Project Evaluation and Review Technique (PERT):

→ PERT stands for "project/programme Evaluation and Review Technique".

→ PERT involves uncertainty into the project completion time.

→ It is a numerical technique used in the projects in which time cannot be estimated accurately such as research and development projects.

→ It is an event oriented network. cost is assumed to be directly proportional to time

Three time estimates are made in PERT:

1. optimistic time (t_o): this is the minimum possible time in which an activity can be completed under the most ideal conditions.

2. pessimistic time (t_p): this is the maximum time required to complete an activity under the worst possible conditions.

3. Most likely time (t_m): this is the time required to complete an activity under normal working conditions. Its value lies between t_o and t_p. It is near to the expected time.

Note: the most likely time (t_m) is based on experience and judgement being based on the time required if the activity is repeated a number of times under

essentially the same conditions

→ this time signifies the most frequently occuring time
it reflects a situation "things are as usual nothing exciting".

Mean Time, Standard Deviation and Variance of an Activity:

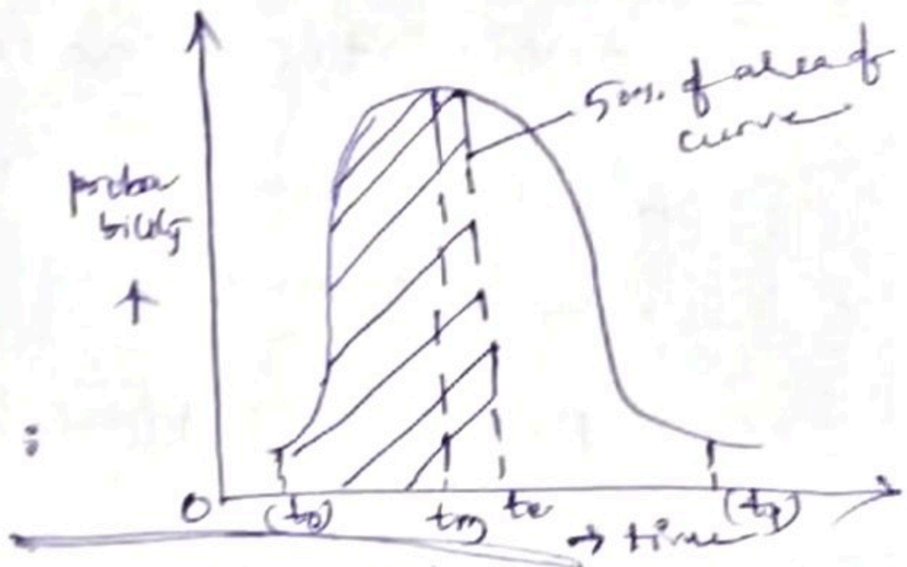
Mean Time (or) expected Time (or) Average Time:

→ in PERT each activity is assumed to follow β-distribution curve of time

→ this is calculated from β-distribution curve of time at which probability of activity is just 50%.
Time taken by various activities follow β-distribution.

→ Hence value of expected time is calculated by weighted average as,

$$te = \frac{to + 4tm + tp}{6}$$



Standard deviation of an activity (σ):

→ this is the measurement of uncertainty, which is approximately one sixth of time range i.e

$$\sigma = \frac{tp - to}{6}$$

→ It can be seen above that 'σ' is affected by relative distance from the most optimistic estimates to the most pessimistic estimates.

→ therefore, wide range in time estimates represents greater uncertainty.

NOTE: In a limiting case, certainty of an activity duration occurs only when the three time estimates coincide, so that the standard deviations and the variance both vanish. consequently the activity duration becomes certain which is the case of CPM. Hence, a PERT is a general case whereas CPM is the particular case of PERT.

Variance of an Activity (σ^2):

→ square of standard deviation is variance of an activity.

→ It is to be noted that higher the uncertainty about a process, greater is the standard deviation and hence greater is the variance of a project.

Central limit theorem:

→ The Theorem states that a project consists of a large number of activities, where each activity has its own mean time (t_e), standard deviation (σ), variance (σ^2) and also its own β-distribution curve.

→ the distribution of time for the project as a whole will approximately be a normal distribution, i.e mean time ∑ expected time of a project is

$t_e = t_{e_1} + t_{e_2} + t_{e_3} + \cdots$ along critical path and the variance is,

$\sigma^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots$ along critical path.

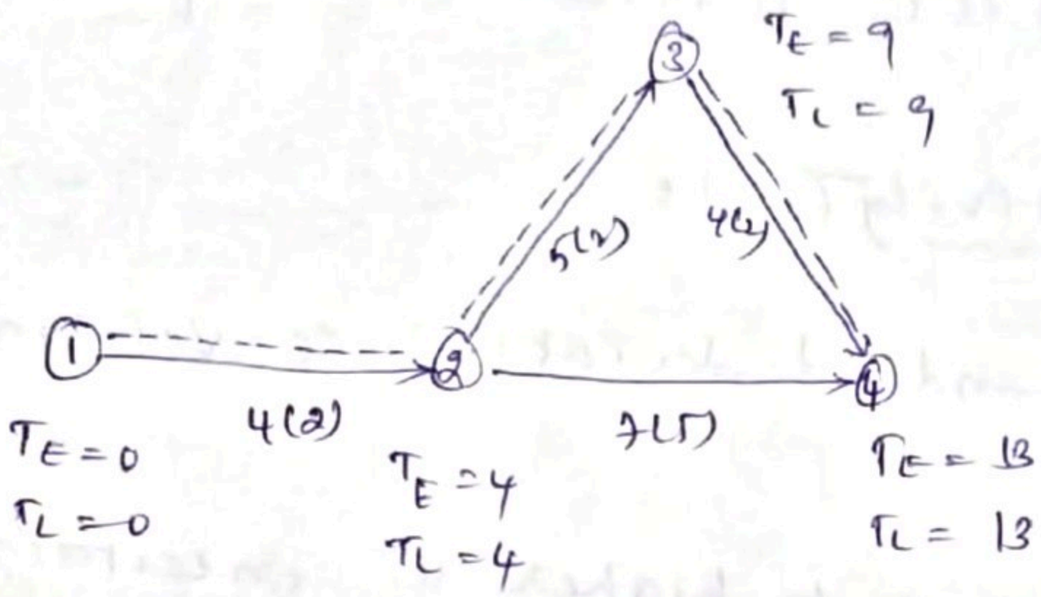
→ Hence standard deviation of the project as a whole

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \cdots}$ along critical path.

Critical path:

→ The time wise longest path is the critical path.

→ In this path, any type of delay in any event will cause delay in the project. These are shown by double lines or dark lines in a network



→ 1-2-3-4 is the critical path of following network

Time computation of events:

1. Earliest expected occurrence time (EOT):

→ The time at which an event is expected to occur earliest.

