ii) Type and grading of aggregate.

(iii) Ratio of fine and coarse aggregate.

(iv) Use of admixtures.

(v) Efficiency of mixing.

Recommended tests to measure workability are

(i) Slump test (ii) Compacting factor test

(iii) Vee Bee consistometer test (iv) Flow test

A Suggested ranges of workability of concrete in accordance of IS : 1199 are given below.

TABLE 1.4 : Workability for Different Placing Conditions (Clause 7.1 of IS 456-2000)

Placing Conditions	Degree of Workability	Slump mm
Blinding concrete, shallow sections, pavements using pavers.	Very low	(0.75-0.8 compaction factor)
Mass concrete, lightly reinforced sections in slabs, beams walls, columns, floors, Hand placed pavements, canal linings, strip footing	Low	25-75
Heavily reinforced sections in slabs, beams, walls, columns.	Medium	50-100
Slip form work, pumped concrete	Medium	75-100
Trench fill. Insitu pilling	High	100-150
Tremie concrete	Very high	Based on flow test

1.3.8 UNIT WEIGHT

The unit weight of concrete depends up on the type of aggregates and amount of voids. The unit weight as specified by the IS : 456-2000 for plain concrete and reinforced concrete are 24 kN/m^3 and 25 kN/m^3 respectively.

1.4 ADVANTAGES OF CONCRETE

The following are the advantages of concrete due to which concrete is extensively used in construction industry.

1. Compressive strength of concrete is very high
2. Concrete can be moulded to any desired shape.
3. The materials for concrete are easily available.
4. It is easy to make.
5. It is durable
6. By proper proportioning of mix, concrete can be made watertight.

7. Its monolithic character gives it more rigidity.
8. It is fire resisting.
9. Its maintenance cost is practically nil.
10. Strength of concrete increases with age.

Disadvantages of Concrete:

1. Tensile strength of concrete is very low and hence plain concrete cannot be used in situations where tensile stresses are developed.
2. Strict quality control has to be maintained during production, placing and compaction.
3. Curing has to be done for at least 14 days and hence time of construction increases.
4. Once the members caste with concrete, it is very difficult to dismantle it.

1.5 CONCRETE MIX PROPORTIONS

The mix proportions shall be selected to ensure the workability of fresh concrete and when concrete is hardened, it shall have the required strength, durability and surface finish.

The main objective of concrete mix proportioning is to select the optimum proportion of the various ingredients of concrete (cement, aggregate, and water), to attain the required workability (when it is fresh), strength and durability (when it is hardened) with minimum cost. It can be done in the following two ways.

(a) Nominal mix concrete

(b) Design mix concrete

1.5.1 NOMINAL MIX CONCRETE

In nominal mix, the proportions of cement, aggregate and water are nominally adopted. Nominal mix concrete is used on small works for routine concrete construction and for concrete *M* 20 and lower. However it requires high cement content. Nominal proportions such as 1:2:3, 1:3:6 etc. have no significance for strength and durability. In these, the quantity of fine aggregate is fixed irrespective of the cement content and the maximum size of aggregate. Hence, the variations in the quality of concrete produced are inevitable. The proportions of materials for nominal mix concrete prescribed by IS : 456 - 2000 are given in Table. 1.3.

TABLE 1.5 : Proportions for Nominal Mix Concrete (Table 9 of IS 456-2000)

Grade Concrete	Total Quantity of Dry Aggregates by Mass Per 50 kg of Cement, kg	Proportion of Fine Aggregate to Coarse Aggregate (by Mass)	Quantity of Water Per 50 kg of Cement Max
M 5	800	Generally 1:2 but subjected to an upper limit of 1:1 1/2 and a lower limit of 1:2 1/2	60
M 7.5	625		45
M 10	480		34
M 15	330		32
M 20	250		30

Note: The proportion of the fine to coarse aggregates should be adjusted from upper limit to lower limit progressively as the grading of fine aggregates becomes finer and the maximum size of coarse aggregate becomes larger.

Example : For an average grading of fine aggregate (Zone-II), the proportions shall be 1:1½, 1:2, 1:2½ for maximum size of aggregate 10 *mm*, 20 *mm* and 40 *mm* respectively.

The proportions for nominal mix concrete generally adopted in practice are given below.

Grade of Concrete	Nominal Mix Proportions (Cement : Fine Aggregate : Coarse Aggregate)
M 5	1 : 5 : 10
M 7.5	1 : 4 : 8
M 10	1 : 3 : 6
M 15	1 : 2 : 4
M 20	1 : 1½ : 3

1.5.2 DESIGN MIX CONCRETE

In design mix, the proportions of cement, aggregate, water and mineral admixtures if any are actually designed. For all important works involving large quantities of concrete, it is preferable to use design mix, which results in considerable economy ensuring the required strength.

The process of mix design consists of selecting optimum proportions of cement, aggregate and water to produce the grade of concrete with the required workability, strength and durability with minimum cost.

Over the years, several mix design methods have been developed, which are listed below.

(i) Minimum voids method
(ii) Maximum density method
(iii) Fineness modulus method
(iv) Road research laboratory (RRL) method
(v) American concrete institute (ACI) method
(vi) I.S Code method

1.6 REINFORCED CONCRETE

Reinforced concrete (RCC) is concrete that contains embedded steel bars, plates, or fibres that strengthen the material.

Concrete has a high compressive strength and a low tensile strength. This led to the invention of reinforced concrete, which is a combination of concrete and steel in which steel provides the tensile strength lacking in concrete. In addition, steel can also carry the compressive load as found in column structures.

The combination of concrete and steel in reinforced concrete produces a structural element that take the advantage of each material. The high tensile capacity of steel is used to overcome the low strength of concrete. On the other hand, the high resistance of concrete against corrosive environment is used to protect steel, which is prone to corrosion. Also both concrete and steel are bonded together very well, thus able to resist the load as a unit. This excellent bond is achieved by chemical adhesion between two materials, the surface roughness and closely spaced ribs along steel.

To produce good reinforced concrete elements, concrete and steel must be of good quality. Mix design for concrete must be prepared such that the produced concrete satisfies the workability, strength, and exposure condition requirements. The steel is produced in factory and normally in good quality. However, make sure that the steels are clean and not corroded.

1.7 STEEL REINFORCEMENT

Steel bars are essentially used in the tension zone of flexural members of concrete to resist the tensile stresses as concrete is weak in tension and in compression members to increase the load carrying capacity.

Steel is used as reinforcement to take up the tensile stresses in RCC construction because of the following reasons.

(a) Its tensile strength is high
(b) It can develop good bond with concrete.

(c) It coefficient of expansion is nearly same as for concrete.

(d) It is easily available.

1.7.1 SIGNIFICANCE/FUNCTIONS OF REINFORCEMENT IN R.C.C

The reinforcement in RCC serves the following different types of functions.

(i) To resist the bending tension in flexural members like slabs, beams and walls of water tanks etc.

(ii) To increase the load carrying capacity of compression members like columns.

(iii) To resist diagonal tension due to shear.

(iv) To resist the effects of secondary stresses like temperature etc.

(v) To reduce the shrinkage of concrete.

(vi) To resist spiral cracking due to torsion.

(vii) To prevent the development of wide cracks in concrete due to tensile strains.

1.7.2 TYPES OF REINFORCEMENT

Reinforcing steel consists of bars usually circular in cross section. The following four types of steel reinforcement are generally used in reinforced concrete construction.

(a) Mild steel and medium tensile steel bars conforming to IS : 432 (Part- I)

(b) High yield strength deformed steel bars (HYSD bars)conforming to IS : 1786

(c) Steel wire fabric conforming to IS : 1566.

(d) Structural steel conforming to Grade-A of IS : 2062

All reinforcement shall be free from loose mill scale, loose rust, oil, mud, and any other substances which reduces bond between steel and concrete. The grades of steel normally used for reinforcement are *Fe* 250, *Fe* 415 and *Fe* 500. In mentioning the grade of steel, *Fe* refers to ferrous metal and the number following it refers to specified guaranteed yield strength in N/mm^2. The stress strain curves for different grades of steel are shown in Fig. 1.1.

Mild Steel Bars (Fe 250) : These plain bars of mild steel are of grade *Fe* 250 conforming to IS : 432 - 1982. It has well defined yield point giving yield stress of 250 N/mm^2 and excellent ductility with high percentage of elongation at failure (> 23 %). The actual and idealized stress strain curve for mild steel is shown in Fig 1.1. The properties of mild steel are given in Table 1.4.

TABLE 1.5 : Properties of Mild Steel (IS 432-1982, Part-I)

Type of Steel	IS Code	Bar Diameter ≤ 20 mm	Minimum Yield Stress N/mm² (f_y)	Minimum Elongation %	Min Ultimate Stress N/mm²
Mild Steel Grade - I	IS 432-1982 Part - I	≤ 20 mm 20 to 50 mm	250 240	23 23	410 410
Mild Steel Grde - II	IS 432-1982 Part - I	≤ 20 mm 20 to 50 mm	225 215	23 23	370 370
Medium tensile steel	IS 432-1982 Part - I	≤ 16 mm 16 to 32 mm > 32 to 50 mm	350 340 330	20 20 20	530 540 510

High Yield Strength Deformed Bars (Fe 415 & Fe 500) : As the name indicates, these HYSD bars have high yield strength but the yield point is not well defined. The yield stress or characteristic stress is given by 0.2 % proof stress. These bars possess ribs, lugs or deformations on their surface due to which the bond strength is improved. The properties of HYSD bars are given in Table 1.6.

TABLE 1.6 : Properties of HYSD Bars (IS 1786-1985)

Property	Fe 415	Fe 500
0.2% proof stress, min N/mm²	415	500
Elongation %, min	14.5	12.0
Ultimate stress, min N/mm²	15 % more than actual 0.2% proof stress	10 % more than actual 0.2 % proof stress

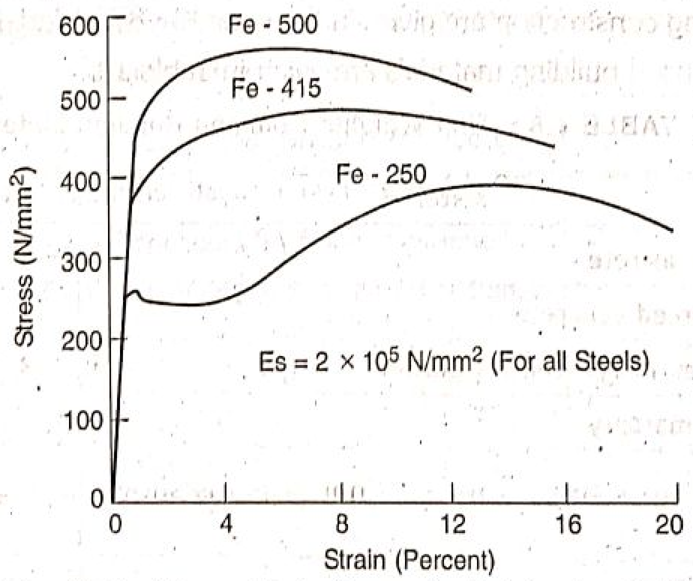


FIG 1.1 : Typical Stress-Strain Curves for Reinforcing Steels

1.7.3 MODULES OF ELASTICITY OF STEEL

The module of elasticity of steel of all grades is taken as $2 \times 10^5\ N/mm^2$ ($200\ kN/mm^2$).

1.7.4 UNIT WEIGHT OF STEEL

The unit weight of steel is $78.5\ kN/m^3$ ($7850\ kg/m^3$). A quicker method to find the weight of bar of circular section is given by the following equation.

$$\text{Weight of bar in kg/m} = \frac{\phi^2}{162.2}$$

Where ϕ = Diameter of the bar in mm

1.8 LOADS

The reinforced concrete structures are designed to resist the following types of loads.

(i) Dead loads

(ii) Live loads or Imposed loads

(iii) Wind loads

(iv) Earthquake forces

(v) Snow loads

(vi) Shrinkage, creep and Temperature effects.

(vii) Other forces and effects

1.8.1 DEAD LOADS

Dead loads are permanent or stationary loads which are transferred to the structure throughout their life span. Dead loads mainly due to self weight of structural members, permanent partitions, fixed equipments and fittings. These loads shall be calculated by estimating the quantity of each material and then multiplying it with the unit weight. The unit weights of various materials used in building construction are given in the code IS : 875 (Part-1) - 1987. The unit weight of commonly used building materials are given in Table 1.8.

TABLE 1.8 : Unit Weights Common Building Materials

S.No.	Material	Unit Weight KN/m^3
1.	Plain concrete	24
2.	Reinforced concrete	25
3.	Brick masonry , cement plaster	20
4.	Stone masonry	24
5.	Wood	8
6.	Steel	78.5
7.	Floor finish	0.6 - 1.2

1.8.2 LIVE LOADS (OR) IMPOSED LOADS

These are the loads that changes with respect to time. Live loads or imposed loads include loads due to the people occupying the floor, weight of movable partitions, weight furniture and materials. The live loads to be taken in design of buildings have been given in IS : 875 (Part - 2) - 1987. Some of the common live loads used in the design of buildings are given in Table 1.8.

TABLE 1.8 : Live loads on floors (IS : 875, Part-2)

S.No.	Type of Floors	Minimum Live Load kN/m^2
1.	Floors in dwelling houses, tenements, hospital wards, hostels and dormitories	2.0
2.	Office floors other than entrance halls, floors of light	2.5 - 4.0 (2.5, when separate storage work rooms facility is provided, other wise 4.0)
3.	Floors of banking halls, office entrance halls and reading rooms	3.0
4	Shops, educational buildings, assembly buildings, restaurants	4.0
5	Office floors for storage, assembly floor space without fixed seating, public rooms in hotels, dance halls and waiting halls	5.0
6.	Ware houses, work shops and factories (a) Light weight loads (b) Medium weight loads (c) Heavy weight loads	 5.0 7.5 10.0
7.	Garages (light-handling vehicles of weight < 25 kN) Garages (heavy- vehicles of weight > 25 kN)	4.0 7.5
8.	Stairs, landings, Balconies and corridors for floors mentioned in 1, but not liable to over crowding Stairs, landings and corridors for floors mentioned in 1, but liable to overcrowding and for all other floors	3.0 5.0
9	Flat slabs, sloped roofs (a) Access provided (b) Access not provided	 1.5 0.75

1.8.3 WIND LOADS

The horizontal load caused by the wind is called as wind loads. It depends up on the velocity of wind and shape and size of the building. Complete details of calculating wind loads on structures are given in IS : 875 (Part - 3) - 1987. Brief idea of these provisions is given below.

(i) Basic wind pressure V_b has to be picked up depending upon the locality of the building.

(ii) To get the design wind velocity Vz the following expression shall be used.

$$V_z = k_1 k_2 k_3 V_b$$

Where k_1 = Risk coefficient

k_2 = Coefficient based on terrain, height and structure size.

k_3 = Topography factor

(iii) The design wind pressure is given by

$$P_z = 0.6\ V_z^2$$

Where P_z is in N/m^2 at height Z and V_z is in m/sec. Up to a height of 30 m, the wind pressure is considered to act uniformly. Above 30 m height, the wind pressure increases.

1.8.4 EARTH QUAKE FORCES

Earthquake shocks cause movement of foundation of structures. Due to inertia additional forces develop on the super structure. The impact of earthquake on structures depends on the stiffness of the structure, stiffness of the soil media, height and location of the structure etc. Accordingly, the country has been divided into several zones depending on the magnitude of the earthquake. Depending on the problem, one of the following two methods may be used for computing the seismic forces.

(i) Seismic coefficient method.

(ii) Response spectrum method.

The details of these methods are prescribed in IS : 1893 code and also in National Building Code of India.

1.8.5 SNOW LOADS

These are important loads for structures located in areas having snow fall, which gets accumulated in different parts of the structure depending on projections, height, slope etc. of the structure. The standard values of snow loads are specified in Part 4 of IS : 875.

1.8.6 SHRINKAGE, CREEP AND TEMPERATURE EFFECTS

Shrinkage, creep and temperature (high or low) may produce stresses and cause deformations like other loads and forces. Hence, these are also considered as loads which are time dependent. The safety and serviceability of structures are to be checked following the stipulations of clauses 6.2.4, 5 and 6 of IS : 456 : 2000 and Part 5 of IS : 875.

1.8.7 OTHER FORCES AND EFFECTS

IS : 456 : 2000 stipulates the following forces and effects to be taken into account in case they are liable to affect materially the safety and serviceability of the structures. The relevant codes as mentioned therein are also indicated below:

- Foundation movement (IS : 1904)
- Elastic axial shortening
- Soil and fluid pressures (IS : 875 - Part - 5)
- Vibration
- Fatigue
- Impact (IS : 875 - Part-5)
- Erection loads (IS : 875 - Part - 2)
- Stress concentration effect due to point of application of load and the like.

1.8.8 COMBINATION OF LOADS

Load combinations have to be taken in accordance to the IS : 875 (Part - 5) - 1987.

1.9 METHODS OF DESIGN OF REINFORCED CONCRETE

The aim of design is to decide the size of the member and amount of reinforcement required, so that the structure will perform satisfactorily during its life period with minimum cost. With an appropriate degree of safety the structure should

(i) Sustain all loads

(ii) Sustain the deformations during and after construction

(iii) Should have adequate durability and

(iv) Should have adequate resistance to misuse and fire.

In any method of design of reinforced concrete structures, the following are the common steps to be followed:

(i) To assess the dead loads and other external loads and forces likely to be applied on the structure,

(ii) To determine the design loads from different combinations of loads,

(iii) To estimate structural responses (bending moment, shear force, etc.) due to the design loads,

(iv) To determine the cross-sectional areas of concrete sections and amounts of reinforcement needed.

Many of the above steps have lot of uncertainties. Estimation of loads and evaluation of material properties are to name a few. Hence, some suitable factors of safety should be taken into consideration depending on the degrees of such uncertainties.

The following three methods have been developed for the design of reinforced concrete structures.

(a) Working stress method (b) Ultimate load method

(c) Limit state method

1.9.1 WORKING STRESS METHOD

Working stress method is based on elastic theory assuming reinforced concrete as elastic material. The stress strain curve of concrete is assumed as linear from zero at the neutral axis to a maximum value at the extreme fiber.

This method adopts permissible stresses which are obtained by dividing ultimate stress by a factor known as factor of safety. For concrete a factor of safety of 3 is used and for steel it is 1.78. This factor of safety accounts for any uncertainties in e... working loads and variation ...

CHAPTER 2

PHILOSOPHY OF LIMIT STATE DESIGN

Chapter Outline

2.1 INTRODUCTION

As discussed in the previous chapter, working stress method gives satisfactory performance of the structure at working loads; it is unrealistic at ultimate state of collapse. Similarly, while the Ultimate load method provides realistic assessment of safety, it does not guarantee the serviceability requirements at working loads. An ideal method is the one which takes in to account not only the ultimate strength of the structure but also the serviceability requirements. Limit state method oriented towards the simultaneous satisfaction of all these requirements.

2.2 PHILOSOPHY OF LIMIT STATE METHOD

A structure may become unfit for use not only when it collapses but when it violates the serviceability requirements such as deflections, cracking etc. A structure is said to have reached its limit state, when the structure as a whole or a part becomes unfit for use, during its expected life. The limit state of a structure is the condition of its being not fit for its intended use.

The philosophy of limit state method of design is to see that the structure remains fit for use throughout its life period by assuring safety against strength and serviceability requirements i.e. the structure will not reach the limit state in its life time. All the relevant limit states have to be considered in the design. The loads and strength of materials are to be estimated by probabilistic approach (characteristic values). The design loads and strengths are derived from the characteristic values through the use of partial safety factors.

2.3 LIMIT STATES

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. Many types of limit states or failure conditions can be specified. The two major limit states which are usually considered are the following.

(i) Limit state of collapse

(ii) Limit state of serviceability

(i) Limit State of Collapse :

It is the limit state at which the structure is likely to collapse. The structure may collapse due to rupture of one or more critical sections or loss of overall stability due to buckling or overturning. This limit state may correspond to

(a) Flexure

(b) Compression

(c) Shear

(d) Torsion

(ii) Limit State of Serviceability :

Limit state of serviceability relate to the performance of the structure at working loads. It is the limit state at which the structure undergo excessive deflection, which adversely affect the finishes causing discomfort to the users and excessive cracking which effects the efficiency or appearance of the structure. This limit state may correspond to

(a) Deflection

(b) Cracking

(c) Other limit states (Vibrations, Fire resistance, Durability)

All relevant limit states have to be considered in the design to ensure adequate degree of safety and serviceability. The structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states.

2.4 CHARACTERISTIC STRENGTH OF MATERIALS

In Limit state design, design should be based on characteristic values for material strengths and applied loads, which take into account the variations in the material strengths and in the loads to be supported. The characteristic values should be based on statistical data if available; where such data are not available they should be based on experience.

The characteristic strength of material is that value of the strength of the material below which not more than 5% of the test results are expected to fall.

For concrete, characteristic strength is compressive strength of 150mm cubes of 28 days in N/mm^2, below which not more than 5% of the test results are expected to fall. Concrete grades are specified based on this strength.

For steel, the minimum yield strength or 0.2% proof strength is taken as the Characteristic strength of steel.

2.5 CHARACTERISTIC LOAD

The maximum working load that the structure has to withstand and for which it has to be designed is called Characteristic load. The characteristic loads are calculated based on statistical analysis.

The characteristic load is defined as that value of load which has 95% probability of not being exceeded during the life of the structure.

The various loads acting on structures consist of dead loads, live loads, wind or earthquake loads etc. These are discussed in sec.1.7. However, the researches made so far fail to estimate the actual loads on the structure. Accordingly, the loads are predicted based on statistical approach, where it is assumed that the variation of the loads acting on structures follows the normal distribution (Fig. 2.1).

Characteristic load should be more than the average or mean load. Accordingly,

Characteristic load = Average or mean load + K (standard deviation for load)

The value of K is assumed such that the actual load does not exceed the characteristic load during the life of the structure in 95 per cent of the cases.

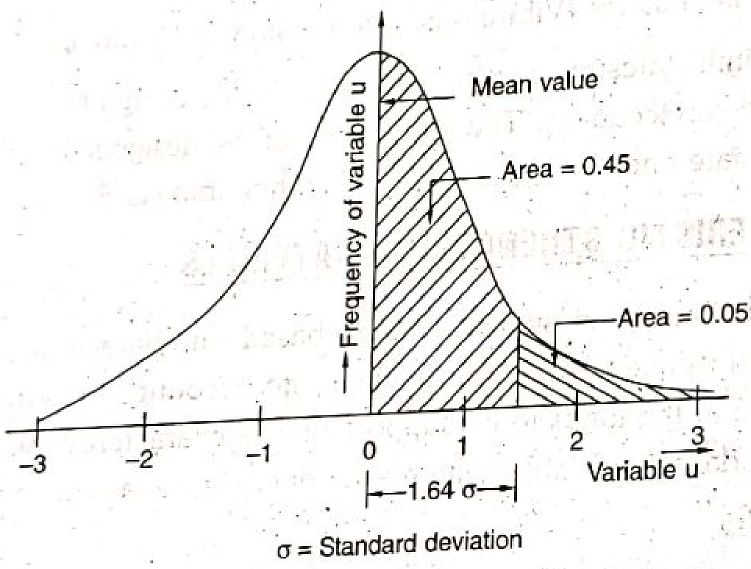


FIG 2.1 : Normal Distribution Curve

2.6 PARTIAL SAFETY FACTORS

The design values are derived from the characteristic values through the use of partial safety factors, one for material strengths and the other for loads.

The characteristic values of loads as discussed in sec. 2.5 are based on statistical data. It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the structures (Fig. 2.1). However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors (γ_f) for loads.

Similarly, the characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test results are expected to fall. However, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material (γ_m), where γ_m depends on the material and the limit state being considered.

2.7 DESIGN STRENGTH OF MATERIALS

The design strength of materials is obtained by dividing the characteristic strength by a factor known as partial safety factor. The partial safety factor takes in to account variation of material strength, local weakness etc.

The design strength of the materials, f_d is given by

$$\text{Design strength} = \frac{\text{characteristic strength}}{\text{partial safety factor}}$$

$$f_d = \frac{f}{\gamma_m}$$

Where f = characteristic strength of the material

γ_m = Partial safety factor appropriate to the material and limit state being considered.

The value of partial safety factors materials as recommended by IS 456-2000 is given in Table 2.1.

Table 2.1 Partial safety factor for material strengths, γ_m as per IS 456-2000

Material	Limit State of Collapse	Limit State of Serviceability
Steel	1.15	1.0
Concrete	1.5	1.0

A higher value of partial safety factor for concrete (1.5) has been adopted because there are greater chances of variation of strength of concrete due to improper compaction, inadequate curing, improper batching & mixing and variations in properties of ingredients. The partial safety factor for steel (1.15) is comparatively lower because the steel for reinforcement is produced in steel plants and commercially available in specific diameters with expected better quality control than that of concrete.

Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes. To take the size effect into account, it is assumed that the concrete in the structure develops a strength of 0.67 times the characteristic strength of cubes. Accordingly, in the calculation of strength employing the limit state of collapse, the characteristic strength (f_{ck}) is first multiplied with 0.67 (size effect) and then divided by 1.5 (γ_m for concrete) to have 0.446 f_{ck} as the maximum strength of concrete in the stress block.

2.8 DESIGN LOADS

Design loads or factored loads, for which the structure is to be designed, are obtained by multiplying the characteristic load with a factor known as partial safety factor. The partial safety factor takes in to account inaccurate assessment of load, unexpected stress redistribution and variation in dimensional accuracy.

The design load, F_d is given by

Design load = characteristic load × Partial safety factor

$$F_d = F.\ \gamma_f$$

Where F = characteristic load

γ_f = Partial safety factor appropriate to the nature of loading and limit state being considered.

The value of partial safety factors for loads as recommended by IS 456-2000 is given in Table 2.2.

Table 2.2 Partial safety factors for loads, γ_f (Table 18 of IS 456-2000)

Load Combination	Limit State of Collapse			Limit State of Serviceability		
	DL	LL	WL	DL	LL	WL
DL + LL	1.5	1.5	—	1.0	1.0	
DL + WL	1.5 or 0.9*	—	1.5	1.0	—	1.0
DL+LL+WL	1.2	1.2	1.2	1.0	0.8	0.8

* This value is to be used when stability against overturning or stress reversal is critical

2.9 DIFFERENCES BETWEEN WORKING STRESS METHOD & LIMIT STATE METHOD

The differences between Working stress method (WSM) and Limit state method (LSM) are mentioned in Table 2.3.

TABLE 2.3 : Differences between WSM and LSM

S.No.	Working Stress Method	Limit State Method
1.	The Stresses in an element is obtained load and compared with design strength. permissible stresses.	The stresses are obtained from design from the working loads and compared with
2.	The method follows linear stress-strain behaviour of both the materials (Elastic approach).	In this method, it follows non linear stress relationship (plastic design) but linear strain relationship.
3.	Material strengths are under estimated to large extent. Factor of safety are used in working stress method.	The material capabilities are not underestimated as much compared WSM. Partial safety factors are used in limit state method.
4.	WSM is stress based method.	LSM is strain based method.
5.	WSM substantially reduces the calculation efforts due to linear stress-strain behaviour.	LSM involves more calculation efforts due to nonlinear stress behaviour.
6.	Average or statistic values are used for loads and material strengths in WSM.	Characteristic values (derived from probabilistic approach which is more realistic and reasonably accurate) are used in case of LSM.
7.	This method yields to uneconomical design.	This method yields to economical design.

REVIEW QUESTIONS

1. Define characteristic strength of concrete. Explain what is meant by M20 mix. **(Oct. 2005)**
2. State the methods of design of reinforced concrete structures. **(March/April. 2019)**
3. Differentiate between Nominal mix concrete and design mix concrete.
4. State the loads to be considered in the design of reinforced concrete elements. **(March. 2006, 2010)**
5. Define the term Limit state. Mention the different types of limit states. **(March/April. 2017 ; April/May. 2011)**
6. Name the three methods recommended by IS 456 for the design of R.C.C members. **(Oct. 2003)**

Unit-1

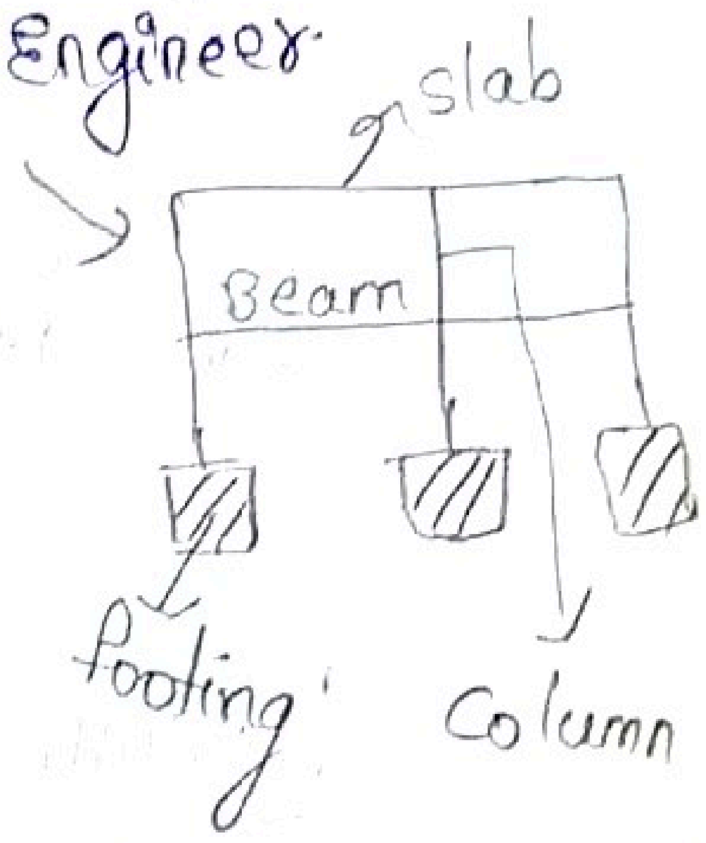
Introduction

Limit state Design

Design → 1.) Functional design - role of Architecture Ex:- No of rooms

2) Structural design - Structural Engineer

Ex:- Beam, Slab.



Design means proportioning structural elements.

Beam undergoes flexure (Bending)

Concrete :- Concrete weak in tension and strong in compression.

Two types

1.) Plain Concrete → Without steel

2) Reinforced Concrete → with steel in Concrete.
which gives strength to particular material.

Design Codes and hand books:-

Basic Code for design:-

IS:456:2000 → Code of practice for plain and reinforced Concrete (Fourth revision)

Loading standards:-

IS 875 - (Part 1-5) Code of practice for design loads (other than earth quakes)

Part-I Dead load, Part-2 Live loads, Earthquake & wind loads (Part-III) + high floors (Part-III), Snow (Part-IV)

Design hand book:-

SP 16:1980 - Design Aids for Reinforced Concrete to IS 456:1978

SP 34 : 1987 — Hand Book on Concrete reinforcement and detailing.

The ~~following are the~~ basical codes and hand books for desigining a Concrete Structure.

Design Consideration:-

Flexure, shear, torsion, axial force (tensile (or) compressive etc) etc.

1.) Safety — Strength, Stability, integrity

2.) Serviceability — implies adequate performance of structure interms of deformation, cracks, Vibrations, fire resistance.

Concrete is only to bare the Compresion load because in foundation Concrete is used

1.7 More Earth quake. (severly) collapses.

Loads:-

Horizontal loads — wind, floods

Vertical loads — Live load, wind

longitudinal loads — Sudden stoping of moving loads.

Loading Standards:-

Dead load + Live Load will comes.

Part-1 Dead load (self weight (or)) (DL):- Static load of buildings (Columns, Beams, walls)
→ Cal. By unit weight (1 kN) x Quantity (Volume l×b×h)

2 Live load (LL):- → Not Static, moving loads, Persons, furniture, Cars.

Imposed load (IL):- Marching, cranes; Cal. percentage of live loads.

3 Wind load (WL):- Due to wind.

4 Earth quake load (EL):-

6 Longitudinal load (LL):-

1.) Working stress method:- (WSM)

2.) Limit State Method (LSM)

1.) Working Stress method:- This method simply uses within elastic theory.

The both steel and Concrete behaves as elastic.

Stress in the material < permissible stress.

Premissible stress = characteristic strength of Concrete (f_{ck})
(or)
yeild strength of steel (f_y)
(or) Ultimate load
/ Factor of Safety.

Perimisible stress < strength of material.

$$\frac{M}{I} = \frac{E}{R} = \frac{f}{y}$$ → Flexure formula.

Factor of Safety

For Concrete - 3 | for IS 456
Steel - 1.8

Stress in Steel is equal to the stress in surrounding Concrete is called

stress Compactibility.

Draw back:- → WSM fails due to different loads. Combinations.

→ It does not considered creep and shrinkage.

→ It is uneconomical design.

Theory of simple Bending

$$\frac{M}{I} = \frac{E}{R} = \frac{f}{y}$$

The design of a structural member is carried out by two methods.

1) Working stress method (WSM):- (or) [elastic theory] (or) modular ratio method. $M = \frac{E \cdot S}{E \cdot C}$

2) Limit State method (LSM) → present

1) Working Stress Method (WSM):- → shape is linear.

→ It is based on elastic theory and assums both steel and Concrete are elastic and obey's Hooke's law

→ The bases of this method is at worst possible combination of working states the stresses in the materials are not exceeded beyond permissible values.

i.e permissible stress < Strength of material [fck (or) fy]

Permissible stress are Calculate by using factor of safety.

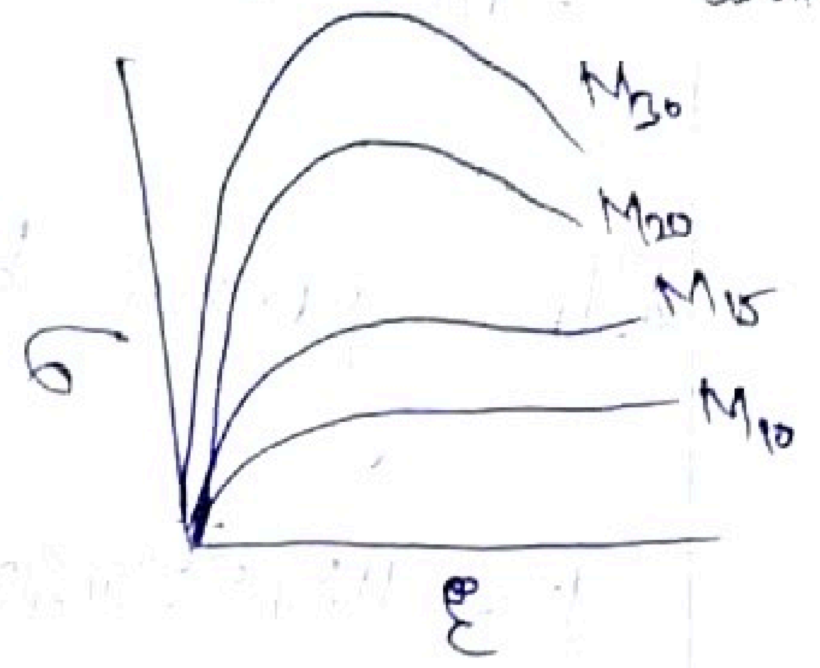
Permissible Stress = Characteristic Strength of material (fck) (or) yeild strength of steel (fy)

Factor of Safety.

As per IS: 456:2000 a factor of Safety 3 is used for Concrete ~~and~~ in Bending Compression and 1.8 is used for ~~Steel~~ yeid Strength of steel.

DrawBacks:- (less life time) (In this only Bear particular loads when extra load apply it does not bear failure will occur)

1.) The assumption Concrete is elastic is not true. Since Concrete is hetrogenous material.



2.) It uses factor of safety for Stresses not for loads.

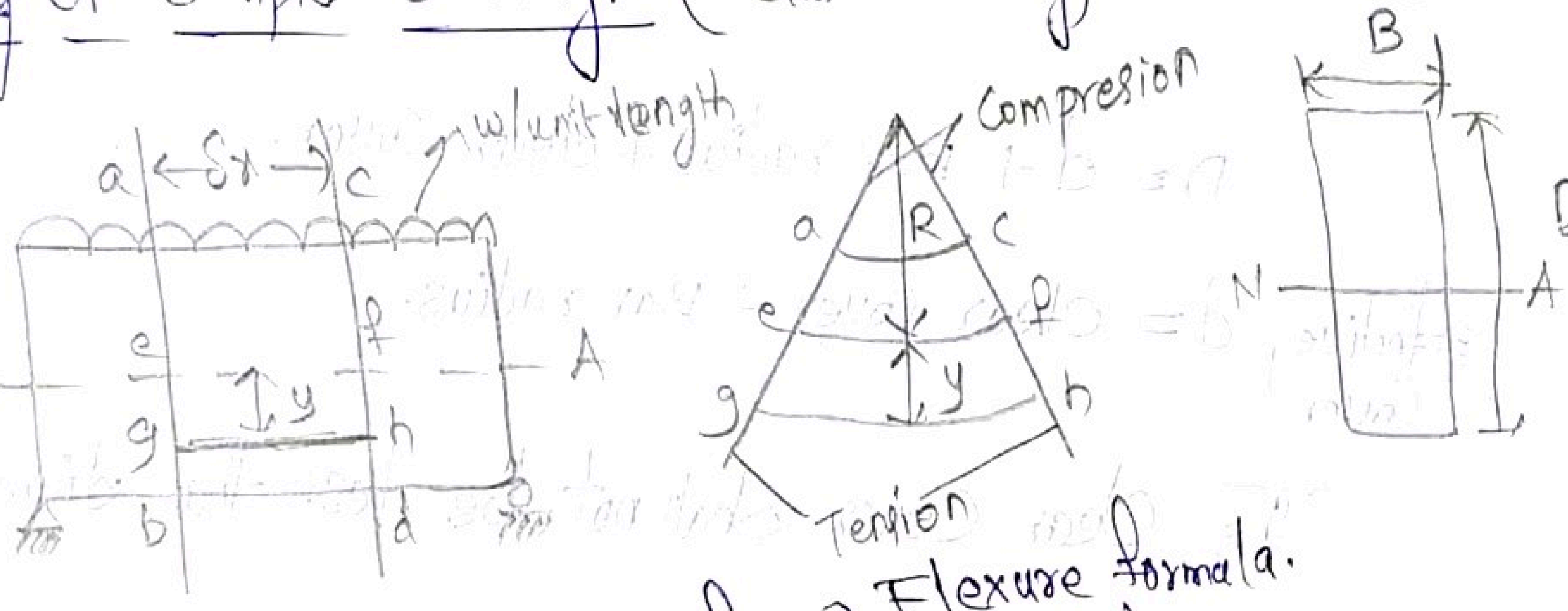
i.e It fails to explain load Combinations and to access failure load.

3.) It doesn't account for Shrinkage and Creep which are time dependent and plastic in nature.

4.) This method gives uneconomical (or) larger Sections.

Assume there is no change in length
R.C = Steel + Concrete.
Strain Steel + Strain Concrete = Strain in R.C.

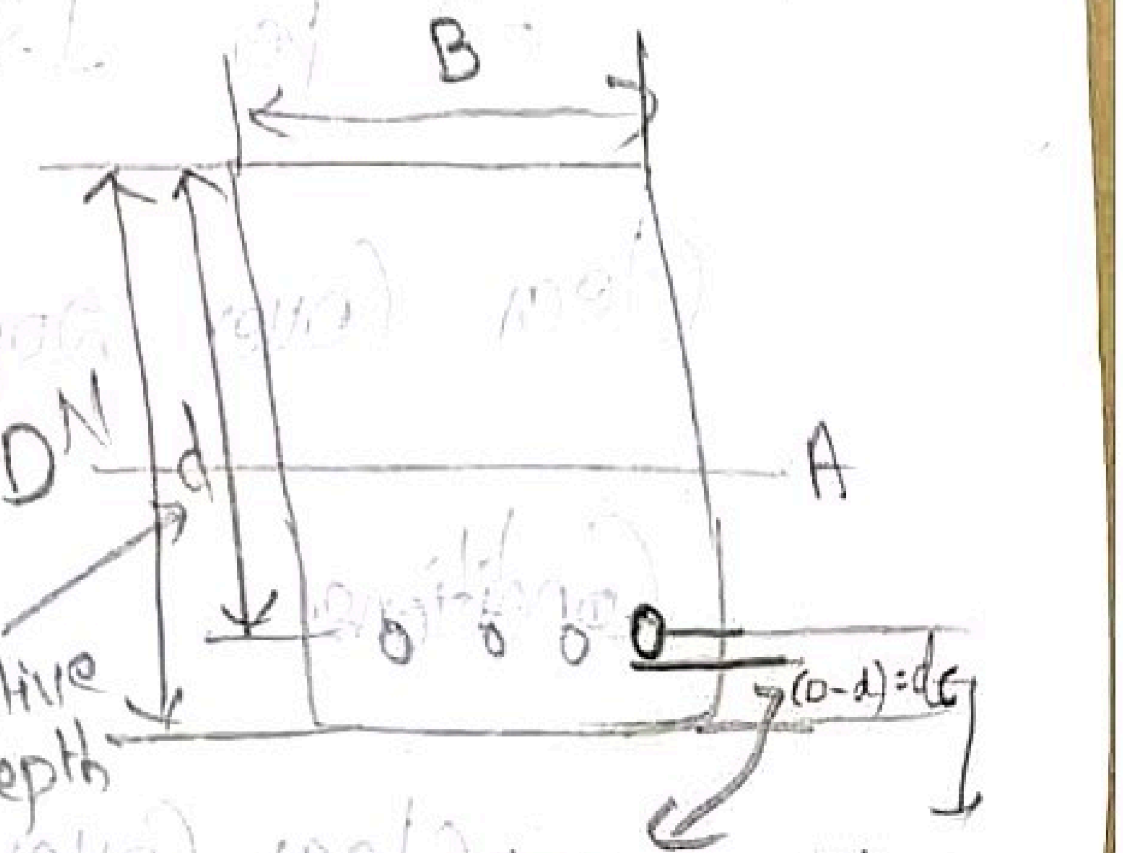
Theory of Simple Bending:- (Elastic theory).



$$\frac{E}{R} = \frac{M}{I} = \frac{f_y}{y}$$ → Flexure formula. (or) Bending equation.

Size of Beam:-

Beam - 25mm
Column - 40 mm
footings - 50 mm



1.) Size of Beam in general depends on Architectural requirements.

2.) The Breadth of the Beam is often restricted

Clear Cover — Effective Cover

D = d + effective cover
= d + Radius Reinforcement + clear Cover.
D = d + $\frac{\phi}{2}$ + clear cover

by wall thickness and depth is restricted by clear Space between beam and floor level.

3) In general B and D are provided as follows.

B = 115, 230, 250, 300, 350mm etc.

D = 230, 250, 300, 350, 375, 400, 450, 500, 600mm etc.

Cover to reinforcement:-

1) Cover is provided to ensure the grip of Concrete over reinforcement. So, that they act as one to resist loads.

2) To prevent (or) protect reinforcement from Corrosion.

$D = d + \text{Bar radius} + \text{clear Cover}$.

effective Cover, $d = \text{Clear Cover} + \text{Bar radius}$.

The Clear Cover shall not be less than diameter (φ) of the bar.

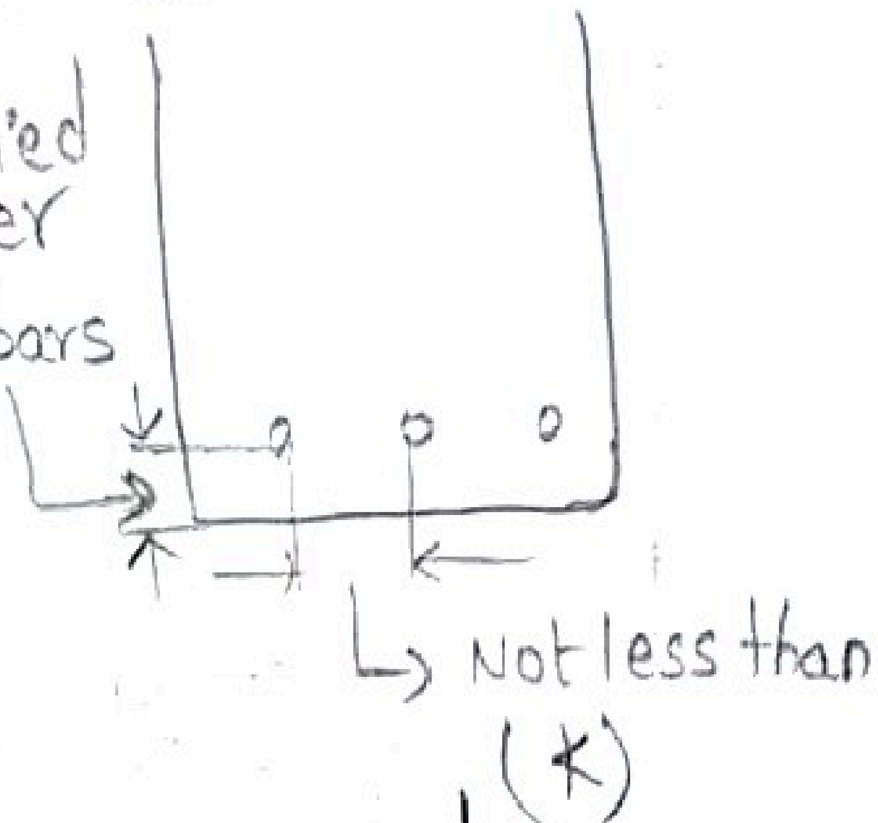
As per IS 456:2000 table No.16 provide a minimum Clear Cover 20mm to the Stirups from the mild exposure Conditions.

∴ Clear Cover to the Main Reinforcement = 20mm + Stirup diameter 8 (or) 10mm.

≃ 30mm.

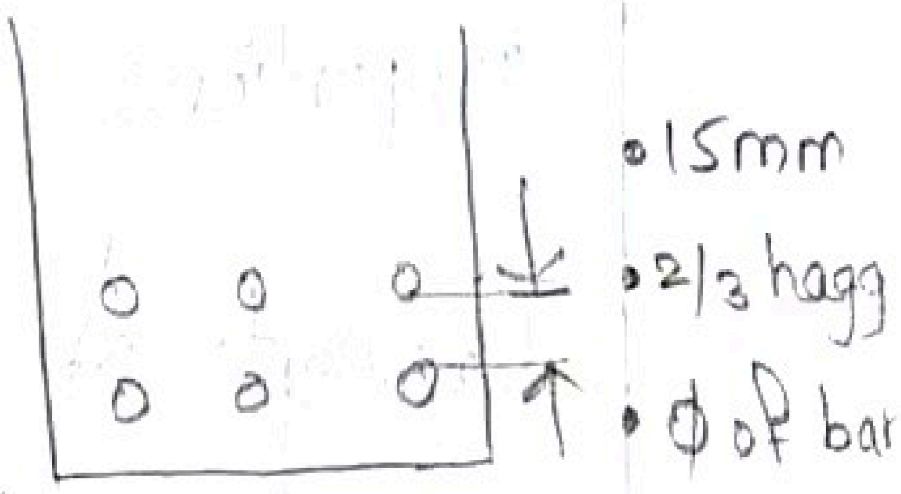
Spacing of bar:-

Bars shall be placed in such a way that they allowed Concrete to enter when placing: (or) A vibrator can be immersed.