→ The time at which an event can be expected to occur earliest.

→ An event occurs when all the activities leading to it are completed.

→ It is generally denoted by T_E. It is calculated by forward path.

$$T_E^j = T_E^i + t_{ij}$$ (when there is only one path)

$$T_E^j = (T_E^i + t_{ij})_{max}$$ (when there are more than one path)

Here T_E^i = EOT of event j

T_E^i = EOT of event i

t_{ij} = expected time of activity.

2. Latest Allowable occurrence Time (LOT):

→ the latest allowable time at which an event must occur to keep the project on schedule.

→ It is generally denoted by T_L. This is calculated through backward path.

$$T_L^i = T_L^j - t_{ij} \quad \text{- when there is only one path.}$$

$$T_L^i = \left(T_L^j - t_{ij}\right)_{min} \quad \text{- when there are more than one path.}$$

where T_L^j = LOT of event j

T_L^i = LOT of event i

t_{ij} = expected time of activity i-j

Note: the latest allowable occurrence time of the finish event is equal to the schedule completion time of the project.

Slack:

→ slack is defined as the difference between latest allowable time (T_L) and earliest expected time (T_E) of an event.

slack for any event $j = T_L^j - T_E^j$

slack for any event $i = T_L^i - T_E^i$

→ slack may be positive, zero or negative

→ when 'slack is greater than zero'. it indicates project is ahead of schedule and availability of excess resources. such events are sub-critical.

→ if slack is zero, it indicates work is on schedule and events are critical. Resources are just adequate.

→ If slack is negative, it indicates work is behind schedule and may cause delay in project completion. events are super critical. extra resources are required.

→ the path having minimum or zero value is the 'critical path' which is also time wise longest path.

Critical path method : (CPM)

→ This is based on deterministic approach in which only one time estimate is made for activity completion.

→ Network diagram in CPM is activity oriented.

→ It is activity oriented network

→ Each activity is represented by arrow and the junctions between the activities represents events.

→ used for repetitive type of project - accurate time and cost estimate for completion of each activity can be made with fair degree of accuracy - deterministic model.

→ Trade of between time & cost

→ used in construction projects like bridges, buildings, dams, canals, etc.

Event Times in CPM :

1. Earliest occurrence time (T_E) :

→ Time at which an event may occur as early as possible

2. Latest allowable time (T_L) :

→ Time at which event may occur as late as possible without delaying the overall project completion time

→ These are similar to PERT and are calculated in the same fashion.

Activity Times in CPM:

1. Earliest start time (EST):

→ It is the earliest possible time at which an activity can be started.

→ for an activity i- j, earliest event time of event i

i.e T_E^i is EST of activity i- j

2. Earliest finish time (EFT):

→ It is the earliest possible time by which an activity can be completed

→ for an activity i-j

$$EFT = EST + t_{ij} = T_E^i + t_{ij}$$

t_{ij} = Activity duration

3. Latest start time (LST):

→ this is the latest possible time at which an activity can be started without delaying the overall project.

$$LST = LFT - \text{Activity duration}$$

$$LST = T_L^j - t_{ij}$$

LFT = latest finish time of activity $i-j = T_L^j$

4. Latest finish time (LFT):

→ this is the latest time by which an operation or activity must be completed without delaying the project.

→ for an activity $i-j$, latest allowable time of head event j

i.e T_{Lj} is LFT of activity $i-j$

Float:

→ It is associated with activity times

→ it is analogous to slack of events in PERT

→ it is the range within which start or finish time of an activity may fluctuate without affecting the project completion time

→ Floats are of following types

1. Total float:

→ the time span by which starting or finishing of an activity can be delayed without delaying the completion of the project

→ It is the maximum available time in excess of the activity completion time.

→ total float is given by f_T.

$$f_T = (T_L^j - T_E^i) - t_{ij}$$

→ total float is given by F_T.

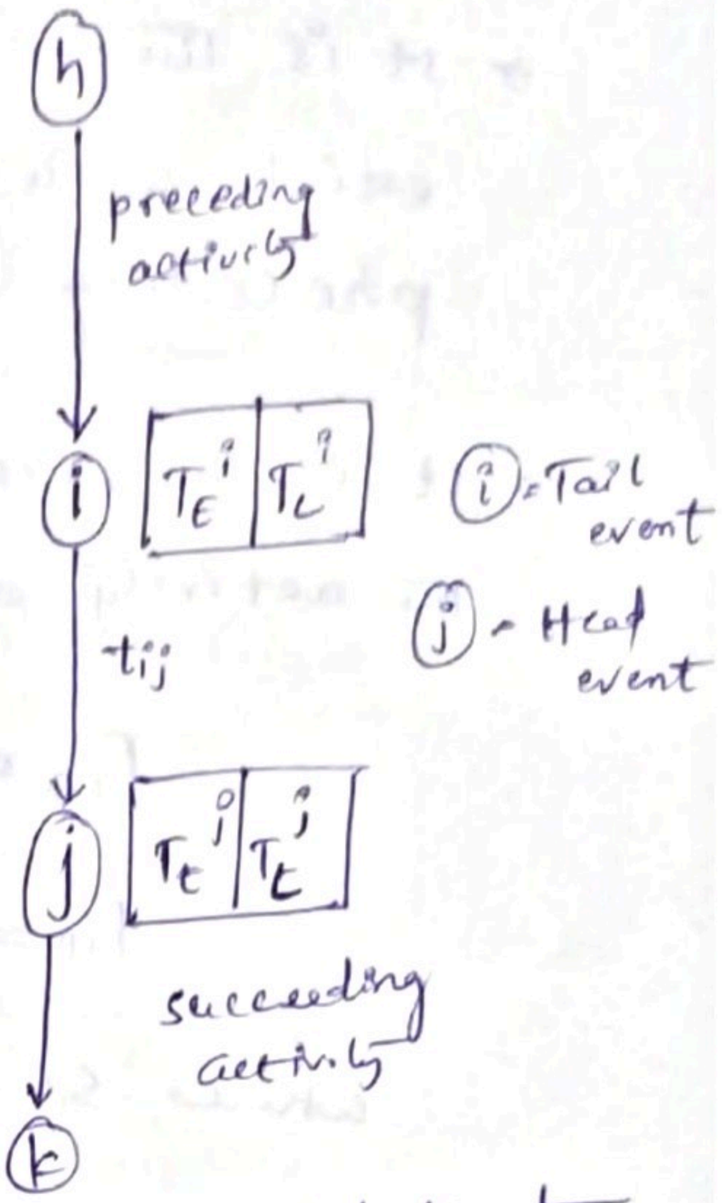
$$f_T = (T_L^j) - (T_E^i + t_{ij})$$

$$f_T = (T_L^j) - (T_E^i + t_{ij})$$

$$F_T = LFT - EFT \quad (or)$$

$$f_T = (T_L^j - t_{ij}) - T_E^i$$

$$f_T = LST - EST$$



→ total float

→ Total float of an activity affects total float of succeeding as well as preceding activities.