The minimum Spacing of bars as ~~follow~~ shown in fig.

• h_{agg} + 5mm
• ϕ of bar.



h_{agg} = Nominal maximum size of aggregate.

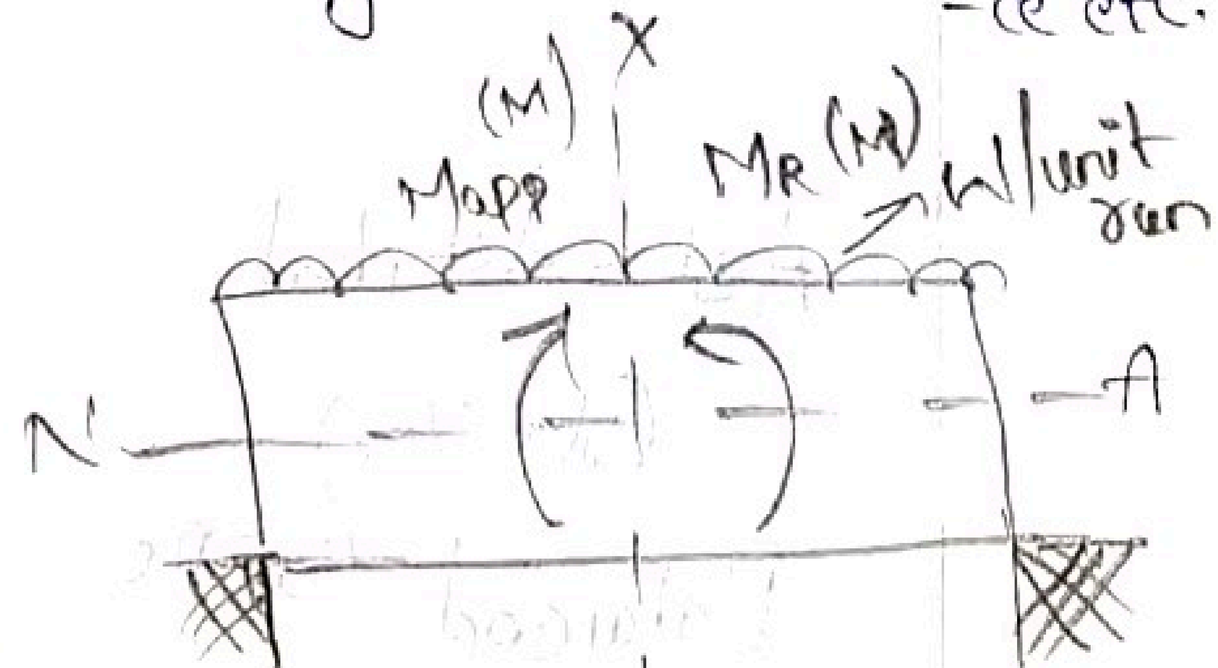
Design requirements of a Beam :-

Mainly the beam is designed for

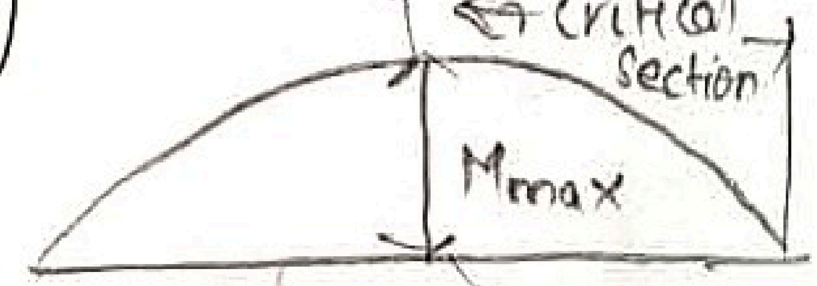
1) Strength → includes flexure, shear, torsion, axial force (tensile (or) Compressive) etc.
2) Serviceability → includes Deflection, cracking, Vibrations, fire resistance etc.

Fundamental of design:-

$$M_{app} \leq M_R\ (M).$$



Let Consider a Simply Supported beam subjected to UDL w/unit run as shown in fig. As the load gradually increases the applied moment also increases. At one particular stage the applied moment becomes maximum. ~~The~~



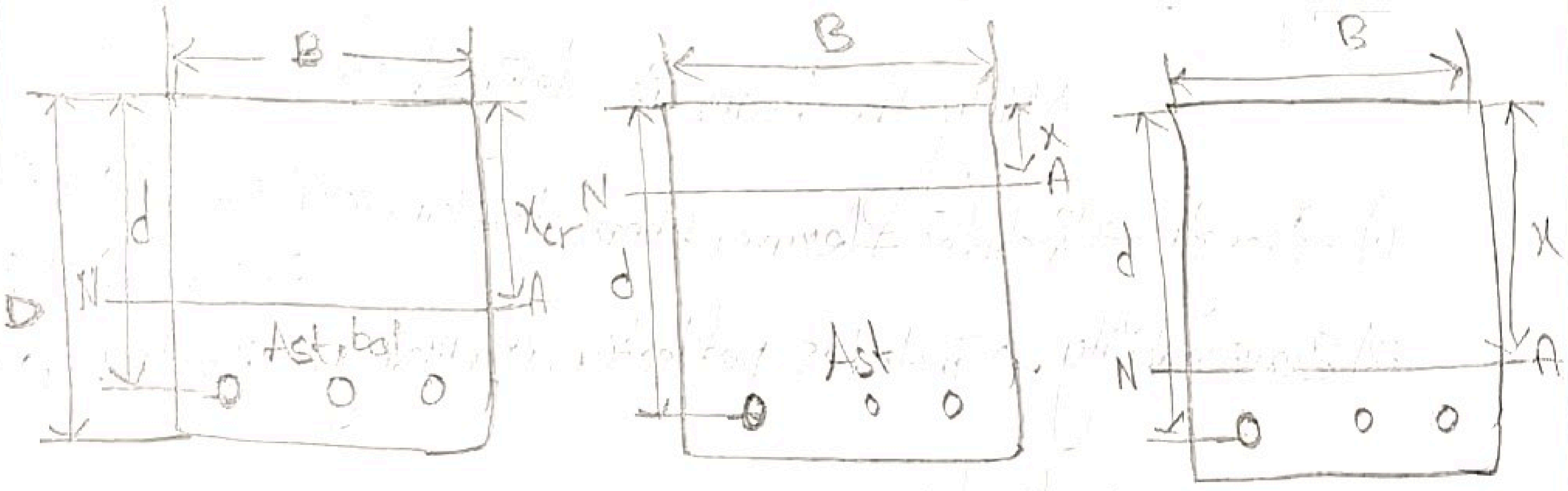
The Section which carries maximum bending Moment is Called "critical Section."

At critical section, $M_{app} \leq M_R$ (M)
(Mapplied) Moment of resistance (or) Strength of Section (or) Capacity of section

If the above Condition is Satisfied the Critical Section remains in its position. The design proportions are ~~addico~~ adequate. This the fundamental Concept of the design.

Types of Beams:- 3 types. Based on sections.

Balanced, underreinforced and over-Reinforced designs (or) Section.



Balanced Section	Under reinforced Section	Over-reinforced Section
$A_{st} = A_{st,bal}$ (bal = balanced)	$A_{st} < A_{st,bal}$	$A_{st} > A_{st,bal}$
$(x = x_{cr})$ $n_a = n_c$	$(x < x_{cr})$ $n_a < n_c$	$(x > x_{cr})$ $n_a > n_c$
	Tension failure.	brittle failure (or) Sudden failure

Balanced design:-

In this design both Concrete and steel reaches & maximum permisible stresses. (design stress) at the Same time and the failure is called

Balanced failure.

In this the steel placed in the section.

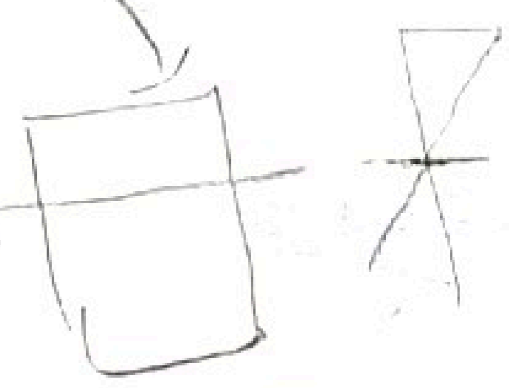
$A_{st} = A_{st,bal}$

Under reinforced design (or) Section :- → gradually failure. Mostly used this only.

In this design the stresses in steel reaches Maximum permissible stress ~~earle~~ earlier than concrete in Compression zone.

As $A_{st} < A_{st,bal}$ and the failure is called Tension failure.

→ Under reinforced design suffers from large deflections and more ~~cracks~~ crack widths.



Over-Reinforced design (or) Section :- → sudden failure.

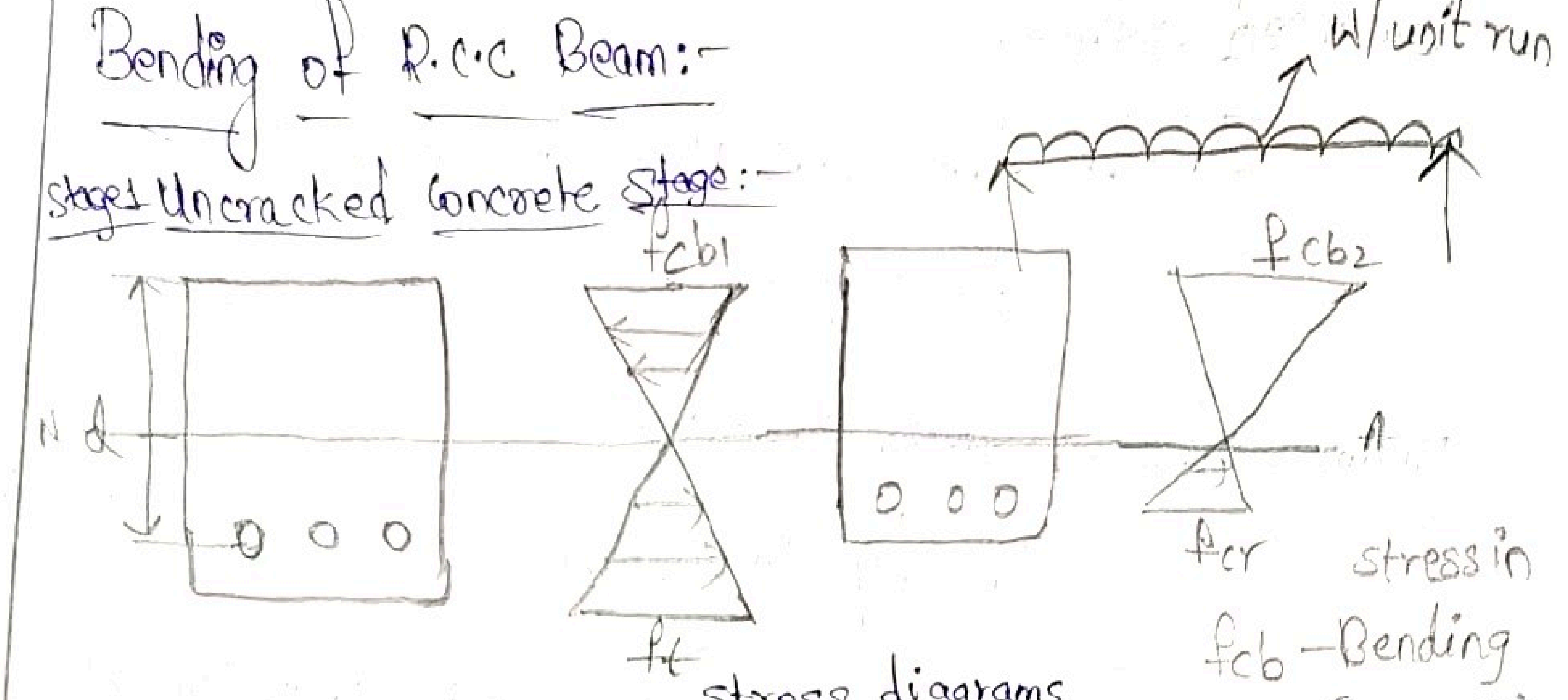
In this design the concrete in compression zone reaches maximum permissible stress earlier than steel as $A_{st} > A_{st,bal}$.

— This type of failure is called brittle failure (or) sudden failure.

→ Normally, structures are designed as under reinforced design because it gives warning to the occupent (or) user by ~~user~~ giving larger deflections and crack widths. Unlike brittle (or) sudden failure of over-reinforced design.

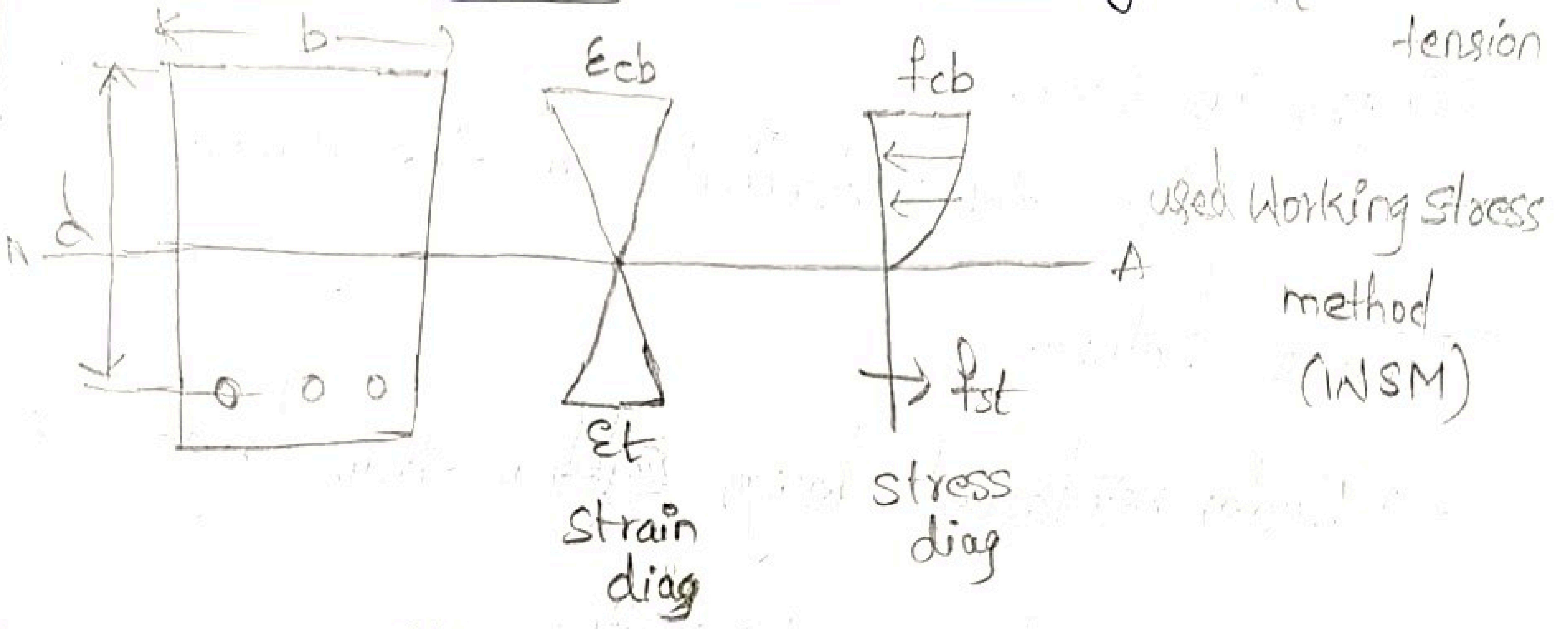
Bending of R.C.C Beam:-

Stage1 Uncracked Concrete Stage:-

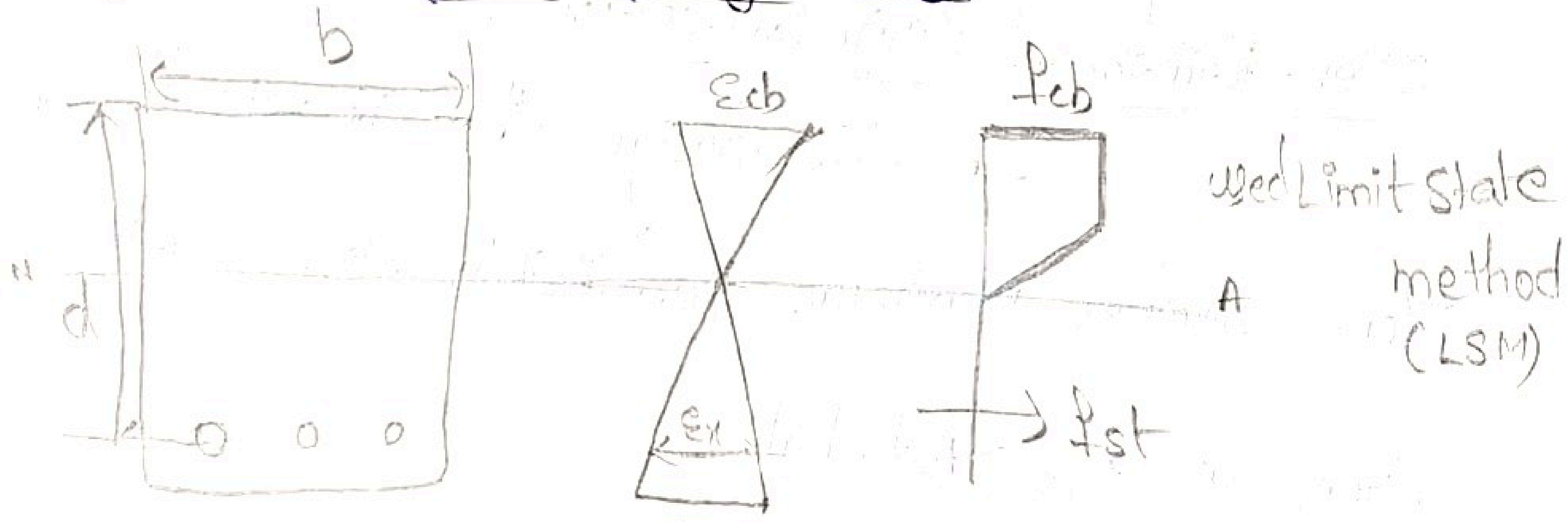


stress diagrams

Stage2 Concrete Cracked elastic stress stage



Stage3 ultimate strength stage



A Beam under flexure passes through above stages before failure.

Stage 2 :- Concrete Cracked Elastic Stage is used for designing Structures in Working stress Method (WSM).

Stage 3:- Ultimate strength stage is use to design Limit state method (LSM)

Permissible Stress:-

Permissible stress = $\frac{\text{Strength of the material } (f_{ck}) \text{ (or) } f_y \text{ yeild strength } (f_{si})}{\text{Factor of Safety}}$

Permissible stress are obtained from Table No. 21 and 22 of IS 456:2000 Code book.

Modular Ratio:- (M)

$E_s = 2\times10^5 \, N/mm^2$

$E_c = 5000\sqrt{f_{ck}}$

$$M = \frac{E.S}{E_C} = \frac{280}{3\sigma_{cbc}}$$

It is the ratio b/w Modulus of elasticity of steel to Modulus of elasticity of concrete.

→ Static Modulus, Where $E_c = 5000\sqrt{f_{ck}}$.

Also E_c is a static Modulus of elasticity. (short-term modulus).

→ Long term modulus, $E_{ce} = \frac{E_c}{1+\theta}$, Where θ = creep coefficent. of elasticity has to be used to determine modular ratio and to consider long term effects like creep.

→ IS 456 Code book gave a value $m = \frac{280}{3\sigma_{cbc}}$,

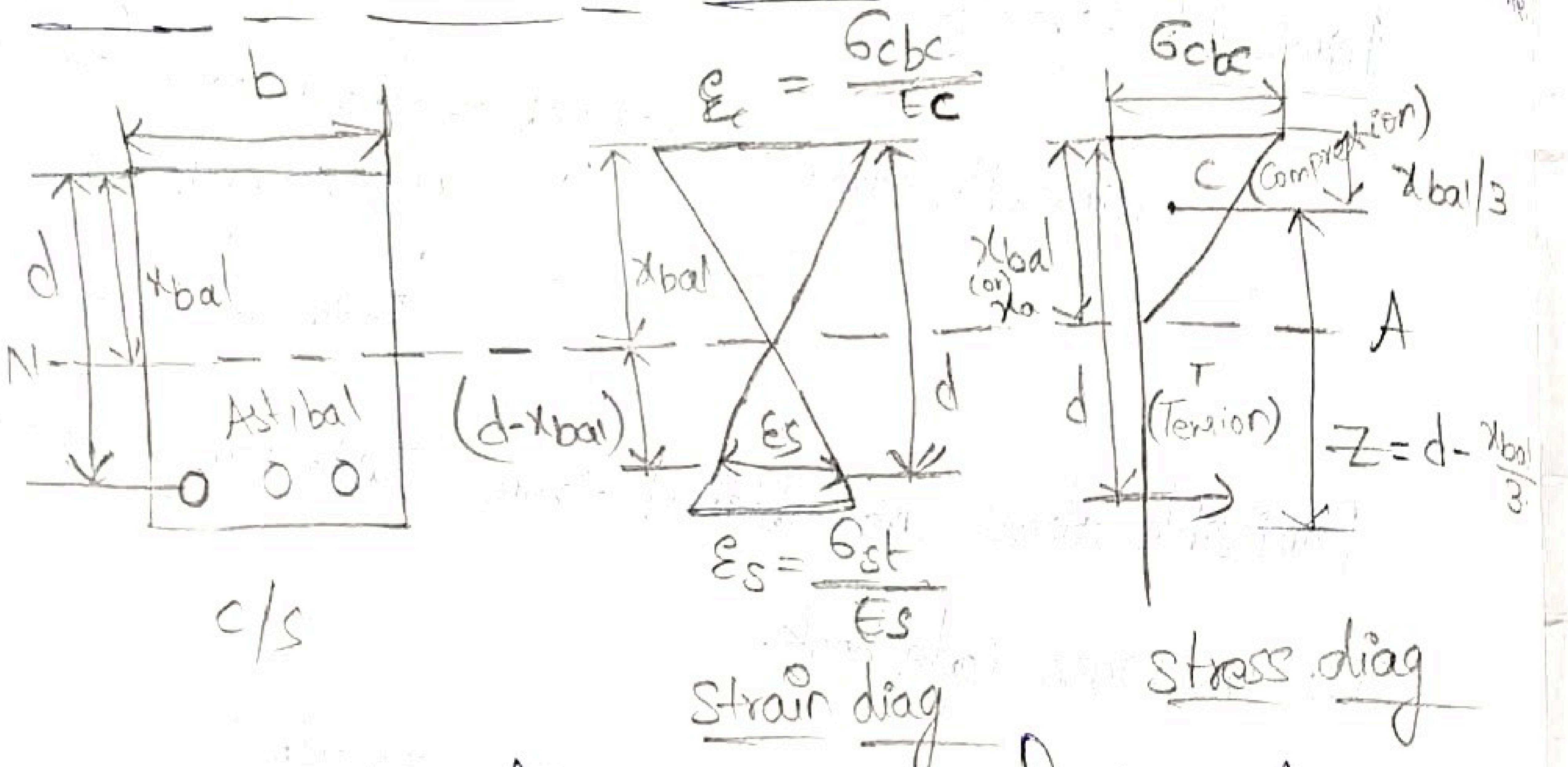
As modular ratio are considering in to long term effects.

$$\therefore M = \frac{E_S}{E_C} = \frac{280}{\sigma_{cbc}}$$

Neutral axis:-
In a Concrete beam through axis, where Stress is equal to zero (or)

Neutral Axis:- An axis which is divided a concrete in to Compression zone and Tension zone.