2. Free float (F_F):

→ the delay which can be made without delaying succeeding activities. It affects only preceding activities.

→ it is denoted by F_F. It is assumed that all activities start as early as possible

→ Free float is given by

$$F_F = (T_E^j - T_E^i) - t_{ij}$$

$$f_F = f_T - S_j$$

where S_j is head event slack

3. Independent float (F_{ID}):

→ It is the minimum excess available time which exists without affecting any of succeeding & preceding activities. It is denoted by F_{ID}.

→ It is the excess of minimum available time over the activity duration.

$$F_{ID} = (T_E^j - T_E^i) - t_{ij}$$

$$F_{ID} = F_F - S_i$$

where S_i is tail event slack

4. Interfering float (F_{INT}): It is smaller to head event slack

$$F_{INT} = S_j = F_T = F_F.$$

Critical path:

→ In CPM analysis, the path along which total floats are zero & minimum is called as critical path. all activities on this path are critical. There can be more than one critical paths.

Subcritical path:

→ It is the path joining all sub critical activities. for a sub critical activity total float is greater than zero i.e

$$F_T > 0$$

Super critical path:

→ It is the path joining all super critical activities.

For a super critical activities total float is less than zero.

$$F_T < 0$$

CPM systems:

→ Mainly two systems are used in CPM analyses.

1. A-O-A system (Activity on arrow system)

→ An activity is graphically represented by an arrow.

→ The tail end and head end of arrow represent start and finish of an activity respectively.

2. A-O-N system (Activity on node system (or) precedence dia)

→ Activity is represented by a circle or a node.
events have no places.

→ Arrows are used only to show the dependency relationship between activity nodes.

→ When two or more activities start parallely then an activity called DEBUT (D_0) is provided at the beginning.

→ Like wise a finish activity (F_0) is provided at the end when more than one activities finish parallely.
Activities D & F has zero duration.

Advantages of A-O-N system

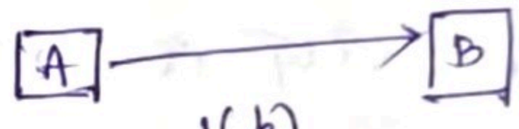
1. A-o-N system eliminates the use of dummy activities
2. It is more helpful for projects having more overlapping activities
3. It is a self sufficient and self-explanatory. All activity times (EST, EFT, LST, LFT) are represented on the diagram.
4. Revisions and modifications are easier
5. pre-operations and post-operations of activities under consideration are distinctly visible.

Examples:

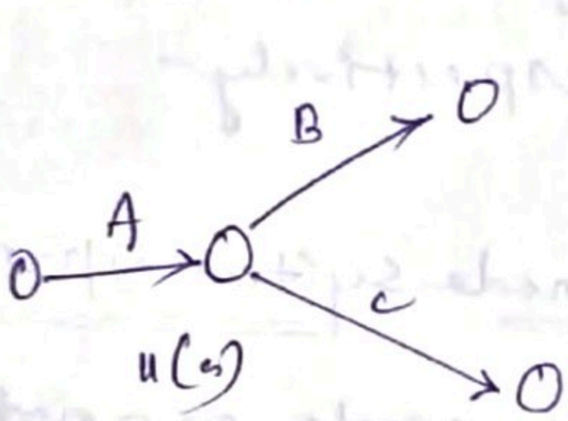
A-o-A

I(a)

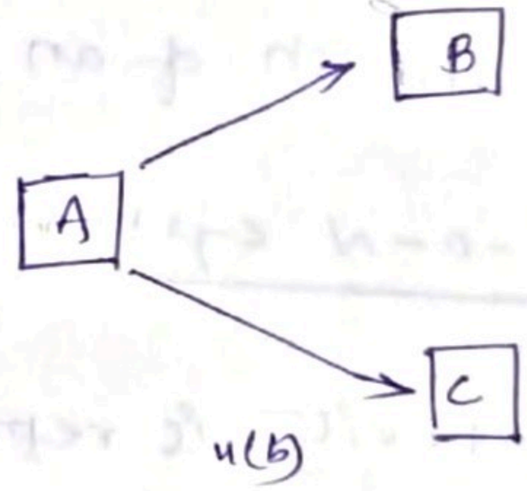
A-O-N



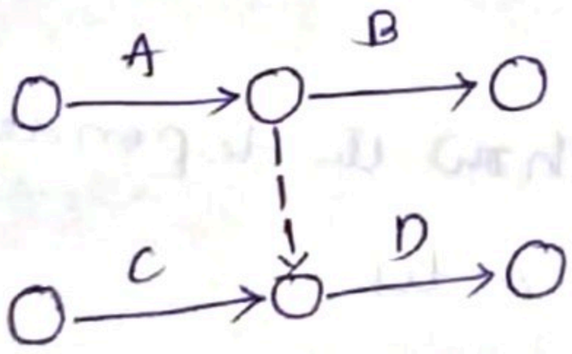
I(b)



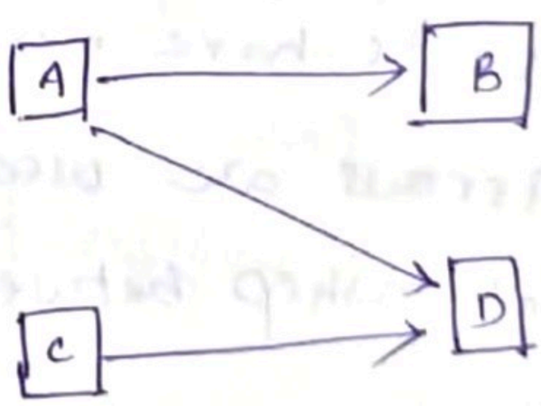
II(a)



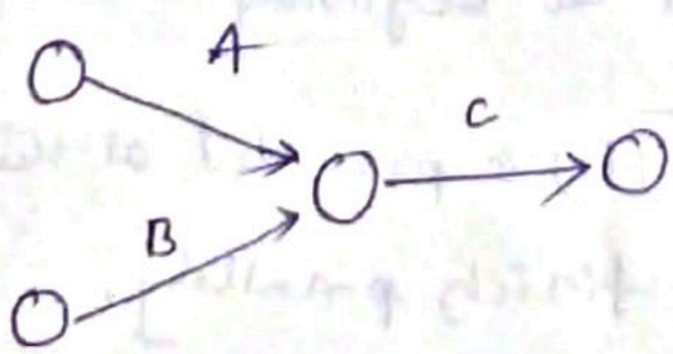
II(b)