Derivation of formulae for balanced Section:-



Let A_{st} = Reinforcement provided for balanced Section.

σ_{st} = Max. permissible stress in steel.

σ_{cb} = Max. permissible stress in Concrete.

E_s = Modulus of Elasticity of steel.

E_c = Modulus of Elasticity of concrete.

ε_c = Strain in Concrete in extreme Compression fiber $= \frac{\sigma_{cbc}}{E_c}$.

ε_s = Strain in Steel.

$$= \frac{\sigma_{st}}{E_s} = \frac{\sigma_{st}}{M \cdot E_c}$$

d = effective depth. (The distance between the extreme Compression fiber to centroid of steel.)

x_{bal} = depth of balanced Neutral axis (distance between extreme Compression fiber to Neutral axis)

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Z = Lever arm. [It is the distance between Centroids of (T) tensile and Compresive (C) forces]

Assumptions in WSM :- (Working stress method)

1.) At any cross section plane sections before bending remain plane after bending.

2.) All tensile stresses are taken up by reinforcement and none by Concrete.

3.) The stress-strain relationship of steel and Concrete under working load is a straight line.

4.) There exist a perfect bond between steel and Concrete.

5.) The modular ratio (M) as the value $280/3\sigma_{cbc}$

σ_{cbc} = permissible compresive stress in building compression.

$$M = \frac{280}{3\sigma_{cbc}}$$

To find NA (x_{bal}).

For linear relationship of strain diagram.

$$\frac{x_{bal}}{d - x_{bal}} = \frac{\frac{\sigma_{cbc}}{E_c}}{\frac{\sigma_{st}}{E_s}}$$

$$= \frac{\sigma_{cbc}}{E_c} \cdot \frac{E_s}{\sigma_{st}}$$

$$= \frac{\sigma_{cbc}}{\sigma_{st}} \cdot \frac{E_s}{E_c}$$

$$\frac{x_{bal}}{d - x_{bal}} = \frac{\sigma_{cbc}}{\sigma_{st}} \cdot M$$

Solving for x_{bal}

$$x_{bal} \times \sigma_{st} = (d - x_{bal}) \times m \cdot \sigma_{cbc}$$

$$x_{bal} \cdot \sigma_{st} = d \cdot m \sigma_{cbc} - m \cdot \sigma_{cbc} \cdot x_{bal}$$

$$x_{bal}\left[\sigma_{st} + m \cdot \sigma_{cbc}\right] = m \cdot d\ \sigma_{cbc}$$

$$x_{bal} = \frac{m \cdot d\ \sigma_{cbc}}{\sigma_{st} + m \cdot \sigma_{cbc}}$$

$$x_{bal} = \frac{m \cdot d\ \sigma_{cbc}}{m\sigma_{cbc}\left[\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} + 1\right]}$$

$$x_{bal} = \frac{d}{\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} + 1}$$

$$x_{bal} = \left(\frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}\right) \cdot d$$

$$\therefore x_{bal} = k \cdot d$$

Where.

$$\therefore k = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

= NA Constant.

(or)

$$x_{bal} = \frac{d}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

$$m = \frac{280}{3\sigma_{cbc}}$$

$$x_{bal} = \frac{d}{1 + \frac{\sigma_{st}}{\sigma_{cbc} \times \frac{280}{3\sigma_{cbc}}}}$$

$$x_{bal} = \frac{d}{1 + \frac{\sigma_{st} \times 3}{280}}$$

$$x_{bal} = \frac{d}{1 + 0.0107\sigma_{st}}$$

To find lever arm (Z):-

$$Z = d - \frac{x_{bal}}{3}$$

Subtituting $x_{bal} = k.d$.

$$Z = d - \frac{k.d}{3}$$

$$Z = d\left[1 - \frac{k}{3}\right]$$

$$Z = d.j$$

Where $j = (1 - k/3)$ = Lever arm Constant.

Total Compressive force $(C) = 1/2 \times (\sigma_{cbc} \times x_{bal}) \times b$.

Total ~~Strain~~ Tensile force $(T) = \sigma_{st} \times A_{st,bal}$.

To find (MR) Moment of resistance:-

$MR = T \times Z$ (or) $C \times Z$

$MR = M_{bal} = C \times Z \rightarrow$ with respect to Compresion.

$$= 1/2 \times \sigma_{cbc} \times x_{bal} \times b \times jd$$

$$= 1/2 \times \sigma_{cbc} \times k.d.b.jd$$

$$= 1/2\ \sigma_{cbc} \times k \times j\ bd^2$$

$$= Q_{bal} \times bd^2$$

where $Q_{bal} = 1/2\ \sigma_{cbc} \times kj$

$MR = M_{bal} = T \times Z \rightarrow$ with respect to Tension

$$= \sigma_{st} \times A_{st} \times jd$$

W_s – load in steel.
W_c – load in Concrete.
E_c – young's modulus of Concrete
E_s – Young's modulus of steel
A_c – Area of Concrete.
A_{sc} – Area of steel in Concrete
f_{sc} – Stress in Steel Concrete.
f_c – Stress in Concrete.

Types of Beams :–

1) Balanced or critical section.

2) Under reinforced section.

3) over reinforced section.

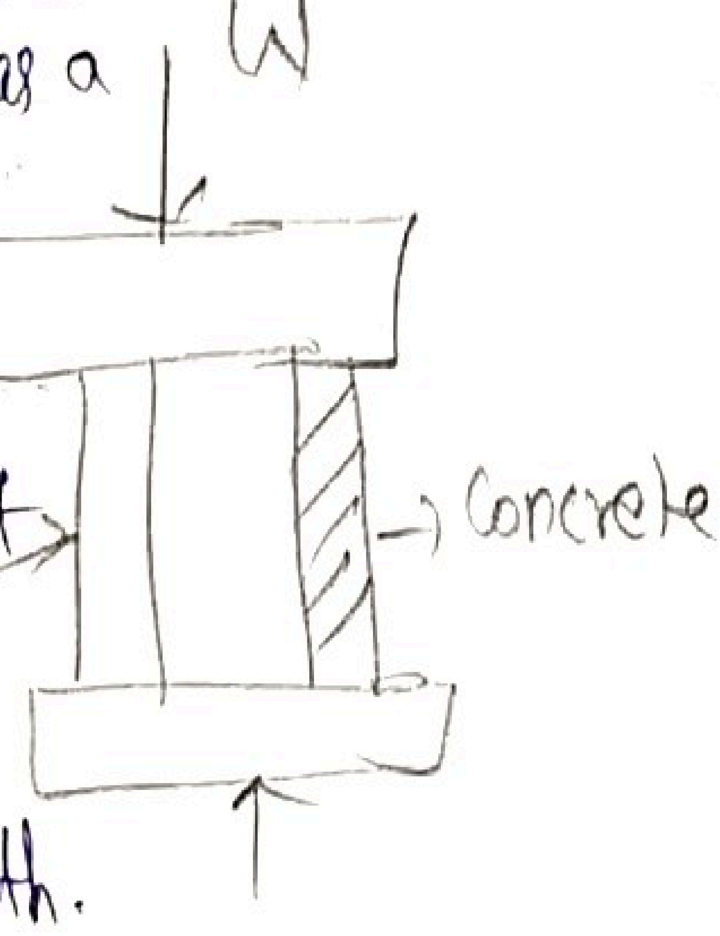
1) ~~Balanced (or) critical Section:–~~

The characteristic property of a reinforced Concrete member is that its Components namely Concrete & steel act together as a single unit as long as they remain in the elastic condition i.e the two Components are bound together so that can be no relative displacement b/w them.

✓ Elastic theory :–

R.C = Concrete + steel.

When subjected to loads.



Assume that there is no change in length.

Strain in steel = strain in Concrete = strain in R.C member.

$$\frac{W_s}{A_{sc}E_s} \rightarrow ① = \frac{W_c}{A_c \cdot E_c} \rightarrow ② = \frac{W_c + W_{sc}}{A_{sc} \cdot E_s + A_c E_c} \rightarrow ③$$

By equating ① & ②. m = modular ratio.

$$\frac{W_s}{A_{sc}E_s} = \frac{W_c}{E_c \cdot A_c}$$

$\frac{W_c}{A_c} = f_c$ Stress

$$\frac{f_{sc}}{E_s} = \frac{f_c}{E_c}$$

$$f_{sc} = \frac{E_s}{E_c} \cdot f_c$$

$\frac{E_s}{E_c} = m$

$$\boxed{f_{sc} = m \cdot f_c}$$

Calculate the load

By comparing ① & ③.

$$\frac{W_s}{A_{sc}E_s} = \frac{W_c + W_{sc}}{A_{sc} \cdot E_s + A_c \cdot E_c}$$

$$\frac{W_s}{A_{sc} \cdot E_s} = \frac{W}{A_c E_c + A_{sc} E_s}$$

$$W_s = \left[\frac{W}{A_c E_c + A_{sc} E_s}\right] A_{sc} \cdot E_s$$

By dividing it with E_c.

$$W_s = \frac{W}{A_c \frac{E_c}{E_c} + A_{sc} \frac{E_s}{E_c}} \cdot \frac{A_{sc} \cdot E_s}{E_c}$$

$$\frac{E_s}{E_c} = m$$

$$W_s = \frac{m \omega A_{sc}}{A_c + m A_{sc}}$$

steel will bear more load when Compare to Concrete.

eq ② & ③.

$$W_c = \frac{W \cdot A_c}{m \cdot A_{sc} + A_c}$$

$W = W_c + W_{sc}$

$$\frac{W_c}{A_c \cdot E_c} = \frac{W_c + W_{sc}}{A_{sc} \cdot E_s + A_c \cdot E_c}$$

Ex:-

$$\frac{W_c}{A_c \cdot E_c} = \frac{W}{A_{sc} \cdot E_s + A_c \cdot E_c}$$

$$W_c = \left[\frac{W}{A_{sc} \cdot E_s + A_c \cdot E_c}\right] A_c \cdot E_c$$

Divide with E_c

$$W_c = \frac{W}{A_{sc} \cdot \frac{E_s}{E_c} + A_c \cdot \frac{E_c}{E_c}} \cdot \frac{A_c \cdot E_c}{E_c}$$

$$W_c = \frac{W \cdot A_c}{A_{sc} \cdot m + A_c}$$

$$W_c = \frac{W \cdot A_c}{m \cdot A_{sc} + A_c}$$

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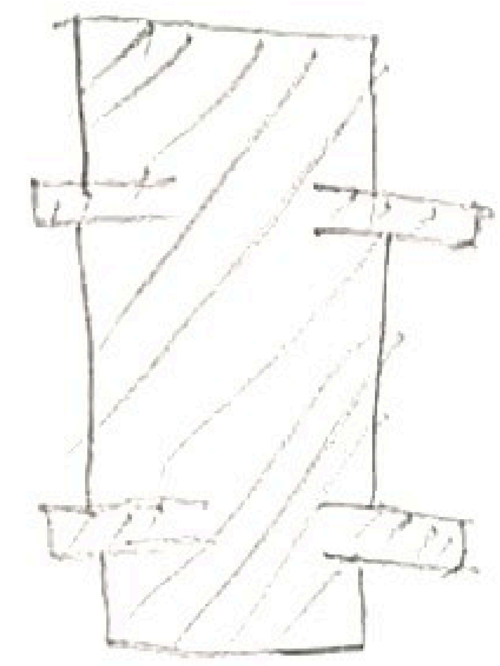
Equivalent Concrete area (or) transformed Concrete area:-

* The strains are equal $[f_{sc} = m f_c]$.

load on steel + load on Concrete = Total load.



$$f_{sc} \cdot A_{sc} + f_c \cdot A_c = W.$$

$$m f_c \cdot A_{sc} + f_c \cdot A_c = W.$$

$$f_c \left[m \cdot A_{sc} + A_c \right] = W.$$

$$f_c = \frac{W}{(m A_{sc} + A_c)}$$

$(m A_{sc} + A_c)$ → Equivalent Concrete area. (A_E)

1.) A Concrete Beam of size 300x300mm Subjected to an a Compresive load of 440kN and it is reinforced with $1257 mm^2$ of steel Supports. Calculate the stress in Concrete and stee take m = 13.33.

Sol:- Given that:-

Size of Concrete Beam, A_c = 300x300mm.

Compresive load, W = 440kN

reinforced, $A_{sc} = 1257 mm^2$

$m = 13.33.$

$f_s, f_{sc} = ?$

$$A_E = m A_{sc} + A_c$$

$$= 13.33 \times 1257 + 300 \times 300$$

$$\boxed{A_E = 106.75 \times 10^3 mm^2}$$

$$f_c = \frac{440 \times 10^3}{106.75 \times 10^3}$$

$$\boxed{f_c = 4.12 \, N/mm^2}$$

$$\% P_{st} = \frac{100 A_{st}}{bd}$$

$$\boxed{f_{sc} = m \cdot f_c}$$

$$f_{sc} = m \cdot f_c = 13.33 \times 4.12$$

$$\boxed{f_{sc} = 54.9 \, N/mm^2}$$

Neutralaxis:- To calculate depth of N.A

Case i:- when we know the stress in the R.c Beam.

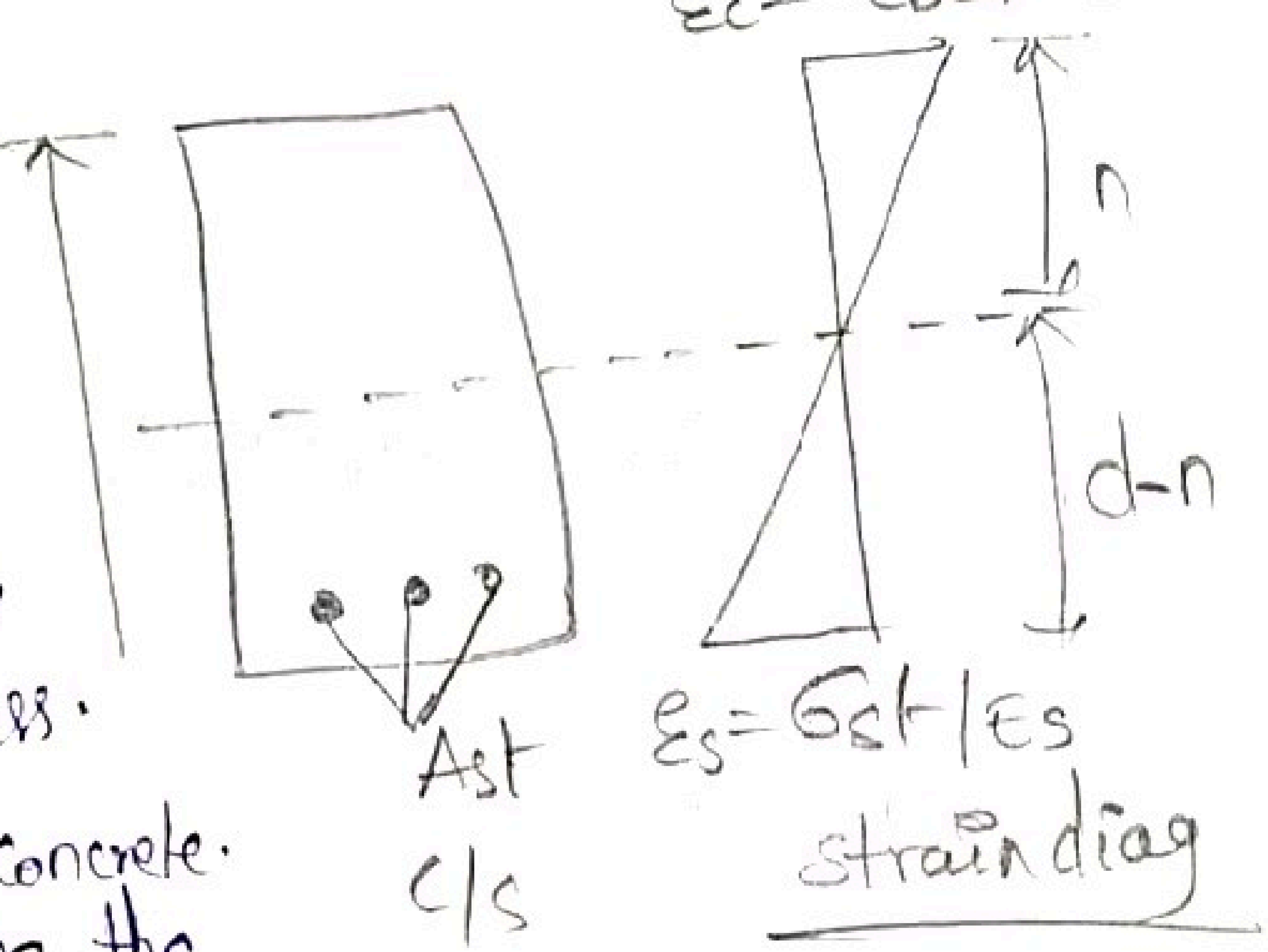
n = Neutral axis depth.

σ_{st} = ~~Comp~~ Permissible tension, Stress.

σ_{cbc} = permissible Compresion Stress.

E_{sc} = Modulus of elasticity of steel.

E_c = Modulus of elasticity of concrete.



for. Cal, of N.A depth, taking the strains along the depth of N.A.

$$\boxed{\frac{\varepsilon_c}{\varepsilon_{cs}} = \frac{n}{d-n}}$$

$$\boxed{\frac{\sigma_{cbc}}{\sigma_{st}/m} = \frac{n}{d-n}}$$

~~$\sigma_{sc} = m \cdot f_c$~~

$$f_s = m \cdot f_c$$

$$\frac{m \, \sigma_{cbc}}{\sigma_{st}} = \frac{n}{d-n}$$

$$\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} = \frac{d-n}{n}$$

$$\frac{\sigma_{st}}{m \cdot \sigma_{cbc}} = \frac{d}{n} - 1$$

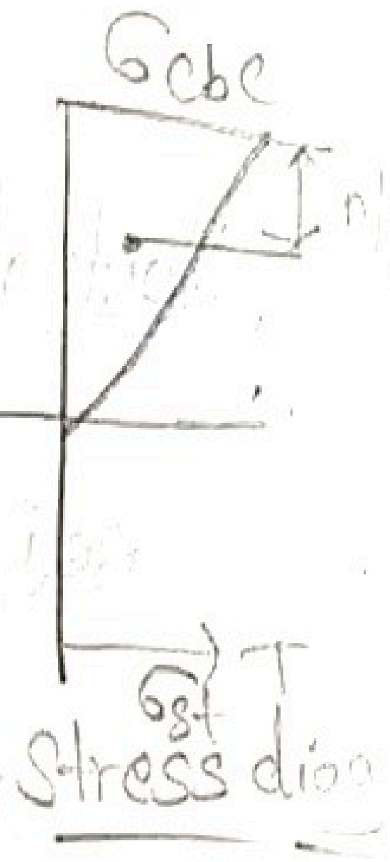
$$\frac{d}{n} = 1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}$$

$$n = \left[\frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}\right] d.$$

$$\boxed{n = k \cdot d.}$$ → Neutral axis depth factor.

Case ii :- When we know the dimensions in the R.C Beam,

→ By taking the internal forces of Concrete and steel.



Force on Compression = Stress x area.

$$= \sigma_{cbc} \times (\tfrac{1}{2} \times b \times n)$$

$$= \frac{\sigma_{cbc}}{2} \times b \times n \rightarrow ①$$

Force on tension $= \sigma_{st} \times A_{st}.$

$$= (m\,\sigma_{cbc})\left(\frac{d-n}{n}\right) A_{st} \rightarrow ②.$$

By eq. ① & ②.

$$\frac{\sigma_{cbc}}{2} \times b \times n = (m\,\sigma_{cbc})\left(\frac{d}{n} - 1\right) A_{st}.$$

$$\frac{bn}{2} = m \cdot A_{st}\left(\frac{d-n}{n}\right)$$

$$\boxed{\frac{bn^2}{2} = m \cdot A_{st}\,(d-n).}$$

$$\frac{bn^2}{2} = m \cdot A_{st} \cdot d - m \cdot A_{st} \cdot n$$

$$\frac{bn^2}{2} + n \cdot m\,A_{st} - m \cdot d\,A_{st} = 0.$$

$$\frac{b}{2} n^2 + (A_{st} \cdot m)\,n - m d\,A_{st} = 0.$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$n = \frac{-m.A_{st} \pm \sqrt{(m.A_{st})^2 - 4 \times (b/2)(-m d A_{st})}}{2 \times (b/2)}$$

$$n = \frac{-m.A_{st} \pm \sqrt{(m.A_{st})^2 + 2bm.d\,A_{st}}}{b}$$

Lever arm :- (Ler distance)

It is the distance between the centroid of compresive and tensile stresses.

$$d - n/3 \qquad n = k.d.$$

$$= d - \frac{k.d}{3}$$

$$= d\left[1 - k/3\right] \qquad J = \text{lever arm Constant.}$$

$$Z = d.J.$$

Moment of resistance :-

M.R = Compresive load × Lever arm.

$$= \frac{\sigma_{cbc}}{2} \times b \times n \times d \times j \qquad n = k.d.$$

$$= \frac{\sigma_{cbc}}{2} \times b \times k \times d \times d \times j$$

$$= \frac{\sigma_{cbc}}{2} \times k \times b d^2 . j$$

$$M.R = Q \cdot bd^2 \qquad Q = \frac{\sigma_{cbc}}{2} \times k.j$$

M.O.R Constant.