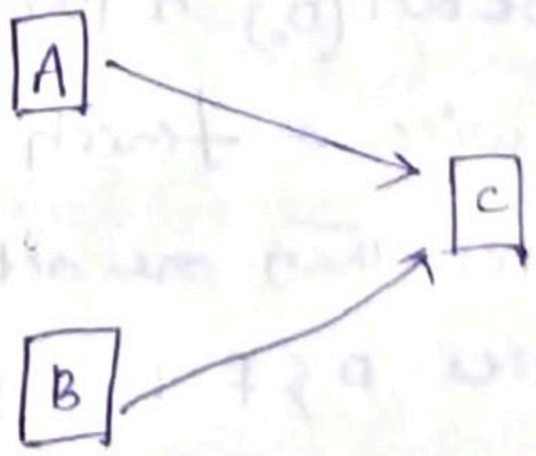


III(a)



III(b)





→ In CPM, time is related to cost and the object is to develop an optimum time - cost relationship.

→ CPM makes use of the cost estimates along with the time estimates and provides a schedule for completing the activities at minimum total cost.

→ the ultimate object of the network technique is not only to bring improvement in planning, scheduling, and control of project but also to asses the possibility of arriving at feasible and desirable time - cost relationship.

→ the overall project duration can be reduced by reducing the duration of only the critical activities in the project network.

→ the duration of such activities may be reduced in two ways.

(a) By deploying more resources for the early completion of such activities.

(b) By relaxing the technical specifications for such activities.

→ In whole of CPM cost model, we will be assuming that project duration is reduced by deploying more resources on critical activities.

→ the optimum duration will be one which gives the most economic cost for completing the project.

→ PCPM, there are two time and cost estimates for each activity. normal estimate and crash estimate.

→ in normal estimate, the emphasis is on cost with time being associated with minimum cost

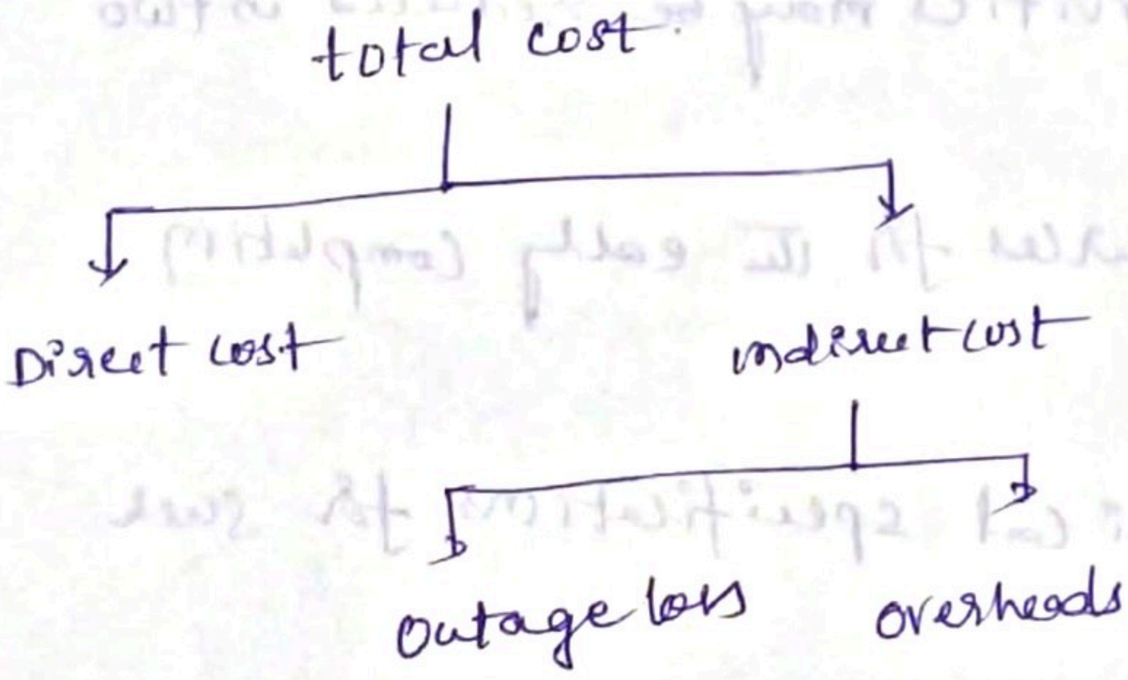
→ the crash estimate involves the absolute minimum time required for the job and the cost necessary to achieve it. Here emphasis is on time

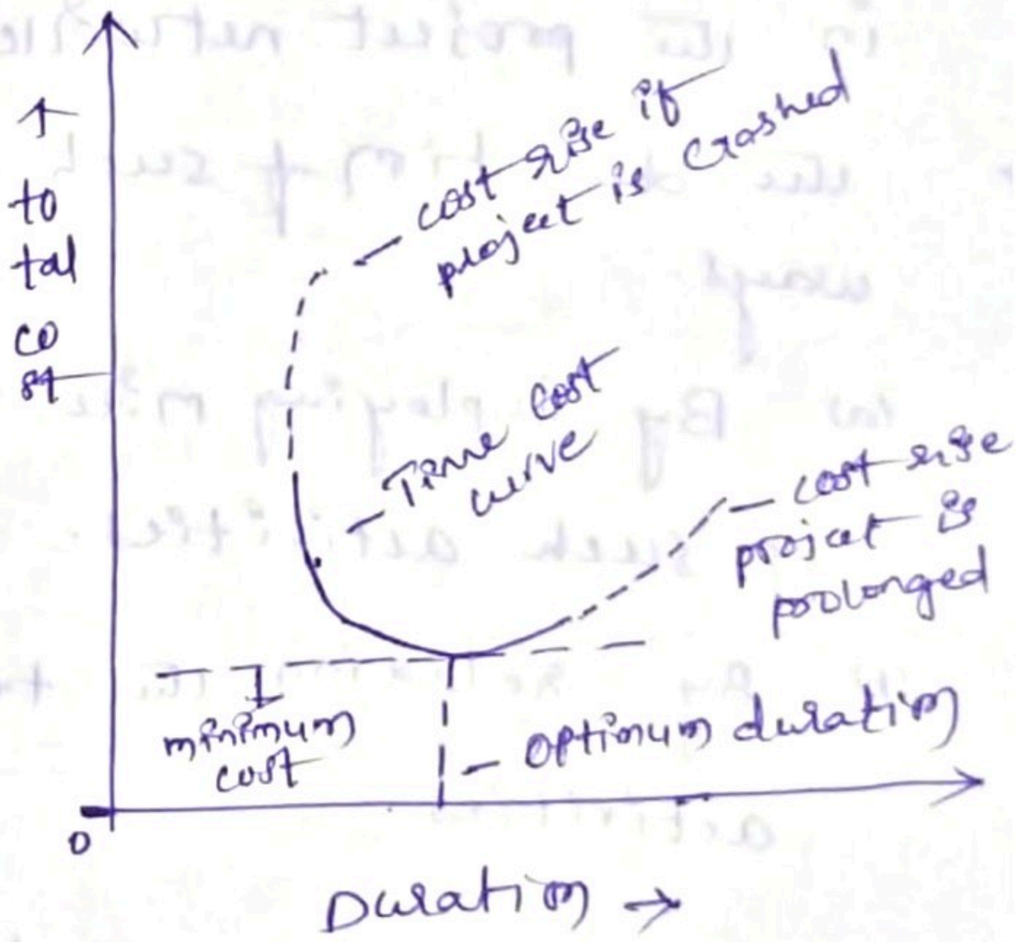
project cost:

→ total project cost is the sum of two separate costs.

(a) the direct cost for accomplishing the work

(b) the indirect cost related to the control & direction of that work, financial overhead, lost production, and the like etc.





→ From graph

(i) if a project goes on indefinitely, the cost will increase.

(ii) the cost will increase if the project is extended.

(iii) cost is minimum at some optimum project duration

components of project cost:

Indirect project cost:

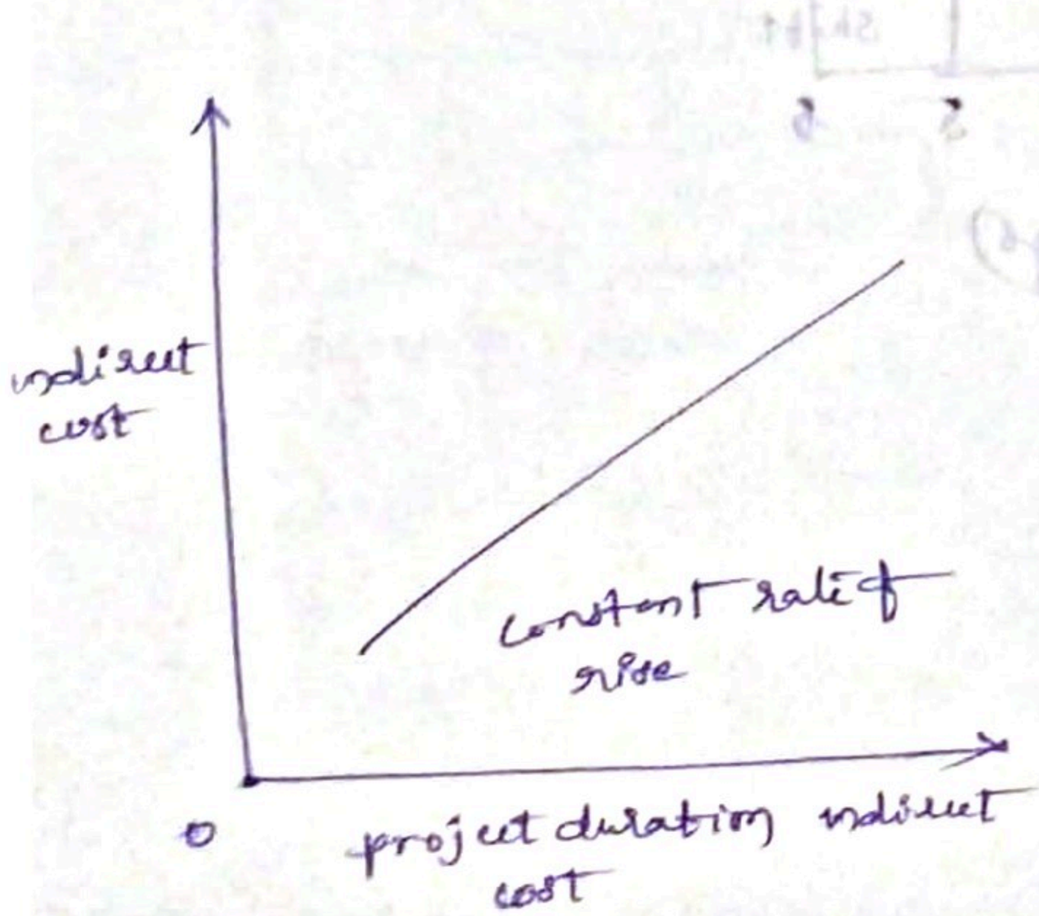
→ Indirect costs on a project are those expenditures which cannot be apportioned (or) clearly allocated to the individual activities of a project, but are assessed as a whole

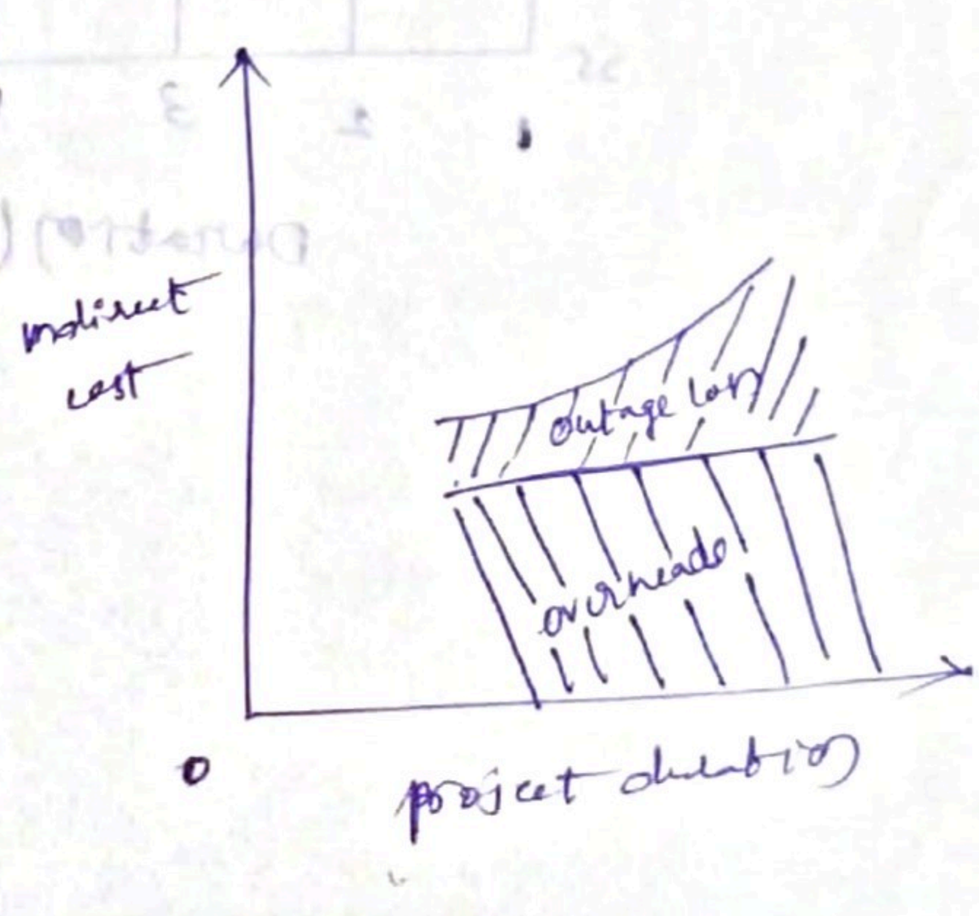
→ The indirect cost includes the expenditure related to administrative and establishement charges, overhead, supervision, expenditure on a central store organization, loss of revenue, lost profits, penalty etc.