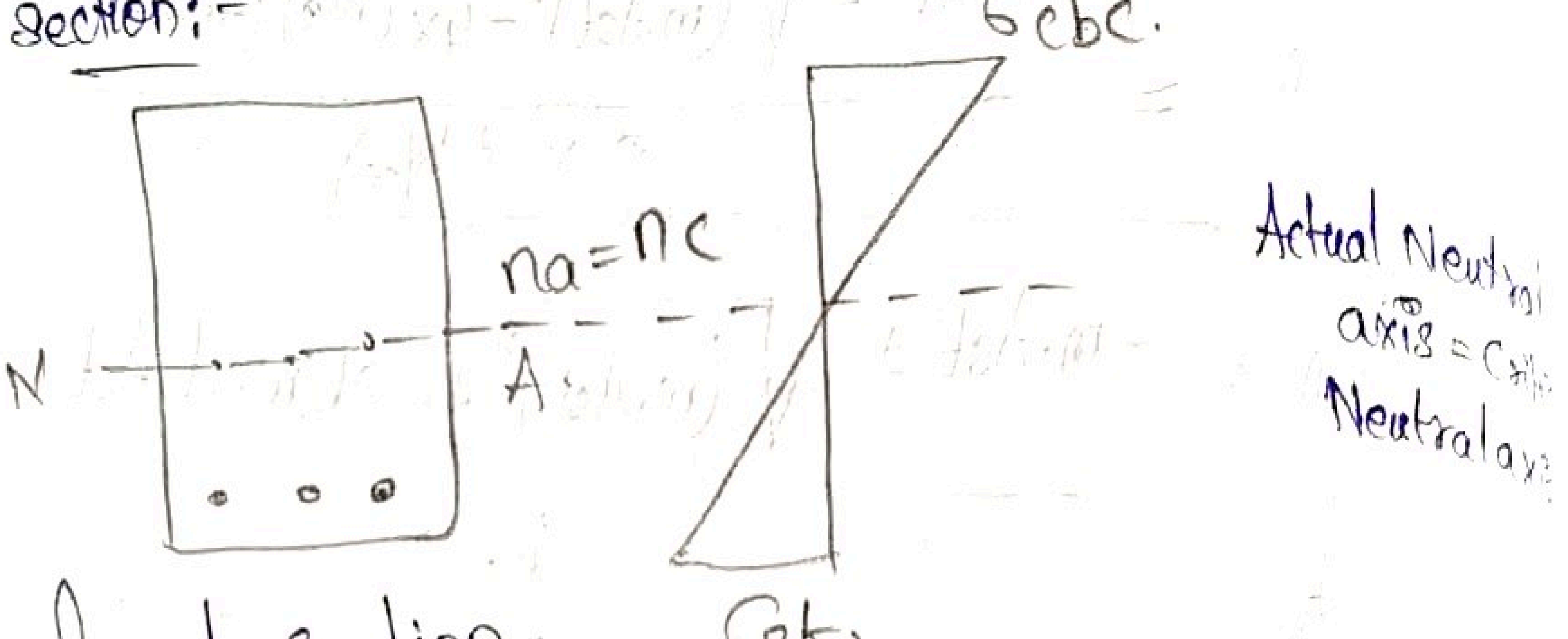
MOR = Tension load × Lever arm.

$$= \sigma_{st} \times A_{st} \times d.J$$

$$A_{st}' = \frac{M}{\sigma_{st} \times J \times d}$$

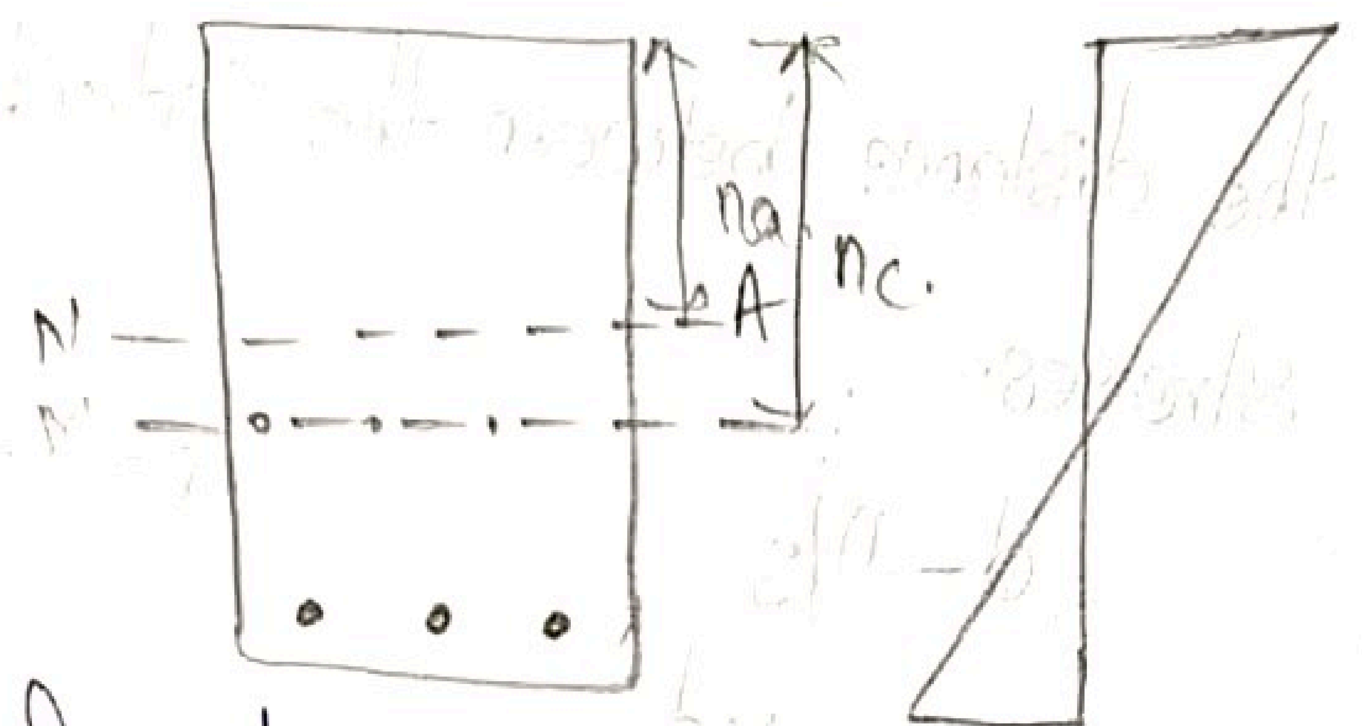
Types of sections:-

1) Balanced section:-

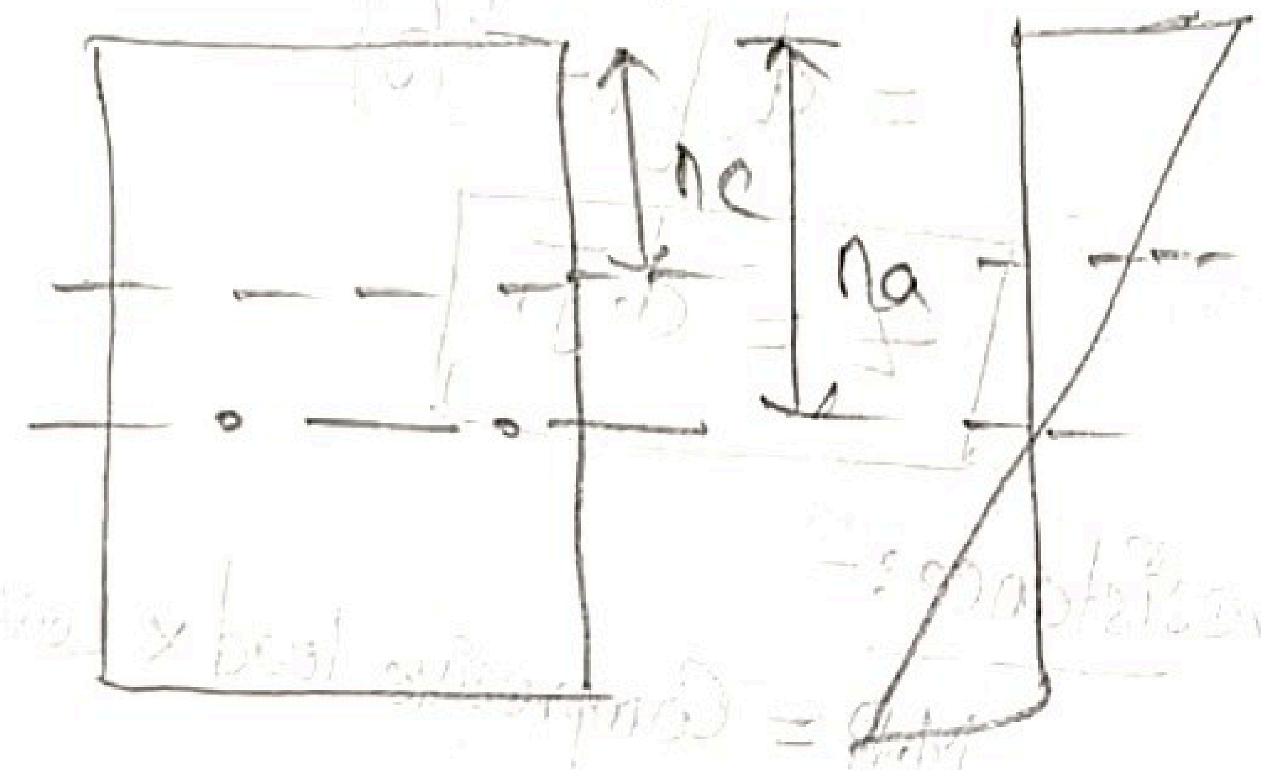


2) under Reinforced section

Mostly used.



3) over-reinforced section:- $n_a < n_c$.



$n_a > n_c$.

① Consider M20 grade of Concrete and Fe415 Steel find Q_c, J_c, K_c and P_c (Percentage of Concrete).

Sol:- Given that grade of Concrete, $f_{ck} = 20 N/mm^2$

Fe415 Steel

Grade of steel, $f_y = 415 N/mm^2$.

find Q_c, J_c, K_c and P_c.

$$K_c = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

$$m = \frac{280}{3\sigma_{cbc}}$$

(Pg No. 80 IS 456 Code)

From table 21, 22 σ_{cbc} & σ_{st}.

$\sigma_{cbc} = 7 N/mm^2$ $\sigma_{st} = 230 N/mm^2$

$$m = \frac{280}{3 \times 7}$$

$$m = 13.33$$

$$K_c = \frac{1}{1 + \frac{230}{13.33 \times 7}}$$

$$\boxed{K_c = 0.289}$$

$$K_c = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

$$J_c = \left[1 - \frac{K_c}{3}\right]$$

$$J_c = \left[1 - \frac{0.289}{3}\right]$$

$$J_c = 0.9036$$

$$\boxed{J_c = 0.904}$$

$$\boxed{Q_c = \frac{\sigma_{cbc}}{2} \times K_c \times J_c}$$

$$= \frac{7}{2} \times 0.289 \times 0.904$$

$$\boxed{Q_c = 0.914}$$

steel = Concrete

$$\sigma_{st} A_{st} = \frac{\sigma_{cbc}}{2} \times b \times n.$$

$$A_{st} = \frac{\frac{\sigma_{cbc}}{2} \times b \times n}{\sigma_{st}}$$

$$\% P_{st} = \frac{100 A_{st}}{bd.}$$

$$P_c = \frac{100 \times \frac{\sigma_{cbc}}{2} \times b \times n}{b d \cdot \sigma_{st}}$$

$$P_c = \frac{100 \sigma_{cbc} \times K_c d}{2 \times \sigma_{st} \times d}$$

$$P_c = \frac{100 \times 7 \times 0.289}{2 \times 230}$$

$$\boxed{P_c = 0.43}$$

$P_c = 0.43 \times 100$

$P_c = 43\%$

2.) For M_{20} grade of concrete $\sigma_{st} = 140 N/mm^2$, $\sigma_{cbc} = 7 N/mm^2$ and Calculate k_c, Q_c and J_c.

M_{20} grade of steel $\sigma_{st} = 140 N/mm^2$

" " of concrete $\sigma_{cbc} = 7 N/mm^2$.

Sol:-

$$K_c = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

$$m = \frac{280}{3\sigma_{cbc}}$$

$$m = \frac{280}{3 \times 7}$$

$m = 13.33$

$$K_c = \frac{1}{1 + \frac{140}{13.33 \times 7}}$$

$K_c = 0.399$

$$J_c = \left[1 - \frac{K_c}{3}\right]$$

$$J_c = \left[1 - \frac{0.399}{3}\right] = 0.867$$

$J_c = 0.867$

$$Q_c = \frac{\sigma_{cbc}}{2} \times K_c \times J_c$$

$$Q_c = \frac{7}{2} \times 0.399 \times 0.867$$

$Q_c = 1.210$

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3) For M_{25} grade of concrete, $\sigma_{st} = 140 N/mm^2$. Calculate Q_c, K_c, J_c.

Sol:-

$\sigma_{cbc} = 8.5 N/mm^2$

$\sigma_{st} = 140 N/mm^2$

$$K_c = \frac{1}{1 + \frac{\sigma_{st}}{m \cdot \sigma_{cbc}}}$$

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 8.5}$$

$$\boxed{m = 10.980}$$

$$K_c = \frac{1}{1 + \frac{140}{10.980 \times 8.5}}$$

$$\boxed{K_c = 0.39}$$

$$J_c = \left[1 - \frac{K_c}{3}\right] = \left[1 - \frac{0.399}{3}\right]$$

$$\boxed{J_c = 0.867}$$

$$Q_c = \frac{\sigma_{cbc}}{2} \times K_c \times J_c$$

$$Q_c = \frac{8.5}{2} \times 0.399 \times 0.867$$

$$\boxed{Q_c = 1.470}$$

Grade of concrete and Steel	K_c	J_c	Q_c
M_{20}, & $Fe = 415 N/mm^2$	0.289	0.904	0.914
M_{20}, $\sigma_{st} = 140 N/mm^2$	0.399	0.867	1.210
M_{25} $\sigma_{st} = 140 N/mm^2$	0.39	0.867	1.470

4.) A reinforced concrete beam 250mm x 475mm over all depth is reinforced with 3 Bars of 16mm dia at an effective depth cover of 50mm use M_{20} grade of concrete and Fe_{415} Steel. Find the depth of the Neutral axis.

Sol:- Given that

Size of R.C Beam = 250mm x 475mm.

dia of bars = 16mm.

No. of Bars = 3

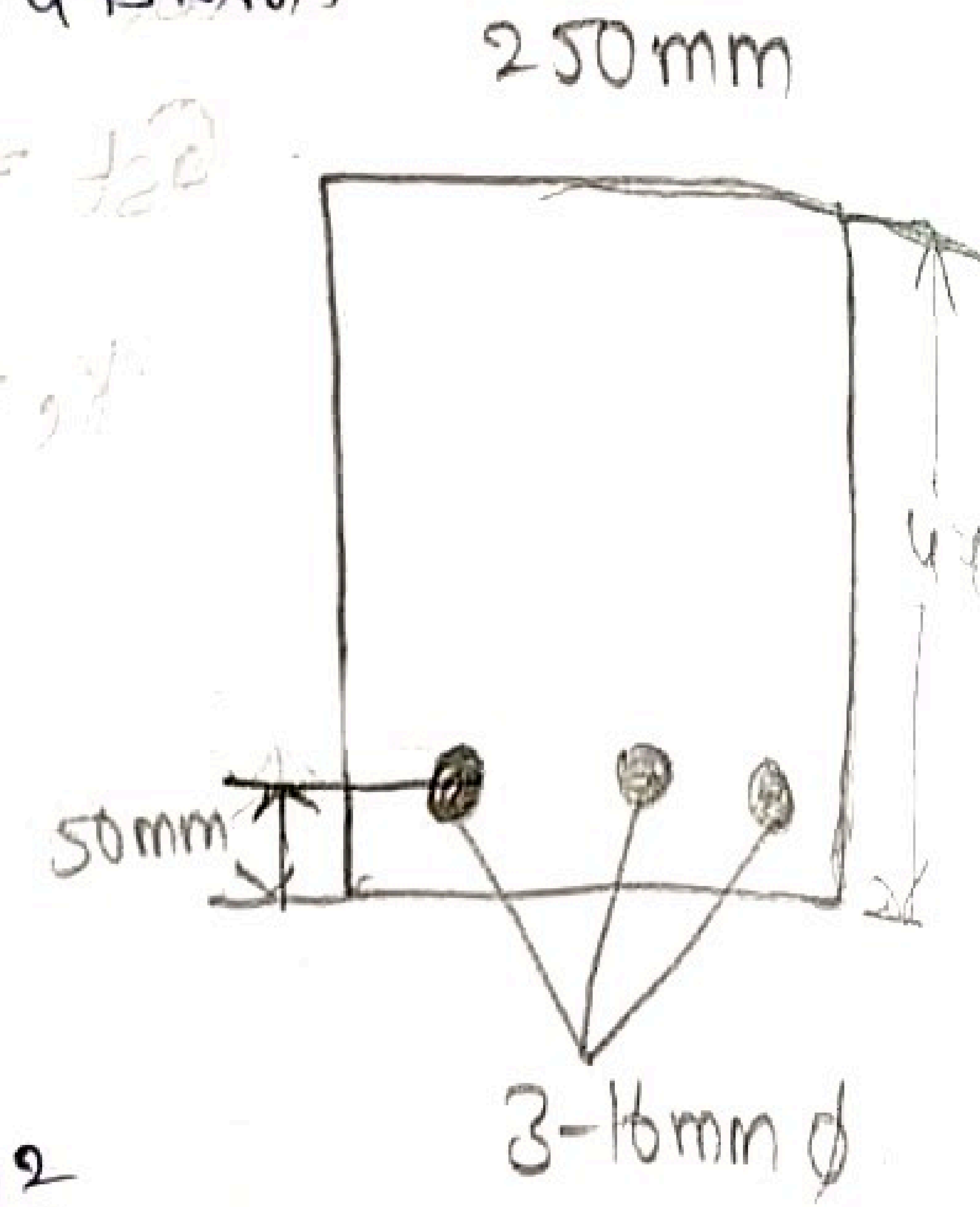
effective covere = 50mm

Grade of Concrete, $f_{ck} = 20 N/mm^2$.

Grade of Steel, $f_y = 415 N/mm^2$.

$A_{st} = n \times \pi/4 \, d^2$.

Area of steel, $A_{st} = 3 \times \pi/4 (16)^2 = 603.185 mm^2$.



$d = D - e$

$d = 475 - 50$

$d = 425mm$.

$m = \frac{280}{3 \sigma_{cbc}}$ $\sigma_{cbc} = 7 N/mm^2$

$= \frac{280}{3 \times 7}$

$\boxed{m = 13.33}$

$\frac{bn^2}{2} = m.A_{st}(d-n)$.

$\frac{250 \times n^2}{2} = 13.33 \times 603.185 (425 - n)$.

$125n^2 = 8040.456 (425 - n)$.

$125n^2 + 8040.456n - 3417193.8 = 0$.

$\boxed{n_a = 136.27mm}$

$n_c = k_c \cdot d$

$= 0.289 \times 425$

$\boxed{n_c = 122.82}$

$n_a > n_c$. over reinforced section

5) A reinforced Concrete Beam 300mm x 550mm over all Depth is reinforced with 4 bars of 20mm diameter at an effective depth 500mm, use M20 grade of Concrete and Fe415 Steel. Calculate the moment of resistance.

Sol: Given that, Size of R.C Beam = 300mm x 550mm.

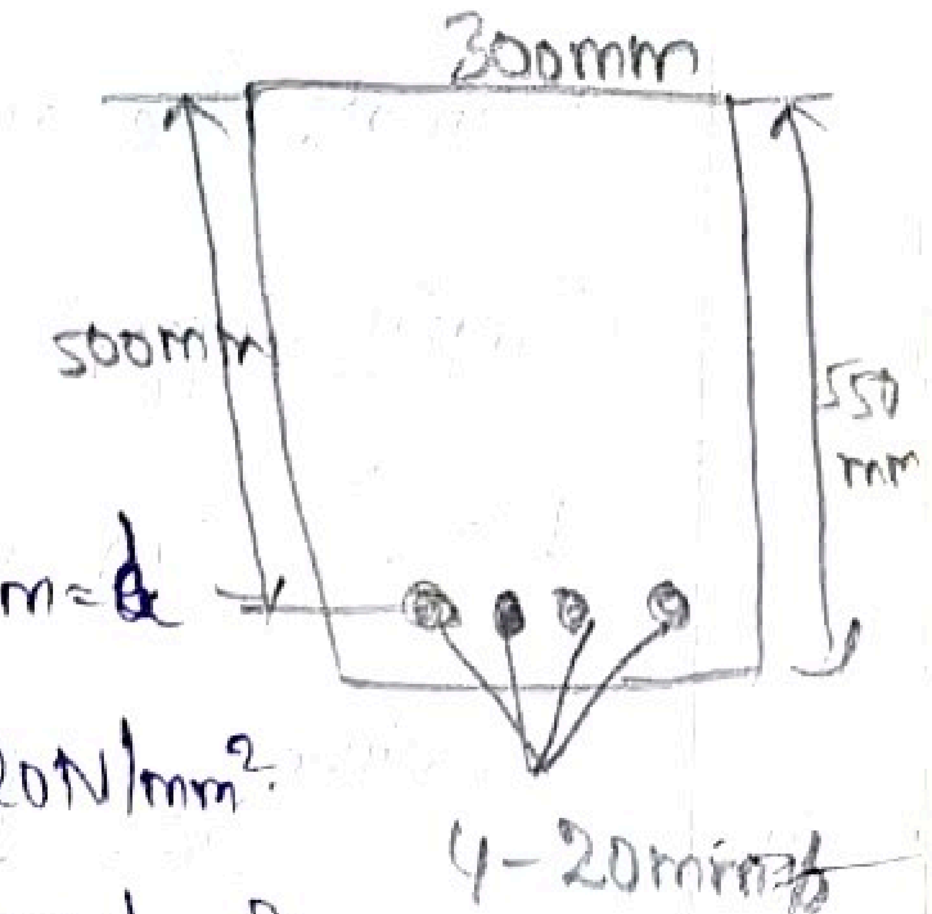
dia of bars = 20mm

No. of Bars = 4.

effective depth = 500mm = d

Grade of Concrete, $f_{ck} = 20 N/mm^2$

Grade of steel, $f_y = 415 N/mm^2$



$A_{st} = n \times \pi/4 \, d^2$

$= 4 \times \pi/4 (20)^2$

$A_{st} = 1256.63 mm^2$

$m = \frac{280}{3 \sigma_{cbc}}$

$= \frac{280}{3 \times 7}$ $\quad \sigma_{cbc} = 7 N/mm^2$ In Code Book Pg. No 81

$\boxed{m = 13.33}$

$\frac{n b^2}{2} = m . A_{st} (d-n)$.

$\frac{n^2 \times 300}{2} = 13.33 \times 1256.63 (500-n)$

$150n^2 + 16750.87n - 8375438.45 = 0$

$\boxed{n_a = 186.96 mm}$ → Position of Netural axis.

$n_c = K_c \cdot d = 0.289 \times 500$ $\quad n_a > n_c$ over reinforced.

$n_c = 144.5 mm$ $\quad M.R = \frac{\sigma_{cbc}}{2} \times b \times n_a \times d \times j$ (or) $\frac{\sigma_{cbc}}{2} \times b \times n (d - \frac{n_a}{3})$.

$K = \frac{1}{1 + \frac{\sigma_{st}}{m \sigma_{cbc}}}$ $\quad = \frac{7}{2} \times 300 \times 186.96 \times 500 \times 0.903$ $\quad j = \left[1 - \frac{k_c}{3}\right]$

$j = \left[1 - \frac{0.289}{3}\right]$

$= \frac{1}{1 + \frac{230}{13.33 \times 7}}$ $\quad M.R = 88.63 \times 10^6$ N-mm $\quad \boxed{j = 0.903.}$

$K = 0.289$ $\quad M.R =$ ~~1.89 KN-m~~. 88.63 KN-m.

$\boxed{M.R = 88.63 KN-m}$

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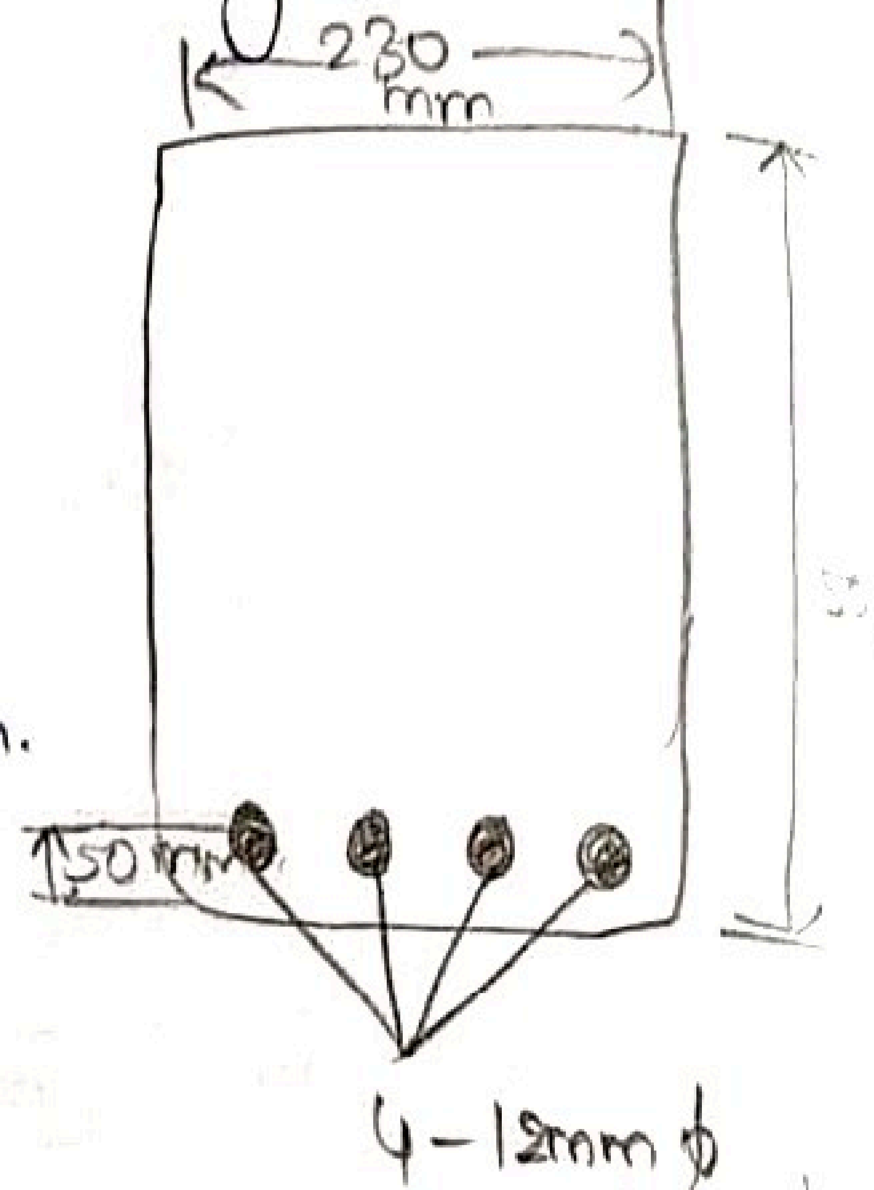
6) A reinforced Concrete Beam 230mm x 500mm depth is reinforced with 4 bars of 12mm dia at an effective Cover of 50mm use M_{20}, Fe_{415} estimate the moment of resistance of the section. If the beam is Simply Supported over a span of 5m. Find the Max. U.D.L on the Beam Can Carry inclusive of its own weight.



Sol:- Given

Size of R.C Beam = 230x500mm.

No. of bars = 4 = n

dia of bar = 12mm.

effective Cover, e = 50mm.

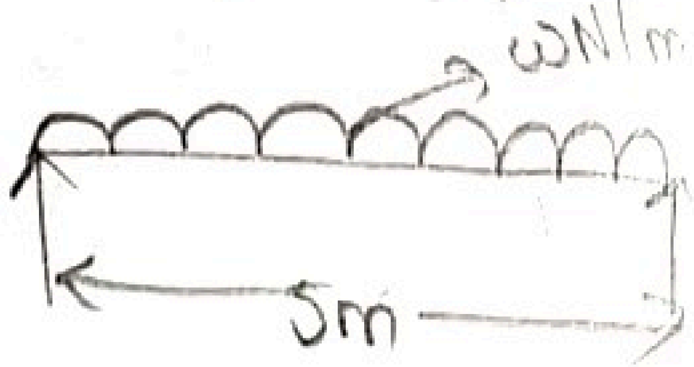
$d = D - e$

$d = 500 - 50$

$d = 450mm.$

Grade of Concrete, $f_{ck} = 20N/mm^2$.

Grade of Steel, $f_y = 415 N/mm^2$.



$A_{st} = n \times \pi/4 d^2 = 4 \times \pi/4 (12)^2$

$A_{st} = 452.38 mm^2$.

$m = \frac{280}{3\sigma_{cbc}}$ → Pg. No 80.

$\sigma_{cbc} = 7 N/mm^2$ for Code book, Pg No. 81. Table 21

$m = \frac{280}{3 \times 7}$

$m = 13.33$

$\frac{bn^2}{2} = m.A_{st}(d-n)$.

$\frac{230}{2} n^2 = 13.33 \times 452.38 (500 - n)$

$115n^2 + 6030.22n - ~~2713599~~ 3015110 = 0.$

$n_a = 137.81mm$

$n_c = K_c . d$

$n_c = 0.289 \times 500$

$n_c = ~~130.05mm~~$ 144.5mm

~~$n_a > n_c$ over ~~under~~ Reinforced section.~~

$n_a < n_c$ under Reinforced Section.

Tension

$M.O.R = \sigma_{st} \cdot A_{st} \cdot d J.$

$= 230 \times 452.38 \times 500 \times 0.904 \left[1 - \frac{K}{3}\right].$

$M.R. = \sout{46.97 \times 10^6}\ 47.02 \times 10^6 \quad \left[1 - \frac{0.289}{3}\right]$

0.90

(or)

$M.R = \sigma_{st} \cdot A_{st}\left(d - \frac{n_a}{3}\right)$

$= 230 \times 452.38\left(500 - \frac{137.81}{3}\right).$

$M.R = 47.24 \times 10^6$ N-mm.

$\boxed{M.R = 47.24 \text{ KN-m.}}$

B.M for U.D.L

$\frac{wl^2}{8} = 47.24 \times 10^3$

$\frac{w(5)^2}{8} = 47.24 \times 10^3$

$w = \frac{47.24 \times 8 \times 10^3}{(5)^2}$

U.D.L $\boxed{w = 15.11 \text{ KN/m}}$

7) Design a reinforced Concrete beam 230mm wide to resist a Bending moment of 30KN-m. Use M_{20} and Fe_{415} Steel.

Sol:- Given that

width of Beam = 230mm

Max B.M = 30KN-m = $30 \times 10^3 \times 10^3$ N-mm.

Grade of concrete, $f_{ck} = 20 N/mm^2$.

Grade of steel, $f_y = 415 N/mm^2$

$Q_c = 0.914$

$J_c = 0.904$

$M.O.R = Q_c b d^2$

$30 \times 10^6 = 0.914 \times 230 \times d^2$

$\boxed{d = 377.76 \text{ mm.}}$

$\sigma_{st} = 230$
Code book
Pg No - 82
Table 22.

$A_{st} = \frac{M}{\sigma_{st} \cdot J \cdot d}$

$A_{st} = \frac{30 \times 10^6}{230 \times 0.904 \times 377.76}$

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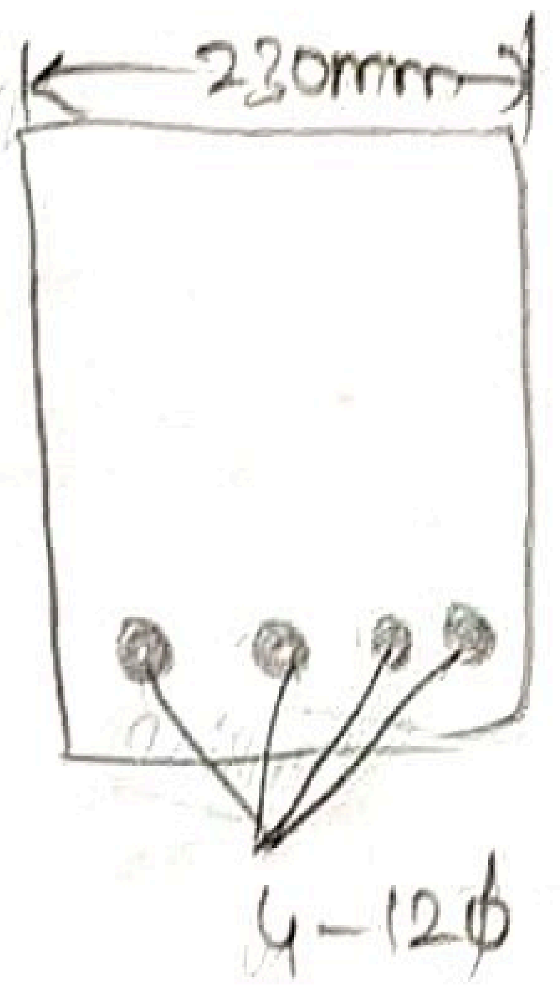
$$A_{st} = 381.95\ mm^2$$

Assuming 12mm dia.



$$A_{st} = n \times \pi/4 (d^2)$$

$$381.95 = n\ \pi/4\ (12)^2$$

$$n = 3.37$$

$$n = 4\ bars$$

A_{st} should not be less. It will be equal, more than

$$A_{st\ provided} = n \times \pi/4 (d^2)$$

$$A_{st} = 4 \times \pi/4 (12)^2$$

$$A_{st} = 452.38\ mm^2$$

8.) Design a rectangular Reinforced Concrete Beam to resist a B.M of 65 KN-m. Use M_{20} grade of Concrete and Fe_{415} Steel and b = 0.5 d.

Sol:- Given that width of beam b = 0.5d.

Max B.M = 65 KN-m = $65 \times 10^3 \times 10^3$ N-mm.

Grade of Concrete, $f_{ck} = 20 N/mm^2$

Grade of Steel, $f_y = 415 N/mm^2$

$$M.O.R = Q_c . b\, d^2$$

$$65 \times 10^6 = 0.914 \times (0.5) d . d^2$$

$Q_c = 0.914$

$J_c = 0.904$

$$d = 521.99\ mm.$$

$$A_{st} = \frac{M}{\sigma_{st} . J . d}$$

$\sigma_{st} = 230$

$$A_{st} = \frac{65 \times 10^6}{230 \times 0.904 \times 521.99}$$

$$A_{st} = 598.90\ mm^2$$

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Assuming 12mm dia. (or) 14,16 upto 20mm

$A_{st} = n \times \pi/4 (d^2)$

$598.90 = n \times \pi/4 (12)^2$

$n = 5.29$

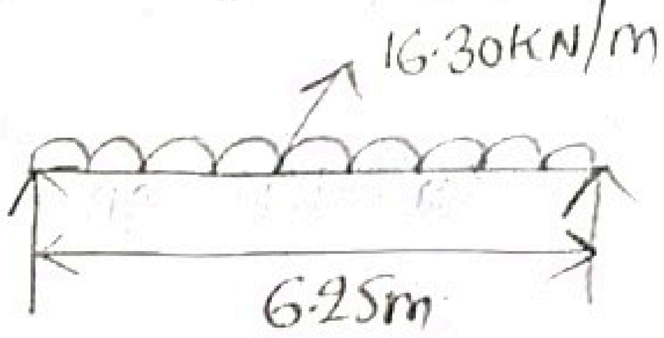
$\therefore n = 6$ bars

A_{st} provided $= n \times \pi/4 (d^2)$

$A_{st} = 6 \times \pi/4 (12^2)$

$A_{st} = 678.58 mm^2$

9) A singly reinforced rectangular concrete Beam 350mm wide has a span of 6.25m and carries an inclusive load of 16.30 KN/m. If the stresses in steel and concrete shall not exceed 230 and 7N/mm². Find the effective depth and Area of Tensile reinforcement. Take m = 13.33.



Sol:- Given that:- width of Beam = 350mm = b

Span $(l) = 6.25m = 6.25 \times 10^3 mm$

U.D.L load $(W) = 16.30 KN/m = 16.30 \times 10^6 N/mm^2$

$m = 13.33$

Permissible stress in concrete, $\sigma_{cbc} = 7 N/mm^2$

Permissible stress in steel, $\sigma_{st} = 230 N/mm^2$

$B.M = \frac{Wl^2}{8} = \frac{16.30 \times (6.25)^2}{8}$

$B.M = 7.958$

$= 79.58 KN\text{-}m.$

for M20, $Q_c = 0.914$

$J_c = 0.904$

$M = Q_c b d^2$

$79.58 \times 10^6 = 0.914 \times (350) d^2$

$d = 498.76mm.$

$A_{st} = \frac{M}{\sigma_{st} \cdot J \cdot d} = \frac{79.58 \times 10^6}{230 \times 0.904 \times 498.76}$

$A_{st} = 767.38 mm^2$

Assuming 12mm dia.

$$Ast = n \times \pi/4 (d^2)$$

$$767.38 = n \times \pi/4 (12)^2$$

$$n = 6.78.$$

$$\boxed{n = 7 \text{ bars.}}$$

$$Ast = n \times \pi/4 (d^2)$$

$$Ast = 7 \times \pi/4 (12^2)$$

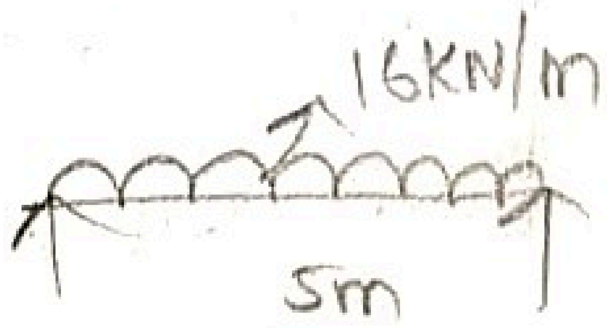
$$\boxed{Ast = 791.68 \text{ mm}^2}$$

10) A singly reinforced concrete beam 300mm wide has an effective depth of 500mm the effective span being 5m. It carries a total load of 160KN/M on the wholespan. It is reinforced with $804mm^2$ of steel. Determine the stresses produced in steel and concrete. Take m = 13.33.

Sol:- Given that wide = 300mm = b. Area of steel, effective depth d = 500mm $Ast = 804mm^2$.

$l = 5m.$

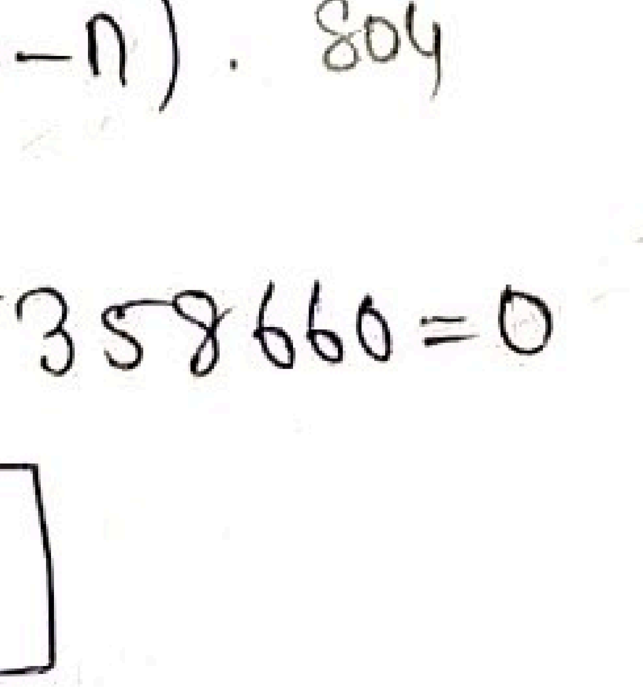
U.D.L w = 160KN/m.



$$M.O.R = \frac{\sigma_{cbc}}{2} b n_a \left[d - \frac{n_a}{3} \right].$$

$$B.M = \frac{wl^2}{8} = \frac{160 \times 10^3 \times 5^2}{8} = ~~187.5 KN\text{-}m~~.$$

$$= 50 KN\text{-}m.$$



$$\frac{n^2 b}{2} = m.Ast (d-n).$$

$$\frac{n^2(300)}{2} = 13.33 \times 804 (500 - n).$$ 804

$$150n^2 + 10717.32n - 5358660 = 0$$

$$\boxed{n_a = 156.65 \text{ mm}}$$

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$$50\times10^6 = \frac{\sigma_{cbc}}{2}\times 300\times 156.63\left[500-\frac{156.63}{3}\right]$$

$$50\times10^6 = \frac{21.04\times10^6}{2}\times\sigma_{cbc}$$

$$\sigma_{cbc} = \frac{50\times10^6\times 2}{21.04}$$

$$\sigma_{cbc} = 4.75\ N/mm^2$$

$$\boxed{\sigma_{cbc} = 4.75\ N/mm^2}$$

M.O.R = σ_{cbc}

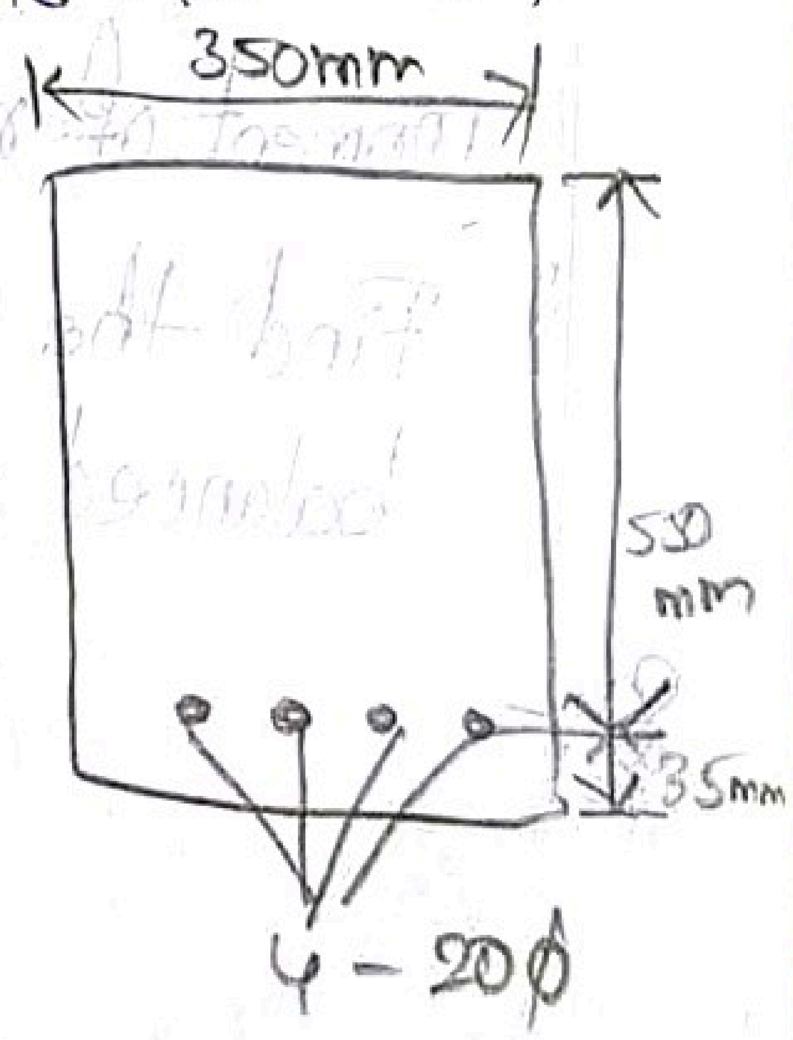
$$M = \sigma_{st}\times A_{st}\left(d-\frac{n_a}{3}\right) \qquad \sigma_{st} = m\cdot\sigma_{cbc}\left(\frac{d-n}{n}\right)$$

$$A_{st} = \frac{M}{A_{st}\left(d-\frac{n_a}{3}\right)} \qquad \sigma_{st} = 13.33\times4.75\left[\frac{500-156.63}{156.63}\right]$$

$$\sigma_{st} = \frac{50\times10^6}{804\left(500-\frac{156.63}{3}\right)} \qquad \boxed{\sigma_{st} = 138.80\ N/mm^2}$$

$$\boxed{\sigma_{st} = 138.87\ N/mm^2}$$

A singly reinforced Beam 350mm×550mm deep has an effective span of 6m and carries a load of 20kN/m. The Beam is reinforced with 4 bars of 20mm dia. at an effective cover of 35mm. Find the Max. stress provided in steel and concrete. Take m=13.33.



wide ⇒ b = 350mm , d = 550 mm

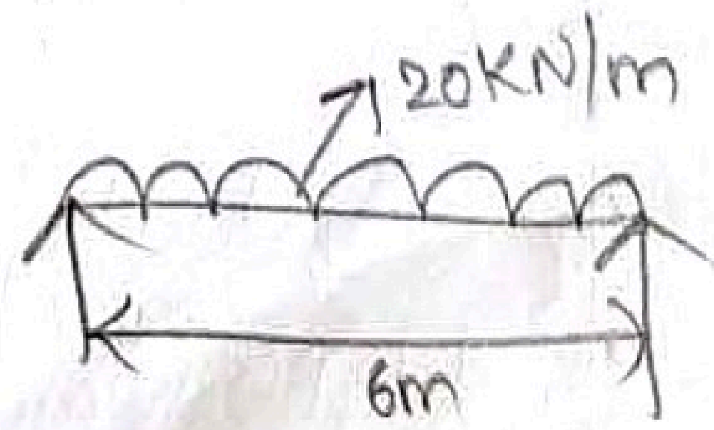
l = 6m

U.D.L, ω = 20kN/m.

$$B.M = \frac{\omega l^2}{8} = \frac{20\times6^2}{8} = 90\ kN\text{-}m$$

$$\frac{n^2 b}{2} = m.A_{st}(d-n)$$

$$\frac{n^2(350)}{2} = 13.33\ A_{st}(550-n)$$



$$A_{st} = n\left(\pi/4\, d\right)^2 = A_{st} = 4\times\pi/4\times(20)^2$$

$$\boxed{A_{st} = 1256.63\ mm^2}$$

$$175n^2 = 13.33\times1256.63(550-n)$$

$$175n^2 + 16750.87n - 9113978.5$$

$$\boxed{n_a = 185.31\ mm}$$

$$M.O.R = \frac{\sigma_{cbc}}{2} \times b \times n_a \left[d - \frac{n_a}{3} \right]$$

$$90 \times 10^6 = \frac{\sigma_{cbc}}{2} \times 350 \times 185.31 \left[550 - \frac{185.31}{3} \right]$$

$$\boxed{\sigma_{cbc} = 5.68\ N/mm^2}$$

$$\sigma_{st} = \sigma_{cbc} \cdot m \cdot \left[\frac{d - n_a}{n_a} \right]$$

$$\sigma_{st} = 5.68 \times 13.33 \left[\frac{550 - 185.31}{185.31} \right]$$

$$\boxed{\sigma_{st} = 149.0\ N/mm^2}$$

$$M = \sigma_{st} \times A_{st} \left(d - \frac{n_a}{3}\right)$$

$$\sigma_{st} = \frac{M}{A_{st}\left(d - \frac{n_a}{3}\right)}$$

12) A Singly reinforced Beam was originally planed to be designed as a balanced Section Later it was desired to increase the moment of resistance of Section by double to the reinforcement. Find the ratio of enhanced moment of resistance to the balanced. Use M_{20} grade of Concrete and Fe_{415} Steel.

Sol:- Given that Grade of Concrete = $20 N/mm^2 = f_{ck}$

Grade of steel = $415 N/mm^2 = f_y$.

Balanced Section

$A_{st} = M / \sigma_{st} \cdot J \cdot d$

Table 22 IS 456.