→ Indirect cost rises with increased duration, considering only overhead and supervision. It is represented by a straight line, with a slope equal to daily overhead.

→ But when there is a loss in profits, due to inability to meet demands (or) due to some penalty, due to delay, a corresponding cost increase must be added to the cost of overheads, producing the curve. such a loss is called outage loss.

→ The total indirect cost curve will then be curved.





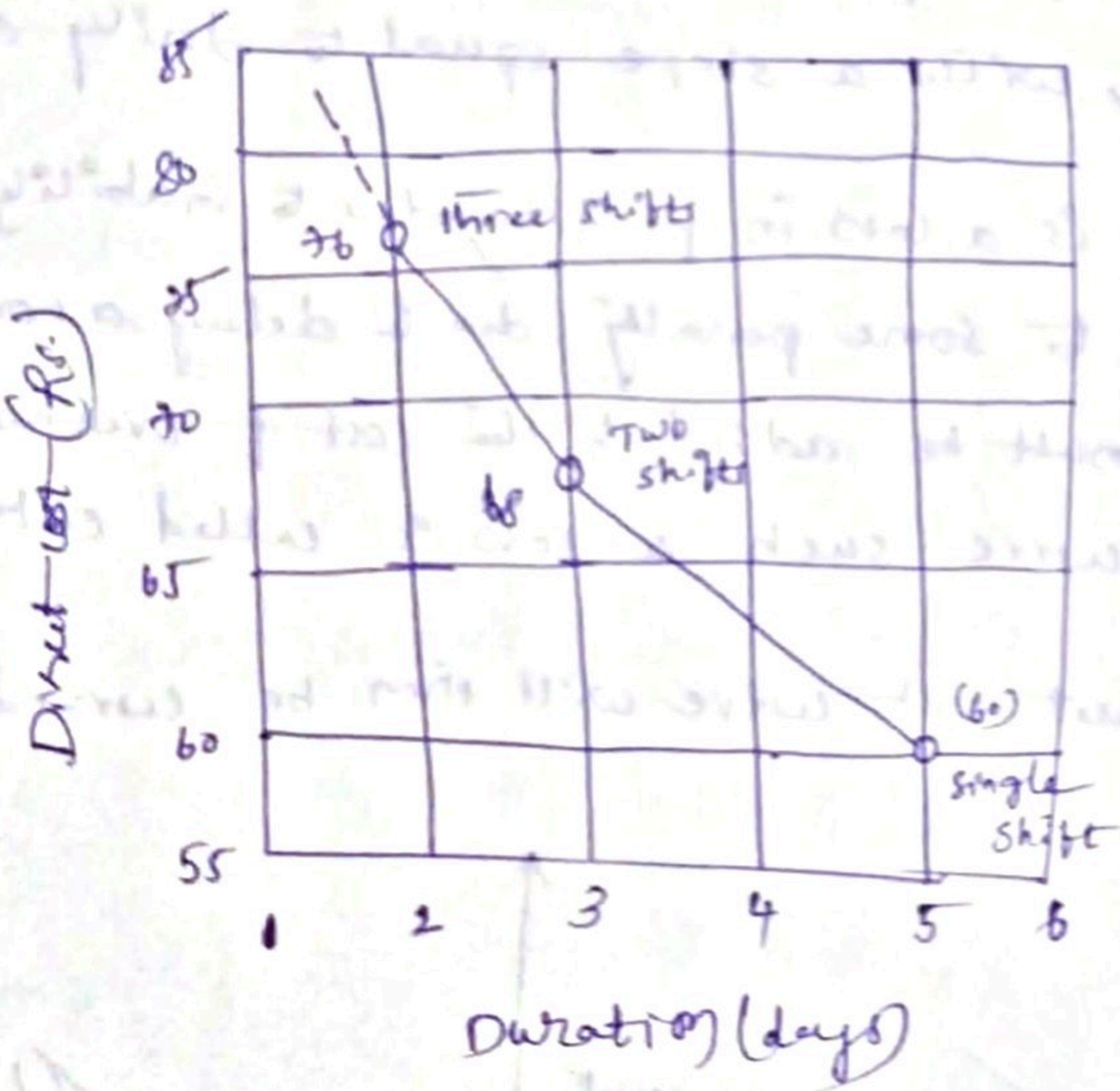
Direct project cost:

→ It is the cost which is directly dependent on the amount of resources involved for completion of activities.

→ It includes labour, material, plants and machining etc

→ To get the same work done in less time we have to increase amount of labour, equipment and time saving material that to at extra charges which simply means increase in direct cost

→ The project has the highest cost corresponding to the crash duration, and has normal cost corresponding to the normal duration.



Normal time (t_n):

Normal time is the standard time that an estimator would usually allow for an activity.