$M = Q b d^2$

$J_c = 0.904$

$Q_c = 0.914$

$$A_{st} = \frac{0.914\ b d^2}{\sigma_{st} \times J\ d}$$

$$A_{st} = \frac{0.914\ b d}{230 \times 0.904}$$

$$A_{st} = 4.39 \times 10^{-3}\ b d.$$

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ii) After doubling the reinforcement the balanced section becomes an over reinforced section.

$$A_{st} = 2 \times 0.0043\, bd.$$

$$A_{st} = 0.0087\, bd$$

$$M.O.R = \frac{\sigma_{cbc}}{2} b n_a \left[d - \frac{n_a}{3}\right].$$

$$\frac{bn^2}{2} = m. A_{st} (d-n).$$

$$\frac{bn^2}{2} = 13.33 \times 0.0087\, bd\, (d-n).$$

$$0.5n^2 = 13.33 \times 0.0087 \times d^2 - 13.33 \times 0.0087\, nd$$

~~$0.5n^2 + 0.1159n - 0.1159d = 0$~~

~~$n_a = 0.37d$~~

$$0.5n^2 + 0.1159nd - 0.1159d^2 = 0.$$

$$\boxed{n = 0.379d}$$

$$M.O.R = \frac{\sigma_{cbc}}{2} b n_a \left(d - \frac{n_a}{3}\right).$$

$$= 7/2\, b\, (0.379d) \left(d - \frac{0.379d}{3}\right)$$

$$M.O.R = 1.158\, bd^2$$

$$\frac{M.O.R}{M_{B.S}} = \frac{1.158\, bd^2}{0.914\, bd^2}$$

$$= 1.266$$

It is worthy to note that by increase of amount of steel by 100 %. The increase of moment of resistance is only 26.6 %.

2) Limit State Method:- ~~In this the~~

Limit state Collapse
- Shear
- Torsion
- Compresion
- flexural

This all should obeys the Construction is prefered.

Limit state Serviceability
- Cracking
- Deflections.

Factor of safety, Concrete → 1·5, Steel → 1·15. — Pg No - 67

Introduction to Limit state method:- The phelosophy of L.S.M of design is to see that the structure remains fit for use throughout its life period by assuring safety against strength and serviceability requirements.

That is the structure will not reach its limit state throughout its life period. A structure is said to have limit state when a structure as a whole (or) a part unfit for use during its expected life time.

Acceptable limits:- These are the acceptable limits for safety and serviceability requirements for the structure before failure occurs.

The two major limit states which are usually considered are:-

1.) Limitstate of Collapse. 2.) Limit State of Serviceability.

1.) **Limit State of Collapse:-** It is a limit state at which the structure is likely to collapse due to rupture of one (or) more critical sections (or) over turning, buckling, loss of over all stability. It may lead to shear, Torsion, Compression and flexural.

2.) **Limit State of Serviceability:-** It relates to the performance of the structure at working loads. It may be due to excessive cracking and deflection. It may lead to deflection, Vibrations, Cracking, other limit states (Fire resistance, Vibrations, durability

Generally the crack width on the surface may be taken as 0.3mm.

Characteristic strength of materials:- It is defined as the value of strength of material below which not greater than 5% of test results are expected to fall.

For Concrete:- It is the compressive strength of 150mm cube of 28days, N/mm^2.

It is denoted by (f_{ck}).

Grade of Concrete	M_{15}	M_{20}	M_{25}	M_{30}
Characteristic strength N/mm^2	15	20	25	30

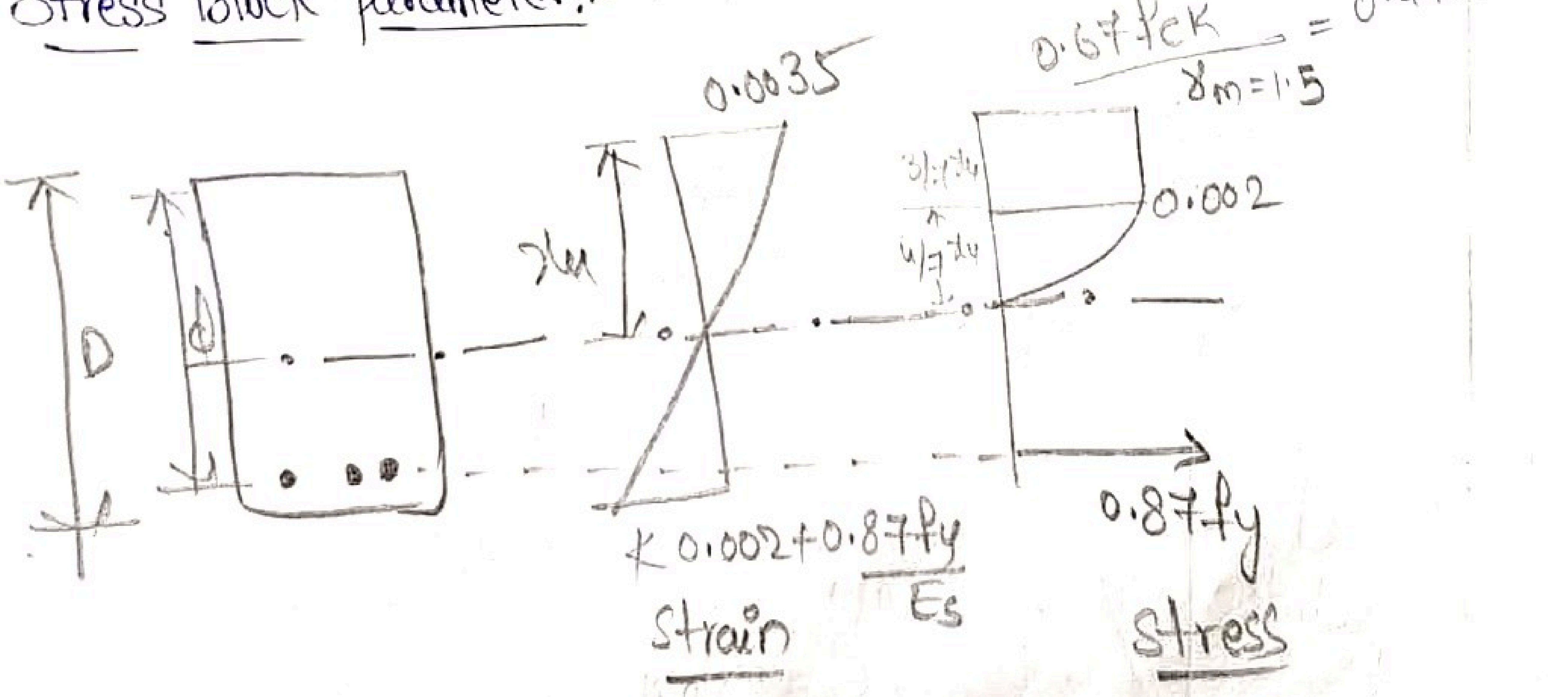
For Steel:- The minimum yeild strength (or) 0.2% of proof stress may be taken as characteristic strength.

Grade of steel	Fe250	Fe415	Fe500
Characteristic strength N/mm^2	250	415	500

It is denoted by (f_y).

~~Characteristic load~~:- characteristic loads:- Pg No - 67.

Stress Block parameter:-



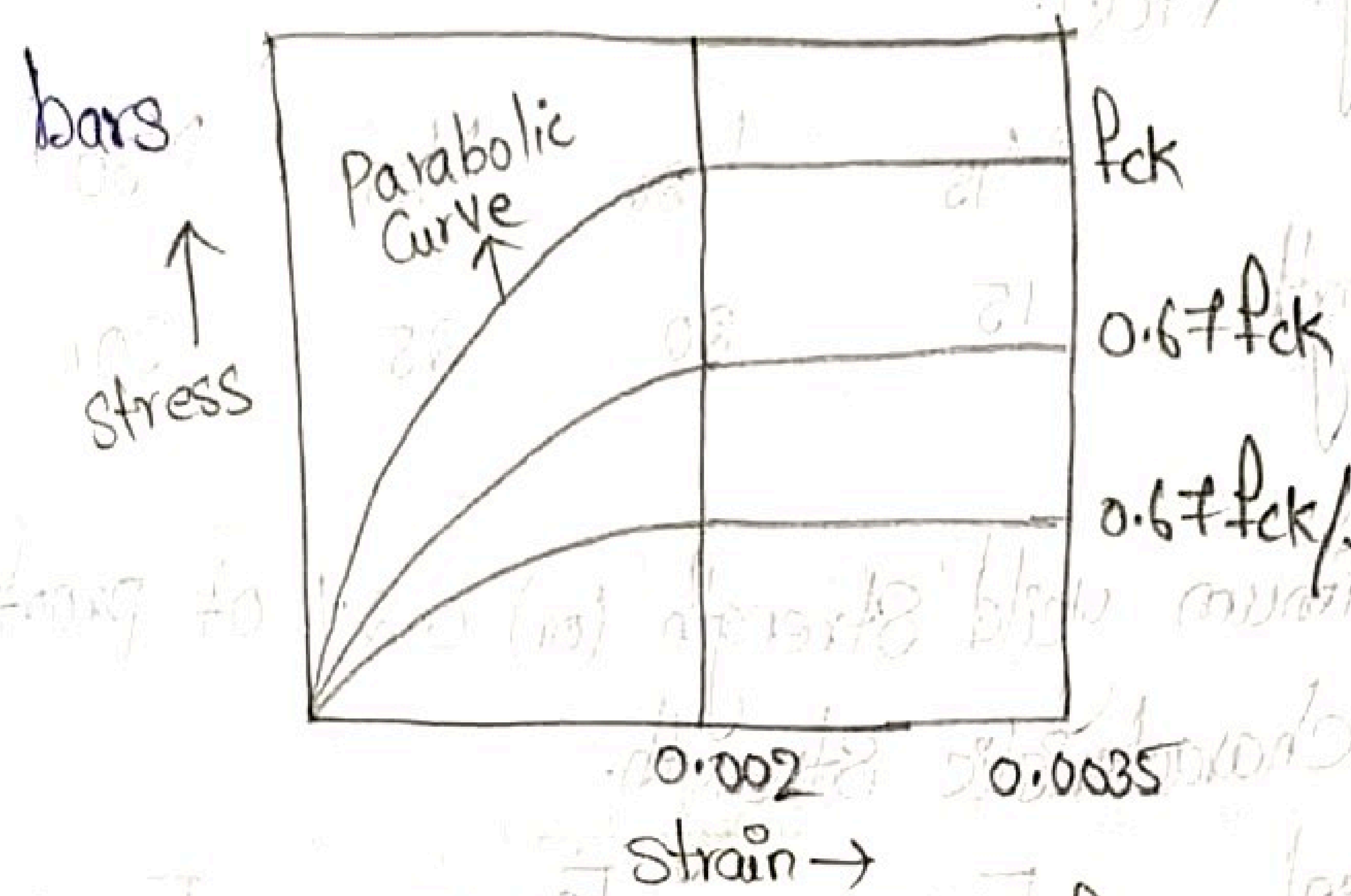
Calculation of Area of Compression:-

C_u = Area of rectangle + Area of parabola.

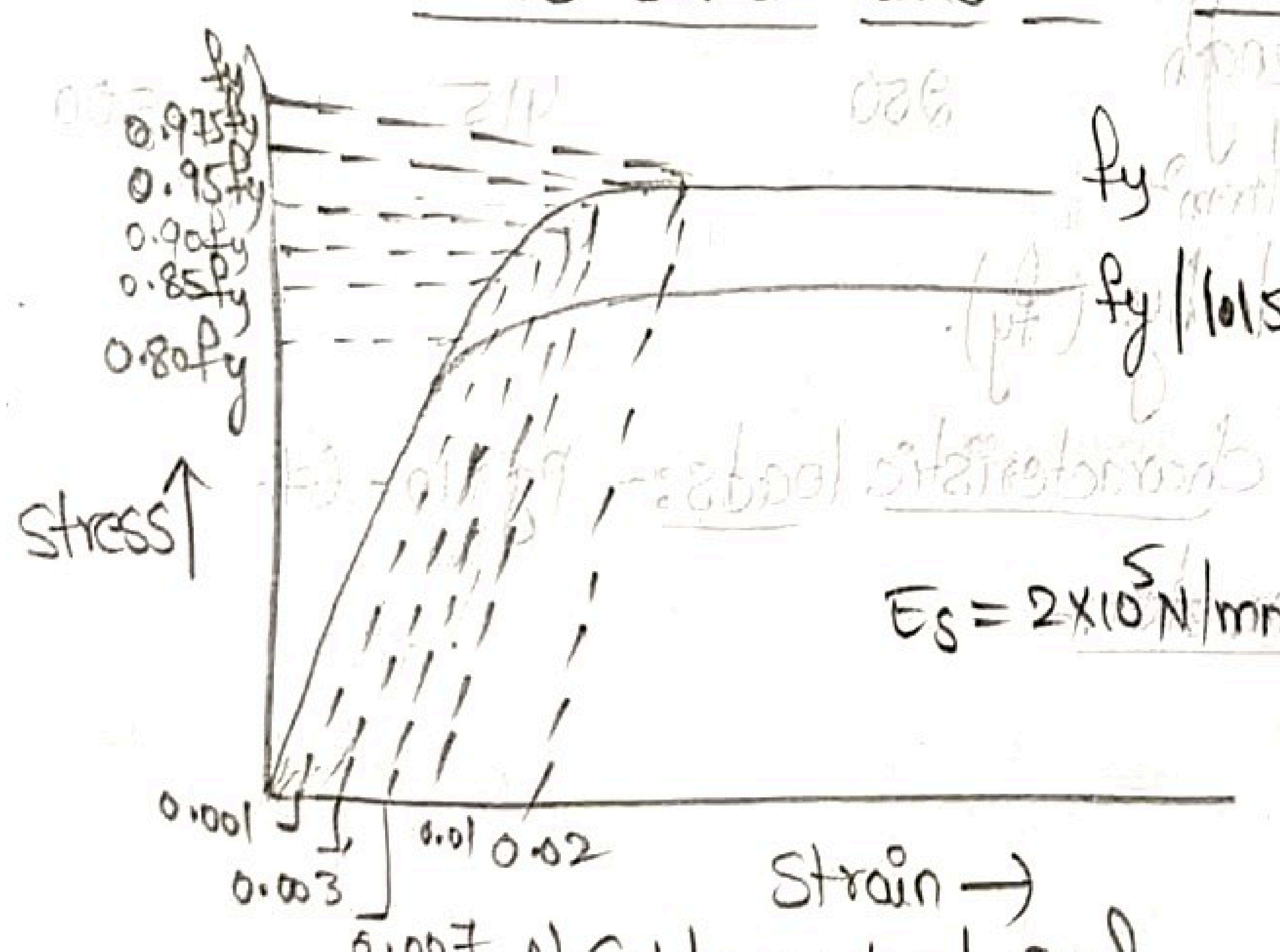
$$= (0.446 f_{ck} \times b) \times 3/7 x_u + 2/3 (0.446 f_{ck}\, b)(4/7 x_u)$$

$$C_u = 0.366 f_{ck}\, b \cdot x_u.$$

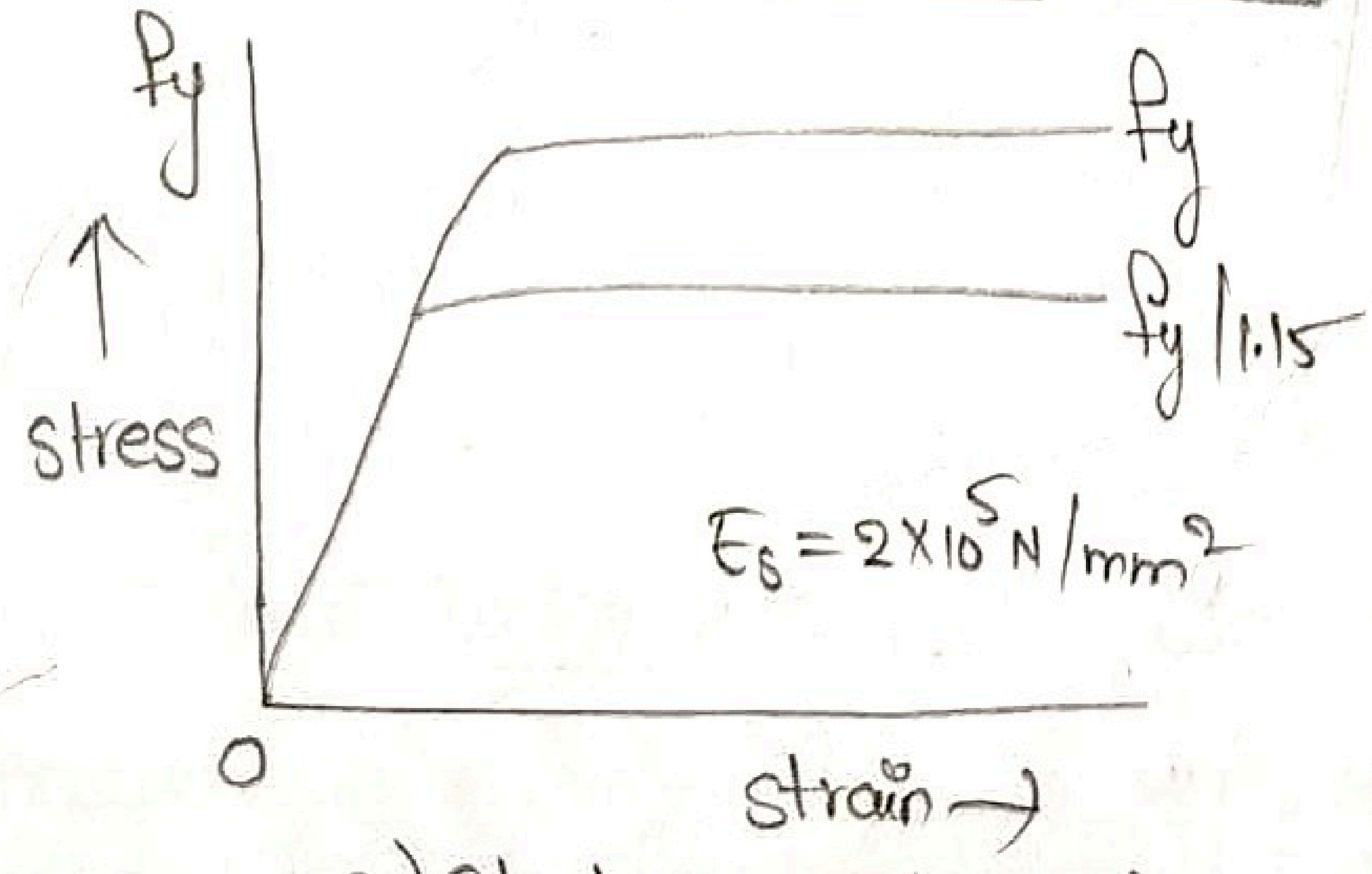
Stress Strain Curves for Cold deformed bars and Mild steel



Stress-Strain Curve for Concrete



A) Cold worked Deformed Bar



B.) Steel Bar with Definite yield point.

Assumptions: Pg No 64, 70

1) plane Sections normal to the axis remain plane after Bending.

2) The tensile strength of Concrete is ignored.

3) The maximum strain in Concrete (Compression) at the outermost fiber is taken as 0.0035. In Bending regardless of the strength of the Concrete.

4) The maximum Strain in the tension reinforcement in the section at failure shall not be less than $0.002 + \frac{f_y}{1.15E_s}$

(or)

$$0.002 + \frac{0.87 f_y}{E_s}$$

5) The relationship between Stress - strain distribution Curve in Concrete is assumed as parabolic.

6) Compressive Strength = 0.67 x characteristic strength.

$= 0.67 \times f_{ck}$.

7) Design Strength = $0.67 f_{ck} / \gamma_m$

γ_m = Partial Safety factor (or) factor of Safety.

for Concrete $\gamma_m = 1.5$

$$= \frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}.$$

8) The Stress in Steel reinforcement is derived from the stress Strain Curve for the type of Steel used as shown in fig.

9) Design Strength $= \frac{f_y}{\gamma_m}$ where $\gamma_m = 1.15$ in Steel

$= 0.87 f_y$

Analysis of a single reinforced Beam:-

If we provide the reinforcement tension zone of Concrete member, then that ~~Single~~ the member is said to be Single reinforced Beam.

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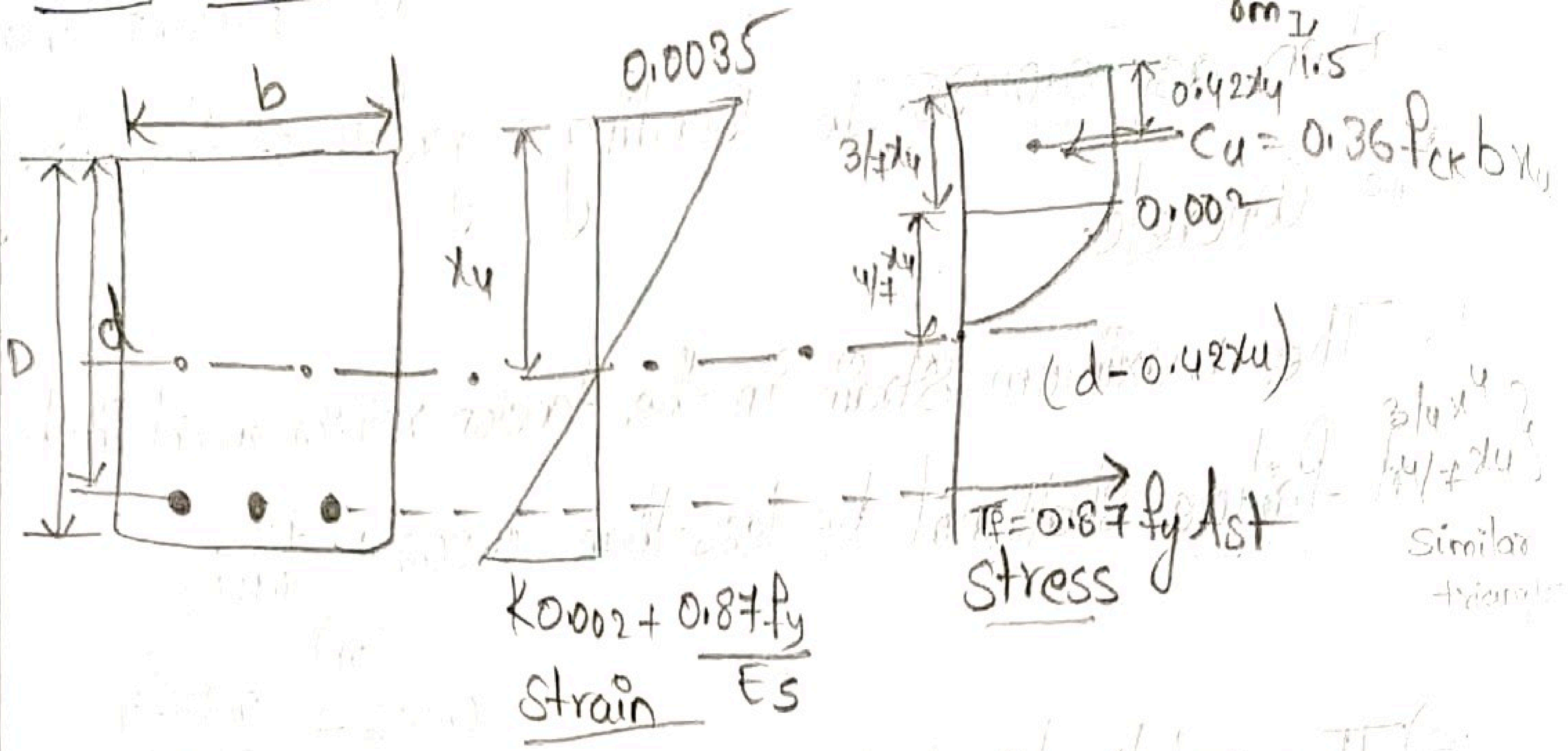
Effective depth:- $D - c = d$

d = overall depth − effective cover.