Crash time (t_c):

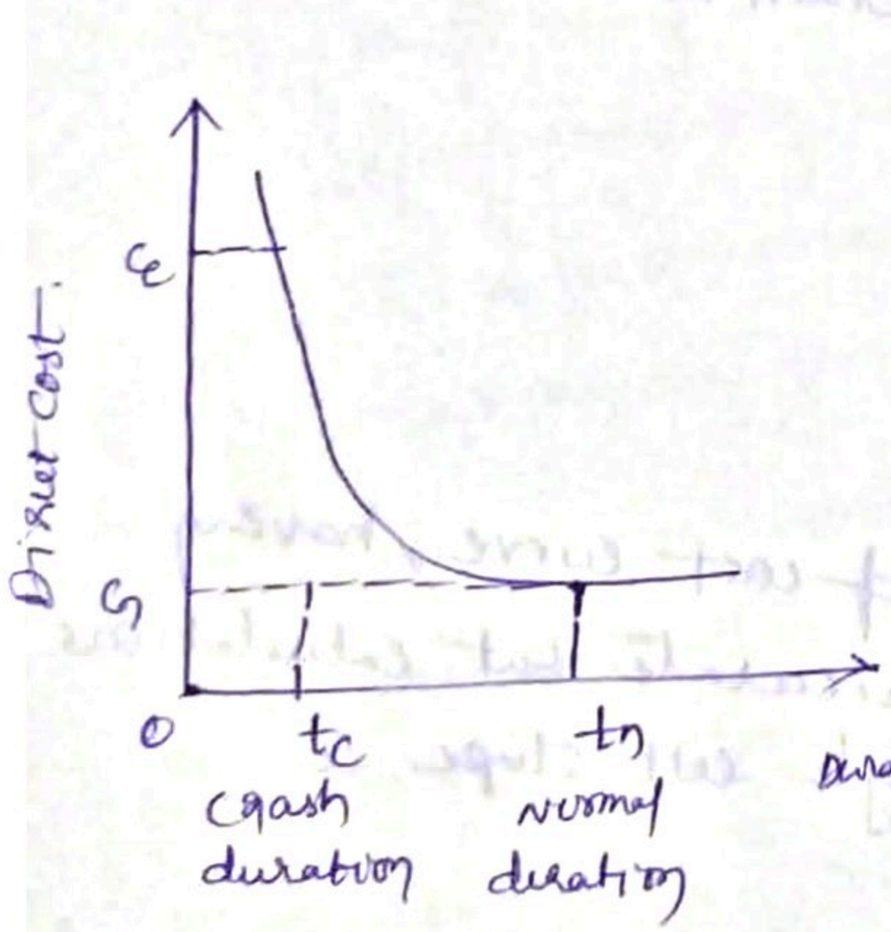
Crash time is the minimum possible time in which an activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortended by any amount of increase in the resources

Normal cost (C_n):

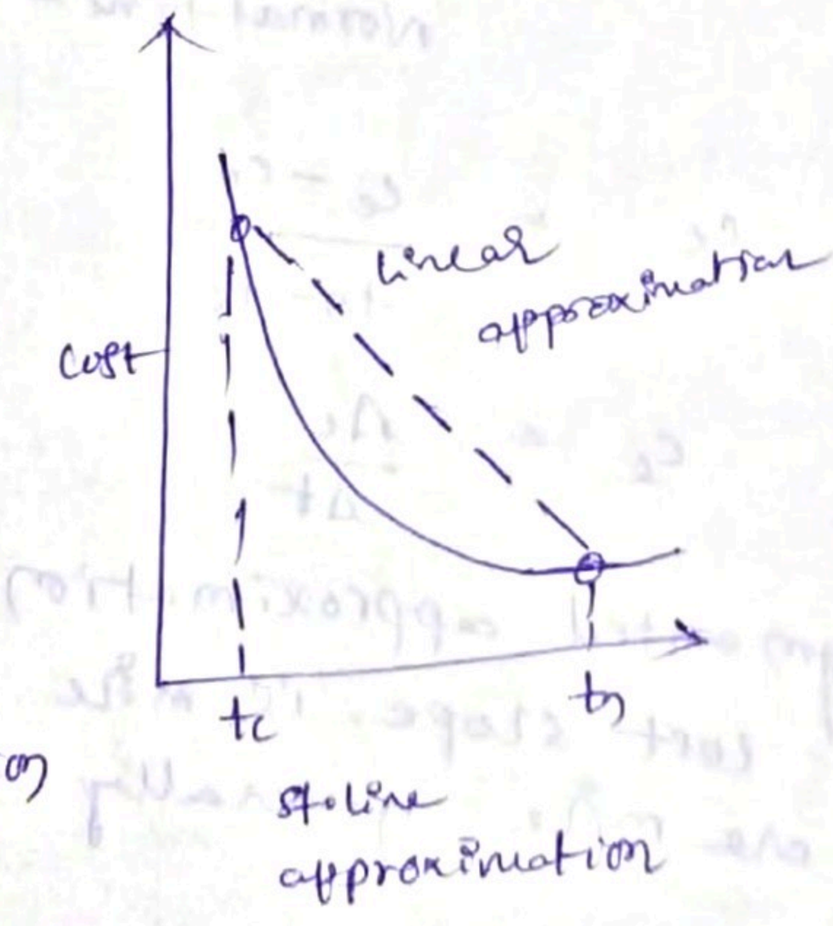
This is direct cost required to complete the activity in normal time duration.

crash cost (C_c):

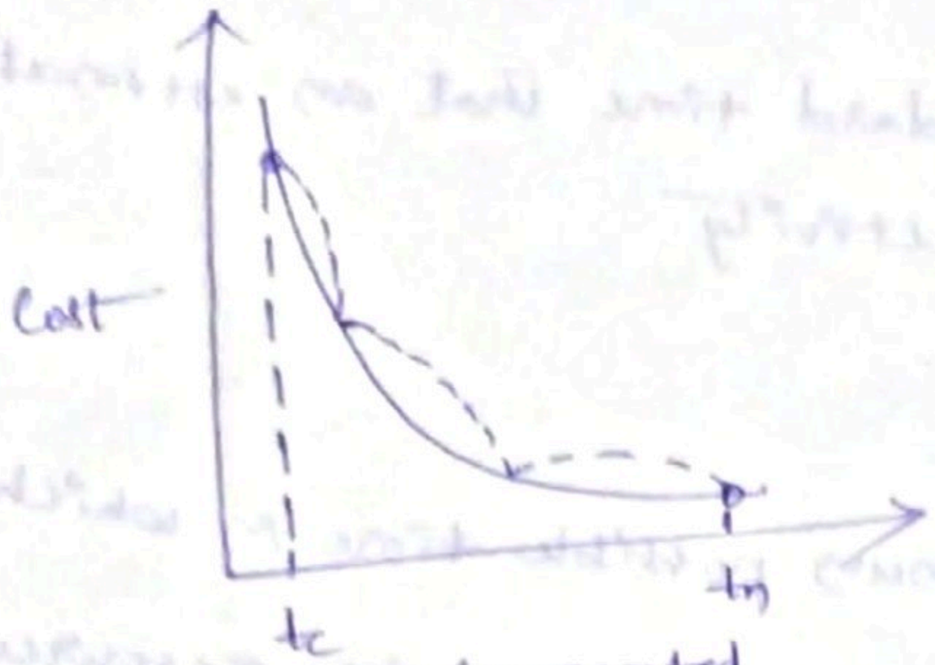
This is the direct cost corresponding to the completion of the activity with in crash time.



Generalized direct cost-time curve



Direct cost curve approximation.



→ st. line (or) segmented approximation of the direct cost curve is helpful in carrying out the cost/project analysis. In such analysis, the cost slope is used.

Cost slope:

→ The cost slope is the slope of the direct cost curve, approximated as st. line.

It is defined as

$$\text{cost slope} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$

$$C_s = \frac{C_e - C_n}{t_n - t_c}$$

$$C_s = \frac{\Delta c}{\Delta t}$$

→ The segmented approximation of cost curve, having multiple cost slope, is more accurate but calculations involved are more, generally single cost slope is assumed.

Resource allocation:

→ All the necessary resources are not available in unlimited quantities

→ Availability of some of the resources may be restricted

→ Availability of manpower and material etc. may be restricted.

→ Availability of funds, credit, capital investment and heavy equipment may be restricted.

→ supervisory, technical and skilled manpower, space and equipment are usually the most important resources that need to be allocated carefully

→ the various activities of the project are to be scheduled in such away that the demand of various resources is more or less uniform all along the project duration.

→ In a network, various activities are involved, and each activity requires some resources to perform it.

→ There may be activities which are to be performed simultaneously, and may require common resources.

→ the requirement of resources to execute these simultaneous activities may exceed the available resources.

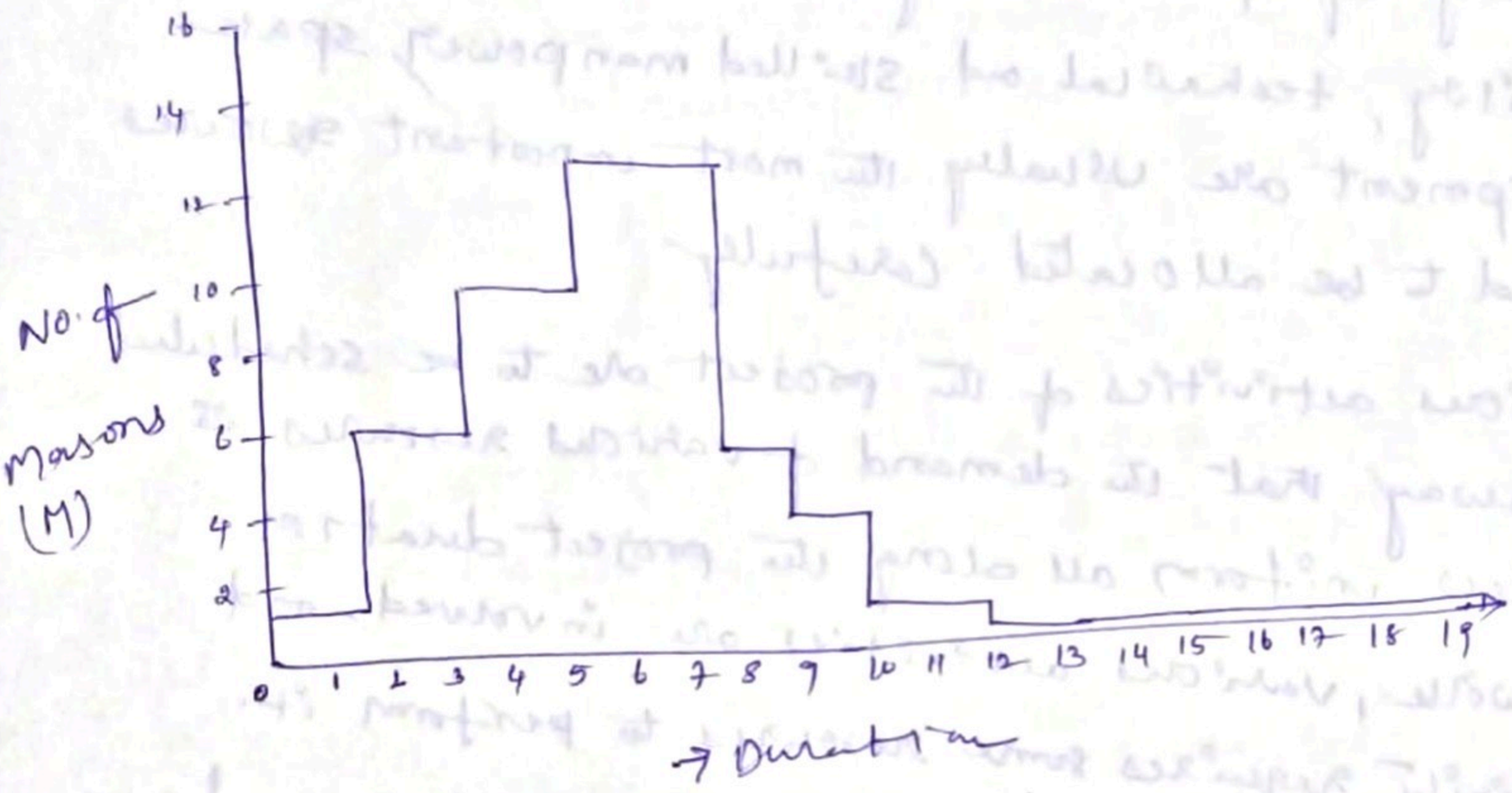
→ However at some other period of the execution of the same project, there may be very few activities which may require these resources.

→ Hence the requirement of particular type of resources may not be uniform during the project duration.

→ This can be best known by plotting the resource usage profiles or histograms.

→ the diagram which shows variation in the requirement of resources with time is called resource usage profile(s) histogram.

→ Resource allocation can be achieved by following two processes.



Resource smoothening:

→ Here resources are considered unlimited.
→ project duration is maintained and critical activities remain unchanged.
→ start time of some of non critical activities are shifted with in their available floats to create uniform demand through out

Resource levelling:

→ Here resources are considered limited. project duration may be changed.
→ Activities are rescheduled to cut down the peak requirement of resources so that it does not cross the limit of resources.

7. Available resources should never be less than the maximum quantity required for any activity of project.

→ Firstly, available floats are used then if needed duration of some activities is increased (or) decreased as per the resource requirement

CPM Updating:

→ the process of reviewing the progress of project execution and redrafting the network according to latest requirement is called 'updating'.

→ During redrafting, scheduled dates are revised. new critical path may emerge and hence project priorities may change.

→ crashing of new critical activities may be required to make project on schedule

→ updating is necessary to compensate for deviations in actual execution of works and original plans.

→ During the process of updating, neither activities are deleted nor new activities added.

When to update:

1- updating should be more frequent for shorter duration projects

2. For larger duration projects, frequency should be increased as project is nearing completion.

3. Whenever major change in the duration of any of activity occurs updating should be done