Stress Block parameters:-

$$\frac{0.67 f_{ck}}{\gamma_m} = 0.446 f_{ck} \quad (\gamma_m = 1.5)$$



By calculation of centroidal distance:-

$$= \frac{(3/7\, x_u)(0.446 f_{ck} b)(1/2 \times 3/7\, x_u) + \left[2/3 (4/7\, x_u)(0.446 f_{ck} b) \times \left[3/7\, x_u + 3/8 (4/7\, x_u)\right]\right]}{0.36 f_{ck}\, b\, x_u}$$

$$= \frac{0.1492}{0.366} x_u$$

$$= 0.42\, x_u$$

Lever arm:-

$$z = (d - 0.42\, x_u)$$

The depth of parabolic part of the stress block.

$0.0035 \rightarrow x_u$

$0.002 \rightarrow ?$

$$\frac{0.002\, x_u}{0.0035} = 4/7\, x_u$$

Depth of Rectangular portion :-

$$x_u - 4/7 x_u$$

$$= \frac{7x_u - 4x_u}{7} = \frac{3}{7} x_u$$

Area of the stress Block:-

Area of rectangle + Area of parabola.

$$= 3/7 x_u \times 0.446 f_{ck} \cdot b + 4/7 \cdot x_u (0.446 f_{ck} b) \times 2/3.$$

$$= 0.366 f_{ck} \, b \cdot x_u.$$

The distance of the centroid of stress Block from the top fiber.

Rectangle + parabola.

$$= \frac{3/7 x_u \times 0.446 f_{ck} \cdot b \times 1/2 \times 3/7 x_u + 2/3 (4/7 x_u)(0.446 f_{ck} \cdot b) \times \left[3/7 x_u + 3/8 \times 4/7 x_u\right]}{0.36 f_{ck} \cdot b \cdot x_u}$$

$$= \frac{0.446 f_{ck} \cdot b \times (3/7)^2 \cdot x_u^2 \times 1/2 + \frac{8}{21} x_u \times 0.446 f_{ck} \cdot b \cdot x_u \left[3/7 + \frac{12}{56}\right].}{0.36 f_{ck} \cdot b \cdot x_u}$$

$$= \frac{0.040 f_{ck} \cdot b \, x_u^2 + 0.1092 f_{ck} \cdot b \cdot x_u^2}{0.366 f_{ck} \cdot b \cdot x_u} = \frac{(0.1492) f_{ck} \cdot b \cdot x_u^2}{0.366 f_{ck} \cdot b \cdot x_u}$$

$$= 0.42 x_u$$

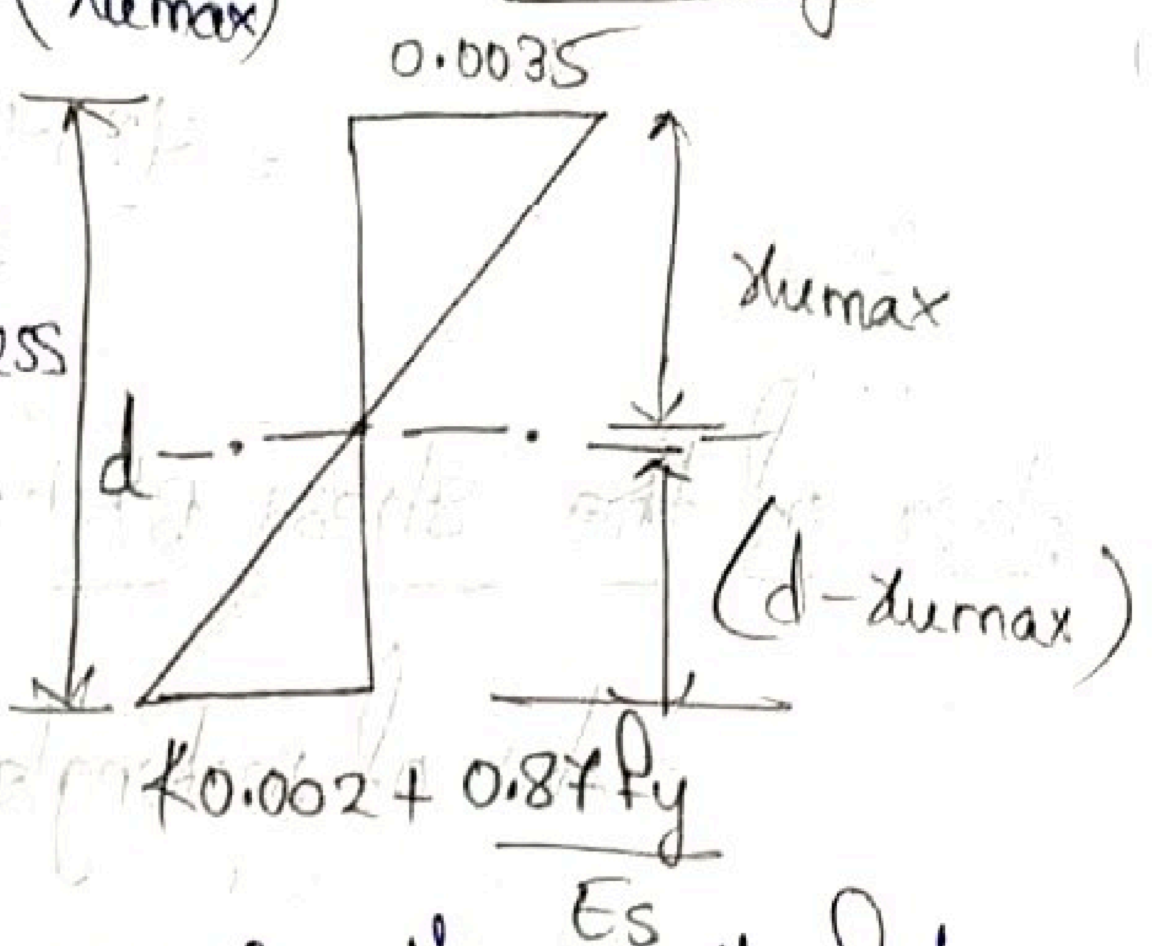
The Compression force $C_u = 0.36 f_{ck} \cdot b \cdot x_u$ acts at a distance of $0.42 x_u$ from the top most fiber.

Lever Arm (z):- $z = d - 0.42 x_u$.

Limiting moment of Resistance:-

Maximum depth of Neutral axis:- (x_{umax})

The max. depth of Neutral axis is limited to ensure the tensile stress will reach its maximum yield stress before the concrete fails in compression thus brittle failure materials.



$$\frac{x_{umax}}{0.0035} = \frac{(d - x_{umax})}{0.002 + \frac{0.87 f_y}{E_s}}$$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + \frac{0.87 f_y}{E_s}} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + \frac{0.87 \times 415}{2 \times 10^5}}$$

put for **Fe415**, $f_y = 415\ N/mm^2$ $E_s = 2 \times 10^5\ N/mm^2$

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + \frac{0.87 \times 415}{2 \times 10^5}}$$

$$x_{umax} = (d - x_{umax}) \times 0.919$$

$$x_{umax} + x_{umax} \times 0.919 = 0.919 d.$$

$$x_{umax} = 0.478 d.$$

$$\boxed{x_{umax} = 0.48 d}$$

Fe 250, $f_y = 250 N/mm^2$. $x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + \frac{0.87 \times 250}{2 \times 10^5}}$

$$x_{umax} = (d - x_{umax}) \times 1.133$$

$$x_{umax} + x_{umax} \times 1.133 = 1.133 d$$

$$\boxed{x_{umax} = 0.531 d}$$

Fe 500, $f_y = 500 N/mm^2$.

$$x_{umax} = \frac{(d - x_{umax}) \times 0.0035}{0.002 + \frac{500 \times 0.87}{2 \times 10^5}}$$

$$x_{umax} = 0.83 (d - x_{umax})$$

$$x_{umax} + 0.83 x_{umax} = 0.83 d$$

$$\boxed{x_{umax} = 0.45 d}$$

fy	xumax/d
250	0.53
415	0.48
500	0.45

pg No. 70 of IS 456

Limiting moment of resistance:-

Since the max. depth of Neutral axis is limited to avoid the brittle failure. The maximum moment of resistance is also limited (M_u limit).

M_u limit for Concrete = $C_u \times Z$.

$= 0.36 f_{ck} \cdot b \cdot x_{ulimit} \times (d - 0.42 x_{ulimit})$

M_{ulimit} for steel = $T_f \times Z$

(or) Maximum $= 0.87 A_{st} f_y \times (d - 0.42 x_{ulimit})$.

For $f_y = 250$

$M_{u\,limit}$ Concrete $= 0.36 f_{ck} \cdot b (0.53 d)(d - 0.42 \times 0.53 d)$.

M_{ulimit} Concrete $= 0.148 f_{ck} \cdot b \cdot d^2$

~~$M_{u\,limit}$ Steel = 0.87 Ast · fy~~

For $f_y = 415$

$= 0.36 f_{ck} \cdot b (0.48 d)(d - 0.42 \times 0.48)$

M_{ulimit} Concrete $= 0.137 f_{ck} \cdot b d^2$.

For $f_y = 500$

$= 0.36 f_{ck} \cdot b (0.45 d)(d - 0.42 \times 0.45)$

M_{ulimit} Concrete $= 0.131 f_{ck} \cdot b d^2$.

Grade of Compound Concrete general M_{ulimit}	Fe 250	Fe 415	Fe 500
General	$0.148 f_{ck} bd^2$	$0.138 f_{ck} bd^2$	$0.133 f_{ck} bd^2$
M20 (f_{ck})	$2.96 bd^2$	$2.76 bd^2$	$2.66 bd^2$
M25	$3.7 bd^2$	$3.45 bd^2$	$3.33 bd^2$

Limiting percentage of steel :-

The percentage of tensile reinforcement corresponding to the limiting moment of resistance is known as limiting percentage of steel.

It is obtained by equating the forces of compression and tension.

$$0.36 f_{ck} \cdot b\, x_{u\,limit} = 0.87 f_y A_{st\,limit}$$

$$A_{st\,limit} = \frac{0.36 f_{ck} \cdot b \cdot x_{u\,limit}}{0.87 f_y}$$

$$A_{st\,limit} = \frac{0.414 f_{ck} \cdot b \cdot x_{u\,limit}}{f_y}$$

limiting percentage of steel, $P_{t\,limit} = \frac{100 A_{st\,limit}}{bd}$

$$= \frac{100 \times 0.414 f_{ck} \cdot b \cdot x_{u\,limit}}{b\,d}$$

$$P_{t\,limit} = \frac{41.4 f_{ck} \cdot x_{u\,limit}}{f_y \cdot d}$$

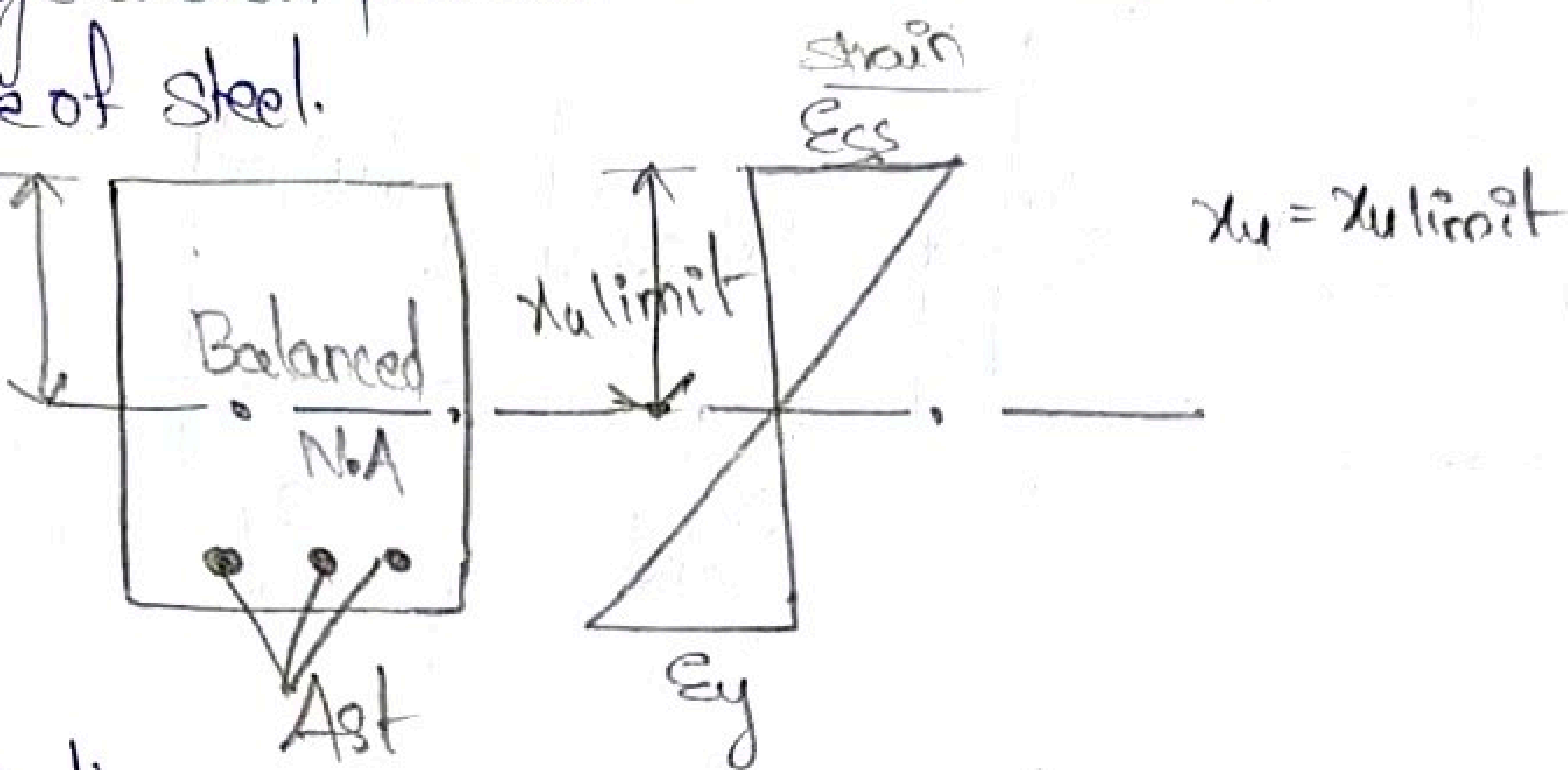
Grade of Concrete	Fe250	Fe415	Fe500
M15	1.32	0.072	0.57
M20	1.76	0.96	0.76
M25	2.02	1.19	0.94

Modes of failures:- (or) Types of sections:

i) A reinforced concrete member considered to have failed when the strain in concrete in extreme fiber reaches its ultimate value. i.e, 0.0035.

1) **Balanced Section:-** When the Max. strains will reach its maximum values simultaneously then that type of section is called balanced section.

The percentage of steel provided in a balanced section is called limiting ~~se~~ percentage of steel.



2) **Under Reinforced Section:-**

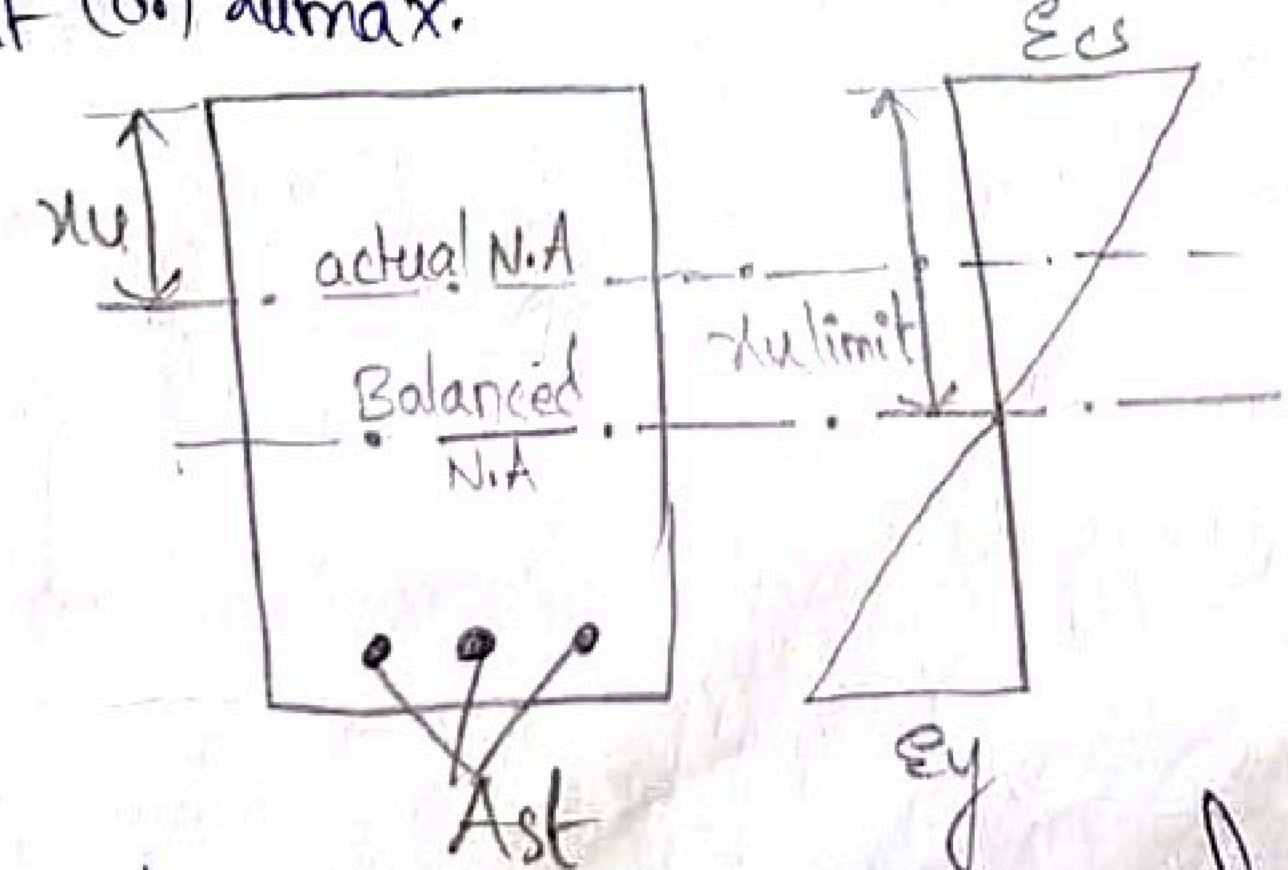
When the amount of steel in a section is less than the required for a balanced section is called Under Reinforced Section.

$x_u < x_{u\,limit}$.

Since $A_{st} < A_{st\,max}$.

In an under reinforced section the strain in concrete does not reaches its maximum value while the strain in steel reaches its max. value. The position of Neutral axis will shift upwards to maintain the section in equilibrium b/w the forces of compression and tension.

Since $x_u < x_{u\,limit}$ (or) $x_{u\,max}$.

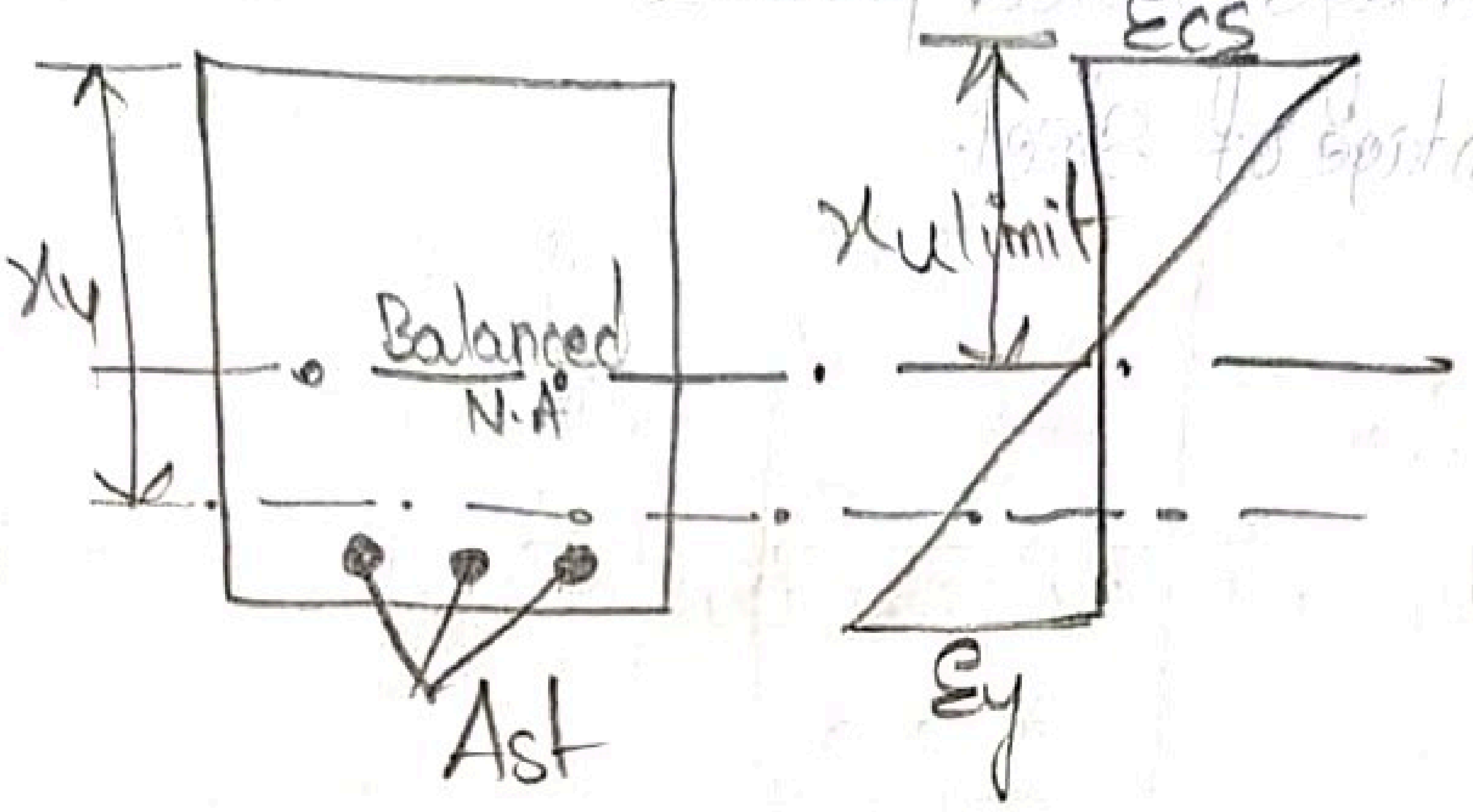


3) **Over-Reinforced Section:-** The percentage of steel provided greater than the required for a balanced section is called as over reinforced section. Since $A_{st} > A_{st\,max}$.

In case of over-reinforced section the

depth of N.A will shift downwards to maintain the structure in Equilibrium b/w the forces of Compression and tension.

Since $x_u > x_{umax}$ (or) x_{ulimit}.



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